UNIT-I

Continuous wave Modulation

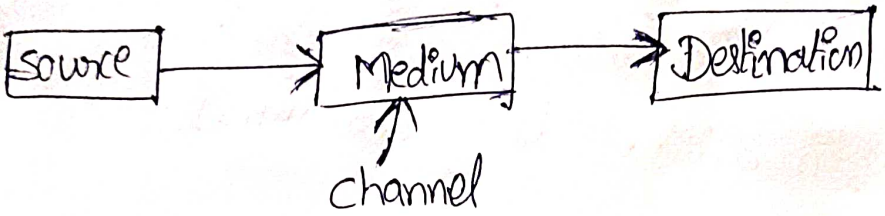
- A Continuous wave is the name given to an early method of Radio Transmission. C.W is also known as "Undamped waves".
  Ex: Laser, Sinusoidal Carrier wave [It is acts as like on and off]
- Information is carried by varying the on and off Period of a signal
- Morse Code is best example for radio Transmission.
- Example of Continuous wave modulation includes Pulse amplitude Modulation [PAM], Pulse duration modulation, Pulse Position modulation [PPM], Pulse width modulation [PWM]

- Communication Process:-
  - It is made up of four key components:
    1) Encoding 2) Medium of Transmission 3) decoding 4) f/b
  - There are also two other factors are sender and the Receiver.
  - The Communication can be used for to transmission of a message from Sender to Receiver. that means it begins with the Sender, and ends with the Receiver.
  - The Importance of communication is immeasurable in the world of business and Personal life.
  - In bussness Communication is must, it shows the difference b/w Success and failure or Profit and loss.

- The Communication begins with the Sender.
- The first step the Sender is faced with involves the encoding Process. that means it translating information into a message in the form of symbols that represent ideas or Concept. such as languages, words or gestures. These all are used for to encode the ideas into messages
- The Sender information is transmit to the channel. The channel they convey the message.
- In the channel there is any effect in the information they have send a/b.
- The Receiver is the individual to whom the message is directed.
- Feed back is the final link in the Chain of the Communication Process.
- After Receiving a message the receiver is responds and signals that response to the Sender.
- Feed back is a key Component in the Communication Process because it allows the Sender to evalute the effectiveness of the message.

Communication Channel:-

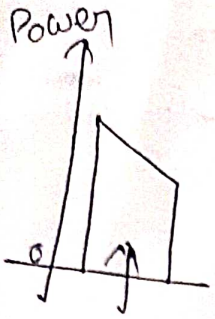
- A Com. channel is the medium used to transport information from one n/w device to another.
  - A channel may be wired channels [it is used for to transport data through wires and cables].
- A Wireless channels transport data from one device to another without the use of cable or wires.
- Wired channels are twisted copper wires [telephone and DSL] Coaxial cable [television and internet], optical fiber, or radio link.



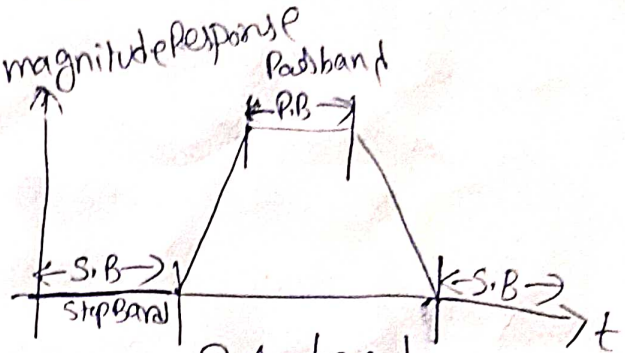
Base band and Passband Signals:-

- Base band signal also known as "low Pass Signal", is a signal in a near zero-frequency range generated from the information source.
- In base band signal the b.w carries only one data sig at a time.
- In base band sig refer to the original transmission sig with no modulation of high frequency carries.
- In baseband refer to the transmission signal before the modulation with freq much lower than the signal modulation.

In base band sig is normally a message signal and only one Communication channel is available at anytime.



Signal baseband



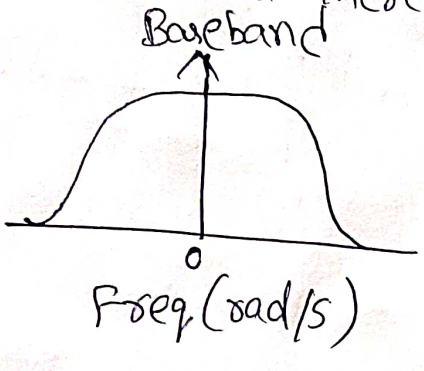
Pass band

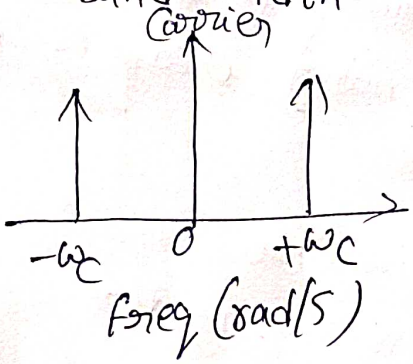
Pass band Signal:-

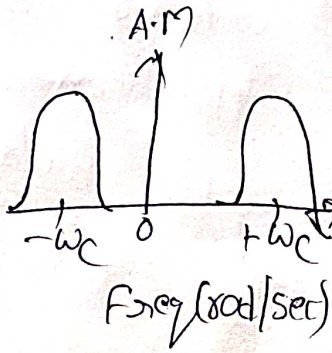
- Pass band Signal Refer to filtered signal or modulated signal in which the freq or Phase of the carrier sig is modulated to transmit the bits.
- It is also a digital data transmission of high freq. Sinusoidal Carrier.
- If a modulated signal is transmitted over a channel it is called Pass band Transmission.
- Pass band Signal are used at a high frequencies for longer distance Transmission.

Freq. Translation of Amplitude Modulation :-

- In the freq. domain amplitude modulation to translate baseband spectrum to a band surrounding the Carrier freq
- Because the baseband spectrum is symmetrical with respect to the Y-axis, this freq. translation results in a factor of 2 increase in band width.



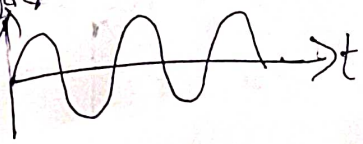




- Freq. Translation is the Process of shifting a signal from one freq to another, without loss of information in the sig
- Always Carrier freq. it not Contain no information.
- The applications of freq. Translation include frequency division multiplexing, improved signal reception and Combined down change, grapings etc.

Need for Modulation:- Modulation is the process of converting data into radio waves by adding information to an carrier sig.

- Modulation is simply widely used as a process in communication system in which a very high freq. carrier wave is used to transmit the low freq message signal so that the transmitted signal continues to have all the information contained in the original message signal.
- Modulation is mainly used for to increase freq. of the mes. signal



Need for modulation;

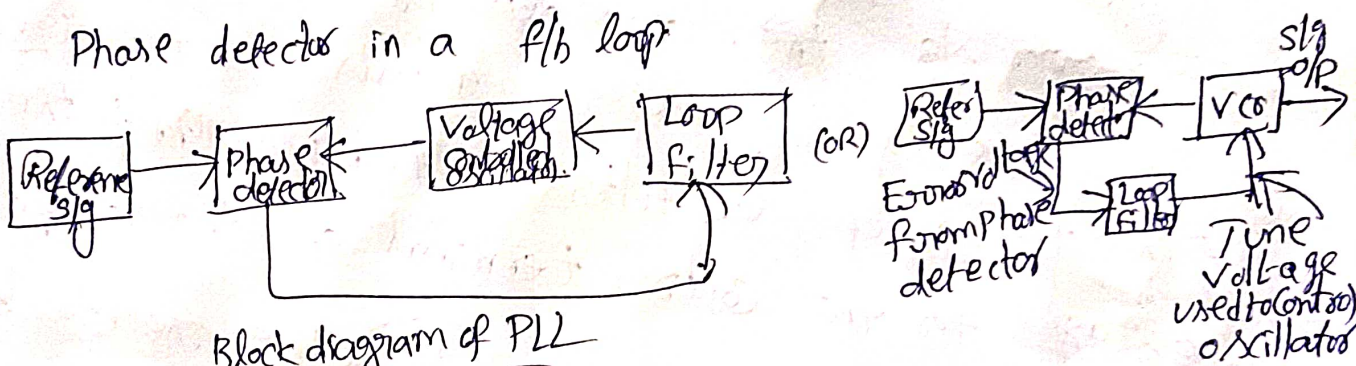
- Reduction in the height of antenna
- Avoid mixing signals
- Increase range of communication
- Multiplexing is possible
- Improves quality of reception.
- Noise Reduction
- To avoid interference.

-> Mainly modulation is used for main reason is the

- In the disturbance of noise.
- After transmitting, our message signal can be attacked by unwanted signals in the channel.
- Message signal can be modified into unwanted signal by noise.
- Because of noise, modulation must be done before transmitting.

PLL Angle Modulation:-

- PLL means Phase Locked Loop
- PLL is an electronic Circuit with a voltage or voltage driven oscillator that Constantly adjusts to match freq. of an i/p sig.
- PLLs are used to generate, stabilize, modulate, demodulate, filter, or recover a signal from a "noisy" communication channel where data has been interrupted.
- PLL is also defined as it is a control sys it generates an op signal whose Phase is related to the phase of i/p signal. Neuronal PLL, Analog or Linear PLL, Digital PLL, software PLL, but the Simplest
- There are several different types of PLL, is an electronic Circuit, consisting of a variable freq. oscillator and Phase detector in a f/b loop



Block diagram of PLL

- PLL is also used for to control different operations in Communication systems, computers and many electronic applications
- The major concept behind the PLL operation is Comparison of the phases of two signals (i.e, Compared the signal Phases of input and output)
- The Phase difference between i/p and o/p signal can be used for Controlling the loop freq.

- In many Comm. Systems, PLL is used for different Purposes.
- The Phase or freq. modulation, it is used as demodulator.
- To Synchronize the the two signals with different frequencies.
- To Remove large noises from tiny signal.
- The basic PLL which Consists of Phase detector, Voltage Controlled Oscillator (VCO), Loop filter.
- The VCO of PLL Produces a Signal and this signal from the VCO is given to the Phase detector.
- The Phase detector is used for to Compare this signal with
- and these Phase detector it Produces an error voltage or difference voltage.
- This error s/g of the Phase detector is fed to the lowPass filter for removing high freq. elements of the signal.
- Then the o/p of loop filter is fed to supply the tuning voltage for the control terminal of the voltage controlled oscillator.
- The tuning voltage is sensed to reduce the Phase difference b/w two signals and the freq. between them.
- Initially PLL does not Lock and the error voltage drags the VCO freq. towards the reference until the error can not be reduced any further and then loop gets locked
- If PLL is locked, then steady state error voltage will be Produced.
- Steady state error represent that there is no Phase difference b/w reference s/g and VCO.
- Thus we can say the freq. of the two signals is exactly same

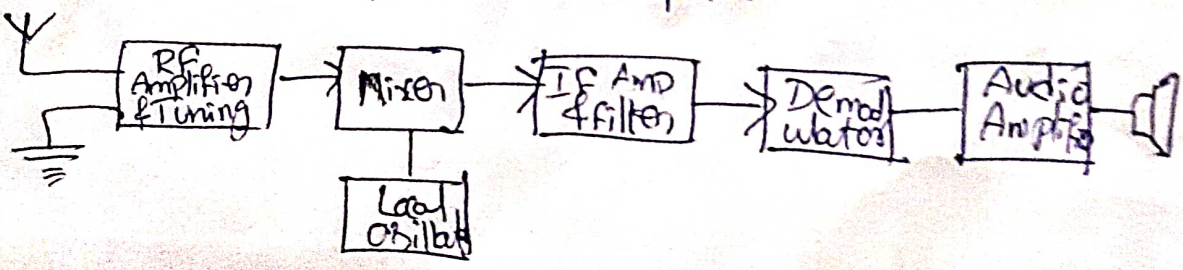
Nonlinear Effects in FM:-

- The Non linearity causes amplitude Modulation of the resultant wave and a distortion in the instantaneous freq.
- In Frequency modulation, the relationship b/w received Power and quality is non-linear.
- For Amplitude Modulation there is a linear relationship b/w the received s/g quality and received s/g Power.
- If the transmitted modulated s/g is affected by non-linearity in the channel. the demodulated signal may become a distorted version of the mes signal.
- The effect of nonlinearity on FM s/g can be used for generating wide band FM signals from narrowband FM.

$$g_{FM}(t) = A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(t)\,dt\right)$$

Superheterodyne Receiversin Angle Modulation:-

- Superheterodyne receiver means it converts Radio signal's RF Frequency to a Standard freq for further Processing.
- This Superheterodyne radio receiver is used in many forms of Radio broadcast Reception, two way radio Communication.
- and it is also useful to have an understanding of the different s/g blocks their functions, and overall signal flow, not only for RF Circuit design, but also from an operational viewpoint.

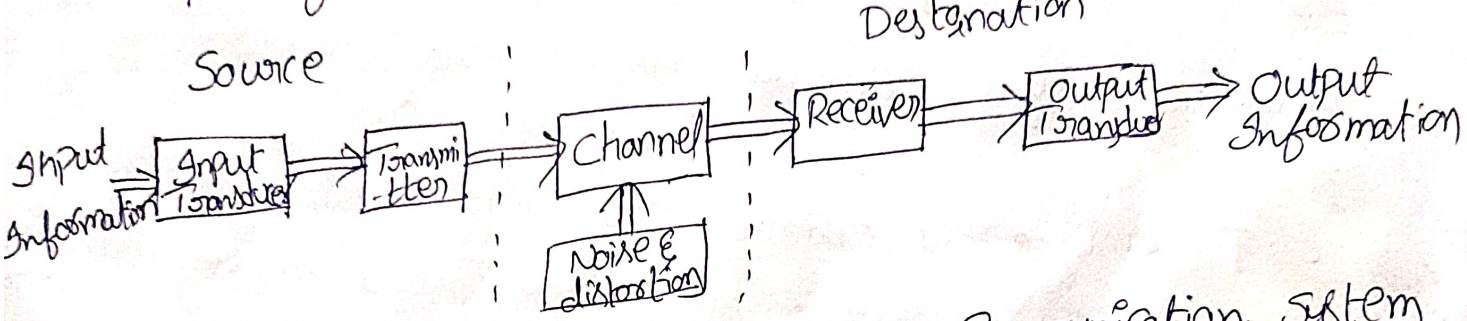


- Signals enter the Receiver from the antenna and are applied to the RF amplifier
- RF amplifier & Tuning is used for to amplifying the signal and remove the image s/g. and also reduce the general level of unwanted signals on other frequencies that are not required.
- The signal are then applied to the mixer along with local oscillator
- Mixer is used for to converted wanted signal into the intermediate freq.
- and then signal is applied to the IF Amp & filter. in this the s/g are amplified and the s/g are filtered.
- The filtering selects signals on one channel against those on the next.
- The adv. of IF filter is it is opposed to RF filtering and these IF filter are used for to designed a fixed freq. This allows for better tuning.
- Variable filters also Provide same level of selectivity with a fixed freq.
- The next block is demodulator. This could be used for amplitude modulation, single side band, freq. modulation,
- These demodulator also used for switch different demodulators in according to the mode being received.
- The final element is audio amplifier this is used for to Process or amplified the demodulated signal.
- These audio amplifier is a straightforward electronic circuit-design. If the audio is applied to the simple headPhones or loud speakers generally
- In a telecommunication the audio freq of the bandwidth about 300Hz to 3.3kHz.

~~3kHz~~ 300Hz to 3kHz

Communication: It is the Process of transferring the information from one place to another. (or) the information exchange b/w two Points.

- The major elements of communication are Transmitter of information, Channel or medium of Communication & Receiver of information.



Block diagram of Electronic Communication System

- The information is generated in source side through the Channel it reaches to destination.
- The channel could be wired line (Copper) or wireless line (atmosphere).
- The Noise and distortion is connected to a channel
- The distortion is the Process of changing the shape of the Communicating signal. Because of this one Channel to convey all frequency, Phase, amplitude information Truthfully from one side to another.
- But Noise is defined as Random, Unwanted interference on transmitted signal.

Ex of C.S:- Internet, Television, Intranet and Extranet, Public Switched Telephone n/w.

- The input transducer converts the non electrical physical into an electrical signal (or) which converts one form of ener another form.
- The Transmitter is to Process the electrical signal from different aspects. Ex:- Radio Broadcasting [In the electrical sig is obtained from sound signal.
- The Channel means it is act as a medium through which the message travels from transmitter to the Receiver.
- They are 2 types of Channels (i) Point to Point Channel (ii) Broad cast channel.
- Ex. of Point to Point channel are wire lines, microwave, optical fibres all these 3 channel Provide a Physical medium for the transmission of signals from one Point to anoth Point.
- In Broadcasting channel Provides a capability where several receiving stations can be reached from a single Tr.
- Ex of Broadcast channel is a Satellite in a geostationary orbit.
- The Noise is greatest effect on the Signal in the Channel
- Receiver it is the main function of it is used for to reproduce the message signal in electrical form from the distorted received sig.
- Destination is the final stage which is used to convert an electrical message signal into its original form.
- Noise is a any electrical sig which interferes with an information signal is called "Noise".

Types of Communication system :-

They are 4 types
(i) Analog Comunication s/m
(ii) Digital " "
(iii) Baseband " "
(iv) Carrier " " [Passband]

- Any two types is needed to specify any Communication s/m.

Analog Communication system :-

- The Analog system Convey the information from the audio, video, and Pictures b/w two Points using Analog s/g.

Ex :- Sinusoidal s/g, Broadcast, telephone | Analog Comm. is used for to Communicated the data as electronic s/g of varying freq or amplitude

Digital C.S :-

- The digital Communication is used for to exchange the information b/w two Points. | (or) In digital technology, data are generated and Processed in two States :- High(1), low is represented by '0'



BaseBand C.S :-

- It is used for to transfer of signals that are not amplified to higher frequencies.
- They helped for transfering signals with near zero freq.

Carrier C.S :-

- It is used for to transfer the information especially voice messages and calls by improving the freq much higher than actual rate.

Difference b/w Analog & Digital Communication system:

Analog	Digital
1) In analog communication analog signal is used for information Transmission.	1) In digital communication the signal is used for information Transmission.
2) In analog. uses analog s/g whose amplitude varies continuously with time from -∞ to ∞	2) In digital com. uses digital signal whose amplitude is of two levels either Low 0, or either High 1.
3) It gets affected by noise highly during Transmission through the channel.	3) It gets affected by noise less during transmission through the channel.
4) In this limited no. of channels are required	4) In this large no. of channels are required.
5) In this error Probability is high.	5) In this error Probability is low
6) In this noise immunity is Poor.	6) In this noise immunity is good.
7) Coding is not Possible	7) Different Coding techniques are Possible
8) Separating Noise & s/g analog Com. is not Possible	8) Separating noise and signal in a digital Communication is Possible.
9) less flexible	9) More flexible
10) In Analog Com. for multiplexing	10) In digital Com for demultiplexing.

UNIT-I

AMPLITUDE MODULATION

Introduction to Noise and Fourier Transform:-

Amplitude:- An Amplitude of the s/g refer to the strength of the signal. (OR) the Process of varying the amplitude of a Carrier s/g in accordance with modulating s/g is known as Amplitude modulation

Modulation:- An original message s/g can't be Transmitted over an outsized distance due to their low freq and amplitude, they are superimposed with high freq. and amplitude wave called Carrier wave.

- i.e, the superimposing a message s/g with a Carrier wave is called modulation and resulting wave gets all the characteristics of mes. signal and thus becomes useful for communicati
- The Resultant wave is to be Transmitted.

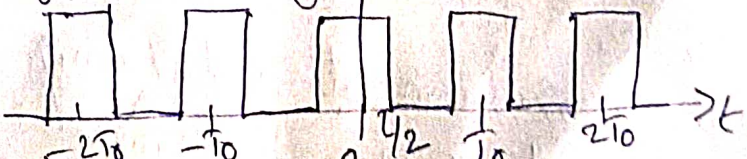
Different types of Modulation (1) Amplitude Modulation
(2) freq. "
(3) Phase "

Noise:- Any electrical s/g which interferes with an information signal is called Noise.

FOURIER TRANSFORM:-

- A Periodic waveform may be expressed as a sum of spectral Components.
- It have finite amplitude and are seperated finite freq intervals $f_0 = 1/T_0$

$T_0 \rightarrow$ energy of the signal in an interval T_0.



- If $T_0 \to \infty$, then the spacing b/w spectral components becomes infinite.
- The freq. of spectral components, the fourier series and discontinuous variable with one-to-one correspond with the integral becomes a continuous variable.
- The fourier series of Periodic wave form is

$$V(t) = \sum_{n=-\infty}^{\infty} V_n e^{j2\pi n f_0 t}$$

$$v(t) = \int_{-\infty}^{\infty} V(f) e^{j2\pi f t} df$$

- The fourier Transform is given by

$$V(f) = \int_{-\infty}^{\infty} V(t) e^{-j2\pi f t} dt$$

$V(f) \to$ amplitude spectral density (or) Fourier transform of $V(t)$

$$V_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} V(t) e^{-j\pi n f_0 t} dt$$

- Let $H(f)$ is a Transfer function. Input signal is $V_i(t)$, Output sig $V_0(t)$ is given by

$$V_0(t) = \int_{-\infty}^{\infty} H(f) V_i(f) e^{j2\pi f t} dt$$

- The fourier Transform $V_0(f) = F[V_0(t)]$

$$F[V_0(t)] = H(f) F[V_i(t)]$$

$$V_0(f) = H(f) V_i(f)$$

- The fourier Transform the s/g should follow Dirichlet's condition
- The fourier Transform is expressed interms of angular freq. ω, since $\omega = 2\pi f$, $d\omega = 2\pi df$

The fourier Transform alternate forms is $V(\omega) = F[V(t)] = \int V(t) e^{-j\omega t} dt$

$$V(t) = F^{-1}[V(\omega)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} V(\omega) e^{j\omega t} d\omega$$

Information signal



∴ Relationship b/w I/P & O/P

Introduction about Noise:

- A Noise is an unwanted s/g which interferes with the original message s/g. (06) Noise is anything that interferes with the communication process b/w the communication speaker and an audience.
- It is random or deterministic.
- This interfering signal is usually random fluctuation in voltage or current.
- types of noise in communication system are electronic noise, thermal Noise, intermodulation noise, cross-talk, impulse noise, shot noise and transit time noise
- Classification of Noise in communication system:-

(1) External noise (a) Atmospheric Noise
(b) Extra terrestrial Noise
(c) Industrial Noise

(2) Internal Noise:- (a) shot Noise
(b) Partition Noise
(c) Flicker Noise or Low freq. Noise
(d) Transit Time Noise (or) High freq Noise
(e) Thermal Noise.

(3) Semantic Noise, But this is not used in comm. s/m

External Noise:-

- It generate a external to a communication s/m
- External Noise can not be analyzed quantitatively
- for a location or Point external noise can not be controlled.
- So, to reduce the effect of external noise, Comm s/m the only was is to shift the Communications/s to another Place or location.

(a) A.N:- Which is also called as static, it provides lightning discharges in thunderstorms which occurs in atmosphere.
- The field strength of atmospheric noise varies approximately inversely with freq.

(b) E.N:- It again can be divided into 2 Noise (1) Solar Noise 2) Cosmic Noise

Solar Noise:- Solar noise is the electrical noise eliminated from the sun.
- There is a regular radiation of noise from the sun.

Cosmic Noise:- Distance stars can be considered as the sun.
- It have high temparature and radiate the noise in the sun.

(c) I.N:-
- It is also called as man-made noise. it is produced in this.

Ex:- automobiles, aircraft ignition, electrical motors, Switch gears, leakage from high v/tg Transmission.

Internal Noise:-

- This type of Noise is generated internally or within the Communication system or Receiver.
- Internal noise is randomly distributed over the entire freq.

(a) Shot noise:- It is used for to arises the active devices due to random behaviour of Charge Carriers.

Ex:- Electron Tubes

(b) Partition Noise:- It is generated in a circuit when a Current has to divide b/w two or more paths.

Ex:- Diode

(c) L.f.N:- Here we can use below a few kHz. in this only Particular type of noise appears.

- The P.S.D of this noise is increases as the freq. decreases. This noise is called flicker noise.

Ex:- Vaccum Tubes

(d) H.f.N:-

- It is generally observed in Semiconductor devices, when the transit time of Charge Carriers Crossing a junction is Comparable with the time period of those.

(e) Thermal Noise:-

- A Thermal Noise is random and is often referred as white noise or Johnson noise. It is a random noise that is generated in a resistor or the resistive Component of a Complex impedance due to molecules atoms electron

eed for Frequency Translation :-

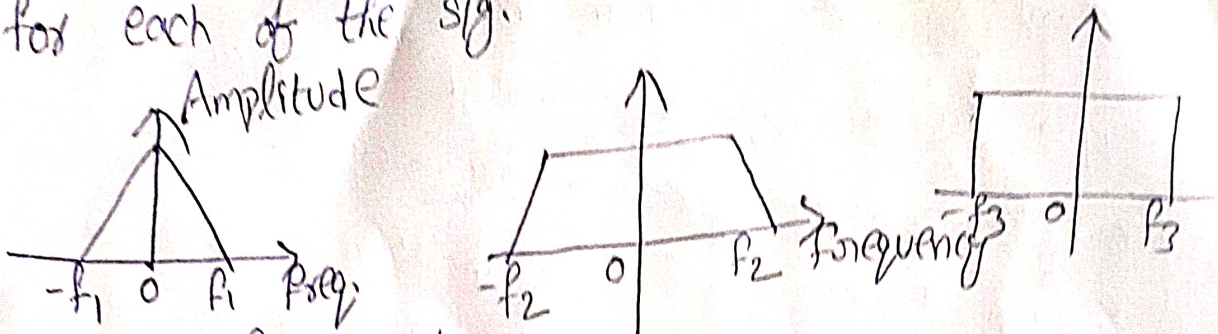
In Processing a signal in a Communication system, to translate the signal from one region in the freq. domain into another Region

- Suppose a signal is band limited, the freq. range extending from a frequency f_1 to a frequency f_2.
- In the Bocess of freq. Translation is one in which original signal is replaced with a new signal range extends from f_1' to f_2'
- Freq. Translation is useful for both analog & digital signals

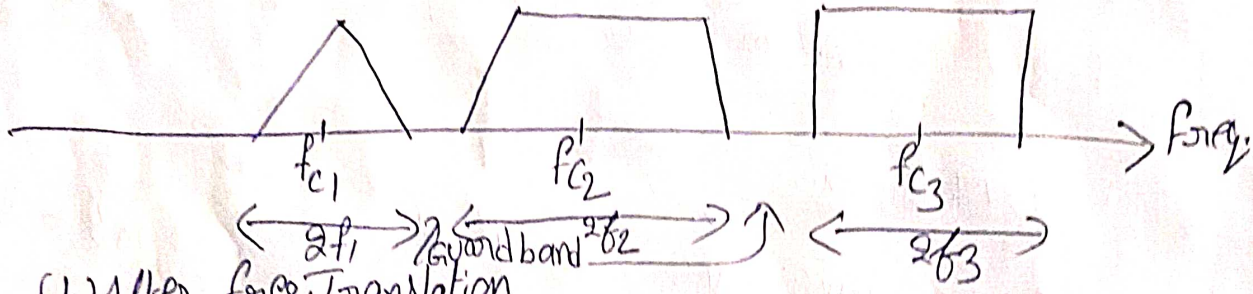
Freq. Multiplexing :-

- Suppose we have take several different signals, all are having the same spectral range.
- Let it b all the s/g are transmitted along a single Communication Channel. But at the receiving end, the signals be separately recoverable.
- The Single channel may be a single Pair of wires or the free space that seperates one radio antenna from another. Such multiple transmissions i.e, multiplexing
- Suppose, one s/g is translated to the freq. rang f_1' to f_2' and second s/g is translated to the freq range f_1'' to f_2''. If these new freq. range do not overlp, then the s/g may be separated at the receiving end. by using bandpass filter then the op of the filters processed to recovered the originals/g

- The below fig. shows how the three s/g having ... spectrum may be freq. multiplexed with separate ... for each of the s/g.



(a) Before freq. Translation



(b) After freq. Translation

Practicability of Antennas:-

- When free space is the communication channel, antenna radiate and receive the s/g.
- The antennas operate effectively only when their dimensions are of the order of the magnitude of wavelength of the s/g being transmitted.
- A signal of freq. 1kHz corresponds to a wavelength of 300,000 m.

Narrow Banding:-

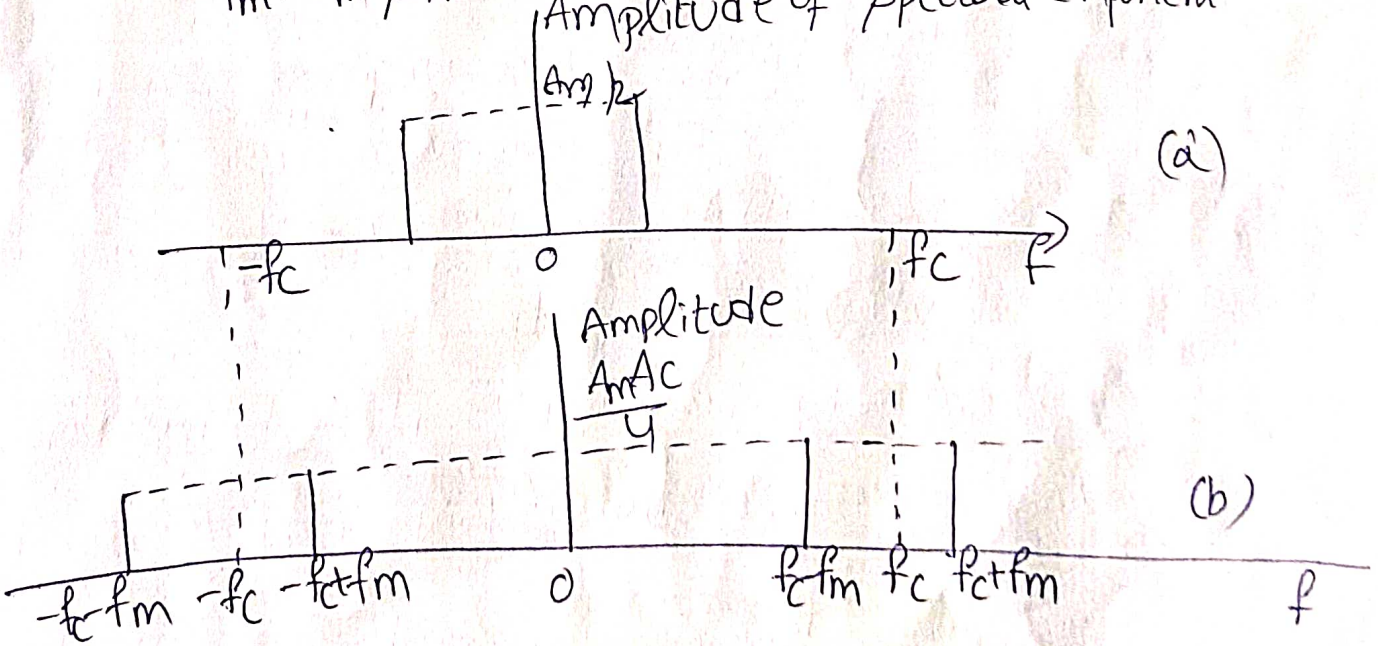
- The Process of freq. Translaton may be used to change a "wide band" signal into an "narrow Band" signal.
- These both bands are used for range of frequencies.

A Method of Freq. Translation:-

- A signal may be translated to a new spectral range by multiplying the signal with an auxiliary sinusoidal sig
- Let us Consider the signal is in sinusoidal waveform is given by

$$V_m(t) = A_m \cos\omega_m t = A_m \cos 2\pi f_m t$$
$$= \frac{A_m}{2}\left(e^{j2\pi f_m t} + e^{-j2\pi f_m t}\right)$$

- where A_m is the constant amplitude
$f_m = \omega_m / 2\pi$ is the freq.



(a) Spectrum pattern of the waveform $A_m \cos\omega_m t$.
(b) Spectral Pattern of the Product waveform $A_m A_c \cos\omega_m t \cos\omega_c t$

- The above diagram consists of two lines, each of ampli located at $f=f_m$ and $f=-f_m$.
- The multiplication of $V_m(t)$ with an auxillary sinusoid

$$V_c(t) = A_c \cos\omega_c t = A_c \cos 2\pi f_c t$$

in which A_c is the constant amplitude and f_c is the fr

$$V_m(t) V_c(t) = A_m A_c \cos\omega_m t \cos\omega_c t$$

$$= \frac{A_m A_c}{4} \left[e^{j(\omega_c+\omega_m)t} + e^{-j(\omega_c+\omega_m)t} + e^{j(\omega_c-\omega_m)t} + e^{-j(\omega_c-\omega_m)t} \right]$$

- In the fig (b) the two original spectral lines have been translated, both in positive freq. direction by amount f_c and also in the negative freq. direction by the same amount.

Amplitude Modulation:-

- Amplitude modulation is a modulation technique used in electronic communication, most commonly used for transmitting messages with a radio wave.
- In amplitude modulation, the amplitude of the wave is varied in proportional to that of the mes. signal.
- (OR) It is the process by varying the amplitude of the carrier s/g in accordance with the modulation is called Amplitude Modulation
- In A.M the source message s/g $m(t)$ that is to be transmitted by the sender to its receiver. and its Fourier transform is denoted as $M(f)$.
- To move the freq. response of $m(t)$ to a new freq. band centered at f_c Hz.
- Now to multiply $m(t)$ by a sinusoidal of freq. f_c such that $s_1(t) = m(t) \cos 2\pi f_c t$
- The basic aim of modulation by having the s/g freq content to be centered at $\pm f_c$ via

$$S_1(f) = \frac{1}{2} M(f - f_c) + \frac{1}{2} M(f + f_c)$$

- The amplitude of the sinusoidal $s_1(t)$ to be proportional to the message s/g. This method is known as Amplitude Modulation.
- Consider a sinusoidal signal

$$s(t) = A(t) \cos[\omega_c t + \phi(t)]$$

- The message s/g can be used to modulate any one of these Parameters [A, f, P] and to carry the information from Tx to

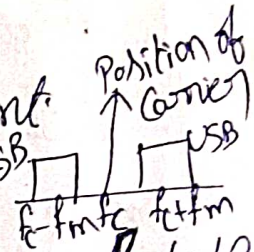
DOUBLE SIDE BAND A.M :- [DSBSC]

- In DSBSC is transmission in which freq. produced by Amplitude mod
- Amplitude modulation is characterized by an information

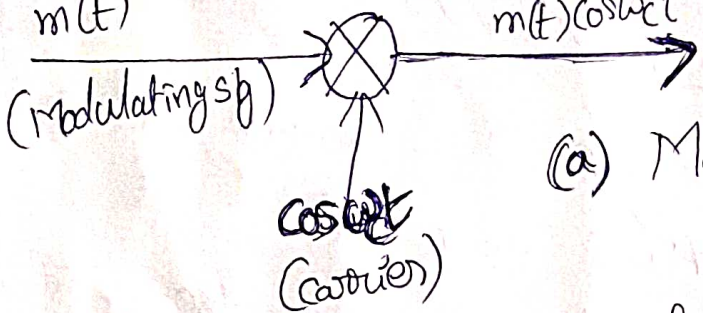
that are symmetrical spaced above carrier below the carrier

with carrier amplitude A(t) that is a linear function of baseband (message) signal m(t).

- DSBSC it has two sidebands of the carrier freq.
- Carrier freq ω_c and θ_c Phase remain constant



- Now we can assume $\theta_c = 0$.
- If the carrier amplitude A is directly proportional to the modulating s/g m(t), then modulated signal is $m(t)\cos\omega_c t$.



(a) Modulator

Applications:

→ Television broadcasting to transmit the two signals
→ Phase shift keying method to transmit the binary data.
- Wireless communication
- It has low power consumption

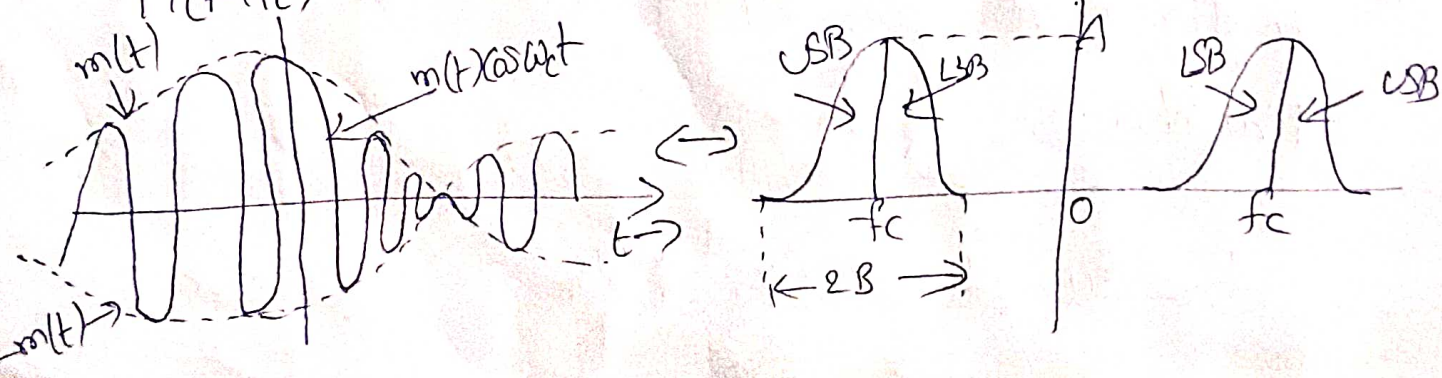
→ Now m(t) is shifts to the carrier freq. Then

$$m(t) \Longleftrightarrow M(f)$$

then $$m(t)\cos 2\pi f_c t \Longleftrightarrow \frac{1}{2}[M(f+f_c)+M(f-f_c)]$$

Here M(f-fc) is M(f) is shifted to the right by fc

M(f+fc) is " " " " left by fc



In above wave it having Bandwidth of $2B$ Hz,
- The modulated s/g centered at $\pm f_c$ Consist of two Parts; one Part lies in outside $\pm f_c$, known as upper side band (USB), and another Part lies inside that is known lower side band (LSB).
- The Message s/g $M(f)$ has an impulse at zero freq. then the modulated s/g doesnot Contain a discrete component of the Carrier freq. f_c. Otherwise the modulation Process does not introduce a Sinusoid at f_c. For this reason it is called as "double Side Band Suppressed Carrier (DSB-SC) modulation".

Demodulation :-

- The DSB-SC modulation translates or shifts the freq. spectrum to the left and right by f_c (i.e, at $+f_c$ and $-f_c$).
- To Recover the original s/g $m(t)$ from the modulated signal, it is necessary to retranslate into its original Position.
- The Process of recovering the signal from the modulating s/g is called as demodulation.
- A LPF is a Circuit that only Passes signals below its cutoff frequency while attenuating $m(t)\cos\omega t$ all the signals above it.

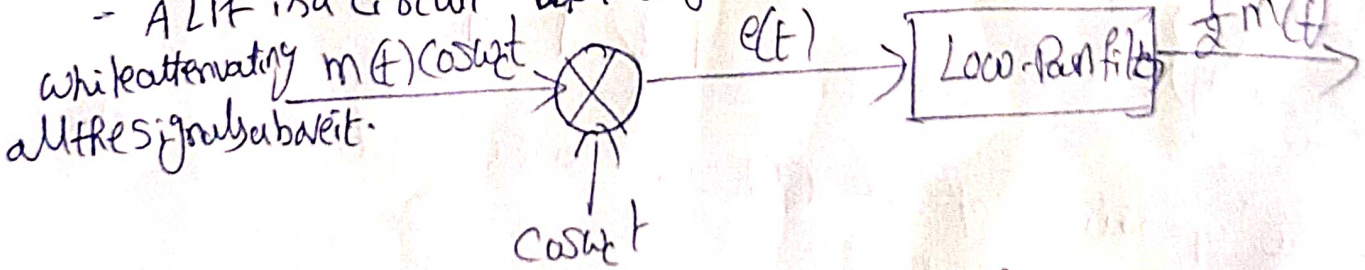


fig: (Demodulator)

- LPF is used for suppressed the s/g.

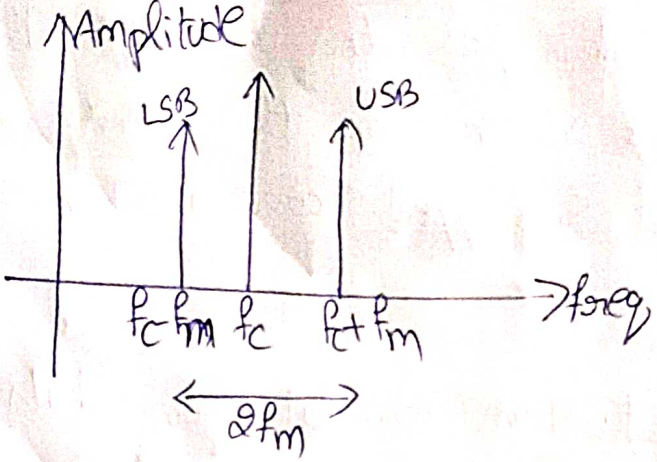
The time domain by observing the signal $e(t)$ is given

$$e(t) = m(t)\cos^2\omega t$$
$$= \frac{1}{2}[m(t) + m(t)\cos 2\omega t]$$

The F.T of the s/g $e(t)$ is $F(f) = \frac{1}{2}M(f) + \frac{1}{4}[M(f+2f_c) + M(f-2f_c)]$

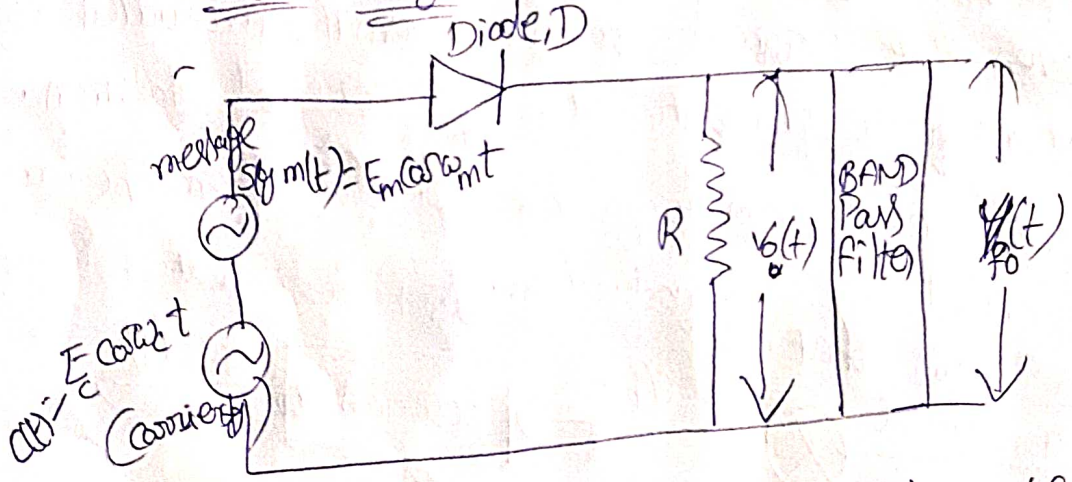
DSB-FC:- Double Side band - Full Carrier:-

- Double-Side band SC transmission is transmission in frequencies Produced by amplitude modulation are symmetrically spaced above and below the Carrier freq.
- The transmission of a signal which contains Carrier along with two side bands can be called as "DSB-FC".



- A Diode is a semiconductor device with two terminals and it allowing the flow of current in one direction only

Circuit Diagram:-



- The operation of the Ckt is based on the switching action of the Diode D
- The Combined i/p s/g is given by

$$V_i(t) = m(t) + c(t)$$
$$= E_m \cos \omega_m t + E_c \cos \omega_c t$$

- The diode converts only when the Combined i/p signal is Positive. $|c(t)| >> |m(t)|$
- A Diode is a semiconductor device it contains two terminals. Anode & Cathode

(or) $|E_c \cos\omega_c t| >> |E_m \cos\omega_m t|$

- A diode is used in rectifiers, voltage Regulators, signal modulator, mixers, oscillators.
- The ON & OFF action of the diode happens Periodically according to the Carrier Pulse. The ON & OFF action is called "Switching Action".
- The Switching function s/g is given by $S(t) = \frac{1}{2} + \frac{2}{\pi}\left[\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \cdots\right]$

The o/p s/g is given by

$$V_o(t) = V_c(t) \times S(t)$$

$$\Rightarrow V_o(t) = \left[m(t) + E_c \cos\omega_c t\right]\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \cdots\right)\right]$$

$$= \frac{m(t)}{2} + \frac{E_c}{2}\cos\omega_c t + \frac{2}{\pi} m(t)\cos\omega_c t + \frac{2E_c}{\pi}\cos^2\omega_c t + \cdots$$

- The Output Passed through a band Pass filter (BPF) having cut off freq. given by

$$\omega_{cut\text{-}off} = \pm\omega_c$$

- After Passing through the BPF, 'multiple of ω_c' freq. components are blocked while 'ω_c' freq. are Passed.
- The final o/p is given by

$$\boxed{V_{fo}(t) = \frac{E_c}{2}\cos\omega_c t + \frac{2}{\pi} m(t)\cos\omega_c t}$$

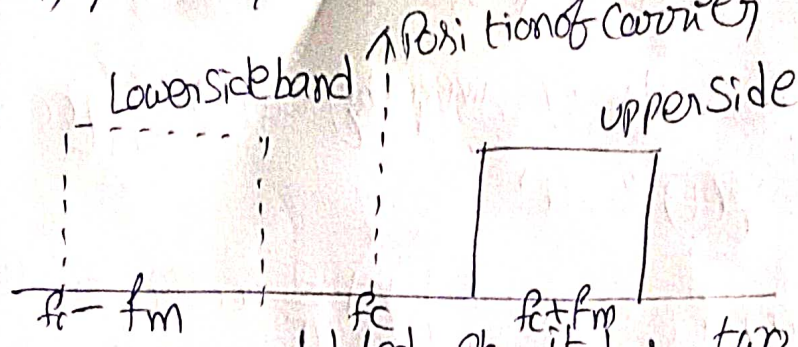
$$= k_1 E_c \cos\omega_c t + k_2 m(t)\cos\omega_c t \quad \left[\because k_1 = \tfrac{1}{2} \text{ and } k_2 = \tfrac{2}{\pi}\right]$$

$$= \underset{s/g}{\text{Carrier}} + \underset{s/g}{\text{DSB-SC}} = \underset{s/g}{\text{DSB-FC}}$$

- A BPF is a arrangement of electronic components that allows only those electric waves lying within a certain range or band of frequencies to Pass and blocks all others.

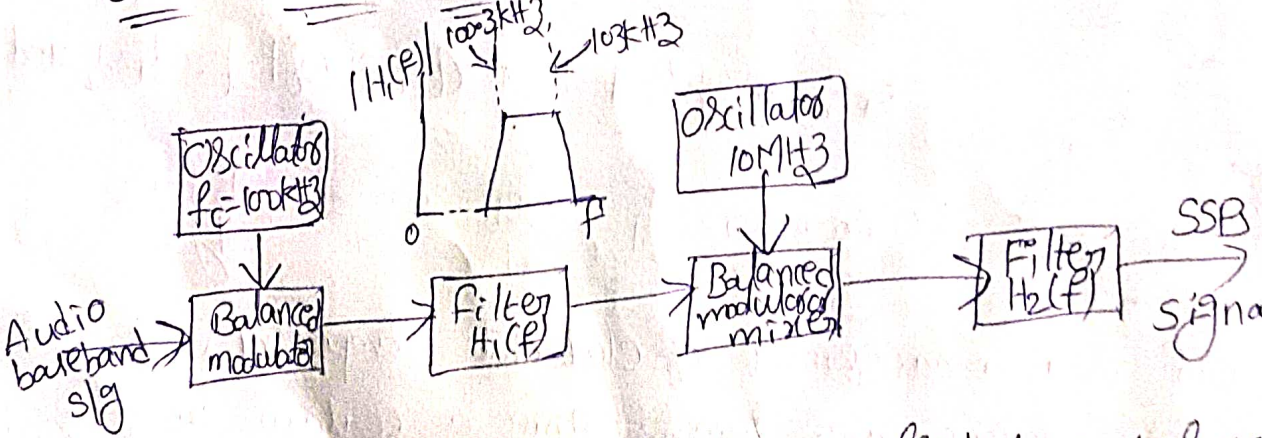
SSB-SC :- Single Side Band Suppressed Carrier :-

- A SSB or SSB SC is a type of modulation used to transmit information, such as an audio s/g by radio waves.



- In The DSBSC modulated s/g it has two side bands. and these two band have contain same information.
- But in SSB-SC modulation it's a process of suppressing one of the sidebands along with the carrier and transmitting a single side band is called 'SSB-SC'. It can be shown in above fig.
- In SSBSC s/m, which transmits a single side band has high Power.

Block diagram of generating SSB s/g



- The Baseband s/g and carrier are applied to a balanced modulator
- The Output of the balanced modulator (s/m) bears both the USB and LSB signals.

- The filter is used to select a frequency range of sideband.
- If the freq. separation of sidebands is twice the freq. of the lowest freq. component of the base band s/g.
- The filter may also used for to suppress the carrier.
- The SSB-SC modulating s/g is $m(t) = A_m \cos(2\pi f_m t)$

  Carrier s/g is $C(t) = A_c \cos(2\pi f_c t)$

Then the eq. of SSBSC wave as

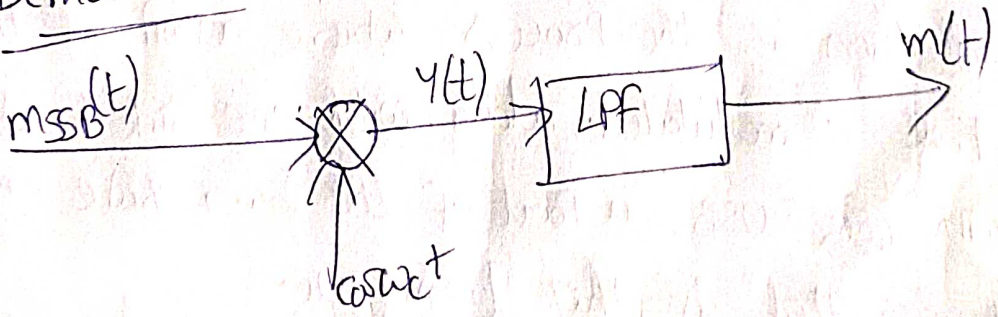
$$S(t) = \frac{A_m A_c}{2} \cos[2\pi (f_c + f_m) t] \quad \text{[for upper band]}$$

(or)

$$S(t) = \frac{A_m A_c}{2} \cos[2\pi (f_c - f_m) t] \quad \text{[for lower sideband]}$$

The B.W of SSBSC modulated wave $= \frac{2 f_m}{2} = f_m$

SSB Demodulation:-



- This is used for to recovery the original s/g from modulating s/g

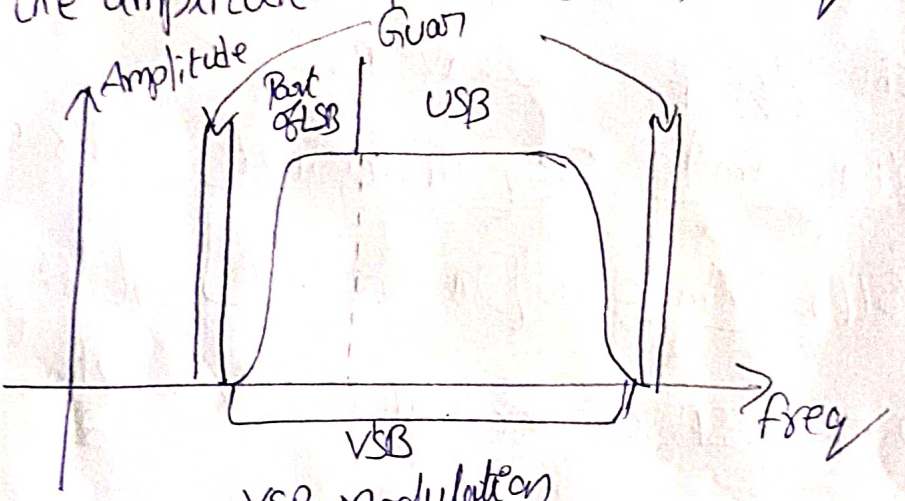
$$m_{SSB}(t) = 0.5 m(t) \cos\omega_c t \pm 0.5 m_h(t) \sin\omega_c t$$

$$y(t) = m_{SSB}(t) \cos\omega_c t = 0.5 m(t) \cos^2\omega_c t \pm 0.5 m_h(t) \sin\omega_c t \cdot \cos\omega_c t$$

$$= 0.25 m(t) [1 + \cos 2\omega_c t] \pm 0.25 m_h(t) \sin 2\omega_c t$$

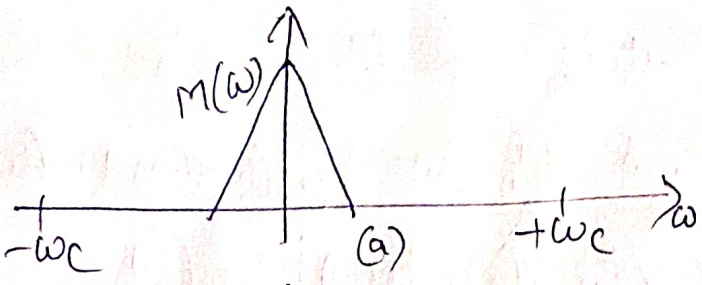
Vestigial Side band (VSB) modulation:-

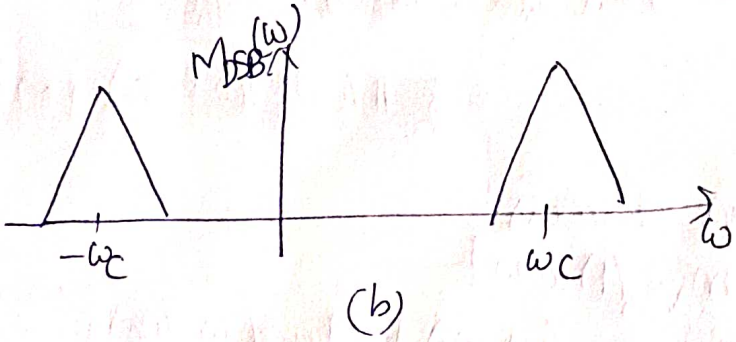
- VSB modulation is something in b/w SSB and DSBSC modulation. Provides at small cost.
- VSB is the Process where the "Vestige" Part of a signal is modulated alongside one side band.
- VSB is a form of AM that encodes the data in a signal by altering the amplitude of the Carrier freq.



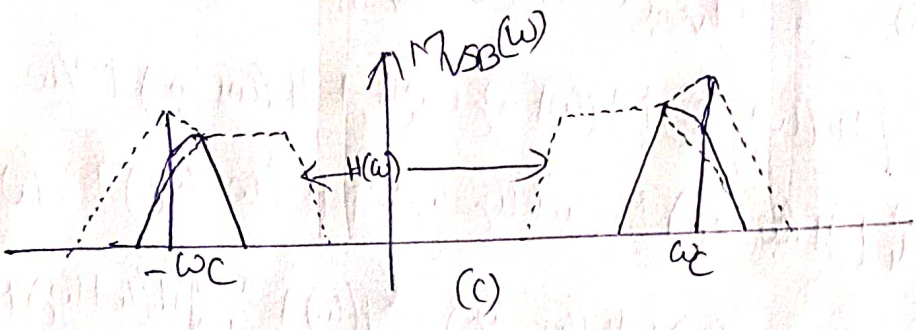
VSB Modulation

- VSB modulation is the Process where a Part of the Sig is called as Vestige is modulated, along with one side band.
- Along with the USB, a Part of the lower side band is also being transmitted in this technique
- A guard band is used in order to avoid the interference
- A VSB used in television transmission.
- A vestigial is nothing but it is something that was once greater or more noticeable.





(b)



(c)

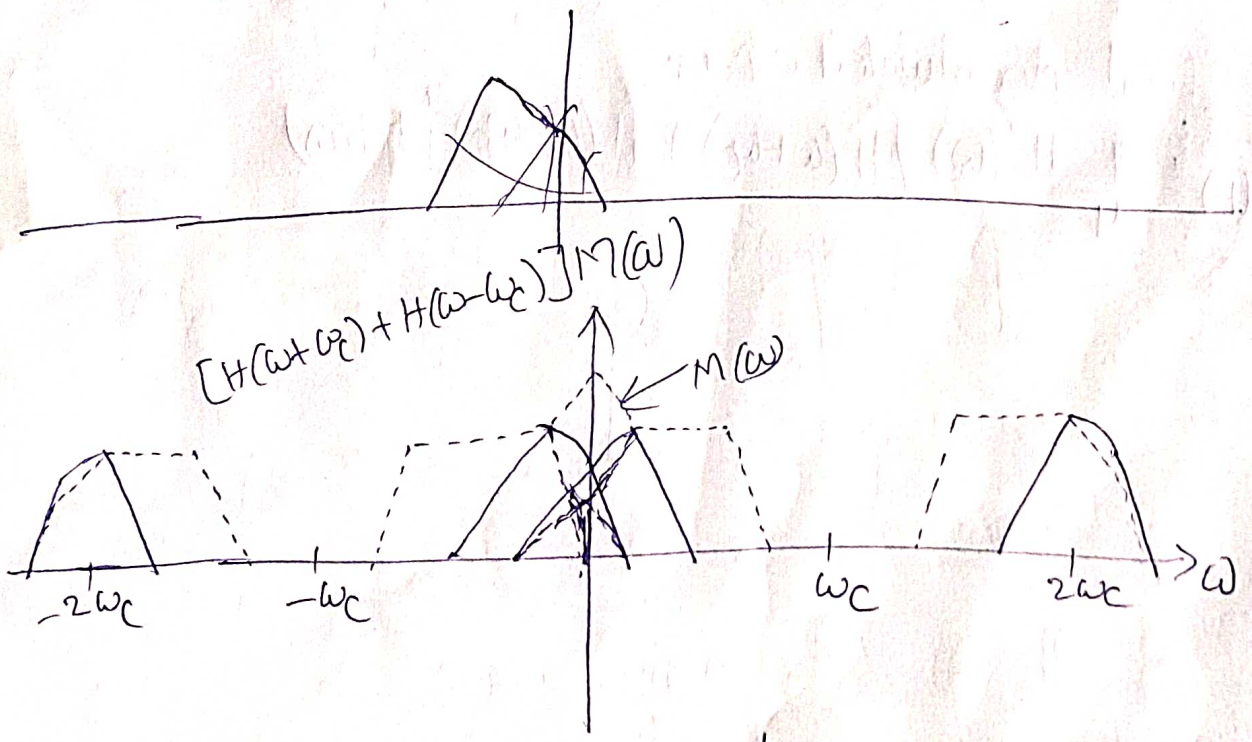


fig VSB modulation & Demodulation
(a) Spectrum of mes. S/g
(b) " " DSB-SC S/g
(c) VSB modulation from DSB-SC modulation
(d) VSB demodulation by coherent detection.

- The VSB signal is generating by filtering the DSB-SC s/g with

DSB-SC s/g spectrum, $M_{DSB}(\omega) = \frac{1}{2}[M(\omega+\omega_c) + M(\omega-\omega_c)]$

Then VSB spectrum, $M_{VSB}(\omega) = H(\omega) M_{DSB}(\omega)$

$= \frac{1}{2} H(\omega)[M(\omega+\omega_c) + M(\omega-\omega_c)]$

- In demodulation Process

$g(t) = m_{VSB}(t) \cos\omega_c t$

$Y(\omega) = \frac{1}{2}[M_{VSB}(\omega+\omega_c) + M_{VSB}(\omega-\omega_c)]$

$= \frac{1}{4}[H(\omega+\omega_c) M(\omega+\omega_c+\omega_c) + H(\omega+\omega_c) M(\omega-\omega_c+\omega_c)]$

$+ \frac{1}{4}[H(\omega-\omega_c) M(\omega+\omega_c-\omega_c) + H(\omega-\omega_c) M(\omega-\omega_c-\omega_c)]$

$= \frac{1}{4}[H(\omega+\omega_c) + H(\omega-\omega_c)] M(\omega) + \frac{1}{4}[H(\omega+\omega_c) M(\omega+2\omega_c)$

$+ H(\omega-\omega_c) M(\omega-2\omega_c)]$

The final demodulated o/p is.

$Y(t) = \frac{1}{4} H_R(\omega)[H(\omega+\omega_c) + H(\omega-\omega_c)] M(\omega)$

Frequency Division Multiplexing:- FDM means multiple signals are combined for transmission on a single comm. line or channel, with each s/g assigned to a different freq. within the main channel.

- FDM is a technique by which the total bandwidth available in a communication medium is divided into a series of non-overlapping freq. bands, each of which is used to carry a separate signals.
- The Amplitude modulation is used for multiplexing.

Eg. The Transmitting many baseband signals $m_1(t), m_2(t) \cdots m_n(t)$ over a common communication channel.

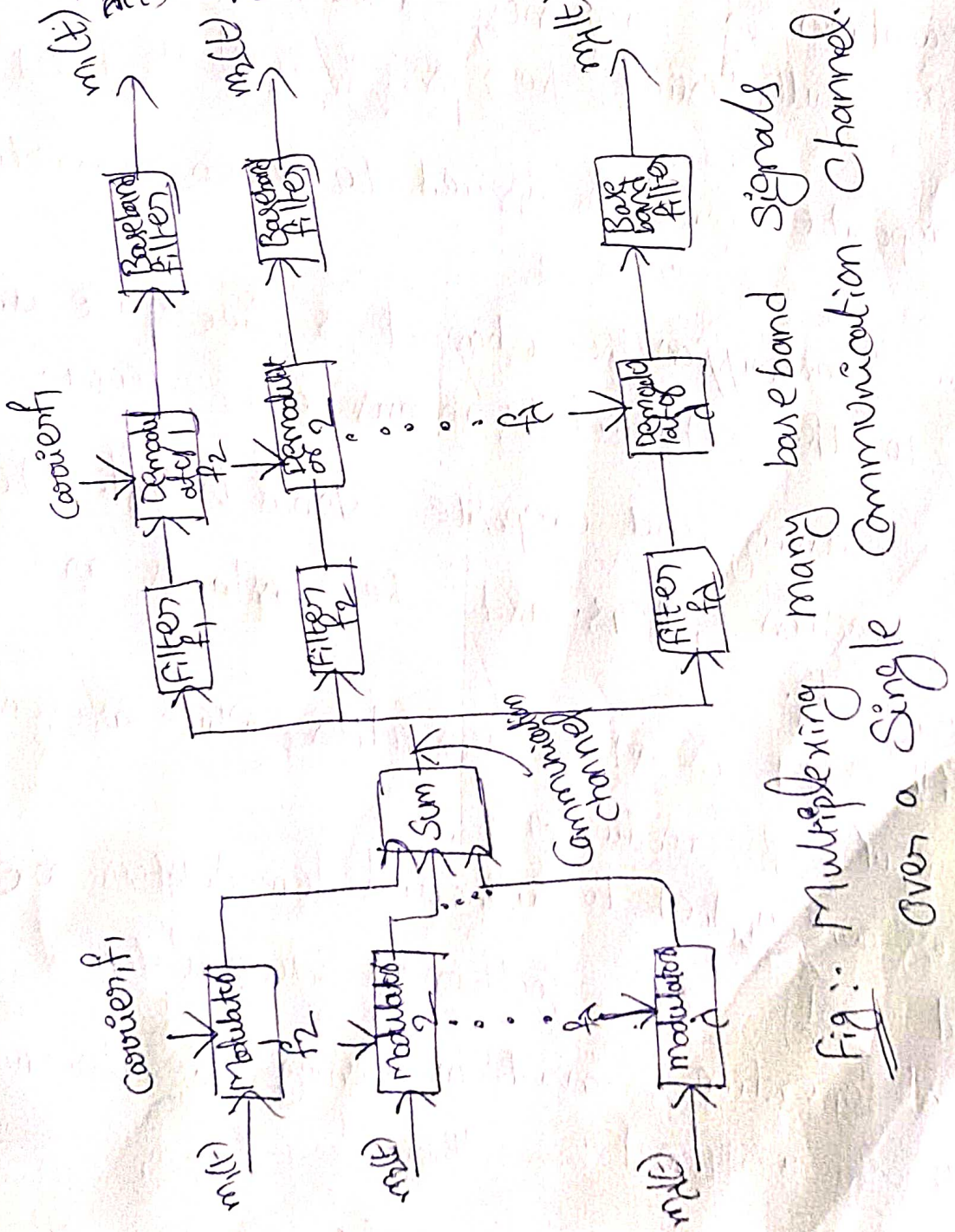


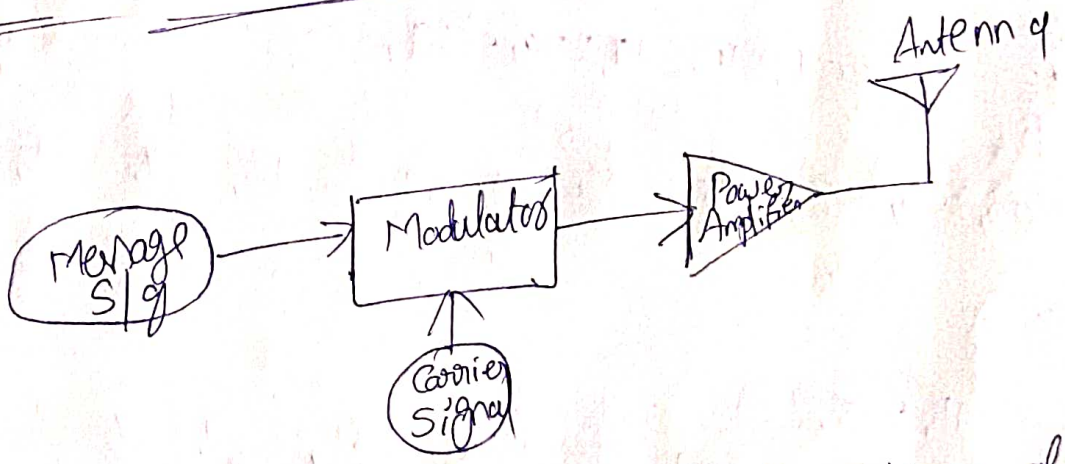
Fig:- Multiplexing many baseband signals over a Single Communication Channel.

- Let us consider baseband signals $m_1(t), m_2(t), \ldots m_N(t)$ each having bandlimited to f_M, that are applied to individual modulators.
- Modulator means a device for modulating the carrier wave.
- Each modulator is supplied with a carrier waveform of frequency $f_1, f_2, \ldots f_N$.
- A filter is a ckt capable of passing (or amplifying) certain freq. while attenuating other freq.
- The individual modulator output signals extend over a limited ranges of ~~modulator~~ output signals. Carrier freq
- The carrier freq. is selected to the spectral range of modulator output signal do not overlap. recovery
- This separation in frequency allows the individuals/g and this multiplexing sys is referred as freq. Multiplexing.
- and Composite signal is applied to common communication channel.
- In Radio Transmission, the channel is free space and coupling to the channel by means of antenna.
- At the Receiving end composite signal is applied to each of bandpass filters. and the passbands are $f_1, f_2, f_3, \ldots f_N$.
- The filter f1 is a band pass filter which passes only at the output of modulator 1.
- Then they are applied to individual demodulators are required only for synchronous demodulator.
- The final operation consists in passing the demodulator o/p through a Base Band Pass filters.

The Baseband filter is low Pass filter with cutoff freq. f_m

- The Baseband filter will Pass, without modification, the baseband signal o/p of the modulator in this sense serves no function in the system described upto the Present Point.

Radio Transmitter And Receiver:-



- We are taking wireless Transmission of AM signal.
- In Radio Transmitter, the baseband sig is translated to a radio freq by a modulation Process which is then amplified by a Power amplifier and Radiated through antenna.
- The freq translation and amplification of the signal can be done in several stages.
- The AM broadcast transmitter occupies 10KHz bandwidth
- if the Carrier signal is of 600KHz freq then it occupies freq range of 595-605 KHz.

- The Oscillator used for Carrier is usually made up Crystal as LC oscillators tend to drift with time are not as stable.
- The Oscillator output passes through a buffer amplifier which increases its power level but more importantly serves as an isolator that prevents variation of load affecting freq. of the oscillator.

III-EEE 17/10/22
ABS RE215, LE→ 202, 209, 210, 219, 220, 239,

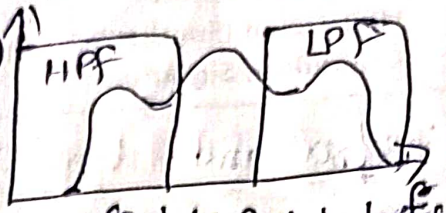
III-EEE 18/10/22
ABS: RE→203, 204, 208, 211, 215 LE→208, 209, 210, 212, 219, 221, 230, 231, 239,

III-EEE 19/10/22
ABS: RE→ 211, 215,
LE→209, 219, 221, 222, 228, 230, 231, 237, 239,

UNIT-IV

Pass Band Transmission Model:-

- Pass Band Transmission is typically used for wireless communication and in band pass filtered channels such as POTS lines. It is also called as PSTN (Public Switched Telephone service) or POT (Plain old Telephone service)



- It also allows for FDM.
- The digital bit stream is converted first into an equivalent base band signal and then to RF signal.
- Pass band Transmission is the transmission after shifting the base band frequencies to some higher freq. range using modulation.

formula of Pass band by using BPF is $f_c = \frac{1}{2\pi\sqrt{LC}}$

Adv:- Pass band digital transmission allows more efficient use of allocated RF bandwidth and flexibility in different base band S/g formats.

Dif of Pass band and Base Band Pass:-

- In B.B it uses low freq. and in this signal that has passed through a Band Pass filter.
- In P.B it uses high freq. and in this signal that falls into the Pass band of a given filter.

sults from both higher and lower frequency sidebands. The symmetrical, so the amplitude variations

UNIT-II

Noise and Pulse Modulations

Receiver Model in Noise:-

→ Receiver noise is directly related to Thermal Noise, dynamic Stress, and GPS [Global Positioning system] receiver.

- Receiver noise is a uncorrelated error source, that means the effect of both multipath and receiver noise are not related to the length of the baseline between GPS receivers.
- In any Communication system, during transmission of the signal, or while receiving the signal some unwanted signal gets enter into the Communication, making it unpleasant for the Rx [Disaggreable or Unlikable] Such a disturbance is called as Noise...
- There are three main types of Receiver noise
  1) Thermal Noise
  2) dark Current Noise
  3) Quantum Noise

1) Thermal Noise:- The electronic Noise generated by the thermal agitation of the electrons inside an electrical Conductor at equilibrium.

2) Dark Current Noise:- It is the statistical variation of dark Current across the sensor.

3) Quantum Noise:- It is also called as quantum mottle, it is the random Process due to fluctuations in the number of Photons reaching the detector from Point to Point.

Noise in AM :-

- A.M noise is simply defined as unwanted amplitude modulation of an F.M Carrier.
- There are 2 types of A.M noise (1) Synchronous Noise
  (2) Asynchronous Noise
- In Asynchronous Noise consists of amplitude modulation of the Carr uncorrelated to the FM modulation of the carrier, typically caused by Power supply or vibration.
- In synchronous noise is refer to the sound recor de in the films in the a.m and F.M of the carrier signa is related.

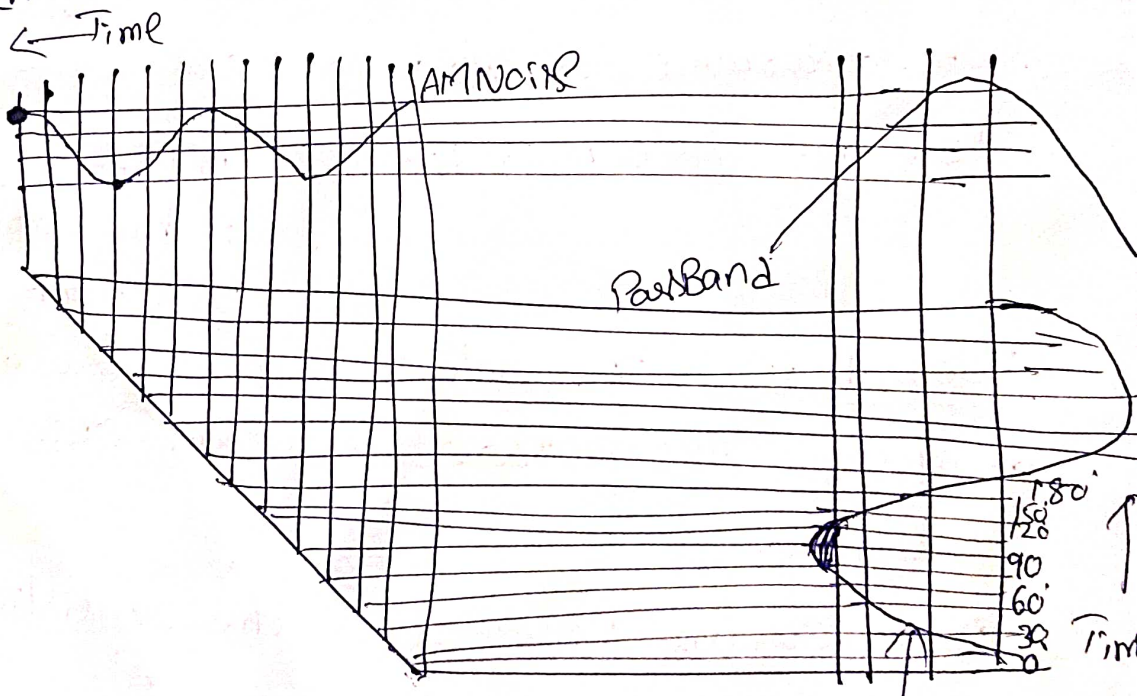


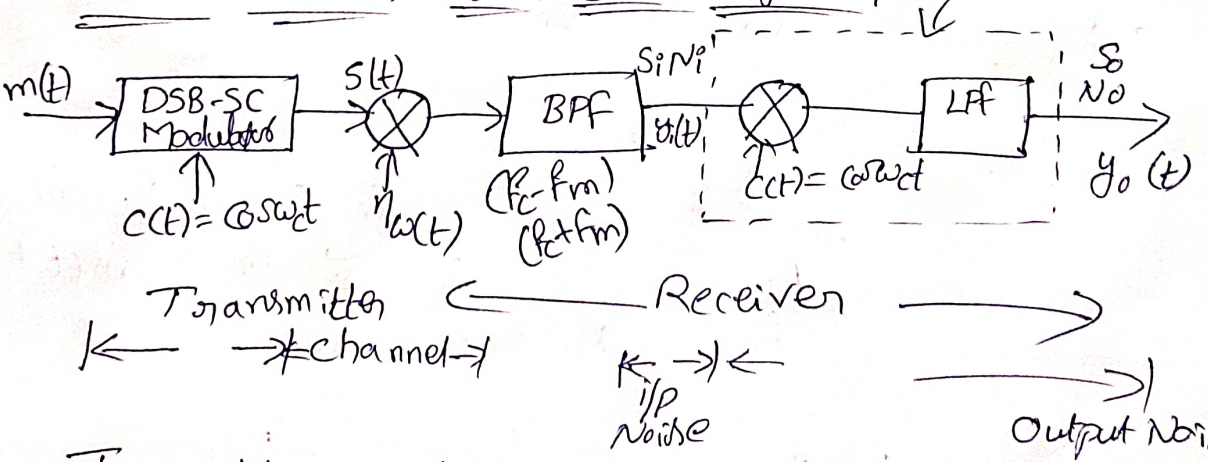
Fig A.M Noise generation

- In above fig. As the F.M carrier freq. shifts with mo in the form of sine wave.
- The Pass band slopes produce a direct variation in the carrier amplitude. These variation are defined by A.M No

- Because Passband is symmetrical, so the amplitude variations results from both higher and lower frequency slopes. The resulting A.M is twice the freq. of the FM modulation.
- If the centre of the Pass band is shifted below the center freq, the resulting A.M is same as F.M.

Noise in DSB:- DSB is used for to transmit two signals.
- DSB Contains two side Band upper and lower side bands.
- DSB is used in Phase shift keying to transmit binary data.
- It is having two types DSB-SC and DSB-FC
- In DSB-SC is a transmission, in which frequency Produced by amplitude modulation are symmetrically spaced above and below the carrier frequency.
- In DSB-FC is a transmission of a signal, which Contains a Carrier along with two side bands Can be termed as DSB-FC.

DSB SC Receiver model Block diagram:-



- Transmitter Contains DSB-SC Modulator whose output $S(t) = m(t) \cdot \cos\omega_c t$

(or) $S(t) = m(t) \cdot \cos(2\pi f_c t)$ $\left[\because f_c = \frac{\omega_c}{2\pi}\right]$

∴ Process ... equivalent

- Noise generated by the channel is considered as white noise $\eta_{w(t)}$ with uniform noise power spectral density $\eta/2$
- The Bandwidth of Band Pass filter is equal to modulated signal bandwidth.
- BPF allows DSBSC signal and Convert white noise into color or band pass noise $\eta_B(t)$

Therefore o/p of Band Pass filter is $y_i(t) = s(t) + \eta_B(t)$

- Synchronous detector is used to extract modulating signal $m(t)$ which contain multiplexed followed by LPF.

i/p signal power is $S_i = m^2(t)/2$

i/p noise power is $N_i = \eta \cdot 2f_m$

Output signal power is $S_o = [m^2(t)/2]^2$

output noise power is $N_o = \eta \cdot \frac{f_m}{2}$

Substituting values in F_{ON}

$$F_{ON} = (S/N)_o / (S/N)_i$$

$$= \frac{[m^2(t)/2]^2}{\eta \cdot \frac{f_m}{2}} \Big/ \frac{m^2(t)}{2} = \frac{m^2(t)}{2 \times \eta \cdot f_m} \times \frac{\eta \cdot 2f_m}{m^2(t)} \times 2$$

$$= 2$$

$$= \frac{m^2(t)}{4} \times \frac{\eta \cdot 2f_m}{\eta \cdot f_m} \times \frac{1}{m^2(t)} \times 2 = 2$$

$$F_{ON} = 2$$

SSB:- (or) SSB-SC:-

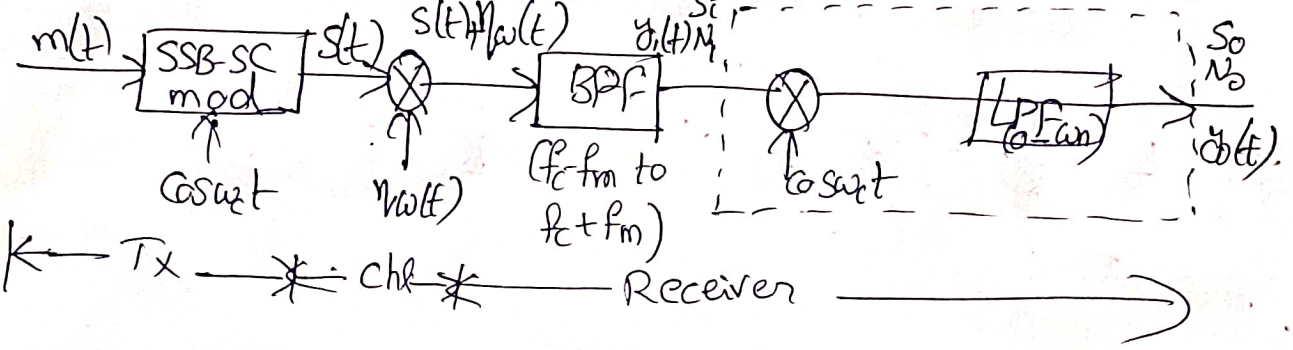
(3)

- In a single side band is a type of modulation used to transmit information along with the single side band and it suppressed the other side band along with carrier signal.

Ex:- Audio signal, radio waves.

Calculation in SSB-SC:-

SSB-SC Signal $S(t) = m(t)\cos\omega_c t \pm \hat{m}(t)\sin\omega_c t$



Output of BPF, $y_i(t) = S(t) + \eta_B(t)$

Band Pass Filter Noise, $\eta_B(t) = n_i(t)\cos\omega_c t - n_q(t)\sin\omega_c t$

Input S/g Power is $S_i = \underline{m^2(t)}$

Input Noise Power $N_i = \eta \cdot f_m$

O/P Signal Power $S_o = m^2(t)/4$

O/P Noise " $N_o = \eta \cdot f_m/4$

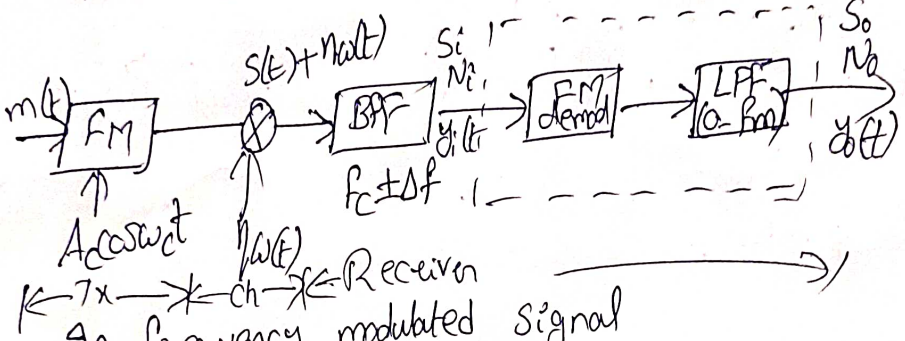
$$FOM = \frac{S_o/N_o}{S_i/N_i} = \frac{m^2(t)/4 / \eta \cdot f_m/4}{m^2(t)/\eta \cdot f_m} = \frac{m^2(t)}{\eta \cdot f_m} \times \frac{\eta \cdot f_m}{m^2(t)} = 1$$

Sampling Process :- ... into equivalent

FM Receivers in Noise :-

- In any communication system, during the transmission of the signal or while Receiving the s/g, some unwanted signal gets introduced into the communication, making it unpleasant for the Receiver. Such a disturbance is called as noise.
- Noise can be reduced in FM Receivers by increasing the frequency deviation.

Noise Calculation in FM System :-



In frequency modulated signal

$$S(t) = A_c \cos\left[\omega_c t + k_f \int m(t)dt\right]$$

Output of BPF is

$$y_i(t) = S(t) + \eta_B(t)$$

$$= A_c \cos\left[\omega_c t + k \int m(t)dt\right] + \eta_B(t)$$

input signal power is, $S_i = \frac{A_c^2}{2}$

Input Noise Power is $N_i = 2\eta \cdot \Delta f$

Output signal Power, $S_o = \gamma^2 k_f^2 \overline{m^2(t)}$

$N_o = 2\eta f_M$

Substi. these values $F_{OM} = \frac{S_o/N_o}{S_i/N_i}$

$$F_{OM} = \frac{3}{4\pi^2} m_f^3$$

where $m_f = \frac{\Delta f}{f_m}$

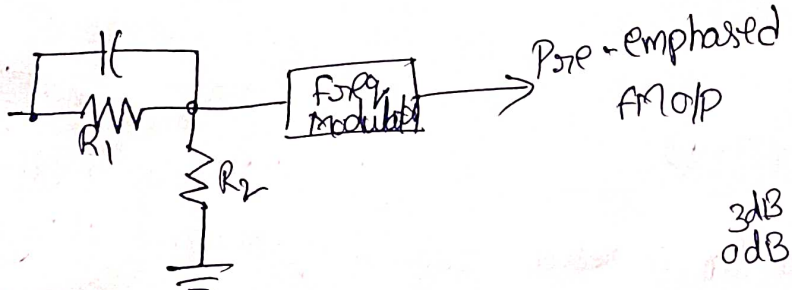
Sampling Process :-

- Process of converting continuous time signals into equivalent "sampling".

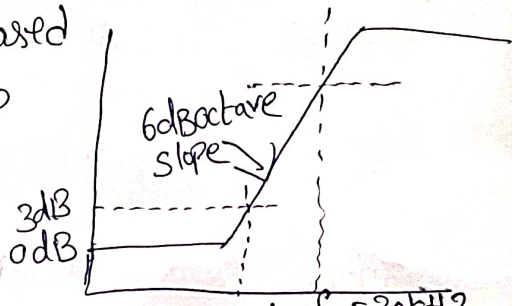
Pre Emphasis and De-emphasis in FM :-

(4)

- Pre Emphasis and De-emphasis circuits only used in freq. modulation
- Pre Emphasis used at transmitter and de emphasis used at Receiver for higher modulating frequencies.
- This higher modulating freq. are used for to increasing the deviation 'δ' and this deviation can be increased by amplitude of the modulating signal at higher frequencies.
- Pre Emphasis works at high freq [2 to 15 kHz] and de emphasis works at low frequency.



(a) Circuit of Preemphasis



(b) Pre emphasis Curve.

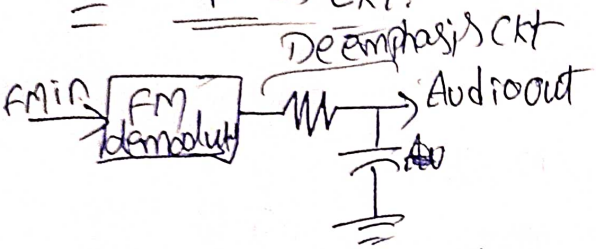
- In above fig. At the transmitter, the modulating signal is passed through a simple network which amplifies high freq more than low freq. components. That is we are taken simple "high Pass filter".
- Here time constant of 75 microseconds (μs) $t = RC$.
- The combination of resistor and capacitor giving time constant.
- Such a circuit cut off freq. f_0 is 2122 Hz.

Sampling Process :-

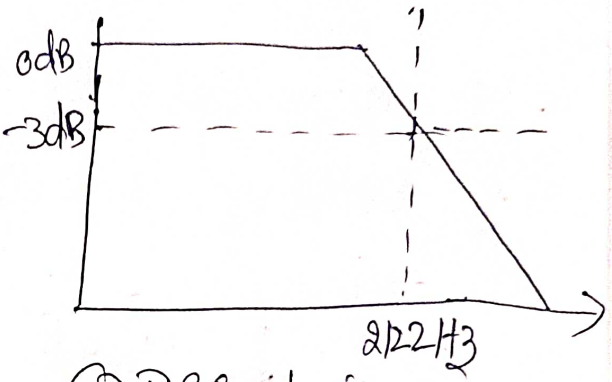
- That is when the freq. higher than 2122Hz will be linearly enhanced
- The o/p amplitude increases with freq at a rate of 6dB per octave.
- Thes Pre emphasis ckt increases then higher freq. signals is also increases.
- This Circuit is used for improving signal to Noise ratio.
- In Preemphasis upper break freq. fu is given by

$$f_u = R_1 + (R_2/2\pi R_1 R_2 C)$$

De emphasis ckt:-

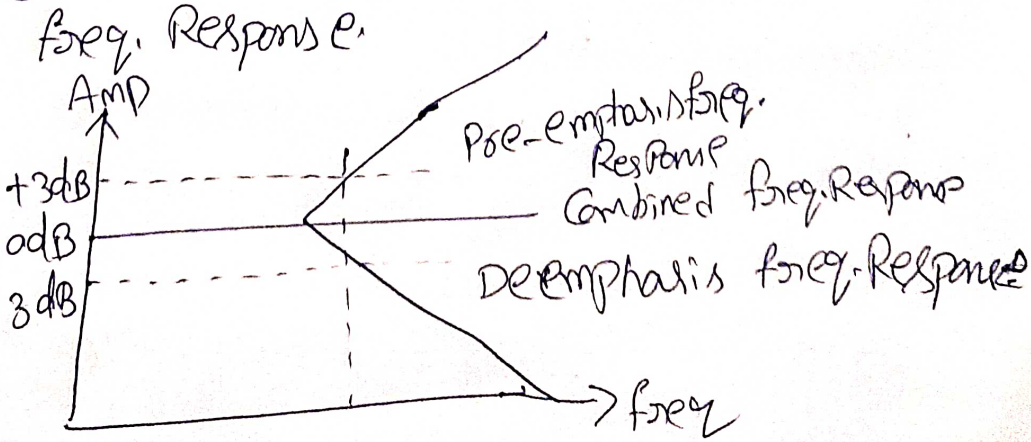


Ckt of Deemphasis



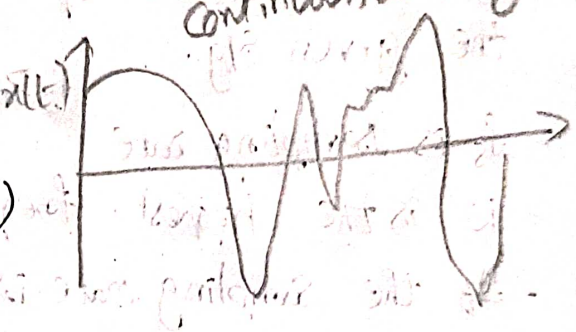
(5) Deemphasis Curve

- This ckt is used for to return the freq. response to its normalled
- This is a simple Low Pass filter with a Constant of 75π C.
- In this cutoff freq. is 2122Hz and causes the signal above this freq to be attenuated at the rate of 6dB per octave
- As a result the Preemphasis at the transmitter is exactly offset by the deemphasis ckt in the receiver, Providing a normal freq. Response.

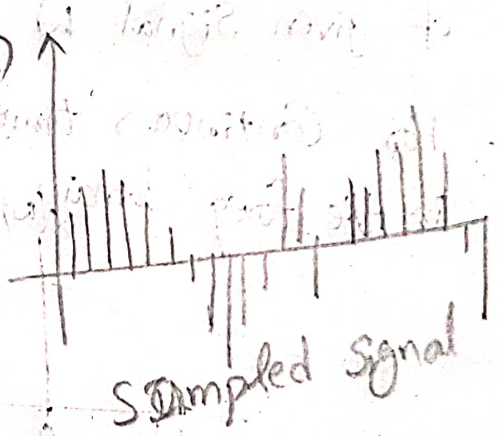


Sampling Process :-

- The Process of converting Continuous time signals into equivalent discrete time signals can be termed as "Sampling".
- In Sampling Process it is not possible to recover the orginal signal from Sampling.
- If Continuous time signal is sampled at regular intervals then $f_s \geq 2f_m$
- A Continuous time signal $x(t)$ and corresponding sampled signal $x_s(t)$



- A Sampling signal is a Periodic train of Pulses, having unit amplitude, sampled at equal intervals of time T_s, which is called as Sampling time.
- Thes data is transmitted at the time instant T_s and carrier signal is transmitted at remaining time.



Sampling rate :-

- To discretize the signals, the gap between the samples should be fixed. That gap can be termed as the Sampling Period T_s.
- The Reciprocal of the Sampling Period is known as Sampling freq. (or) Sampling rate f_s.

$$f_s = \frac{1}{T_s}$$

$f_s \rightarrow$ Sampling freq

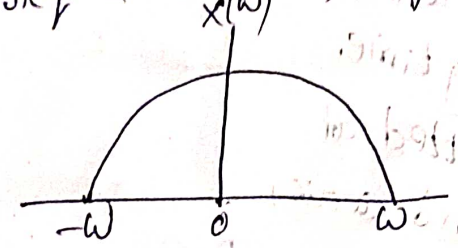
$T_s \rightarrow$ Sampling Period

Pulse

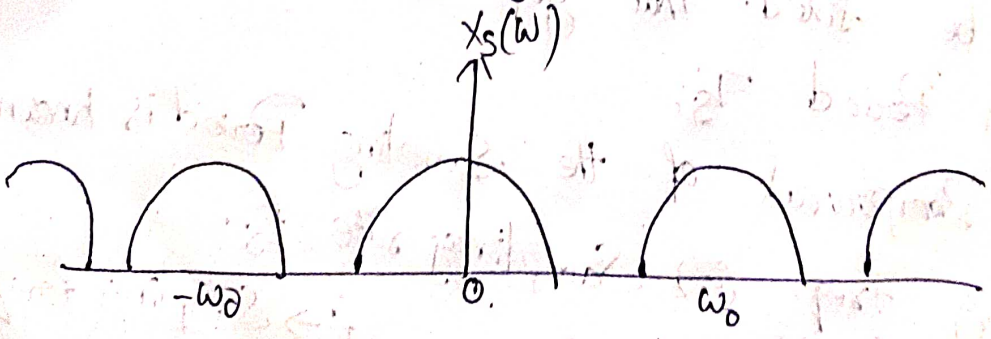
PA

- P

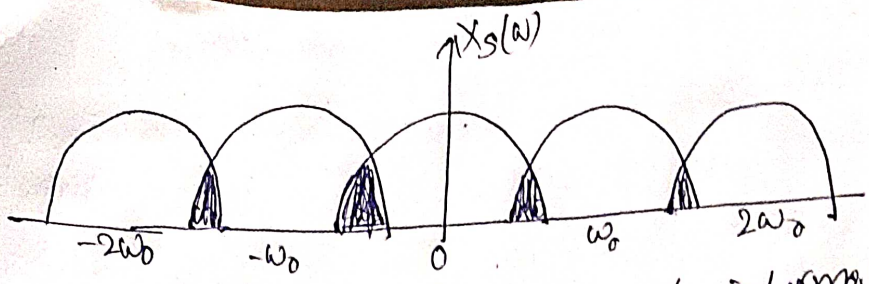
- The Sampling rate should be such as that the data in message s/g should be lost nor it should get overlapped.
- A Sampling Theorem states that, "a Signal can be exactly reproduced if it is sampled at the rate f_s, which is greater than or equal to twice the max. freq of the given s/g.. $f_s \geq 2W$ (or) $f_s \geq 2f_m$
- f_s is Sampling rate
- W is the highest freq. of the given signal
- If the sampling rate is equal to twice the max. freq. of given Signal W then it is called Nyquist rate.
- For Continuous time Signal $x(t)$, which is band limited in the freq. domain is represent as shown in below fig.



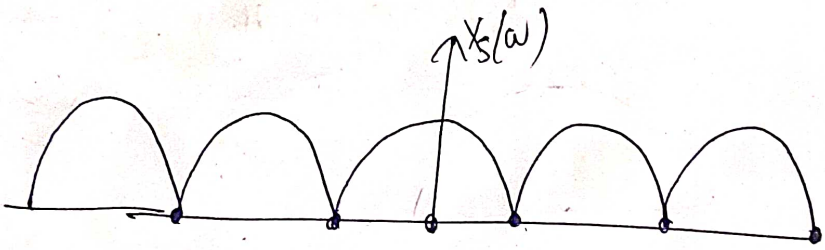
- If the Signal is Sampled above the Nyquist rate, then the original Signal can be recovered. i.e, the Sample Signal is higher than the $2f_m$ (or) $2W$ in freq. domain
- If the Sample Signal



- If the same Signal is Sampled at a rate less than 2w, then the Sampled Signal look like below fig



- In above fig. the overlapping of information is called Aliasing.
- Aliasing means high freq. Component of a signal, is taking the identify of a low frequency Component in the sampled version.
- If the sampling signal is equal to twice the highest frequency of agiven signal w,



In this case, the signal can be recovered without any loss, Hence this is good Samplingrate.

Pulse width Modulation:-

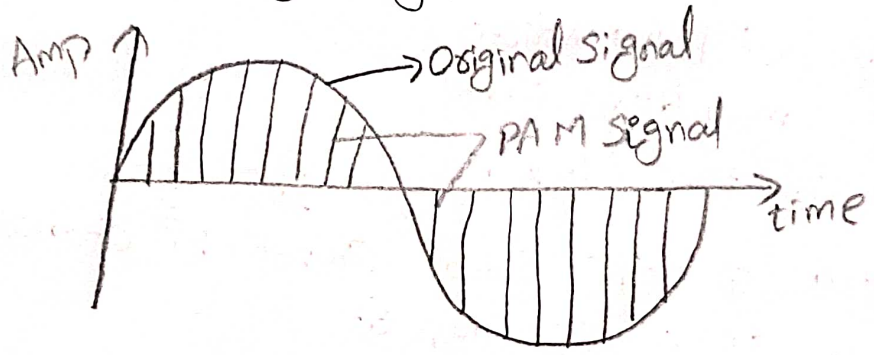
(8) (7)

PAM:-

- PAM is a form of signal modulation where the message information is encoded in the amplitude of a series of signal Pulses.

(OR)

- It is a analog Pulse modulation in which the amplitudes of train of Carrier Pulse are varied according to the Samplevalue of the message signal.
- Demodulation is Performed by detecting the amplitude level of the Carrier at every single Period.



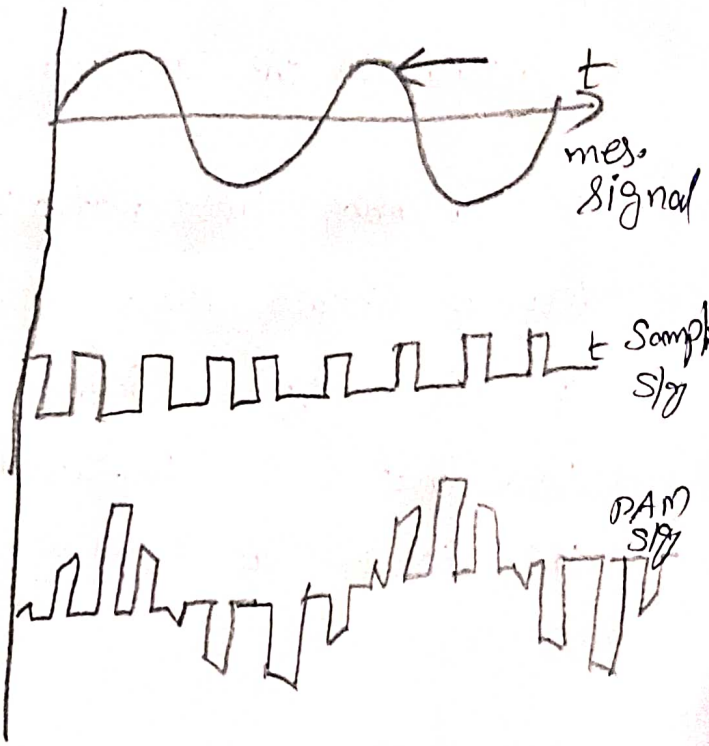
Adv :-

- PAM allows data to be transmitted more effectively, efficiently, and quickly using Conventional Copper wires in greater volume.
- F.M available are infinite. PAM formulas can be used for to increased data over existing networks.

PAM :-

- In PAM is a technique in which the amplitude of each Pulse is Controlled by the instantaneous amplitude of modulating signal.
- In this modulation signal is Sampled at regular intervals and each Sample is made Proportional to the amplitude of the signal.

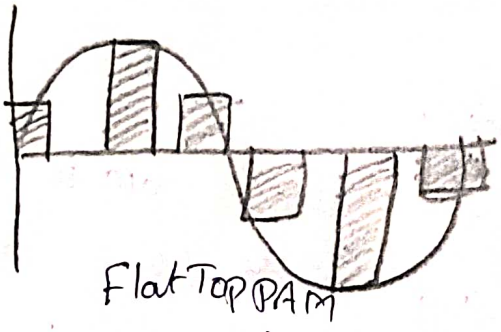
- This technique transmits the data by encoding the amplitude of a series of signal pulses.
- There are two types of Sampling techniques for transmitting a signal using PAM. They are
  1) Flat Top PAM
  2) Natural PAM.



Flat Top PAM:-

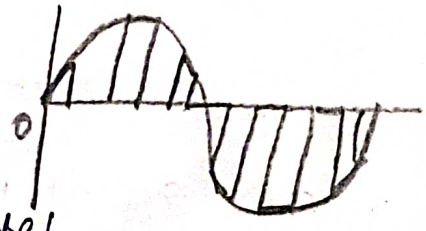
- The amp. of each Pulse is directly Proportional to modulating signal amplitude at the time of Pulse.
- The amplitude of the Signal Cannot be changed with Respect to the analog Sg to be Sampled.

PAM Signal



Natural PAM:-

The amplitude of each Pulse is directly Proportional to modulating Signal amplitude at the time of pulse occurence.



In Pulse modulation, the unmodulated Carrier Signal is a Periodic train of Signals. So the Pulse train can be described like the following.

$$c_p(t) = \sum_{k=-\infty}^{\infty} A \, \text{rect}\left(\frac{t - kT_s}{\tau}\right)$$

A → Unmodulated Pulseamp.
τ → Pulse width.

PAM eq. is

$$up(t) = q_0 + \sum_{n=1}^{\infty} a_n \cos\frac{2\pi nt}{T_S}$$

$$= q_0 + q_1 \cos\frac{2\pi nt}{T_S} + q_2 \cos\frac{4\pi nt}{T_S} + \cdots$$

Demodulation of PAM:

- For Demodulation PAM signal is fed to the low Pass filter.
- LPF it eliminates the high freq. ripples and generates the demodulated signal.
- This signal is then applied to the inverting amplifier to amplify its signal level to the demodulated o/p.



Demodulation of PAM signal

TDM: Bandwidth Noise Trade Off:

- Bandwidth has the fixed quantity and neither can be changed
- So Channel capacity is directly Proportional to the Signal Noise Ratio.
- When the channel capacity is changed by the signal noise ratio to occupy the smallest system or the allocated bandwidth.
- So B.W is directly Proportional to noise.

$$C = B\log_2\left(1+\frac{S}{N_0B}\right) \Rightarrow B\log_2(1+SNR)$$

- The Capacitor of Band limited, Power limited of Continuous channel is given by information Capacitor theorem.
- The Capacitor of Band limited Power limited Continuous Channel is depends on both B.w and signal to noise.

Capacity of Noiseless Channel:-

- Here Noise Power = 0

For Noiseless channel is $C = B\log(1+SNR)$

$= B\log(1+\infty)$ [$\because SNR=\infty$]

$C=\infty$

- So the Capacity of noiseless Channel will be infinity.

Capacity of Infinite B.w Channel:-

- Here B.w ↑, Noise Power also ↑, SNR ↓

$$\frac{S}{N} = SNR$$

$$C = B\log_2\left(1+\frac{S}{N_0B}\right)$$

$$= \frac{S}{N_0}\frac{N_0B}{S}\log_2\left(1+\frac{S}{N_0B}\right)$$

$$= \frac{S}{N_0}\cdot\log_2\left(1+\frac{S}{N_0B}\right)^{\frac{N_0B}{S}}$$

$$C = \frac{S}{N_0}\log_2\left(1+\frac{S}{N_0B}\right)^{1/\frac{S}{N_0B}}$$

$B\to\infty \quad C\to C_\infty$

$$\boxed{C_\infty = \lim_{B\to\infty} C}$$

we know that $\frac{1}{x}\log(1+x) = \log(1+x)^{1/x}$

Let $x = \frac{S}{N_0B}$ then

$B\to\infty, \; x\to 0$

$$C_\infty = \frac{S}{N_0}\lim_{x\to 0}\log(1+x)^{1/x}$$

$x = \frac{S}{N_0B}$

In PWM, the width of the Carrier pulse is varied according to the modulated s/g or mes. signal.

UNIT-II

(1)

PULSE MODULATION

Definition:- A Pulse modulation is a type of modulation in which the s/g is transmitted in the form of Pulses.

- In Pulse modulation is used to transmit analogue information.
- In this Continuous s/g are ~~sig~~ Sampled at regular intervals.
- By using analog modulation Pulse modulation is mainly classified into 3 types.
  1) Pulse Amplitude Modulation
  2) Pulse width/Pulse duration modulation
  3) Pulse Position modulation.

PAM:- It is a form of Signal modulation where the message information is encoded in the amplitude of a Series of Signal Pulses. (or) It is an analog Pulse modulation scheme in which the amplitude of a Carrier Pulse is varied according to the sample value of message s/g.

PWM:- It is a method of reducing the avg. Power delivered by an electrical s/g.

- In this the avg. voltage fed to the load is Controlled by turning the switch b/w supply and load on and off at a fast rate.

PPM:- It is a form of signal modulation in which M message bit are encoded by transmitting a single Pulse in one of 2^M Possible required time shifts.

Difference b/w PAM, PPM, PWM :-

- In PAM, the amplitude of the Pulses Carrier s/g varies according to the amplitude of the analog modulating s/g.
- In PWM, the width of the Pulses of Carrier wave varies according to the modulating s/g.
- In PPM, the Position of the Pulses shows variations according to the modulating s/g.

- In PAM, Immunity towards noise is low
- In PWM " " " is high
- In PPM, " " " is high
- In PAM, Signal to noise ratio is low
- In PWM, " " " " moderate
- In PPM, " " " " Comparatively high
- In PAM, the band width depends on the Pulse width
- In PWM, " " " " on rise time of Pulse
- In PPM " " " " " " "
- In PAM, band width requirement is low.
- In PWM, " " " is high
- In PPM, " " " " high
- In PAM, Synchronization b/w Tx and Rx is not needed
- In PWM " " " " "
- In PPM, " " " " needed

Sampling Theoram for Low Pass Signal:- Ames. stg originator from analog & digital source.

(2)

Sampling:- The Process of Converting Continuous time s/g to discrete time s/g is called sampling.

Theorem:- Sampling theorem states that it is not Possible to recover the original s/g from the sampled one.

- if a Continuous s/g is Sampled with a frequency less than twice the maximum freq. of the original s/g or message s/g.

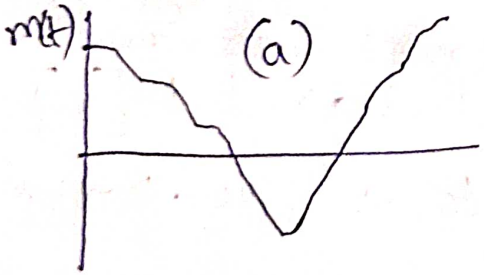
that is $f_s \leq 2f_m$

Sampling theoram for low Pass s/g:-

- For a bandlimited Continuous time s/g, if the Sampling Instants are sufficiently close then the s/g may be reconstructed by using LPF

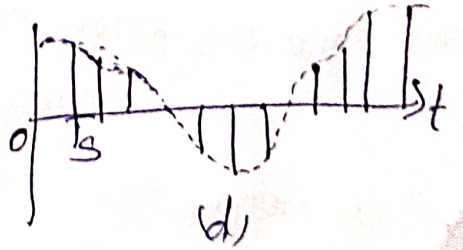
- Let m(t) be a signal which is bandlimited its highest freq. Component is f_M.
- Let m(t) be determined at regular intervals separated by times $T_s \leq 1/2f_m$
- the signal is periodically sampled every T_s seconds. these Samples is $m(nT_s)$.
- Where n is an integer.

T_s is Sampling time.









(a) A s/g m(t) which is to be Sampled

(b) Sampling function s(t) consists of a train of very narrow unit amplitude Pulse

(c) The Sampling operation is Performed in a multiplier

(d) The Samples of s/g m(t)

- In above diagram baseband s/g m(t) which is to sampled.
- A periodic train of pulses S(t) of a unit amplitude and a Period T_S as shown in fig(b).
- The pulses are having arbitarily narrow, having a width a
- The two signals m(t) and S(t) are applied to a multipliers.
- In fig(c). The output is a Product of S(t)m(t).
- In fig(d). to be the signal m(t) is sampled at each pulse
- when a pulse occurs, the multiplier o/p has the same value as does m(t), and at all other times the multiplier output is zero.
- The signal S(t) is Periodic with Period T_S and has the fourier expansion is

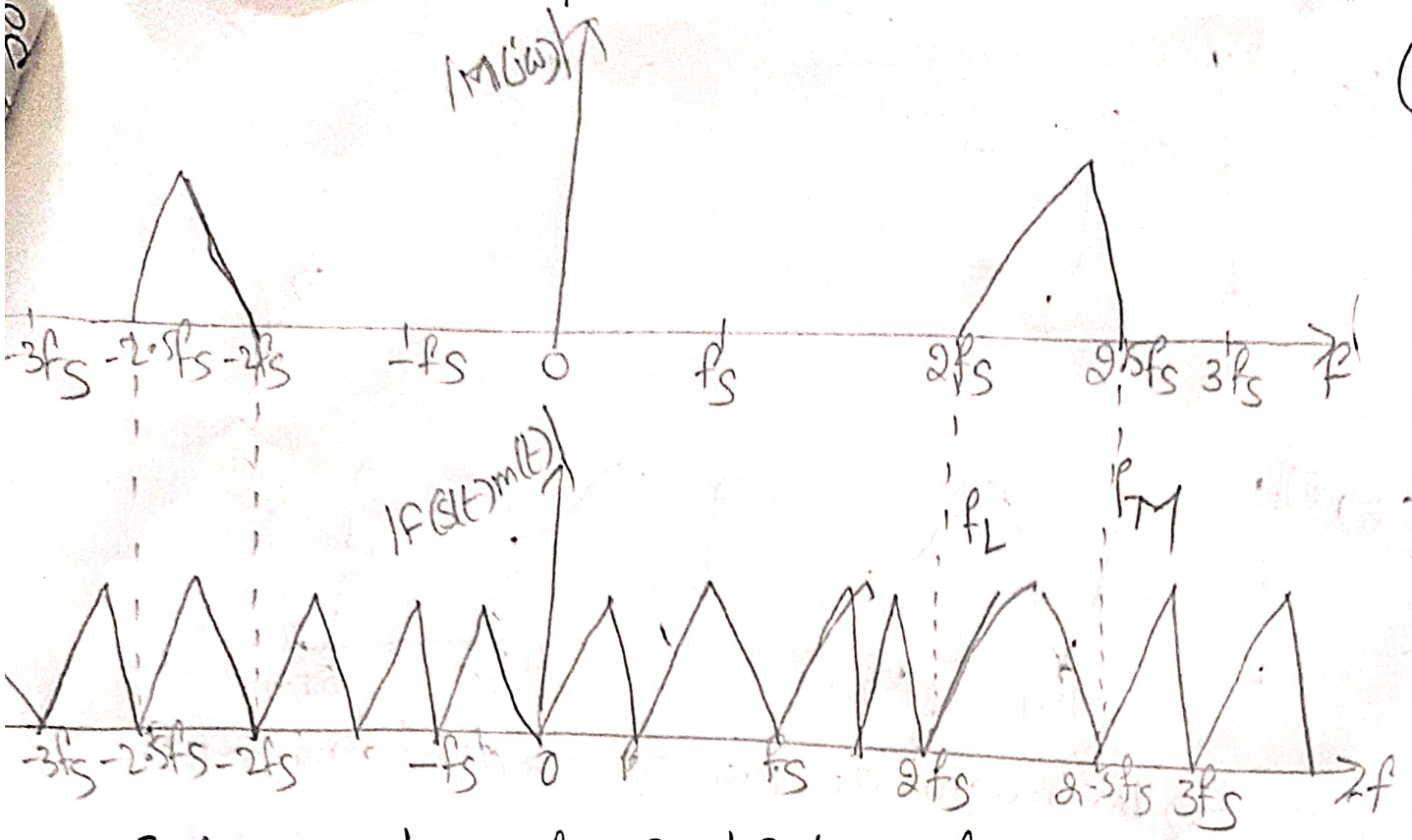
$$S(t) = \frac{dt}{T_S} + \frac{2dt}{T_S}\left[\cos 2\pi\frac{t}{T_S} + \cos 2\times 2\pi\frac{t}{T_S} + \cdots\right]$$

for the case $T_S = 1/2f_m$, the Product S(t)m(t) is

$$S(t)m(t) = \frac{dt}{T_S}m(t) + \frac{dt}{T_S}\left[2m(t)\cos 2\pi(2f_m)t + 2m(t)\cos 2\pi(4f_m)t + \cdots\right]$$

- In this first term is m(t) and second term is Product of m(t) and sinusoidal freq
- Let the s/g m(t) have a spectral density $M[j\omega] = F[m(t)]$ then the m(t) is band limited to the freq. range below f_m
- Then m(t) is first extends from 0 to f_m.
- Then the second term is symmetrical about the freq $2f_m$ and it extends from $2f_m - f_m = f_m$ to $2f_m + f_m = 3f_m$

(3)



Fig(a):- spectrum of Band Pass signal

(b):- spectrum of Sampled band Pass signal.

- In fig(b) it is a two sided spectral Pattern of a signal m(t) with fourier transform M(jω). So n=2
- f_L coincides with the second harmonic of the sampling freq. while the Sampling freq. is exactly $f_S = 2(f_M - f_L)$ and fig(b) is a spectral Pattern of the sampled Signal s(t)m(t).
- The Product of m(t) and the spectral Component in s(t) of freq. $f_S = 1/T_S$.
- The spectrum of m(t) extends over the first half of the freq. interval b/w sampling freq. from 2.0 f_S to 2.5 f_S.
- As a result, there is no spectrum overlap, and signal recovery is Possible.

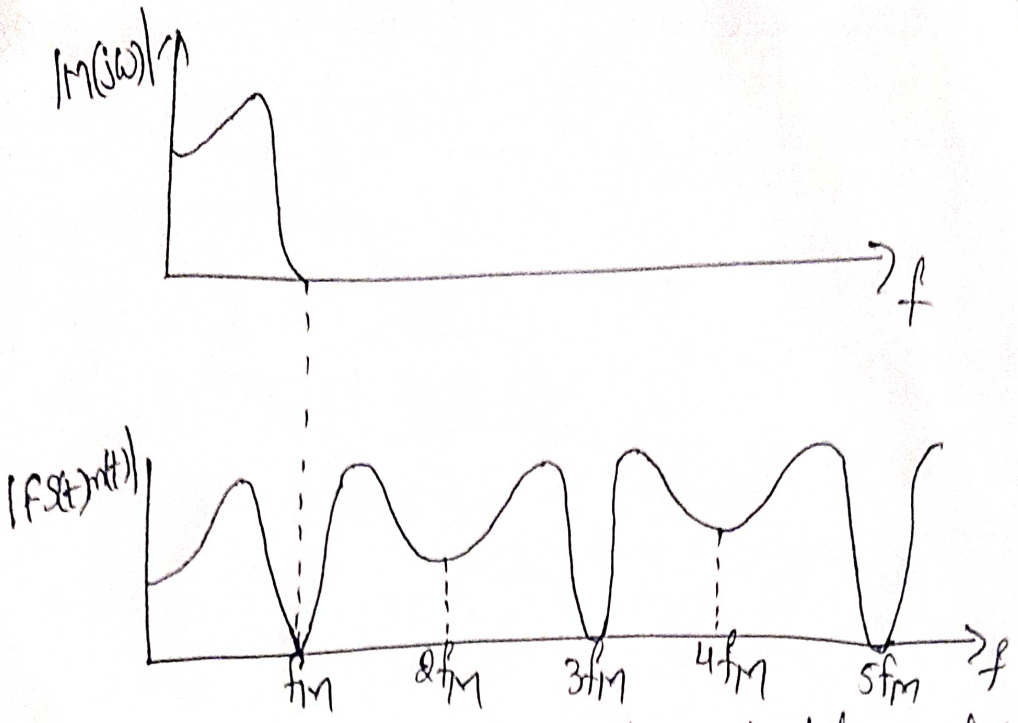


Fig: (a) The magnitude plot of spectral density of a s/g band limited to f_M.
(b) plot of amplitude of Spectrum of sampled s/g

Sampling of Band Pass S/g:-

→ This rate of sampling rate is appropriate to a signal that is supported on the interval $(-\pi, \pi)$ which is described by base band

- Consider a signal m(t) have a highest freq. Component is f_M.
- In Band Pass signal the sampling freq. f_S must be no less than $f_S = 2f_M$ only if the lowest freq. spectral component of m(t) is $f_L = 0$.
- If the $f_L \neq 0$ in this time Sampling freq. need be no longer than $f_S = 2(f_M - f_L)$
- For ex, the s/g extends from 10.0 to 10.1MHz, the signal may be recovered from samples taken at a freq.

$$f_S = 2(10.1 - 10.0) = 0.2\text{MHz}$$

- In fig(a) to establishing the sampling theorem for band Pass signals, let us select Sampling freq. $f_S = 2(f_M - f_L)$
- and also assume the freq. f_L is an integral multiple of f_S that is $f_L = n f_S$ → where n is integer.

- The S/g is divided into ~~UNIT-III~~ Positive spectrum, negative spectrum

(4)

PULSE MODULATION

- In negative spectrum we get $(N-1)$ of the Sampling waveform.

$$(N-1)f_S - f_L \leq f_L$$

$$Nf_S - f_M \geq f_M$$

So that $B = f_M - f_L$ we have

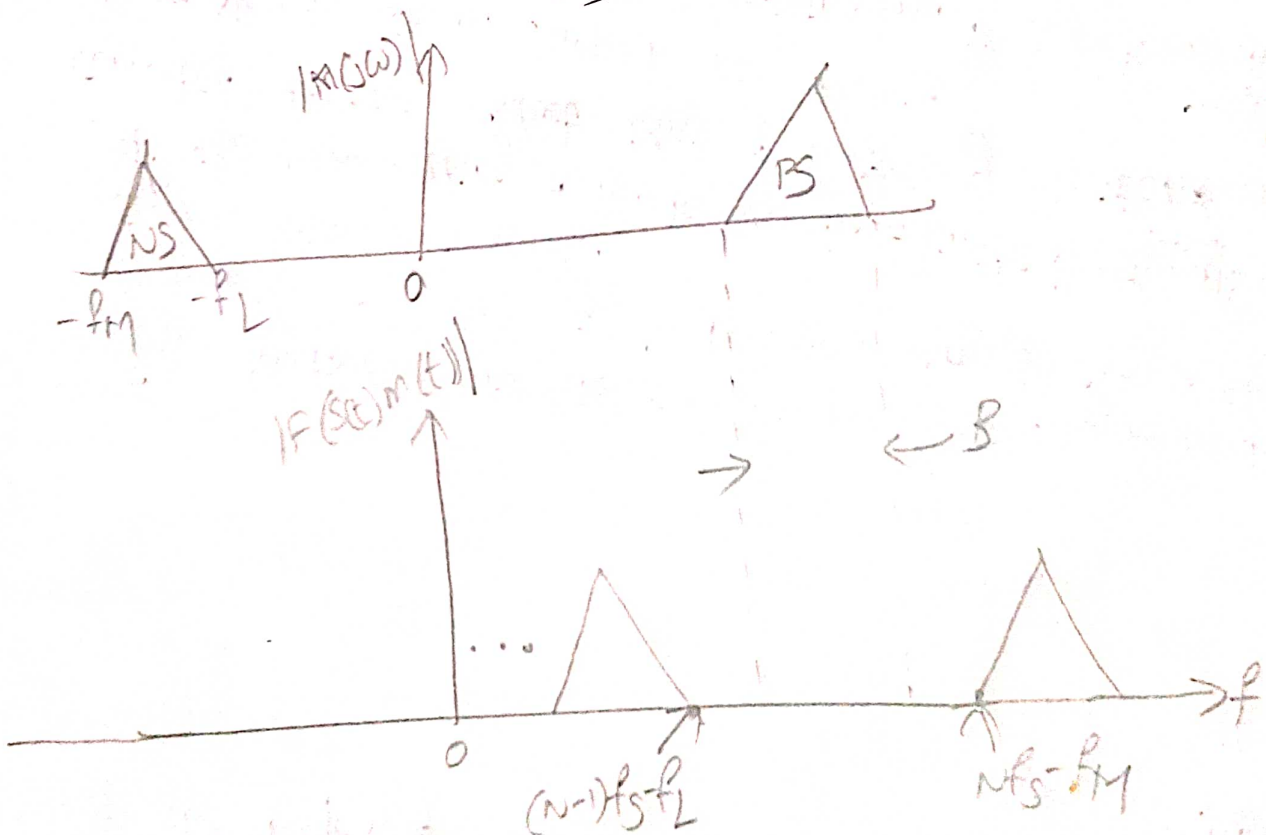
$$(N-1)f_S \leq 2(f_M - B) \rightarrow ①$$

$$Nf_S \geq 2f_M \rightarrow ②$$

If we let $k = f_M/B$, then the eq ① & ② becomes

$$f_S \leq 2B\left[\frac{k-1}{N-1}\right]$$

$$f_S \geq 2B\left[\frac{k}{N}\right]$$



(b)

Fig(a) Spectrum of Band Pass S/g.
(b) Spectrum of NS shifted by the (N-1)st and the Nth harmonic of the Sampling waveform

Pulse Amplitude Modulation for TDM:-

- PAM is a form of modulation the amplitude of carrier is varied accordance with the modulating signal(or)mes.
- This PAM Modulation technique is used for sampling Princip for the Purpose of time division multiplexing.

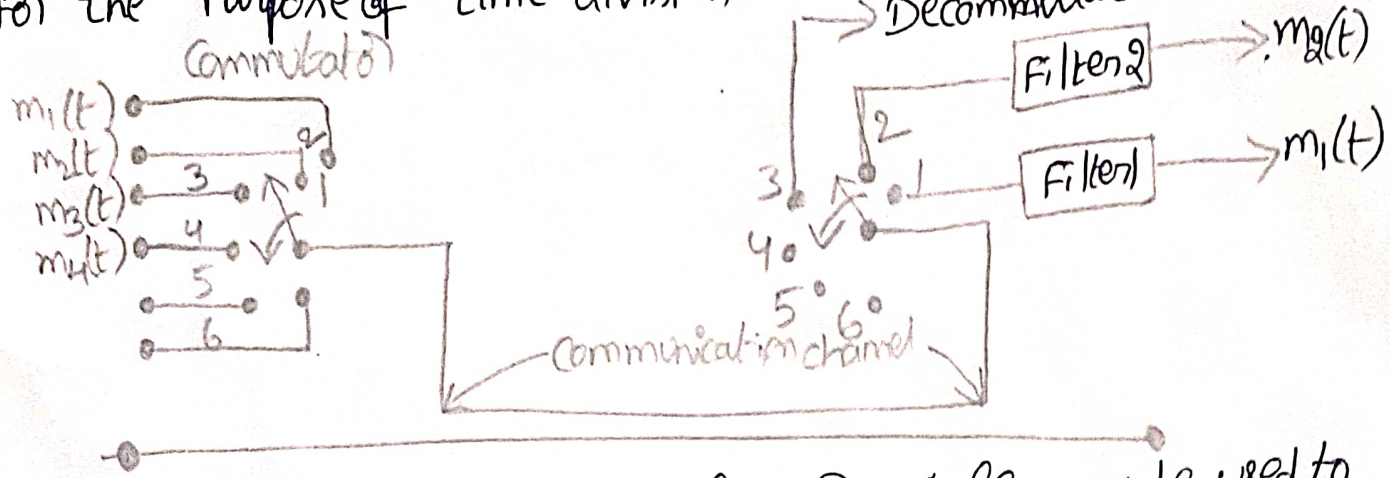


Fig:- Illustrating how the sampling Principle may be used to transmit a number of bandlimited signals over a single communications channel.

- At the transmitting a number of bandlimited signals are connected to the contact Point of a rotary switch.
- For ex, voice signals limited to 3.3kHz.
- The rotary switch at the receiving end is synchronism with the switch at the sending end.
- The two switches make contact simultaneously at similarly bandlimited.
- With each revolution of the switch, one sample is taken of each i/p signal and Presented to the correspondingly numbered contact of the receiving end switch.
- The train of samples, terminal 1 in the receiver, pass through low Pass filter1, and the filter output, the original signal $m_1(t)$ appears reconstructed.

to the modulated s/g of mes. Signal.

- if f_m is the highest spectral component present in any of the input signal, the switches must take $2f_m$ revolution per second.
- When the signals to be multiplexed vary slowly with time, so that sampling rate is slow, mechanical switches may be employed.
- When the switching speed required is outside the range of mechanical switches, electronic switching ckt may be employed.
- The switching mechanism, corresponding to the switch at the left is a sampler the signals, is called "Commutator".
- The switching mechanism which performs the function of the switch at the right is called "Commutator".
- The Commutator samples is used for combined the samples,
- While decommutator is used for to separates the samples to individual signals maybe reconstructed.
- The Interlacing of the samples that allows multiplexing.

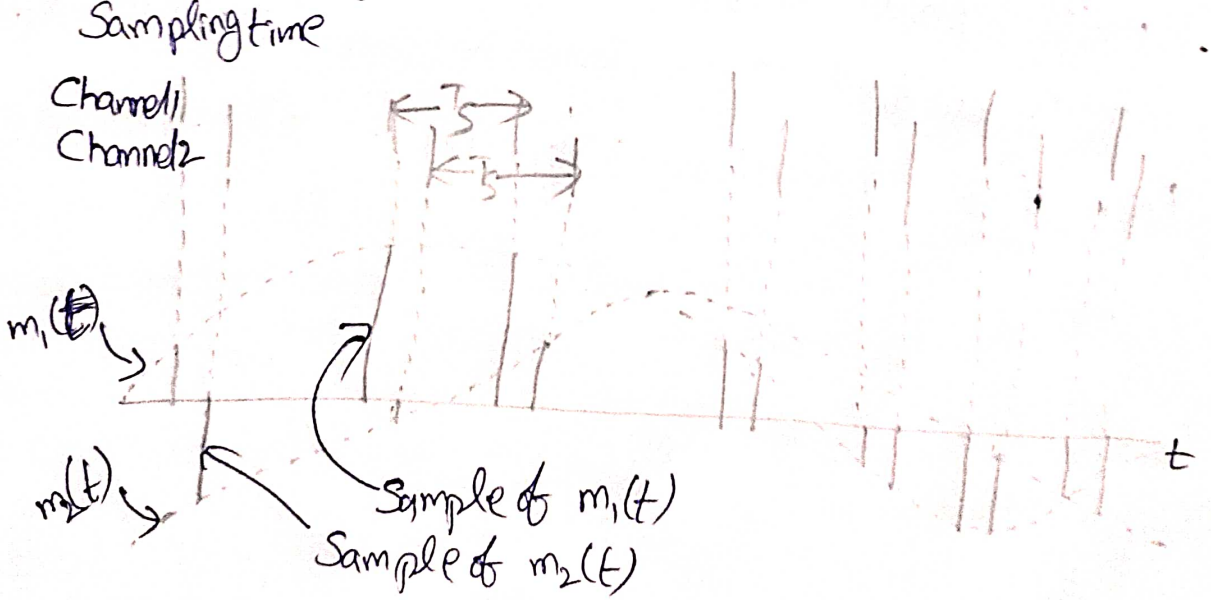
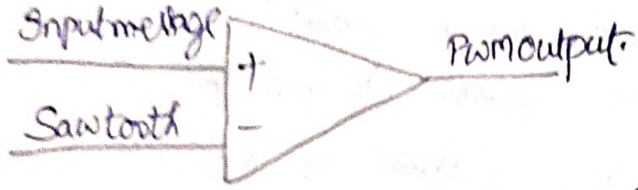


Fig:- The Interlacing of two baseband signal.

- In above fig are considered two signals $m_1(t)$ & $m_2(t)$. The $m_1(t)$ is sampled with a interval of T_s. with time.
- the sig $m_2(t)$ is sampled are taken at a time different from the sampling time of $m_1(t)$
- Then the train of Pulses corresponding to the each sig are modulated in amplitude in accordance with the sig. This of sampling is called "PAM".
- Multiplexing of several PAM signals is possible because the various signals are separately recoverable.
- Hence this system is an ex. of TDM system.
- If the multiplexed sig are transmitted directly, over a pair of wires, no further sig processing need be undertak
- To transmit TDM-PAM signal from one antenna to the another.
- ~~Amplt~~ would be necessary amplitude modulate or fre modulate a high freq. Carrier with the TDM-PAM sig.
- Then it is referred as PAM-AM or PAM-FM

Pulse width Modulation:

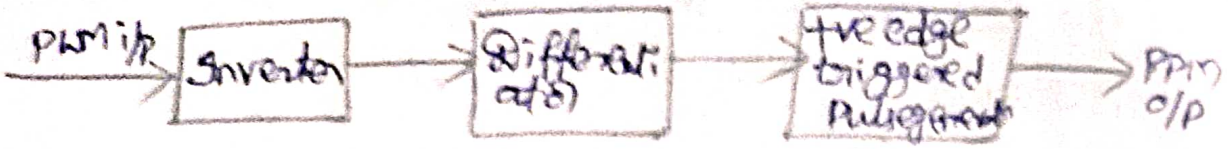
- In PWM, the width of the Carrier pulse is varied according to the modulated s/g of mes. Signal.
- The message s/g is modulated with the width of the Pulse.
- In PPM, the position of fixed width Pulse in each sample Period is modulated by mes. Signal.
- PWM application is motor control, in delivery of Power is regulated by width of the Pulse.
- PWM and PPM are known as Pulse Time Modulation or PTM
- The Simplest method for PWM generation is



(a) PWM generation by a Comparator

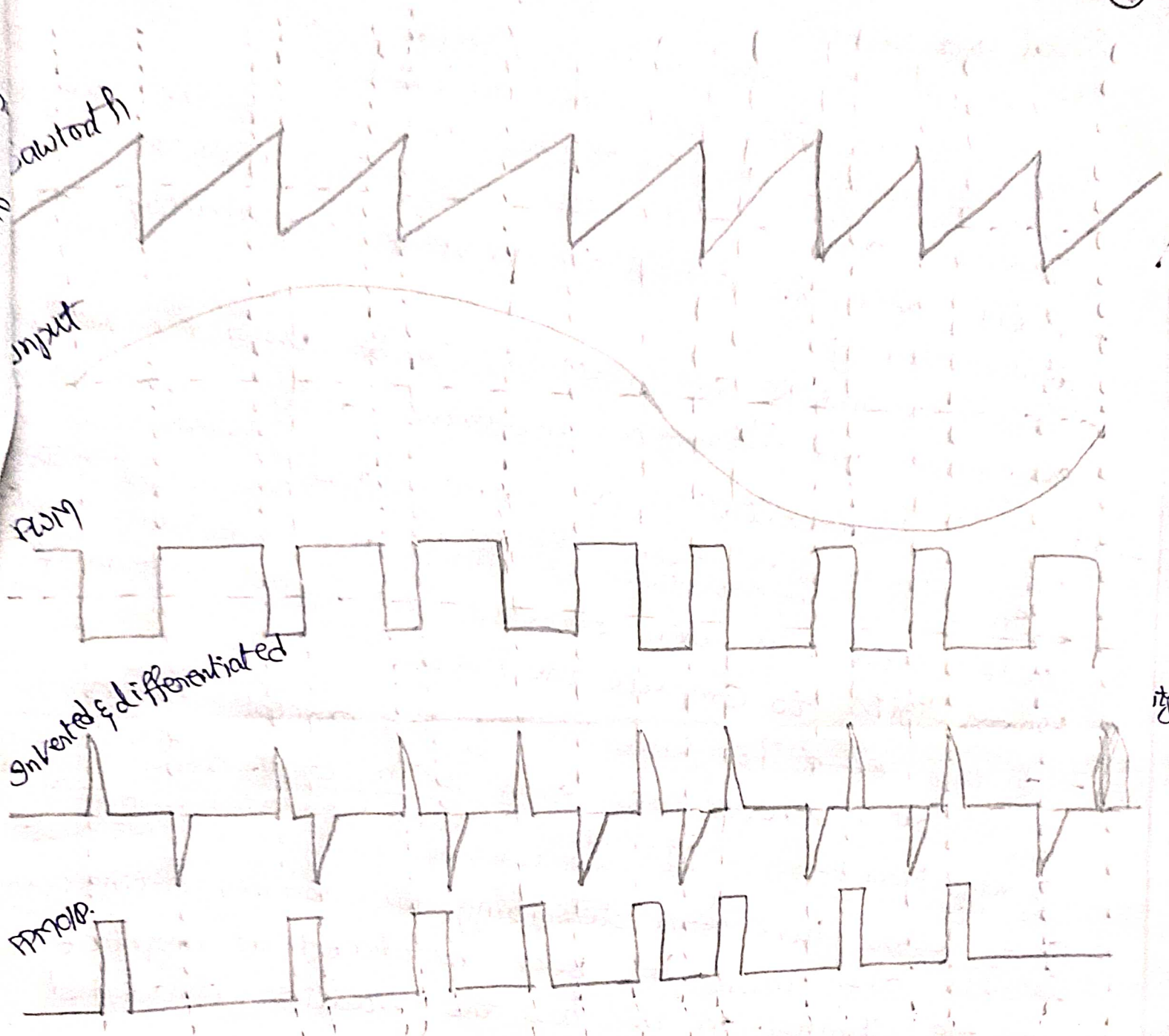
- In above fig. we have used a Comparator, One i/p is input message signal. and other by a Sawtooth s/g
- Sawtooth s/g are used to operate Carrier freq.
- The maximum of input s/g should be less than Sawtooth s/g then we will get PWM output.
- When the Sawtooth is minimum, and which is less than the minimum of input s/g, then the +ve input of Comparator is at higher Potential and hence Comparator Output is Positive
- When the Sawtooth s/g rises with a fixed Slope and crosses the input Signal, the -ve input of Comparator is at higher Potential and Comparator output will be -ve
- The width of Pulse generated at the output is directly Proportional to amplitude of mes. s/g

PPM generation from PWM



- The PWM o/p is again given to the PWM input of the s
- Inverter is used for to reverses polarity of the Pulses
- If it will have +ve Spikes where in PWM s/g Pulse was going from HIGH to Low
- It will have -ve Spikes where in PWM s/g Pulse was going from Low to High.
- If it is +ve edge triggered then It Produce (or) generates a Pulses of fixed width.
- When a +ve Spike appears, it will get original PWM signal
- Here the fixed width Pulses is directly Proportional to amplitude of mes s/g.
- This type of method is known as direct method.
- Indirect method means, the PAM s/g from i/p mes is generated to which Sawtooth like added.
- Here we are use a IC555 with RC differentiator is used for to generate a PWM s/g.

Principle of PWM and PPM generation.



- In PPM outputs where position of pulses are sample Period carry the input message information.
- In the demodulation of PWM and PPM. Here we start with positive edge and stop it negative edge.
- But here width are different and with different heights in each cycle which is directly proportional to the pulse width of the modulating sig.

- and this s/g is passed through the LPF ... signal and demodulation is done.
- But In PPM demodulation it start with positive edge ends with positive edge of the next pulse.
- The delay b/w pulses decides the height of the generated and closely to the modulating message
- A LPF after the filters the envelop information demodulated s/g.
- Transistor and RC combinations can be used for both generation and filtering to implement a demodulator con...

→ Digital Representation of Analog Signal: A digital representation sound is a sequence of numbers that represent the fluctuating voltage of the analog s/g

- In digital representation of analog s/g we need amplitude axis and then represent that signal in terms of binary digits to complete the process.
- In this ~~ex~~ of for understanding this concept from quantization noise & quantization error. Digital Sampling creates a digital representation by taking measurements of instantaneous amplitudes at equally spaced intervals.

quantization of s/g:-

- Once a s/m have been describing for communicating over long channel is one noise has been introduced at any place along the channel. In this how the situation is modified by using operation of quantization.

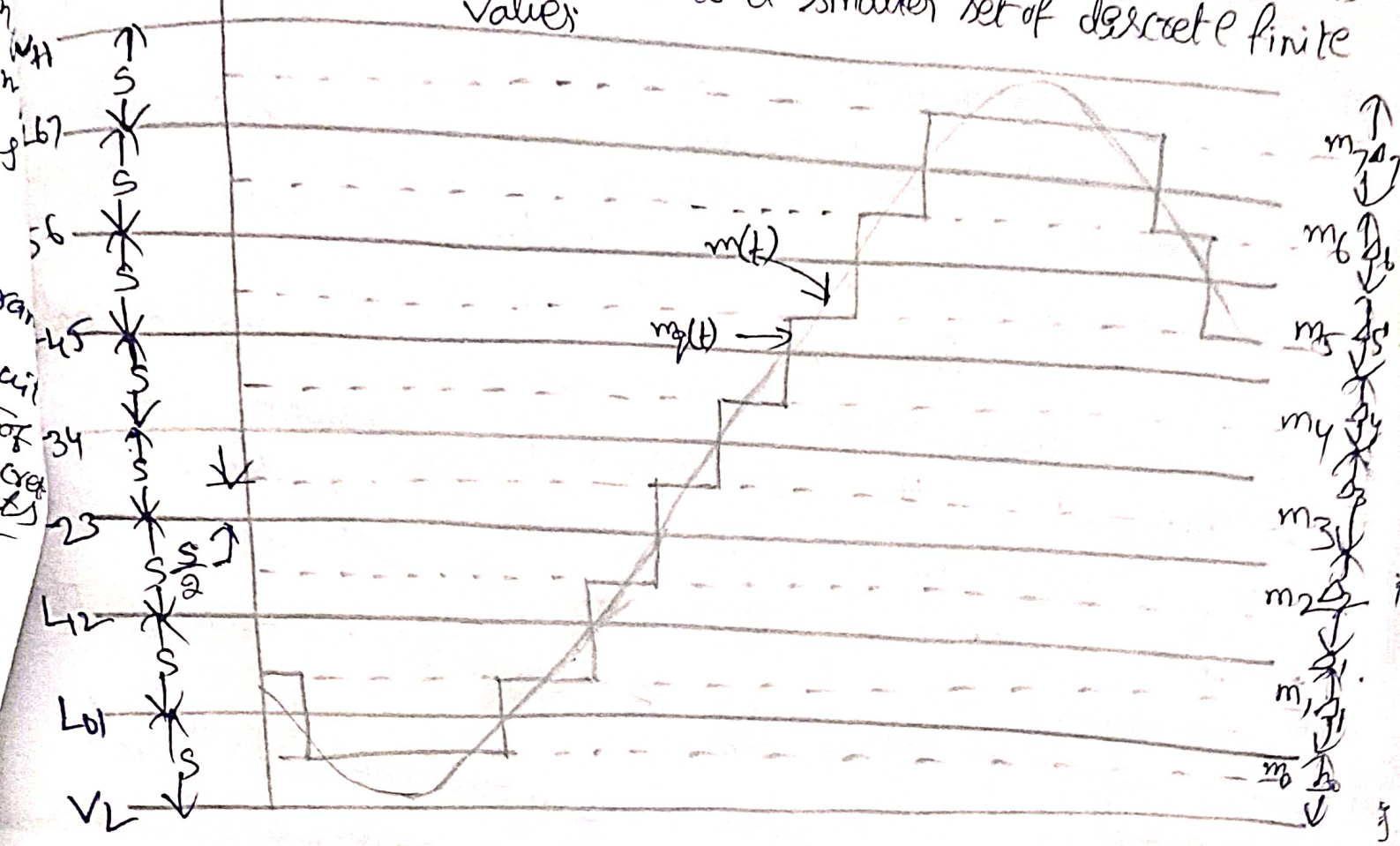
Quantization This is used for to minimize the error through process known as quantizing

- When quantizing s/g $m(t)$, we create a new signal $m_q(t)$ which is an approximation to $m(t)$
- The quantized s/g $m_q(t)$ has the great merit, in large measure, separable from additive noise

quantization, it is the process of mapping input values from a large set to output values in a smaller set, often with a finite number of elements.

- The operation of quantization is represented in below fig.

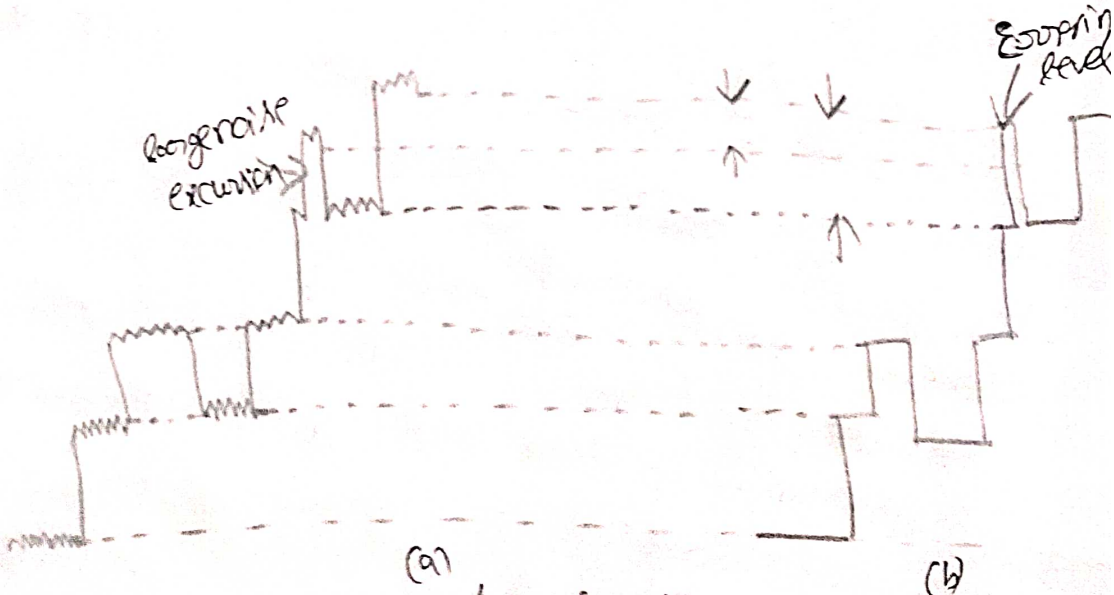
$m(t), m_q(t)$ → Quantization Process is the mapping of continuous infinite values to a smaller set of discrete finite values.



f: The operation of quantization

- Q.P is used for to reduce computation demand and increasing Power efficiency
- A signal $m(t)$ whose ranges from V_L to V_H. and we have divided this total range into M equal intervals each of size S.
- S is called step size. is $S = \frac{(V_H - V_L)}{M}$
- In above fig in which $M=8$,
- In the centre of each of these steps we locate quantization level $m_0, m_1, m_2, m_3 \ldots m_7$
- The quantized signal $m_q(t)$ is generated in the following way:
  whenever $m(t)$ is in the range Δ_0
  - and the signal $m_q(t)$ will maintains the constant level m_0
  - Whenever $m(t)$ is in the range Δ_1, $m_q(t)$ maintains the constant level m_1 and so on.
- The signal $m_q(t)$ will at all times be found at one of the level $m_0, m_1, \ldots m_7$
- This is used in simulation and embedded computing.

- The transition in $m_q(t)$ from $m_q(t) = m_0$ to $m_q(t) = m_1$
- when $m(t)$ passes the transition level L_0, which is mid
m_0 & m_1.
- At every instant of time $m_q(t)$ has the value of the quantized level to which $m(t)$ is closest
- $m_q(t)$ does not change at all with time or step sizes. The
- The quantization error $m(t) - m_q(t)$ has magnitude which equal or less than $s/2$.
- The quantized signal is an approximation to the original
- The quality of the sig is improved by reducing the sizes steps, there by increasing the number of allowable level
- the quantization signal can also reduced the effect of additive noise.
- By decreasing the spacing of repeaters, we decrease the $m_q(t)$
- If noise Power is decreases then the Probability P_q of the Error is also decreases.
- If P_q is reduced then step size 'S' is increased.
- If 'S' is increased then the difference b/w $m(t) - m_q(t)$ is called quantization noise.



(a) A quantized signal with added noise
(b) The sig after requantization.

The quantized signal is shown in above fig(a)
allowable quantizer o/p levels are indicated by dashed lines.
separated by amount S.
The o/p of quantizer shown in fig(b).
- The quantizer o/p is the level to which the input is closest.
- When the noise does exceed S/2 then the error level does occur.
- The method of signal quantization, the effect of additive noise can be reduced.
- By decreasing noise then Probability P_q of error.
- Because of P_q is reduced then it increases the step size S.
- The difference b/w $m(t)$ & $m_q(t)$ due to error caused by additive noise and quantization noise.

Quantization Error:

⇒ The quantized signal and the original signal from which it was derived differ from one another in random manner.
- This difference or error may be viewed as noise due to quantization Process and is called quantization error.
- When you calculate mean square quantization error $\overline{e^2}$ where e is the diff. b/w original and quantized signal.
- Let us divide the total Peak-to-Peak range of the mes signal m(t) into M equal voltage intervals each of magnitude S volts
- At the center of each voltage interval we locate quantization level $m_1, m_2, \ldots m_M$
- The dashed level represents instantaneous value of the mes. signal m(t) at a time t.
- m(t) happens to be closest to the level m_k, then the quantizer op will be m_k, the voltage corresponding to that level.

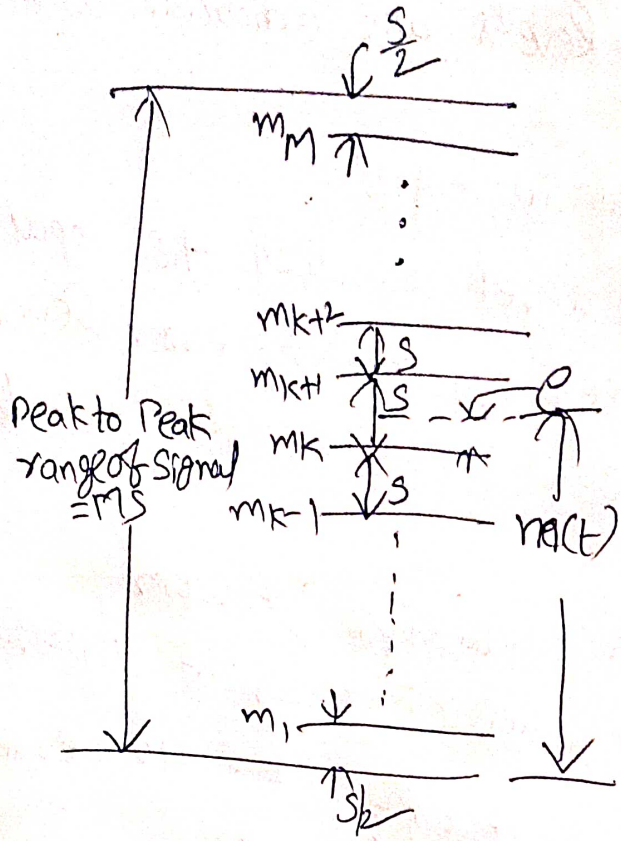


Fig:- A range of voltage over which signal m(t) makes Excursions is divided into M quantization ranges of each size

UNIT-IV

DIGITAL MODULATION

Binary Amplitude Shift Keying:-

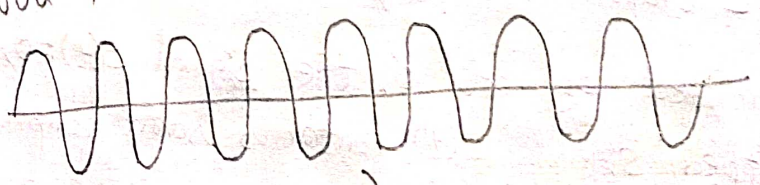
Introduction of Digital Modulation:-

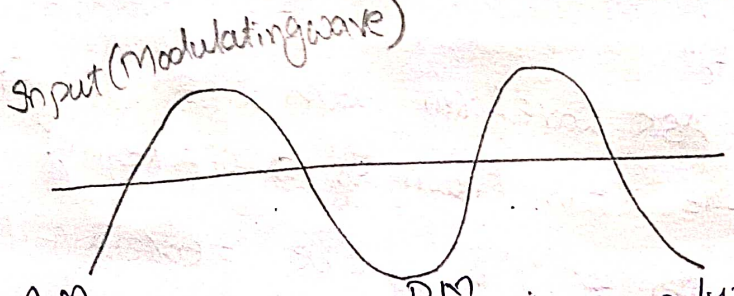
A Digital modulation is the process of encoding digital information signal into the amplitude, phase or frequency of the transmitted signal. (OR)

Mes The modulation method in which discrete signals are used to modulate carrier waves and remove noise from the waves is known as Digital Modulation.

- Theyare 3 types of digital modulation.
  1) Amplitude Shift keying [ASK]
  2) Frequency shift keying [FSK]
  3) Phase " " [PSK]

- Digital modulation provides increased data capacity, enhanced information security and speedier systemaccessibility through high quality transmission.
- DM also used for to reduces communication noise.
- DM is used in CableTV and broadcast systems.





A.M

- In analog modulation s/g can be represent any value within a range.
- Can produce a s/g that carries changing information.
- It is difficult to separate s/g from noise

D.M

- In D.M. s/g can only represent one of a discrete values
- Produce a s/g whose value is change at specific interval of time.
- It is easy to separted s/g from noise.

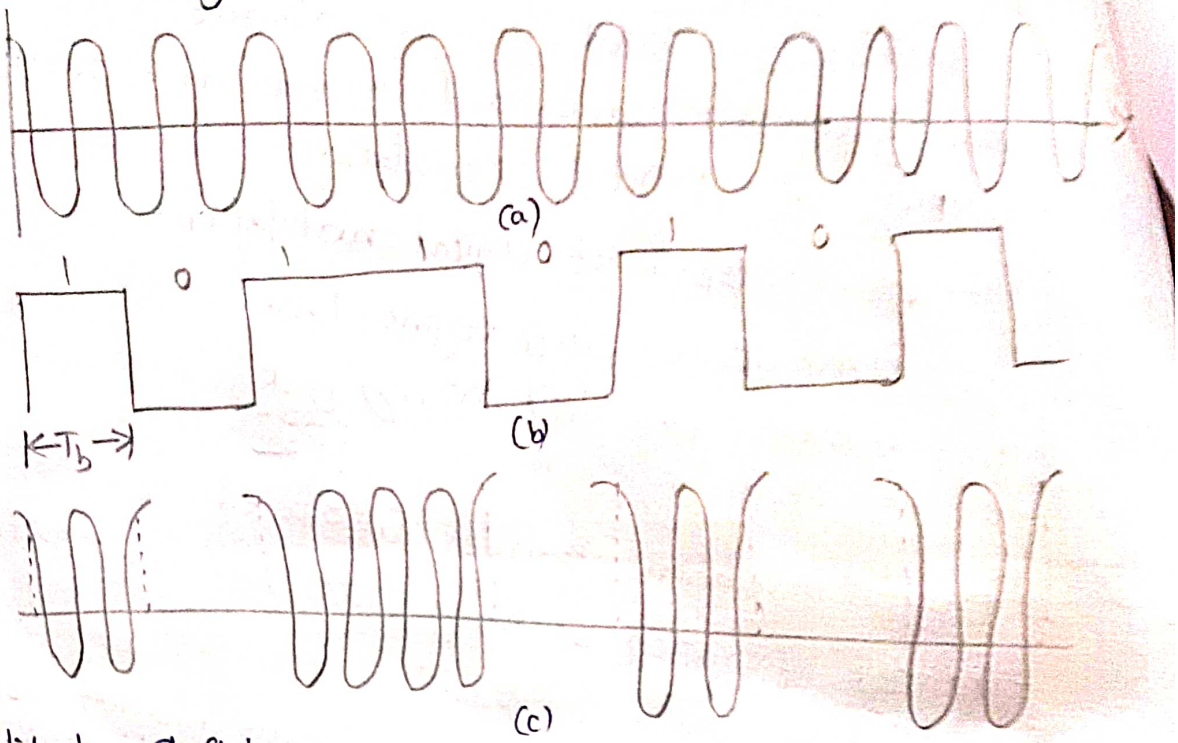
BASK:-

- ASK or on-off keying is the simplest digital modulation
- In this method, there is only one unit energy carrier on or off depending upon the input binary sequence.

Expression and waveforms for BASK:-

- The ASK wave form may be represented as

$$S(t) = \sqrt{2P_S}\cos(2\pi f_c t)$$



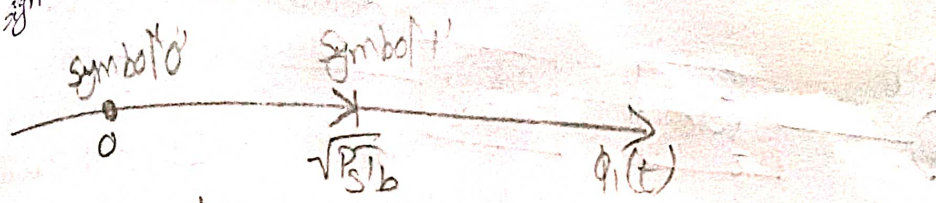
Amplitude Shift keying waveforms
(a) Un modulated carrier
(b) NRZ Unipolar bit sequence, (c) ASK waveform

- To Transmitted Symbol '0', the signal $S(t)=0$, i.e, no signal is transmitted/ signal $S(t)$ contains some complete cycle of carrier freq. f_c
- Hence, the ASK waveform looks like ON-OFF of the signal. Therefore it is known as ON-OFF keying
- In above fig symbol '1' is can be represented as

$$S(t) = \sqrt{P_S T_b}\sqrt{2/T_b}\cos(2\pi f_c t) = \sqrt{P_S T_b}\,\phi_1(t)$$

- This means that there is only one carrier function $\phi_1(t)$.

signal space diagram will have two points on $\phi_1(t)$



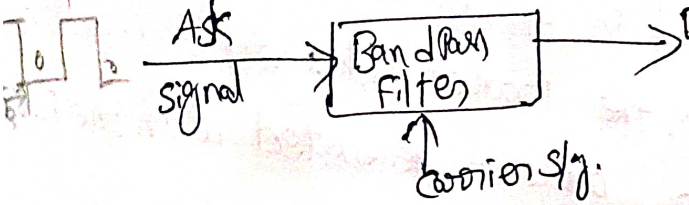
Signal space diagram of ASK.

one will be at zero and other will be at $\sqrt{E_b}$.

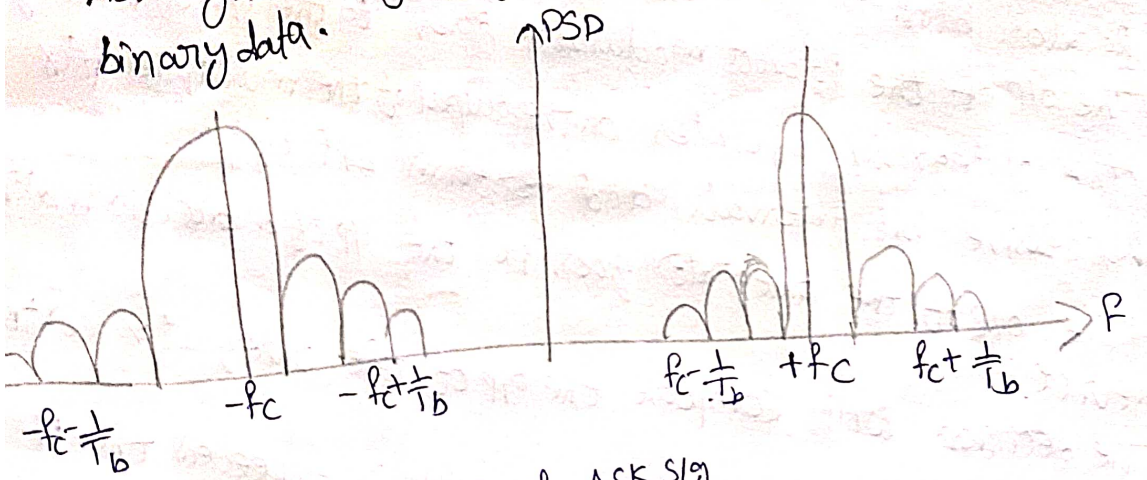
Then the distance b/w two signal points is

$$d = \sqrt{E_b} - 0 = \sqrt{E_b}$$

Generation of ASK



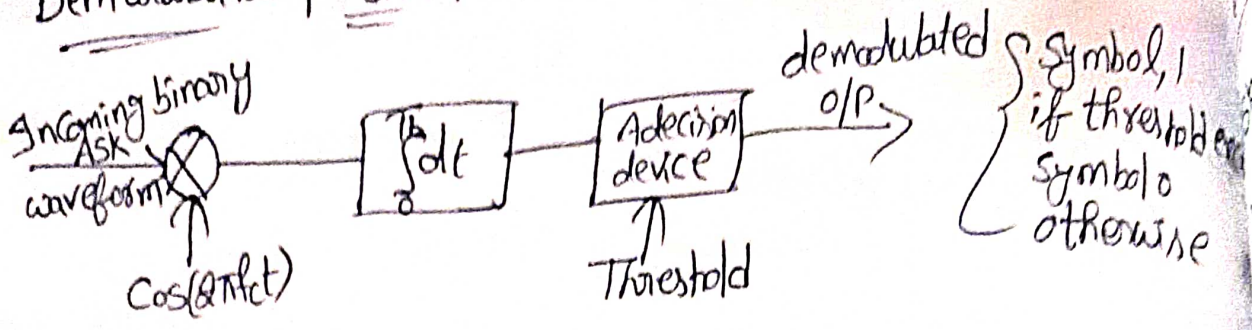
- ASK signal may be generated by simply applying the incoming binary data.



PSD of ASK s/g

- The binary data and sinusoidal carrier to the two inputs of balanced modulator then the resulting o/p will be ASK waveform
- In PSD of BASK, which is basically product of binary seq. and carrier s/g has a PSD.
- The baseband on-off signal is shifted with the freq. domain by $\pm f_c$.
- A BPF is a electronic device or ckt that allows s/g b/w two specific frequencies to pass, but that discriminates against s/g at other frequencies.

Demodulation of BASK:-



- The demodulation of binary ASK waveform can be achieve with the help of Coherent detector.
- It consist of Product modulator which is followed by an integrator and decision making device.
- The incoming ASK signal is applied to one i/p of the Produc modulator. other input is the Product modulator is supplied with sinusoidal Carrier which is generated with the help o localoscilator.
- The o/p of the Product modulator goes to the i/p of the Integra
- The Integrator operates on the output of the multiplier for successive bit intervals and Performs LPF action.
- The o/p of the integrator goes to the i/p of a decisionmaking device.
- The decision will compare the o/p of the Integrator with the Threshold.
- A decision is when the threshold is exceeded then it get symbol1 otherwise it is symbol 0'.
- A coherent detection makes the use of linear operation
- In this method, we assumed that local Carrier is in Perfect synchronization with the Carriers used in Transmitter.

Phase shift keying: [BPSK]

- In BPSK the transmitted signal is a sinusoid of fixed amplitude. It has one fixed phase when the data is at one level and when the data is at other level the phase is different by 180°.
- The below fig. is a BPSK modulated waveform signal and corresponding digital mes.

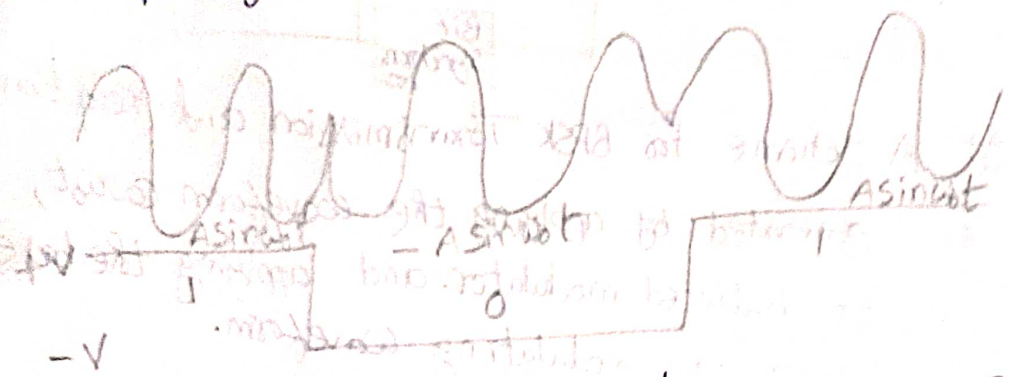


Fig:- BPSK modulated signal.

- If the sinusoid is of amplitude A, it has a Power $P_S = \frac{1}{2}A^2$

$$P_S = \frac{1}{2}A^2$$

$$A = \sqrt{2P_S}.$$

- Thus the Transmitted signal is

$$V_{BPSK}(t) = \sqrt{2P_S}\cos(\omega_c t)$$

$$V_{BPSK}(t) = \sqrt{2P_S}\cos(\omega_c t + \pi)$$

$$= \sqrt{2P_S}\cos(\omega_c t)$$

- In BPSK the data b(t) is of binary digits with voltage level +1V and -1V.
- when b(t) = 1V, then it is logic level 1
- when b(t) = -1V then it is logic level 0

Then $V_{BPSK}(t)$ can be rewrite as

$$V_{BPSK} = b(t)\sqrt{2P_S}\cos\omega_c t$$

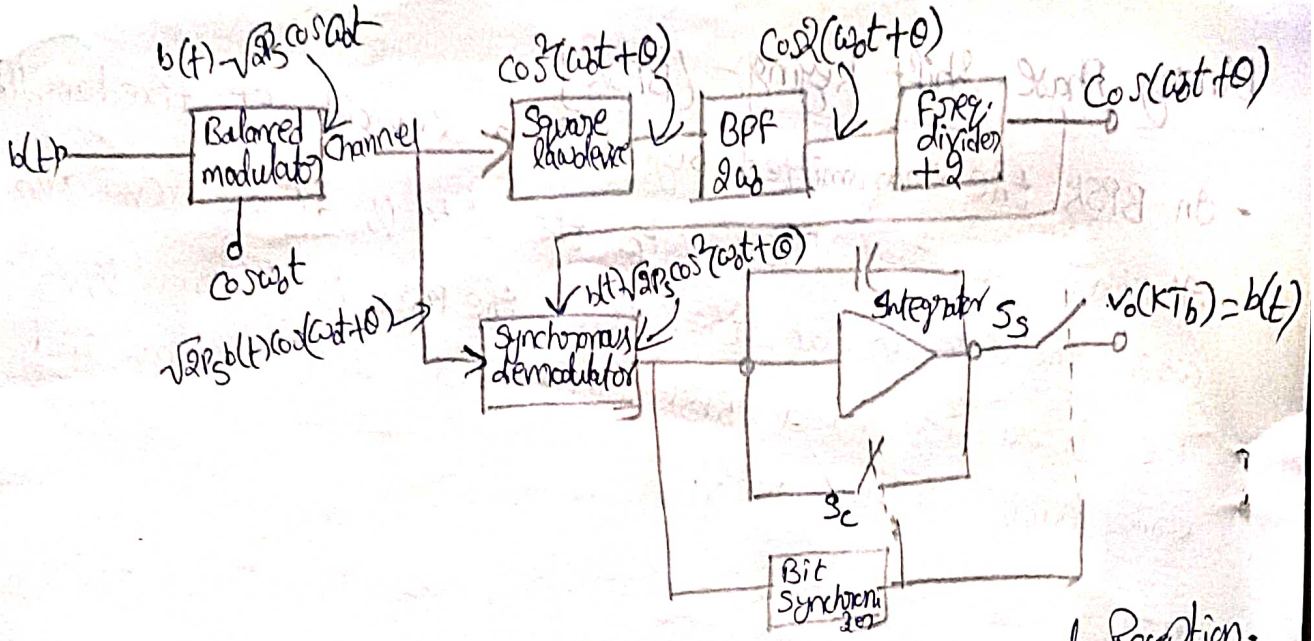


Fig:- A scheme for BPSK Transmission and Reception.

- A BPSK s/g is generated by applying the waveform $\cos\omega_c t$, as a carrier, to balanced modulator. and applying the base band signal $b(t)$ as the modulating waveform.
- BPSK can be thought of an AM signal.
- BPSK more Power efficient than ASK.
- The Received s/g has the form

$$r_{BPSK}(t) = b(t)\sqrt{2P_s}\cos(\omega_0 t + \theta)$$

$$= b(t)\sqrt{2P_s}\cos\omega_0(t + \theta/\omega_0)$$

$\theta \rightarrow$ nomial fixed phase shift

$\theta/\omega_0 \rightarrow$ Time delay

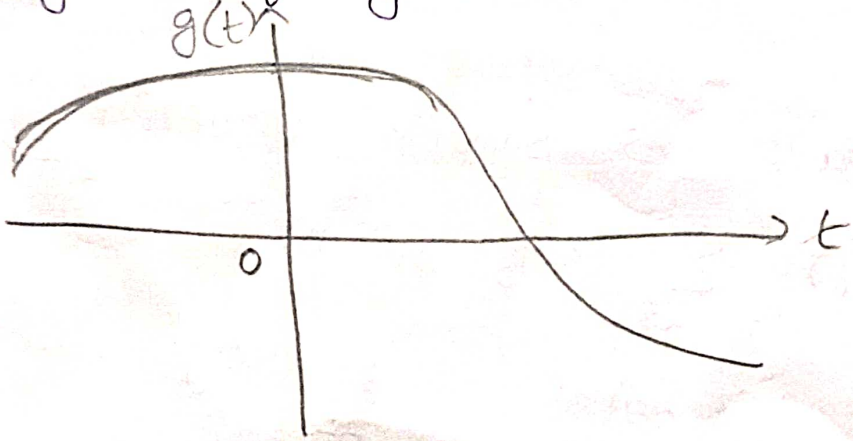
- The original data $b(t)$ is recovered in the demodulator.
- The demodulation technique called Synchronous demodulation
- The Received signal is squared to generate the s/g

$$\cos^2(\omega_c t + \theta) = \frac{1}{2} + \frac{1}{2}\cos 2(\omega_c t + \theta)$$

- The dc component is removed by band Pass filter whose baseband is centered around $2f_0$ and
- A freq. divider is used to regenerate the waveform $\cos[\omega_c t + \theta]$

1. Sampling Process:

- Sampling Process described as a Time domain
- Sampling is an operation in digital signal Processor and digital Communication.
- Sampling Process is an analog signal that is Converted into Corresponding sequence of Samples that are Uniformly in time.
- Sequence of Samples defines as original Analog Signal
- Consider an arbitary signal $g(t)$ of finite energy, the segment of Signal(A.S) becomes like this.



- If the Sample signal $g(t)$ is at Uniform rate at T_s seconds.
- If we obtained the infinite sequence of Samples T_s and denoted by $\{g(nT_s)\}$
  → where n is the integer values.
  → T_s is the Sampling Period.

and its reciprocal is $\boxed{f_s = \frac{1}{T_s}}$

- Let $g_\delta(t)$ denoted the signal obtained by sequence of delta functions spaced T_s second the sequence of numbers $\{g(nT_s)\}$ as show

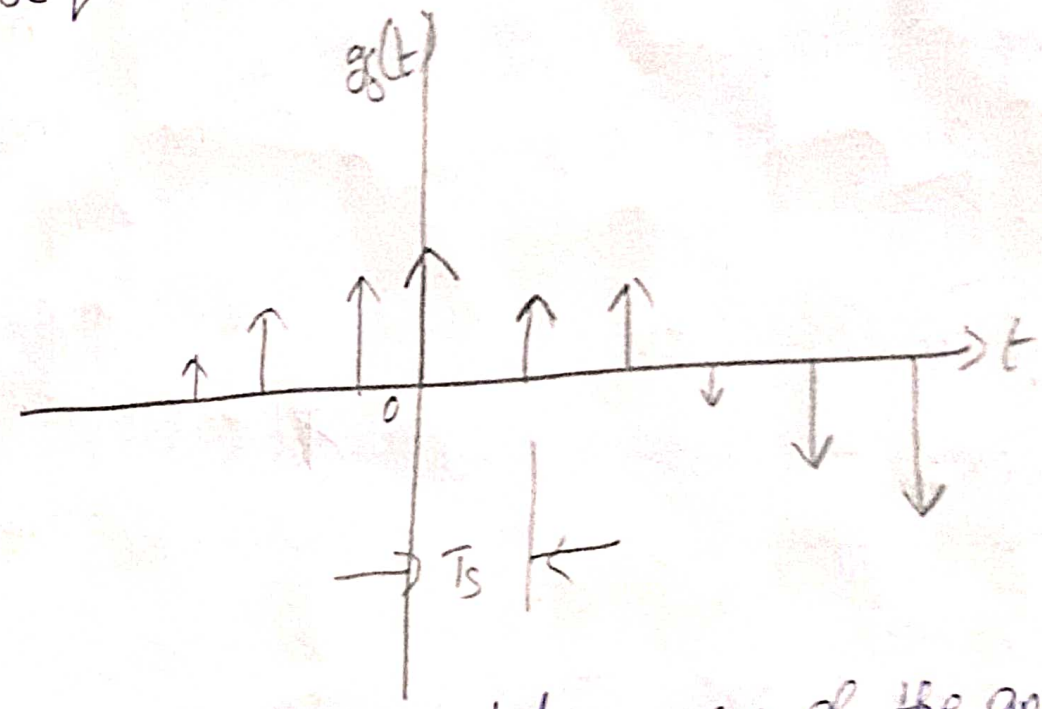


fig.b : Instanteneous sampled version of the analog sig

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t-nT_s)$$

$g_\delta(t) \rightarrow$ Sampled Signal $\quad n \rightarrow$ Integer values

$\delta(t-nT_s) \rightarrow$ delta function at time $t = nT_s$

- These delta fun. approximately closed to the rectang Pulse of duration Δt
- and amplitude $g(nT_s)/\Delta t$

Using fourier Transform we get

$$g_\delta(t) \rightleftharpoons f_s \sum_{m=-\infty}^{\infty} G(f-mf_s)$$ $\rightarrow$ It is Continuous time Signal of finite energy

→ F.T of original signal $g(t)$

→ Sampling rate.

e F.T of delta fun. is equal to $\exp(-j2\pi nfT_s)$

Let $G_s(f)$ denote fourier Transform of $g_s(t)$.

$$G_s(f) = \sum_{n=-\infty}^{\infty} g(nT_s) \exp(-j2\pi nfT_s)$$ → This is called discrete time fourier Transform.

- Suppose the sampling Period $T_s = \frac{1}{2W}$

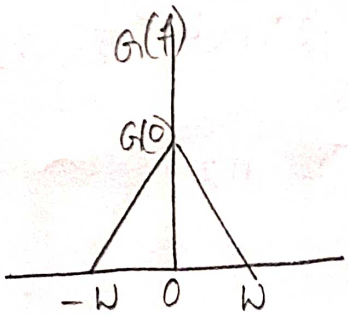
$$G_s(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(\frac{-j2\pi nf}{2W}\right)$$

$$G_s(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(\frac{-j\pi nf}{W}\right)$$

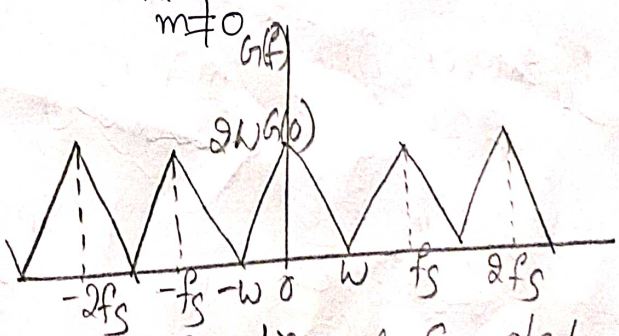
- If the signal $g(t)$ is a band limited with no freq. Component higher than W Hertz. Then the F.T $G(f)$ is zero

The fourier Trans. of $g_s(t)$ may also be Expressed as

$$G_s(f) = f_s G(f) + f_s \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} G(f - mf_s)$$



(a) Spectrum of Strictly band limited Signal $g(t)$



(b) Spectrum of Sampled version of $g(t)$ for Sampling Period $T_s = \frac{1}{2W}$

They are 2 conditions 1. $G(f) = 0$ for $|f|$

2. $f_s = 2W$

From above Eq. we get

$$G(f) = \frac{1}{2W} G_g(f), \quad -W < f < W$$

From above 2 Eq. we get

$$G(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(\frac{-j\pi n f}{2W}\right) \quad -W < f < W$$

This Eq. is refer to the F.T of $G(f)$ to a Signal is uniquely determined by using discrete Time F.T.

- If $G(f)$ is Inverse F.T then signal $g(t)$ is determined by $g\left(\frac{n}{2W}\right)$ for $-\infty < n < \infty$

The formula for Inverse F.T.

$$g(t) = \int_{-\infty}^{\infty} G(f) \exp(j2\pi ft)\, df$$

$$= \int_{-W}^{W} \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(\frac{-j\pi nf}{W}\right) \exp(i2\pi ft)\, df$$

By Interchanging

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{1}{2W} \int_{-W}^{W} \exp\left[j2\pi f\left(t - \frac{n}{2W}\right)\right] df$$

By Integrating we get

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{\sin(2\pi Wt - n\pi)}{(2\pi Wt - n\pi)}$$

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \sin(2Wt - n), \quad -\infty < t < \infty \Rightarrow$$

This is the Equation for Interpolation formula for reconstructing the original S/g $g(t)$ from Sample values ~~$g(n/2W)$~~ $g(n/2W)$

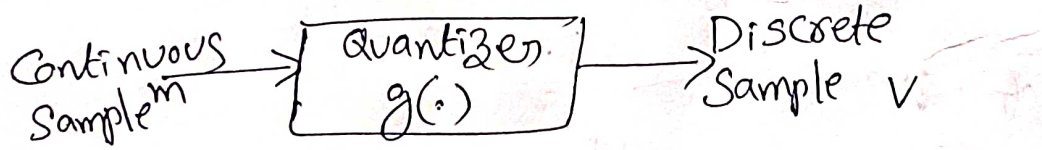
ling Theoram for Strictly band limited Signal a 2 Equal Parts

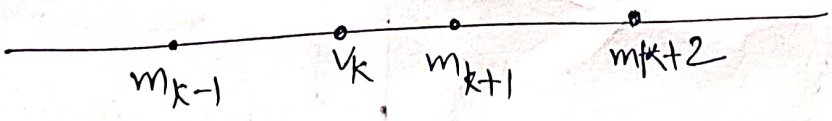
(i) A band limited s/g of a finite Energy which has no freq. Component higher than W hertz, then the values of a Signal at time separated by $\frac{1}{2W}$ seconds

(ii) A band limited s/g of finite Energy which has no freq. Component higher than W Hertz, its Sample is taken at the rate of 2W Sampled Per second.

2. Quantization Process:-

- A Continuous Signal Such as voice, amplitude and its samples have a Continuous amplitude range.



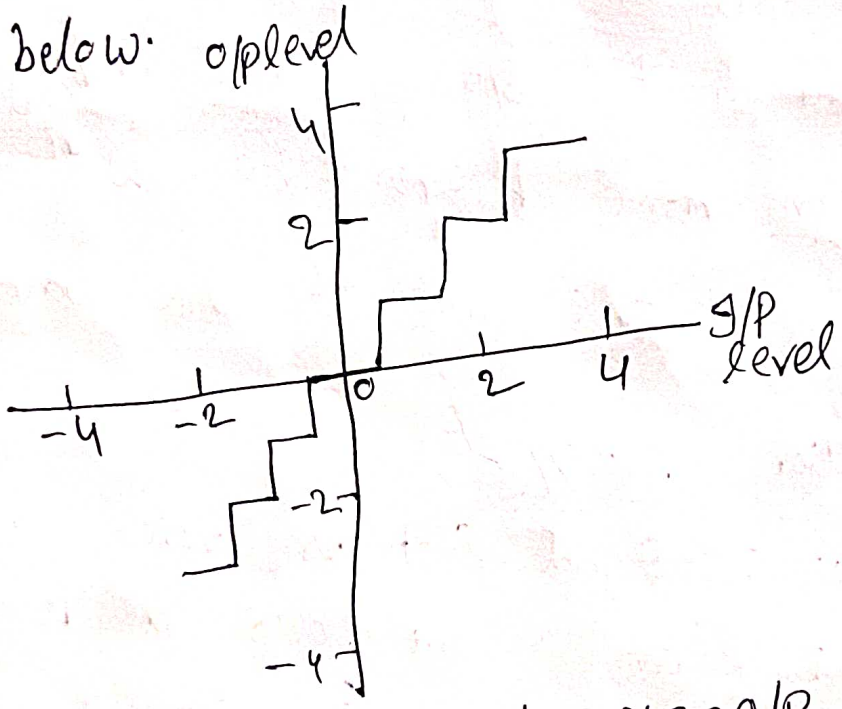


Description of memoryless quantizer.

- Amplitude quantization is defined as the Process of transforming the Sample amplitude $m(nT_s)$ of a message signal $m(t)$ at time $t = nT_s$ into)

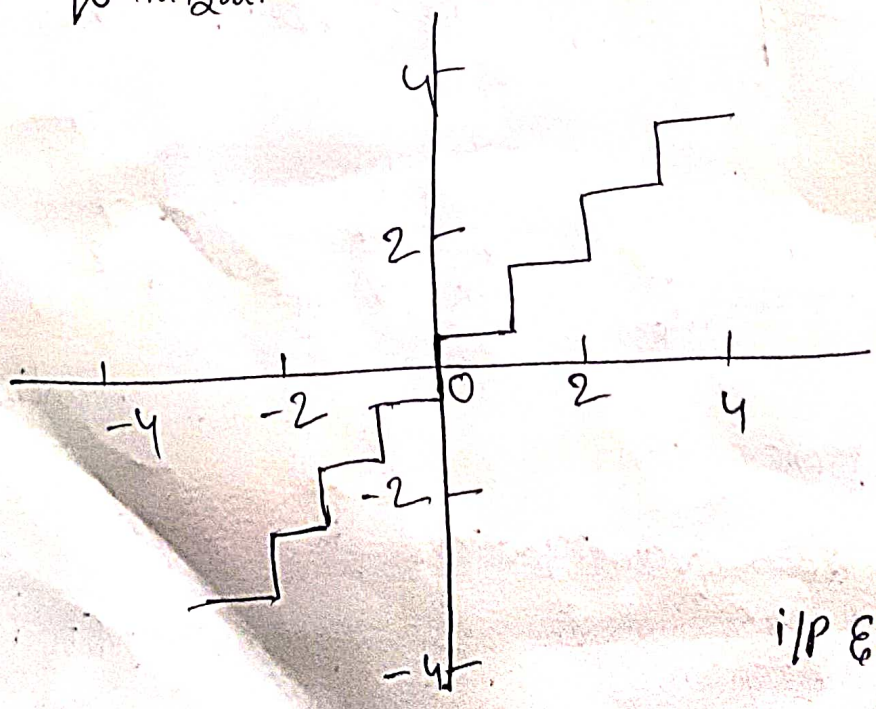
discrete amplitude v(nTs).

- Quantization Process is a memoryless and
- When dealing memoryless quantizers, we simplify notation by droping the time inder.
- The Symbol m in place of m(nTs) as indicat shown in below.



fig(a) midtread i/p & o/p characteristic.

- 2 Types of quantization (i) mid tread and (ii) midraise



i/p & o/p Characteri

fig(b) midrise

the Signal Amplitude m is specified the index k then the Paratial cell is

$$P_k : \{m_k < m \leq m_{k+1}\} \qquad k = 1, 2, \ldots L$$

L → Total no. of amplitude levels

m_k → discrete amplitudes, $k = 1, 2 \ldots L$

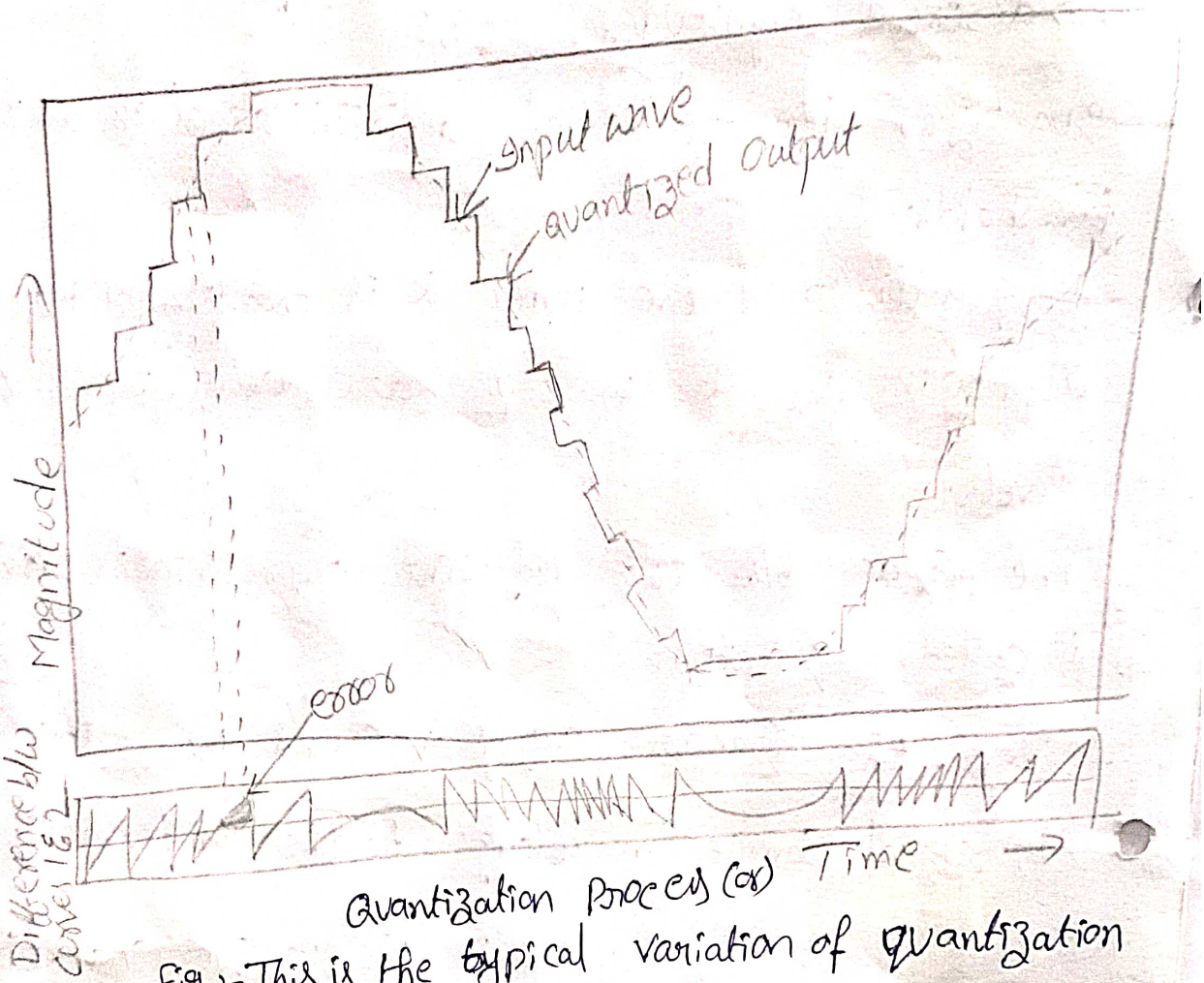
- quantizer i/p are called decision level or decision thresholds.
- At quantizer o/p the index k is transformed into amplitude v_k, $k = 1, 2, \ldots L$ are called representation levels.
- The space b/w two adjacent representation level is called quantum.
- Then the quantizer output v equals v_k if the i/p signal Sample 'm' belongs to the interval P_k

$V = g(m)$ → This is the quantizer Characteristic

- Quantizer Can be Uniform or non Uniform type
- If it is Uniform, the representation levels are Uniformly spaced. otherwise it is non uniform.
- If it is non Uniform, the

Quantization Noise:-

- The use of quantization introduce a error the difference b/w input signal m and signal v. The error is called "quantization



Quantization Process (or)

Fig:- This is the typical variation of quantization noise varies with time.

- Let us consider quantizer i/p 'm' be the samplevalue of a zero mean random variable 'M' and discrete random variable 'V'
- Let The quantization error is denoted by random variable 'Q'

$$q = m - V$$

(or)

$$Q = M - V$$

i/p m of Continuous amplitude ranges $(-m_{max}, m_{max})$.

We find the step-size of quantizer is given by

$$\Delta = \frac{2 \cdot m_{max}}{L} \quad \rightarrow (1)$$

L → Total no. of representation levels

- The Probability density function of the quantization error Q as follows

$$f_Q(q) = \begin{cases} \frac{1}{\Delta}, & -\frac{\Delta}{2} < q \leq \frac{\Delta}{2} \\ 0 & \text{otherwise} \end{cases} \quad \rightarrow (2)$$

- The mean of quantization error being zero, its variance σ_Q^2 is same as mean square value.

$$\sigma_Q^2 = E[Q^2]$$

$$= \int_{-\Delta/2}^{\Delta/2} q^2 f_Q(q)\, dq \Rightarrow \frac{1}{\Delta}\int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} q^2\, dq = \frac{1}{\Delta} \times \frac{q^3}{3}\Big|_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}}$$

$$= \frac{1}{3\Delta} \times 2\left(\frac{\Delta}{2}\right)^3$$

$$= \frac{1}{3\Delta} \times 2\,\frac{\Delta^3}{8} = \frac{\Delta^2}{12}$$

by applying eq condition $\sigma_Q^2 = \frac{\Delta^2}{12}$ ∴

- Let R is Consider as bits per Sample then

$$L = 2^R \rightarrow (3) \qquad R = \log_2 L$$

Sub (3) in (1)

$$\Delta = \frac{2\, m_{max}}{2^R} \quad \rightarrow (4)$$

Sub (4) in $\sigma_Q^2 = \frac{\Delta^2}{12} = \left(\frac{2\, m_{max}}{2^R}\right)^2 \cdot \frac{1}{12} = \frac{4 m_{max}^2}{2^{2R}} \cdot \frac{1}{12}$

$$= 4 \times \frac{m_{max}^2}{12} \cdot 2^{-2R} = \frac{1}{3} m_{max}^2 \, 2^{-2R}$$

$$\boxed{\sigma_Q^2 = \frac{1}{3} m_{max}^2 \, 2^{-2R}}$$

Condition for Optimality of Scalar

- Consider message signal $m(t)$ and it is from $-A \leq m \leq A$
- The boundaries of Partition cell is a set number $m_1, m_2, \ldots m_{L+1}$ Here $L \rightarrow$ cells
- It has 3 Condition.

$$m_1 = -A$$
$$m_{L+1} = A$$
$$m_k \leq m_{k+1} \quad \text{for } k = 1, 2, \ldots L$$

The kth Partial Cell is defined by

$$P_k : m_k < m \leq m_{k+1} \quad \text{for } k = 1, 2 \ldots L$$

- A quantizer Consist of two components
- For Encoder the set of Partiation cell $\{P_k\}_{k=1}^{L}$ is located at the Transmitter.
- For Decoder the Set of representation levels $\{v_k\}_{k=1}^{L}$ is located at the Receiver.

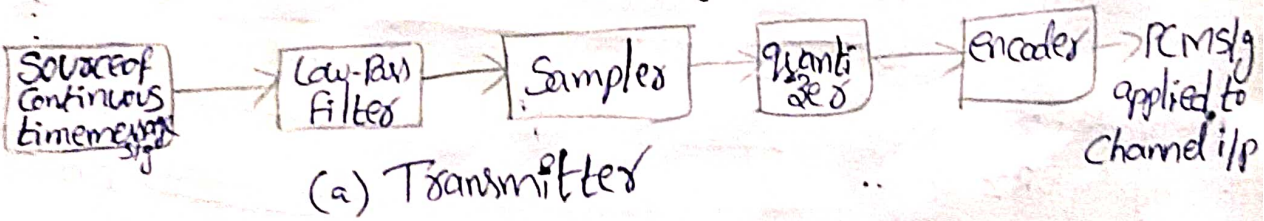
Companding: It is nothing but the sample version of message signal is then quantized, thereby providing digital representation of signal that is discrete in time and amplitude.

- A nonuniform quantizer is equivalent to passing base band signal through compressor. That compressed law is called as μ-law. which is defined by

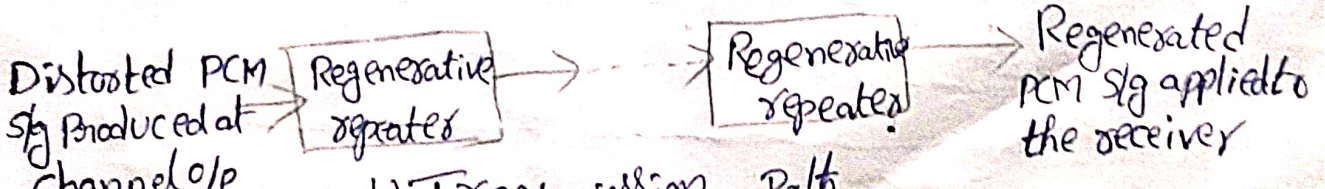
$$|V| = \frac{\log(1+\mu|m|)}{\log(1+\mu)}$$

PULSE Code Modulation :- (PCM)

- In PCM a message is represented by a sequence of code Pulses, which is representing the signal in discrete in both time and amplitude.
- The basic operation for PCM are Sampling, Quantizing and encoding.



(a) Transmitter



(b) Transmission Path



(c) Receiver

Fig: Basic elements of a PCM system

- Low Pass filter:- A filter that Passes Signals lower than Cut off frequency and attenuates signals higher than cut off freq.

- Sampler:- The message S/g is sampled with a rectangular Pulses So it is closely to the instantaneous Sampling Process.

- Quantization:- The message S/g is quantized thereby Provide new representation of the signal is discrete in both time and amplitude.

Reconstruction filter:- This is called anti-imaging filter used for to Construct analog S/g from a digital I/P (or) Sampled data

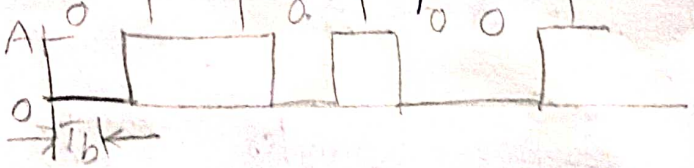
- **Encoder:-** Encoding is the Process to translate discrete set of samples to the more appropriate form of a Signal

(or) **Here** we are given 2^N inputs and given n outputs

Regenerative repeater:- This is used for digital Transmission. which S digital Signals are amplified, reshaped, and Retransmitted

Line Codes:-

- This is used for electrical representation of binary data stream.

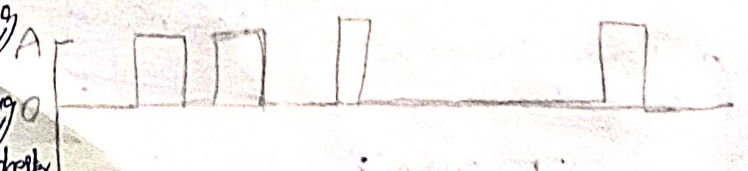


(a)

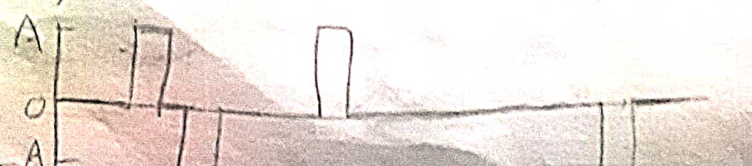
Line Code for
(a) Unipolar NRZ S/g
(b) Polar NRZ signaling
(c) Unipolar RZ Signaling
(d) Bipolar RZ Signaling
(e) Split-Phase or Manchester Code



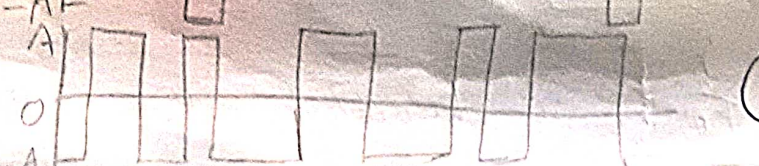
(b)



(c)



(d)



(e)

Five important line codes:

Polar non return zero (NRZ) signaling

Polar non return zero (NRZ) signaling

(i) U.N.R.Z Signaling:-

Symbol 1 represent Pulse amp is tramitted.

Symbol 0 represent Switch off the pulse

This line code is also called as on-off Signaling.

(ii) P.N.R.Z. Signaling:-

Here Symbol 1 and 0 represented by transmitting Pulse of amplitudes $+A$ & $-A$

Adv: This line code is easy to generate

disadv: The Power spectrum of the signal is large near to zero freq.

(iii) Unipolar return to zero (RZ) Signaling:-

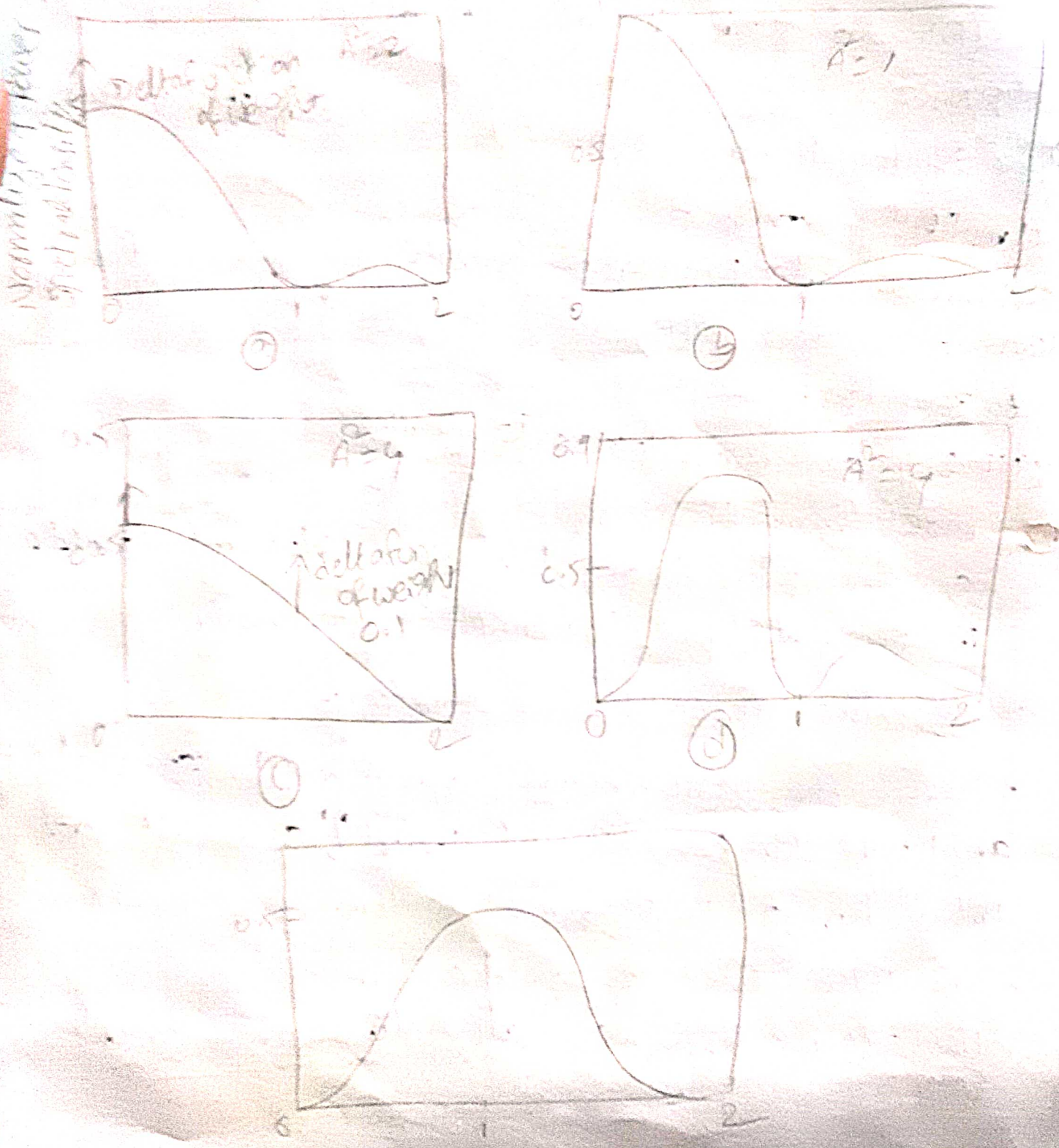
- Here Symbol 1 is represented by a rectangular Pulse of amplitude A and half Symbol width and Symbol is represented by transmitting no Channel.

(iv) Bipolar return to zero Signaling:-

- Here the Positive and negative Pulses of equal amplitude ($+A$ & $-A$) are used symbol '1' and with each Pulse having half Symbol width.
- no Pulse used for Symbol 0
- This code also called as alternate mark inversion (AMI)

(v) Split-Phase (Manchester code)

- Here Positive Pulse of amplitude, negative Pulse of amplitude $-A$. But in half symbol wide.
- For '0' represents the Pulses are reversed.



Encoding:-

method is used for to encode the information terms of Signal Transitions.

- Here a Transition is used to designate symbol '0' in the incoming binary data stream, while no transition is used to designate symbol 1.
- Whether the Transition is occured or not to checkout that one by simply compare the binary information to the Polarity of adjacent of binary symbols.
- A differential encoding it requires a reference bit before initiating the encoded Process.

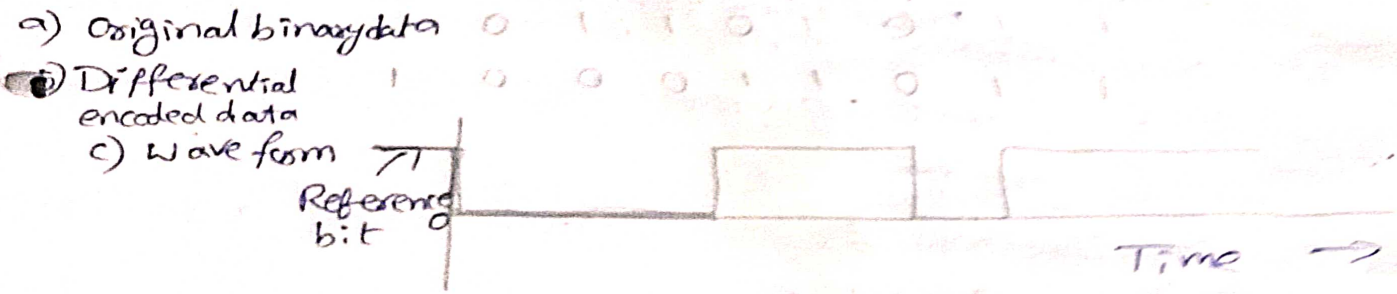


fig :- a & b & c

Regeneration :-

- The most important features of PCM system is the ability to control the effect of distortion and noise Produced by PCM transmitting signal through a channel.
- For reconstruct the PCM signal by means of Regenerative Repeaters.

- They are 3 equal Parts in regenerative
(i) Equalization (ii) Timing (iii) decision making.

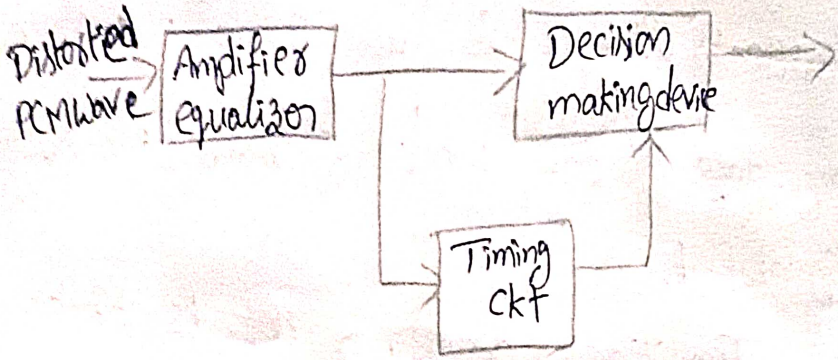


Fig:- Block diagram of Regenerative Repeater.

- Equalization is received pulse so it compensate the effect of ~~Pulse~~ amplitude and phase distortion Produced by non ideal characteristics of the channel.
- The timing is used for to Provide the Periodic Pulse. Here Sampling is equalized Pulse where the signal-to noise ratio is max.
- Decision making device is used for whether recived symbol is 0 or 1 · on the basis of this threshold is exceded or not.
- If the Threshold is exceeded, means Pulse is representing Symbol 1
otherwise it is symbol '0'.
- In ~~Timing~~ each of the Sample is Compared with Threshold in Decision making.

Encoding:-

Decoding:- The Decoding Process to generate the Pulse the amplitude of which is the linear Sum of all Pulses in the code word, and each Pulse by its Place value is $(2^0, 2^1, 2^2, \ldots 2^{R-1})$ in the code.

- Where R is the number of bits Per Sample.

Filtering:-

- The final operation is in the Receiver the message s/g by Passing Decoder o/p through a low Pass reconstruction filter whose Cutoff freq. is equal to the message band width.

Noise Consideration on PCM System:-

The PCM system it Performs two major sources of Noise.

1) Channel Noise:- Which is introduced anywhere between the transmitter output and receiver i/p. Channel noise is always Present, once the equipment is switched on.

2) Quantization Noise:- It is introduced in transmitter and is carried all the way along to the receiver o/p Unlike Channel noise, quantization noise is signal dependent in the sense it disappears when the message s/g is switched off.

- Once the PCM system is operation start then automatically these two sources of noise is appeared.
- The main effect of Channel noise is introduce bit error into the received signal.
- For binary PCM system, the presence of bit error causes the symbol 1 is to be mistaken and by symbol '0' or vicevers
- The effect of Channel noise can be made Practically negligible in PCM system.
- But quantization noise is essentially used for to designer Control.

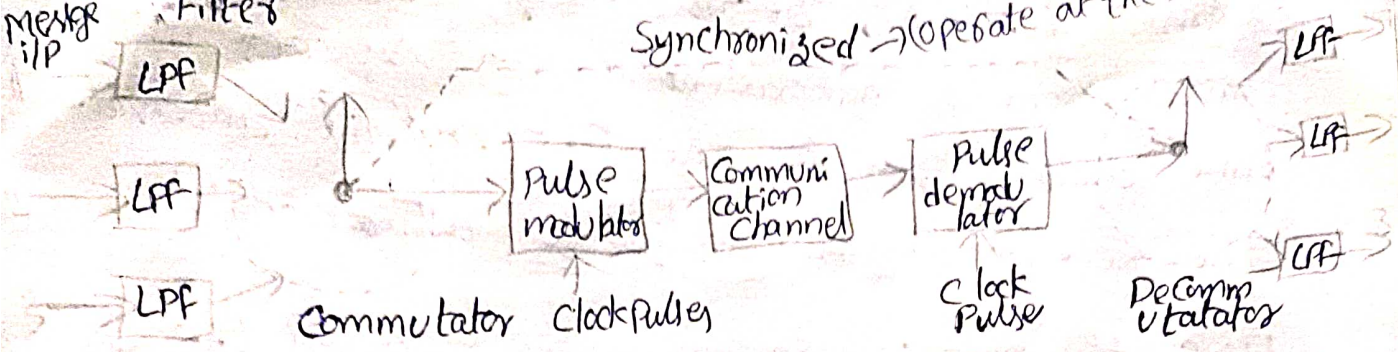
Time Division Multiplexing

- A Sampling Theorem Provides the transmitting the information Contained in a band limited ~~Sig~~ message signal m(t) as sequence of Samples m(t) at uniform rate that is slightly higher than Nyquist rate.
- The important Process of Sampling is Conservation of time.
- A TDM is used for digital signals.
- TDM which enables the joint utilization of Common communication Channel by Plurality of independent message Sources without mutual interference among them.
- TDM is a method of transmitting & Receiving independent ~~Sig~~ over a Common Signal Path by means of synchronized switches at each end of transmission line.

- TDM multiple data sig are transmitted over a common channel in different time slots is known as TDM

Low Pass (Anti-aliasing) filter

Synchronized → Operate at the same time



Block Diagram of TDM System.

- The message sig is transferred to the low pass filter in LPF to remove the freq. that are non essential to signal representation.
- The LPF o/p is applied to Commutator.
- It is (Commutator) is a electrical switch or electrical motor (or) Motor is switching b/w Phases
- The Commutator is implemented by using electronic Switching Circuit.
- They are 2 function in Commutator.
  (i) To take narrow sample of each of the N input messages at a rate f_s that is slightly higher than $2W$, where W is the cutoff freq.
  (ii) to sequentially interleave N samples inside the Sampling Interval T_s.
- The multiplexed sig is applied to Pulse modulator the Purpose of Pulse modulator is to transform the multiplexed sig into a suitable for transmission over the Common channel.
- P.M means amplitude is modulated in order to carry the inform the modulating signal from a Pulse modulator

- The use of TDM it is introduces a bandwidth expansion factor 'N', because squeeze N samples derived from N independent message sources into a time that is equal to the one sampling interval.
- The Receiving s/g is applied to Pulse demodulator.
- Here the narrow samples produced at the Pulse demodulator o/p are distributed to the appropriate low Pass reconstruction filter by means of decommutator.
- In this way synchronization is implemented depends the method of Pulse modulation used to transmit the multiplexed sequence of samples.
- The TDM system is highly sensitive in the common channel, that is variation of amplitude with freq. (or) Phase with freq.

Synchronize switches means to generate matching freq.

Synchronization:-

- In application of PCM, Theyare different multiplex message source by time division, each source have individual journey from Transmitter to receiver.
- The message source may be dropped or reinserted in time division multiplex system.
- As the number of independent message source is increased the time interval allated in each source has to be reduced.
- So the time interval is equal to the reciprocal of sampling rate.
- There is one possible Procedure is there to Synchronize the transmitter and receiver clock is to set code element or Pulse at the end of frame.
- Frame Patterns it should be either 1 or 0
- To establish the absense of Synchronizing Pulse, the receiver Pulse is set back by one code element and the next code element is observed. This searching Procedure is repeated until Synchronizing Pulse is detected.

Delta Modulation :- (D.M)

D.M is an analog to digital & digital-analog conversion used for Transmission of voice information.

- In, delta Modulation, an incoming message Signal is oversample (i.e, at a rate much higher than nyquist rate) then increase the correlation between adjacent samples of the signal.
- DM Provides a Staircase approximation to the oversampled version of the message signal. oversampling & under Sampling in data analysis to adjust the class distribution of dataset.

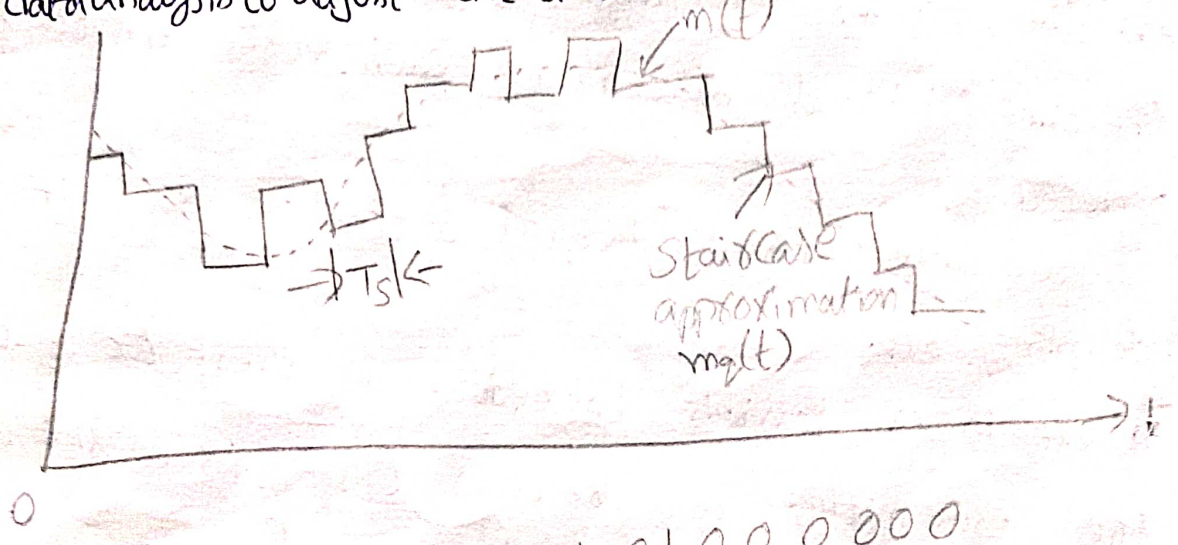


fig:- Delta Modulation

In above diagram it shows the difference b/w i/p and approximation is quantized into only two levels, namely $\pm\Delta$ that corresponds to positive and negative differences.

- The approximation falls below the signal it is increased by $\Delta(+\Delta)$, If the approximation is above the signal it is decreased by $\Delta(-\Delta)$.
- If we find the staircase approximation remains within $\pm\Delta$ of the input s/g.
- If the oversampling means the B.W of the signal is Perfectly reconstructed when the Sampling signal is at the Nyquist rate or above it.

- Let $m(t)$ denote the input signal and $m_q(t)$ denote the staircase approximation.
- Then the digital signal Processing is

$$m[n] = m(nT_s) \qquad n = 0, \pm 1, \pm 2 \text{ - - - -}$$

Where $T_s \rightarrow$ Sampling Period

$m(nT_s) \rightarrow$ Sampling of the signal $m(t)$ at time $t = nT_s$

- In D.M the following are relation in the discrete time

$$e[n] = m[n] - m[n-1]$$

$$e_q[n] = m_q[n] - m_q[n-1] \longrightarrow (1)$$

$$e_q = \Delta \operatorname{Sgn}(e[n]) \longrightarrow (2)$$

$$m_q[n] = m_q[n-1] + e_q[n] \rightarrow (3)$$

$e_q \rightarrow$ Error signal representing difference b/w Present Sample $m[n]$ of the i/p signal and latest approximation $m_q[n-1]$

- $e_q[n]$ is the quantized version of $e[n]$.
- $\operatorname{Sgn}(\cdot)$ is the Signum function.

$m_q[n] \rightarrow$ coded to Produce the DM signal.

we know $f_s = \frac{1}{T_s}$

→ The delta modulation is generated by applying Sample version of incoming message s/g to Δ modulator that involves comparator, quantizer and Accumulat

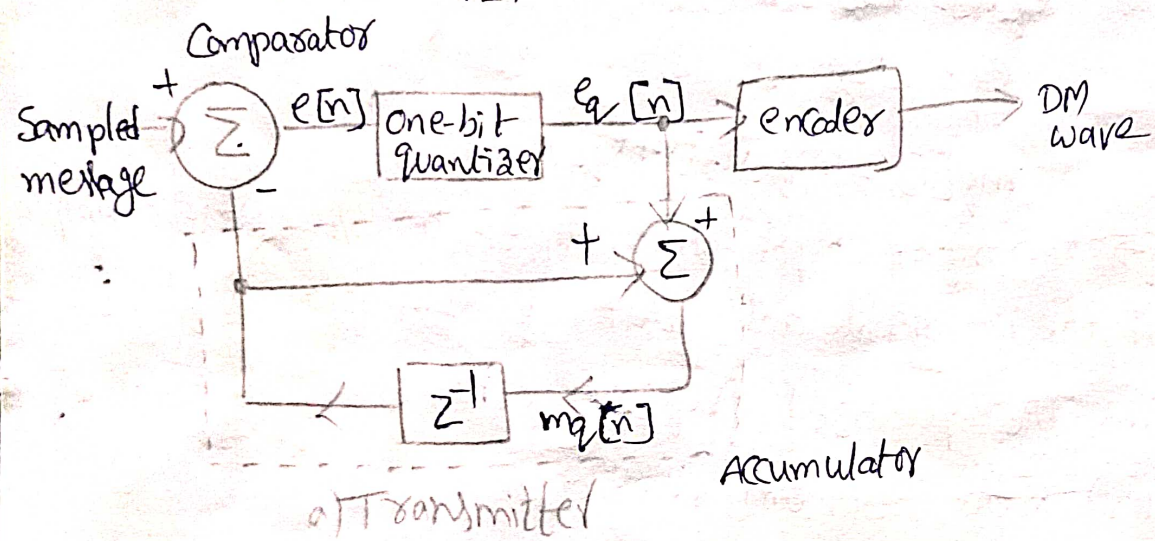
- The block labled z^{-1} inside the accumulator repre unit delay, delay is equal to one Sampling Period.

From eq ① & ③ the Comparator Computes the difference b/w its two inputs.

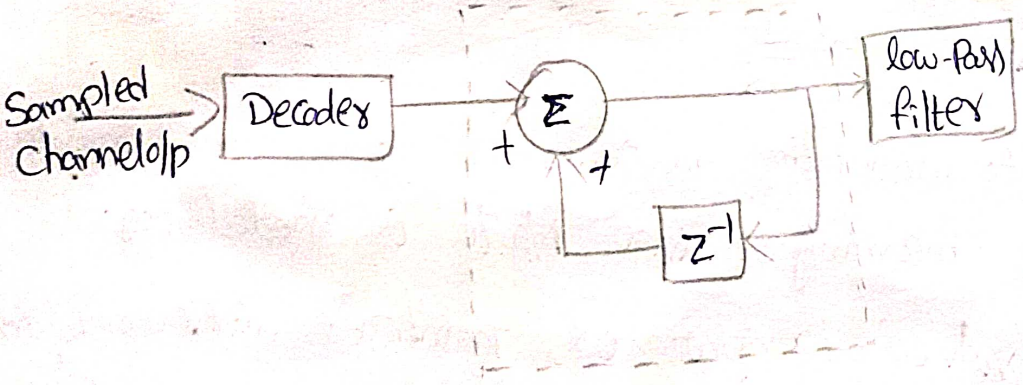
- A quantizer Consist of a hard limiter with an i/p-o/p relation that is scaled version of Signum function.
- The quantizer o/p is applied to accumulator, Producing as

$$m_q[n] = \Delta \sum_{i=1}^{n} \text{sgn}[e[n]]$$

$$= \sum_{i=1}^{n} e_q[i]$$



a) Transmitter



b) Receiver

A.L is a accumulates Something it is similar to rechargeable electrical Battery.

- The sampling instant nTs, the Accumulator increments the approximation by a step Δ in a positive or negative direction
- If the i/p sample m[n] is greater than approximation $m_q[n]$ positive increment is applied to the approximation.
- If the input sample is smaller negative increment -Δ is applied to approximation.
- Fig(b) is the staircase approximation $m_q(t)$ is reconstructed by passing positive or negative pulses produced the decoder o/p.
- D.M have two types of quantization error: Slope overload distortion and granular noise.

The quantization error is q[n]

$$m_q[n] = m[n] + q[n]$$

From eq(1) the i/p to quantizer is

$$e[n] = m[n] - m[n-1] - q[n-1]$$

Differential Pulse Code Modulation:

- When a voice or video signal is sampled at a rate higher than Nyquist rate thus is done in PCM, then the resulting sampled signal is found in high degree of correlation b/w adjacent samples
- In PCM highly samples are encoded resulting encoded signal contains redundant information
- That means the transmission of information result to generate the encoding process. By removing this before encoding, we obtain a more efficient coded signal, which is basic idea in differential PCM.
- Suppose a Baseband signal $m(t)$ is sampled at the rate $f_s = 1/T_s$ to produce the sequence $\{m[n]\}$ sampled at T_s seconds.
- The signal $m(t)$ Provides the differential quantization scheme.

$$e[n] = m[n] - \hat{m}[n]$$

which is the difference b/w unquantized i/p sample $m[n]$ and Prediction of it denoted by $\hat{m}[n]$

- By encoding quantizer o/p, we obtain a variant of PCM known as differential Pulse code Modulation (DPCM)

The quantizer o/p is

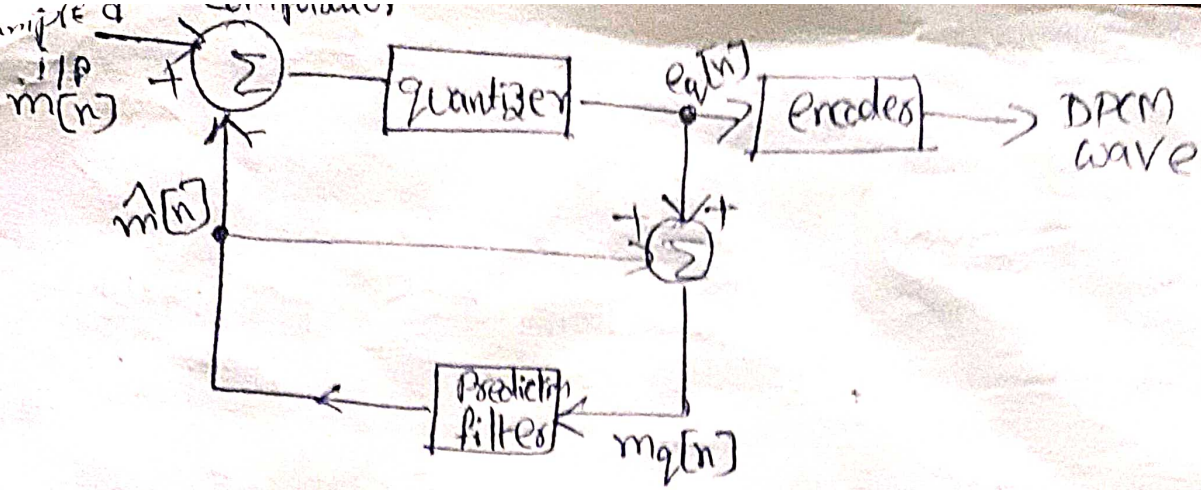
$$e_q[n] = e[n] + q[n] \rightarrow (1)$$

$$m_q[n] = \hat{m}[n] + e_q[n] \rightarrow (2)$$

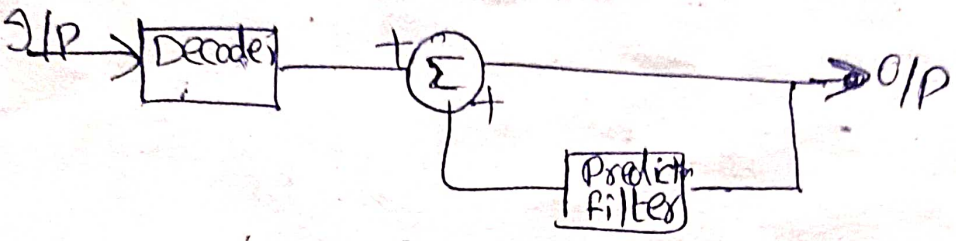
Sub (1) in (2)

$$m_q[n] = \hat{m}[n] + e[n] + q[n]$$

$$m_q[n] = m[n] + q[n]$$



DPCM (a) Transmitter



(b) Receiver

- The Prediction filter it shows difference b/w i/p Sample quantized Sample $m_q[n]$ and i/p Sample m[n] and quantization error q[n].
- fig(b) Decoder to reconstruct the quantized error
- The quantized i/p is reconstructed from decoder o/p using Prediction filter.
- The D.M and DPCM having a feedback.
- DPCM and DM have a slope overload distortion taker place.
- DPCM Suffers from quantization noise.
- P.F are a family of estimation techniques

Processing gain:

- The output signal to noise ratio of DPCM is

$$(SNR)_o = \frac{\sigma_M^2}{\sigma_Q^2}$$

$\sigma_M^2 \rightarrow$ variance of original input sample $m[n]$

$\sigma_Q^2 \rightarrow$ the variance of quantization error.

The product of two factors is

$$(SNR)_o = \left(\frac{\sigma_M^2}{\sigma_E^2}\right)\left(\frac{\sigma_E^2}{\sigma_Q^2}\right)$$

$\sigma_E^2 \rightarrow$ the variance of prediction error.

$$= G_p (SNR)_Q$$

signal to quantization noise ratio is

$$(SNR)_Q = \frac{\sigma_E^2}{\sigma_Q^2}$$

Processing gain G_p is

$$G_p = \frac{\sigma_M^2}{\sigma_E^2}$$

If G_p is greater than unity, it represents gain in signal to noise ratio.

Sreelekha XEROX

Name of the Student : S. Nikitha

Subject : DCS - III unit

Semister/Course :

XEROX , JOB WORKS, COLOUR XEROX, LAXER PRINTIN
PROJECTS, SPIRAL BINDING, LAMINATION

Near R.T.C. Bustand.
Old Hero Honda Show Room, Kadapa

Matched Filter :- ①

→ detection of a pulse signal of known waveform that is immersed in additive white noise, the device used for the optimum detection of such a pulse involves the use of a linear time-invariant filter known as a "matched filter". M.F is used in radar. Here a known s/g is sent out, and reflected s/g is examined for a common elements of the outgoing channel.

→ which is so called because its impulse response is matched to the pulse signal. O.L.F is used for to measured the matrices & noises (or) to derive optimal values for filter coefficients

LTI is uniquely specified by impulse response

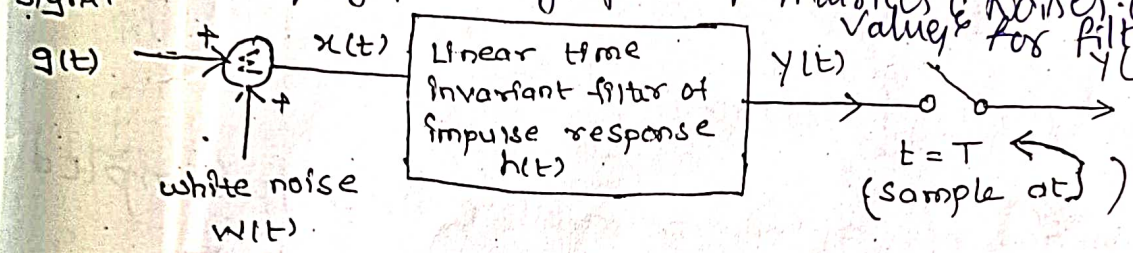


fig :- Linear receiver

$x(t) = g(t) + w(t) \rightarrow ①$

$g(t) \rightarrow$ pulse signal, which represents a binary symbol 1 (or) 0 in a DCS.

$w(t) \rightarrow$ sample function of white noise process of zero mean and PSD $N_0/2$.

∵ Filter is linear, the resulting o/p may be expressed as

$$\boxed{y(t) = g_0(t) + n(t)} \rightarrow ②$$

At $t = T$, the instantaneous power in the output signal $g_0(t)$ considerably greater than $n(t)$

i.e., at $t = T$ for $g_0(t)$,

$$\boxed{g_0(T) >> n(t)} \rightarrow$$ such type of filter we have to design.

→ peak pulse signal to noise ratio

$$\boxed{\eta = \frac{|g_0(T)|^2}{E[n^2(t)]}} \rightarrow ③$$

M.F is a optimal linear filter for max. signal to noise ratio in the presence of additive stochastic no

→ The requirement is to specify the impulse respons
$h(t)$ of the filter such that the o/p SNR is meaxi

→ $G(f)$ is F.T of $g(t)$
$H(f)$ is F.T of $h(t)$ [frequency response of the filter]

$\therefore$ F.T of $\boxed{g_0(t) = H(f)G(f)} \Rightarrow g_0(t) = H(f)G(f)$.

$g_0(t)$ is IFT of $G_0(f)$

$$\therefore g_0(t) = \int_{-\infty}^{\infty} H(f)G(f)\, e^{j2\pi ft} \cdot df$$

→ In the absence of channel noise when filter o/p sample
at $t = T$, we have

$$= |g_0(T)|^2 = \left| \int_{-\infty}^{\infty} H(f)\, G(f)\, e^{j2\pi fT} \cdot df \right|^2 \rightarrow (4)$$

→ In the presence of noise,

→ The PSD of o/p noise = PSD of i/p noise times squared the magnitude res

$$\boxed{S_N(f) = \frac{N_0}{2} |H(f)|^2}$$

$\therefore$ Average power of the o/p noise $n(t)$ is

$$\rightarrow E[n^2(t)] = \int_{-\infty}^{\infty} S_N(f) \cdot df = \int_{-\infty}^{\infty} \frac{N_0}{2} |H(f)|^2 \cdot df$$

$$\therefore E[n^2(t)] = \int_{-\infty}^{\infty} \frac{N_0}{2} |H(f)|^2 \cdot df \rightarrow (5)$$

$$\therefore \eta = \frac{|g_0(T)|^2}{E[n^2(t)]} = \frac{\left| \int_{-\infty}^{\infty} H(f)\, G(f)\, e^{j2\pi fT} \cdot df \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 \cdot df}$$

→ our problem is to find, for a given $G(f)$ the particular form of frequency response $|H(f)|$ of the filter that makes "η" maximum.

Immersed in means involves oneself deeply in a particular activi
PSD: Power Spectral density (ranges from -2.2 to -2.7 mA)
ite noise is a random sig having equal intensity at differe
freq

According to Schwarz's Inequality

$$\left| \int_{-\infty}^{\infty} \phi_1(x)\, \phi_2(x)\, dx \right|^2 \le \int_{-\infty}^{\infty} |\phi_1(x)|^2 \cdot dx \int_{-\infty}^{\infty} |\phi_2(x)|^2 \cdot dx$$

the equality holds, if and only if

$$\boxed{\phi_1(x) = K\, \phi_2^*(x)}$$

K → arbitrary constant

* → complex conjugation.

By setting the numerator, according to Schwarz's Inequality

$$\phi_1(x) = H(f), \quad \phi_2(x) = G(f)\, e^{-j2\pi fT}$$

$$\left| \int_{-\infty}^{\infty} H(f)\, G(f)\, e^{j2\pi fT} \cdot df \right|^2 \le \int_{-\infty}^{\infty} |H(f)|^2 \cdot df \int_{-\infty}^{\infty} |G(f)|^2 \cdot df$$

$$\therefore \quad \eta \le \frac{\int_{-\infty}^{\infty} |H(f)|^2 \cdot df \int_{-\infty}^{\infty} |G(f)|^2 \cdot df}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 \cdot df}$$

$$\boxed{\eta \le \frac{2}{N_0} \int_{-\infty}^{\infty} |G(f)|^2 \cdot df}$$

→ doesn't depend on the frequency response of filter but only on the signal energy and noise PSD.

→ According to Schwarz's Inequality

'η' will maximum, when

$$H_{opt}(f) = K \cdot G^*(f)\, e^{-j2\pi fT}$$ then

$$\boxed{\eta_{max} = \frac{2}{N_0} \int_{-\infty}^{\infty} |G(f)|^2 \cdot df}$$

Applying IFT to H(f)

$$h_{opt}(t) = K \int_{-\infty}^{\infty} G^*(f)\, e^{-j2\pi fT} \cdot e^{j2\pi ft} \cdot df$$

$$= K \int_{-\infty}^{\infty} G(-f)\, e^{-j2\pi f(T-t)} \cdot df$$

$$= K \int_{-\infty}^{\infty} G(f)\, e^{j2\pi f(T-t)} \cdot df$$

$$= K \cdot g(T-t)$$

$$\boxed{h_{opt}(t) = K\, g(T-t)}$$

t shows that the impulse response of the timum filter, except for scaling factor, is a ne-reversed and delayed version of i/p gnal $g(t)$, that is, it is "matched" to the i/p gnal $g(t)$.

$K g(T-t)$

Properties of Matched Filter :-

The Signal to noise ratio of the matched filter depends only upon the ratio of the signal energy to the psd of white noise at the filter input.

i.e., $\eta_{max} = \frac{2E}{N_0} = \frac{E}{(N_0/2)}$ (4)

Proof : To demonstrate this property, consider a filter matched to a known signal $g(t)$. The inverse transform of resulting matched filter output $g_0(t)$

$$\therefore G_0(f) = H_{opt}(f) G(f)$$

$$= K G^*(f) e^{-j2\pi fT} \cdot G(f)$$

$$G_0(f) = K |G(f)|^2 \cdot e^{-j2\pi fT}$$

Applying IFT, at $t = T$

$$g_0(T) = \int_{-\infty}^{\infty} G_0(f) \cdot e^{j2\pi fT} \cdot df$$

$$= K \int_{-\infty}^{\infty} |G(f)|^2 \cdot e^{-j2\pi fT} \cdot e^{j2\pi fT} \cdot df$$

$$g_0(T) = K \int_{-\infty}^{\infty} |G(f)|^2 \cdot df$$

According to Rayleigh's energy theorem, the integral of squared magnitude spectrum of a pulse signal with respect to frequency is equal to signal energy E.

$$\therefore E = \int_{-\infty}^{\infty} |G(f)|^2 \cdot df \rightarrow (1)$$

$$\therefore \boxed{g_0(T) = KE} \rightarrow (I)$$

We know that the maximum peak pulse signal noise ratio of a matched filter is

$$\eta_{max} = \frac{2}{N_0} \int_{-\infty}^{\infty} |G(f)|^2 \cdot df \quad \rightarrow (2)$$

put eqn(1) in eqn(2)

$$\therefore \eta_{max} = \frac{2}{N_0} E$$

$$\boxed{\eta_{max} = \frac{E}{(N_0/2)}} = \frac{2E}{N_0}$$

Hence it is proved.

(2) The maximum signal component occurs at $t = T$ and has magnitude E, i.e., energy of the signal

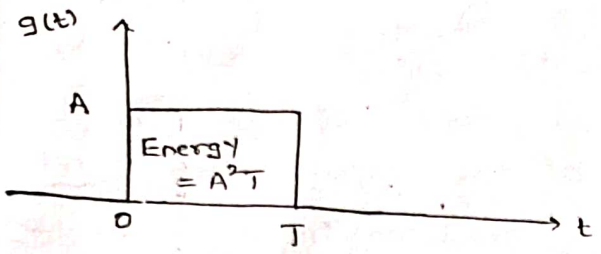
Proof:- From the above property, the maximum signal component at $t = T$, is from eqn(1) is

$$g_0(T) = KE$$

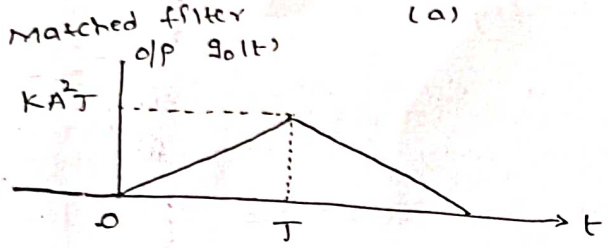
when $K = 1$

$$\therefore \boxed{g_0(T) = E}$$

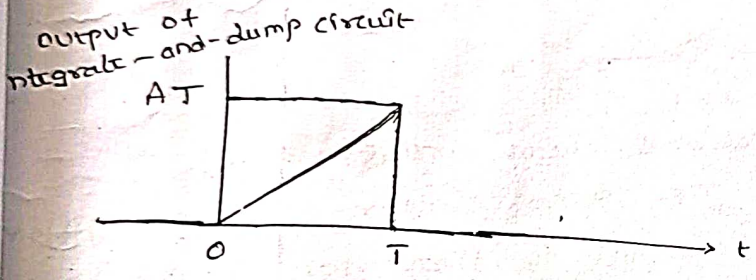
Matched Filter for Rectangular pulse :-



(a)



(b)



(c)

fig :- (a) Rectangular pulse (b) Matched filter output (c) Integrator and Dump circuit output.

Consider a signal $g(t)$ in the form of a rectangular pulse of amplitude A and duration T as shown in fig (a). In this example, the impulse response $h(t)$ of the matched filter has exactly the same waveform as the signal itself..

The output signal $g_o(t)$ produced in response to the input signal $g(t)$ has a triangular

waveform as shown in fig(b). The maximum v
the output signal $g_0(t)$ is equal to $k A^2 T$,
is the input of energy of the input signal g
scaled by the factor k; the maximum value
at t=T, as indicated in fig(b).

For the special case of a rectan
pulse, the matched filter may be implemente
using a circuit known as the Integrate-and-du
circuit, as shown below.

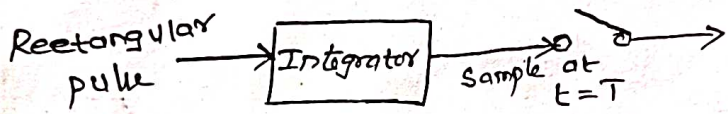


fig:- Integrate-and-dump circuit

The integrator computes the area under the recte
pulse, and the resulting output is then sampled
time t=T, where 'T' is the duration of the pulse
Immediately after t=T, the integrator is res
to its initial condition: hence the name of the circ

fig(c) shows the output of integrate-and-d
circuit for the rectangular pulse as shown in fig
we see that for $0 \leq t \leq T$, the output of the ci
has the same waveform as that appearing at the
output of the matched filter.

ERROR RATE DUE TO NOISE (OR)

PROBABILITY OF ERROR :- It means the rate of occurance of error in a hypothetical infinite repetition of the Procedure.

To proceed with the analysis, consider a binary PCM System based on polar NRZ signaling. The channel noise is modeled as additive white Gaussian noise $W(t)$ of zero mean and power spectral Density of $N_0/2$.

In the signal interval $0 \le t \le T_b$, the received signal is then written as follows :

$$x(t) = \begin{cases} +A + W(t) & ; \text{ symbol 1 was sent} \\ -A + W(t) & ; \text{ symbol 0 was sent.} \end{cases}$$

Given the noisy signal $x(t)$, the receiver is required to make a decision in each signaling interval as to whether the transmitted symbol is 1 or 0. The structure of the receiver used to perform this decision-making process is shown in below figure :

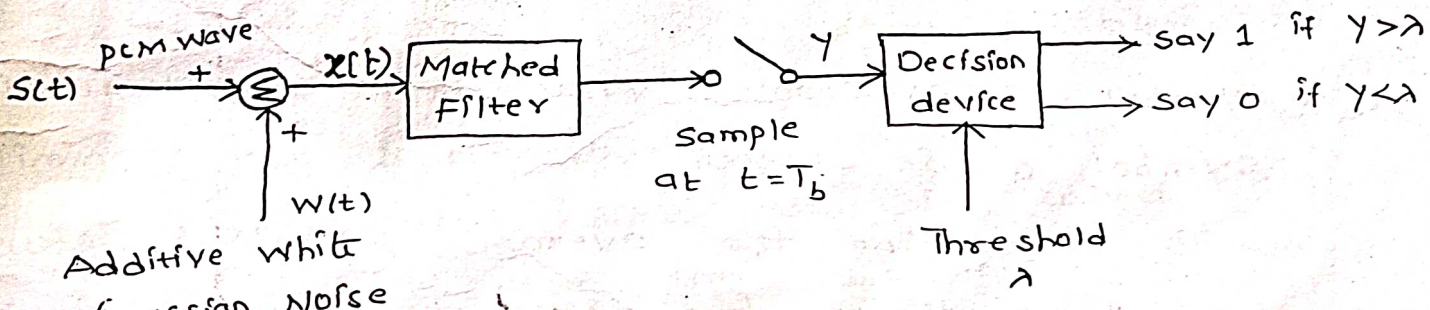


fig : Receiver for baseband transmission of binary-encoded PCM wave using polar NRZ

ERROR RATE :- The Measurement of the effectiveness of a Communication Channel.

The above figure consists of a matched fil
followed by a sampler, and then finally a decisi
device.

Let 'y' denote the sample value obtained a
end of a signaling interval. The sample value
is compared to a present threshold 'λ' in th
decision device. If the sample value exceeded t
threshold value, the receiver makes a decision
favor of symbol 1; if not, a decision is ma
in favor of symbol '0'.

We adopt the convention that when the sam
value 'y' is exactly equal to the threshold 'λ',
the receiver just makes a guess as to which s
was transmitted.

There are two possible kinds of error to
considered:

1) symbol '1' is choosen when a '0' was transmi
2) symbol '0' is choosen when a '1' was transmi

To determine the average probability of
we consider these two situations separately.

1) Suppose that symbol '0' was transmitted

Then, $x(t) = -A + w(t)$; $0 \le t \le T_b$

the matched filter o/p, sampled at $t = T_b$

$$y = \int_0^{T_b} x(t) \cdot dt$$

$$Y = \int_0^{T_b} (-A + W(t)) \cdot dt$$

$$= \int_0^{T_b} -A \cdot dt + \int_0^{T_b} W(t) \cdot dt$$

$$= -A \int_0^{T_b} dt + \int_0^{T_b} W(t) \cdot dt$$

$$= -A [t]_0^{T_b} + \int_0^{T_b} W(t) \cdot dt$$

$$= -A T_b + \int_0^{T_b} W(t) \cdot dt$$

$$\boxed{Y = -A + \frac{1}{T_b} \int_0^{T_b} W(t) \cdot dt}$$

The random variable 'y' is Gaussian distributed with a mean of $-A$. The variance of random variable 'y' is

$$\sigma_y^2 = E[(Y + A)^2]$$

$$= \frac{1}{T_b^2} E\left[\int_0^{T_b} W(t) \cdot dt \int_0^{T_b} W(u) \cdot du \right]$$

$$= \frac{1}{T_b^2} \left[\int_0^{T_b} \int_0^{T_b} E[W(t) W(u)] \, dt \cdot du \right]$$

$$= \frac{1}{T_b^2} \left[\int_0^{T_b} \int_0^{T_b} R_W(t,u) \, dt \, du \right]$$

$R_W(t,u)$ is the autocorrelation function of the white noise $W(t)$. Since $W(t)$ is white noise with power spectral density $N_0/2$, we have

$$\boxed{R_W(t,u) = \frac{N_0}{2} \delta(t-u)}$$

$$\therefore \quad \sigma_y^2 = \frac{1}{T_b^2}\left[\int_0^{T_b}\int_0^{T_b} \frac{N_0}{2}\,\delta(t-u)\,dt\,du\right]$$

$$= \frac{N_0}{2T_b^2}\left[\int_0^{T_b} (1)\cdot du\right].$$

$$= \frac{N_0}{2T_b^{\not 2}}\left[\not{T}_b\right]$$

$$\therefore \quad \boxed{\sigma_y^2 = \frac{N_0}{2T_b}}$$

The conditional probability density function of random variable 'y', given that symbol '0' sent.

$$f_y(y/0) = \frac{1}{\sqrt{2\pi\sigma_y^2}}\, e^{\left[\frac{-(y-A_y)^2}{2\sigma_y^2}\right]} \quad ; -\infty$$

$$f_y(y/0) = \frac{1}{\sqrt{\not{2}\pi\left(\frac{N_0}{\not{2}T_b}\right)}}\, e^{\left[-\frac{(y+A)^2}{\not{2}\cdot(N_0/\not{2}T_b)}\right]}$$

$$\boxed{f_y(y/0) = \frac{1}{\sqrt{\pi N_0/T_b}}\, e^{\left[\frac{-(y+A)^2}{N_0/T_b}\right]}}$$

This function is plotted in fig (a), Let P_{10} denotes the conditional probability of error, that symbol '0' was sent. This probability is defined by the shaded area under the curve $f_y(y/0)$ from the threshold to infinity.

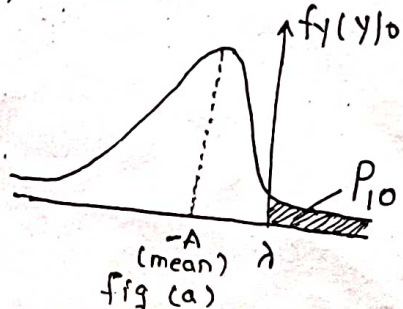


fig (a)

$P_{10} = p(y > \lambda / \text{symbol } 0 \text{ was sent})$

$$P_{10} = \int_{\lambda}^{\infty} f_y(y|0) \cdot dy$$

$$\therefore P_{10} = \frac{1}{\sqrt{\pi N_0/T_b}} \int_{\lambda}^{\infty} e^{\left[\frac{-(y+A)^2}{N_0/T_b}\right]} \cdot dy$$

Here, Introduce the definition of "Complementary error function".

$$\boxed{\text{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_{u}^{\infty} e^{(-z^2)} \cdot dz}$$

To reformulate the conditional probability of error P_{10} in terms of the complementary error function, we first define a new variable.

$$z = \frac{y + A}{\sqrt{N_0/T_b}}$$

Limits

Upper limit if $y = \infty$ then $z = \infty$.

Lower limit if $y = \lambda$ then $z = \frac{A+\lambda}{\sqrt{N_0/T_b}}$

$$dz = \frac{dy}{\sqrt{N_0/T_b}} \Rightarrow dy = dz\left(\sqrt{N_0/T_b}\right)$$

$$\therefore P_{10} = \frac{1}{\sqrt{\pi N_0/T_b}} \int_{(A+\lambda)/\sqrt{N_0/T_b}}^{\infty} e^{(-z^2)} \cdot dz\left(\sqrt{N_0/T_b}\right)$$

$$\therefore \boxed{P_{10} = \frac{1}{\sqrt{\pi}} \int_{(A+\lambda)/\sqrt{N_0/T_b}}^{\infty} e^{(-z^2)} \cdot dz}$$

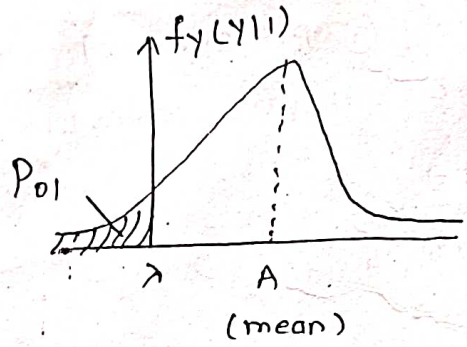
$$\therefore \quad \boxed{P_{10} = \frac{1}{2}\,\text{erfc}\left(\frac{A+\lambda}{\sqrt{N_0/T_b}}\right)}$$

Symbol 1 was transmitted:

This time the random variable 'y' h
mean $+A$ and variance $N_0/2T_b$. Therefore,
conditional probability density function $f_y(y|$

$$\boxed{f_y(y|1) = \frac{1}{\sqrt{\pi N_0/T_b}}\, e^{\left[\frac{-(y-A)^2}{N_0/T_b}\right]}}$$

This function $f_y(y|1)$ is plotted in below fig



$$\therefore \quad P_{01} = \int_{-\infty}^{\lambda} f_y(y|1)\cdot dy$$

$$P_{01} = \frac{1}{\sqrt{\pi N_0/T_b}} \int_{-\infty}^{\lambda} e^{\left[\frac{-(y-A)^2}{N_0/T_b}\right]} \cdot dy$$

the variable, $z = \dfrac{A-y}{\sqrt{N_0/T_b}}$

$$\therefore \quad P_{01} = \frac{1}{\sqrt{\pi}} \int_{(A\to\lambda)/\sqrt{N_0/T_b}}^{\infty} e^{(-z^2)} \cdot dz$$

$$\boxed{P_{01} = \frac{1}{2}\,\text{erfc}\left(\frac{A-\lambda}{\sqrt{N_0/T_b}}\right)}$$

Hence, the average probability of symbol error in the receiver is given by

$$P_e = P_0 P_{10} + P_1 P_{01}$$

$$= \frac{P_0}{2} \operatorname{erfc}\left(\frac{A+\lambda}{\sqrt{N_0/T_b}}\right) + \frac{P_1}{2} \operatorname{erfc}\left(\frac{A-\lambda}{\sqrt{N_0/T_b}}\right)$$

For the special case when symbols 1 and 0 are equiprobable, we have

$$\boxed{P_1 = P_0 = \frac{1}{2}}$$

Then optimum threshold is

$$\boxed{\lambda_{opt} = 0}$$

$$\therefore P_e = \frac{1}{4} \operatorname{erfc}\left(\frac{A}{\sqrt{N_0/T_b}}\right) + \frac{1}{4} \operatorname{erfc}\left(\frac{A}{\sqrt{N_0/T_b}}\right)$$

$$\boxed{P_e = \frac{1}{2} \operatorname{erfc}\left(\frac{A}{\sqrt{N_0/T_b}}\right)}$$

Now, the transmitted signal energy per bit is

$$E_b = A^2 T_b$$

$$A = \sqrt{E_b/T_b}$$

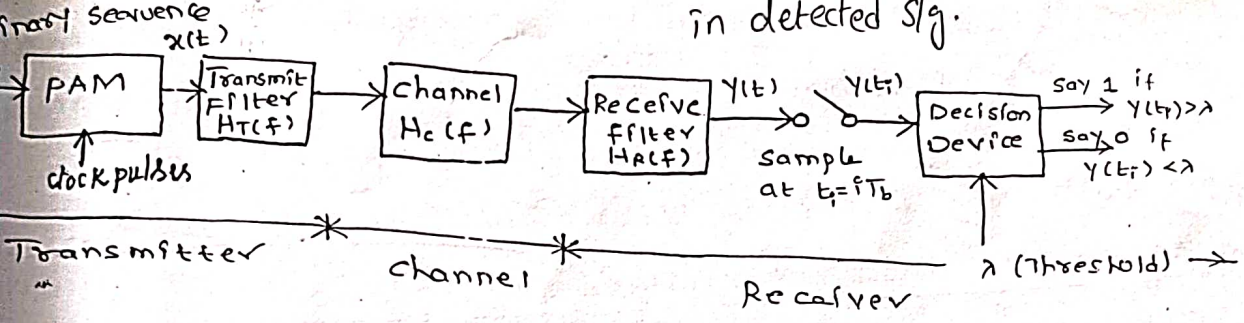
$$\therefore \boxed{P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

∴ The average probability of error depends only on the transmitted signal energy per bit to the noise spectral Density.

Intersymbol Interference :-

(9) ISI is a form of a distortion of a S/g one symbol is interface with subsequent symbol.

Consider a binary baseband PAM system shown in below figure : ISI is used for to identify errors in detected S/g.



The input binary sequence $\{b_k\}$ is applied to the PAM. The PAM generates the pulse waveform $x(t)$. This waveform can be represented mathematically as.

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} A_k \, g(t - kT_b)} \rightarrow (1)$$

Here, T_b is the duration of each i/p binary bit,

$g(t)$ is the shaping pulse, and

$$A_k = \begin{cases} +a & \text{if } b_k = 1 \\ -a & \text{if } b_k = 0 \end{cases}$$

Since polar NRZ signaling scheme is considered.

The signal $x(t)$ is then passed through the transmitting filter. The transmitting filter combines all the necessary transmitting circuits & systems. The combined transfer function of transmitting filter is $H_T(f)$. The signal is then passed through the channel having transfer function $H_c(f)$.

PAM:- It is a signal modulation where the information is encoded in the amplitude of a series of signal pulses

The channel delivers the signal to the receiv filter. It consists of all the necessary recei filter circuits and systems. The combined t function of receiving filter is $H_R(f)$. The outp of receiving filter is $y(t)$. This $y(t)$ is noisy replica of the transmitted signal $x(t)$.

→ The signal $y(t)$ is sampled synchronously w the transmitter. The sampling instants are t_i= These sampling instants are synchronous to th clock pulses at the transmitter.

The sampled signal $y(t_i)$ is then given to the decision device.

→ The decision device compares the i/p signal wit threshold 'λ'. Then the decision is take as follows:

If $y(t_i) \geq \lambda$ select symbol '1'
$y(t_i) < \lambda$ select symbol '0'.

ISI problem:

The receiver filter output is written as:

$$y(t) = \mu \sum_{k=-\infty}^{\infty} A_k p(t - kT_b) + n(t)$$

'μ' is the scaling factor.
$p(t)$ is the shape of pulse different from $g(t)$
$n(t)$ is the additive white Gaus

In figure, observe that $A_k g(t)$ is the signal applied to the input of cascade connection of transmitting filter, channel and receiving filter. The output of this cascade connection is $\mu A_k p(t)$.

$$\therefore \mu A_k p(t) = A_k g(t) * h(t) \quad \text{(*)}$$

Convolution in time domain is transformed into multiplication in the frequency domain.

$$\mu A_k p(f) = A_k G(f) H(f)$$

where, $H(f)$ is the transfer function of overall system,

$$\therefore H(f) = H_T(f) H_c(f) H_R(f)$$

$$\therefore \mu A_k p(f) = A_k G(f) H_T(f) H_c(f) H_R(f)$$

The receiving filter o/p $y(t)$ is sampled at $t_i = i T_b$, then

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} A_k p[t_i - k T_b] + n(t_i)$$

$$= \mu \sum_{k=-\infty}^{\infty} A_k p[i T_b - k T_b] + n(t_i)$$

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} A_k p[(i-k) T_b] + n(t_i)$$

If $i = k$,

$$y(t_i) = \mu A_i p(0) + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} A_k p[(i-k) T_b] + n(t_i)$$

$p(0)$ is normalized such that, $p(0) = 1$

$$\therefore \boxed{y(t_i) = \mu A_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} A_k p[(i-k)T_b] + n(t_i)} \quad i=0$$

Comments :-

→ The first term in above equation is μA_i, it the contribution of ith transmitted bit.

→ The second term represents the residual eff of all other bits transmitted before and afte the sampling instant "t_i".

→ In the absence of channel noise and ISI, second term will not be present in above e

$$\therefore \boxed{y(t_i) = \mu A_i}$$

→ observe that it is decoded correctly in the absence of ISI and channel noise.

Remedy :-

→ The ISI can be reduced by proper desig of pulse spectrum $G(f)$, transmit filter receive filter $H_R(f)$ and channel $H_c(f)$.

→ It has proved that the function which pro a zero ISI is a "sinc function". Hence, instead of a rectangular pulse if we too a sinc pulse then ISI can be reduced zero. This is known as Nyquist

SI Definition :-

: The overlapping of adjacent symbols with the required symbol is known as Intersymbol Interference.

(OR)

ISI can be defined as the overlapping of the tails of other pulses over a particular pulse.

Nyquist Criterion for Distortionless Baseband Binary Transmission (OR) Optimum Transmission

(OR)

Nyquist pulse Shape criterion : Nyquist Criterion (1) it requires sampling freq. be at least twice the highest freq. Contains inasig. The problem in the baseband binary ransmission is to determine the frequency response f transmft, channel and receive filter so as to reconstruct the original binary sequence $\{b_k\}$.

The receiver does this by extracting and then decoding the corresponding coefficients of data sequence $\{a_k\}$. $\{b_k\}$

Extracting :- Extracting involves sampling the output $y(t)$ at time $t_i = iT_b$.

$$\boxed{y(t_i) = \mu A_i}$$

ecoding :- It requires that the weighted pulse contribution $A_k p[(i-k)T_b]$ for $i=k$ be free from ISI.

Nyquist Pulse Shape Criterion means when Satisfy the Communication Channel (including tx and Rx) then result no intersymbol interference. Here timedomain is equal to freq.domain

Pulse Shape Used in a Communicatication.
Extracting means remove the effort) or force
Decoding means Converting Code into Plain text or any format.

This is possible if the received pulse is controlled such that

$$p[(i-k)T_b] = \begin{cases} 1, & \text{for } i = k \\ 0, & \text{for } i \neq k \end{cases} \rightarrow (1)$$

If $p(t)$ satisfies the above condition, receiver output $y(t_i)$ in the absence of c noise is $\boxed{y(t_i) = \mu A_i}$.

which implies zero ISI. Here, the two condit eq(1) ensure perfect reception in the absence

From a design point of view, it is informa transform the equation (1) into frequency dom

Frequency Domain

Consider the sequence of samples $p[nT_b]$, $n = 0, \pm 1, \pm 2, \ldots$ the samples are taken at th interval T_b.

Sampling in time domain produces periodicity frequency domain.

$$\boxed{P_\delta(f) = f_b \sum_{n=-\infty}^{\infty} P(f - n f_b)} \rightarrow (2)$$

$P_\delta(f)$ represents the Spectrum of $p[nT_b]$
$P(f)$ represents the Spectrum of $p(t)$

In the equation (2), (3) $P_\delta(f)$ is the fourier transform of an infinite periodic sequence of delta function of period T_b, which are weighted with (12) amplitudes of $p(t)$

$$\therefore \boxed{P_\delta(t) = \sum_{m=-\infty}^{\infty} p(mT_b)\,\delta(t-mT_b)} \rightarrow (3)$$

$$\therefore P_\delta(f) = \int_{-\infty}^{\infty} \left[\sum_{m=-\infty}^{\infty} p(mT_b)\,\delta(t-mT_b)\right] e^{-j2\pi ft}\,dt$$

$$= \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} p[(i-k)T_b]\,\delta(t-(i-k)T_b)\, e^{-j2\pi ft}\,dt \quad \text{for } m=i-k$$

From Nyquist criterion.

$$P_\delta(f) = \begin{cases} \int_{-\infty}^{\infty} p(0)\,\delta(t)\, e^{-j2\pi ft}\,dt & \text{for } i=k \\ 0 & \text{for } i \neq k \end{cases}$$

$$\therefore P_\delta(f) = \int_{-\infty}^{\infty} p(0)\,\delta(t)\, e^{-j2\pi ft}\,dt$$

for normalized condition, $p(0)=1$.

$$\therefore P_\delta(f) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j2\pi ft}\,dt$$

$$\therefore \boxed{P_\delta(f) = 1}$$

$$\therefore \text{eq}(2) \Rightarrow f_b \sum_{n=-\infty}^{\infty} p(f-nf_b) = 1$$

$$\therefore \boxed{\sum_{n=-\infty}^{\infty} P(f - nf_b) = T_b}$$

→ The above expression is the Nyquist criterion
distortionless baseband binary transmission in
absence of noise.

→ The frequency function $P(f)$ eliminates ISI
samples taken at T_b, $P(f)$ refers to overall
incorporating the transmit filter, channel and
receive filter.
//

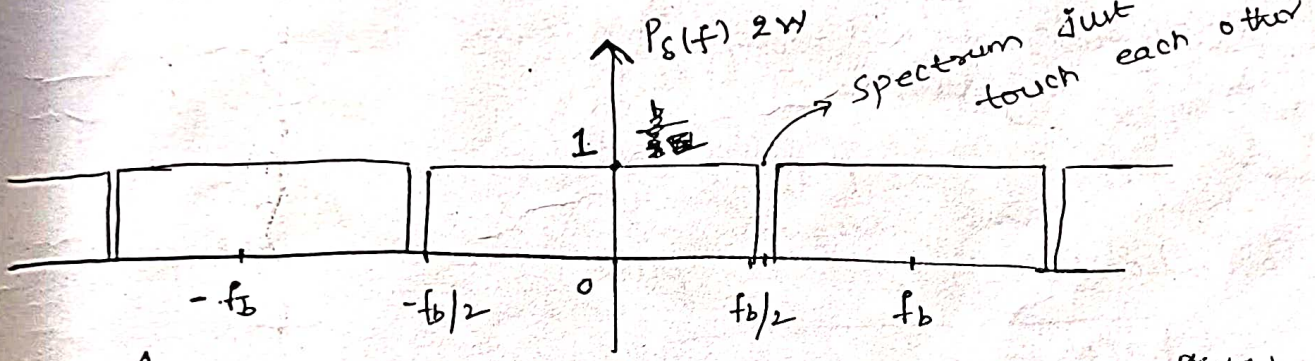
eal Nyquist Channel:

Consider.

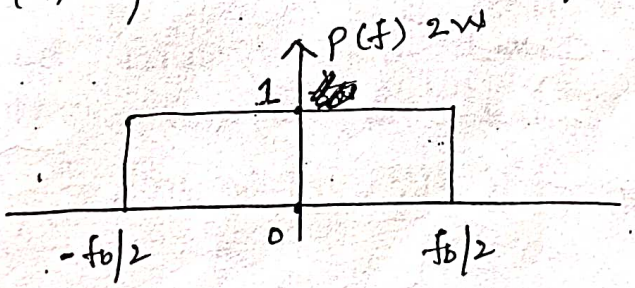
$$\sum_{n=-\infty}^{\infty} P(f - n f_b) = T_b$$

he simplest form of satisfying the above equation is to pecify the frequency function $P(f)$ to be in the form of a rectangular function as

$$P(f) = \frac{1}{f_b} \text{rect}\left(\frac{f}{f_b}\right)$$



(a) Spectrum of a sampled signal



(b) Spectrum of $p(t)$

Where rect(f) stands for a rectangular function of unit amplitude, and the overall system bandwidth "W" is defined by

$$W = \frac{f_b}{2} = \frac{1}{2T_b}$$

$$\therefore \quad P(f) = \begin{cases} \frac{1}{2W}, & -W < f < W \\ 0, & |f| > W \end{cases}$$

$$\therefore \quad \boxed{P(f) = \frac{1}{2W} \, rect\left(\frac{f}{2W}\right)}$$

To obtain $p(t)$ from $P(f)$; Inverse fourier transfo[illegible] of above ~~equation~~ function can be obtained from standard fourier transform pair as

$$sinc(f_b t) \underset{I.F.T}{\overset{F.T}{\longleftrightarrow}} \frac{1}{f_b} rect\left(\frac{f}{f_b}\right)$$

$$\therefore \quad p(t) = sinc(f_b t)$$

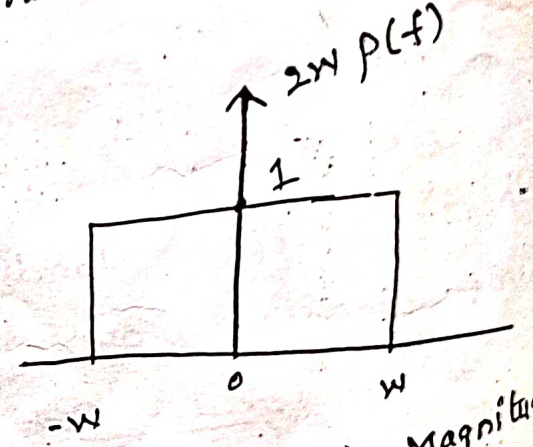
The bandwidth of the pulse "W" $= \frac{f_b}{2}$, then

$$\boxed{p(t) = sinc(2W t)}$$

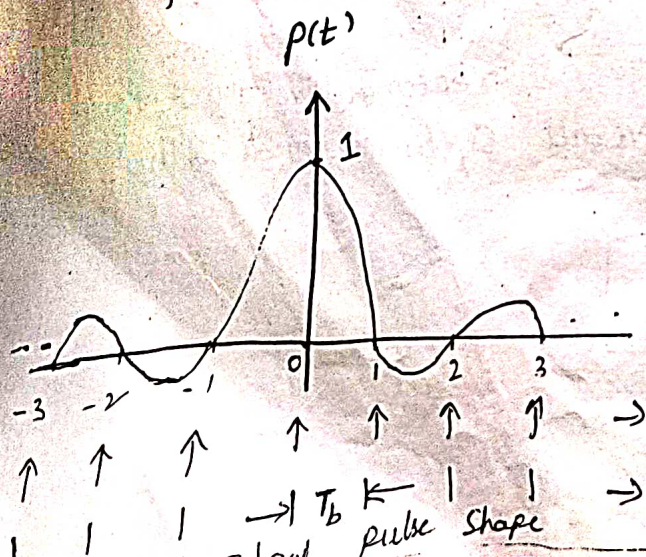
Here, "W" is called the Nyquist bandwidth.

"$f_b = 2W$" is called the Nyquist rate

→ Nyquist Bandwidth is the minimum transmission [illegible] for zero ISI.

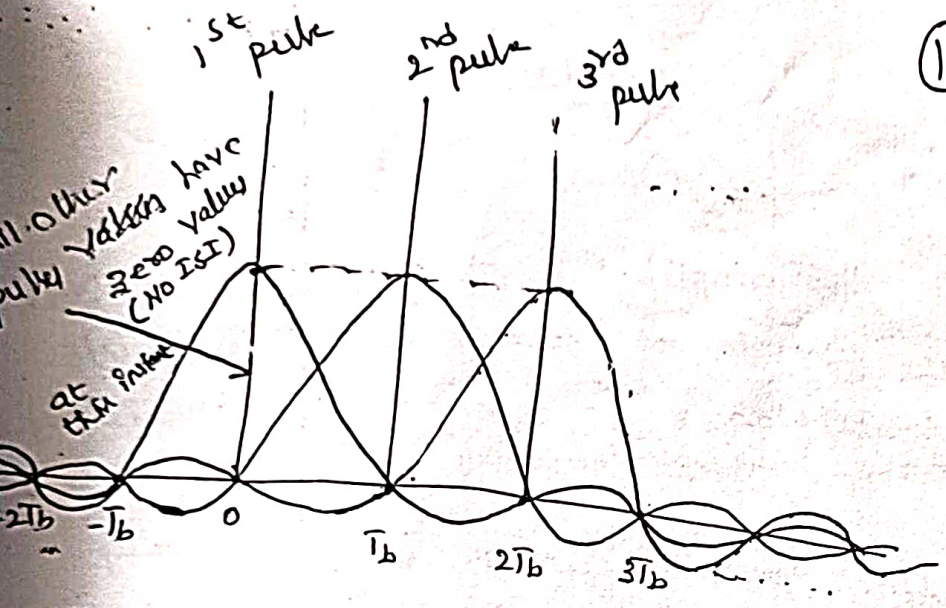


(a) Ideal Magnitu[illegible] Respon[illegible]



→ Sampling instants

→ Sampling intervals



$p(t-nT_b)$

II

(2)

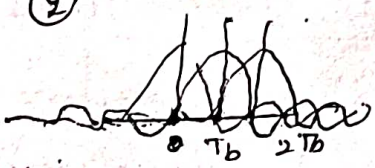


fig:- Sinc pulses for baseband transmission produces zero ISI

The above figure shows the three sinc pulses transmitted successfully. One sinc pulse is transmitted for one bit. Hence the sinc pulses are transmitted at $t=0, \pm T_b, \pm 2T_b, \ldots$

In the above figure, observe that at $t=0$, all the sinc pulses have zero value except the transmitted pulse. Hence ISI will be zero.

In general we can say that ISI will be zero if we sample the pulses at $t=0, T_b, 2T_b, \ldots$

Ideal sinc pulse is not possible:

Just now we have seen that sinc pulse is ideal for zero ISI in baseband transmission. Let us see whether transmission of exact sinc pulse is physically possible.

We know that. $p(t) = \text{sinc}(2Wt)$

Possible Difficulties :-

(1) It is necessary that the amplitude characteristics of should be flat from $-W$ to W and zero outside t band. But, abrupt transition at $\pm W$ is not physi realizable.

2) Hence the ideal sinc pulse cannot be physically generated, even though it eliminates ISI Complet
→ Due to discontinuity of $P(f)$ at $\pm W$, there is practically no margin of er in sampling times of the re end.

Raised cosine channel :-

→ Due to these problems, it is practically not possible to use the sinc funct as it is.

→ But, the practical difficulties can be over-come b increasing the bandwidth from "W" to an adjustab value between W and 2W.

→ The raised cosine spectrum is given as follows:

$$P(f) = \begin{cases} \frac{1}{2W} & \text{for } 0 \le |f| < f_1 \\ \frac{1}{4W}\left\{1 + \cos\left[\frac{\pi(|f| - W)}{2W - 2f_1}\right]\right\} & \text{for } f_1 \le |f| \le 2 \\ 0 & |f| \ge 2W - f_1 \end{cases}$$

$\frac{\pi(|f| - W)}{2W - 2f_1}$

→ In the above equation, the middle term represent gradual roll off from f_1 to $2W - f_1$. The freque f_1 and nyquist bandwidth W are related by t roll off factor i.e.,

$$\alpha = 1 - \frac{f_1}{W}$$

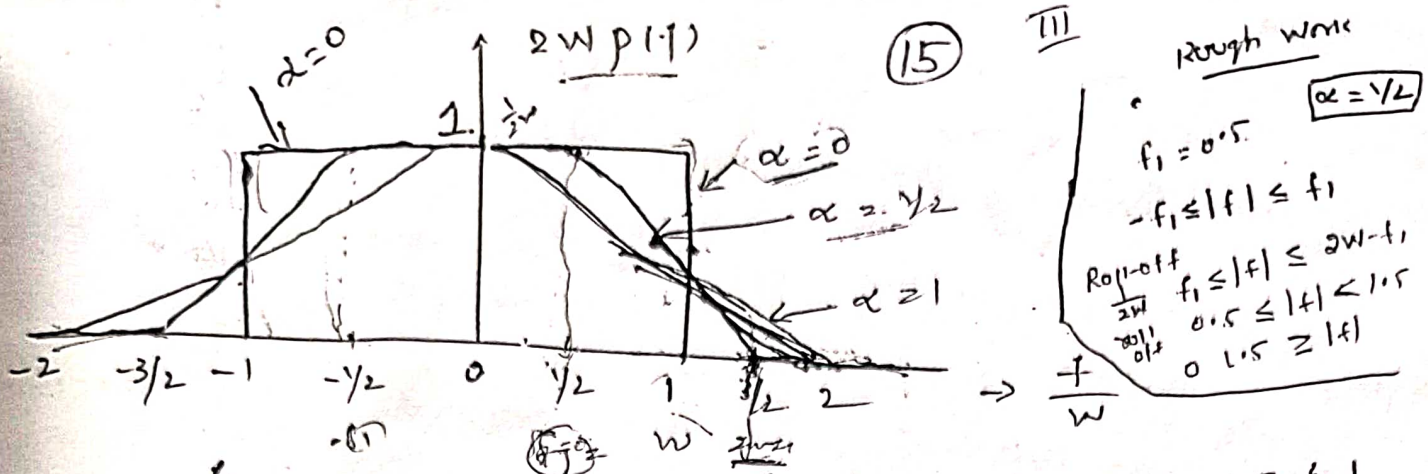


fig:- Raised cosine Spectrum for $\alpha = 0, 0.5$ & 1

Frequency Response

→ Time domain for raised cosine Spectrum is

$$p(t) = \text{sinc}(2Wt) \frac{\cos(2\pi\alpha Wt)}{1 - 16\alpha^2 W^2 t^2}$$



Comments :-

→ (1) The above pulse function is the product of two functions. The first, i.e., sinc(2Wt) is the sinc pulse. The second factor decreases as $\frac{1}{|t|^2}$ for large 't'.

Because of the second factor, the abrupt transitions in the frequency domain are avoided.

The first factor, i.e; sinc function makes p(t) zero at the sampling instants of $t = 0, \pm T_b, \pm 2T_b, \ldots$

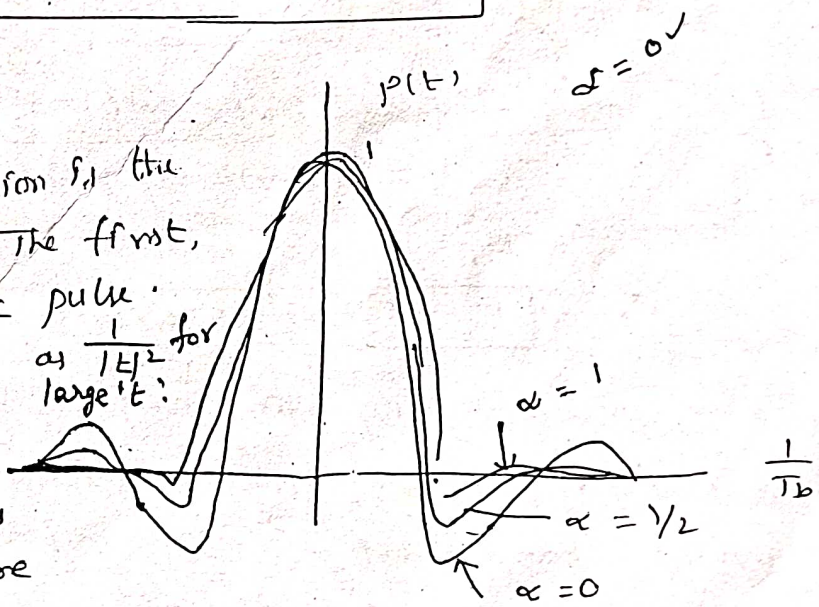


fig :- Time response

(2) Figure shows the pulse p(t) for $\alpha = 0$, 0.5 and 1. This pulse is normalized on both the scales. Observe that at $\alpha = 0$, it becomes sinc pulse.

Bandwidth Required :-

The Raised cosine spectrum has non-zero values upto $2W - f_1$.

$\therefore$ Bandwidth required for raised cosine spectrum will be $\boxed{B_T = 2W - f_1}$

$\boxed{\alpha = 1 - \frac{f_1}{W}} \Rightarrow \alpha = \frac{W - f_1}{W}$

$\Rightarrow W\alpha = W - f_1$

$f_1 = W - W\alpha$

$\underline{f_1 = W(1-\alpha)}$

$f_1 \rightarrow$ Nyquist bandwi

$W \rightarrow$ utilized bandwi

$\therefore B_T = \underline{2W - W(1-\alpha)}$

$= 2W - W + \alpha W$

$B_T = W + \alpha W$

$\therefore \boxed{B_T = W(1+\alpha)}$ $\alpha \leq 2W$

$1.5W$

$\alpha = 1$

$B_T = 2W$

$\alpha = 0$

$B_T = W$

For $\alpha = 0$, $B_T = W$, this is the case of Ideal

→ As 'α' increases, the bandwidth "W" increases abo Nyquist bandwidth. $W \quad 2W$

→ When $\alpha = 1$, $B_T = 2W$.

i.e., the bandwidth required by the raised cosine Spectrum is double of the Nyquist bandwidth.

Correlative-Level coding :- This is the digital communication often adopt so called correlative level coding. This is mainly used for we observed that raised cosine spectrum control ISI. eliminates ISI at the cost of increased bandwidth. The ideal baseband transmission signaling rate (bit rate) is 2W. But such system is not physically realizable.

Correlative level coding allows the signaling rate of 2W in the channel of bandwidth W. This is made physically possible by allowing ISI in the transmitted signal in a controlled manner. This ISI is known to the receiver. Hence effects of ISI are eliminated at the receiver. This scheme is also known as Partial-response signaling.

The correlative level coding is implemented by Duo-binary signaling and Modified-Duo-binary signaling.

Duo-binary Signaling scheme (Class-I partial Response) :-

The word "duo" means to double the transmission capacity of the binary system.

Consider a binary i/p sequence $\{b_k\}$ consisting of uncorrelated binary symbols 1 and 0 each having a bit duration of T_b. The sequence is applied to PAM producing a two-level sequence of polar NRZ, whose amplitude is defined as,

$$a_k = \begin{cases} +1 & \text{if } b_k = 1 \\ -1 & \text{if } b_k = 0 \end{cases}$$

When this sequence is applied to a duo-binary encoder as shown in below fig, it is converted into a three level output, namely, -2, 0 and

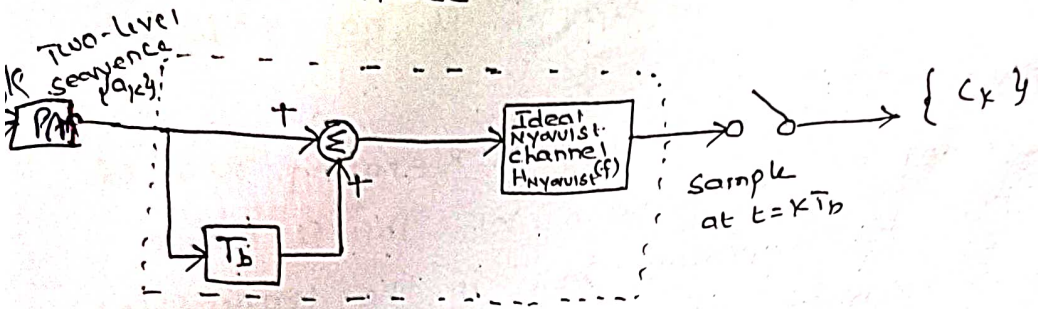


fig :- Duo-binary Encoder

→ The output of duo-binary encoder can be exp as

$$c_k = a_k + a_{k-1}$$

→ Because of the duo-binary signaling scheme the uncorrelated sequence $\{a_k\}$ of two level pu is converted to correlated sequence of thre level pulses. This introduces ISI in the sign in an artificial (controlled) manner to redu bandwidth.

Frequency Response

→ In the above figure, there is a delay elem "T_b", the frequency response of delay element is $e^{-j\pi f T_b}$. Hence, frequency response of delay-line filter will be $1 + e^{-j\pi f T_b}$

→ The delay line filter is connected in cascade wit ideal Nyquist channel, hence overall frequency response of this scheme will be

$$H_I(f) = H_{Nyquist}(f)\left[1 + e^{-j\pi f T_b}\right].$$

$$H_I(f) = H_{Nyquist}(f)\left[e^{-j\pi f T_b} e^{j\pi f T_b} + e^{-j\pi f T_b} e^{-j\pi f T_b}\right]$$

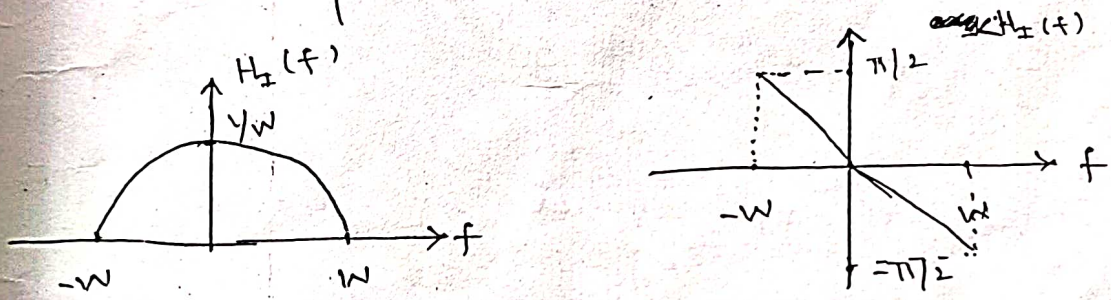
$$= H_{Nyquist}(f)\left[e^{j\pi f T_b} + e^{-j\pi f T_b}\right] e^{-j\pi f T_b}$$

$$= H_{Nyquist}(f)\; 2\cos(\pi f T_b)\, e^{-j\pi f T_b}$$

We know that,

$$H_{Nyquist}(f) = \begin{cases} \frac{1}{2W} & \text{for } -W < f < W \\ 0 & \text{for } |f| > W \end{cases}$$

$$\therefore H_I(f) = \begin{cases} \frac{1}{W}\cos(\pi f T_b)\, e^{-j\pi f T_b} & \text{for } -W < f < W \\ 0 & \text{for } |f| > W \end{cases}$$



(a) Magnitude Response (b) phase Response

fig :- Frequency Response

Reconstruction :- Let $\hat{a}_k$ represents the estimate of a_k. Then we obtain $\hat{a}_k$ as, $c_k = \hat{a}_k + \hat{a}_{k-1}$

$$\therefore \boxed{\hat{a}_k = c_k - \hat{a}_{k-1}}$$

This shows that if c_k is received with error, then $\hat{a}_k$ will have error. This error will propagate in the output sequence.

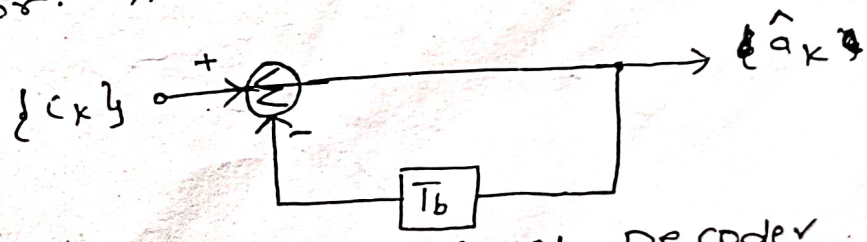


fig:- Duo-binary Decoder

Example :- Consider a binary data stream 001101001 and determine the o/p of duo-bi encoder and decoder.

	K=-1	K=0	K=1	K=2	K=3	K=4	K=5	K=6	K=7	K=8
$\{b_k\}$		0	0	1	1	0	1	0	0	1
a_k	1	-1	-1	+1	+1	-1	+1	-1	-1	+1
a_{k-1}		1	-1	-1	+1	+1	-1	+1	-1	-1
c_k		0	-2	0	+2	0	0	0	-2	0
$\hat{a}_{k-1}$		1	-1	-1	+1	+1	-1	+1	-1	-1
$\hat{a}_k$	-1	-1	+1	+1	-1	+1	-1	-1	+1	
$\{b_k\}$	0	0	1	1	0	1	0	0	1	

Hence, the input binary sequence and output binary sequence are same. If the estimation is exact, otherwise error takes place and this error will propogate in the output sequence.

inary Encoder With precoder (Differential Encoder)

recoder is used in duo-binary encoder to d error propogation. The below figure shows block diagram of duo-binary encoder with coder. Precoder Error Propagation (or Propagation of Uncertainity) is used for to measurement of errors.

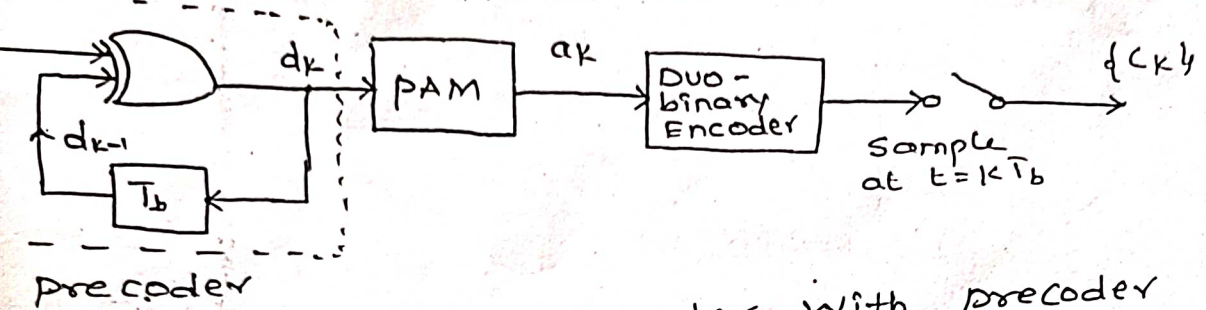


fig:- Duo-binary Encoder with precoder

From the above figure, the output of the de precoder is $\boxed{d_k = b_k \oplus d_{k-1}}$

$$d_k = \begin{cases} 1 & \text{if either } d_{k-1} \text{ or } b_k \text{ is 1 (not both)} \\ 0 & \text{otherwise.} \end{cases}$$

This is normal exclusive-or operation.

The sequence $\{d_k\}$ is applied to PAM. Then the output sequence of PAM with polar NRZ signaling scheme will be

$$a_k = \begin{cases} +1 & \text{if } d_k = 1 \\ -1 & \text{if } d_k = 0 \end{cases}$$

The sequence $\{a_k\}$ is then applied to duo-binary encoder. Then the duo-binary encoder output is $\boxed{c_k = a_k + a_{k-1}}$.

Reconstruction (Decoder) :- The below figure shows the block diagram of a duo-binary decoder. Rectifier means it is a device converts an alternating current (a bi-directional) into a direct current (DC)

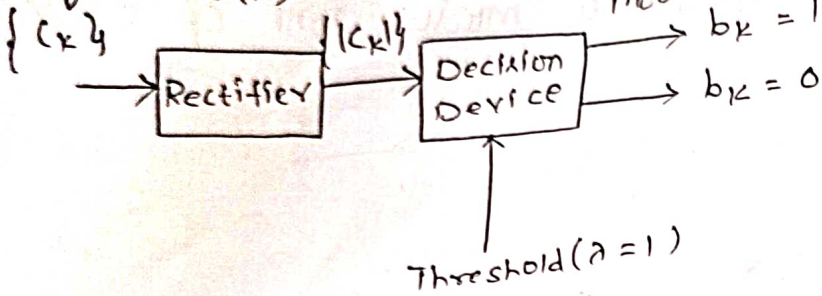


fig:- Detector of Duo-binary encoder with p

If $|C_k| < 1$ take $b_k = 1$

$|C_k| > 1$ take $b_k = 0$

→ Here note that, the output "b_k" depends up the present value of 'C_k'. This shows that th will be no propagation of error in this system

Example: (Precoded Duo-binary encoder & Decoder)

	K=-1	K=0	K=1	K=2	K=3	K=4	K=5	K=6	K=7
b_k		0	1	0	1	1	1	0	1
d_k	1	1	0	0	1	0	1	1	0
d_{k-1}		1	1	0	0	0	0	1	1
a_k		+1	+1	-1	+1	-1	+1	+1	-1
a_{k-1}		+1	+1	-1	-1	+1	-1	+1	+1
C_k		+2	0	-2	0	0	0	+2	0
b_k		0	1	0	1	1	1	0	1

Note:- Therefore, error propagation will avoid b using precoded duo-binary scheme thus no erro

Modified Duo-binary Signaling scheme
(class - IV partial response)

→ In the duo-binary signaling scheme the freaquency response $H_I(f)$, of the transmitted pulse is non-zero at origin. This is considered to be an undesirable feature in some applications, since many communication channels cannot transmit a DC component. This deficiency can be correct by class IV partial response (or) Modified Duo-binary signaling scheme.

Encoder :-

→ The below figure shows the modified duo-binary signaling scheme

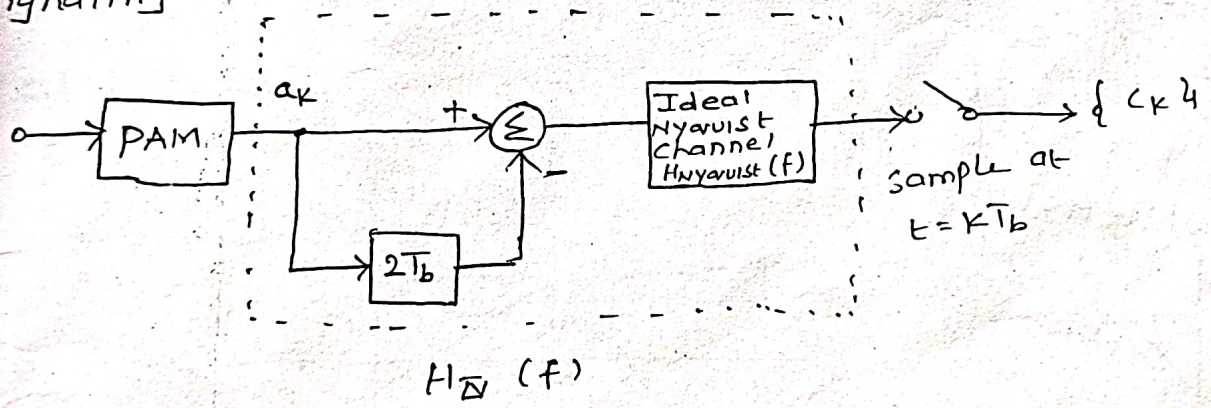


fig :- Modified Duo-binary Encoder

→ The output of the modified Duo-binary encoder is,

$$c_k = a_k - a_{k-2}$$

→ Hence, again a three-level seaquence is generated, with $a_k = \pm 1$, we find that c_k takes one of three values, namely, -2, 0 and $+2$.

→ The overall frequency response of $H_{IV}(f)$ is

$$H_N(f) = H_{Nyquist}(f)\left[1 - e^{-j4\pi fT_b}\right]$$

$$= H_{Nyquist}(f)\left[e^{-j2\pi fT_b} \cdot e^{j2\pi fT_b} - e^{-j2\pi fT_b} \cdot e^{-j2\pi fT_b}\right]$$

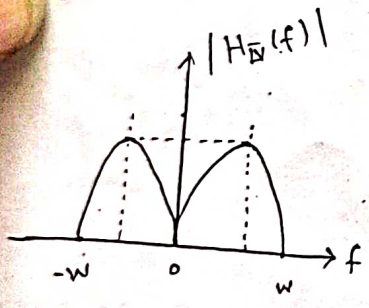
$$= H_{Nyquist}(f) \cdot \left[e^{+j2\pi fT_b} - e^{-j2\pi fT_b}\right] e^{-j2\pi fT_b}$$

$$= H_{Nyquist}(f)\ 2j\ \sin(2\pi fT_b)\ e^{-j2\pi fT_b}$$

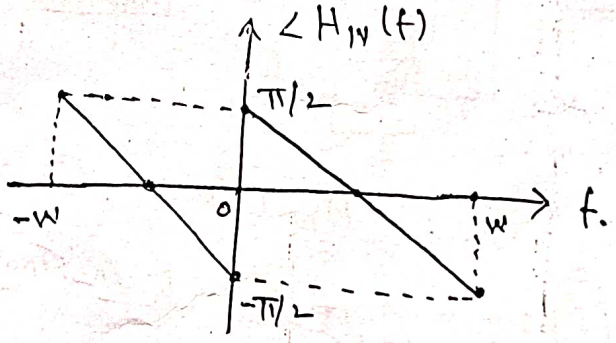
we know that

$$H_{Nyquist}(f) = \begin{cases} \frac{1}{2W} & \text{for } -W < f < W \\ 0 & \text{for } |f| > W \end{cases}$$

$$\therefore H_{IV}(f) = \begin{cases} \frac{j}{W} \sin(2\pi fT_b)\, e^{-j2\pi fT_b} & \text{for } -W < f \\ 0 & \text{for } |f| > \end{cases}$$



(a) Magnitude Response



(b) phase Response

fig:- Frequency Response

Note:- A useful feature of modified duo-binary Coder is the fact that its output has no DC component at origin.

construction :- Let $\hat{a}_k$ represents the estimate of a_k. Then we can obtain

$$c_k = \hat{a}_k - \hat{a}_{k-2}$$

$$\therefore \boxed{\hat{a}_k = c_k + \hat{a}_{k-2}}$$

The below figure shows that the modified duo binary decoder

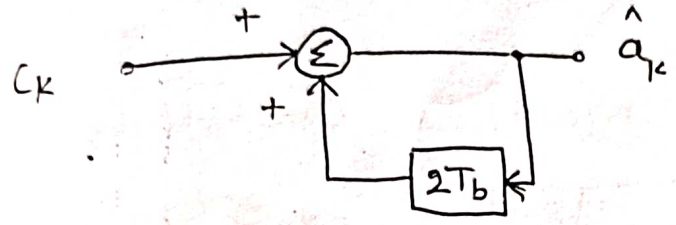


fig :- Decoder of Modified Duo-binary Encoder

Modified Duo-binary Encoder with precoder :-

→ To eliminate the possibility of error propagation in the modified duo-binary system, we use a precoding procedure as shown below

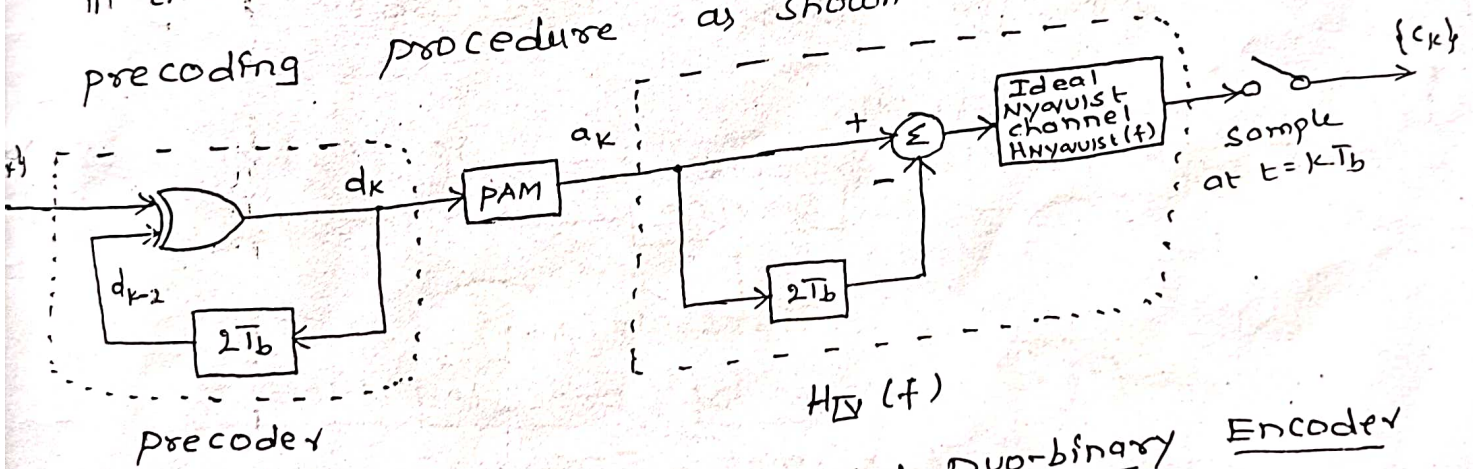


fig :- Precoded Modified Duo-binary Encoder

→ The output of precoder in the figure is

$$d_k = b_k \oplus d_{k-2}$$

→ The output of precoder is passed to PAM, then

$$a_k = \begin{cases} +1 & \text{if } d_k = 1 \\ -1 & \text{if } d_k = 0 \end{cases}$$

→ output of precoded Modified Duo-binary encoder is

$$c_k = a_k - a_{k-2}$$

The sequence will be correlated with the levels -2, 0 and $+2$.

Detector (Decoder or Receiver or Reconstruction) :-

The below figure shows the detector of precoded Modified Duo-binary encoder.

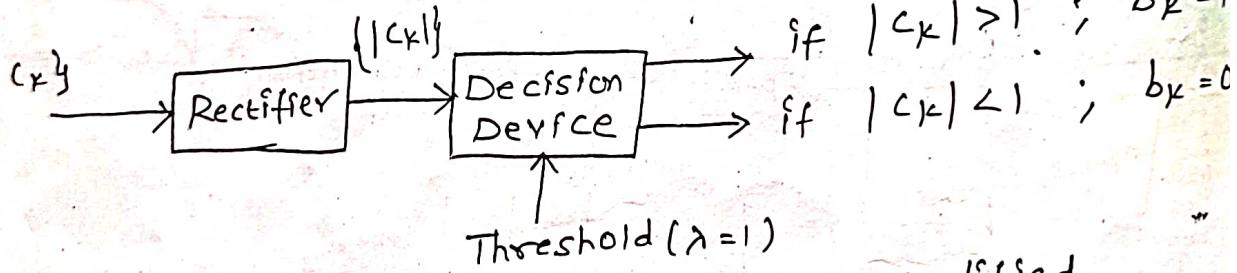


fig:- Dector of precoded Modified Duo-binary Encoder

If $|c_k| > 1$ take $b_k = 1$

$|c_k| < 1$ take $b_k = 0$.

→ The output sequence depends only on the present value of "c_k", hence no estimation required.

Example [Modified Duo-binary Encoder & Decoder with precoder]

		0	0	1	1	0	1	0	0	1
+1	+1	1	1	0	0	0	1	0	1	1
		+1	+1	1	1	0	0	0	1	0
+1	+1	+1	+1	−1	−1	−1	+1	−1	+1	+1
		+1	+1	+1	+1	−1	−1	−1	+1	−1
		0	0	−2	−2	0	+2	0	0	+2
		0	0	1	1	0	1	0	0	1

→ From the above example, it is clear that output sequence $\{b_k\}$ depends only on the present value of $|c_k|$.

Generalized Form of Correlative-Level coding
OR
Partial-Response Signaling

The duo-binary & Modified duo-binary techniques have correlation span of 1 binary digit and 2 binary digits respectively. It is straight-forward matter to generalize these two techniques to other scheme, which are known collectively as correlative-level coding (or) partial-Response Signaling schemes.

This generalization is shown in below figure. It involves the use of tapped-delay line filter with tap-weights : $W_0, W_1, W_2, \ldots$

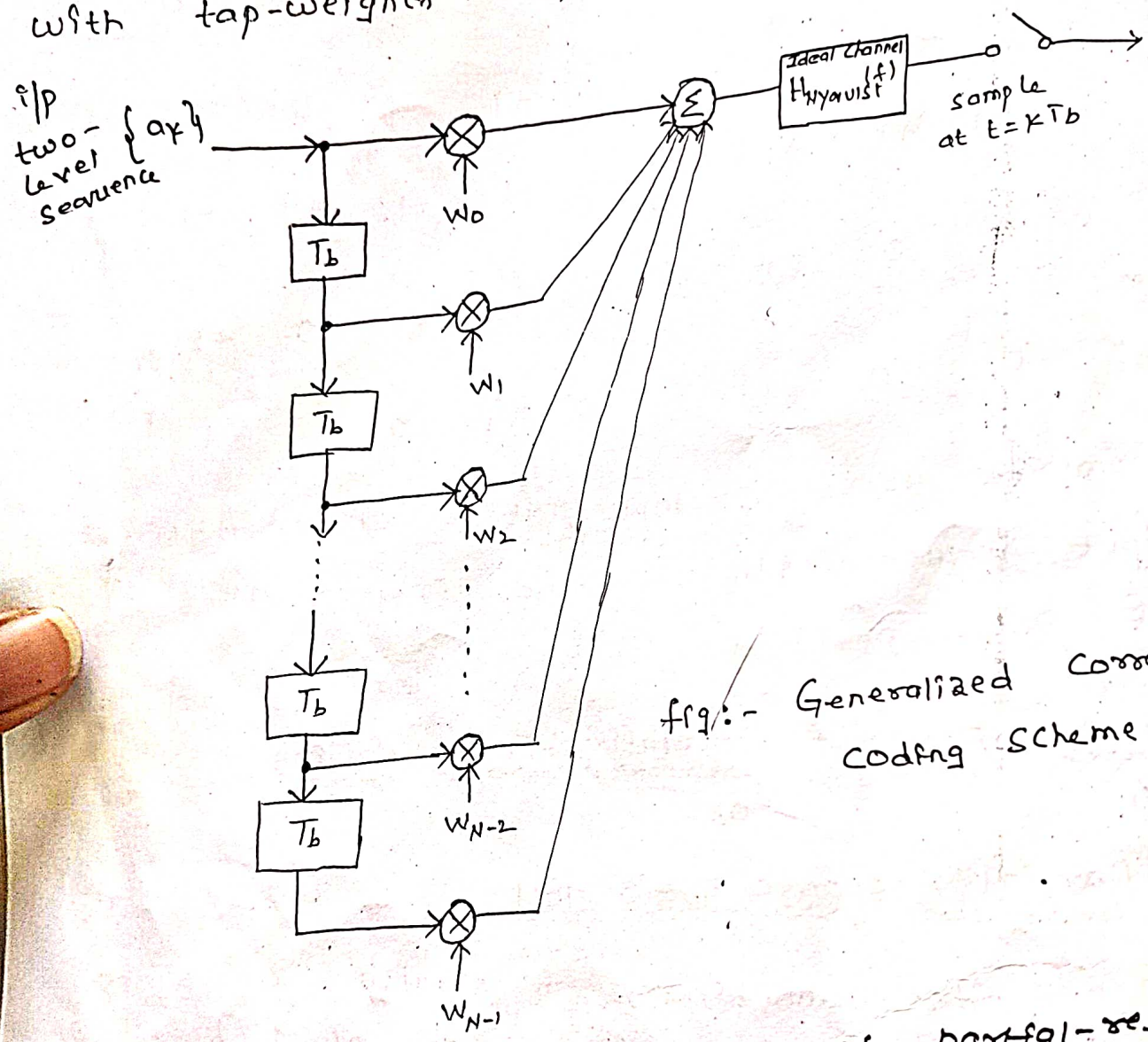


fig:- Generalized correlative coding scheme

→ Specifically, different classes of partial-response signaling schemes may be achieved by using a weighted linear combination of N ideal Nyquist (sinc) pulses, as shown by

$$h(t) = \sum_{n=0}^{N-1} W_n \operatorname{sinc}\left[\frac{t}{T_b} - n\right]$$

below table presents the appropriate choice
ge weights
top

pe of class	N	W_0	W_1	W_2	W_3	W_4	Comments
I	2	1	1				Duo-binary coding
II	3	1	2	1			
III	3	2	1	-1			
IV	3	1	0	-1			Modified Duo-binary
V	5	-1	0	2	0	1	

from the above table.

or Duo-binary coder,

$$W_0 = +1$$
$$W_1 = +1 \quad \text{and}$$
$$W_n = 0 \quad \text{for } n \geq 2.$$

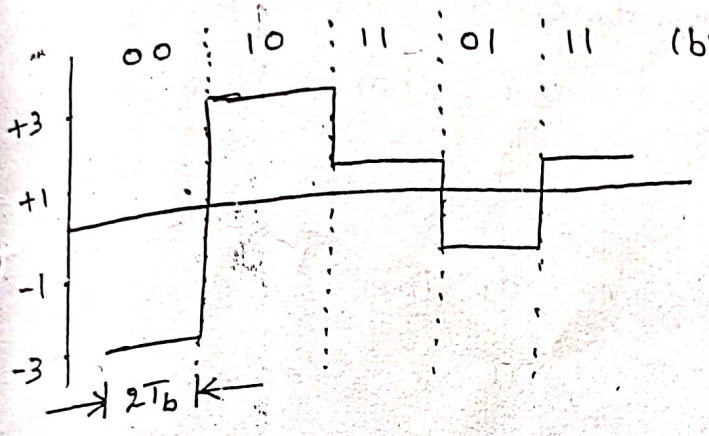
For Modified Duo-binary Coder,

$$W_0 = +1$$
$$W_1 = 0$$
$$W_2 = -1 \quad \text{and}$$
$$W_n = 0 \quad \text{for } n \geq 3.$$

Useful characteristics of partial-Response signaling may be:-
Binary data transmission accomplished at a rate
closer to Nyquist rate.
Different spectral shapes can be produced, appropriate
for the application at hand.

Baseband M-ary PAM Transmission :-

In the baseband binary transmission, the PAM produces binary pulses, that is, pulses with one of two amplitude levels. On the otherhand, in a baseband M-ary PAM system, the PAM produce one of M possible amplitude levels with $M > 2$.



(a) Waveform

Dibit	Amplitude
00	-3
01	-1
11	+1
10	+3

(b) Representation of four (4) possible dibits, based on Gray encoding

→ In the case of a quaternary ($M = 4$) system, the binary data sequence is 00 10 11 01. The fig (a) is based on the electrical representation for each of four possible dibits (pair of bits) given in fig(b).

→ We can also observed that in the case of quaternary PAM system. for example, the four possible symbols may be identified with the dibits, 00, 01, 10 & 11. We thus see that each symbol represent 2 bits of information, and 1 baud is equal to 2 bits/sec.

→ In an M-ary PAM system, 1 baud is equal to $\log_2 M$ bits/sec, and the symbol duration 'T' of the M-ary PAM system is

$$T = T_b \log_2 M \text{ sec}$$

EYE PATTERN :- An experimental tool for evaluating the combined effect of ISI and channel noise on overall system performance.

Def :- Eye pattern is defined as the synchronized superimpose of all possible realizations, viewed within a particular signaling interval.

Eye pattern is a pattern displayed on the screen of CRO. The shape of this pattern resembles with the shape of human eye. Therefore it is called as an eye pattern.

Eye pattern is obtained on the CRO by applying the received signal to vertical deflection plate (Y-pattern) of the CRO and a sawtooth wave at the horizontal deflection plates (X-pattern) at a rate ($\frac{1}{T_b}$), as shown below.

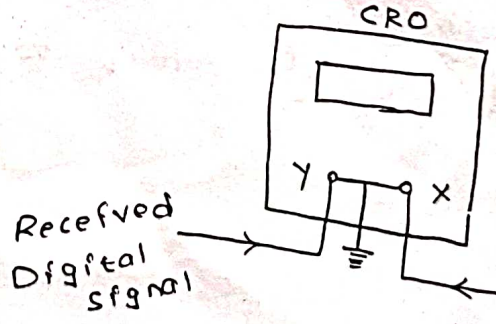
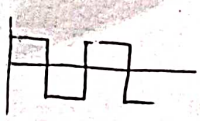


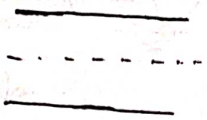
fig :- oscilloscope connection

sider a binary data as shown below and
ir pattern on CRO

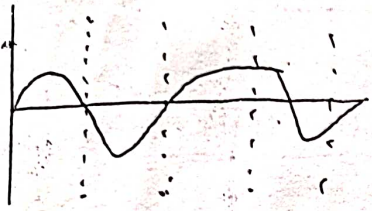
(24)

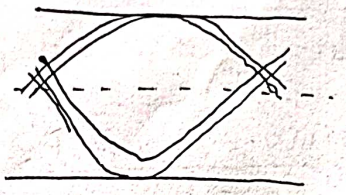


CRO

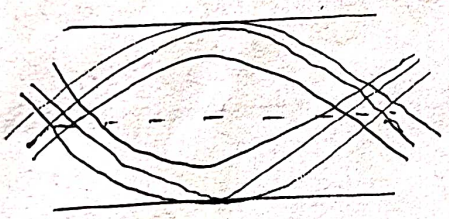


he received signal in the presence of ISI and
hannel noise is



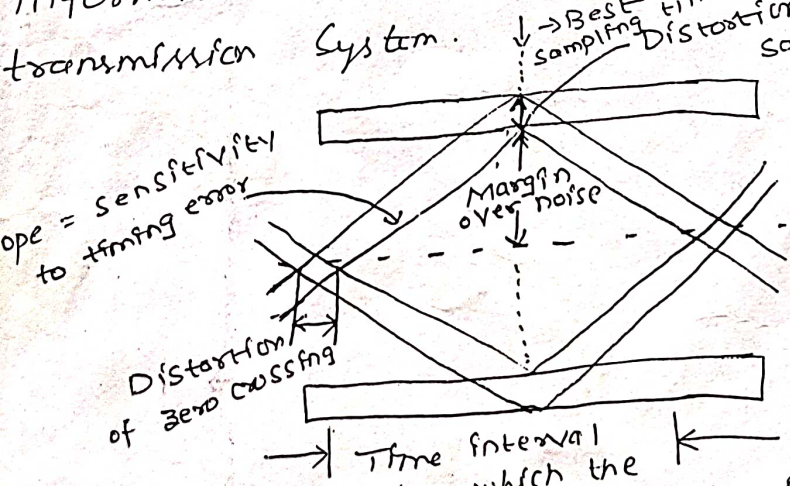


If more number of bits are transmitted, then



The interior region of the eye pattern is called an Eye Opening.

Eye pattern provides the great deal of useful information about the performance of a data transmission system.



Information Obtained from Eye pattern:-

→ The width of the eye opening defines the time interval over which the received wave can be sampled without an error due to ISI.

The best time for sampling is when the eye is open widest.

→ The sensitivity of the system to the timing error is determined by the rate of closure of the eye.

→ The height of eye opening at a specified sampling time defines the margin over noise.

→ When the effect of ISI is severe, the eye is completely closed and it is impossible to avoid errors due to the combined effect of ISI and noise in the system.

ISI

Nyquist

Matched

UNIT-III

QAM [Quadrature Amplitude Modulation]:-

- It is a method of combining two Amplitude modulation (AM) signals into single channel.
- By using this approach its double of effective B.W.
- QAM is also used with Pulse AM (PAM) in digital systems, like Wireless application.
- QAM is a family of analog mod. as well as digital mod.
- QAM is similar to DSBSC, generating two side Bands symmetric to the carrier freq.
- and it will send message sig to the same channel.
- The Ex. of QAM in analog modulation is PM [Phase Mod.]
- and in case of digital modulation is [PSK]
- Quadrature refers to the phase shift of 90 degrees
- It means four and is derived from 360 then degree/4 = 90 degr
- In this types of mod. both cosine and sine wave are used.

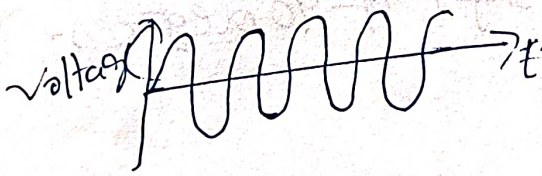
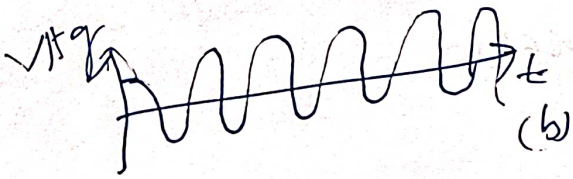


Fig (a):- Sine wave



(b) Cosine wave

- It can be used to vary the amplitude in analog modulation and two digital bit streams using ASK.

QAM Modulator:-

- Here we are consider two messg s/g over the same channel.
- The first mes. signal is sent by multiplying with cosine s/g

$m_1(t)$ = First message signal

$\cos\omega_c t$ = The Carrier s/g.

Product = $m_1(t)\cos\omega_c t$

Whly second mes s/g is sent in quadrature. The mes. s/g is multiplied $\sin\omega_c t$

$m_2(t)$ = 2nd mes. s/g $\sin\omega_c t$ = Second Carrier s/g

Product = $m_2(t)\sin\omega_c t$

- The Product of both s/g is added

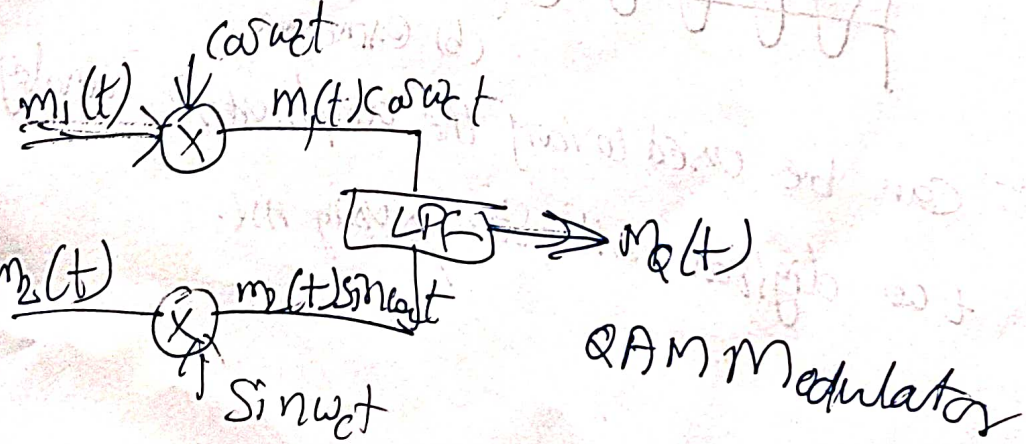
$$M_Q(t) = m_1(t)\cos\omega_c t + m_2\sin\omega_c t$$

- Under 90° degrees means that both Carriers s/g differ by a phase shift of 90 degrees.

$\cos(90-\omega_c t) = \sin\omega_c t$

(or)

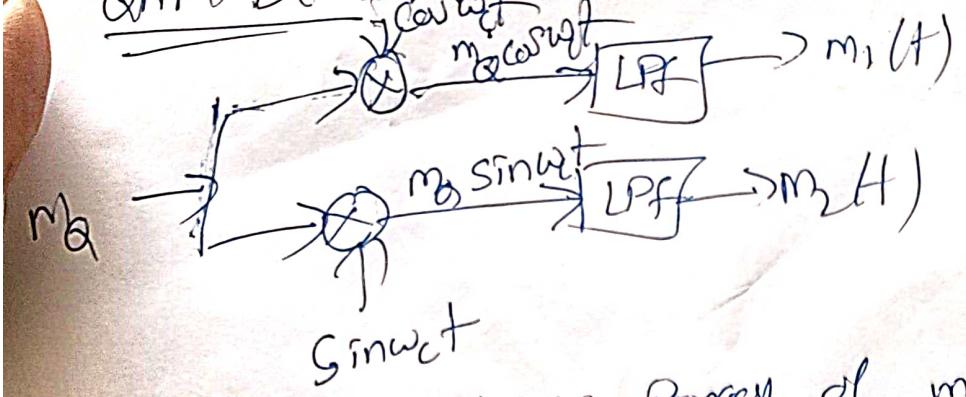
$\sin(90-\omega_c t) = \cos\omega_c t$



QAM Modulator

QAM Demod:

UNIT - III



- Demod. is a reverse Process of mod. It recovers original s/g from the Product to make it available to the Receiver.
- Demod. is based on Coherent detection method.
- The Carrier is multiplied with O/P of modulator and then applied to LPF

$$m_Q \cos\omega_c t = [m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t] \times \cos\omega_c t$$

$$m_Q \sin\omega_c t = [m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t] \times \sin\omega_c t$$

ML and MAP Decoding:-

ML Decoding: Maximum likelihood decoding is technique used to determine the most likely transmited message in a digit comm system, based on received s/g and statical models of noise and Interference.

- MAP Decoding:- To estimate Channel i/p that maximizes the Posterior Probability of Channel o/p

$$P_e = \sum_{i=1}^{M} P_i \left[P(\hat{m} \neq m_i | m_i) \right]$$

$m_i \rightarrow$ transmitted signal

$\hat{m} \rightarrow$ estimation of m_i

$P(\hat{m} \neq m_i | m_i)$ Conditional error probability given that i^{th} symbol was sent.

Geometric Representation of Signals :-

* The essence of geometric representation of signals is,

to represent any set of M energy signals $[s_i(t)]$ as linear combinations of N orthonormal basis functions where $N \leq M$.

* Let there may be M number of energy signals $s_1(t), s_2(t), \ldots s_M(t)$, each of duration T seconds, we write $[\phi_j(t) \Rightarrow j = 1 \text{ to } N]$ → set of orthonormal basis functions

$$s_i(t) = \sum_{j=1}^{N} s_{ij} \phi_j(t) \quad \begin{cases} 0 \leq t < T \\ i = 1, 2, \ldots M \end{cases} \rightarrow (1)$$

(s_{ij} : coefficients of $s_i(t)$)

Where the coefficients of the expansion are defined by

$$s_{ij} = \int_0^T s_i(t) \phi_j(t) \, dt, \quad \begin{cases} i = 1, 2, \ldots M \\ j = 1, 2, \ldots N \end{cases} \rightarrow (2)$$

The real-valued basis-functions $\phi_1(t), \phi_2(t), \ldots \phi_N(t)$ are orthonormal by which we mean

$$\int_0^T \phi_i(t)\,\phi_j(t)\,dt = \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

Where δ_{ij} = kronecker delta.

When $i = j$,

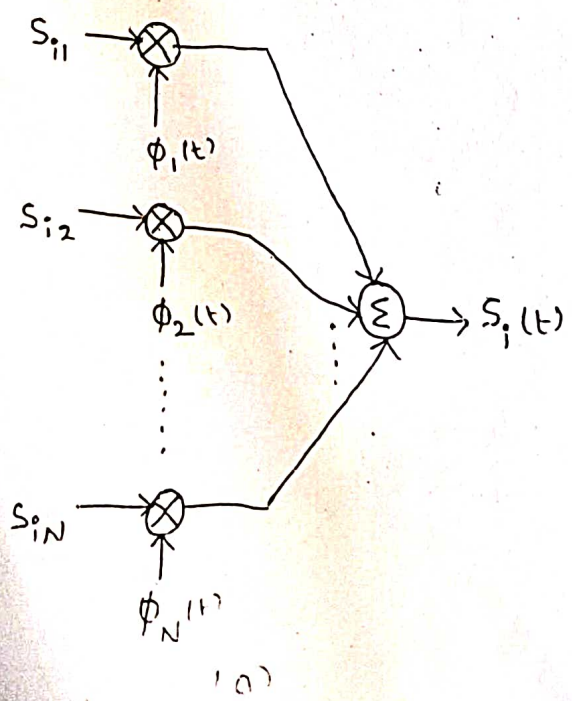
$$\int_0^T \phi_i(t)\,\phi_i(t)\cdot dt = \int_0^T \phi_i^2(t)\cdot dt = E_i$$

→ This condition states that each basis function is normalised to have unit energy.

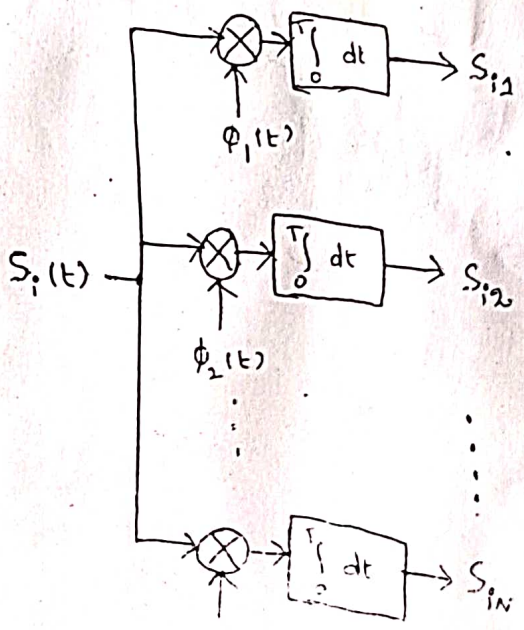
When $i \neq j$

$$\int_0^T \phi_i(t)\,\phi_j(t)\cdot dt = 0$$

→ This condition states that the basis functions $\phi_1(t), \phi_2(t), \ldots \phi_N(t)$ are orthogonal with respect to each other over the interval $0 \leq t < T$.



Synthesizer for generating signal $S_i(t)$.



Analyzer for generating the Vectors S_{ij}

M-ARY Quadrature Amplitude

Signal space :- N-diemensional Euclidean Space is called the signal space.

(3)

Example : Two-diemensional Signal Space with Three symbols

* Let us consider the vector Space representation of $M = 3$ message symbols with the help of $N = 2$ orthonormal basis functions. Then

$$S_i(t) = \sum_{j=1}^{N} S_{ij}\phi_j(t), \quad i = 1, 2, 3$$

$$S_i(t) = \sum_{j=1}^{2} S_{ij}\,\phi_j(t)$$

$$S_i(t) = S_{i1}\,\phi_1(t) + S_{i2}\,\phi_2(t)$$

$$S_1(t) = S_{11}\,\phi_1(t) + S_{12}\,\phi_2(t) \rightarrow S_1(t) = \begin{bmatrix} S_{11} \\ S_{12} \end{bmatrix}\begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix}$$

$$S_2(t) = S_{21}\,\phi_1(t) + S_{22}\,\phi_2(t)$$

$$S_3(t) = S_{31}\,\phi_1(t) + S_{32}\,\phi_2(t)$$

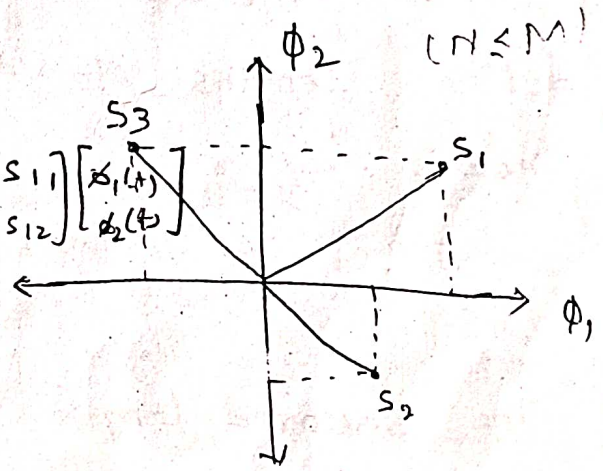


fig :- Geometric Representation of message vectors.

$$\vec{S}_1 = \begin{bmatrix} S_{11} \\ S_{12} \end{bmatrix} \quad \vec{S}_2 = \begin{bmatrix} S_{21} \\ S_{22} \end{bmatrix} \quad \& \quad \vec{S}_3 = \begin{bmatrix} S_{31} \\ S_{32} \end{bmatrix}$$

* The figure shows three message vectors S_1, S_2 and S_3. The position of these vectors depend upon the coefficients S_{11}, S_{12}, S_{21}, .. etc. Here ϕ_1 and ϕ_2 are perpendicular to each other, since they are orthogonal.

Then $\phi_1 - \phi_2$ signal space is called Euclidean Space. The figure shows two-dimensional Euclidean

Length or Absolute Value or Norm of a Vector :

Consider the vector S_i, which is completely determined by its coefficients. i.e.,

$$S_i = \begin{bmatrix} S_{i1} \\ S_{i2} \\ \vdots \\ S_{iN} \end{bmatrix} \quad i = 1, 2, \ldots M.$$

* In an N-dimensional Euclidean Space, we may define "Lengths of vectors" and "angles between vectors".
* Length of the vector S_i is denoted by the symbol $\|S_i\|$.
* The squared-length of any signal vector S_i is defined to be the inner product or dot product of S_i with itself,

$$\therefore \|S_i\|^2 = S_i^T S_i \quad \text{[Inner (or) Dot product]}$$

$$= \begin{bmatrix} S_{i1} & S_{i2} & S_{i3} & \cdots & S_{iN} \end{bmatrix} \begin{bmatrix} S_{i1} \\ S_{i2} \\ S_{i3} \\ \vdots \\ S_{iN} \end{bmatrix}$$

$$= S_{i1}^2 + S_{i2}^2 + S_{i3}^2 + \cdots + S_{iN}^2$$

$$\boxed{\|S_i\|^2 = \sum_{j=1}^{N} S_{ij}^2 \quad , \quad i = 1, 2, \ldots M}$$

Relation between Signal Energy and its Vector

* The energy of a signal $S_i(t)$ of duration T seconds is

$$\boxed{E_i = \int_0^T S_i^2(t)\, dt}$$

Synthesizer for ...

$$\int_0^T \left[\sum_{j=1}^{N} \left(\sum_{k=1}^{N} s_{ik}\, \psi_k(t) \right) \right] \cdot dt$$

Re-arranging the order of Summation & Integration

$$E_i = \sum_{j=1}^{N} \sum_{k=1}^{N} s_{ij}\, s_{ik} \int_0^T \phi_j(t)\, \phi_k(t) \cdot dt$$

We know that

$$\int_0^T \phi_j(t)\, \phi_k(t) \cdot dt = \begin{cases} 1 & \text{for } j = k \\ 0 & \text{for } j \neq k \end{cases}$$

with the above result,

$$E_i = \begin{cases} \sum_{j=1}^{N} \sum_{k=1}^{N} s_{ij}\, s_{ik} & \text{if } j = k \\ 0 & \text{if } j \neq k \end{cases}$$

$$\therefore \boxed{E_i = \sum_{j=1}^{N} s_{ij}^2} \quad \text{for } j = k$$

$$\therefore \boxed{E_i = \| s_i \|^2} \quad (\text{or}) \quad \boxed{E_i = s_i^T \cdot s_i}$$

* The signal energy is equal to Squared length of the signal vector.

Euclidean Distance :-

* The Euclidean distance between the two signal vectors s_i and s_k is

$$\boxed{\| s_i - s_k \|^2 = d_{ik}^2}$$

$$E_{ik} = \| S_i - S_k \|^2 = \sum_{j=1}^{N} (S_{ij} - S_{kj})^2$$

$$E_{ik} = \int_0^T [S_i(t) - S_k(t)]^2 \cdot dt$$

- The angle θ_{ik} between two vectors S_i & S_k is,

$$\cos \theta_{ik} = \frac{S_i^T S_k}{\| S_i \| \| S_k \|}$$

The "cosine" of the angle "θ_{ik}" is equal to the inner product of these two vectors divided by the product of their individual norms.

- The two vectors S_i and S_k are thus orthogonal or perpendicular to each other if their inner product $S_i^T S_k$ is zero. In which case $\boxed{\theta_{ik} = 90^\circ}$

14/09/15

413, 440, 409, 414, 419.

422, 425, 426, 427, 428,

429, 436, LE 402

Hemanth ...

Synthesizer for you.....

GRAM-SCHMIDT ORTHOGONALIZATION PROCEDURE :-

Purpose of this procedure:

* We know that any signal vector can be represented in terms of orthonormal basis functions $\phi_1(t), \phi_2(t), \ldots \phi_N(t)$.

* Gram-Schmidt Orthogonalization procedure is the tool to obtain the orthonormal basis functions "$\phi_j(t)$"

procedure :- Suppose we have a set of M energy signals denoted by $S_1(t), S_2(t), \ldots S_M(t)$.

Starting with $S_1(t)$ chosen from this set arbitrarily the first basis function is defined by

$$\phi_1(t) = \frac{S_1(t)}{\sqrt{E_1}} \rightarrow (1)$$

where E_1 is the energy of the signal $S_1(t)$.

We have,

$$S_1(t) = \sqrt{E_1}\ \phi_1(t)$$

we know that, $S_1(t) = S_{11}\ \phi_1(t)$ } on comparing

$\therefore S_{11} = \sqrt{E_1}$ and $\phi_1(t)$ has unit energy.

Next, using the signal $S_2(t)$, we define the coefficient "S_{21}" as $\rightarrow S_{ij} = \int_0^T S_i(t)\phi_j(t)\,dt$

$$S_{21} = \int_0^T S_2(t) \cdot \phi_1(t) \cdot dt \rightarrow (2)$$

Let $g_2(t)$ be a new function which is given as:-

$$g_2(t) = s_2(t) - s_{21}\phi_1(t) \quad \rightarrow (3)$$

This function is orthogonal to $\phi_1(t)$ over the interval $0 \le t < T$. The second basis function is defined as.

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{E_{g2}}} \quad \rightarrow (4)$$

where $E_{g2} = \int_0^T g_2^2(t) \cdot dt$ is the energy of $g_2(t)$.

To prove that $\phi_2(t)$ has unit energy.

$$\int_0^T \phi_2^2(t) \cdot dt = \int_0^T \left[\frac{g_2(t)}{\sqrt{E_{g2}}}\right]^2 \cdot dt$$

$$= \frac{1}{E_{g2}} \int_0^T g_2^2(t) \cdot dt$$

$$= \frac{1}{E_{g2}} \cdot E_{g2} = 1$$

$\therefore$ $\phi_2(t)$ has unit energy.

To prove $\phi_2(t)$ and $\phi_1(t)$ are orthonormal.

$$\int_0^T \phi_1(t)\,\phi_2(t) \cdot dt = \int_0^T \left(\frac{s_1(t)}{\sqrt{E_1}}\right)\left(\frac{g_2(t)}{\sqrt{E_{g2}}}\right) \cdot dt$$

$$= \frac{1}{\sqrt{E_1 E_{g2}}} \int_0^T s_1(t)\, g_2(t) \cdot dt$$

In above equation observe that the

Product of two terms $S_1(t)$ and $g_2(t)$. We

the two symbols are not present at a time

Hence the product $S_1(t) \cdot g_2(t) = 0$ (6)

$$\boxed{\int_0 \phi_1(t)\,\phi_2(t)\cdot dt = 0}$$

∴ The two basis functions are orthonormal.

Generalized equations for orthonormal Basis Functions

$$\cancel{\boxed{\phi_j(t) = \frac{g_j(t)}{\sqrt{E_{g_j}}}}}, \quad j = 1, 2, \ldots$$

~~where~~ $g_j(t)$ is

$$g_j(t) = S_j(t) < \boxed{\phi_i(t) = \frac{g_i(t)}{\sqrt{E_{g_i}}}} \quad i = 1, 2, \ldots N$$

where

$$\boxed{g_i(t) = S_i(t) - \sum_{j=1}^{i-1} S_{ij}\,\phi_j(t).}$$

and the coefficients S_{ij} are given by

$$\boxed{S_{ij} = \int_0^T S_i(t)\,\phi_j(t)\cdot dt} \quad \begin{cases} j = 1, 2, \ldots i-1 \\ i = 1, 2, \ldots M \end{cases}$$

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413, 440, 408, 423, 430, 436,

Hemanth Reddy

Consider the signals $S_1(t)$, $S_2(t)$, & $S_3(t)$ given below.

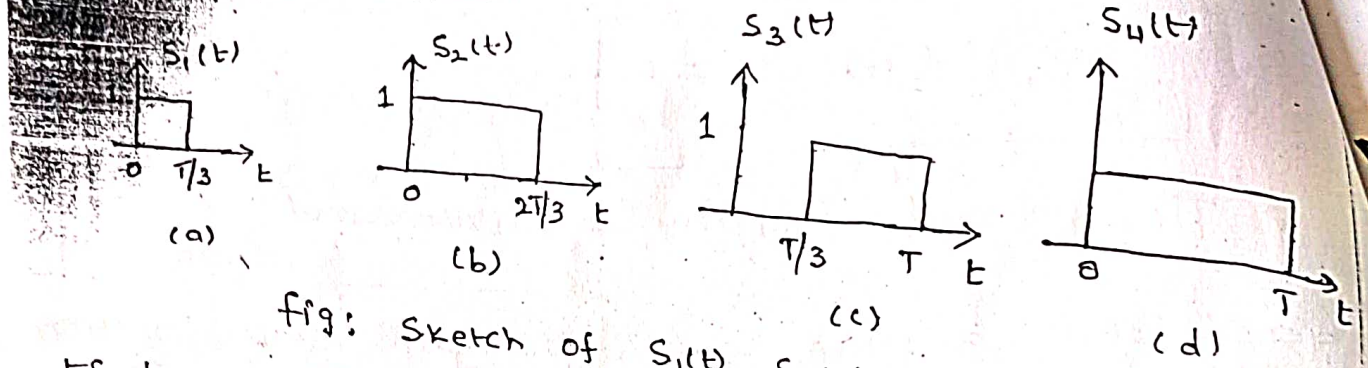


fig: Sketch of $S_1(t)$, $S_2(t)$, $S_3(t)$ & $S_4(t)$.

Find an orthonormal basis for these set of signals using Gram-Schmidt Orthogonalization procedure.

Sol: Here clearly $S_4(t) = S_1(t) + S_3(t)$. This mean all four signals are not linearly Independent. Gram-Schmidt orthogonalization procedure is carried out for a subset which is linearly independent. Here $S_1(t)$, $S_2(t)$ and $S_3(t)$ are linearly independent hence we will determine orthonormal basis.

To obtain $\phi_1(t)$:

$$\phi_1(t) = \frac{S_1(t)}{\sqrt{E_1}}$$

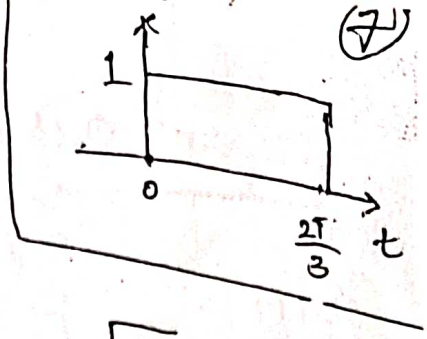
$$E_1 = \int_0^{T/3} S_1^2(t)\,dt = \int_0^{T/3} 1\,dt = T/3$$

$$\therefore \quad \phi_1(t) = \frac{1}{\sqrt{T/3}} \quad \text{for} \quad 0 \le t \le T/3$$

$$\phi_1(t) = \begin{cases} \sqrt{\frac{3}{T}} & \text{for } 0 \le t \le T/3 \\ 0 & \text{elsewhere.} \end{cases}$$

To obtain $\phi_2(t)$:

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{E_{g_2}}}$$

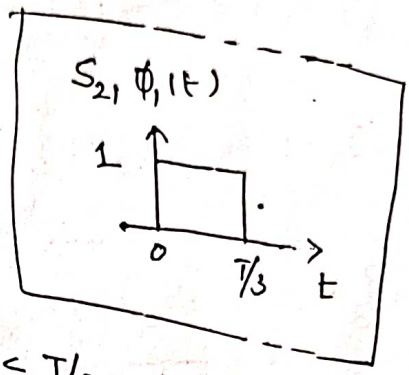


$$g_2(t) = s_2(t) - s_{21}\phi_1(t)$$

$$s_{21} = \int_0^T s_2(t)\cdot\phi_1(t)\cdot dt$$

$$= \int_0^{T/3} (1)\cdot\sqrt{\frac{3}{T}}\cdot dt = \sqrt{\frac{3}{T}} \times \frac{T}{3} = \sqrt{\frac{T}{3}}.$$

$$\therefore \boxed{s_{21} = \sqrt{\frac{T}{3}}}$$



$$s_{21}\phi_1(t) = \sqrt{\frac{T}{3}}\cdot\sqrt{\frac{3}{T}} = 1.$$

$$\therefore s_{21}\phi_1(t) = \begin{cases} 1 & \text{for } 0 \le t \le T/3 \\ 0 & \text{elsewhere.} \end{cases}$$

$$\therefore g_2(t) = s_2(t) - s_{21}\phi_1(t)$$

$$g_2(t) = \begin{cases} 0 & \text{for } 0 \le t \le T/3 \\ 1 & \text{for } T/3 \le t \le 2T/3 \\ 0 & \text{for } t > 2T/3. \end{cases}$$

$$E_{g_2} = \int_{T/3}^{2T/3} g_2^2(t)\cdot dt = \int_{T/3}^{2T/3} 1\cdot dt = T/3$$

$$\therefore \phi_2(t) = \frac{1}{\sqrt{T/3}} = \sqrt{\frac{3}{T}}$$

$$\therefore \boxed{\phi_2(t) = \begin{cases} \sqrt{\frac{3}{T}} & \text{for } T/3 \le t \le 2T/3 \\ 0 & \text{elsewhere} \end{cases}}$$

Obtain $\phi_3(t)$

$$\phi_i(t) = \frac{g_i(t)}{\sqrt{E_{gi}}}$$

$$g_i(t) = s_i(t) - \sum_{j=1}^{i-1} s_{ij}\,\phi_j(t)$$

$$g_3(t) = s_3(t) - \sum_{j=1}^{2} s_{3j}\,\phi_j(t)$$

$$\boxed{g_3(t) = s_3(t) - s_{31}\phi_1(t) - s_{32}\phi_2(t)}$$

$$s_{31} = \int_0^{T/3} s_3(t)\,\phi_1(t)\cdot dt = 0.$$

$$\therefore \boxed{s_{31}\,\phi_1(t) = 0}$$

$$s_{32} = \int_{T/3}^{2T/3} s_3(t)\,\phi_2(t)\cdot dt = \int_{T/3}^{2T/3} (1)\sqrt{\frac{3}{T}}\cdot dt = \sqrt{\frac{3}{T}}\cdot\frac{T}{3} = \sqrt{\frac{T}{3}},$$

$$T - \frac{2T}{3} = \frac{3T-2T}{3} = T/3.$$

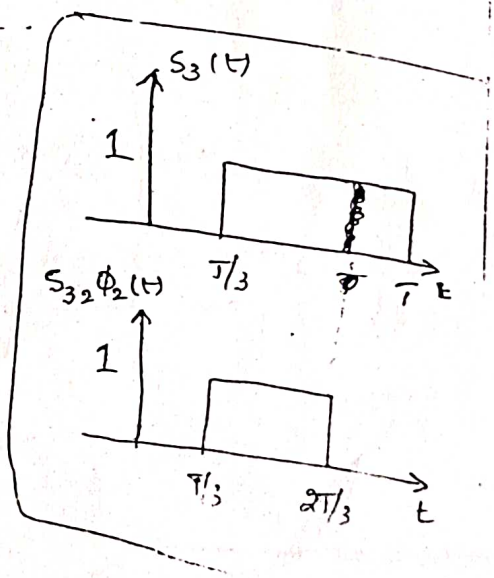
$$\therefore s_{32}\phi_2(t) = \sqrt{\frac{T}{3}}\cdot\sqrt{\frac{3}{T}} \quad \text{for } \frac{T}{3} \le t \le \frac{2T}{3}$$

$$\therefore s_{32}\phi_2(t) = \begin{cases} 1 & \text{for } T/3 \le t \le 2T/3 \\ 0 & \text{elsewhere.} \end{cases}$$

$$\therefore g_3(t) = s_3(t) - s_{32}\phi_2(t)$$

$$g_3(t) = \begin{cases} 0 & \text{for } 0 \le t < 2T/3 \\ 1 & \text{for } 2T/3 \le t \le T \\ 0 & \text{for } t > T \end{cases}$$

$$E_{g3} = \int_{2T/3}^{T} (1)\cdot dt = \int_{2T/3}^{T} dt = T/3$$



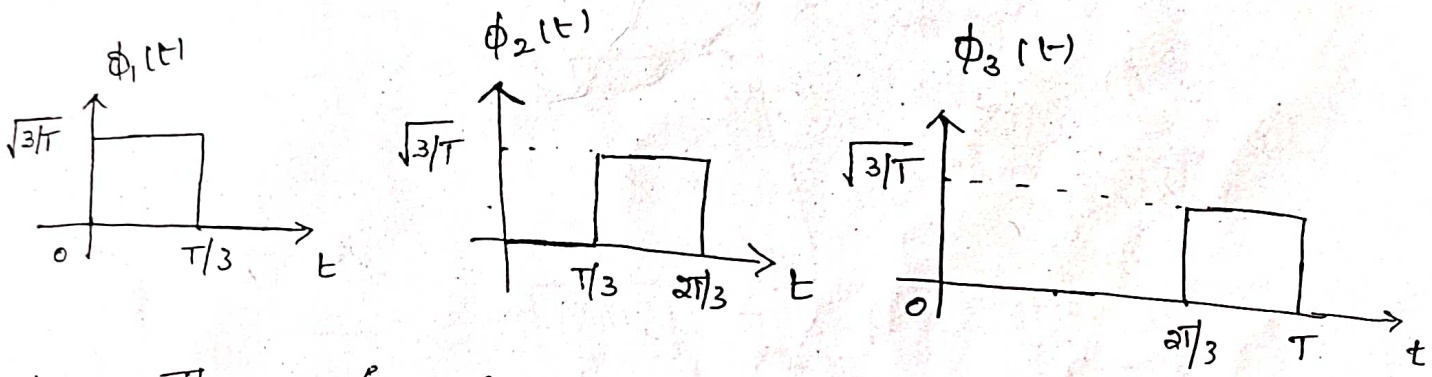
M-ARY Quadrature Amplitude

(8)

$$\phi_3(t) = \frac{g_3(t)}{\sqrt{E_{g3}}}$$

$$= \begin{cases} \frac{1}{\sqrt{T/3}} & \text{for } \frac{2T}{3} \le t \le T \\ 0 & \text{elsewhere} \end{cases}$$

$$\phi_3(t) = \begin{cases} \sqrt{\frac{3}{T}} & \text{for } 2T/3 \le t \le T \\ 0 & \text{elsewhere.} \end{cases}$$



* These functions are orthonormal and any of $S_1(t)$, $S_2(t)$, $S_3(t)$ and $S_4(t)$ can be expressed as a linear combination of above basis functions.

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413, 440, 403, 404, 406, 408, 409, 411,

412, 413, 414, 415, 422, 426,

428, 430, 432,

Ghose, Hemanth

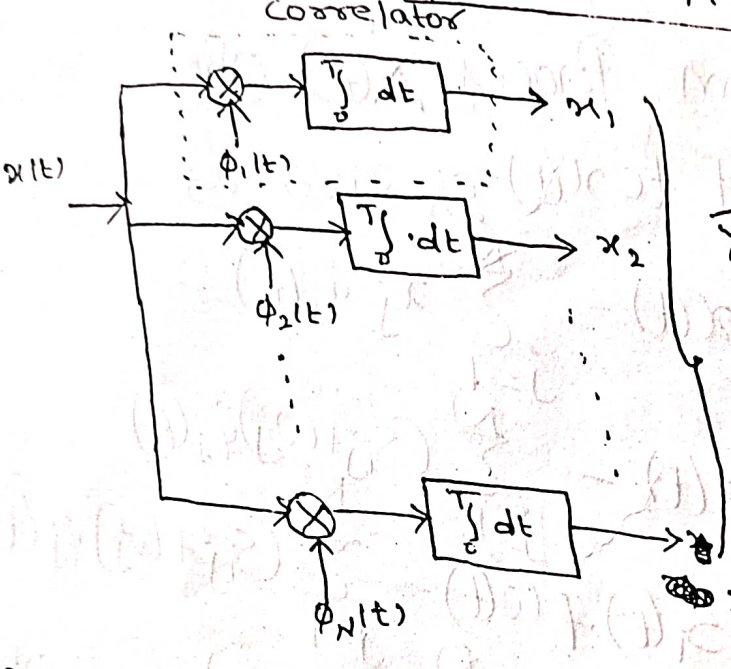
Version of the Continuous AWGN Channel into a Vector Channel :-

(9)

AWGN: Additive White Gaussian Noise

Suppose that the i/p to the bank of N product integrators or correlators as shown in below figure is

$$x(t) = s_i(t) + w(t) \rightarrow (1)$$



$$x_j = s_{ij} + w_j$$

$$x_j = \int_0^T x(t)\phi_j(t)dt \quad j = 1, 2, \cdots N$$

where $w(t)$ is a sample function of White Gaussian noise process $W(t)$ of zero mean & PSD $N_0/2$.

Let the o/p of correlator x_j, say, is the sample value of a random variable X_j, as shown by.

$$x_j = \int_0^T x(t)\,\phi_j(t)\cdot dt \qquad j = 1, 2, \cdots N$$

$$x_j = s_{ij} + w_j \rightarrow (2) \qquad j = 1, 2, \cdots N$$

The first term "s_{ij}" in eqn (2) is determined by defined

$$s_{ij} = \int_0^T s_i(t)\,\phi_j(t)\cdot dt \rightarrow (3)$$

and the Second term "w_j" is defined by

$$w_j = \int_0^T w(t)\,\phi_j(t)\,dt \rightarrow (4)$$

Statistical Characterization of the Correlator output

$x(t)$ denote the random process,

AWGN:- It is a basic noise model used in information theory to the effect of many random process occur in nature. It provides uniform power across the freq. band for the information system.

Consider a new random process $x'(t)$ whose a sample fun. $x'(t)$ and it is related to $x(t)$ as given as

$$x'(t) = x(t) - \sum_{j=1}^{N} x_j \phi_j(t) \qquad x_j = s_{ij} + w_j$$

$$= x(t) - \sum_{j=1}^{N} (s_{ij} + w_j)\phi_j(t)$$

$$= s_i(t) + w(t) - \sum_{j=1}^{N} (s_{ij} + w_j)\phi_j(t)$$

$$= w(t) - \sum_{j=1}^{N} w_j \phi_j(t) \qquad \text{where } s_i(t) = s_j(t) = 0$$

$$= w'(t)$$

∴ The Sample function $x'(t)$ depends only on the channel noise $w(t)$.

Statistical Characterization of the Correlator Outputs

Let X_j denote the random variable whose sample value is represented by the correlator output x_j, $j = 1, 2, \ldots$

Let W_j denote the random variable represented by the sample value w_j produced by the jth correlator in response to the White Gaussian Noise component w(t).

By the deffnition, the random variable W_j has zero mean, because the noise process W(t) has zero mean.

∴ The mean of X_j depends only on s_{ij}, as shown by

$$\mu_{X_j} = E[X_j]$$
$$= E[s_{ij} + W_j]$$
$$= s_{ij} + E[W_j] \quad [\because E(W_j) = 0]$$
$$= s_{ij}$$

$$\boxed{\mu_{X_j} = s_{ij}}$$

The Variance of X_j, $\sigma^2_{X_j} = E[(X_j - s_{ij})^2]$

$$= E[W_j^2]$$

$$\therefore \sigma^2_{X_j} = \int_0^T \int_0^T \phi_j(t)\, \phi_j(u)\, E[W(t) W(u)]\, dt\, du$$

$$= \frac{N_0}{2} \int_0^T \int_0^T \phi_j(t)\, \phi_j(u)\, \delta(t-u)\, dt\, du$$

$[\because E[W(t)W(u)] = R_W(t,u) = \frac{N_0}{2}\delta(t-u)]$

$$= \frac{N_0}{2} \int_0^T \phi_j(u)\, \phi_j(u)\, du \qquad \left[\because \int_{-\infty}^{\infty} g(t)\, \delta(t-t_0)\, dt = g(t_0)\right]$$

$$\boxed{\sigma^2_{X_j} = \frac{N_0}{2}} \qquad \left[\because \int_0^T \phi_j^2(u)\, du = \text{unit energy}\right]$$

∴ Each X_j is a Gaussian random variable with mean s_{ij} and variance $N_0/2$. Then PDF of X_j is

$$f_{X_j}(x_j | m_i) = \frac{1}{\sqrt{\pi N_0}}\, e^{\left[-\frac{(x_j - s_{ij})^2}{N_0}\right]} \quad \begin{matrix} i = 1, 2 \\ j = 1, 2, \ldots \end{matrix}$$

We may state that the AWGN channel of figure $s_i(t) \rightarrow \otimes \rightarrow x(t)$

ersion of the Continuous AWGN Channel into a
tor channel :-

onsider the input to the bank of N product
rators or correlators is $x(t)$, is given as

$$x(t) = s_i(t) + w(t) \quad \begin{cases} 0 \le t \le T \\ i = 1, 2, \ldots M \end{cases} \rightarrow (1)$$

$(t) \rightarrow$ received signal [i/p to correlators]
$(t) \rightarrow$ transmitted signal
$(t) \rightarrow$ a sample function of white Gaussian noise process
of zero mean and PSD of $N_0/2$.

e output of correlator "x_j" is the sample value of
andom variable X_j, as shown by

$$x_j = \int_0^T x(t)\, \phi_j(t)\, dt$$

$$x_j = s_{ij} + w_j \quad , \quad j = 1, 2, \ldots N \rightarrow (2)$$

: first component "s_{ij}" is

$$s_{ij} = \int_0^T s_i(t)\, \phi_j(t)\, dt$$

and

: Second component w_j, is the sample value of
random variable W_j that arises because of the
resence of the channel noise $w(t)$; it is defined
:

$$w_j = \int_0^T w(t)\, \phi_j(t)\, dt$$

LIHOOD FUNCTIONS:

The conditional probability Density functions $f_x(x|m_i)$, $i = 1, 2, \ldots M$, are the very characterizati of an AWGN channel.

Their derivation leads to a functional dependence on the observation vector x, given the symbol m_i.

At receiver we have the exact opposite situati "Given", the observation vector x and the requirem is to estimate the message symbol m_i, that is responsible for generating x.

To emphasize this, the idea of a likelihood function, is introduced, denoted by $L(m_i)$ and defined by

$$L(m_i) = f_x(x|m_i) \qquad i = 1, 2, \ldots M$$

Log-likelihood function denoted by $l(m_i)$ and defined by

$$l(m_i) = \log L(m_i) = \log \left[f_x(x|m_i) \right]_{i=1,2,\ldots M}$$

$$\therefore \quad l(m_i) = \frac{-1}{N_0} \sum_{j=1}^{N} (x_j - s_{ij})^2 \quad , \quad i = 1, 2, \ldots M$$

by ignoring the constant term $-(N/2)\log(\pi N_0$

$$\therefore \log[f_x(x|m_i)] = \log \left[\frac{1}{\sqrt{\pi N_0}} \; e^{\left[-\frac{1}{N_0} \sum_{j=1}^{N} (x_j - s_{ij})^2 \right]} \right]$$

$$= \left[-\frac{(Y_2)}{(-Y_2)} \log(\pi N_0) \right] + \left[\frac{-1}{N_0} \sum_{j=1}^{N} (x_j - s_{ij})^2 \right] \qquad i = 1, 2, \ldots M$$

Ignoring $\log(\pi N_0)$

COHERENT DETECTION OF SIGNALS IN NOISE:

(OR)

MAXIMUM LIKELIHOOD DECODING:

Coherent detection originate from RF sig. Here RF sig can be translated or demodulated.

Some of the assumptions in this decoding are:

One of the M - signals transmitted in each duration of T seconds.

Assume all M signals (or symbols) with equal probability

$$P_r(m_i) = \frac{1}{M}, \quad i = 1, 2, \ldots M \quad \text{for all "t"}$$

AWGN of zero-mean with a power spectral density of $N_0/2$.

If $X(t)$ denotes the received random process with sample function $x(t)$, $0 \le t \le T$, we may write

$$X(t) = S_i(t) + w(t) \qquad 0 \le t \le T \text{ \& } i = 1, 2, \ldots M \quad \rightarrow (1)$$

Then the JOB of Receiver is:

To make "best estimate" of transmitted signal $[S_i(t)]$ upon receiving $x(t)$.

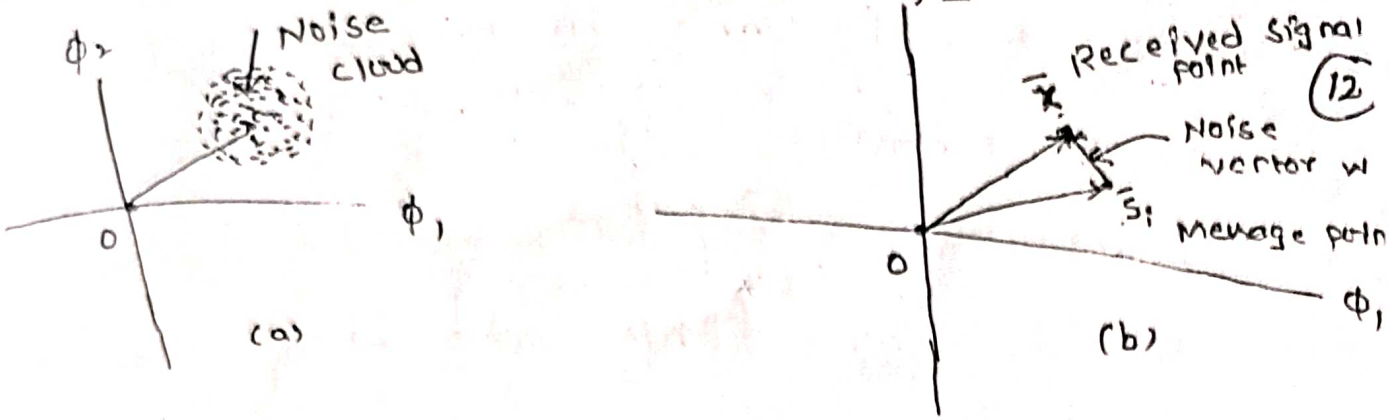
Constellation:

We may represent $S_i(t)$ by a point in Euclidean space of dimension $N \le M$. We refer to this point as the transmitted signal point or message point.

Collection of M signal (or message) points in Signal Space is called a Signal Constellation.

Based on the Observation vector $\vec{x}_j$ we may represent the received signal $x(t)$ by a point in the same Euclidean space used to

ransmitted signal. We refer this point as the
ved signal point.



(12)

. Illustrating the effect of noise depicted in (a) on the location of the received signal point, depicted in (b).

Anoisevector means it is a random vector it contains Probability distribution with zeromeans & finite variance

The relationship between the observation vector $\bar{x}$ and the signal vector s_i is shown in fig (b)., from this we could identify a "noisy vector" w."

Signal Detection problem: Detection problem stated "Given the observation vector $\bar{x}$, perform a mapping from $\bar{x}$ to an estimate $\hat{m}$ of the transmitted symbol, m_i, in a way that would minimize the probability of error in the decision-making process."

"Maximum Likelihood Detection scheme provides a general solution to this problem". It leads to an Optimum Receiver.

Maximum Likelihood Detection principle :-

Suppose that, given the observation vector $\bar{x}$, we make the decision $\hat{m} = m_i$. The probability of error in this decision, denoted by $P_e(m_i|\bar{x})$

$$P_e(m_i|\bar{x}) = P(m_i \text{ not sent}|\bar{x}) \rightarrow (2)$$
$$= 1 - P(m_i \text{ Sent}|x)$$

Optimum Decision rule to minimize the probability of error in mapping is:

Set $\hat{m} = m_i$ if

$$P(m_i \text{ sent} | \bar{x}) \geq P(m_k \text{ sent} | \bar{x}) \quad \text{for all } k \neq i \quad \rightarrow (3)$$

here $k = 1, 2, \ldots M$. This decision rule is referred as the maximum a posteriori probability (MAP) rule

The condition of eqn(3) may be expressed more explicitly interms of a priori probabilities of the transmitted signals and interms of the likelihood functio

USING BAYE's RULE: We can Restate MAP RULE AS:

$$\beta(m_i|\bar{x}) \cdot \beta(\bar{x}) = P_r(\bar{x}|m_i) P(m_i).$$

- $\beta(m_i|\bar{x})$: A posteriori probability of m_i given $\bar{x}$
- $\beta(\bar{x})$: Joint probability of $\bar{x}$ (independent of i)
- $P_r(\bar{x}|m_i)$: A priori probability of $\bar{x}$ given m_i
- $P(m_i) = \frac{1}{M}$

Hence eqn(3)

$$\begin{cases} \text{Set } \hat{m} = m_i \text{ if} \\ P_r(\bar{x}|m_k) \text{ is maximum for } k = i \quad \rightarrow (4) \end{cases}$$

↖ "likelihood Function"

An equivalent form of eqn(4) is

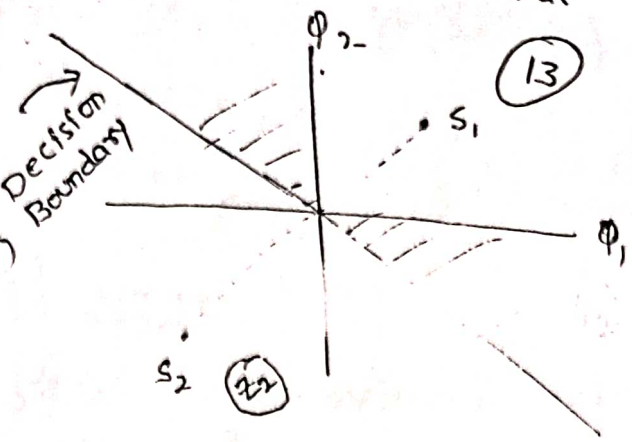
$$\begin{cases} \text{Set } \hat{m} = m_i \\ \ln(m_k) \text{ (or) } \ln[P_r(\bar{x}|m_k)] \text{ is maximum for } k = i. \end{cases} \quad \rightarrow (5)$$

This decision rule is referred as 'max. likelihood' Rule.

clue from eq(5), we decide the signal space Decision Regions "Z_i", $i = 1, 2, \dots M$. such that

...ivation vector "$\bar{x}$" lies region Z_i if

$l(m_k) = P_r(\bar{x} / m_k)$ is max. $\rightarrow$ (6)



...om the likelihood function defined, for an AWGN Channel.

An equivalent statement of eq(5) & eq(6) is

Received vector $\bar{x}$ lies inside decision region Z_i.

$$\sum_{j=1}^{N} (\bar{x}_j - S_{kj})^2 \text{ is minimum for } k = i \rightarrow (7)$$

W.K.T

$$\sum_{j=1}^{N} (\bar{x}_j - S_{kj})^2 = \| \bar{x} - S_k \|^2$$

$\therefore$ eq (7) can be restated as.

Received vector $\bar{x}$ lies inside Z_i if

$\| \bar{x} - S_k \|$ is minimum for $k = i$ $\rightarrow$ (8)

Eq. (8) states that the "maximum likelihood decision rule is simply to choose the message point closest to the received signal point.

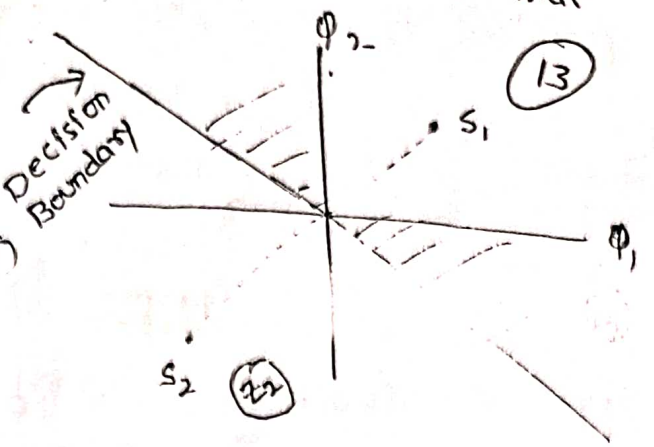
clue from eqn (5), we decide the signal space

Decision Regions "Z_i", $i = 1, 2, \ldots M$, such that

vation vector "$\bar{x}$" lies

region Z_i if

$l(m_k) = P_r(\bar{x}/m_k)$ is max. → (6)



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$\sum_{j=1}^{N} (\bar{x}_j - s_{kj})^2$ is minimum for $k = i$ → (7)

W.K.T

$$\sum_{j=1}^{N} (\bar{x}_j - s_{kj})^2 = \|\bar{x} - s_k\|^2$$

∴ eqn (7) can be restated as.

Received vector $\bar{x}$ lies inside Z_i if

$\|\bar{x} - s_k\|^2$ is minimum for all $k = i$ → (8)

Eqn. (8) states that the "maximum likelihood decision rule is simply to choose the message point closest to the received signal point.

$$(\bar{x}_j - s_{kj})^2 = \sum_{j=1}^{N} \bar{x}_j^2 - 2\sum_{j=1}^{N} \bar{x}_j \cdot s_{kj} + \sum_{j=1}^{N} s_{kj}^2$$

Independent of signal s_k, it can be ignored.

Inner product of x & s_k

Energy of the transmitted signal s_k.

her equivalent form of the optimum Decision rule

Observation vector $\bar{x}$ lies in region z_i if $\rightarrow (9)$

$$\left(\sum_{j=1}^{N} \bar{x}_j s_{kj} - \frac{1}{2} E_k \right)$$ is maximum for $k = i$.

: Correlation Receiver Implements the maximum elihood Decision Rule of eqn (9) by:

First determining $\bar{x}$ with a correlation detector

b) Computing the metric of eqn (9) & taking decision in a vector receiver.

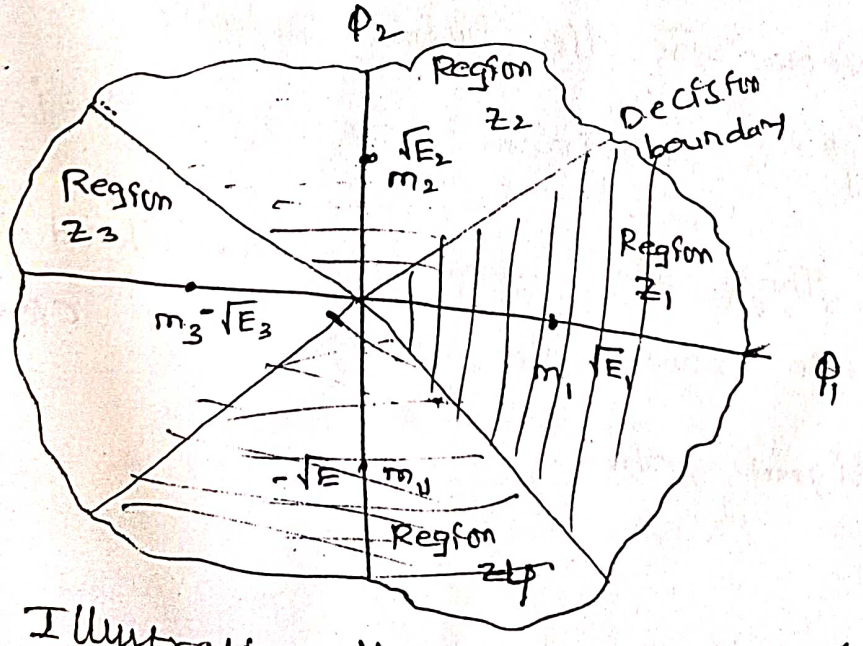


Fig:- Illustrating the partitioning of the observation space into decision regions for the case when $N=2$ & $M=4$.

RELATION RECEIVER : It consists of two sub system

The correlation Detector is shown in below figur

It consists of a bank of M product-Integrators or correlators, supplied with a corresponding set of coherent orthonorma basis functions $\phi_1(t), \phi_2(t), \ldots \phi_N(t)$. This bank of correlators operates on the received signal $x(t)$, $0 \le t \le T$ to produce the observation vector $\bar{x}$.

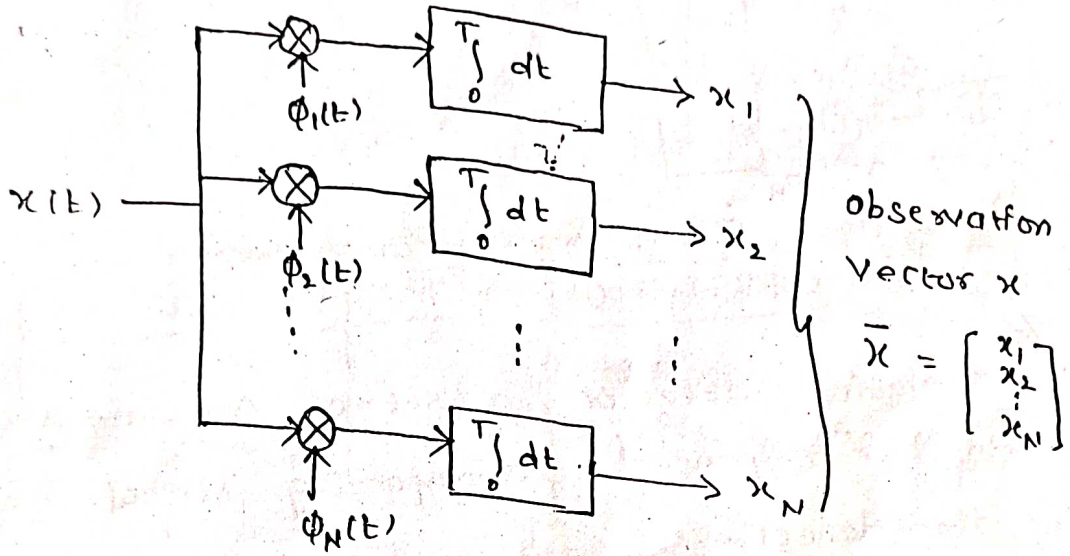


fig : correlation detector

2) Signal transmission Decoder : It is implemented in the form of a maximum-likelihood decoder that operates on the observation vector, to produce an estimate $\hat{m}$, of the transmitted signal (or symbol) m_i, $i = 1, 2, \ldots M$, in a way that would minimize the average probability of symbol error.

The 'N' elements of the observation vector 'x' are first multiplied by the corresponding 'N' elements of each of the signal vector $s_1, s_2, \ldots s_M$ and the resulting products are successfully summed in accumulator to

t, the inner products are corrected for the fact that transmitted signal energies may be unequal. Finally, the largest in the resulting set of numbers is selected.

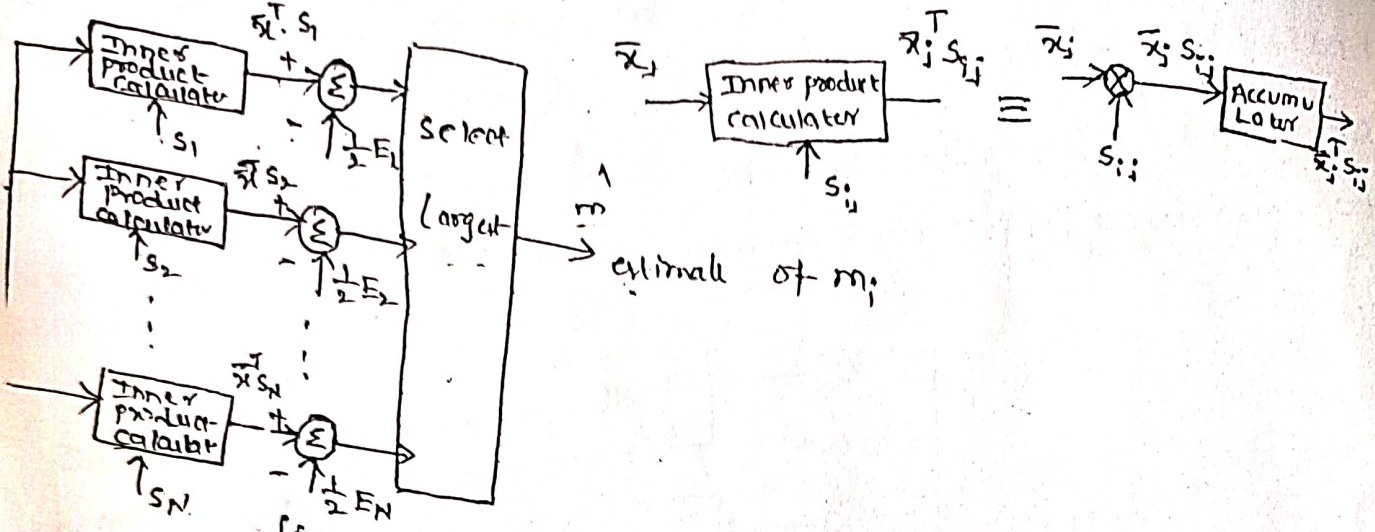


fig:- Signal transmission decoder

EQUIVALENCE OF CORRELATION & MATCHED FILTER RECEIVERS:

In correlation detector, a set of correlators are used. Alternatively, we may use a corresponding set of matched filters to build the detector.

To demonstrate the equivalence of a correlator & a matched filter, consider a linear time-invariant filter with impulse response $h_j(t)$. With received signal $x(t)$ used as the filter input, the resulting filter output, $y_j(t)$, is defined by the convolution integral:

$$y_j(t) = \int_{-\infty}^{\infty} x(\tau)\, h_j(t-\tau)\, d\tau \quad \rightarrow (1)$$

i.e. the impulse response of matched filter, matched to an input signal "$\phi_j(t)$" is a time reversed and delayed version of the input $\phi_j(t)$.

$$\therefore \boxed{h_j(t) = \phi_j(T-t)}$$

$$\therefore y_j(t) = \int_{-\infty}^{\infty} x(\tau)\, \phi_j(T-t+\tau)\, d\tau$$

LTI produces o/p's only from any i/p signal. LTI has time dependent sm function that is not a direct function of time. It depends only indirectly in time domain.

applying this output at time $t = T$, we get

$$\therefore \gamma_1(T) = \int_0^T x(\tau)\, \phi_1(T - \tau + \tau)\, d\tau$$

$$\boxed{\gamma_1(T) = \int_0^T x(\tau)\, \phi_1(\tau)\, d\tau}$$

∴ From the above expression we can say that the output of correlator detector is equivalent to the output of a corresponding matched filter.

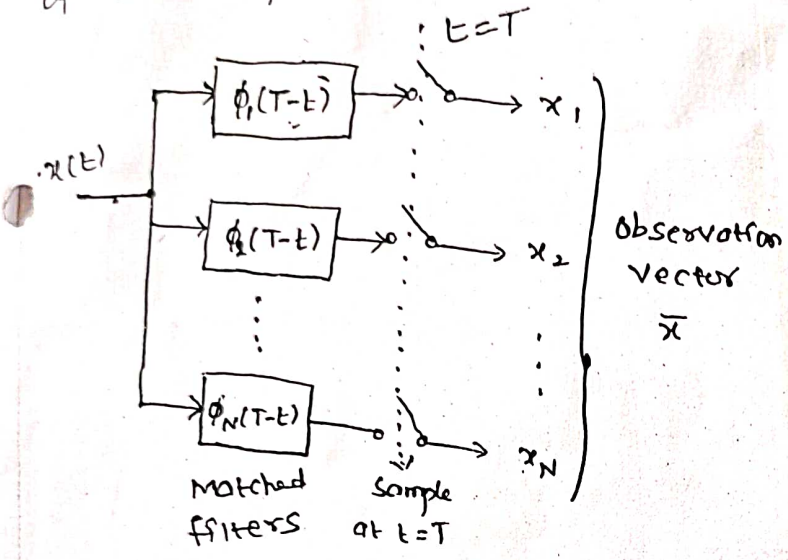


fig:- Detector part of matched filter

M.F:- It is the optimal linear filter for max. signal-to-noise ratio (SNR) in the presence of Additive Stochastic noise. M.F are used in radars. In this impulse response is matched to input Pulse signals.

Probability of ERROR :- An error occurs whenever the received signal point represented by $\bar{x}$ does not fall inside region Z_i associated with the message point represented by S_i. The average probability of symbol error, P_e is

$$P_e = \sum_{i=1}^{M} P_i\, P(\bar{x} \text{ does not lie in } Z_i \mid m_i \text{ sent})$$

$$= \frac{1}{M} \sum_{i=1}^{M} P(\bar{x} \text{ does not lie in } Z_i \mid m_i \text{ sent}) \quad \text{--- (4)}$$

$$= 1 - \frac{1}{M} \sum_{i=1}^{M} P(\bar{x} \text{ lies in } Z_i \mid m_i \text{ sent}) \rightarrow (1)$$

∴ $\bar{x}$ lies in the sample value of random vector $\bar{X}$, we

follows:

$$P_e = 1 - \frac{1}{M} \sum_{i=1}^{M} \int_{z_i} f_x(x|m_i)\, dx \quad \rightarrow (2)$$

r an N-dimensional observation vector, the integral
n eqn (2) is likewise N dimensional.

Regenerative repeater CKT (M-ARY QAM)

Unit-III

SIGNAL SPACE ANALYSIS

(1)

S. Nikitha V. Hari Priya

→ Introduction :

Generic Digital Communication System :-

* A message source emits one symbol every 'T' seconds, with the symbols belonging to an alphabet of M symbols denoted by $m_1, m_2, \ldots m_M$.

* In any event, a priori probabilities $P_1, P_2, \ldots P_1$ specify the message source output. In the absence of priori information, it is customary to assume that the M symbols of the alphabet are equally likely.

* Then the probability of symbol 'm_i' is

$$P_i = P(m_i)$$
$$= \frac{1}{M} \quad \text{for } i = 1, 2, 3, \ldots M$$

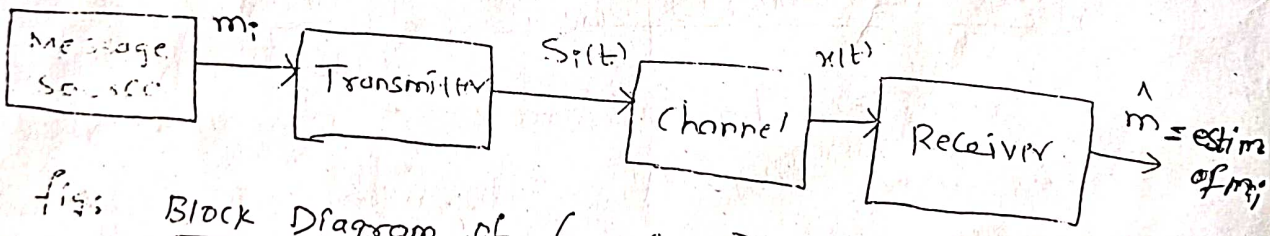


fig: Block Diagram of Generic Digital Communication Systems

* The transmitter takes the message source output m_i and codes it into a distinct signal $S_i(t)$ suitable for transmission over the channel.

* Most important $S_i(t)$ is a real-valued [illegible] with finite energy.

$$E_i = \int_0^T s_i^2(t) \cdot dt \qquad i = 1, 2, \ldots M$$

* Channel is assumed to have two characteristics:

1. Channel is linear
2. channel noise, $w(t)$, is the sample function of a zero-mean white Gaussian noise process.

* Then $x(t)$ is

$$x(t) = s_i(t) + w(t), \quad \begin{cases} 0 \le t < T \\ i = 1, 2, \ldots M \end{cases}$$

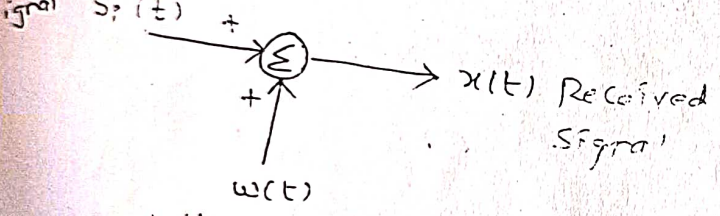


fig. AWGN model of a channel

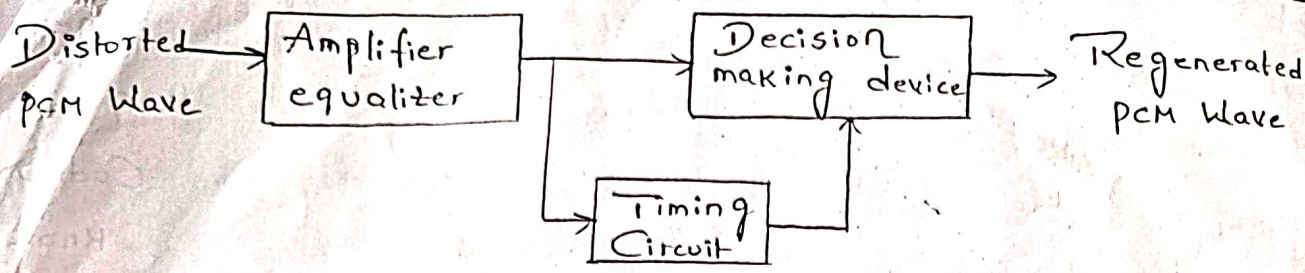
* The receiver has the task of observing the received signal $x(t)$ for a duration of T second, and making a best estimate of the transmitted signal $s_i(t)$ or equivalently the symbol m_i. However, owing to the presence of channel noise, this decision making process is statistical in nature with the result that the receiver will make occasional errors.

The requirement is therefore to design the optimum receiver

M_ forget a given value of ... Modulation:_ (M-ARY QAM)

- Regenerative repeater CKT

(3)



Block diagram of regenerative repeater.

Equilizer

It shapes the received pulses so as to compensate the amplitude and phase distortion produced by the channel.

Timing Circuit

It provides the periodic pulse train, derived from received pulses for sampling condition.

Decision Making

The function of this device is to compare the measured value with the reference value.

→ If the measured value with the referenced value lo threshold value. Symbol '1' is transmitted otherwise '0' is transmitted.

→ Ideally regenerated signal is exactly same as the signal originally transmitted.

→ The unavoidable presence of channel noise & interference causes the regenerated to make wrong decision there by introducing bit error into the regenerated signo

→ If the spacing b/w the sequence of pulses deviated from its assigned value. a jitter is introduced which causes distortion.

Decoder

The first Operation in the receiver is to rege the received pulses the Clean pulses are then regrouped into Code words and decode the Codew into a Quantized PAM Signals, this process is Know as decoding process.

Reconstruction filter

The final Operation in the receiver is to recover the message Signal by passing the o/p a decoder through a low pass reconstruction filte whose Cutoff frequency is equal to the message bandwidth 'W'.

$$\frac{d(w)}{dw} = \frac{\log(1+w)}{w}(1+w)$$

$$|V| = \frac{\log(1+w(m))}{\log(1+m)}$$

IIEEE-B - Absentees

RE → 450, 456, 462, 466, 468, 473,

LE 419, 422, 428, 429, 433,

17/7/23

for a given value of μ, the reciprocal slope of the compression curve, which defines the quantum steps, is given by the derivative of $|m|$ w.r.to $|v|$ i.e,

$$\frac{d|m|}{d|v|} = \frac{\log(1+\mu)}{\mu}(1+\mu|m|)$$

∴ the μ-law is neither strictly linear not strictly logarithmic, but it is approximately linear at low i/p levels corresponding to $\mu|m| << 1$, and approximately at high i/p levels to $\mu|m| >> 1$

$$|v| = \begin{cases} \frac{A|m|}{1+\log|A|}, & 0 \le |m| \le \frac{1}{A} \\ \frac{1+\log(A|m|)}{1+\log A}, & \frac{1}{A} \le |m| \le 1 \end{cases}$$

The uniform quantizer corresponds to $A=1$. the derivative of $|m|$ and $|v|$ is given by

$$\frac{d|m|}{d|v|} = \begin{cases} \frac{1+\log A}{A}, & 0 \le |m| \le 1/A \\ (1+A)|m|, & \frac{1}{A} \le |m| \le 1 \end{cases}$$

Encoding

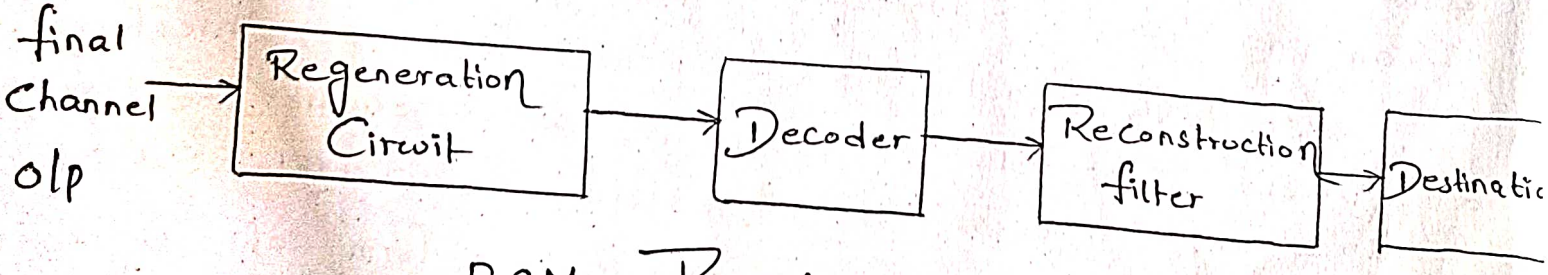
In Combining the processes of Sampling and quantization, the Specification of a Continuous message Signal becomes limited to a discrete Set of values, but not in the form of best Suited to a transmission over a telephone line (or) radio path. To exploit the advantages of Sampling and quantizing for the purpose of making the transmitted Signal more robust to noise, interference and Other Channel impairements, we require the use of an encoding process to translate the discrete Set of Sample values to a more appropriate form of Signals for representing each of this discrete Set of value particular arrangement discrete values is Called Codes one of the discrete events in a Code Called Code

Ordinal no. of representating level	binary number
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
10	1 0 1 0
11	1 0 1 1
12	1 1 0 0
13	1 1 0 1
14	1 1 1 0
15	1 1 1 1

Binary number System for R = 4 bits / Sample

PCM Receiver

The basic Operations in the receiver are regeneration of impaired Signals, decoding and reconstruction of the train of the quantized Samples, as shown in fig. regeneration also occurs an intermediate points along the transmission path.



PCM Receiver

M-ARY Quadrature Amplitude Modulation:- (M-ARY QAM)

- M-ary PAM, which is one dimensional.
- M-ary QAM is a two-dimensional
- In M-ary PAM it involves two orthogonal Pass band basis functions.

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t) \qquad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t) \qquad 0 \le t \le T$$

- Let ith message Point S_i in the (ϕ_1, ϕ_2) Plane be denoted by $(a_i d_{min}/2,\ b_i d_{min}/2)$
- Where d_{min} is distance b/w two message Points. & a_i & b_i are integers.

Where $i = 1, 2 \cdots M$

- Let $(d_{min}/2) = \sqrt{E_0}$
- Where E_0 is the energy of the signal
- Then the transmitted M-ary QAM signal for symbol i then it is defined by

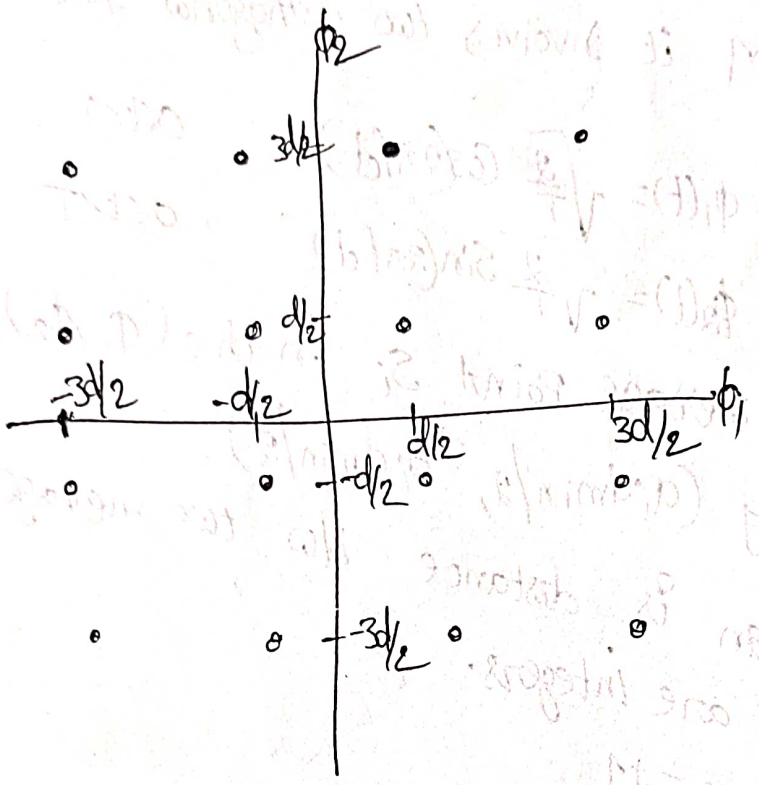
$$S_i(t) = \sqrt{\frac{2E}{T}}\, a_i \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}}\, b_i \sin(2\pi f_c t) \qquad 0 \le t \le T$$

- The signal $S_i(t)$ consists of two Phase quadrature Carriers with each one being modulated by a set of discrete amplitudes. Hence the name is said to be Quadrature amplitude Modulation.
- QAM is divided into 2 Constellation.

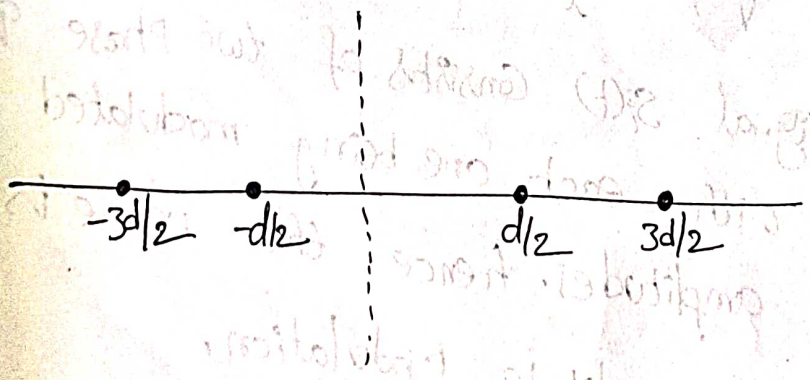
i) Square Constellation:- Here the number of bits per symbol is even.

(ii) Cross Constellation: where the number of bits per symbol is odd.

Ex:- If $M=16$



(a) Signal space diagram of M-ary QAM for $M=16$



(b) Signal Space diagram of the Corresponding 4-PAM Signal.

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Test Name:
Candidate Name:
Hall-Ticket Number:
Candidate Signature:

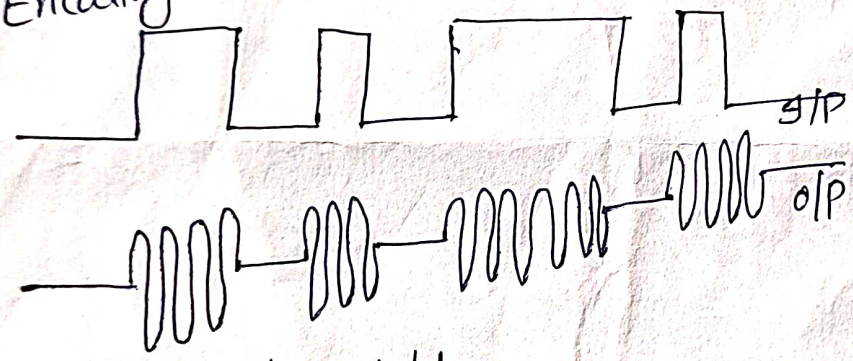
Date:
Test Center:
Hall/Lab:
Invigilator Signature:

UNIT-V

Digital Modulation & Information Schemes

Digital Modulation:-

- It is the Process of encoding a digital Information signal into a amplitude, Phase, or freq. of the Transmitted Signal.
- The encoded Process will effect the band width
- Encoding means Convert into Coded form or Convert into information or Instruction into Particular form.



Coherent digital modulation:-

- A Coherent digital modulation scheme requires a refence s/g at the Receiver that is in Phase coherence with the Corrie of the transmitted Signal. [which may be Surpressed]
- Coherent means its a logical or Consistent.
- C.D.M. Technique which employ Coherent detection.
- The Coherent Modulation is a technique that uses modulation of amplitude and Phase of light, as well as transmission across two Polarizations to enable more information across the optical fiber.

... s/g from another is a Change in

UNIT-II

PASS BAND DATA TRANSMISSION

- In a Baseband Pulse Transmission, data stream represented by in the form of a discrete Pulse amplitude modulated (PAM) signal that is transmitted low Pass channel.
- In a digital baseband Transmission, the incoming data is modulated onto a Carrier (Usually Sinusoidal) s/g with a fixed frequency.
- The Communication Channel used for Pass band data transmission may be a micro wave radio link, a Satellite Channel.
- The Application of Passband data transmission are in the design of Passband line code for use on digital Subscriber loops.
- The Modulation Process making the Transmission Possible involves switching the Amplitude, frequency or Phase of a Sinusoidal Carrier.
- They are Three basic Signaling Schemes in Pass band data Transmission.
  1) ASK (Amplitude Shift keying)
  2) FSK (Freq. Shift keying)
  3) PSK (Phase Shift keying)

- Sinusoidal Carrier used by the modulator to s/g from another is a change in

Pass-band Transmission Model

- Here a message source is used for to emits one symbol seconds, with the symbols belonging to an alphabet 'M' symbols, which is denoted by $m_1, m_2, m_3, \ldots m_M$.
- The Proor Probabilities $P(m_1), P(m_2) \ldots P(m_M)$ specify the message outputs.
- When M symbols of the alphabet are equally likely

$$P_i = P(m_i) = \frac{1}{M} \text{ for all } i.$$

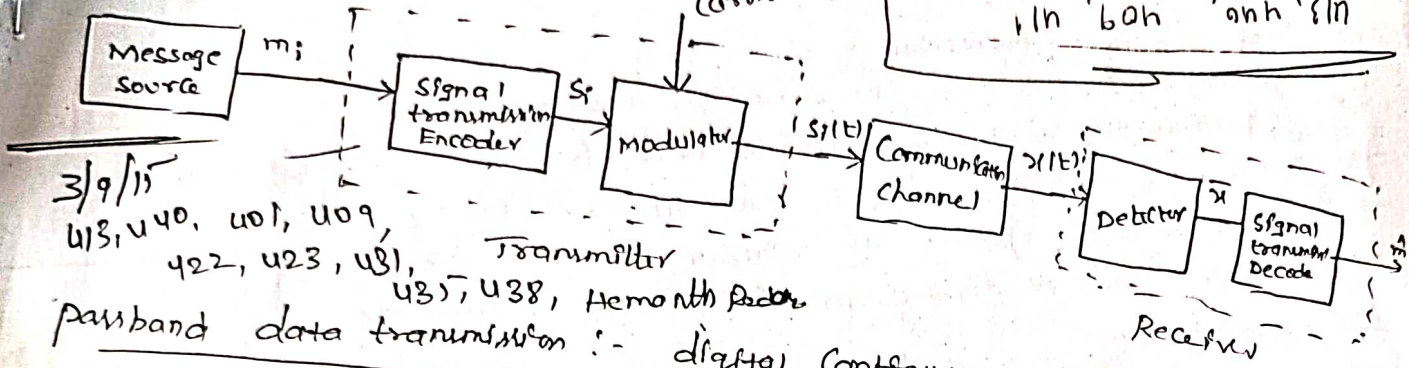
- The M-ary o/p of the message source is present to s/g transmission encoder, it produces a corresponding vector & madeup of N real elements.
- One such set for each of M symbols of source alphabet the dimension of N is less than or equal to M.
- The vector s_i is input, the modulator construct a distinct s/g $s_i(t)$ of duration T seconds.
- The signal $s_i(t)$ is an energy s/g.

$$E_i = \int_0^T s_i^2(t)\,dt \quad i = 1, 2 \ldots M$$

- The Band pass communication channel, the Tx to Rx is assumed to have two characterstics.
  1) The channel is linear
  2) The channel is noise $w(t)$ it contains Gaussian noise process of zero mean and P.S.D of $N_0/2$
- In the Receiver which consists of detector followed by signal Transmission decoder.
  1) It reverse the operations performed in the transmit[illegible] waves [illegible] the effect of channel noise [illegible]

UNIT-II S. Nikitha

PASSBAND DATA TRANSMISSION



3/9/15

U18, U40, U01, U09, U22, U23, U31, U35, U38, Hemanth Reddy

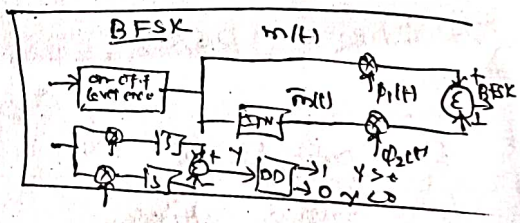
Passband data transmission :- Digital Continuous Wave Modulation

→ The data is transmitted by digital modulating high frequency carrier. [Suitable over long distance]

Baseband data transmission :- [digital data is transmitted directly over the channel without modulation].

Types of P.B.M

1) PSK : Digital data modulate phase of the carrier
2) FSK : Frequency of the Carrier
3) ASK : Amplitude of the Carrier.



Reception (Detection) Types

Coherent (synchronous) Detection : Local Carrier generated at the receiver is phase locked with the Carrier at the transmitter.

Noncoherent (Envelope) Detection :- The receiver Carrier need not be phase locked with transmitter Carrier. It is simple but it has higher probability of error.

Requirements :

→ Maximum data transmission rate / Min. B.W
→ Min. prob. of Symbol error
→ Min. Transmitted power
→ Min. Channel BW

→ Max. Resistance to Interfering Signals
→ Min. CKT Complexity

Advantages :

- Long distance transmission
- Analog Channels, can be used for transmission
- Multiplexing techniques can be used for bandwidth conservation.
- problems such as ISI & crosstalk are absent.
- It can take place over wireless channels also.
- Large no. of modulation techniques are available

Disadvantages :-

1. System is complex.
2. Not suitable for short distance communication

413, 440, 409, 411, 428, 430, LE 401, 402 Hemanth Reddy

(Q) Advantages & Disadvantages of Digital Modulation Schemes :

→ 1) Coherent PSK

1) BPSK
- Error probability
- Generation & Detection
- Power spectrum

2) QPSK 3) BFSK

2) M-array PSK, M-array QAM.

3) Non-coherent Orthogonal Modulation Schemes.

OPSK, BFSK.

Generation & Detection.

Comparison of all

BPSK

$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

$$S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi)$$

$$S_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

$$0 \le t \le T_b$$

$$\boxed{f_c = \frac{n}{T_b}}$$

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)$$

$$S_1(t) = \sqrt{E_b}\, \phi_1(t)$$

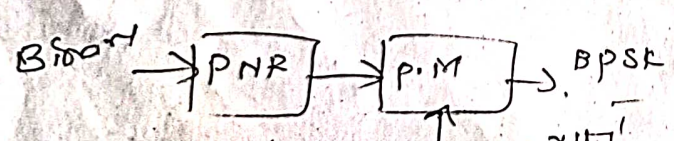
$$S_2(t) = -\sqrt{E_b}\, \phi_1(t)$$

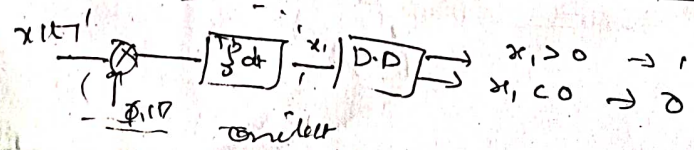
$$S_{11} = +\sqrt{E_b}$$

$$S_{21} = -\sqrt{E_b}.$$

BFSK

$$\boxed{f_i = \frac{n+i}{T_b}}$$





1) It reverse the operations

Coherent Modulation Schemes:- It is a Technique that uses a Coherent Phase-Shift Keying Modulation of the amplitude & Phase.

Binary PHase-Shift Keying (BPSK):-

In a coherent binary PSK system, the pair of signals $s_1(t)$ and $s_2(t)$ are used to represent binary symbols 1 and 0, respectively, is defined by

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \rightarrow (1)$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \rightarrow (2)$$

where $0 \le t \le T_b$, and E_b is transmitted signal energy per bit. The Carrier frequency f_c is $f_c = \frac{n}{T_b}$, 'n' is some fixed integer.

A pair of sinusoidal waves that differ only in a relative phase-shift of 180 degrees, as defined in eqn (1) and eqn (2) are referred to as "antipodal signals".

In the case of binary PSK, there is only one basis function of unit energy, from the pair of eqn (1) & (2). namely,

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t), \quad 0 \le t \le T_b \rightarrow (3)$$

* Transmitted signals $s_1(t)$ and $s_2(t)$ in terms of $\phi_1(t)$ as follows:

$$s_1(t) = \sqrt{E_b}\, \phi_1(t), \quad 0 \le t \le T_b \rightarrow (4)$$

and

$$s_2(t) = -\sqrt{E_b}\, \phi_1(t), \quad 0 \le t \le T_b \rightarrow (5)$$

∴ The Coordinate of message point are

$$s_{11} = \int_0^{T_b} s_1(t)\, \phi_1(t) \cdot dt = +\sqrt{E_b} \rightarrow (6) \qquad s_{21} = \int_0^{T_b} s_2(t)\, \phi_1(t) \cdot dt = -\sqrt{E_b} \rightarrow (7)$$

The message point corresponding to $S_1(t)$ is located at $S_{11} = +\sqrt{E_b}$, and the message point corresponding to $S_2(t)$ is located at $S_{21} = -\sqrt{E_b}$.

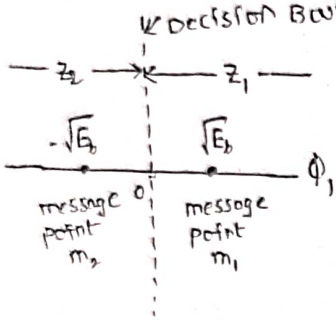


fig:- Signal constellation or signal-space diagram for coherent binary PSK System

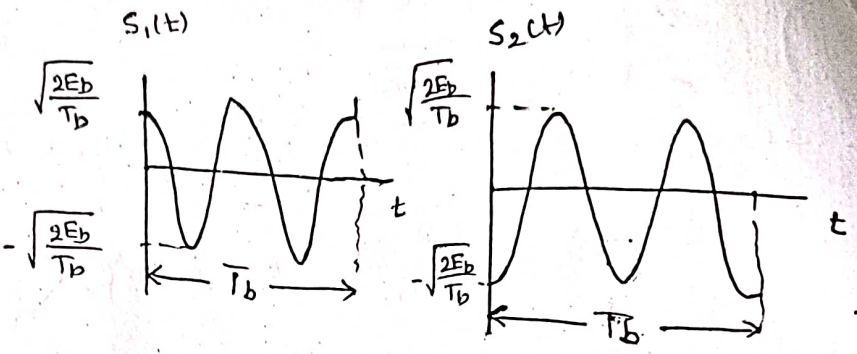
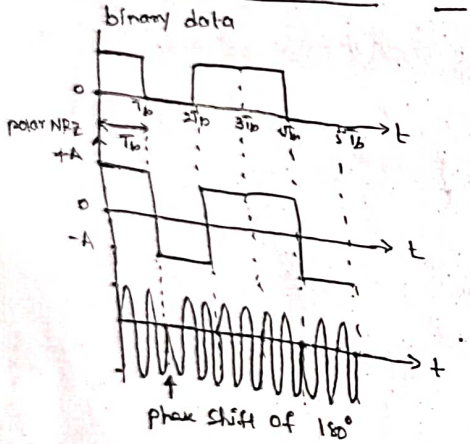


fig:- Example of antipodal signals $S_1(t)$ and $S_2(t)$, for $n = 2$.

Graphical Representation of BPSK Signals



Generation and Detection of coherent BPSK signal :-

- To generate a binary PSK signal, represent the input binary sequence in polar form with symbols 1 and 0 represented by constant amplitude levels of $+\sqrt{E_b}$ and $-\sqrt{E_b}$, respectively.
- This signal transmission encoding is performed by a polar NRZ level encoder.
- The encoder output and a sinusoidal carrier $\phi_1(t)$ are applied to a product modulator. The desired binary PSK wave is obtained at the modulator output.

Probability Error of BPSK :-

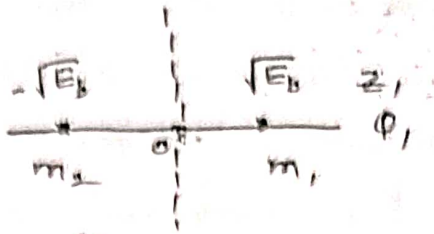


Fig :- Signal Space of BPSK.

* From the figure, the decision boundary is constructed at the midpoint of two message points. From these, decision regions Z_1 and Z_2 are marked.

* The decision rule is now simply to decide that signal $s_1(t)$ was transmitted if the received signal point falls in region Z_1, and decide that signal $s_2(t)$ was transmitted if the received signal point falls in region Z_2. There are two kinds of error possibilities:

* Suppose signal $s_2(t)$ is transmitted, but the noise is such that the received signal point falls inside region Z_1 and so the receiver decides in favor of signal $s_1(t)$. Alternatively, signal $s_1(t)$ is transmitted, but the noise is such that the received signal point falls inside region Z_2 and so the receiver decides in favor of signal $s_2(t)$.

* To calculate the probability of making an error of the first kind, we note from fig. that the decision region associated with symbol 1 or signal $s_1(t)$ is described by

$$Z_1 : 0 < x_1 < \infty$$

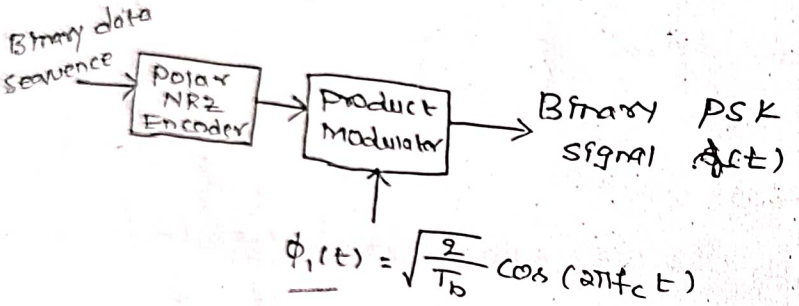
where the observable element x_1 is related to the received signal $x(t)$ by

$$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt \quad \rightarrow (1)$$

Coherent detection of Bpsk signals :-

* To detect the original binary sequence of 1's and 0's we apply the noisy psk signal $x(t)$ to a correlator which is also supplied with a locally generated coherent reference signal $\phi_1(t)$.

* The correlator output, x_1, is compared with a threshold of zero volts. If $x_1 > 0$, the receiver decides in favor of symbol 1. On the other hand, if $x_1 < 0$, it decides in favor of symbol 0. If x_1 is exactly zero, the receiver makes a random guess in favor of 0 or 1.



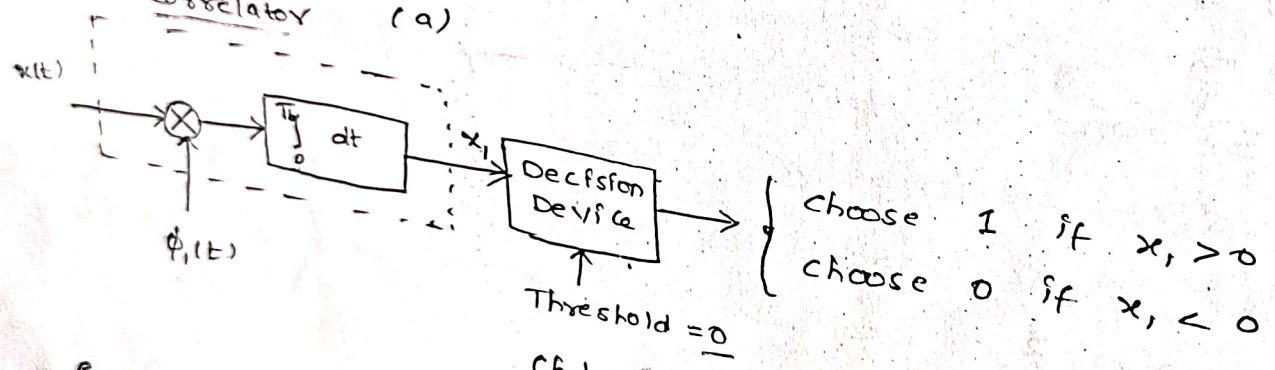


fig :- Block diagrams for
(a) Binary PSK transmitter
(b) Coherent Binary PSK receiver.

waveforms and the effect of

Coherent Frequency-Shift Keying :-

BINARY FSK: In a binary FSK system, symbols 1 and 0 are distinguished from each other by transmitting one of two sinusoidal waves that differ in frequency by a fixed amount. A typical pair of sinusoidal waves is described by

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t), & 0 \le t \le T_b \\ 0, & \text{elsewhere} \end{cases} \rightarrow (I)$$

where, $i = 1, 2$ and E_b is the transmitted signal energy per bit, the transmitted frequency is $f_i = \frac{n_0 + i}{T_b}$ → (II) for some fixed integer "n" and $i = 1, 2$.

Thus symbol "1" is represented by $s_1(t)$ and symbol '0' by $s_2(t)$. The FSK signal described here is known as Sunde's FSK. This form of digital modulation is an example of continuous-phase frequency-shift keying (CPFSK).

The orthonormal basis function is

$$\phi_j(t) = \begin{cases} \sqrt{\frac{2}{T_b}} \cos(2\pi f_j t), & 0 \le t \le T_b \\ 0, & \text{elsewhere} \end{cases} \rightarrow (III)$$

where $i = 1, 2$, $j = 1, 2$. Then the coefficient s_{ij} is defined by

$$s_{ij} = \int_0^{T_b} s_i(t)\,\phi_j(t)\,dt = \int_0^{T_b} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_j t)\,dt$$

$$s_{ij} = \begin{cases} \sqrt{E_b}, & i = j \\ 0, & i \neq j \end{cases} \rightarrow (IV)$$

The two message points are defined by

$$\underline{s}_1 = \begin{bmatrix} s_{11} \\ s_{12} \end{bmatrix} = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix}$$

$$\underline{s}_2 = \begin{bmatrix} s_{21} \\ s_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix}$$

with the Euclidean distance between them equal to $\sqrt{2E_b}$.

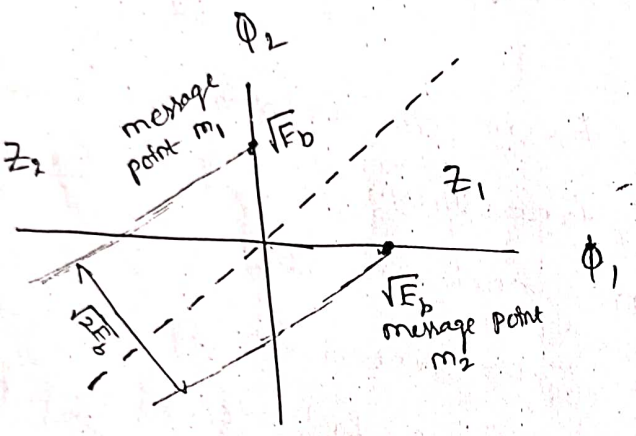
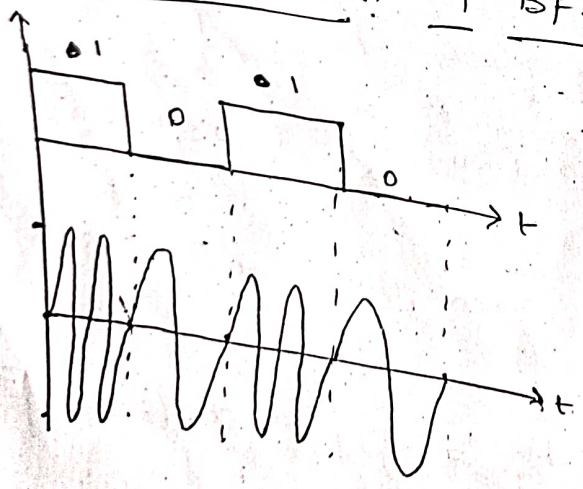


fig:- Signal Constellation (or Signal Space diagram of binary FSK.

Graphical Representation of BFSK Signals



neration & Detection of BFSK Signals :-

To generate a binary FSK, we may use the scheme shown in below fig (a).

The incoming binary data sequence is first applied to on-off level encoder. By using an inverter in the lower channel as in fig (a), we in effect make sure that when we have symbol 1 at the input, the oscillator with frequency f_1 in the upper channel is switched on, and while the oscillator with frequency f_2 in the lower channel is switched off, with the result that 'f_1' is transmitted. Conversely, when we have symbol 0 at the input, the oscillator in the upper channel is switched off and the oscillator in the lower channel is switched on, with the result that frequency 'f_2' is transmitted.

In the transmitter, assume that two oscillators are synchronized, so that their outputs satisfy the requirements of the two orthonormal basis functions $\phi_1(t)$ & $\phi_2(t)$. The frequency of the modulated wave is shifted with a continuous phase, in accordance with the input binary wave.

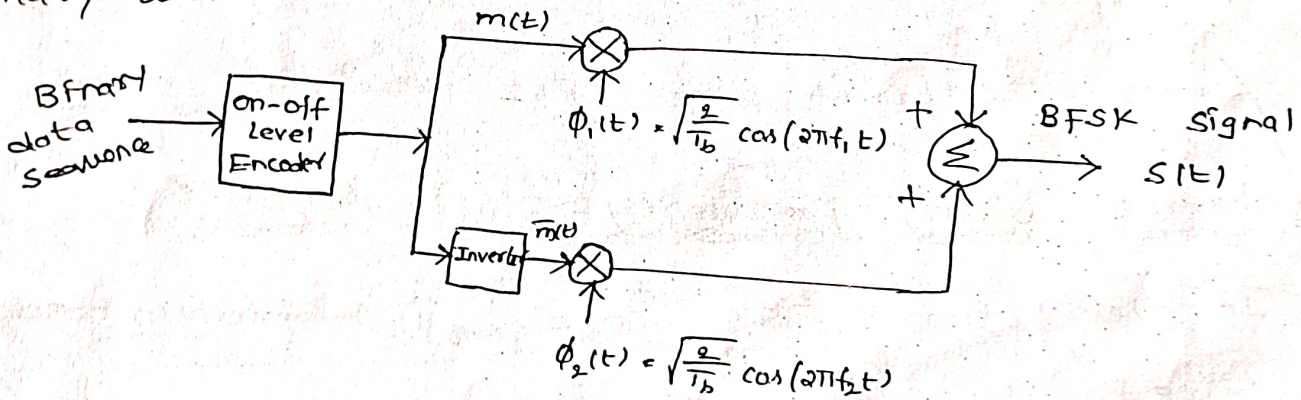


fig (a) :- Generation of BFSK Signal

Coherent Detection of BFSK Signal :-

* To detect the original binary sequence, we may use the receiver shown in below fig (b).
* It consists of two correlators with a common input, which are supplied with locally generated coherent reference signals $\phi_1(t)$ and $\phi_2(t)$. The correlator outputs are then subtracted, one from the other, and the resulting difference, Y, is compared with a threshold of zero volts. If $Y > 0$, the receiver decides in favor of 1. On the other hand, if $Y < 0$, it decides in favor of 0. If Y is exactly zero, the receiver makes a random guess in favor of 1 or 0.

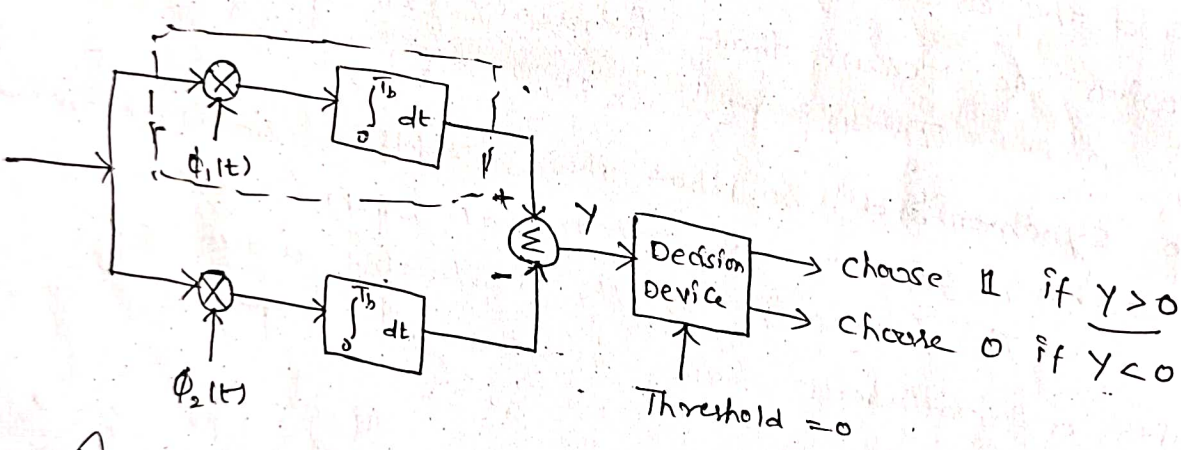


Fig (b) :: Coherent binary FSK receiver

Non-Coherent Binary Frequency-Shift Keying :-

In the binary FSK case, the transmitted signal is defined by

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t), & 0 \le t \le T_b \\ 0, & \text{elsewhere} \end{cases} \rightarrow (1)$$

where the carrier frequency equals one of two possible values, f_1 and f_2. The transmission of frequency f_1 represents symbol 1, and the transmission of frequency f_2 represents symbol 0.

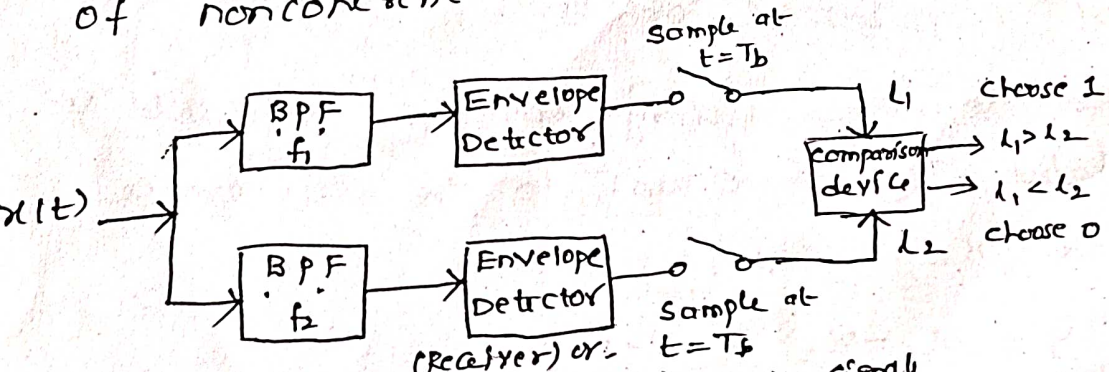
The non-coherent receiver of BFSK is shown in the below figure: The receiver consists of a pair of band pass (matched filters) followed by envelope detectors.

The filter in the upper path is matched to f_1 (or) $\cos(2\pi f_1 t)$ and the filter in lower path is matched to $\cos(2\pi f_2 t)$ or f_2, and in both cases $0 \le t \le T_b$.

The resulting envelope detector outputs are sampled at $t = T_b$, and their values are compared. The envelope samples of the upper and lower paths are l_1 and l_2 respectively.

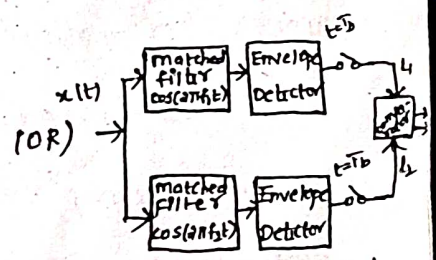
Then if $l_1 > l_2$, the receiver decides in favor of symbol 1, and if $l_1 < l_2$, the receiver decides in favor of symbols 0. If $l_1 = l_2$, the receiver simply makes a guess in favor of symbol 1 or 0.

The non-coherent BFSK is a special case of noncoherent orthogonal modulation with $T = T_b$, $E = E_b$.



(Receiver) or.

fig:- Non-coherent Detection of BFSK signals



figs- Non-coherent Receiver for the detection of BFSK signals.

probability Error of Binary FSK

* The observation vector 'x' has two elements x_1 & since two correlators, are defined by,

$$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\cdot dt \quad \rightarrow (1)$$

$$x_2 = \int_0^{T_b} x(t)\,\phi_2(t)\cdot dt \quad \rightarrow (2)$$

where $x(t) = s_1(t) + w(t)$, when symbol 1 was transmitted

and $x(t) = s_2(t) + w(t)$, when symbol 0 was transmitted,

where $w(t)$ is the sample function White Gaussian Noise process of zero mean and power spectral Density of $N_0/2$.

* From the signal constellation diagram, we find that, the observation space is partitioned into two decision regions, Z_1 and Z_2 as shown in fig.

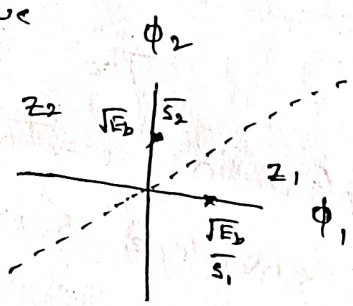


fig:- Signal Constellation of BFSK

* The receiver decides in favor of symbol 1 if the received signal point represented by the observation vector 'x' falls inside region Z_1. This occurs when $x_1 > x_2$. If on the other hand, we have $x_1 < x_2$, the observation vector 'x' falls inside region Z_2. On the decision boundary we have $x_1 = x_2$. Then the receiver makes a random guess in favor of symbol '1' or '0'.

* Consider a new Gaussian random variable y whose sample value 'y' is equal to the difference between x_1 and x_2.

i.e; $$y = x_1 - x_2 \quad \rightarrow (3)$$

Mean value of random variable y depends on which binary symbol was transmitted.

Symbol 1

$$E[Y/1] = E[X_1/1] - E[X_2/1]$$
$$= +\sqrt{E_b}$$

Symbol 0

$$E[Y/0] = E[X_1/0] - E[X_2/0]$$
$$= -\sqrt{E_b}.$$

* The variance of the random variable 'y' is independent of which binary symbol was transmitted. Since the random variable X_1 & X_2 are statistically independent, each with a variance equal to $N_0/2$.

$$\therefore Var[Y] = Var[X_1] + Var[X_2]$$
$$= N_0/2 + N_0/2$$
$$= N_0.$$

* Suppose, symbol '0' was transmitted, then the conditional probability Density Function of 'y' is given by

$$f_y(y/0) = \frac{1}{\sqrt{2\pi N_0}} e^{\left[-\frac{(y+\sqrt{E_b})^2}{2N_0}\right]}$$

* The probability of error, when symbol '0' way transmitted is

$$P_{10} = \int_0^\infty f_y(y/0) \cdot dy$$

$$= \frac{1}{\sqrt{2\pi N_0}} \int_0^\infty e^{\left[-\frac{(y+\sqrt{E_b})^2}{2N_0}\right]} \cdot dy$$

$$\frac{y+\sqrt{E_b}}{\sqrt{2N_0}} = z$$

$$z^2 = \frac{(y+\sqrt{E_b})^2}{2N_0}$$

Lower limit

if $y = 0$

$z = \sqrt{E_b/2N_0}$

Upper limit

If $y = \infty$

then $z = \infty$.

$$dz = \frac{1}{\sqrt{2N_0}} \cdot dy$$

$$\boxed{dy = \sqrt{2N_0} \cdot dz}$$

$$\therefore P_{10} = \frac{1}{\sqrt{2\pi N_0}} \int_{\sqrt{E_b/2N_0}}^{\infty} e^{(-z^2)} \cdot \sqrt{2N_0} \cdot dz$$

$$= \frac{\sqrt{2N_0}}{\sqrt{2\pi N_0}} \int_{\sqrt{E_b/2N_0}}^{\infty} e^{(-z^2)} \cdot dz$$

$$= \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/2N_0}}^{\infty} e^{(-z^2)} \cdot dz$$

$\because$ $\boxed{erfc(y) = \frac{2}{\sqrt{\pi}} \int_y^{\infty} e^{(-z^2)} dz}$

$$\boxed{P_{10} = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)}$$

Similarly, we may show that P_{01} as

$$\boxed{P_{01} = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)}$$

$\therefore$ Average probability $P_e = P_0 P_{10} + P_1 P_{01}$

$$\therefore \boxed{P_e = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)} \quad \because P_0 = P_1 = \frac{1}{2}$$

$\therefore$ BER of BFSK is $\frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$

Quadriphase - Shift Keying (QPSK) :-

(OR)

Quadrature phase - shift keying (QPSK)

(OR)

Quaternary phase - shift keying (QPSK) :-

* It is a bandwidth - conserving (Protect) modulation scheme known as coherent quadriphase - shift keying, which is an example of quadrature - carrier multiplexing.

* In QPSK, as with BPSK, information carried by the transmitted signal is contained in the phase. In particular, the phase of the carrier takes on one of four equally spaced values, such as $\pi/4$, $3\pi/4$, $5\pi/4$ & $7\pi/4$. For this set of values, we may define the transmitted signal as

$$s_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right], & 0 \le t \le T \\ 0, & \text{elsewhere} \end{cases} \rightarrow (1)$$

where $i = 1, 2, 3, 4$; 'E' is the transmitted signal energy per symbol, i.e., $E = 2E_b$ and T is the symbol duration, i.e., $T = 2T_b$. The carrier frequency f_c equals n_c/T for some fixed integer n_c.

* Each possible value of the phase corresponds to a unique dibit.

For example, choose the Gray-encoded set of dibits: 10, 00, 01 and 11, where only a single bit is changed from one dibit to the next.

* Equation (1) can be redefine as

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos\left[(2i-1)\frac{\pi}{4}\right] \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\left[(2i-1)\frac{\pi}{4}\right] \sin(2\pi f_c t)$$

where $i = 1, 2, 3, 4$. Based on this representation, we can make the following observation.

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t), \quad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t), \quad 0 \le t \le T$$

TABLE:- Signal space characterization of QPSK

Signal vector	Gray-encoded Input Dibit b_0 b_e	Phase of QPSK Signal (radians)	Coordinate of Message point s_{i1}	s_{i2}
s_1	1 0	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
s_2	0 0	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$
s_3	0 1	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
s_4	1 1	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$

* There are four message points, and the associated signal vectors are defined by

$$s_i = \begin{bmatrix} \sqrt{E}\cos[(2i-1)\pi/4] \\ -\sqrt{E}\sin[(2i-1)\pi/4] \end{bmatrix}, \quad i = 1, 2, 3, 4.$$

Signal - space diagram of coherent BPSK system

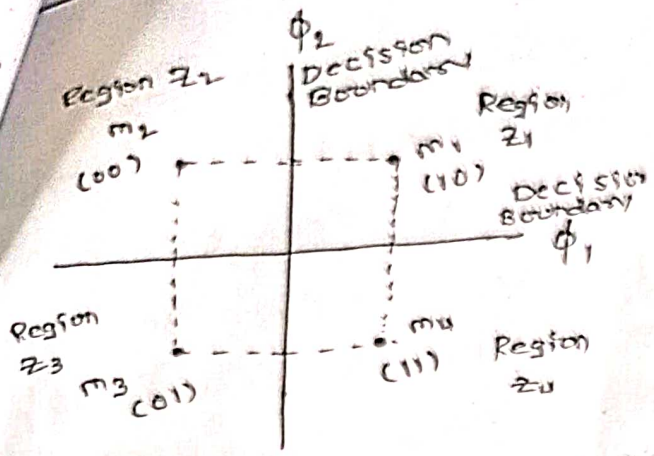


fig:- Signal constellation of BPSK (where phase angles increase in an anti-clockwise direction)

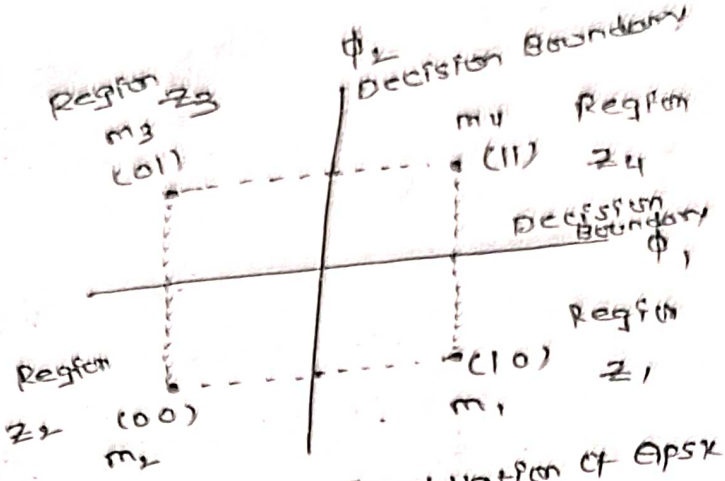


fig:- Signal Constellation of BPSK (where phase angles increase in a clock-wise direction).

Graphical Representation of BPSK

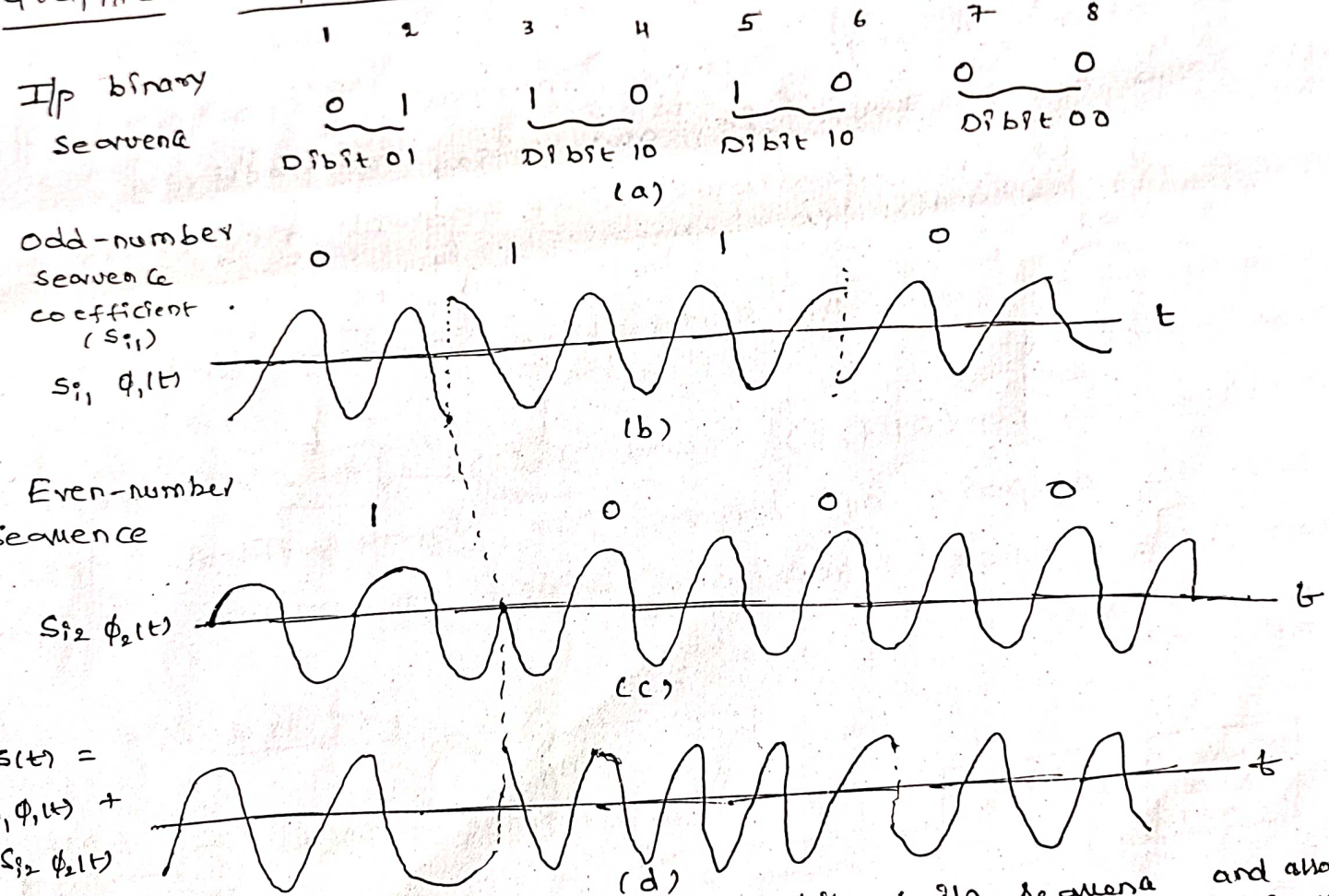


Fig: (a) I/p binary sequence (b) Odd-numbered bits of I/p sequence and associated BPSK wave (c) Even-numbered bits of I/p sequence & associated BPSK wave (d) BPSK wave

Generation and Detection of coherent QPSK signals

The below figure shows a block diagram of a typical QPSK transmitter. The incoming binary data sequence is first transformed into polar form by a nonreturn-to-zero level encoder. Thus, symbols 1 and 0 are represented by $+\sqrt{E_b}$ (or) $+\sqrt{\frac{E}{2}}$ and $-\sqrt{\frac{E}{2}}$ (or) $-\sqrt{E_b}$, respectively.

This binary wave is next divided by means of a demultiplexer into two separate binary waves consisting of the odd- and even-numbered input bits. These two binary waves are denoted by $b_o(t)$ and $b_e(t)$.

Note that in any signaling interval, the amplitudes of $b_o(t)$ and $b_e(t)$ equal s_{i1} and s_{i2}, respectively, depending on the particular dibit that is being transmitted.

The two binary waves $b_o(t)$ and $b_e(t)$ are used to modulate a pair of quadrature carriers (or) orthonormal basis functions:

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t) \quad \text{and} \quad \phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t)$$

The result is a pair of binary PSK signals. Finally, the two binary PSK signals are added to produce the desired QPSK signal.

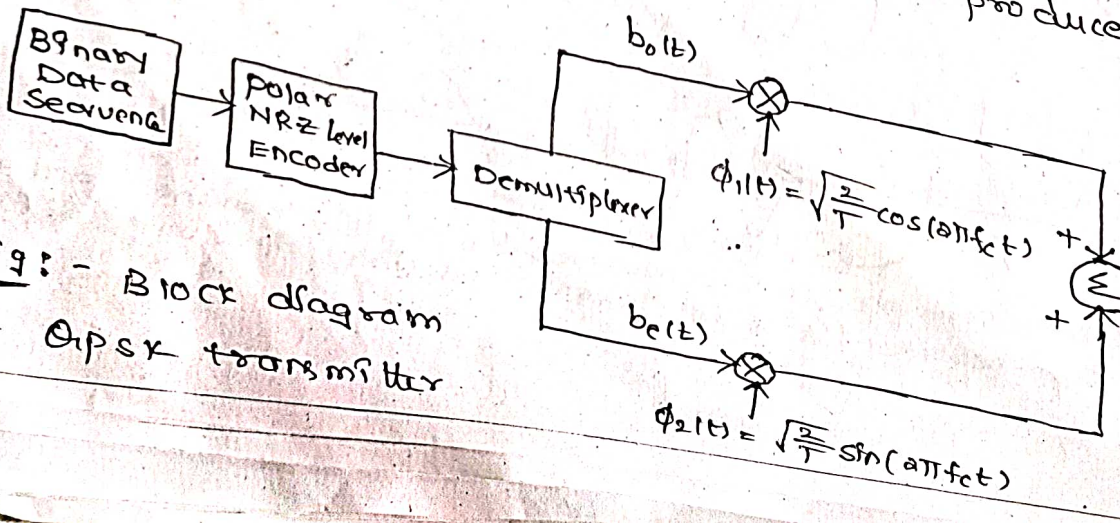


Fig:- Block diagram of QPSK transmitter

Coherent QPSK Receiver :-

The QPSK receiver consists of a pair of correlators with a common input and supplied with a locally generated pair of coherent reference signals $\phi_1(t)$ and $\phi_2(t)$.

The correlator outputs x_1 & x_2, produced in response to the received signal $x(t)$. The observation elements x_1 & x_2 are compared with a threshold of zero. If $x_1 > 0$, the decision is made in favor of '1' for the in-phase channel output, but if $x_1 < 0$, a decision is made in favor of symbol '0'. Similarly, if $x_2 > 0$, a decision is made in favor of symbol 1 for the quadrature channel output, but if $x_2 < 0$, a decision is made in favor of symbol '0'.

Finally, these two binary sequences at the in-phase and quadrature channel outputs are combined in a multiplexer to reproduce the original binary sequence.

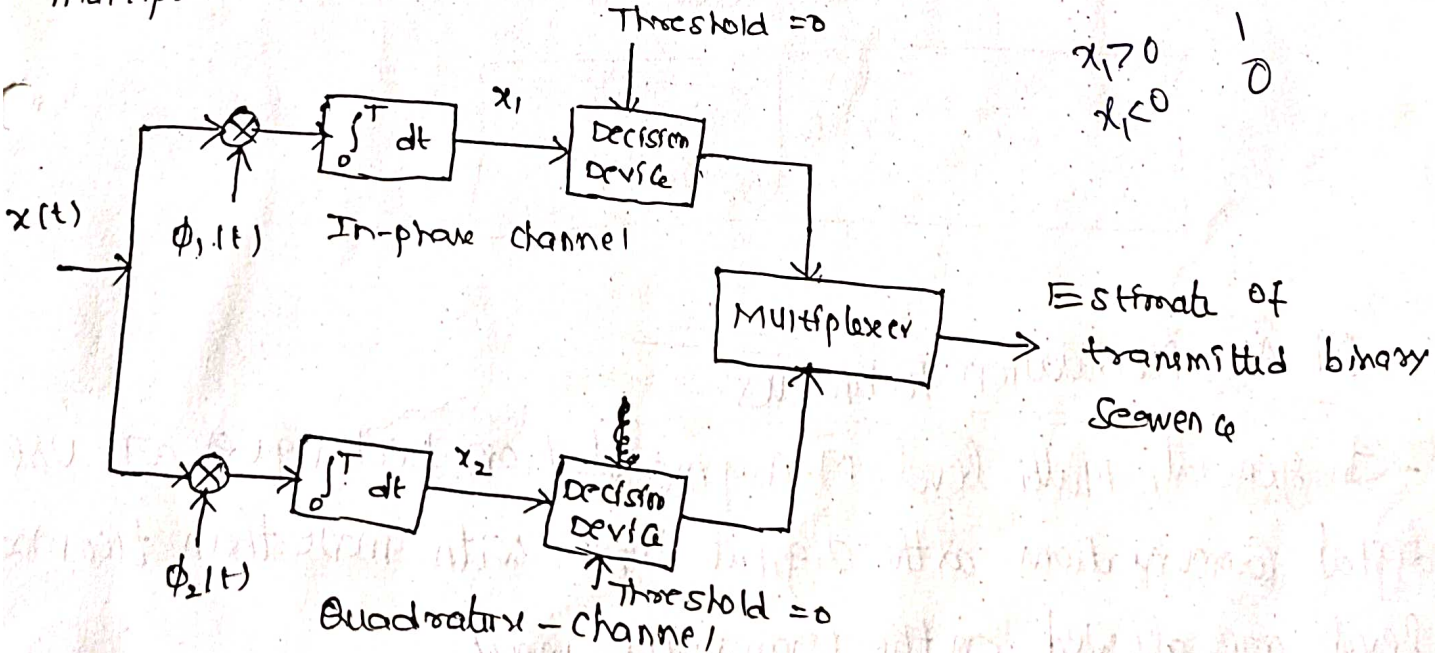


fig:- Coherent QPSK Receiver.

obability of error of QPSK:-

Received sgl is $x(t) = s_i(t) + w(t)$, $i = 1,2,3,4$ ...

Observation elements: $x_1 = \int_0^T x(t)\phi_1(t)\,dt$, $x_2 = \int_0^T x(t)\phi_2(t)\,dt$

we know that $s_i(t) = \sqrt{\frac{2E}{T}}\left[\cos(2i-1)\frac{\pi}{4}\right]\cos(2\pi f_c t) - \sin(2i-1)\frac{\pi}{4}\sin(2\pi f_c t)\Big]$

The inphase channel o/p x_1 & the quadrature channel o/p x_2 may be viewed as individual o/p's of two Coherent BPSK systems. Then the avg. P_e in each channel is

$$P_{BPSK1} = P_{BPSK2} = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right).$$

$\therefore$ Avg. probability of error for QPSK is

$$P_{QPSK} = P_e = \frac{1}{2}\cdot\frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right) + \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right) \cdot \frac{1}{2}$$

(P_{BPSK1}) (P_{BPSK2})

$\{\because$ In QPSK sgl. coefficients are $\sqrt{\frac{E}{2}}$, hence in place of E in BPSK we have $\sqrt{\frac{E}{2}}$.$\}$

$$= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right).$$

$$\therefore P_e = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right), \text{ since } E_b = 2E_b \text{ for QPSK;}$$

$$\Rightarrow P_e = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{2E_b}{2N_0}}\right)$$

$$\therefore P_e = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

M-Ary Modulation Techniques:

- In general, multi-level M-Ary modulation techniques are used in digital communications as the digital inputs with more than two modulation level are allowed on the transmitter input.
- They are Three types of techniques: ~~M-ary Ask (M-ASK)~~ M-ary PSK (M-PSK) ~~M-ary FSK (M-~~

M-ary Equation is $N = \log_2 M$

Constellation of M-ary PSK

M-ARY PSK :-

In M-ARY PSK, in each signaling interval of duration T, one of the M possible signals is sent.

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{2\pi}{M}(i-1)\right), \quad i = 1, 2, \ldots M \rightarrow (1)$$

where, E is the signal energy per symbol.

f_c is carrier frequency, i.e., $f_c = \frac{n_c}{T}$, for some fixed integer of n_c.

and 'T' is the symbol duration.

Eqn (1) can be exponded by some trigonometric function.

$$\rightarrow (2)$$

$$S_i(t) = \sqrt{\frac{2E}{T}} \left[\cos\left[\frac{2\pi}{M}(i-1)\right] \cos(2\pi f_c t) - \sin\left[\frac{2\pi}{M}(i-1)\right] \sin(2\pi f_c t) \right]$$

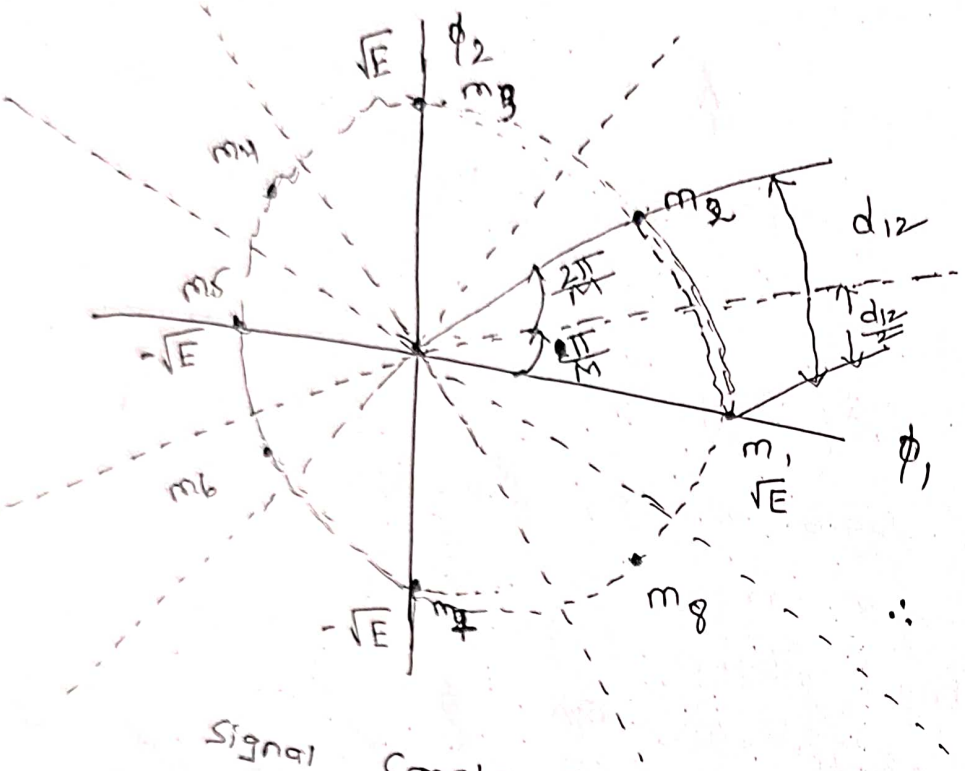
then the quadrature carriers, (or) orthonormal basis functions $\phi_1(t)$ & $\phi_2(t)$, given as

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \rightarrow (3) \text{ \&}$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t) \rightarrow (4)$$

Example :- Consider $M = 8$. which is a octaphase-shift keying.

From the signal space, diagram we note that the signal-space diagram is circularly symmetric. Therefore, we may determine the average probability of symbol error for M-ary PSK [i.e., $M = 8$] based on union boundary probability.



$$\sin\left(\frac{\pi}{M}\right) = \frac{d_{12}/2}{\sqrt{E}}$$

$$\boxed{d_{12} = 2\sqrt{E}\sin\left(\frac{\pi}{M}\right)}$$

$$\therefore d_{12} = d_{18} = 2\sqrt{E}\sin\left(\frac{\pi}{M}\right)$$

Signal Constellation of M-ary PSK ($M = 8$)

$$S_{ij} = \begin{bmatrix} \sqrt{E} & \cos\frac{2\pi}{M}(i-1) \\ -\sqrt{E} & \sin\frac{2\pi}{M}(i-1) \end{bmatrix} \quad \begin{matrix} i = 1, 2, \ldots M \\ j = 1, 2. \end{matrix} \quad (1)$$

Probability error of M-ary PSK : ($M = 8$)

Since the signal constellation (or) signal space diagram is union boundary. Then the average probability of symbol error is

$$P_e \leq \sum_{\substack{k=1 \\ k \neq i}}^{8} \frac{1}{2}\operatorname{erfc}\left(\frac{d_{ik}}{2\sqrt{N_0}}\right)$$

$\because$ $d_{12} = d_{18} = 2\sqrt{E}\sin\left(\frac{\pi}{M}\right)$

$\therefore$ P_e (or) $BER = \frac{1}{2}\operatorname{erfc}\left[\frac{2\sqrt{E}\sin\left(\frac{\pi}{8}\right)}{2\sqrt{N_0}}\right]$

$$BER = \frac{1}{2}\operatorname{erfc}\left[\sqrt{\frac{E}{N_0}}\sin\left(\frac{\pi}{M}\right)\right]$$

(OR)

$$P_e \simeq \operatorname{erfc}\left[\sqrt{\frac{E}{N_0}}\sin\left(\frac{\pi}{M}\right)\right].$$

$N = \log_2 M$

$T = T_b \log_2 M$

(OR)

$T = N T_b$

$E = E_b \log_2 M$

$E = N E_b$

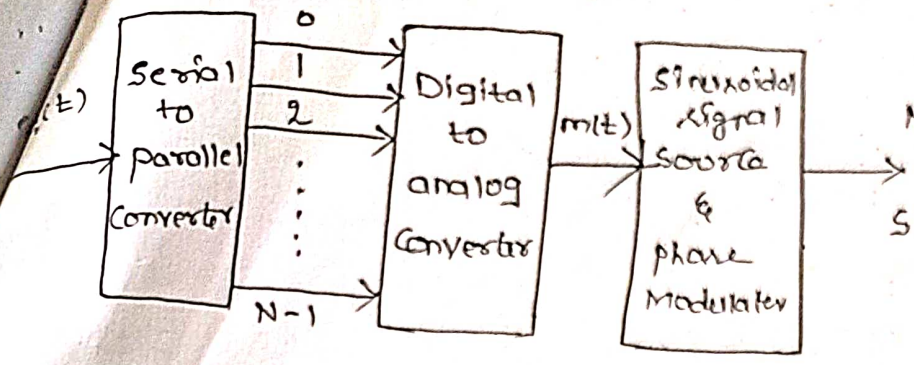


fig :- Transmitter block diagram

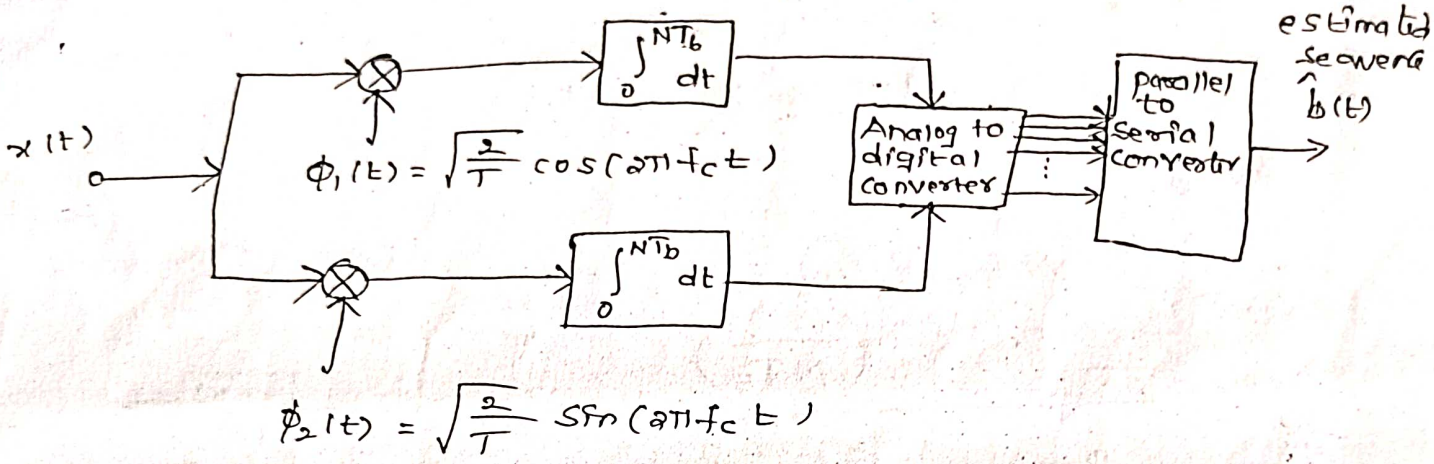


fig :- M-ary psk Receiver.

Power Spectral Density of M-ary PSK :-

The symbol duration of M-ary psk is defined by

$T = T_b \log_2 M$

W.K.T the Baseband power Spectral Density of BPSK is

$S_B(f) = 2E \, \text{sinc}^2(fT)$

$= 2 E_b \log_2 M \, \text{sinc}^2 [f T_b \log_2 M]$

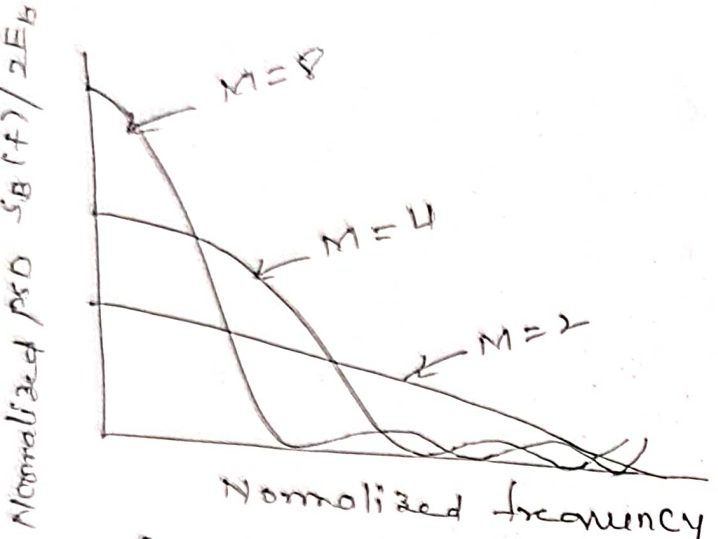


fig :- PSD of M-ary PSK signals, M = 8, 4, 2.

Bandwidth of M-ary PSK :-

W.K.T,

$$B.W = 2f_b$$
$$= 2\left(\frac{1}{NT_b}\right)$$

$$\boxed{B.W = \frac{2f_b}{N}}$$

Comparision of PSK Modulation techniques :-

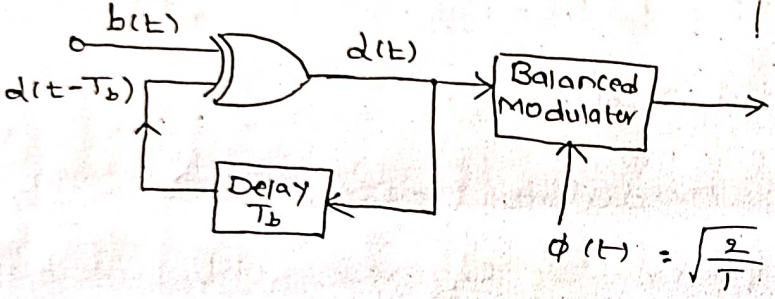
	BPSK	QPSK	M-ary PSK
B.W	$2f_b$	$2\left(\frac{f_b}{2}\right)$	$\frac{2f_b}{N}$
distance	$2\sqrt{E_b}$	$2\sqrt{E_b}$	$2\sqrt{E}\sin\left(\frac{\pi}{M}\right)$
E	E_b	$2E_b$	NE_b, where $N = \log_2 M$
T	T_b	$2T_b$	$T = T_b \log_2 M$
S(t)	$\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \pi)$	$\sqrt{\frac{2E_b}{T_b}}\cos[(2\pi f_c t) + (2i-1)\frac{\pi}{4}]$	$\sqrt{\frac{2E}{T}}\cos\left[2\pi f_c t + [i-1]\frac{2\pi}{M}\right]$
BER	$\frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$	$\simeq \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$ $= \text{erfc}\left(\sqrt{E_b/N_0}\right)$	$\simeq \text{erfc}\left[\sqrt{\frac{E}{N_0}}\sin\left(\frac{\pi}{M}\right)\right]$

(Differential phase-shift keying) :-

DPSK is a non-coherent version of PSK. It eliminates the need of coherent reference signal at the receiver by combining two basic operations at the transmitter: (1) differential encoding of the input binary wave and (2) phase-shift keying, hence the name, DPSK.

Principle :- The i/p sequence of binary bit is modified such that the next bit depends upon the previous bit. Therefore in the receiver the previous received bits are used to detect the present bit.

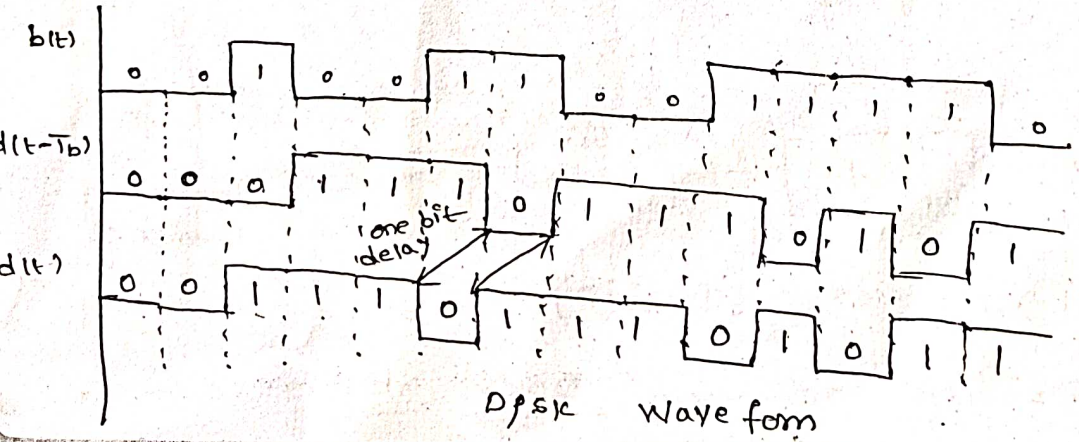
DPSK Transmitter



$$S(t) = \pm \sqrt{\frac{2E}{T}} \cos(2\pi f_c t)$$
$$= \pm \sqrt{2P} \cos(2\pi f_c t)$$

fig:- Block diagram of DPSK transmitter

The input sequence b(t), output sequence is d(t) and d(t-Tb) is the previous output delayed by one bit period. Depending upon values of b(t) and d(t-Tb) exclusive OR gate generates the output sequence d(t).



DPSK Wave form

From the DPSK waveform, it is clear that $d(t-T_b)$ is the delayed version of $d(t)$ by one bit period T_b.

$$\therefore \quad \boxed{d(t) = b(t) \oplus d(t-T_b)}$$

Assume $d(t-T_b) = 0$, initially.

Important Conclusions:

1) Output sequence $d(t)$ changes level at the beginning of each interval in which $b(t) = 1$, and it does not changes level when $b(t) = 0$.

2) When $b(t) = 0$, $d(t) = d(t-T_b)$ &
When $b(t) = 1$, $d(t) = \overline{d(t-T_b)}$

3) The sequence $d(t)$ modulate sinusoidal carrier.

4) The phase of the carrier is changed only if $b(t) = 1$.

* In BPSK, phase of the carrier changes on both the symbol '1' and '0'. Where as in DPSK phase of the carrier changes only on symbol '1'. This is the main difference between BPSK & DPSK.

DPSK Receiver :-

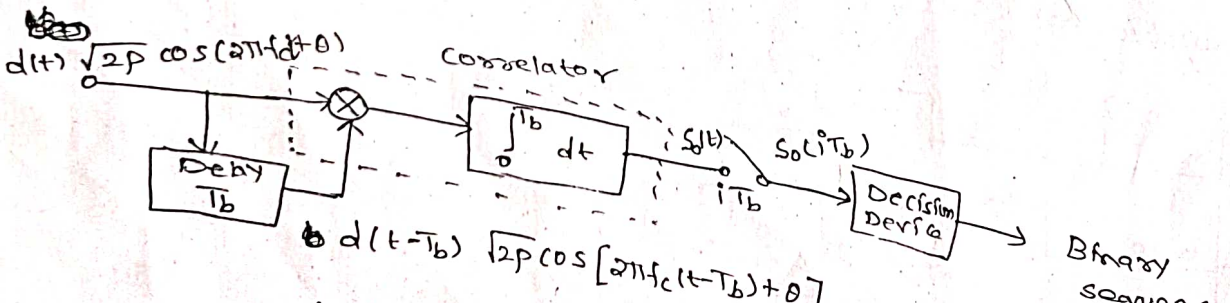


fig :- Non-orthogonal DPSK receiver.

ration of the Receiver :-

During the transmission, the DPSK signal undergoes some phase shift θ. Therefore the signal received at the i/p of the receiver is

$$x(t) = d(t) \sqrt{2P} \cos(2\pi f_c t + \theta)$$

Multiplier output: The signal is multiplied with its delayed version by one bit.

∴ o/p of the multiplier is

$$= d(t) \sqrt{2P} \cos(2\pi f_c t + \theta)\, d(t-T_b) \sqrt{2P} \cos[2\pi f_c (t-T_b) + \theta]$$

$$= d(t)\, d(t-T_b)\, 2P \cos(2\pi f_c t + \theta) \cos[2\pi f_c (t-T_b) + \theta]$$

$$\because \cos A \cos B = \frac{\cos(A+B) + \cos(A-B)}{2}$$

$$d(t)\, d(t-T_b)\, P\left[\cos[(2\pi f_c t + \theta) + (2\pi f_c (t-T_b) + \theta)] + \cos[2\pi f_c t + \theta - 2\pi f_c t + 2\pi f_c T_b - \theta]\right]$$

$$= d(t)\, d(t-T_b)\, P\left[\cos 2\pi f_c T_b + \cos[2\pi f_c t + \theta + 2\pi f_c t - 2\pi f_c T_b + \theta]\right]$$

$$= d(t)\, d(t-T_b)\, P\left[\cos(2\pi f_c T_b) + \cos[4\pi f_c t - 2\pi f_c T_b + 2\theta]\right]$$

$$d(t)\, d(t-T_b)\, P\left[\cos(2\pi f_c T_b) + \cos\left[4\pi f_c \left(t - \frac{T_b}{2}\right) + 2\theta\right]\right]$$

$$\because f_b = \frac{1}{T_b} \qquad f_c = \frac{n}{T_b}$$

$$\boxed{f_c \cdot T_b = n}$$

$$= d(t)\,d(t-T_b)\,P\left[\underbrace{\cos 2\pi n}_{1} + \cos\left[4\pi f_c\left(t-\frac{T_b}{2}\right)+2\theta\right]\right]$$

$\because \cos 2\pi n = 1$;

$$= d(t)\,d(t-T_b)\,P + d(t)\,d(t-T_b)\,P\cos\left[4\pi f_c\left(t-T_b/2\right)+2\theta\right]$$

Integrator :- The above signal is given to the integrater.

In the ith bit interval, the integrater o/p can be written as

$$S_o(iT_b) = d(iT_b)\,d[(i-1)T_b]\,P\int_{(i-1)T_b}^{iT_b} \cdot dt +$$

$$P\,d(iT_b)\,d[(i-1)T_b]\int_{(i-1)T_b}^{iT_b}\cos\left[4\pi f_c\left(t-\frac{T_b}{2}\right)+2\theta\right]\cdot dt.$$

The integration of the second term will be zero. Since it is integration of carrier over one bit duration.

$$\therefore S_o(iT_b) = d(iT_b)\,d[(i-1)T_b]\,P[iT_b-(i-1)T_b]$$

$$= d(iT_b)\,d[(i-1)T_b]\,P[T_b]$$

$$\boxed{S_o(iT_b) = d(iT_b)\,d[(i-1)T_b]\,PT_b}$$

The product $d(iT_b)\,d[(i-1)T_b]$ can be used to determine the transmitted bit $b(t)$.

$\because$ From DPSK waveform, w.k.T, $d(t) = d(t-T_b)$ then $b(t) = 0$

$\therefore d(t)\,d(t-T_b) = 1V$, then $b(t) = 0$

if $d(t) = \overline{d(t-T_b)}$ then $b(t) = 1$

$\therefore d(t)\,d(t-T_b) = -1V$ then $b(t) = 1$

$B.W = 2f_b$

$= \frac{2}{2T_b}$ $\because f_b = \frac{1}{2T_b}$

$\boxed{B.W = f_b}$

Advantage: (1) need not require carrier at receiver (2) B.W < BPSK

Disadvantage: -

Decision Device :-

$$S_o(iT_b) = \begin{cases} -PT_b, & b(t) = 1 \quad \& \\ +PT_b, & \text{then } b(t) = 0 \end{cases}$$

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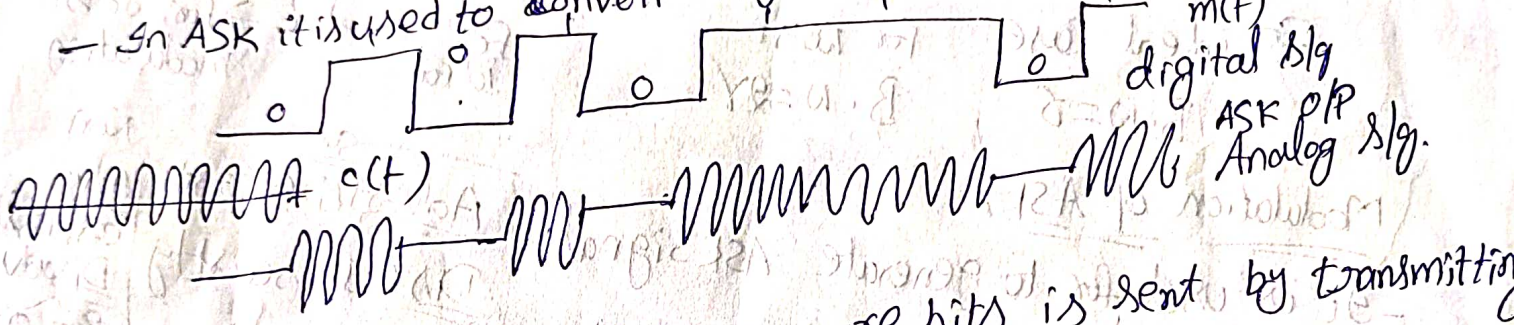
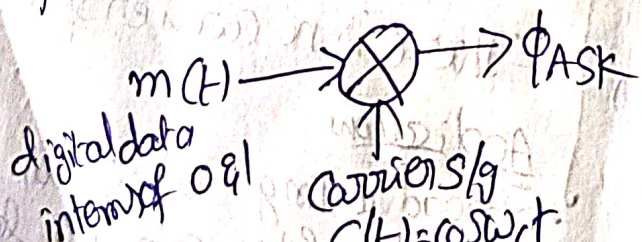
ASK (Amplitude Shift keying): It is one type of digital modulation.

- ASK is a form of amplitude modulation that represents digital data as variations in the amplitude of a carrier wave.
- In ASK it is used to convert digital data into analog form.



- In ASK system, one or more bits is sent by transmitting a fixed amplitude carrier wave at a fixed frequency for a specific time duration.
- A digital modulation uses a finite number of signals to represent digital data.
- In ASK it uses a finite number of amplitudes.

Defi ASK: The Amplitude of the carrier s/g is varies with respect to amplitude of the message signal.



- For binary symbol '1' it is referred as BASK or ON/OFF Shift keying

Bandwidth of ASK:-

$\phi_{ASK} = m(t)\cos\omega_c t$

r [Baud rate]

$\Rightarrow B.W \propto$ Baud Rate

$\Rightarrow B.W = (1+d)r = (1+d)\frac{R}{n}$ [R → data rate]

[n → no. of bits required for sample

d → factor for modulation & filtering process

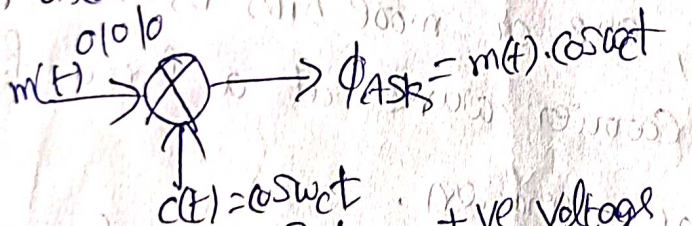
Range of d is (0, 1)

For ideal ← 0, → For worst modulation

For ideal case $B.W = r$, For worst $B.W = 2r$

Modulation of ASK:-

- It is used for to generate ASK signal,



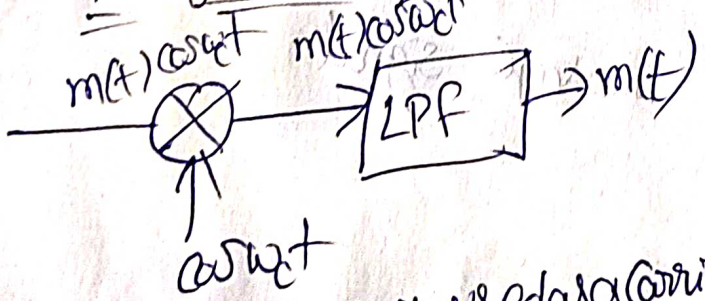
where, $m(t) = \begin{cases} 1 & \text{+ve voltage} \\ 0 & \text{No voltage} \end{cases}$

Sy	Non
Adv: It is efficient	Adv: cost is low
Dis: It is costly	Disadv: Performance is poor with less SNR received s/g.

Demodulation of ASK:-

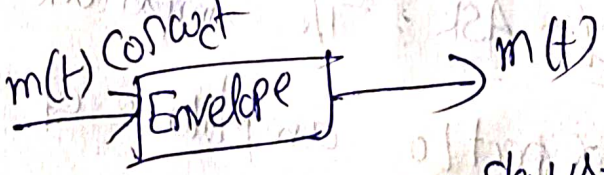
This is having 2 Technique
1) Synchronous (coherent)
2) Non Synchronous (Non coherent)

For Synchronous



- In this oscillator is used as a carrier and its cost is high.

For Non Synchronous



- There is no carrier s/g used here

Application.

→ Broad casting of signal

→ In optical fiber com. for laser intensity modulation it can be used

M-Ary Modulation Techniques:-

- M-Ary in the sense multilevel-Ary modulation.
- In M-Ary transmission is a type of digital modulation where instead of transmitting one bit at a time, two or more bits are transmitted simultaneously.
- M-Ary is used for data Transmission.

M-Ary PSK Modulation:-

- M-Ary phase shift keying or MPSK is a modulation where data bits select one of M-Phase shifted versions of the carrier to transmit the data.
- In MPSK all bits are having same amplitude and freq, but different phase.

the condition for M-ary PSK is $M = 2^n$

and symbol duration $T = nT_b$

and $B.W \propto \frac{1}{nT_b}$

- The use of M-ary PSK provides a reduction in transmission bandwidth by a factor by a factor $\boxed{n = \log_2 M}$

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Invigilator Signature: P_7

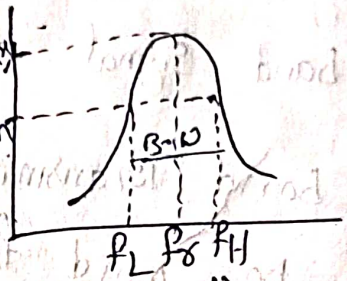
Power Spectra and Bandwidth Efficiency:-

Power: The Power efficiency of digital modulation is measured by E_b/N_0 ratio required at the receiver for a certain bit error rate.

B.W Efficency:- It is measured by ratio of the information rate to the Bandwidth.

- B.W is a information density or spectral density.

B.W formula is $\Delta f = f_H - f_L = f_2 - f_1 = 2\pi LR$



B.W Efficiency is defined as

(Payload bytes carried) ÷ (total bytes carried)

- Efficiency of modulation when it comes to transmit data.

$$B\eta = \frac{\text{transmission bit rate (bps)}}{\text{minimum Bandwidth (Hz)}}$$

...g and Frequency Synchronization:-

...ing synchronization is the Process by which a receiver ...e determines the correct instants of time at which to sample the incoming signal.

- Freq. synchronization is the Process by which a receiver adapts the freq and phase of its local carrier oscillator with those of the received signal.

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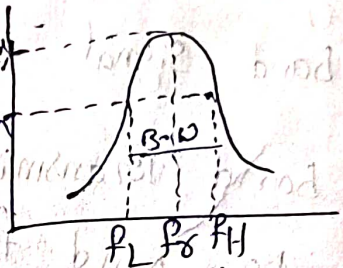

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- Efficiency of modulation when it comes to transmit data.

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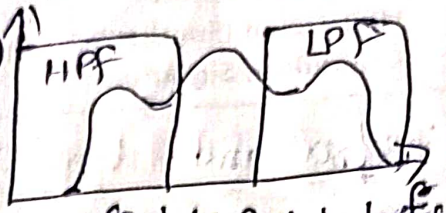
Timing and Frequency Synchronization:-

- Timing synchronization is the Process by which a receiver node determines the correct instants of time at which to sample the incoming signal.
- Freq. synchronization is the Process by which a receiver adapts the freq and phase of its local carrier oscillator with those of the received signal.

UNIT-IV

Pass Band Transmission Model:-

- Pass Band Transmission is typically used for wireless communication and in band pass filtered channels such as POTS lines. It is also called as PSTN (Public Switched Telephone service) or POT (Plain old Telephone service)



- It also allows for FDM.
- The digital bit stream is converted first into an equivalent base band signal and then to RF signal.
- Pass band Transmission is the transmission after shifting the base band frequencies to some higher freq. range using modulation.

formula of Pass band by using BPF is $f_c = \frac{1}{2\pi\sqrt{LC}}$

Adv:- Pass band digital transmission allows more efficient use of allocated RF bandwidth and flexibility in different base band S/g formats.

Dif of Pass band and Base Band Pass:-

- In B.B it uses low freq. and in this signal that has passed through a Band Pass filter.
- In P.B it uses high freq. and in this signal that falls into the Pass band of a given filter.