UNIT-1

Transmission line parameters

⇒ **Current Distortion Effect:** When a conductor carrying a steady direct current [DC] it will distribute the current uniformly over the entire cross-section of the conductor.

In practice, the alternating current [AC] doesn't distribute uniformly over the cross section of the conductor but is distorted due to

1. skin effect.
2. proximity effect
3. spirality effect.

※ **Skin Effect:-** The tendency of alternating current to concentrate near the surface of a conductor is known as skin effect.

Due to the skin effect the effective area of cross-section of the ~~current~~ conductor through which current flows over ~~the~~ c.s. is reduced.

Skin effect depends on following factors: Standard wire has less skin effect as compared to Hollow conductors

1. Nature of material.
2. Diameter of wire - increases with the increase of diameter of wire.
3. frequency - increases with the increase in frequency
4. Shape of wire - less for standard conductors than the solid conductors.
5. Distance between the conductors.
6. Resistivity & permeability of material

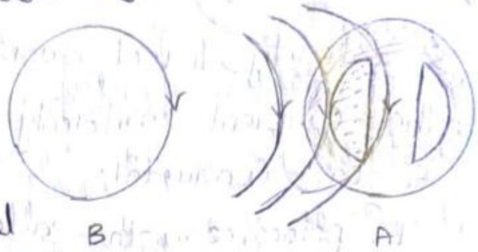
→ Note: Skin effect is negligible when supply frequency is low (50Hz) and conductor diameter is small (<1km).

※ **proximity Effect :-** Non-Uniformity of current in the cross sectional conductor is also observed in the case of proximity effect and is similar to that of skin effect. proximity effect is more in standard conductors.

The alternating magnetic flux in a conductor caused by the current flowing in a neighbouring coil gives rise to circulating current which cause the apparent increase in the resistance of a conductor. This phenomenon is called proximity effect.

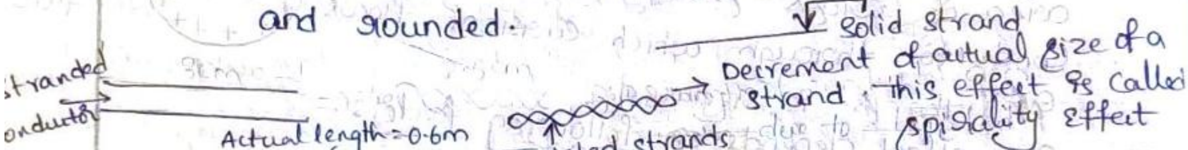
proximity effect is influenced by:

1. Size of the conductor.
2. frequency of supply
3. Resistivity of material.
4. permeability of material
5. Distance between conductors.



* **Spirality Effect:** The magnitude of the effect depends on the size, method of construction of the conductor (i.e., standard & composite).
But this effect at normal frequency is less & can be ignored in non-magnetic conductor.

Eg: Take a two strands & that two strands are twisted together. In order to easily transport the conductors, the strands are twisted and rounded.



⇒ The actual length of the each strand is 0.6m. But the length of the twisted strand is less than 0.6m.

Inductance:

Inductance is a measure of the amount of magnetic flux produced for a given electric current.
It is defined as the flux linkages per unit ampere & is denoted by 'L'.

$$L = \frac{\text{flux linkages}}{\text{Current}} = \frac{\psi}{I} \text{ (Henry)} = \frac{\psi_{int} + \psi_{ext}}{I}$$

for a transmission line, the inductance depends upon the material used & the dimensions of the conductor
where ψ - flux linkages (weber-turns)
I - Current (ampere)

This is true to provide flux linkages of the ckt vary linearly with current.

In other words this means that the magnetic ckt has constant permeability.

If permeability is not constant we have to make use of second fundamental equation, namely

$$e = L\frac{di}{dt} \text{ volt's}$$

where, L = Constant of proportionality

e = induced voltage (v)

$\frac{di}{dt}$ = rate of change of current (Amps/sec)

Mutual inductance b/w the two ckt's is defined as flux linkages of one ckt per current in the other ckt.

i.e., $M_{12} = \frac{\psi_{12}}{I_2}$ H

where,

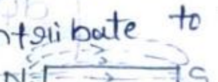
ψ_{12} = flux linkages with ckt (wb, turns)

I_2 = current in ckt

Flux linkages due to a single current carrying conductor:

Consider a long straight cylindrical conductor of radius 'r' meter's and carrying a current 'I' amperes (rms)

This current will set up magnetic field. The magnetic lines of force will exist inside the conductor as well as outside the conductor. Both these fluxes will contribute to the inductance of the conductor.



⇒ **Inductance of a conductor due to internal flux:**

Internal flux & External flux Calculate [illegible] total flux [illegible]

* **Ampere's Law:** According to ampere's law, mmf (amp-turns) around any closed path equals current enclosed by the path. The current enclosed by the path is I_x → mmf = I_x

$B = \frac{\phi}{A}$ [T (wb/m²)]

$H = \frac{mmf}{l}$ $\left[\frac{A\text{-}T}{m^2}\right]$

mmf = H × l

$$mmf = \underbrace{H_x}_{\text{field intensity}} \times \underbrace{2\pi x}_{\text{circumference}}$$

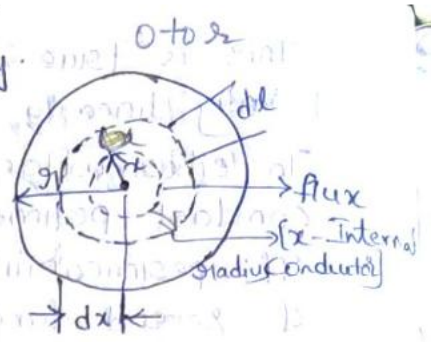
$$H_x \times 2\pi x = I_x$$

The magnetic field intensity at a point 'x' meter's from the center is given by

for conductor L here we take cylindrical conductor, so, circumference of conductor = $2\pi x$

$$H_x = \frac{I_x}{2\pi x} \rightarrow (1)$$

Assuming a uniform current density



$$I_x = \frac{\pi x^2}{\pi r^2} \cdot I$$ → Total conductor area

$$I_x = \frac{x^2}{r^2} \times I$$

Sub I_x in ①

$$H_x = \frac{x^2}{r^2} I \times \frac{1}{2\pi x} = \frac{x}{2\pi r^2} I \; [AT/m]$$

If $\mu = \mu_0 \mu_r$ is the permeability of the conductor, then flux density at the considered point is given by

permeability $\mu = \frac{B}{H}$, $B = \mu H$

$$B_x = \mu_0 \mu_r H_x \quad wb/m^2$$

$$= \frac{\mu_0 \mu_r x}{2\pi r^2} \cdot I = \frac{\mu_0 x}{2\pi r^2} I \; (wb/m^2)$$

$$B_x = \frac{\mu_0 x}{2\pi r^2} I \; (wb/m^2) \rightarrow ②$$ $\because \mu_r = 1$; permeability of air = 1

flux $d\phi$ through a cylindrical shell of radial thickness 'dx' and axial length 1m is given by

$B = \frac{\phi}{A}$, $\phi = BA$

$$d\phi = B_x \times 1 \times dx \overset{\text{from ②}}{=} \frac{\mu_0 x I}{2\pi x^2} dx \; (wb/m)$$

dx part I_x current flow ലെ small flux produce ചെയ്യും.

This current links with current 'I_x' only

∴ flux linkages per meter of the conductor is

$$d\psi = \frac{\pi x^2}{\pi r^2} d\phi = \frac{\mu_0 x I}{2\pi r^2} dx \times \frac{\pi x^2}{\pi r^2}$$ $\boxed{d\psi = I_x \, d\phi}$

$$\boxed{d\psi = \frac{\mu_0 I x^3}{2\pi r^4} dx} \; (wb\text{-}turns/m)$$

Total flux linkages from centre upto the conductor surface is

$$\psi_{int} = \int_0^r \frac{\mu_0 I x^3}{2\pi r^4} dx = \frac{\mu_0}{2\pi r^4} \left[\frac{x^4}{4}\right]_0^r = \frac{\mu_0}{2\pi r^4} \cdot \frac{r^4}{4} = \frac{\mu_0}{8\pi}$$

integral flux 0 - r

$\int x^3 dx = \frac{x^4}{4}$

* $$\boxed{\psi_{int} = \frac{\mu_0 I}{8\pi}} \; (Wb\text{-}turns / metre\ length)$$

where, $\mu_0 = 4\pi \times 10^{-7}$ H/m

$$\psi_{int} = \frac{4\pi \times 10^{-7}}{8\pi} I = \frac{10^{-7} I}{2} \; (wb\text{-}T/m)$$

$$\boxed{\psi_{int} = 0.5 \times 10^{-7} I} \; (wb\text{-}T/m)$$

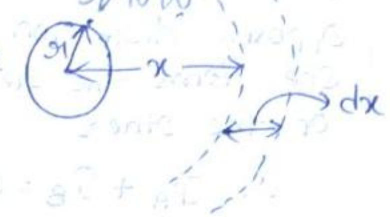
∴ Inductance of a conductor due to internal flux linkages

$$\boxed{L_{int} = \frac{\psi_{int}}{I} = 0.5 \times 10^{-7} \ H/m}$$

⇒ Inductance of a Conductor due to external flux:

The external flux linkages of the conductor is shown in fig:

The external flux extends from the surface of the conductor to infinity



The field intensity at a distance 'x' m (from centre) outside the conductor is given by

$$H_x = \frac{I}{2\pi l} \ (AT/m)$$

mmf = I

$H = \frac{mmf}{l}$

$H = \frac{I}{(2\pi l)}$

flux density, $B_x = \mu_0 H_x = \frac{\mu_0 I}{2\pi x}$ (wb/m^2)

Now, flux $d\phi$ through a cylindrical shell of thickness 'dx' and axial length 'l' metre

$B = \mu_0 \mu_r H$

$\mu_r = 1$

$$d\phi = B_x dx \cdot 1 = \frac{\mu_0 I}{2\pi x} dx \ (wb)$$

$B = \frac{\phi}{A}$

$\phi = BA$

$A = l \times b$

The flux $d\phi$ links all the current in conductor once (and only once (flux encloses only one conductor)

∴ flux linkages,

$$d\psi = d\phi = \frac{\mu_0 I}{2\pi x} dx \ (wb\text{-turns})$$

Total flux linkages of the conductor from surface to infinity

$$\boxed{\psi_{ext} = \int_r^{\infty} \frac{\mu_0 I}{2\pi x} dx} \ (wb\text{-Turns})$$

Overall flux linkages:

$$\psi = \psi_{int} + \psi_{ext}$$

$$= \frac{\mu_0 I}{8\pi} + \int_r^{\infty} \frac{\mu_0 I}{2\pi x} dx$$

$$= \frac{\mu_0 I}{8\pi} + \frac{\mu_0 I}{2\pi} \int_r^{\infty} \frac{dx}{x}$$

$$\because \int \frac{1}{x} dx = \log x$$

$$= \frac{\mu_0 I}{2\pi} \left[\frac{1}{4} + \int_r^{\infty} \frac{1}{x} dx\right]$$

$$\cancel{= \frac{\mu_0 I}{2\pi} \left(\frac{1}{4} + (\log x)_r^{\infty}\right)}$$

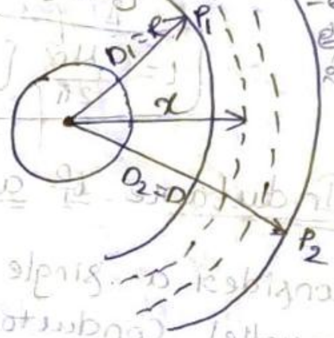
$$\cancel{= \frac{\mu_0 I}{2\pi} \left(\frac{1}{4} + \log\right.}$$

$$\psi = \frac{\mu_0 I}{2\pi}\left[\frac{1}{4} + \int_{r}^{\infty}\frac{dx}{x}\right] \text{ wb turn's/m length}$$

If External flux tends from surface of a Conductor from distance D_1 to D_2, then

let us Consider flux linkages of an isolated Conductor which lie between two points P_1 and P_2 distance D_1 and D_2 respectively from the centre of the conductor.

Since, the flux paths are concentric circles around the conductor, all the flux between P_1 and P_2 lies within concentric cylindrical surfaces passing through points P_1 & P_2. Similar to the above procedure.



Total flux linkages from the surface of the conductor of a distance D_1 to D_2 is

$$\psi_{ext} = \int_{D_1}^{D_2} \frac{\mu_0 I}{2\pi l}\, dx$$

$\int \frac{1}{x} dx = \log x$

$$= \frac{\mu_0 I}{2\pi}\left[\int_{D_1}^{D_2} \frac{1}{x}\, dx\right]$$

$$= \frac{\mu_0 I}{2\pi}\left[\log x\right]_{D_1}^{D_2} = \frac{\mu_0 I}{2\pi}\left[\log D_2 - \log D_1\right]$$

$$\boxed{\psi_{ext} = \frac{\mu_0 I}{2\pi}\left[\log \frac{D_2}{D_1}\right]} \ \omega_b\text{-T/m}$$

$\because \log A - \log B = \log \frac{A}{B}$

$\therefore$ Inductance

Overal flux linkages:

$$\psi = \psi_{int} + \psi_{ext}$$

$$= \frac{\mu_0 I}{8\pi} + \frac{\mu_0 I}{2\pi}\left[\log \frac{D_2}{D_1}\right]$$

from fig
$D_1 = D$
$D_2 = R$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{4} + \log \frac{D_2}{D_1}\right]$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{4} + \log \frac{D}{R}\right]$$

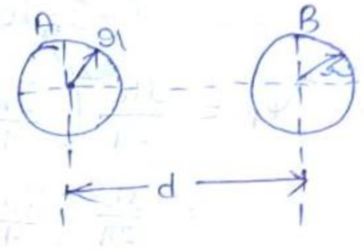
$$\boxed{\psi = \frac{\mu_0 I}{2\pi}\left[\frac{1}{4} + \log \frac{D}{R}\right]} \ (\text{wb-T/m})$$

$$\therefore L = \frac{\psi}{I}$$

$$\boxed{L_{ext} = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \ln\frac{D}{R}\right]} \quad (\text{wb-T/m})$$

⇒ Inductance of a single phase two-wire line:

Consider a single phase overhead line consisting of two parallel conductors A and B spaced 'd' meters apart as shown in fig



Conductors A and B carry the same amount of current (i.e., $I_A = I_B$), but in the opposite direction because one forms the return ckt of the other. $I_A = -I_B$

$$\therefore I_A + I_B = 0$$

flux linkages with conductor A due to it's own current

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \int_r^{\infty}\frac{dx}{x}\right] \longrightarrow ①$$

flux linkages with conductor A due to I_B. $\int_r^{\alpha} \frac{\mu_0 I_B\, dx}{2\pi x}$ $L = \frac{\psi}{I}$

$$= \frac{\mu_0 I_B}{2\pi}\int_d^{\alpha}\frac{dx}{x} \longrightarrow ②$$

Total flux linkages with conductor A is

$$\psi_A = \text{Eq } ① + \text{Eq } ②$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \int_r^{\infty}\frac{dx}{x}\right] + \frac{\mu_0 I_B}{2\pi}\int_d^{\alpha}\frac{dx}{x}$$

$$= \frac{\mu_0}{2\pi}\left\{\left(\frac{1}{4} + \int_r^{\alpha}\frac{dx}{x}\right)I_A + \left(\int_d^{\alpha}\frac{dx}{x}\right)I_B\right\}$$

$$= \frac{\mu_0}{2\pi}\left[I_A\left\{\frac{1}{4} + (\log x)_r^{\alpha}\right\} + I_B\left\{(\log x)_d^{\alpha}\right\}\right]$$

$$= \frac{\mu_0}{2\pi}\left\{I_A\left[\frac{1}{4} + (\log_e \alpha - \log_e r)\right] + I_B\left[\log_e \alpha - \log_e d\right]\right\}$$

$$= \frac{\mu_0}{2\pi}\left\{I_A\left[\frac{1}{4} + \log_e \alpha - \log_e r\right] + I_B\left[\log_e \alpha - \log_e d\right]\right\}$$

$$= \frac{\mu_0}{2\pi}\left\{\frac{I_A}{4} + \log_e \alpha \cdot I_A - \log_e r \cdot I_A + \log_e \alpha \cdot I_B - \log_e d \cdot I_B\right\}$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + \log_e \infty (I_A + I_B) - I_A \log_e r - \log_e d \cdot I_B\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + \log_e \infty (0) - I_A \log_e r - I_B \log_e d\right]$$

$\because I_A + I_B = 0$ or $I_A = -I_B$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + 0 - I_A \cdot \log_e r - I_B \cdot \log_e d\right]$$

$$\because I_A = -I_B$$

$$-I_B \log_e d = I_A \log_e d$$

$$\therefore \psi_A = \frac{\mu_0}{2\pi}\left(\frac{I_A}{4} - I_A \log_e r + I_A \log_e d\right]$$

$$\cancel{= \frac{\mu_0}{2\pi}\left(\frac{I_A}{4}\right.}$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} - \log_e r + \log_e d\right]$$

$\left[\because \log a - \log b = \log \frac{a}{b}\right]$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \log_e\left(\frac{d}{r}\right)\right]$$

$\therefore$ Inductance of conductor A,

$$L_A = \frac{\psi_A}{I_A}$$

$$L_A = \frac{\mu_0 \cancel{I_A}}{2\pi}\left[\frac{1}{4} + \log_e\left(\frac{d}{r}\right)\right] \cdot \frac{1}{\cancel{I_A}}$$

$$L_A = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e\left(\frac{d}{r}\right)\right] \text{ (H/m)}$$

$$= \frac{\cancel{4}^{2}\pi \times 10^{-7}}{\cancel{2}\pi}\left[\frac{1}{4} + \log_e\left(\frac{d}{r}\right)\right]$$

$$= 2 \times 10^{-7}\left(\frac{1}{4} + \log_e\left(\frac{d}{r}\right)\right)$$

$$\boxed{L_A = 10^{-7}\left[\frac{1}{2} + 2\log_e\left(\frac{d}{r}\right)\right]} \text{ (H/m)}$$

loop inductance $= 2L_A$ H/m

$$= 2 \times 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{d}{r}\right] \text{ H/m}$$

$$= 10^{-7}\left[1 + 4\log_e \frac{d}{r}\right] \text{ H/m}$$

$$\boxed{\therefore \text{ loop inductance} = 10^{-7}\left[1 + 4\log_e\left(\frac{d}{r}\right)\right]} \text{ (H/m)}$$

* Expression in alternate form:

The expression for the inductance of a conductor can be put in a concise form.

$$L_A = 10^{-7}\left[\frac{1}{2} + 2\log_e\left(\frac{d}{r}\right)\right] \text{ H/m}$$

$$= 2\times10^{-7}\left[\frac{1}{4} + \log_e\frac{d}{r}\right] = 2\times10^{-7}\left[\log_e e^{1/4} + \log_e\frac{d}{r}\right]$$

$$= 2\times10^{-7}\log_e\frac{d}{r e^{-1/4}}$$

If we put $re^{-1/4} = r'$, then

$$\boxed{L_A = 2\times10^{-7}\log_e\left(\frac{d}{r'}\right) \text{ H/m}}$$

The radius r' is that of a fictitious conductor assumed to have no internal flux but with the same inductance as the actual conductor of radius r.

The quantity $e^{-1/4} = 0.7788$, so that

$$\boxed{r' = re^{-1/4} = 0.7788r}$$

The term r' is called "Geometric Mean Radius" (GMR) of the wire.

Note: $r' = 0.7788r$ is applicable to only solid round conductor

Loop inductance $= 2L_A$

$$= 2\times2\times10^{-7}\log_e\left(\frac{d}{r'}\right) \text{ H/m}$$

⇒ Inductance of a 3-φ Overhead line:

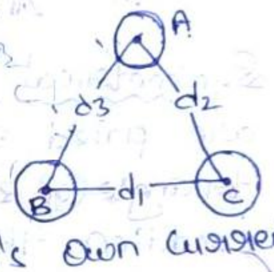


fig: shows the three conductors A, B and C of a 3-φ line carrying currents I_A, I_B and I_C respectively.

Let d_1, d_2 and d_3 be the spacings between the conductors as shown.

flux linkages with conductor A due to it's own current

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \int_{r}^{\infty}\frac{dx}{x}\right] \longrightarrow (1)$$

flux linkages with conductor A due to current I_B

$$= \frac{\mu_0 I_B}{2\pi}\int_{d_3}^{\infty}\frac{dx}{x} \longrightarrow (2)$$

flux linkages with conductor A due to current I_C

$$= \frac{\mu_0 I_C}{2\pi}\int_{d_2}^{\infty}\frac{dx}{x} \longrightarrow (3)$$

Total flux linkages with Conductor A

$$\psi_A = \text{Eq}(1) + \text{Eq}(2) + \text{Eq}(3)$$

$$\psi_A = \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \int_{r}^{\infty}\frac{dx}{x}\right] + \frac{\mu_0 I_B}{2\pi}\int_{d_3}^{\infty}\frac{dx}{x} + \frac{\mu_0 I_C}{2\pi}\int_{d_2}^{\infty}\frac{dx}{x}$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} + \int_{r}^{\infty}\frac{dx}{x}\right)I_A + \left(\int_{d_3}^{\infty}\frac{dx}{x}\right)I_B + \left(\int_{d_2}^{\infty}\frac{dx}{x}\right)I_C\right]$$

$$\psi_A = \frac{\mu_0}{2\pi}\left\{\left[\frac{1}{4} + [\log_e x]_{r}^{\infty}\right]I_A + [\log_e x]_{d_3}^{\infty} I_B + [\log_e x]_{d_2}^{\infty} I_C\right\}$$

$$= \frac{\mu_0}{2\pi}\left\{\left[\frac{1}{4} + (\log_e \infty - \log_e r)\right]I_A + I_B(\log_e \infty - \log_e d_3) + I_C(\log_e \infty - \log_e d_2)\right\}$$

$$= \frac{\mu_0}{2\pi}\left\{\left[I_A\left(\frac{1}{4} - \log_e r\right) - I_B \log_e d_3 - I_C \log_e d_2\right] + \left[\log_e \infty\,(I_A + I_B + I_C)\right]\right\}$$

$\because I_A + I_B + I_C = 0$

$$\therefore \psi_A = \frac{\mu_0}{2\pi}\left\{\left[\left(\frac{1}{4} - \log_e r\right)I_A - I_B \log_e d_3 - I_C \log_e d_2\right] + [\log_e \infty (0)]\right\}$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right)I_A - I_B \log_e d_3 - I_C \log_e d_2\right]$$

The total flux linkages per metre length of conductor 'A' is

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right)I_A - I_B \log_e d_3 - I_C \log_e d_2\right]$$

(i) Symmetrical spacing:- If the three conductors A, B and C are placed symmetrically at the corners of an equilateral triangle of side d, then $d_1 = d_2 = d_3 = d$. Under such condition the flux linkages with conductor A become,

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right)I_A - I_B \log_e d_3 - I_C \log_e d_2\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right)I_A - I_B \log_e d - I_C \log_e d\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right)I_A - (I_B + I_C)\log_e d\right] \quad \because I_B + I_C = -I_A$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} + \log_e\left(\frac{1}{r}\right)\right)I_A - (-I_A)\log_e d\right]$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \log_e\left(\frac{1}{r}\right) + \log_e d\right]$$

$$\boxed{\therefore \psi_A = \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \log\left(\frac{d}{r}\right)\right] \text{ wb-T/m}}$$

Inductance of conductor A,

$$L_A = \frac{\psi_A}{I_A} \ H/m$$

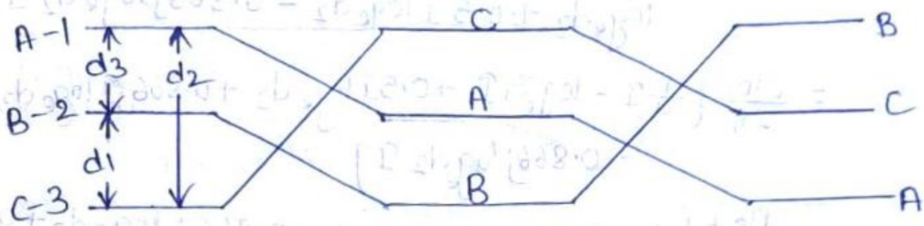
$$L_A = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e\left(\frac{d}{r}\right)\right] \ H/m$$

$$L_A = \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e\left(\frac{d}{r}\right)\right] \ H/m$$

$$\boxed{\therefore L_A = 10^{-7}\left[0.5 + 2\log_e\left(\frac{d}{r}\right)\right] \ H/m}$$

Similarly the expressions for inductance are the same for conductor B and C.

(ii) Unsymmetrical Spacing :- when 3-φ line conductors are not equidistant from each other, the conductor spacing is said to be unsymmetrical.



Under such conditions, the flux linkages and inductance of each phase are not the same. A different inductance in each phase results in unequal voltage drop in the three phases even if the currents in the conductors are balanced.

Therefore, the voltage at the receiving end will not be same for all phases. In order that voltage drops are equal in all conductors, we generally inter change the positions of the conductors at regular intervals along the line so that each conductor occupies the original position of every other conductors over an equal distance. Such an exchange of positions is known as "transposition".

Fig shows the 3-φ transposed line having unsymmetrical spacing. let each of the three sections is 1m in length. Assume balance conditions i.e., $I_A + I_B + I_C = 0$, let the line currents be:

$$I_A = I(1+j0)$$
$$I_B = I(-0.5 - j0.866)$$
$$I_C = I(-0.5 + j0.866)$$

the total flux linkages per metre length of conductor A is

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d_3 - I_C \log_e d_2\right]$$

putting the values of I_A, I_B & I_C we get

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I(1+j0) - \log_e d_3\, I(-0.5 - j0.866) - \log_e d_2\, I(-0.5 + j0.866)\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} I - \log_e r\, I + 0.5 I \log_e d_3 + 0.866 j \log_e d_3 + 0.5 I \log_e d_2 - 0.866 j \log_e d_2 I\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4} I - \log_e r\, I + 0.5 I \log_e d_3 + 0.866 j I \log_e d_3 + 0.5 I \log_e d_2 - 0.866 j \log_e d_2 I\right]$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{4} - \log_e r + 0.5 \log_e d_3 + 0.866 j \log_e d_3 + 0.5 \log_e d_2 - 0.866 j \log_e d_2\right]$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{4} - \log_e r + 0.5 \log_e d_2 d_3 + 0.866 j (\log_e d_3 - \log_e d_2)\right]$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{4} - \log_e r + \log_e \sqrt{d_2 d_3} + j 0.866 \log_e\left(\frac{d_3}{d_2}\right)\right]$$

$$\psi_A = \frac{\mu_0 I}{2\pi}\left(\frac{1}{4} + \log_e\left(\frac{\sqrt{d_2 d_3}}{r}\right) + j\, 0.866 \log_e\left(\frac{d_3}{d_2}\right)\right]$$

∴ Inductance of conductor A is

$$L_A = \frac{\psi_A}{I_A} = \frac{\psi_A}{I}$$

$$L_A = \frac{\mu_0 \cancel{I}}{2\pi}\left[\frac{1}{4} + \log_e\left(\frac{\sqrt{d_2 d_3}}{r}\right) + j 0.866 \log_e\left(\frac{d_3}{d_2}\right)\right] \times \frac{1}{\cancel{I}}$$

$$\boxed{\therefore L_A = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e\left(\frac{\sqrt{d_2 d_3}}{r}\right) + j 0.866 \log_e\left(\frac{d_3}{d_2}\right)\right]} \text{ H/m}$$

Similarly inductance of conductor B and C will be

$$\boxed{\therefore L_B = 10^{-7}\left[\frac{1}{2} + 2\log_e\left(\frac{\sqrt{d_3 d_1}}{r}\right) + j 1.732 \log_e\left(\frac{d_1}{d_3}\right)\right] \text{ H/m}}$$

$$\boxed{L_C = 10^{-7}\left[\frac{1}{2} + 2\log_e\frac{\sqrt{d_1 d_2}}{r} + j1.732\,\log_e\left(\frac{d_2}{d_1}\right)\right]} \; H/m$$

Inductance of each line conductor

$$L = \frac{1}{3}\left[L_A + L_B + L_C\right]$$

On solving

$$L = \left[\frac{1}{2} + 2\log_e\frac{\sqrt[3]{d_1 d_2 d_3}}{r}\right]\times 10^{-7} \; H/m$$

$$= \left[0.5 + 2\log_e\frac{\sqrt[3]{d_1 d_2 d_3}}{r}\right]\times 10^{-7} \; H/m$$

By comparing the formula of inductance of an unsymmetrical spacing transposed line with that of symmetrically spaced line, the inductance of each line conductor in the two cases will be equal if $d = \sqrt[3]{d_1 d_2 d_3}$. The distance 'd' is known as "Equivalent Equilateral Spacing" for unsymmetrically transposed line.

Self GMD: (D_s)

GMD stands for "Geometrical Mean Distance". Consider the expression for inductance per conductor per metre.

$$\text{Inductance/conductor/m} = 2\times10^{-7}\left[\frac{1}{4} + \log\frac{d}{r}\right]$$

$$= 2\times10^{-7}\times\frac{1}{4} + 2\times10^{-7}\log\frac{d}{r}$$

In the above expression, the term $2\times10^{-7}\times(1/4)$ is the inductance due to flux within the solid conductor. If we replace the original solid conductor by an equivalent hollow cylinder with extremely thin walls, the current is confined to the conductor surface and internal conductor flux linkage would be almost zero. Consequently, inductance due to internal flux would be zero and the term $2\times10^{-7}\times\frac{1}{4}$ shall be eliminated. The radius of this equivalent hollow cylinder must be sufficiently smaller than the physical radius of the conductor to allow room for enough additional flux to compensate for the absence of internal flux linkages.

$$\text{Inductance/conductor/m} = 2\times10^{-7}\log_e\frac{d}{r_1}$$

$$= 2\times10^{-7}\log_e\frac{d}{D_s}$$

where, D_s = GMR or self GMD = $0.7788r$

Note:- Self GMD of a conductor depends upon size and shape of the conductor and is independent of the spacing b/w the conductors.

Mutual GMD:- The mutual GMD is the geometrical mean of the distances from one conductor to the other and must be between the largest and smallest such distance. Infact mutual GMD simply represents the equivalent geometrical spacing.

⇒ The mutual GMD b/w two conductors (assuming that spacing b/w conductors is large compared to the diameter of each conductor) is equal to the distance b/w the centres i.e.,

$$D_m = \text{spacing b/w conductors} = d$$

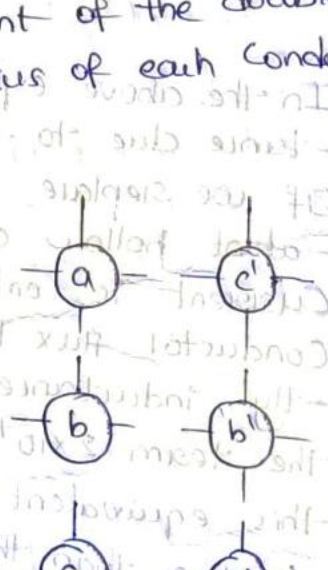
⇒ for a single ckt 3-φ line, the mutual GMD is equal to the equivalent equilateral spacing i.e., $(d_1 d_2 d_3)^{1/3}$

$$D_m = (d_1 d_2 d_3)^{1/3}$$

⇒ The principle of geometrical mean distances can be employed to 3-φ double circuit lines.

Consider the conductor arrangement of the double ckt shown in fig. Suppose the radius of each conductor is 'r'.

Self GMD of conductor $= 0.7788r$



Self-GMD of combination aa' is

$$D_{s1} = (D_{aa} \times D_{aa'} \times D_{a'a'} \times D_{a'a})^{1/4}$$

Self-GMD of combination bb' is

$$D_{s2} = (D_{bb} \times D_{bb'} \times D_{b'b'} \times D_{b'b})^{1/4}$$

Self GMD of combination cc' is

$$D_{s3} = (D_{cc} \times D_{cc'} \times D_{c'c'} \times D_{c'c})^{1/4}$$

Equivalent Self-GMD of one phase

$$D_s = (D_{s1} \times D_{s2} \times D_{s3})^{1/3}$$

The value of D_s is the same for all the phases as each conductor has the same radius

Mutual GMD b/w places A and B is

$$D_{AB} = (D_{ab} \times D_{ab'} \times D_{a'b} \times D_{a'b'})^{1/4}$$

Mutual GMD b/w phases B and C is

$$D_{BC} = (D_{bc} \times D_{bc'} \times D_{b'c} \times D_{b'c'})^{1/4}$$

Mutual GMD b/w phases C and A is

$$D_{CA} = (D_{ca} \times D_{ca'} \times D_{c'a} \times D_{c'a'})^{1/4}$$

Equivalent Mutual GMD

$$D_m = (D_{AB} \times D_{BC} \times D_{CA})^{1/3}$$

Note:- Mutual GMD depends only upon the spacing and is substantially independent of the exact size and shape & orientation of the conductor.

Inductance formulas in term's of GMD:

i) Single phase line: Inductance/conductor/m $= 2\times10^{-7} \log_e \frac{D_m}{D_s}$

where, $D_s = 0.7788r$

D_m = spacing b/w conductor $= d$

ii) Single circuit 3-ϕ line:

Inductance/phase/m $= 2\times10^{-7} \log_e \frac{D_m}{D_s}$

where, $D_s = 0.7788r$

$D_m = (d_1 d_2 d_3)^{1/3}$

iii) Double circuit 3-ϕ line:

Inductance/phase/m $= 2\times10^{-7} \log_e \frac{D_m}{D_s}$

where,

$D_s = (D_{s1} D_{s2} D_{s3})^{1/3}$ &

$D_m = (D_{AB} \times D_{BC} \times D_{CA})^{1/3}$

problem's on loop Inductance:-

① A single phase line has two parallel conductors 2m apart. The diameter of each conductor is 1.2cm. Calculate the loop inductance/km of the line.

Sol Spacing, $d = 2\,m = 2\times100\,cm$

Radius $= \frac{d}{2} = \frac{1.2}{2} = 0.6\,cm$

loop inductance/m of the line $= 10^{-7}\left(1 + 4\log_e \frac{d}{r}\right)$ H

$= 10^{-7}\left(1 + 4\log_e \frac{200}{0.6}\right)$

$= 2.424 \times 10^{-6}$ H

loop Inductance per km of the line

$= 2.423 \times 10^{-6} \times 1000 H$

$= 242.36 \mu H$

(2) A single phase transmission line has two parallel Conductors 3m apart, the radius of each conductor being 1cm. Calculate the loop inductance / km length of the line if the material of the conductor is

(i) copper (ii) steel with relative permeability of 100

Sol Given:

Distance b/w the Conductor, $d = 3m = 300cm$

radius, $r = 1 cm$

loop inductance $= 10^{-7}(\mu_r + 4\log_e \frac{d}{r})$ H/m

i) Copper : $\mu_r = 1$

loop inductance/km $= 10^{-7}(1 + 4\log_e \frac{d}{r})$ H $\times 1000$

$= 10^{-7}(1 + 4\log_e \frac{300}{1}) \times 1000$

$= 2.38 mH$

ii) Steel : $\mu_r = 100$

loop inductance /km $= 10^{-7}(100 + 4\log_e \frac{d}{r}) \times 1000$

$= 10^{-7}(100 + 4\log_e (\frac{300}{1})) \times 1000$

$= 12.28 mH$

(3) find the inductance/km of a 3-φ transmission line using 1.24cm diameter conductor when these are placed at the corners of an equilateral triangle of each side 2m.

Sol Given: $d = 2m = 200cm$

$r = \frac{d}{2} = \frac{1.24}{2} = 0.62 cm$

Loop inductance / phase/km $= 10^{-7}(1 + 4\log_e \frac{d}{r}) \times 1000$

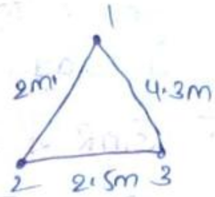
$= 10^{-7}(1 + 4\log \frac{200}{0.62}) \times 1000$

$= 2.311 mH$

(4) the three Conductors of a 3-φ line are arranged at the corners of a triangle of sides 2m, 2.5m and 4.5m Calculate the ~~inductors~~ inductance /km of the line with when the Conductors are regularly transposed.

The diameter of each Conductor is 1.24cm

Sol $D_{12} = 2m$, $D_{23} = 2.5m$, $D_{31} = 4.5m$



$r = \frac{1.24}{2} = 0.62cm$

$D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}} = \sqrt[3]{2 \times 2.5 \times 4.5}$

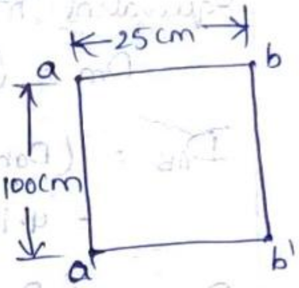
$D_{eq} = 282\,Cm$

$L/ph/Km = 10^{-7}\left(0.5 + 4\log_e \frac{282}{0.62}\right) \times 1000$

$= 2.448 \times 10^{-3} H$

(5) Two Conductors of a 1-φ line, each of 1cm diameter, are arranged in a vertical plane with one conductor mounted 1m above the other. A second identical line is mounted at the same length as the first and spaced horizontally 0.25m apart from it. The two upper and the two lower Conductors are connected in parallel. Determine the inductance /km of the resulting double ckt line.

Sol fig shows the 1-φ double ckt line. conductors aa' from One connection and Conductors bb' from the return Connection. The conductor radius,

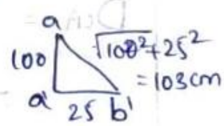


$r = 1/2 = 0.5cm$

GMR of conductor $= 0.7788\,r$

$= 0.7788 \times 0.5$

$= 0.389cm$



Self GMD of aa' combination is

$D_s = (D_{aa} \times D_{aa'} \times D_{a'a'} \times D_{a'a})^{1/4}$

$= (0.389 \times 100 \times 0.389 \times 100)^{1/4} = 6.2cm$

Mutual GMD b/w a & b is

$D_m = (D_{ab} \times D_{ab'} \times D_{a'b} \times D_{a'b'})^{1/4}$

$= (25 \times 103 \times 103 \times 25)^{1/4} = 50.74cm$

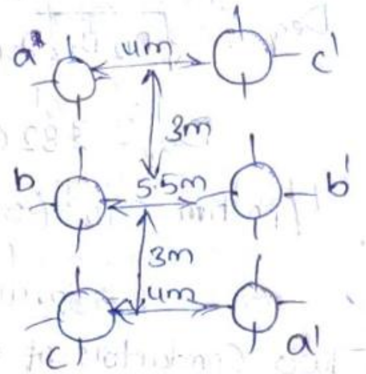
Inductance/Conductor/m $= 2 \times 10^{-7} \log_e \frac{D_m}{D_s} = 2 \times 10^{-7} \times \log_e \frac{50.74}{6.2}$

$= 0.42 \times 10^{-6} H$

loop Inductance/km $= 2 \times 0.42 \times 10^{-6} \times 1000 = 0.84mH$

⑥ find the inductance/phase/km of double ckt of 3-ϕ line shown in fig. The conductor are transposed and are of radius 0.75cm each. The phase sequence is ABC

Sol GMR of a conductor $= 0.75 \times 0.7788 = 0.584\,cm$



$D_{ab} = \sqrt{3^2+(0.75)^2} = 3.1m$

$D_{ab'} = \sqrt{(3^2)+(4.75)^2} = 5.62m$

$D_{aa'} = \sqrt{6^2+4^2} = 7.21m$

$D_{s1} = (D_{aa} \times D_{aa'} \times D_{a'a'} \times D_{a'a})^{1/4}$

$= (0.584 \times 7.21 \times 0.584 \times 7.21 \times 10^{-4})^{1/4}$

$= 0.205m = D_{sm}$

$D_{s2} = (D_{bb} \times D_{bb'} \times D_{b'b'} \times D_{b'b})^{1/4}$

$= [(0.584 \times 10^{-2}) \times 5.5 \times (0.584 \times 10^{-2}) \times 5.5]^{1/4}$

$= 0.18m$

$D_s = (0.205 \times 0.18 \times 0.205)^{1/3} = 0.195m$

Equivalent mutual GMD is

$D_m = (D_{AB} \times D_{BC} \times D_{CA})^{1/3}$

$D_{AB} = (D_{ab} \times D_{ab'} \times D_{a'b} \times D_{a'b'})^{1/4} = (3.1 \times 5.62 \times 5.62 \times 3.1)^{1/4}$

$= 4.17m = D_{BC}$

$D_{CA} = (D_{ca} \times D_{ca'} \times D_{c'a} \times D_{c'a'})^{1/4} = (6 \times 4 \times 4 \times 6)^{1/4}$

$= 4.9m$

$D_m = (4.17 \times 4.17 \times 4.9)^{1/3} = 4.4m$

Inductance/ph/m $= 2 \times 10^{-7} \log_e 4.4/0.195 = 0.623 \times 10^{-3}\,mH$

$= 0.623mH$

Electric potential:-

- The electric potential at a point due to a charge is the work done in bringing a unit positive charge from infinity to that point.

i) potential at a charged single conductor:- Consider a long straight conductor A of radius r meters. Let the conductor operate at such a potential (V_A) that charg

Q_A/m exists on the conductor.

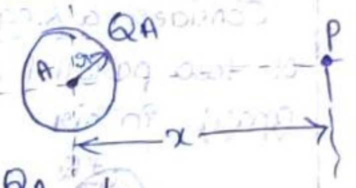
The electric intensity E at a distance x from the centre of the conductor in air is given by.

$$E = \frac{Q}{2\pi x \epsilon_0} \text{ volts/m}$$

where, Q_A = charge/m length

ϵ_0 = permitivity of free space $= 8.8 \times 10^{-12}$ f/m

Gauss's law:- the net electric flux through any closed surface $= \frac{1}{\epsilon}$ flux net Electric charge with in the closed surface.

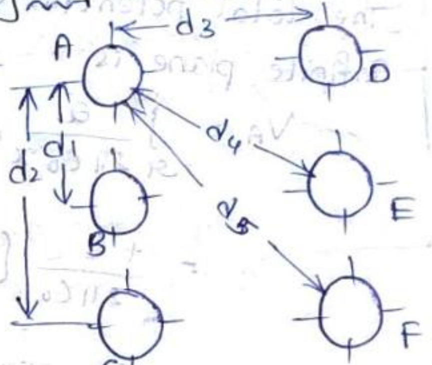


potential difference b/w A & infinity distant neutral plane, $V_A = \int_r^\infty E\,dx = \int_r^\infty \frac{Q_A}{2\pi x \epsilon_0}\,dx$

$$\boxed{V_A = \frac{Q_A}{2\pi\epsilon_0}\int_r^\infty \frac{dx}{x}}$$

net Electric flux through any closed surface equal to charge with in the closed surface

ii) potential at a conductor in a group of charged conductor:-



Consider a group of long straight conductor A, B, C etc., operating at potentials such that charges Q_A, Q_B, Q_C etc coulomb/m length exists on the respected conductor.

potential at A due to it's own charge $Q_A = \int_r^\infty \frac{Q_A}{2\pi x \epsilon_0}\,dx \rightarrow (1)$

potential at conductor A due to charge $Q_B = \int_{d_1}^\infty \frac{Q_B}{2\pi x \epsilon_0}\,dx \rightarrow (2)$

potential at conductor A due to charge $Q_C = \int_{d_2}^\infty \frac{Q_C}{2\pi x \epsilon_0}\,dx \rightarrow (3)$

Overall p.d b/w conductor A & infinite neutral phase is

$$= \int_r^\infty \frac{Q_A}{2\pi x \epsilon_0} + \int_{d_1}^\infty \frac{Q_B}{2\pi x \epsilon_0}\,dx + \int_{d_2}^\infty \frac{Q_C}{2\pi x \epsilon_0}\,dx$$

$-\log_e^r$

$\log_e r^{-1}$

$\log_e \frac{1}{r}$

$$= \frac{1}{2\pi\epsilon_0}\left[Q_A(\log_e^\infty - \log_e^r) + Q_B(\log_e^\infty - \log_e^{d_1}) + Q_C(\log_e^\infty - \log_e^{d_2})\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_1} + Q_c \log_e \frac{1}{d_2} + \log_e \infty (Q_A + Q_B + Q_c)\right]$$

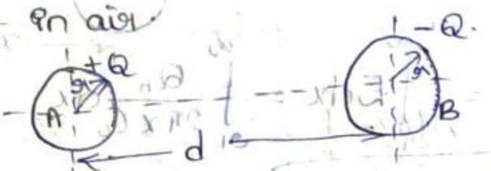
$\because Q_A + Q_B + Q_c = 0$

$$= \frac{1}{2\pi\epsilon_0}\left[Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_1} + Q_c \log_e \frac{1}{d_2} + \log_e \infty (0)\right]$$

$$V_A = \frac{1}{2\pi\epsilon_0}\left[Q_A \log_e \frac{1}{r} + Q_B \log \frac{1}{d_1} + Q_c \log_e \frac{1}{d_2}\right]$$

⇒ Capacitance of a single phase Two-wire line:-

Consider a single phase overhead transmission line Consisting of two parallel conductor A and B spaced 'd' metres apart in air.



Suppose the radius of each conductor is 'r' m. Let their respective charge be +Q & -Q. Coulomb's per metre length.

The total potential difference b/w conductor A and neutral infinite plane is

$$V_A = \int_r^\infty \frac{Q}{2\pi\epsilon_0 x}\,dx + \int_d^\infty \frac{-Q}{2\pi\epsilon_0 x}\,dx$$

$$= \frac{+Q}{2\pi\epsilon_0}\left[\log_e \frac{\infty}{r} - \log_e \frac{\infty}{d}\right]$$

$$= \frac{Q}{2\pi\epsilon_0}\left[\log_e \frac{d}{r}\right] \text{ volts.}$$

Similarly, potential difference b/w conductor B & neutral infinite plane

$$V_B = \int_r^\infty \frac{-Q}{2\pi\epsilon_0 x}\,dx + \int_d^\infty \frac{Q}{2\pi\epsilon_0 x}\,dx$$

$$= \frac{-Q}{2\pi\epsilon_0}\left[\log_e \infty - \log_e r - \log_e \infty + \log_e d\right]$$

$$= \frac{-Q}{2\pi\epsilon_0}\left[\log_e \frac{d}{r}\right] \text{ volts.}$$

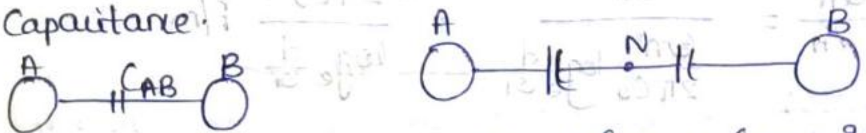
Both these potentials are w.r.t the same neutral plane. Since the unlike charges attract each other the p.d b/w the conductors is

$$V_{AB} = 2V_A = \frac{2Q}{2\pi\epsilon_0}\log_e\frac{d}{r}$$

Capacitance, $C_{AB} = \frac{Q}{V_{AB}} = \frac{\cancel{Q}}{\frac{\cancel{2}\cancel{Q}}{\cancel{2}\pi\epsilon_0}\log_e\frac{d}{r}}$ F/m

$$\therefore C_{AB} = \frac{\pi\epsilon_0}{\log_e\frac{d}{r}} \text{ F/m} \longrightarrow (1)$$

Capacitance to Neutral: equation (1) gives the capacitance b/w the conductors of a two wire line, often it is defined to know the capacitance b/w one of the conductor and a neutral point b/w them.
Since, the potential of a mid-point b/w the conductor is zero, the potential difference b/w each conductor & the ground or neutral is half the pd b/w the conductors.
Thus, the capacitance to ground or capacitance to neutral for the two-wire line is twice the line-to-line capacitance.



$C_{AN} = 2C_{AB}$; $C_{BN} = 2C_{AB}$

∴ Capacitance to neutral

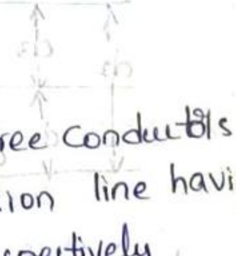
$$C_N = C_{AN} = C_{BN} = 2C_{AB}$$

$$\therefore C_N = \frac{2\pi\epsilon_0}{\log_e\frac{d}{r}} \text{ F/m} \longrightarrow (2)$$

Note:- Equation (2) applies only to a solid round conductor.

⇒ Capacitance of a 3-φ Overhead line :-

i) Symmetrical Spacing: fig shows the three conductors A, B and C of the 3-φ overhead transmission line having charges Q_A, Q_B, Q_C per/m length respectively let the conductor be equidistant (d m) from each other.



potential difference b/w conductor A & infinite neutral plane is given by

$$V_A = \int_r^{\infty}\frac{Q_A}{2\pi\epsilon_0 x}dx + \int_d^{\infty}\frac{Q_B}{2\pi\epsilon_0 x}dx + \int_d^{\infty}\frac{Q_C}{2\pi\epsilon_0 x}dx$$

$$= \frac{1}{2\pi\epsilon_0}\left[Q_A(\log_e \infty - \log_e r) + Q_B(\log_e \infty - \log_e d) + Q_C(\log_e \infty - \log_e d)\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[Q_A \log_e \infty + Q_B \log_e \infty + Q_C \log_e \infty + Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d} + Q_C \log_e \frac{1}{d}\right]$$

$\boxed{\therefore Q_A + Q_B + Q_C = 0}$

$$= \frac{1}{2\pi\epsilon_0}\left[Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d} + Q_C \log_e \frac{1}{d}\right]$$

$Q_B + Q_C = -Q_A$

$$= \frac{1}{2\pi\epsilon_0}\left[Q_A \log_e \frac{1}{r} + (Q_B + Q_C)\log_e \frac{1}{d}\right]$$

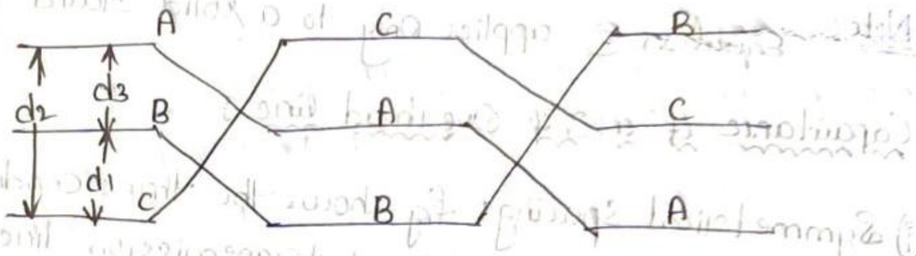
$$V_A = \frac{1}{2\pi\epsilon_0}\left[Q_A \log_e \frac{1}{r} - Q_A \log_e \frac{1}{d}\right] = \frac{Q_A}{2\pi\epsilon_0}\left[\log_e \frac{1}{r} - \log_e \frac{1}{d}\right]$$

$$V_A = \frac{Q_A}{2\pi\epsilon_0} \log_e \frac{d}{r} \text{ volts}$$

$$C_A = \frac{Q_A}{V_A} = \frac{Q_A}{\frac{Q_A}{2\pi\epsilon_0}\log_e \frac{d}{r}} = \frac{2\pi\epsilon_0}{\log_e \frac{d}{r}} \text{ F/m}$$

Similarly the expression for capacitance are the same for Conductor B and C.

ii) Unsymmetrical Spacing: fig shows a 3-ϕ transposed line having unsymmetrical spacing. let us assume balanced conditions $Q_A + Q_B + Q_C = 0$



potential of 1st position $V_1 = \frac{1}{2\pi\epsilon_0}\left(-Q_A \log_e r - Q_B \log_e d_3 - Q_C \log_e d_2\right)$

$$V_2 = \frac{1}{2\pi\epsilon_0}\left[-Q_A \log_e r - Q_B \log_e d_1 - Q_C \log_e d_3\right]$$

$$V_3 = \frac{1}{2\pi\epsilon_0}\left[-Q_A \log_e r - Q_B \log_e d_1 - Q_C \log_e d_2\right]$$

$$\therefore V_A = \frac{1}{3}(V_1 + V_2 + V_3)$$

$\because Q_A+Q_B+Q_C=0$

$$= \frac{1}{3\times 2\pi\epsilon_0}\left[Q_A \log_e\frac{1}{r^3} + (Q_B+Q_C)\log_e\frac{1}{d_1d_2d_3}\right]$$

$$= \frac{1}{6\pi\epsilon_0}\left[Q_A \log_e\frac{1}{r^3} + (\underbrace{Q_B+Q_C}_{-Q_A})\log_e\frac{1}{d_1d_2d_3}\right]$$

$$= \frac{1}{6\pi\epsilon_0}\left[Q_A \log_e\frac{1}{r^3} - Q_A\log_e\frac{1}{d_1d_2d_3}\right]$$

$$= \frac{Q_A}{6\pi\epsilon_0}\log_e\frac{d_1d_2d_3}{r^3} = \frac{1}{3}\times\frac{Q_A}{2\pi\epsilon_0}\log_e\frac{d_1d_2d_3}{r^3}$$

$$= \frac{Q_A}{2\pi\epsilon_0}\log_e\left(\frac{d_1d_2d_3}{r^3}\right)^{1/3} = \frac{Q_A}{2\pi\epsilon_0}\log_e\frac{\sqrt[3]{d_1d_2d_3}}{r}$$

$$\boxed{C_A = \frac{Q_A}{V_A} = \frac{2\pi\epsilon_0}{\log_e\frac{\sqrt[3]{d_1d_2d_3}}{r}}\ F/m}$$

Problem:-

1. A 1-ϕ transmission line has two parallel conductors 3m apart, radius of each conductor being 1cm calculate, the capacitance of the line /km. Given that $\epsilon_0 = 8.854\times10^{-12}$ F/m

Sol: Given: $r = 1cm$, $d = 3m = 300cm$

$$C = \frac{\pi\epsilon_0}{\log_e d/r} = \frac{\pi\times 8.854\times10^{-12}}{\log_e\left(\frac{300}{1}\right)} = 0.4875\times10^{-12}\ F/m$$

$$C = 0.4875\times10^{-12}\ F/km.$$

2. A 3-ϕ overhead transmission line has its conductors arranged at the corners of an equilateral triangle of 2m side. Calculate the capacitance of each line conductor per km. Given the diameter of each conductor is 1.25cm

Sol: $r = \frac{1.25}{2} = 0.625cm$

$d = 2m = 200cm$

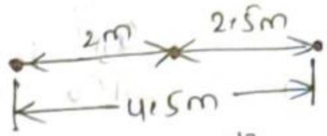
$$C = \frac{2\pi\epsilon_0}{\log_e d/r} = \frac{2\pi\times 8.854\times10^{-12}}{\log_e\frac{200}{0.625}} = \cancel{0.0096\times10^{-6}\ F/km}$$

$$= 9.6442\times10^{-12}\ F/m$$

$$= 9.64\times10^{-9}\ F/km = 0.0096\ \mu F/km$$

(3) A 3-φ, 50Hz, 66kV Overhead line conductor are placed in a horizantal plane as shown in fig. The conductors diameter is 1.25cm. If the line length is 100km, Calculate (i) capacitance /phase (ii) charging ~~current~~ Capacitance/ phase assuming Complete transposition of the line.

Sol- $f = 50Hz$



$V_{ph} = 66kV = 66\times10^3 V$

$D = 1.25$

$r = \frac{1.25}{2} = 0.625cm$

$\epsilon_0 = 8.854\times10^{-12}$

(i) line to neutral capacitance = $\frac{2\pi\epsilon_0}{\log_e \frac{d}{r}} = \frac{2\pi\times8.854\times10^{-12}}{\log_e(\frac{282}{0.625})}$

$d = \sqrt[3]{d_1 d_2 d_3} = \sqrt[3]{2\times2.5\times4.5} = 2.82\times100 = 282cm$

line to neutral Capacitance = $0.0091 \mu F/km$

line to neutral Capacitance for 100km, $C = 0.0091\times100$

$C = 0.91\mu F$

ii) $I_c = \frac{V_{ph}}{X_c} = \frac{66,000}{\sqrt{3}} \times 2\pi fc$

$= \frac{66,000}{\sqrt{3}} \times 2\pi\times50\times0.91\times10^{-6} = 10.89A$

(4) 3-φ transmission line having three Conductors each having a radius of 2cms. find out GMD and GMR

(i) when Conductors are equally spaced at a distance of 8m

(ii) when the conductors are horizantally spaced and distance b/w then is 8mt's.

Sol- (i) Radius = 2cm's = $\frac{2}{100} = 0.02m$

no: of Conductors = 3

$GMD = \sqrt[3]{d_1 d_2 d_3}$

$d_1 = d_2 = d_3 = d = 8m$

$$\boxed{GMD = \sqrt[3]{8\times8\times8} = 8}$$

$GMR_1 = GMR_2 = GMR_3 = 0.7788\,r$

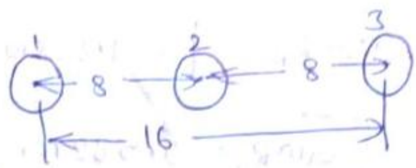
$r' = 0.7788 \times 0.02 = 0.016m$

$GMR = (GMR_1 \times GMR_2 \times GMR_3)^{1/3}$

$= \sqrt[3]{0.016\times0.016\times0.016}$

$$\boxed{\therefore GMR = 0.016m}$$

(ii) Horizantally spaced.



$GMD = \sqrt[3]{GMD_1 \cdot GMD_2 \cdot GMD_3}$

$GMD_1 = (8\times16)^{1/2} = \sqrt[2]{128} = 11.314m$

$GMD_2 = (8\times8)^{1/2} = \sqrt[2]{64} = 8$

$GMD_3 = (8\times16)^{1/2} = 11.314$

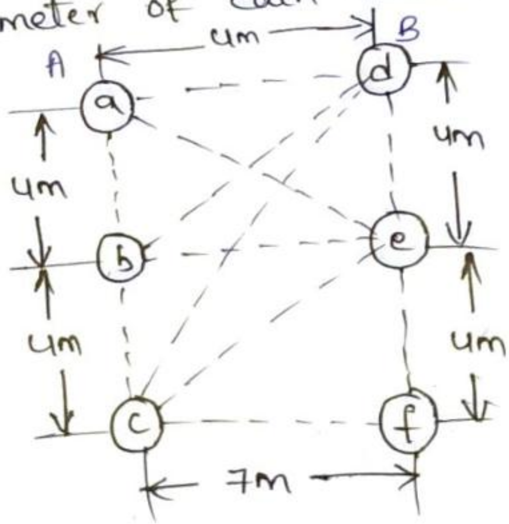
$GMD = \sqrt[3]{11.314\times8\times11.314}$

$$\boxed{\therefore GMD = 10.1m}$$

$GMR = (GMR_1 \cdot GMR_2 \cdot GMR_3)^{1/3}$

$$\boxed{GMR = 0.016m}$$

5. findout GMD and GMR of individual ckt's inductance of individual ckt and total inductance of ckt for a given configuration which is shown in below fig, diameter of each conductor is 2cm



$d = 2\,cm$

$r = \frac{d}{2} = \frac{2}{2} = 1\,cm = 0.01\,m$

GMD = ? GMR = ?

GMR of phase-A

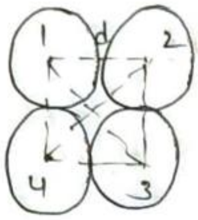
$$GMR_A = \left[(r' \times 4 \times 8)(r' \times 4 \times 4)(r' \times 4 \times 8)\right]^{1/9}$$

$$r' = 0.7788 \times r = 0.7788 \times 0.01 = 7.788 \times 10^{-3}$$

$$GMR_A = \left[(7.788 \times 10^{-3} \times 4 \times 8)(7.788 \times 10^{-3} \times 4 \times 4)(7.788 \times 10^{-3} \times 4 \times 8)\right]^{1/9}$$

$$\boxed{GMR_A = 0.582\,m}$$

⑥ find GMR of given configuration which is having centre to centre distance is 'd'.



Sol-

$$GMR_1 = (r' d_{12} d_{14} d_{13})^{1/4}$$

$$GMR_2 = (r' d_{21} d_{23} d_{24})^{1/4}$$

$$GMR_3 = (r' d_{32} d_{34} d_{31})^{1/4}$$

$$GMR_4 = (r' d_{42} d_{41} d_{43})^{1/4}$$

$$d_{24} = \sqrt{d+d} = \sqrt{2}d$$

$$d_{31} = \sqrt{d+d} = \sqrt{2}d$$

$$GMR = \left[(r' d \cdot \sqrt{2}d \cdot d)(r' \cdot d \cdot d \cdot \sqrt{2}d)(r' \cdot d \cdot d \cdot \sqrt{2}d)(r' \cdot d \cdot \sqrt{2}d \cdot d)\right]^{1/6}$$

$$= \left[(r')^4 (d\sqrt{2}d\ d)^4\right]^{1/6}$$

$$= \left[(0.788 \times r)^4 (d \cdot \sqrt{2}d \cdot d)^4\right]^{1/6}$$

UNIT-II

PERFORMANCE OF TRANSMISSION LINES

Introduction:

The performance of transmission line is decided based on the following parameter's.

⇒ Transmission Efficiency
⇒ Voltage Regulation
⇒ power flow (power capacity of the line)

* Efficiency of transmission line is defined by the ratio of receiving end to the sending end power

$$\eta = \frac{P_R}{P_S} \times 100$$

The power obtained at the receiving end of a transmission line is generally less than the sending end power due to losses in the line resistance.

$$\% \eta = \frac{\text{receiving end power}}{\text{Sending end power}} \times 100$$

$$= \frac{V_R I_R \cos\phi_R}{V_S I_S \cos\phi_S} \times 100$$

where V_R, I_R & $\cos\phi_R$ are the receiving end voltage, current and power factor ; V_s, I_s and $\cos\phi_s$ are the sending end power voltage, current and power factor.

* Voltage Regulation: when a transmission line is carrying current, there is a voltage drop in the line due to resistance and inductance of the line.

The result is that receiving end voltage (V_R) of the line is less than the sending end voltage (V_S). This voltage drop ($V_S - V_R$) in the line is expressed as a percentage of receiving end voltage V_R and is called voltage regulation.

"The difference in voltage at the receiving end of a transmission line b/w no load and full load is called "Voltage regulation" and is expressed as a percentage of the receiving end voltage.

$$\% \text{ Voltage Regulation} = \frac{V_S - V_R}{V_R} \times 100$$

The important considerations in the design and operation of a transmission line are -

⇒ Determination of voltage drop.

⇒ line losses &

⇒ Efficiency of transmission line.

⇒ Classification of Overhead transmission Line:

Depending upon the manner in which capacitance is taken into account, the overhead transmission lines are classified as:

1) Short transmission line.
2) Medium transmission line.
3) Long transmission line.

① Short transmission line:-

* when the length of an overhead transmission line is upto 80km and the line voltage is comparatively low i.e., <20kV it is usually considered as a short transmission line.
* Due to smaller length and lower voltage, the capacitance effects are small and hence can be neglected.
* while studying the performance of short transmission line, Only resistance and inductance of the line are taken into account.

② Medium transmission line:-

* when the length of an overhead transmission line is about 80-160km and the line voltage is moderately high i.e., >20KV<100KV, it is considered as a medium transmission line.
* Due to sufficient length and voltage of the line, the capacitance effects are taken into account
* for purpose of calculations, the distributed capacitance of the line is divided and lumped in the form of condensers shunted across the line at one or more point's.

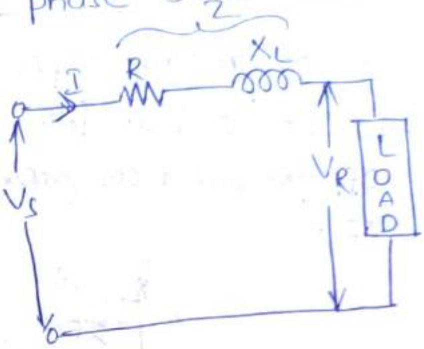
③ Long transmission line

* when the length of an overhead transmission line is more than 150km and line voltage is very high i.e., >100KV, it is considered as a long transmission line.
* for the treatment of such a line, the line constants are considered uniformly distributed over the whole length of the line & rigorous methods are employed for solution.

⇒ Performance of 1-φ short transmission lines

The effects of line capacitance are neglected for a short transmission line.

∴ the equivalent ckt of a single phase short transmission line is shown in fig①



let, I = load current

R = loop resistance i.e, resistance of both conductors

X_L = loop reactance

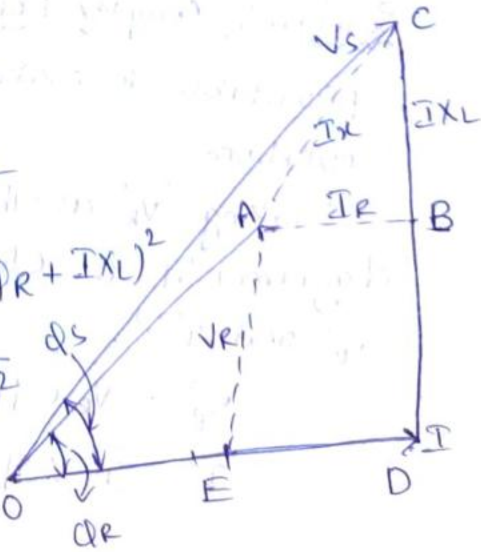
V_R = Receiving end voltage

$\cos\phi_R$ = receiving end power factor (lagging)

V_s = sending end voltage

$\cos\phi_s$ = sending end power factor

The phasor diagram of the line for lagging power factor is shown in fig②



from right angle triangle ODC

$$(OC)^2 = (OD)^2 + (DC)^2$$

$$V_s^2 = (OE+ED)^2 + (DB+BC)^2$$

$$V_s^2 = (V_R\cos\phi_R + IR)^2 + (V_R\sin\phi_R + IX_L)^2$$

$$V_s = \sqrt{(V_R\cos\phi_R + IR)^2 + (V_R\sin\phi_R + IX_L)^2}$$

(ii) Sending end power-factor

$$\cos\phi_s = \frac{OD}{OC} = \frac{V_R\cos\phi_R + IR}{V_s}$$

(iii) power delivered = $V_R I_R \cos\phi_R$

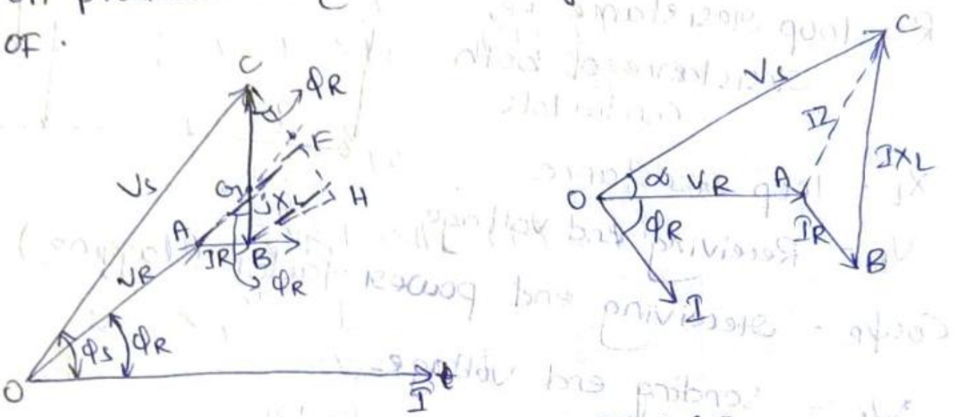
line losses = I^2R

$$\%\ \text{Transmission Efficiency} = \frac{\text{power delivered}}{\text{power sent out}} \times 100$$

power sent out = $V_R I_R \cos\phi_R + I^2R$

$$\boxed{\%\eta = \frac{V_R I_R \cos\phi_R}{V_R I_R \cos\phi_R + I^2R} \times 100}$$

An approximate expression for the sending end voltage V_s can be obtained as follows. Draw perpendicular from B and C on OA produced as shown in fig. Then OC is nearly equal to OF.



$$OC = OF = OA + AF = OA + AG + GF$$

$$= OA + AG + BH$$

$$V_s = V_R + IR\cos\phi_R + IX_L\sin\phi_R$$

Solution in Complex Notation: If it is often convenient and profitable to make the line calculations in complex notation.

Taking $\vec{V_R}$ as the reference phasor, draw the phasor diagram, It is clear that $\vec{V_s}$ is the phasor sum of $\vec{V_R}$ and $\vec{I}\vec{Z}$.

$$\vec{V_R} = V_R + j0$$

$$\vec{I} = \vec{I}\angle -\phi_R = I[\cos\phi_R - j\sin\phi_R]$$

$-\phi_R$ Because of lagging nature

$$\boxed{I_s = I_R}$$

$\vec{Z} = R + jX_L$

$$\boxed{\vec{V_S} = \vec{V_R} + \vec{I}\vec{Z}}$$

$= (V_R + j0) + I(\cos\phi_R - j\sin\phi_R)(R + jX_L)$

$= V_R + IR\cos\phi_R + jIX_L\cos\phi_R - jR\sin\phi_R I - j^2 IX_L\sin\phi_R$

$V_S = (V_R + IR\cos\phi_R + IX_L\sin\phi_R) + j(IX_L\cos\phi_R - IR\sin\phi_R)$

$V_S = \sqrt{(V_R + IR\cos\phi_R + IX_L\sin\phi_R)^2 + (IX_L\cos\phi_R - IR\sin\phi_R)^2}$

The second term under the root is quite small and can be neglected with reasonable accuracy. Therefore, approximate expression for V_S becomes:

$V_S = V_R + IR\cos\phi_R + IX_L\sin\phi_R$

The following points may be noted:

i) Approximate formula for $V_S = V_R + IR\cos\phi_R + IX_L\sin\phi_R$ gives fairly correct results for lagging power factor. However appreciable error is caused for leading power factors. Therefore approximate expression for V_S should be used for lagging p.f only.

ii) The solution in complex notation is in more presentable form.

* **Effect of load powerfactor on Regulation and efficiency:**

The regulation and efficiency of a transmission line depend to a considerable extent upon the power factor of the load.

1. **Effect on regulation:** The expression for voltage regulation of a short transmission line is given by:

$$\%\ \text{Voltage regulation} = \frac{IR\cos\phi_R + IX_L\sin\phi_R}{V_R} \times 100 \quad [\text{lag p.f}]$$

$$\boxed{V_R < V_S}$$

$$\%\ \text{Voltage regulation} = \frac{IR\cos\phi_R - IX_L\sin\phi_R}{V_R} \times 100 \quad [\text{lead p.f}]$$

$$\boxed{V_R > V_S}$$

The following ~~perm.~~ conclusions can be drawn from the above expression:

i) when the load p.f is lagging or unity or such leading that $IR\cos\phi_R > IX_L\sin\phi_R$ then the voltage regulation is positive i.e., receiving end voltage V_R will be less than the sending end voltage V_s.

2) for a given V_R and I, the voltage regulation of the line increases with the decrease in power factor for lagging loads.

3) When the load p.f is leading to this extent that $IX_L\sin\phi_R > IR\cos\phi_R$ then voltage regulation is negative i.e., the receiving end voltage V_R is more than the sending end voltage V_s.

4) For a given V_R and I, the voltage regulation of the line decreases with the decrease in p.f for leading loads.

2. Effect on transmission efficiency: The power delivered to the load depends upon the power factor.

$$P = V_R I\cos\phi_R \quad (\text{for } 1\text{-}\phi)$$

lag er ajat drop $\uparrow$
load " drop $\downarrow$
$V_s > V_R$

$$I = \frac{P}{V_R\cos\phi_R}$$

$V_R > V_s$ ferranti effect
leading power factor occur

$$P = 3V_R I\cos\phi_R \quad (\text{for } 3\text{-}\phi)$$

capacitor bank
power factor $\uparrow$
current $\downarrow$
$I^2R \downarrow$

$$\uparrow I = \frac{P}{3V_R\cos\phi_R \downarrow}$$

$\cos\phi\uparrow - \eta_{Tr}\uparrow$
$\cos\phi\downarrow - \eta_{Tr}\downarrow$

It is clear that in each case for a given amount of power to be transmitted (P) and receiving end voltage (V_R), the load current I is inversely proportional to the load p.f $\cos\phi_R$. Consequently, with the decrease in load p.f., the load current and hence the line losses are increased. this leads to the conclusion that transmission efficiency of a line decreases with the decrease in the load power factor and vice versa.

1) A 1-ϕ overhead transmission line delivers 1100 kW at 33kV at 0.8 p.f lagging the total resistance and inductive reactance of a line are 10Ω and 15Ω

respectively. Determine: (i) Sending end voltage (ii) Sending end power-factor and (iii) transmission efficiency.

Sol

KW = 1100 KW

$V_R = 33 \times 10^3$ V

$\cos\phi_R = 0.8$ lag $\Rightarrow \sin\phi_R = 0.6$

(i) $V_s = ?$

(ii) $\cos\phi_s = ?$

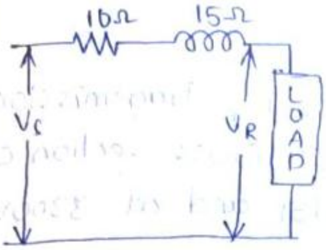
(iii) $\eta = \% ?$

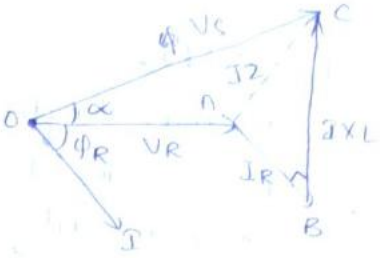
$\cos\phi_R = 0.8$

$\phi_R = \cos^{-1}(0.8)$

$= 36.869°$

Total line impedance, $\bar{Z} = R + jX_L = 10 + j15$





$\vec{V_R} = V_R + j0 = 33 \times 10^3$ V

$\vec{I} = I(\cos\phi_R - j\sin\phi_R)$

$= 41.67(0.8 - j0.6)$

$\vec{I} = \cancel{I \angle \cos\phi} = 33.33 - j25$

$I = \dfrac{KW \times 10^3}{V_R \cos\phi_R} = \dfrac{1100 \times 10^3}{33 \times 10^3 \times 0.8}$

$I = 41.67$ A

(i) $\vec{V_s} = \vec{V_R} + \vec{I}\vec{Z}$

$= 33000 + \cancel{41.67(0.8 - j0.6)}(10 + j15)$ $(33.33 - j25)$

$= 33000 + 333.3 - j250 + j500 + 375 = 33708.3 + j250$

$V_s = \sqrt{(33708.3)^2 + (250)^2} = 33709$ V

(ii) Angle between V_s and V_R is

$\alpha = \tan^{-1}\left(\dfrac{250}{33708.3}\right) = 0.42°$

Sending end power factor angle

$\phi_s = \phi_R + \alpha = 36.87 + 0.42$

$\boxed{\phi_s = 37.29°}$

$\cos\phi_s = \cos(37.29°) = 0.796$ lag

(iii) Line losses $= I^2R = (41.67)^2 \times 10 = 17.364$ KW

Output = 1100 KW

power sent = Output + loss

$= 1100 \times 10^3 + 17.364 \times 10^3$

$= 1117.364$ KW

Transmission $\eta = \dfrac{\text{power delivered}}{\text{power sent}} \times 100 = \dfrac{1100}{1117.36} \times 100$

$= 98.44\%$

$V_s = V_R + IR\cos\phi_R + IX_L\sin\phi_R$

$= 33000 + 41.67 \times 10 \times 0.8 + 41.67 \times 15 \times 0.6$

$\therefore V_s = 33708.39V$

$\cos\phi_s = \dfrac{V_R\cos\phi_R + IR}{V_s} = \dfrac{33000 \times 0.8 + 41.67 \times 10}{33708.39}$

$\therefore \cos\phi_s = 0.7958$

② What is the max length in km for a 1-ϕ transmission line having copper conductor of $0.775\ cm^2$ cross section over which 200KW at unity power factor and at 3300V are to be delivered & the efficiency of transmission is 90%. Take specific resistance at $1.725\ \mu\Omega$ cm.

Sol- $\cos\phi_R = 1$

Cross section area of copper $= 0.775 cm^2$

~~cos~~ Receiving end power $= 200 \times 10^3 W$

Transmission efficiency $= 90\% = 0.9$

Sending end power $= \dfrac{200 \times 10^3}{0.9} = 2,22,222W$

line losses $= 2,22,222 - 200000$

$= 22,222W$

Line current, $I = \dfrac{KW \times 10^3}{V_R\cos\phi_R} = \dfrac{200 \times 10^3}{3300 \times 1} = 60.6A$

line losses $= 2I^2R$

$R = \dfrac{\text{line loss}}{2I^2} = \dfrac{22222}{2 \times (60.6)^2}$

$R = 3.025\Omega$

$R = \dfrac{\rho l}{a}$

$l = \dfrac{Ra}{\rho} = \dfrac{3.025 \times 0.775}{1.725 \times 10^{-6}}$ $\quad \rho = 1.725 \times 10^{-6}\ \Omega$.

$l = 1.36 \times 10^6 cm$

$= 13.6 km$

(3) An overhead ~~ts~~ 3-φ transmission line delivers 5000kW at 22kV at 0.8pf lag. The resistance and reactance of each conductor is 4Ω and 6Ω respectively. Determine (i) Sending end voltage (ii) percentage regulation (iii) transmission efficiency.

Sol

$kW = 5000kW$ $\quad V_s = ?$

$V_R = 22kV = 22\times10^3 V$ $\quad \% V_R = ?$

$\cos\phi_R = 0.8\,lag$ $\quad V_R = \frac{22000}{\sqrt3} = 12700$ $\quad \eta_{Tr} = ?$

$\phi_R = 36.87^\circ$

$\sin\phi_R = 0.6\,lag$ $\quad I = \frac{kW\times10^3}{V_R\cos\phi_R} = \frac{5000\times10^3}{22\times10^3\times0.8}$

$R = 4\Omega$

$X_L = 6\Omega$ $\quad I = 164A$

$\vec{V_R} = V_R + j0 = 22\times10^3 V$ $\quad Z = R + jX_L = 4+j6$

$\vec{I} = I(\cos\phi_R - j\sin\phi_R)$

$= 164(0.8 - j0.6)$

$\vec{I} = 131.2 - j98.4$

(i) Sending end voltage $V_s = \vec{V_R} + \vec{I}\vec{Z} = 22\times10^3 + (131.2 - j98.4)(4+j6)$

$V_s = 13815.2 + j393.6$

$V_s = \sqrt{(13815.2)^2 + (393.6)^2} = 13820.8V$

Line value of $V_s = \sqrt3 V_s = \sqrt3\times13820.8 = 23938V$

(ii) % Voltage Regulation $= \frac{V_s - V_R}{V_R}\times100$

$= \frac{13820.8 - 127000}{127000}\times100$

$= 8.825\%$

(iii) line losses $= 3I^2R = 3\times(164)^2\times4 = 322.752kW$

∴ Transmission efficiency $= \frac{5000}{500 + 322.752}\times100$

$= 93.94\%$

(4) Estimate the distance over which a load of 15000kW at a p.f of 0.8 lag can be delivered by a 3-φ transmission line having conductors each of resistance 1Ω/km. The

Voltage at the receiving end is to be 132 kV and the loss in the transmission is to be 5%

Sol- line Current,

$$I = \frac{\text{power delivered}}{\sqrt{3} \times \text{line Voltage} \times \text{power factor}} = \frac{1500 \times 10^3}{\sqrt{3} \times 132 \times 10^3 \times 0.8}$$

$I = 82A$

Line losses = 5% of power delivered

$= 0.05 \times 15000 = 750 kW$

Line losses $= 3I^2R \Rightarrow 750 \times 10^3 = 3 \times 82^2 \times R$

$$R = \frac{750 \times 10^3}{3 \times (82)^2} = 37.18 \Omega$$

Resistance of each Conductor per km is 1Ω

length of line = 37.18 km

(5) A 3-φ line delivers 3600 kW at p.f 0.8 lag to a load. If the sending end voltage is 33 kV, determine (i) the receiving end voltage (ii) line current (iii) transmission efficiency. The resistance and reactance of each conductor are 5.31 Ω and 5.54 Ω respectively.

Sol- Resistance of each conductor = 5.31 Ω

Reactance of each conductor $X_L = 5.54 \Omega$

$\cos\phi_R = 0.8$ lag

$V_s/ph = 33000/\sqrt{3} = 19{,}052 V$

$$I = \frac{\text{power delivered}/ph}{V_R \cos\phi_R} = \frac{1200 \times 10^3}{V_R \times 0.8}$$

$$I = \frac{150 \times 10^5}{V_R} \longrightarrow (1)$$

(i) $V_s = V_R + IR\cos\phi_R + IX_L \sin\phi_R$

$$19052 = V_R + \frac{15 \times 10^5}{V_R} \times 5.31 \times 0.8 + \frac{15 \times 10^5}{V_R} \times 5.54 \times 0.6$$

$V_R^2 - 19052 V_R + 1{,}13{,}58{,}000 = 0$

$V_R = 18{,}435 V$

∴ line voltage at the receiving end $= \sqrt{3} \times 18{,}435 =$

$= 31{,}903 V$

$= 31.93 kV$

(ii) line Current, $I = \frac{15\times10^5}{V_R} = \frac{15\times10^5}{18,435} = 81.36A$

(iii) line losses, $= 3I^2R = 3\times(81.36)^2\times5.31 = 105.447kW$

$\therefore$ Transmission efficiency $= \frac{3600}{3600+105.447}\times100$

$= 97.15\%$

(6) A short transmission line with an impedance of $(6+j8)\Omega$ per phase has sending and receiving end voltages of 120KV and 110KV respectively for some receiving end load at a p.f of 0.9 lag. Determine (i) power output and (ii) sending end power factor.

Sol Resistance of each conductor, $R = 6\Omega$

Reactance of each conductor, $X_L = 8\Omega$

load power factor, $\cos\phi_R = 0.9$ lag $\Rightarrow \sin\phi_R = 0.435$

$V_R = \frac{110\times10^3}{\sqrt3} = 63508V$

$V_S = \frac{120\times10^3}{\sqrt3} = 69282V$

$V_S = V_R + IR\cos\phi_R + IX_L\sin\phi_R$

$69282 = 63508 + I\times6\times0.9 + I\times8\times0.435$

$I = \frac{5774}{8.88} = 650.2A$

(i) Output power $= \frac{3V_RI\cos\phi_R}{1000} = \frac{3\times63508\times650.2\times0.9}{1000}$

$= 1,11,490KW$

(ii) $\cos\phi_S = \frac{V_R\cos\phi_R + IR}{V_S} = \frac{63508\times0.9+650.2\times6}{69282} = 0.88$ lag.

(7) A, 3-ϕ, 50Hz, 16km long overhead line supplies 1000kW at 11KV 0.8pf lag. The line resistance is 0.03Ω/ph/km and line inductance is 0.7mH/ph/km. Calculate the sending end voltage, voltage regulation and efficiency of transmission.

Sol $R = 0.03\times16 = 0.48\Omega$

$X_L = 2\pi fL = 2\pi\times50\times16 = 3.52\Omega$ $\quad I = \frac{1000\times kW}{3V_R\cos\phi_R}$

$V_R = \frac{11\times10^3}{\sqrt3} = 6351V$ $\quad I = \frac{1000\times10^3}{3\times6351\times0.8} = 65.6A$

$\cos\phi_R = 0.8$ lag $\Rightarrow \sin\phi_R = 0.6$ lag

$V_S = V_R + IR\cos\phi_R + IX_L\sin\phi_R$

$V_S = 6515V$

% Voltage Regulation $= \frac{V_S - V_R}{V_R}\times100 = \frac{6515-6351}{6351}\times100 = 2.58\%$

line losses $= 3I^2R = 3\times(65.6)^2\times0.48 = 6.2kW$

Input power = Output power + losses

$= 1000 + 6.2 = 1006.2\ kW$

Transmission efficiency $= \frac{\text{Output power}}{\text{Input power}} \times 100 = \frac{1000}{1006.2} \times 100$

$\eta_{Trans} = 99.38\%$

(8) A 3-φ load of 2000kVA, 0.8pf is supplied at 6.6kV, 50Hz by means of a 33kV transmission line 20km long and 33/6.6kV step-down transformer. The resistance and reactance of each conductor are 0.4Ω and 0.5Ω/km respectively. The resistance and reactance of transformer primary are 7.5Ω and 13.2Ω while those of secondary are 0.35Ω & 0.65Ω respectively. find the voltage necessary at the sending end of transmission line when 6.6kV is maintained at the receiving end. Determine also the sending end power factor and transmission efficiency.

Sol: Resistance of conductor $= 0.4 \times 20 = 8\Omega$

Reactance of conductor $= 0.5 \times 20 = 10\Omega$

Equivalent Resistance of transformer = primary R + 0.35 $(33/6.6)^2$ (secondary Resistance)

$= 7.5 + 0.35\,(33/6.6)^2$

$= 7.5 + 8.75 = 16.25\Omega$

Equivalent Reactance of transformer = primary reactance + secondary Reactance $(33/6.6)^2$

$= 13.2 + 0.65\,(33/6.6)^2$

$= 29.45\Omega$

Total Resistance of line and transformer

$R = 8 + 16.25 = 24.25$

$X_L = 10 + 29.45 = 39.45$

$V_R = \frac{33000}{\sqrt{3}} = 19052V$

$I = \frac{2000 \times 10^3}{\sqrt{3} \times 33000} = 35A$

$V_S = V_R + IR\cos\phi_R + IX_L\sin\phi_R$

$= 19052 + 35 \times 24.25 \times 0.8 + 35 \times 39.45 \times 0.6$

$V_S = 20.559kV$

Sending end line voltage $= \sqrt{3} \times 20.559 = 35.6kV$

$\cos\phi_S = \frac{V_R\cos\phi_R + IR}{V_S} = \frac{19052 \times 0.8 + 35 \times 24.25}{20559} = 0.7826$ lag

line loss $= \frac{3I^2R}{1000} = \frac{3 \times 35^2 \times 24.25}{1000} = 89.12kW$

Output power = 2000KVA × 0.8 = 1600KW

∴ Transmission efficiency = $\frac{1600}{1600+89.12} \times 100 = 94.72\%$

⇒ Medium Transmission lines: In short transmission line calculations, the effect of the line capacitance are neglected because such lines have smaller length and low voltages. However, as the length & voltage of the line increases, the capacitance gradually becomes of greater importance. Therefore in order to obtain reasonable accuracy in the medium transmission line, the calculation of line capacitance must be taken into consideration.

The capacitance is uniformly distributed over the entire length of the line. In order to make the calculations simple, the line capacitance is assumed to be lumped or concentrated in the form of capacitors shunted across the line at one or more points. So, the line capacitance gives reasonable accurate results.

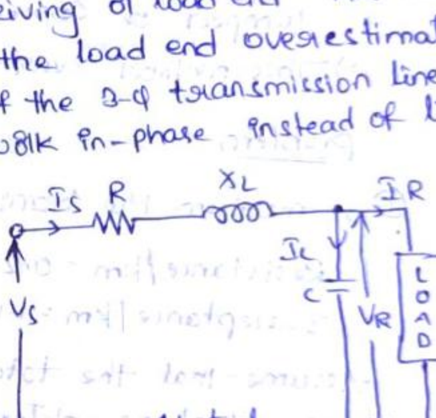
The most commonly used methods are:

1) Nominal T-method. } localised capacitor method's
2) End condenser method.
3) Nominal π method.

To improve p.f of the Transmission line we connect capacitors across the load.

load end or Capacitance method

(1) ⇒ End condenser Method: In this method the capacitance of the line is lumped or concentrated at the receiving or load end. This method of localising the line capacitance at the load end overestimates the effect of capacitance. One phase of the 3-φ transmission line is shown as it is more convenient to work in-phase instead of line-to-line values.



let I_R = load current/ph

R = Resistance per phase

X_L = inductive reactance per phase

C = Capacitance per phase

$\cos\phi_R$ = receiving end power factor (lagging)

V_s = sending end Voltage per phase

Taking the receiving end voltage $\vec{V_R}$ as the reference

$\vec{V_R} = V_R + j0$ → Receiving end vtg

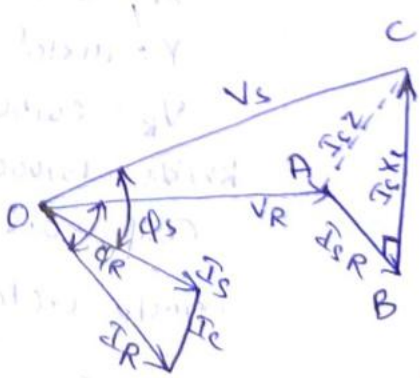
$\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$ → load current

$\vec{I_C} = j\vec{V_R}\omega C = j2\pi f C\vec{V_R}$

$I = \frac{V}{R} = \frac{V_R}{\frac{1}{X_C}}$

$X_C = \frac{1}{2\pi f C}$

The sending end current I_s is the phasor sum of load & Capacitance current



$\overline{I_s} = \overline{I_R} + \overline{I_c}$

$= I_R(\cos\phi_R - j\sin\phi_R) + j\,2\pi f C V_R$

$= I_R\cos\phi_R - jI_R\sin\phi_R + j\,2\pi f C V_R$

$= I_R\cos\phi_R + j(2\pi f C V_R - I_R\sin\phi_R)$

Voltage drop / phase $= \overline{I_s}\,\overline{Z} = \overline{I_s}(R+jX_L)$

Sending end voltage $\vec{V_s} = \vec{V_R} + \vec{I_s}\,\vec{Z} = \vec{V_R} + \vec{I_c}(R+jX_L)$

The magnitude of sending end voltage V_s can be calulated

$$\%\text{ Voltage Regulation} = \frac{V_s - V_R}{V_R} \times 100$$

$$\%\text{ Voltage Transmission efficiency} = \frac{\text{power delivered/phase}}{\text{power sent/phase}} \times 100$$

$$= \frac{\text{power delivered/phase}}{\text{power delivered/phase + losses/phase}} \times 100$$

$$= \frac{V_R I_R \cos\phi_R}{V_R I_R \cos\phi_R + I_s^2 R} \times 100$$

Limitations / drawbacks :

1) there is a considerable error about 10% in calculations because the distributed capacitance has been assumed to be lumped or concentrated

2) this method overestimate the effect of line capacitance. Because the capacitance connected across the load.

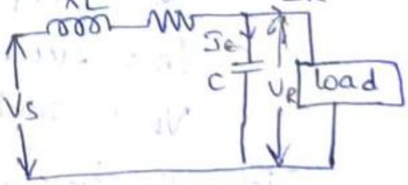
problem

① A medium 1-ϕ transmission line 100km has the following constants

Resistance/km = 0.25Ω Reactance/km = 0.8Ω

susceptance/km = 14×10^{-6} siemen Receiving end line voltage = 66,000V

Assume that the total line capacitance is localised at the receiving end alone, determine the line is delivering 15,000kV at 0.8 p.f lag. Draw the phasor diagram to illustrate your calulations?



Sol

~~1km = 10~~ length, l = 100km

$R = 0.25\Omega/km = 0.25\times100 = 25\Omega$

$X_L = 0.8\Omega/km = 0.8\times100 = 80\Omega$

$Y = 14\times10^{-6}$ siemen

$V_R = 66{,}000V$

KW = 15,000V

$\cos\phi_R = 0.8$ lag

$\sin\phi_R = 0.6$ lag

$$I_R = \frac{KW\times10^3}{V_R\cos\phi_R} = \frac{15{,}000\times10^3}{66{,}000\times0.8}$$

$I_R = 284A$

$\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 284(0.8 - j0.6) = 227 - j170$

$\bar{I_c} = j2\pi f c V_R = jY \times V_R = j14 \times 10^{-6} \times 100 \times 66{,}000 = j92$

$\frac{170}{\frac{92}{78}}$

i) $\vec{I_s} = \vec{I_R} + \vec{I_c} = 227 - j170 + j92 = 227 - j78$

$I_s = \sqrt{(227)^2 + (78)^2} = 240A$

ii) Voltage drop $\vec{I_s}\vec{Z} = 240\angle(227 - j78)(25 + j80) = 11{,}915 + j16210$

$\vec{V_s} = \vec{V_R} + \vec{I_s}\vec{Z}$

$= 66{,}000 + 11{,}915 + j16210 = 77{,}915 + j16210$

$V_s = \sqrt{(77915)^2 + (16210)^2} = \underline{79583V}$

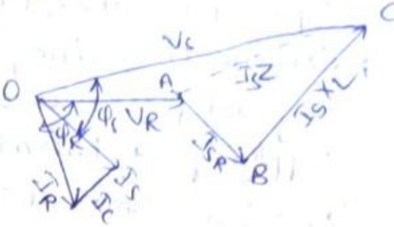
iii) Voltage Regulation $= \frac{V_s - V_R}{V_R} \times 100 = \frac{79583 - 66000}{66000} \times 100 = \underline{20.58\%}$

iv) phase angle b/w V_R & I_R

$\theta_1 = \tan^{-1}\left(\frac{-78}{227}\right) = -18.96°$

phase angle b/w V_R & V_s

$\theta_2 = \tan^{-1}\left(\frac{16210}{77915}\right) = 11.50°$



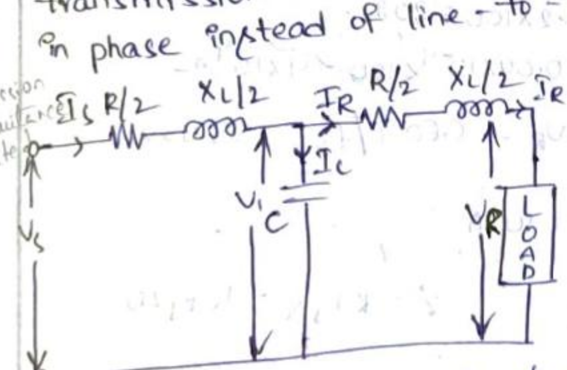
$\phi_s = 18.96 + 11.50 = 30.46$

$\cos\phi_s = \cos(30.46) = 0.86\,lag$

middle condenser method

(2) ⟹ **Nominal T-Method:** In this method the line capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on its either side as shown in fig. Therefore in this arrangement full charging current flows over half the line. one phase of 3-φ transmission line is shown as it is advantageous to work in phase instead of line-to-line values.

for low or short transmission line capacitance is neglected



I_R = load current /phase

X_L = Inductive reactance /phase

$\cos\phi_R$ = Receiving end power factor (lag)

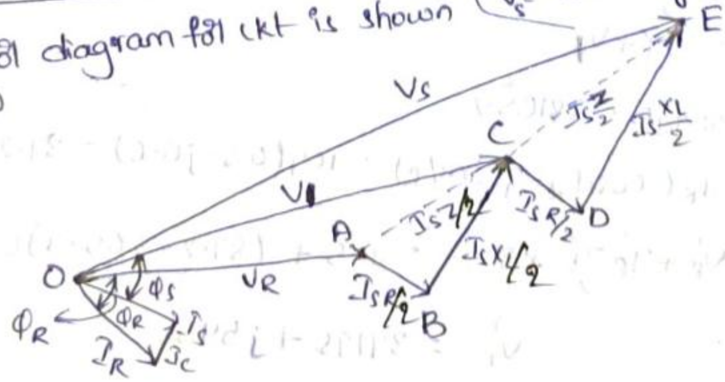
V_1 = Voltage across capacitor C

R = Resistance /phase

C = capacitance /phase

V_s = sending end voltage /phase

the phasor diagram for ckt is shown below



Receiving end voltage, $\vec{V_R} = V_R + j0$

load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$

Voltage across C $\vec{V_1} = \vec{V_R} + \vec{I_R}\vec{Z}/2$

$$= V_R + I_R(\cos\phi_R - j\sin\phi_R)\left(\frac{R}{2} + j\frac{X_L}{2}\right)$$

Capacitive current $\vec{I_C} = j\omega C\vec{V_1} = j2\pi f C\vec{V_1}$ leading to V_1

Sending end current $\vec{I_S} = \vec{I_R} + \vec{I_C}$

Sending end voltage $\vec{V_S} = \vec{V_1} + \vec{I_S}\frac{\vec{Z}}{2} = \vec{V_R} + \vec{I_S}\left(\frac{R}{2} + j\frac{X_L}{2}\right)$

① A 3-φ 50Hz Overhead transmission line 100km long has the following constants

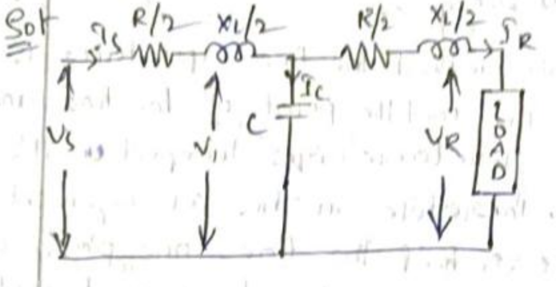
Resistance/km/ph = 0.1Ω

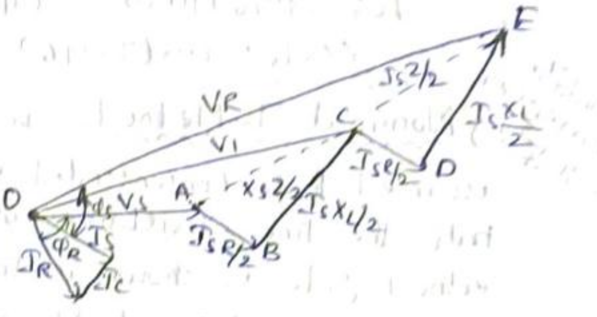
Inductive reactance/km/ph = 0.2Ω

Capacitance susceptance/km/ph = 0.04×10^{-4} siemen

Determine ① sending end current ② sending end voltage ③ Sending end power factor and ④ Transmission efficiency when supplying a balanced load of 10,000 kw at 66kV, pf 0.8 lag. Use Nominal T-method.

Sol





Total resistance/phase, $R = 0.1 \times 100 = 10\Omega$

Total reactance/phase, $X_L = 0.2 \times 100 = 20\Omega$

Capacitive susceptance, $Y = 0.04 \times 10^{-4} \times 100 = 4 \times 10^{-4}$ S

Receiving end voltage/phase, $V_R = 66000/\sqrt{3} = 38105V$

$$I_R = \frac{10000 \times 10^3}{\sqrt{3} \times 66 \times 10^3 \times 0.8} = 109A$$

$\cos\phi_R = 0.8$, $\sin\phi_R = 0.6$

$\vec{Z} = R + jX_L = 10 + j20$

$I_C = jY + V_1$

$\vec{V_R} = V_R + j0 = 38105V$

$\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 109(0.8 - j0.6) = 87.2 - j65.4$

$\vec{V_1} = \vec{V_R} + \vec{I_R}\frac{\vec{Z_1}}{2} = 38105 + (87.2 - j65.4)(5 + j10)$

$\vec{V_1} = 39195 + j545$

Charging Current, $\vec{I_c} = j4\times10^{-4}(39{,}195+j545) = -0.218+j15.6$

Sending end Current, $\vec{I_s} = \vec{I_R}+\vec{I_c}$

$= (87.2-j65.4)+(-0.218+j15.6)$

$= 87.0-j49.8 = 100\angle{-29°47'}A$

$\therefore \vec{I_s} = 100A$

(ii) Sending end voltage, $\vec{V_s} = \vec{V_1}+\vec{I_s}\,Z/2 = (39195+j545)+(87-j49.8)(5+j10)$

$\therefore \vec{V_s} = 40128+j1170 = 40145\angle{1°40'}V$

Line value of sending end voltage $= 40145\times\sqrt{3} = 69.533kV$

(iii) θ_1 = angle b/w $\vec{V_R}$ & $\vec{V_s} = 1°40'$

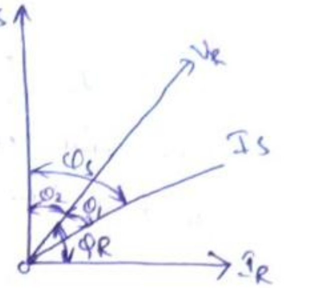
θ_2 = angle b/w $\vec{V_R}$ & $\vec{I_s} = 29°47'$

ϕ_s = angle between $\vec{V_s}$ and $\vec{I_s} = \theta_1+\theta_2$

$= 1°40' + 29°47' = 30°87'$

Sending end power factor, $\cos\phi_s = \cos(30°87')$

$= 0.853$ lag



(iv) Sending end power $= 3V_sI_s\cos\phi_s = 3\times40{,}145\times100\times0.853 = 10273.11$ kW

power delivered = 10,000kW

$\therefore$ Transmission Efficieny $= \dfrac{10{,}000}{10273.11}\times100 = 97.34\%$

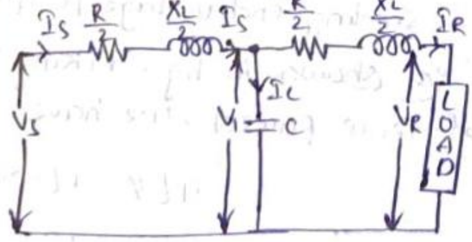
② A 3-ϕ, 50Hz transmission line 100km long delivers 20MW at 0.9p.f lagging and at 110kV. The resistive and reactance of the line per phase per km are 0.2Ω and 0.4Ω respectively, while capacitance admittance is 2.5×10^{-6} siemen/km/phase. calculate (i) the current and voltage at the sending end (ii) efficiency of transmission. Use nominal T-method.

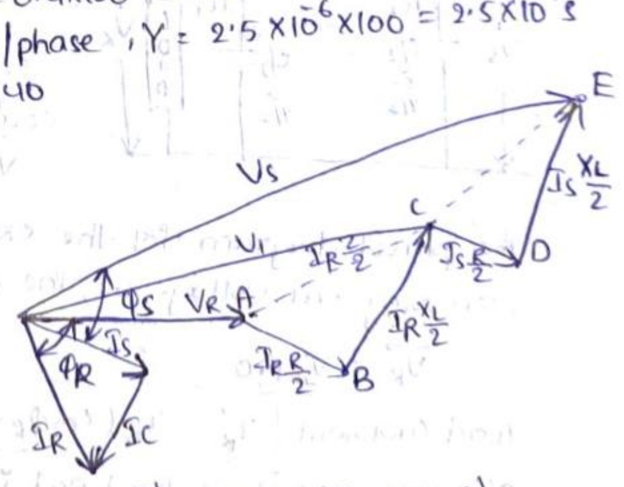
Sol: Total resistance/phase, $R = 0.2\times100 = 20\Omega$

Total reactance/phase, $X_L = 0.4\times100 = 40\Omega$

Total capacitance admittance/phase, $Y = 2.5\times10^{-6}\times100 = 2.5\times10^{-4}S$

phase impedance, $\vec{Z} = 20+j40$





$V_R = 110\times10^3/\sqrt{3} = 63508V$

$I_R = \dfrac{20\times10^6}{\sqrt{3}\times110\times10^3\times0.9} = 116.6A$

$\cos\phi_R = 0.9$, $\sin\phi_R = 0.435$

(i) $\vec{V_R} = V_R+j0 = 63508V$

$\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$

$= 116.6(0.9-j0.435)$

$= 105-j50.7$

Voltage across, C $\Rightarrow \vec{V_1} = \vec{V_R} + \vec{I_R}\vec{Z}/2 = 63508 + (105 - j\,507)(10+j20)$

$$\vec{V_1} = 65572 + j1593$$

Charging current, $\vec{I_C} = jY\vec{V_1} = j2.5\times10^{-4}\times(65572+j1593)$

$$= -0.4 + j16.4$$

$$\vec{I_s} = \vec{I_R} + \vec{I_C} = (105 - j50.7) + (-0.4 + j16.4) = 110\angle{-18^\circ 9'}\,A$$

$$\therefore \vec{I_s} = 110A$$

$$\vec{V_s} = \vec{V_1} + \vec{I_s}\vec{Z}/2 = (65572+j1593) + (104.6 - j34.3)(10+j20)$$

$$= 67304 + j3342$$

$$V_s = \sqrt{(67304)^2 + (3342)^2} = 67387V$$

Line value of sending end voltage $= 67387\times\sqrt{3} = 116717V$

ii) Total line losses for the three phases $= 3I_s^2 R/2 + 3I_R^2 R/2$

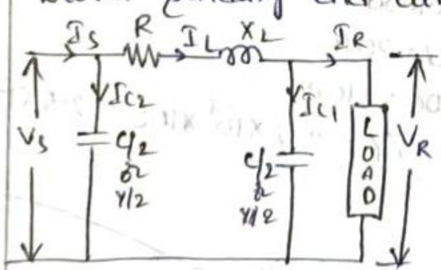
$$= 3\times110^2\times10 + 3\times(116.6)^2\times10$$

$$= 0.770MW$$

$\therefore$ Transmission efficiency $= \frac{20}{20+0.770}\times100 = 96.29\%$

Split condenser method

(3) $\Rightarrow$ Nominal π-method : In this method, capacitance of each conductor [i.e., line to neutral] is divided into two halves; one half being lumped at the sending end and the other half at the receiving end. It is obvious that capacitance at the sending end has no effect on the line drop. However, it's charging current must be added to line current in order to obtain the total sending end current.



let I_R = load current / phase

R = Resistance / phase

X_L = Inductive reactance / phase

C = capacitance per phase

$\cos\phi_R$ = receiving end power factor

V_s = sending end voltage per phase

The phasor diagram for the ckts is shown in fig. Taking the receiving end voltage as the reference phasor, we have

$$\vec{V_R} = V_R + j0$$

$I\angle{-\phi} = I(\cos\phi - j\sin\phi)$

load current $\boxed{\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)}$

$\omega = 2\pi f c$

Charging current at load end is

$$\vec{I_{C_1}} = j\omega(C/2)\vec{V_R} = j\pi f C\vec{V_R}$$

voltage across capacitor

or $I_{C1} = V_{C1}\frac{Y}{2} = V_R\frac{Y}{2}$

line current $\vec{I_L} = \vec{I_R} + \vec{I_{C_1}}$

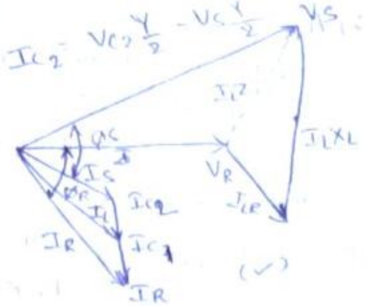
Sending end voltage, $\vec{V_s} = \vec{V_R} + \vec{I_L}\vec{Z} = \vec{V_R} + \vec{I_L}(R + jX_L)$

Charging current at the sending end is

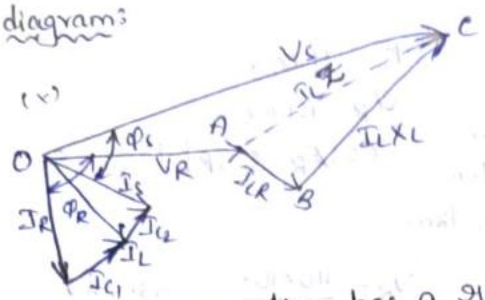
$\vec{I_{c2}} = j\omega(C/2)\vec{V_s} = j\pi f C\vec{V_s}$ or $I_{c2} = V_s \frac{Y}{2}$

Y = shunt admittance

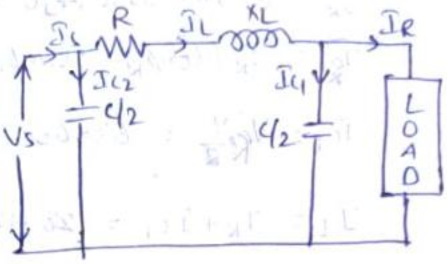
$\therefore \vec{I_s} = \vec{I_L} + \vec{I_{c2}}$



Phasor diagram:



(2) A 3-ϕ, 50Hz, 150Km line has a resistance, inductive reactance and capacitive shunt admittance of 0.1Ω, 0.5Ω and 3×10^{-6} S per km per phase. If the line delivers 50MW at 110KV and 0.8pf lagging, determine the sending end voltage and current. Assume a nominal π ckt for the line.



Sol

$R = 0.1\times150 = 15\Omega$

$X_L = 0.5\times150 = 75\Omega$

$Y = 3\times10^{-6}\times150 = 45\times10^{-5}$ S

$V_R = 110\times10^3/\sqrt{3} = 63508$ V

$I_R = \dfrac{50\times10^6}{\sqrt{3}\times110\times10^3\times0.8} = 328$ A

$\cos\phi_R = 0.8$; $\sin\phi_R = 0.6$

$\vec{V_R} = V_R + j0 = 63508$ V

$\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 328(0.8 - j0.6) = 262.4 - j182.5$

$\vec{I_{c1}} = \vec{V_R}\, j\frac{Y}{2} = 63508\times j\frac{45\times10^{-5}}{2} = j14.3$

$\vec{I_L} = \vec{I_R} + \vec{I_{c1}} = (262.4 - j196.8) + j14.3 = 262.4 - j182.5$

$\vec{V_s} = \vec{V_R} + \vec{I_L}\vec{Z} = \vec{V_R} + \vec{I_L}(R + jX_L)$

$= 63508 + (262.4 - j182.5)(15 + j75)$

$= 81131 + j16942.5 = 82881\angle 11°47'$ V

Line to line sending end voltage $= 82881\times\sqrt{3} = 143,550$ V

$= 143.55$ kV

Charging current at the sending end is

$I_{c2} = j\vec{V_s}\, Y/2 = (81131 + j16942.5)j\dfrac{45\times10^{-5}}{2}$

$= -3.81 + j18.25$

$\vec{I_s} = \vec{I_L} + \vec{I_{c2}} = (262.4 - j182.5) + (-3.81 + j18.25)$

$= 306.4$ A

Sending end current, $\vec{I_s} = 306.4$ A

3) A 3-ϕ, 50Hz, 150Km line has resistance, inductive reactance, Capacitive shunt admittance of 0.1Ω, 0.5Ω, 3×10^{-6}/km/ph of the line delivers 50mw at 110KV and 0.8 p.f lagging. Determine the Sending end voltage and Current. Assume nominal-π Ckt for the line.

Sol Given data $f = 50Hz$ $P_1 = 50mw$

$l = 150Km$ $V_R = 110KV$

$R = 0.1 \Omega/km$ $\cos\phi_R = 0.8pf$ lag

$X_L = 0.5\Omega/km$

$Y = 3\times10^{-6} \mho/km$

$R = 0.1\times150 = 15\Omega$ $V_R = \frac{110\times10^3}{\sqrt{3}} = 63.5KV$

$X_L = 0.5\times150 = 30\Omega$

$Y = 3\times10^{-6}\times150 = 4.5\times10^{-4} \mho$ $I_R = \frac{P}{3\cos\phi_R V_R} = \frac{50\times10^6}{3\times63.5\times10^3} = \cancel{0.984A}$ 328A

$\vec{V_R} = V_R + j0$

$\vec{V_R} = \cancel{63.5KV}$ $63508 + j0$

$\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 328(0.8 - j0.6) = 26.24 - j196.9A$

$I_{C_1} = V_R\frac{Y}{2} = 63508 \cdot \frac{\cancel{3\times10}4.5\times10^{-4}}{15} = 1.905A$

$I_L = I_R + I_{C_1} = \cancel{26.328+1.905 = 329.905A}$ $(26.24 - j196.9) + (1.905 + j0)$
$= (28.145 - j196.9)$

$I_{C_2} = V_S\frac{Y}{2}$

$V_S = V_R + I_L Z = V_R + I_L(R + jX_L) = 63508 + (28.145 - j196.9)(15 + j30)$

$= (69837.175 - j2109.15)V$

$I_{C_2} = jV_S\frac{Y}{2} = j(69837.175 - j2109.15)\cdot\frac{4.5\times10^{-4}}{2}$

$= -3.81 + j18.25$

$I_S = I_L + I_{C_2} = (262.4 - j182.39) + (-3.81 + j18.25)$

$= 258.59 + j64.14$

$= \cancel{112.79}$

3) A 100km long, 3-ϕ 50Hz transmission line has line constants resistance/ph/km is 0.1Ω inductive reactance/ph/km = 0.5Ω, admittance /ph/km is 10×10^{-6} If the line supplies line load of 20mw at 0.9pf at 66kv at the receiving end. Calculate the nominal-π method (i) Sending end power factor (ii) Regulation (iii) Transmission efficiency

$R = 0.1\times100 = 10\Omega$ $P_R = 20mw$

$X_L = 0.5\times100 = 50\Omega$ $\cos\phi_R = 0.9$

$Y = 10\times10^{-6}\times100 = 10^{-3}℧$ $V_R = 66KV$

(i) $\cos\phi_s = ?$

(ii) $\% V_R = ?$

(iii) $\eta = ?$

$V_R = \frac{66000}{\sqrt3} = 38.105\times10^3 = 38.105KV$

$I_R = \frac{P}{\sqrt3 V_R \cos\phi} = \frac{20\times10^6}{\sqrt3\times66\times10^3\times0.9} = \cancel{349}\ 194.39A$

$\cos\phi_R = 0.9$

$\sin\phi_R = 0.435$

$I_{c1} = j\frac{Y}{2}V_R$

$\boxed{I_{c1} = j19.052A}$

$I_{c2} = j\frac{Y}{2}V_s$

$\vec{I_R} = I_R[\cos\phi_R - j\sin\phi_R] = 194.34[0.9 - j0.435]$

$\boxed{\vec{I_R} = 174.906 - j84.538}$

$\boxed{\vec{V_R} = V_R + j0 = 38105 + j0}$

$\vec{V_s} = \vec{V_R} + \vec{I_L}Z = \vec{V_R} + \vec{I_L}(R + jX_L)$

$= (38105 + j0) + (174.906 - j65.486)(10 + j50)$

$I_L = I_R + I_{c1}$

$= (174.906 - j84.538) + (j19.052)$

$\boxed{I_L = 174.906 - j65.486}$

$\cancel{\vec{V_s} = 41558.36 + j240.44}$

$\boxed{\vec{V_s} = 43128.35 + j8090.4}$

$= \cancel{-42524.38 \angle 7192.46}$

$= 43880.2 \angle 10.62$

$I_{c2} = j\frac{Y}{2}V_s$

$= \left[j\frac{10^{-3}}{2}\right][43128.35 + j8090.4]$

$\boxed{I_{c2} = -4.045 + j21.56}$

$\vec{I_s} = I_L + I_{c2}$

$= (174.906 - j65.486) + (-4.045 + j21.56)$

$\vec{I_s} = 704.384 + j4035.86 = \cancel{172.07 \angle 683}\ 4096.86 \angle \cancel{80.09}\ 14.46$

$\phi_s = 10.62 + \cancel{80.09}\ 14.46$

$= \cancel{90.71}\ 25.11$

(i) $\cos\phi_s = \cos(\cancel{90.71}\ 25.11) = 0.9$

(ii) $V/t\ Reg = \frac{V_s - V_R}{V_R} = \frac{43082.2 - 38105}{38105}\times100 = 0.1306 = 13.06$

(iii) Sending end power $= 3V_s I_s \cos\phi_s$

$= 3\times43082.2\times177.5\times0.9 = 20.64MW$

$\eta = \frac{20.64}{20}\times100 = 95.2$

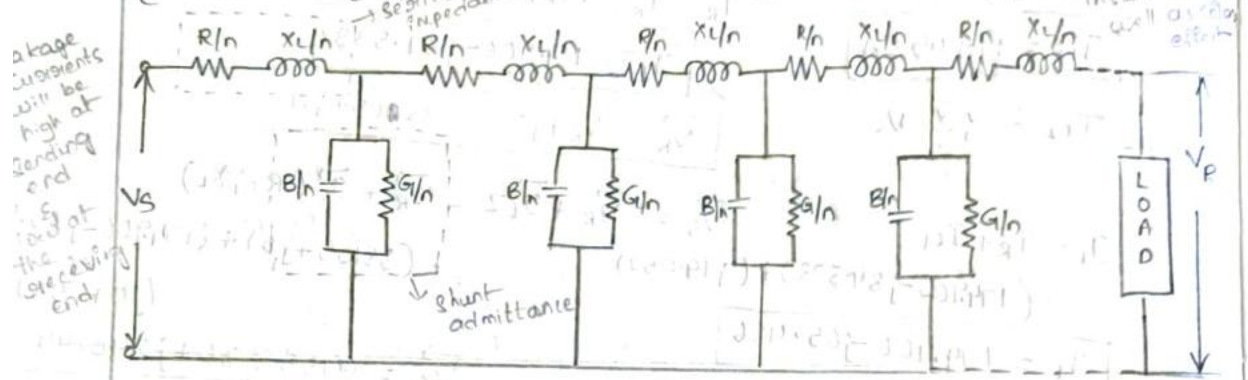
4) ~~END~~ ~~CONDENSER~~ ~~METHOD~~ >160km ≈ 250km [100kV]

⇒ Long Transmission line :- If an assumption of lumped Circuit is applied to long transmission line (having length excess of about 160km) it is found that serious errors are introduced in the performance calculations

∴ in order to obtain fair degree of accuracy in the performance calculations of long lines, the line constants are considered as uniformly distributed throughout the length of the line.

Rigorous mathematical treatment is required for the solution of such lines.

B → leakage susceptance
G → leakage conductance



~~Ckt~~ Ckt :- for 3-ϕ long transmission line on a phase neutral basis.

The whole line length is divided into 'n' sections, each section having line constants $\frac{1}{n\text{th}}$ of those for the whole line

The following points may be noted :-

→ The line constants are uniformly distributed over the entire length of line as is actually the case.

⇒ The resistance and inductive reactance are the series elements

⇒ The leakage susceptance (B) and leakage conductance are shunt elements

The leakage susceptance is due to the fact that capacitance exists between line and neutral.

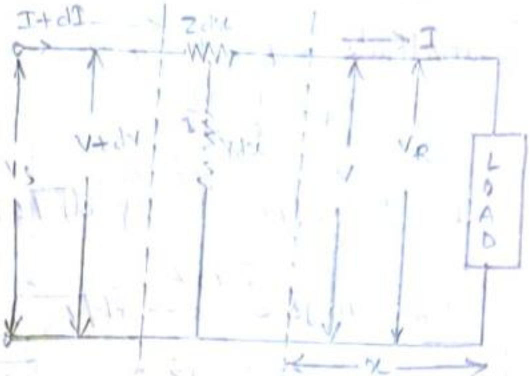
The leakage conductance takes into account the energy losses occurring through leakage over the insulators or due to corona effect between conductors.

$$\boxed{\text{Admittance} = \sqrt{G^2+B^2}}$$

⇒ The leakage current through shunt admittance is maximum at the sending end of the line and decreases continuously as the receiving end of the ckt is approached at which point its value is zero.

Analysis of Long Transmission line [Rigorous Method]

fig shows one phase and neutral connections of a 3-ϕ line with impedance and shunt admittance of the line uniformly distributed



Consider a small element in the line of length dx situated at a distance x from the receiving end

let, z = series impedance of the line per unit length

y = shunt admittance of the line " "

V = Voltage at the end of element towards receiving end.

V+dv = Voltage at the end of element towards sending end.

I+dI = Current entering the element dx

I = current ~~leaving~~ leaving the element dx.

Then for the small element dx,

zdx = series impedance

ydx = shunt impedance

Obviously, $dv = Izdx$

$$\frac{dv}{dx} = Iz \longrightarrow (1)$$

Now, the current entering the element is I+dI whereas the current leaving the elements is I. The difference in the current flows through shunt admittance of the element of elements

dI = current through shunt admittance

$dI = Vydx$

$$\frac{dI}{dx} = Vy \longrightarrow (2)$$

Differentiate eq (1) we get

$$\frac{d^2v}{dx^2} = z\frac{dI}{dx}$$

sub (2) in above equation

$$\frac{d^2v}{dx^2} = (Vy)z \Rightarrow \frac{d^2v}{dx^2} - Vyz = 0 \longrightarrow (3)$$

$$(D^2 - yz)V = 0$$

$$D^2 = yz \Rightarrow D = \pm\sqrt{yz} \begin{cases} m_1 = \sqrt{yz} \\ m_2 = -\sqrt{yz} \end{cases}$$

The solution for differential equation is $y = ae^{xt} + be^{yt}$

$$V = Ae^{m_1 t} + Be^{m_2 t}$$

we are doing w.r.t 'x'

$$V = Ae^{\sqrt{yz}\,x} + Be^{-\sqrt{yz}\,x} \longrightarrow (4)$$

$$\boxed{\frac{d}{dx}(e^{ax}) = ae^{ax}}$$

differentiate eq (4) w.r.t x we get

$$\frac{dV}{dx} = A\sqrt{YZ}\, e^{\sqrt{YZ}\cdot x} + B(-\sqrt{YZ})\, e^{-\sqrt{YZ}\cdot x} \longrightarrow ⑤$$

Substitute eq ① in eq ⑤

Eq① $\frac{dV}{dx} = IZ$ $\quad I = \frac{1}{Z}\frac{dV}{dx}$

$$I = \frac{1}{Z}\frac{dV}{dx}$$

$$I = \frac{1}{Z}\left[A\sqrt{YZ}\cdot e^{\sqrt{YZ}\cdot x} + B(-\sqrt{YZ})e^{-\sqrt{YZ}\cdot x}\right]$$

$$\frac{\sqrt{YZ}}{Z} = \frac{\sqrt{Y}\sqrt{Z}}{\sqrt{Z}\sqrt{Z}}$$

$$IZ = A\sqrt{YZ}\, e^{(\sqrt{YZ})x} + B(-\sqrt{YZ})\, e^{(-\sqrt{YZ})x}$$

$$I = A\sqrt{\frac{Y}{Z}}\, e^{\sqrt{YZ}\cdot x} - B\sqrt{\frac{Y}{Z}}\, e^{(-\sqrt{YZ})x} \longrightarrow ⑥$$

where

$\gamma = \sqrt{YZ}$ → propagation constant

$Z_c = \sqrt{\frac{Z}{Y}}$ → characteristic equation

from equation ④

$$V = Ae^{\gamma x} + Be^{-\gamma x} \longrightarrow ⑦$$

from equation ⑥

$$I = \frac{A}{Z_c}e^{\gamma x} - \frac{B}{Z_c}e^{-\gamma x} \longrightarrow ⑧$$

The characteristic impedance is also called as surge impedance

$$Z_c = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

for long length line $R=0, G=0$

$$Z_c = \sqrt{\frac{L}{C}}$$

Two constants A & B are to be determined, hence two boundary conditions should be known as mentioned previously the receiving end voltage current are known.

$\therefore$ At $x=0$ $\quad \left.\begin{matrix} V = V_r \\ I = I_r \end{matrix}\right\} \longrightarrow ⑨$

substitute eq ⑨ in equation ⑦ & eqation ⑧, we get

$$V_r = Ae^{\gamma(0)} + Be^{-\gamma(0)} \longrightarrow ⓐ$$

$$V_r = A+B \longrightarrow ⑩ \qquad B = V_r - A \rightarrow ⓐ$$

$$I_r = \frac{A}{Z_c}e^{\gamma(0)} - \frac{Be^{-\gamma(0)}}{Z_c}$$

$I_rZ_c = A - B$

$I_rZ_c + B = A$

$B = V_r - I_rZ_c - B$

$2B = V_r - I_rZ_c$

$$I_r = (A-B)\frac{1}{Z_c} \longrightarrow ⑪$$

$$B = \frac{V_r - I_rZ_c}{2}$$

By solving ⑩ and ⑪ we get

sub in ⓐ

$$\boxed{\begin{aligned} A &= \frac{V_r + I_rZ_c}{2} \\ B &= \frac{V_r - I_rZ_c}{2} \end{aligned}} \longrightarrow ⑫$$

$V_r - A = \frac{V_r - I_rZ_c}{2}$

$V_r - \frac{V_r - I_rZ_c}{2} = A$

$A = \frac{2V_r - V_r + I_rZ_c}{2}$

Substitute A and B values in equation ⑦ & ⑧, we get

$\frac{V_r + I_rZ_c}{}$

(7) ⇒ $V = \left[\frac{V_r + I_r Z_c}{2}\right]e^{\gamma x} + \left[\frac{V_r - I_r Z_c}{2}\right]e^{-\gamma x}$ ⟶ (12)

$$V = \frac{V_r}{2}e^{\gamma x} + \frac{I_r Z_c}{2}e^{\gamma x} + \frac{V_r}{2}e^{-\gamma x} - \frac{I_r Z_c}{2}e^{-\gamma x}$$

$$V = \frac{V_r}{2}\left[e^{\gamma x} + e^{-\gamma x}\right] + \frac{I_r Z_c}{2}\left[e^{\gamma x} - e^{-\gamma x}\right]$$

$$V = V_r \cosh\gamma x + I_r Z_c \sinh\gamma x$$

$\cosh x = \frac{e^{x}+e^{-x}}{2}$

$\sinh x = \frac{e^{x}-e^{-x}}{2}$

(8) ⇒ $I = \frac{1}{Z_c}\left[\frac{V_r + I_r Z_c}{2}e^{\gamma x} - \frac{V_r - I_r Z_c}{2}e^{-\gamma x}\right]$

$$= \frac{1}{Z_c}\left\{V_r\left(\frac{e^{+\gamma x} - e^{-\gamma x}}{2}\right) + I_r Z_c\left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right)\right\}$$

$$= \frac{1}{Z_c}\left\{V_r \sinh\gamma x + I_r Z_c \cosh\gamma x\right\}$$

$$I = \frac{V_r}{Z_c}\sinh\gamma x + I_r \cosh\gamma x$$

Rewrite the equations $x = l$, $V = V_s$, $I = I_s$

$V_s = V_r \cosh\gamma l + I_r Z_c \sinh\gamma l$ ⟶ (13)

$I_s = \frac{V_r}{Z_c}\sinh\gamma l + I_r \cosh\gamma l$ ⟶ (14)

Compare equation (13) and (14) with ABCD equations

i.e., $V_s = AV_r + BI_r$

$I_s = CV_r + DI_r$

Then $A = \cosh\gamma l$ $\quad C = \frac{1}{Z_c}\sinh\gamma l$

$B = Z_c \sinh\gamma l$ $\quad D = \cosh\gamma l$

Reciprocity $AD - BC = 1$

Symmetry $A = D$

Reciprocity
$$AD - BC = 1$$
$$\cosh\gamma l \cdot \cosh\gamma l - \cancel{Z_c}\sinh\gamma l \cdot \frac{1}{\cancel{Z_c}}\sinh\gamma l = 1$$
$$\cosh^2\gamma l - \sinh^2\gamma l = 1$$

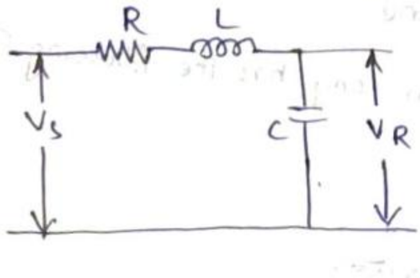
satisfied

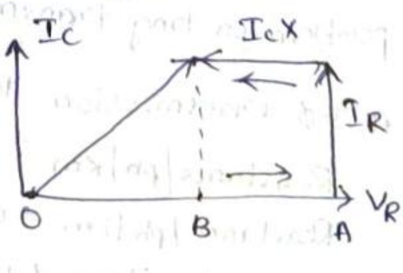
$\cosh^2\gamma l = \sinh^2\gamma l$ } Both satisfied

Symmetry $A = D$

Ferranti effect: When a long transmission line is operating under no-load or lightly load condition, the receiving end voltage is greater than the sending end voltage, due to the presence of capacitance this effect is called ferranti effect

$$\boxed{V_R > V_S}$$





$$OA - OB = BA$$
$$OA - OB = I_c X$$
$$= V_R \frac{Y}{2} \cdot X$$
$$= V_R \frac{(2\pi f C)(2\pi f L)}{2} = \frac{V_R}{2}(2\pi f)^2 LC$$

$$\boxed{I_c = V_R \frac{Y}{R} = V_R \frac{C}{R}}$$

$$OA - OB = \frac{V_R}{2}(2\pi f)^2 LC$$

Charging Current: Due to the presence of the capacitance in the long transmission line (100km more than) the current will be produced that current is called as charging current

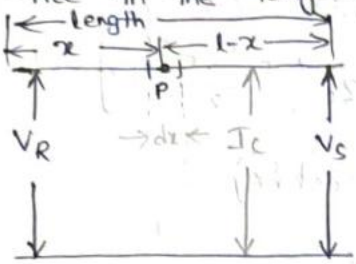


fig: power loss in the transmission line due to charging current, I_c.

Charging current at this point 'P'.

$$I_p = \frac{I_c}{l}(l - x)$$

Resistance per unit length R/l

Resistance for small length, $dx = I_p^2 \frac{R}{l} dx$

loss occur in the small element $dx = I_p^2 \frac{R}{l} dx$

Total loss of the entire line

$$\int_0^l I_p^2 \frac{R}{l} dx = \int_0^l \frac{I_c^2}{l^2}(l-x)^2 \frac{R}{l} dx = \frac{I_c^2 R}{l^3}\int_0^l (l-x)^2 dx$$

$$= \frac{I_c^2 R}{l^3}\left[\frac{(l-x)^3}{3}\right]_0^l (-1) = \frac{I_c^2 R}{l^3}\left[\left(\frac{(l-l)^3}{3} - \frac{(l-0)^3}{3}\right)\right](-1)$$

$$\int \frac{(l-x)^3}{3} dx \qquad \frac{(l-x)^3}{3}\frac{d}{dx}(-x) \qquad \left[\frac{(l-x)^3}{3}\right]_0^l (-1)\frac{(l-x)^3}{3}$$

$$= \frac{I_c^2 R}{l^3}\left(-\frac{l^3}{3}\right)(-1)$$

$$= \frac{I_c^2 R}{l^3} \cdot \frac{l^3}{3} = \frac{I_c^2 R}{3}$$

Total loss of the entire line is "$\frac{I_c^2 R}{3}$".

problem on long transmission line

1) A 3-ϕ transmission line 200km long has the following constants:

Resistance/ph/km = 0.16Ω

Reactance/ph/km = 0.25Ω

Shunt admittance/ph/km = 1.5×10^{-6} S

Calculate by using rigorous method the sending end voltage & Current when the line is delivering a load of 20MW at 0.8pf lagging. The receiving end voltage is kept constant at 110KV.

Sol: Given: Resistance/ph $R = 0.16 \times 200 = 32\Omega$

Reactance/ph $X_L = 0.25 \times 200 = 50\Omega$

Shunt admittance/ph, $Y = 1.5 \times 10^{-6} \times 200 = 0.0003 \angle 90^\circ$

Series impedance/ph, $Z = R + jX_L = 32 + j50 = 59.36 \angle 57.3$

$\therefore Z = 59.4 \angle 58$

Sending end voltage

$$\boxed{V_S = V_R \cos\sqrt{YZ} + I_R \sqrt{\frac{Z}{Y}} \sin\sqrt{YZ}}$$

$$\sqrt{YZ} = \sqrt{0.0003 \angle 90^\circ \times 59.36 \angle 57.3} = -0.0150 + j0.0096 = 0.0178 \angle 147.38$$

Generalised Circuit Constants of a Transmission line

In any four terminal network the input voltage and input current can be expressed interms of output voltage and output current.

∴ The input voltage (V_s) and input current (I_s) of a 3-ϕ transmission line can be expressed as

$$\vec{V_s} = A\vec{V_R} + B\vec{I_R} \longrightarrow (a)$$

$$\vec{I_s} = C\vec{V_R} + D\vec{I_R} \longrightarrow (b)$$

where, V_s = Sending end voltage/ph

I_s = " " Current /ph

V_R = Receiving end voltage/ph

I_R = " " Current

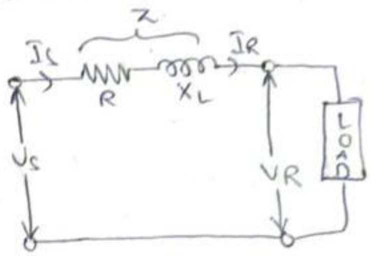
A, B, C, D are generalised constants

⇒ for a given transmission line $\vec{A} = \vec{D}$

⇒ $AD - BC = 1$

⇒ A & D are dimensionless B and C are ohm's & siemen respectively

(i) **Short transmission line:** In short transmission line the effect of line capacitance is neglected. So the line is considered to have series impedance



Here $I_s = I_R \longrightarrow (1)$

$V_s = V_R + I_R Z \longrightarrow (2)$

Compare (1) & (2) equations with (a) & (b)

we get

$$\boxed{A = 1,\ B = Z,\ C = 0,\ D = 1}$$

Conditions

symmetry $A = D$ [satisfy]
$1 = 1$

Reciprocity $AD - BC = 1$
$1\times1 - 0\times Z = 1$ [satisfy]
$1 = 1$

(ii) **Medium Transmission line:**

⟫ **Nominal 'T' method:** In this method the whole line to neutral capacitance is assumed to be considered at the middle of the line

$\because V_1 = V_R + I_R \frac{Z}{2}$

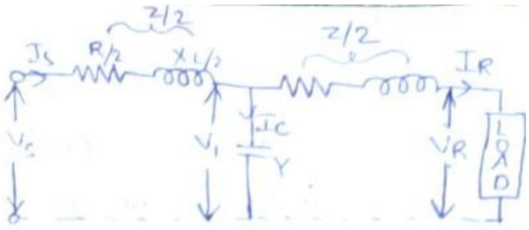
Here, $V_s = V_1 + I_s \frac{Z}{2}$

$V_s = V_R + I_R \frac{Z}{2} + I_s \frac{Z}{2}$

V_S= $I_S = I_C + I_R \rightarrow (i)$

$I_C = I_S - I_R = V_1 Y$

$I_C = Y\left(V_R + I_R \frac{Z}{2}\right) \rightarrow (ii)$



Sub (ii) in (i)

$$I_S = I_R + YV_R + YI_R\frac{Z}{2}$$

$$I_S = I_R\left(1+\frac{YZ}{2}\right) + V_R Y \longrightarrow (2)$$

Sub I_S in V_S

$$V_S = V_R + I_R\frac{Z}{2} + I_S\frac{Z}{2} = V_R + I_R\frac{Z}{2} + \left[I_R(+YV_R + YI_R\frac{Z}{2}\right]\frac{Z}{2}$$

$$= V_R + I_R\frac{Z}{2} + \left(I_R + V_R Y + I_R\frac{YZ}{2}\right)\frac{Z}{2}$$

$$= V_R\left(1+\frac{YZ}{2}\right) + I_R\left[\frac{Z}{2}+\frac{Z}{2}+\frac{YZ^2}{4}\right]$$

$$V_S = V_R\left(1+\frac{YZ}{2}\right) + I_R\left(Z+\frac{YZ^2}{4}\right) \longrightarrow (1)$$

Compare (1) and (2) with (a) and (b), we get

$V_S = AV_R + BI_R$ $\quad\quad$ $V_S = V_R\left(1+\frac{YZ}{2}\right) + I_R\left(Z+\frac{YZ^2}{4}\right)$

$I_S = CV_R + DI_R$ $\quad\quad$ $I_R = V_R(Y) + I_R\left(1+\frac{YZ}{2}\right)$

$A = 1+\frac{YZ}{2}$ $\quad\quad$ $C = Y$

$B = Z+\frac{YZ^2}{4}$ $\quad\quad$ $D = 1+\frac{YZ}{2}$

Conditions

$A = D \Rightarrow 1+\frac{YZ}{2} = 1+\frac{YZ}{2}$ [satisfy] $\longrightarrow$ Symmetry

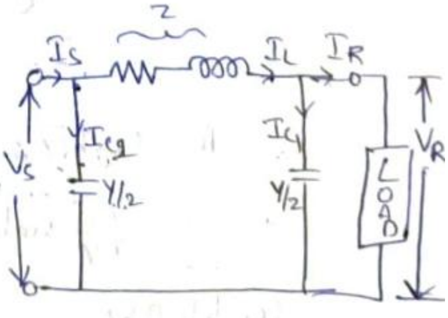
$AD - BC = 1 \Rightarrow \left(1+\frac{YZ}{2}\right)\left(1+\frac{YZ}{2}\right) - Y\left(Z+\frac{YZ^2}{4}\right) = 1$

$$1+\cancel{\frac{YZ}{2}}+\cancel{\frac{YZ}{2}}+\cancel{\frac{Y^2Z^2}{4}}-\cancel{YZ}-\cancel{\frac{Y^2Z^2}{4}}=1$$

$$1+\cancel{YZ}-\cancel{YZ}=1$$

$1 = 1$ [satisfy] $\rightarrow$ Reciprocity

→ Nominal 'π' Method: In this method the line-to-neutral capacitance is divided into two halves: one is concentrated at load end and the other half is at sending end



Here, $I_S = I_L + I_{C2}$ $\quad\quad$ $I_{C2} = V_S Y/2$

$I_L = I_R + I_{C1}$ $\quad\quad$ $I_{C1} = V_R\frac{Y}{2}$

$I_L = I_R + V_R\frac{Y}{2}$

$I_S = I_R + V_R\frac{Y}{2} + V_S\frac{Y}{2}$

$V_S = V_R + I_L Z = V_R + I_R Z + V_R\frac{YZ}{2} = V_R\left(1+\frac{YZ}{2}\right) + I_R Z \longrightarrow (1)$

$I_S = I_R + V_R\frac{Y}{2} + V_R\frac{Y}{2} + I_R\frac{YZ}{2} + V_R\frac{Y^2Z}{4}$

$$I_s = V_R\left(\frac{Y}{2}+\frac{Y}{2}+\frac{Y^2Z}{4}\right)+I_R\left(1+\frac{YZ}{2}\right)$$

$$I_s = V_R\left(Y+\frac{Y^2Z}{4}\right)+I_R\left(1+\frac{YZ}{2}\right) \longrightarrow ②$$

Compare ① and ② with ⓐ and ⓑ

$$V_s = AV_R + BI_R \longrightarrow ⓐ$$

$$I_s = CV_R + DI_R \longrightarrow ⓑ$$

$$V_S = V_R\left(1+\frac{YZ}{2}\right)+ZI_R \longrightarrow ①$$

$$I_s = V_R\left(Y+\frac{Y^2Z}{4}\right)+\left(1+\frac{YZ}{2}\right)I_R \longrightarrow ②$$

$A = 1+\frac{YZ}{2}$ $\qquad C = Y+\frac{Y^2Z}{4}$

$B = Z$ $\qquad D = 1+\frac{YZ}{2}$

Conditions:

1) $A = D \Rightarrow 1+\frac{YZ}{2} = 1+\frac{YZ}{2}$ (satisfy) $\rightarrow$ symmetry

2) $AD-BC = 1$

$$\left(1+\frac{YZ}{2}\right)\left(1+\frac{YZ}{2}\right)-Z\left(Y+\frac{Y^2Z}{4}\right) = 1+\frac{YZ}{2}+\frac{YZ}{2}+\frac{Y^2Z^2}{4}-YZ-\frac{Y^2Z^2}{4}$$

$$= 1+\cancel{YZ}+\cancel{\frac{Y^2Z^2}{4}}-\cancel{YZ}-\cancel{\frac{Y^2Z^2}{4}}$$

$$\left(1+\frac{YZ}{2}\right)\left(1+\frac{YZ}{2}\right)-Z\left(Y+\frac{Y^2Z}{4}\right) = 1 \quad \text{(satisfy)} \rightarrow \text{Reciprocity}$$

iii) Long transmission line - Rigorous method:

Here,
$$V_s = V_R\cosh\sqrt{YZ} + I_R\sqrt{\frac{Z}{Y}}\sinh\sqrt{YZ}$$
$\leftarrow$ 400 to 600Ω

$\leftarrow$ 40 to 60Ω Under ground

$$I_s = V_R\sqrt{\frac{Y}{Z}}\sinh\sqrt{YZ} + I_R\cosh\sqrt{YZ}$$

Natural power

Compare it with standard equations

$$V_S = AV_R + BI_R$$

$$I_s = CV_R + DI_R$$

$A = \cosh\sqrt{YZ}$ $\qquad C = \sqrt{\frac{Y}{Z}}\sinh\sqrt{YZ}$

$B = \sqrt{\frac{Z}{Y}}\sinh\sqrt{YZ}$ $\qquad D = \cosh\sqrt{YZ}$

Conditions,

1) $A = D$ (satisfy) $\rightarrow$ symmetry

2) $AD-BC = 1$ $\qquad [\cosh\sqrt{YZ}][\cosh\sqrt{YZ}]-\left[\sqrt{\frac{Y}{Z}}\sinh\sqrt{YZ}\right]\left[\sqrt{\frac{Z}{Y}}\sinh\sqrt{YZ}\right]=1$

$\cosh^2\sqrt{YZ} - \sinh^2\sqrt{YZ} = 1$ (satisfy)

$1 = 1 \longrightarrow$ Reciprocity

Surge Impedance

Surge impedance (Z_c) of a line is defined as the square root of Z/Y i.e.,

$$\boxed{Z_c = \sqrt{\frac{Z}{Y}}}$$

where, Z is the series impedance $(R+jX)$ and
Y is the shunt admittance $(G+jB)$

Surge impedance is the characteristic impedance of a loss-less line.

for a line having negligible resistance and having no shunt leakage i.e., $R=0$ & $G=0$ then

$$Z_c = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+jX}{G+jB}} = \sqrt{\frac{0+j\omega L}{0+j\omega C}} = \sqrt{\frac{j\omega L}{j\omega C}}$$

$\boxed{Z_c = \frac{L}{C}}$ which is pure resistance.

It has a value of 400 to 600Ω for an overhead line of 40 to 60Ω for an underground cable.

The surge impedance of a line may be measured interms of Z_{oc} and Z_{sc} where these are impedance measured at the sending end with receiving end open-circuited and short-circuited respectively.

for a transmission line.

$$V_S = AV_R + BI_R \longrightarrow (1)$$

$$I_S = CV_R + DI_R \longrightarrow (2)$$

Case:1

When the line at the receiving end is open-circuited $I_R = 0$, equation (1) & (2) becomes.

$$Z_{oc} = \frac{V_S}{I_S}$$

$$V_S = AV_R$$

$$I_S = CV_R$$

$$\therefore Z_{oc} = \frac{V_S}{I_S} = \frac{AV_R}{CV_R} = \frac{A}{C} \longrightarrow (3)$$

Case: 2

Similarly, when the line at receiving end is short-circuited, $V_r = 0$ then eq ① & ② becomes

$$Z_{SC} = \frac{V_R}{I_R},$$

$$V_s = B I_r$$
$$I_s = D I_r$$

$$\therefore Z_{sc} = \frac{V_s}{I_s} = \frac{B I_r}{D I_r} = \frac{B}{D} \longrightarrow ④$$

Multiplying ③ & ④, we get

$$Z_{oc} Z_{sc} = \frac{A}{C} \cdot \frac{B}{D}$$

WKT $A = D$ for a bilateral network

$$Z_{oc} Z_{sc} = \frac{B}{C} = \frac{Z}{Y} \longrightarrow (i)$$

WKT $Z_c = \sqrt{\frac{Z}{Y}}$ or $Z_c^2 = \frac{Z}{Y} \longrightarrow (ii)$

Compare (i) and (ii)

$$Z_c^2 = Z_{oc} Z_{sc}$$

$$Z_c = \sqrt{Z_{oc} Z_{sc}}$$

⇒ Surge impedance loading: This is defined as the load that can be delivered by the line having no resistance, the load being at unity power factor. The power transmitted under these condition is

$$\boxed{P_r = \frac{V_r^2}{Z_0} \text{ MW}} \longrightarrow (i)$$

where V_r is the receiving end line voltage in kV and Z_0 is the surge impedance of the line in ohm's.

P_r is known as the surge impedance loading also called Natural power of the line.

for $Z_0 = 400\Omega$

$$P_r = \frac{V_r^2}{Z_0} = \frac{V_r^2}{400}$$

$$P_r = 2.5 V_r^2 \text{ kW}$$

Surge impedance loading can be used for the comparison of loads that can be carried on the lines at different

Voltages

In order to increase the power transmitted through a long transmission line either value of receiving and voltage is to be increased or more than one transmission line can run in parallel. The latter method is however very costly.

Thus from equation (i), in order to increase P_r either V_r is to be increased or Z_0 is to be decreased.

i) Increase in Voltage, V_r: Now-a-days the trend is for higher and higher voltages so that this is the most widely adopted method to increase the power limit for heavily log transmission lines. But there are some practical difficul-ties in this method and it is expensive.

ii) Decrease of surge impedance: Since the spacing b/w the conductors can't be decreased much, it being depen-dant on the line voltages and corona etc., the value of Z_0 cannot be varied as such.

But some artificial means are employed to decrease Z_0 for a lossless transmission line $Z_0 = \sqrt{\frac{L}{C}}$ and $\gamma = j\omega\sqrt{LC}$

$\gamma = j\beta$. We know that γ is the propagation constant and β is the phase shift. The latter determines torque angle of between V_s and V_r and hence system stability.

To ~~determine~~ decrease Z_0 either L is decreased using series capacitors or C is increased using shunt capacitors

UNIT-III

Mechanical Design of Transmission line

Introduction:- The overhead line conductors should be supported on the poles of towers in such a way that currents from conductors do not flow to earth through supports i.e., line conductors must be properly insulated from supports. This is achieved by securing line conductors to supports with the help of insulators.

The insulators provide necessary insulation between line conductors and supports and thus prevent any leakage current from conductors to earth.

Properties of insulators:

* ⇒ High mechanical strength in order to withstand conductors load, wind load etc.
* ⇒ High electrical resistance of insulator material in order to avoid leakage currents to earth.
* ⇒ High relative permitivity of insulator material in order that dielectric strength is high.
* ⇒ The insulator material should be non-porous, free from impurities and cracks otherwise the permitivity will be lowered.
* ⇒ The ratio of puncture strength to flashover.

The most commonly used material for insulators of overhead line is porcelain, but glass, stealite and special composition materials and are also used to a limited extent.

porcelain is produced by firing at a high temperature a mixture of koolen, feldspar and quartz. It is stronger mechanically than glass, gives less trouble from leakage and is less effected by changes in temperature.

Types of insulators:

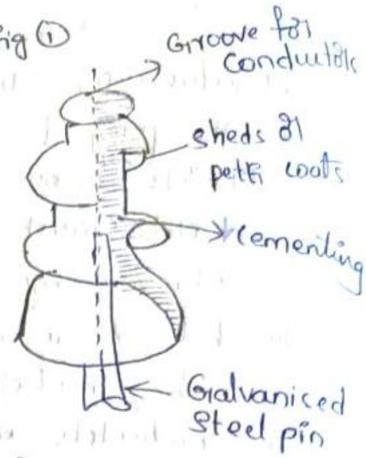
There are several types of insulators but the most commonly used are

1) pin type.
2) suspension type.
3) strain.
4) shackle.

1) Pin type insulators The part section of a pin type insulator is shown in fig ①. The pin type insulator is secured to the cross arm on pole.

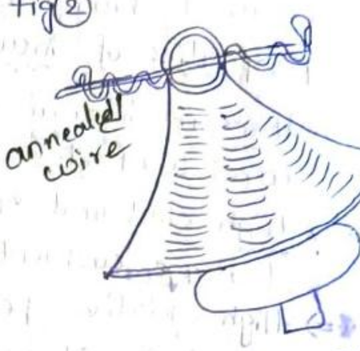
There is a groove on the upper end of the insulator for housing the conductor. The conductor passes through this groove and is bounded by the annealed wire of the same material as the conductor shown in fig ②

fig ①



Pin type insulators are used for transmission & distribution of electric power at Voltages upto 33KV. Beyond operating Voltage of 33KV, the pin type insulators become too bullay and hence uneconomical.

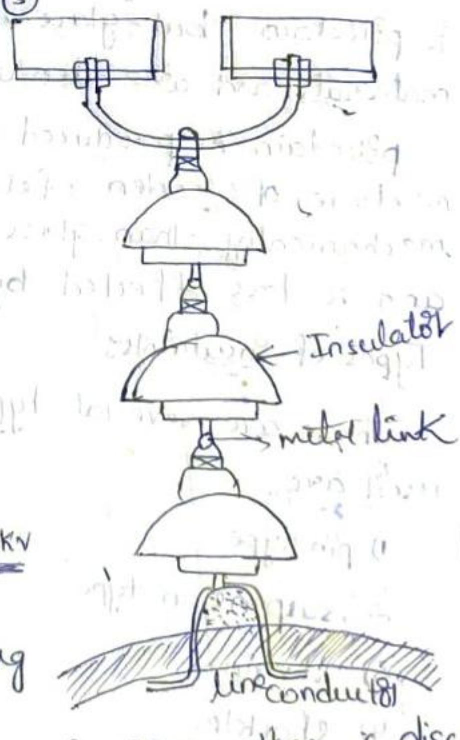
fig②



2) Suspension type insulators:

The cost of pin type insulator increases rapidly as the working Voltage is increased. Therefore, this type of insulator is not economical beyound 33KV. for high Voltages (>33KV) it is a usual practice to use suspension type insulators shown in fig ③

fig③



They consist of a number of porcelain discs connected in series by metal links in the form of a string. The conductor is suspended at the bottom end of this string while the other end of the string is secured to the cross-arm of the tower. Each unit of disc is designed for low Voltage, say 11KV. The no: of discs in series would obviously depend upon the working Voltage.

for instance, of the working Voltage is 66KV, then 6 discs in series will be provided on the string.

Advantages

* Suspension type insulators are cheaper than pin type insulators for voltages beyond 33kV
* Each unit or disc of suspension type insulator is designed for low voltage, usually 11kV. Depending upon working voltage, the desired no: of discs can be connected in series.
* If any one of the disc is damaged, the whole string doesn't become useless because the damaged disc can be replaced by the second one.
* The suspension arrangement provides greater flexibility to the line. The connection at the cross arm is such that insulator string is free to swing in any direction and can takeup the position where mechanical stresses are minimum.
* Incase of increased demand on the transmission line, it is found more satisfactory to supply the greater demand by raising the line voltage than to provide another set of conductors. The adition insulation required for raised voltage can be easily obtained in the suspension arrangement by adding the desired number of discs.
* The suspension type insulators are generally used with steel towers. As the conductors are run below the earthed cross-arm of the tower, therefore this arrangement provides practical protection from lightning.

3) **Strain Insulators**: When there is a dead end of the line or there is corner or sharp curve, the line is subjected to greater tension. In order to relieve the line of excessive tension, strain insulator are used.

For low voltage lines (<11kV) shackle insulators are used as strain insulators; however for high voltage transmission line strain insulator consists of an assembly of suspension insulators as shown in fig ④

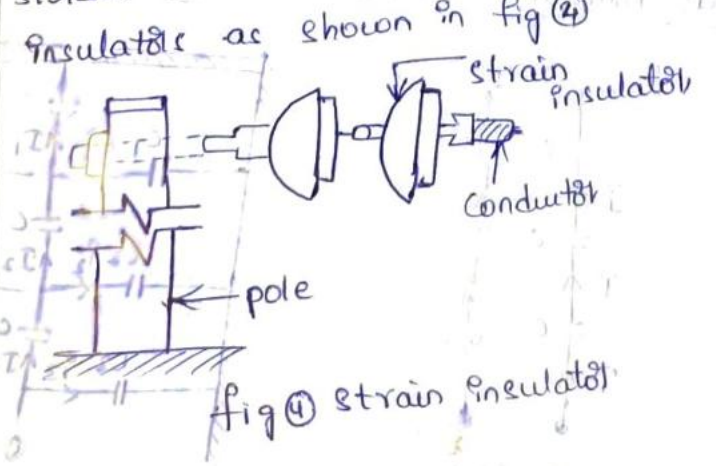
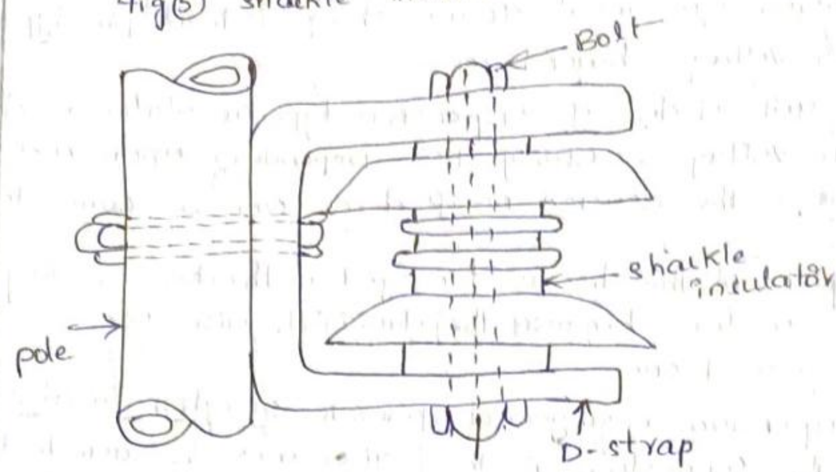


fig ④ strain insulator

fig⑤ shackle insulator.



The disc of strain insulators are used in the vertical plane. when the tension in lines is exceedingly high, as at long river spans, two or more strings are used in parallel.

Shackle Insulators: In early days, the shackle insulators were used as strain insulators. But now-a-days, they are frequently used for low voltage distribution lines. Such insulators can be used either in horizontal position or in a vertical position. they can be directly fixed to the pole with a bolt or to the cross arm. as shown in fig⑤ the conductor in the groove is fixed with a soft binding wire.

⇒ **potential distribution over suspension Insulator string:**

A string of suspension insulator consists of a number of porcelain discs connected in series through metallic link's the fig(a) shows 3-disc string of suspension insulators. The porcelain position of each disc is in between two metal links. Therefore, each disc form a capacitor C as shown in fig⑥. This is known as mutual capacitance or self capacitance.

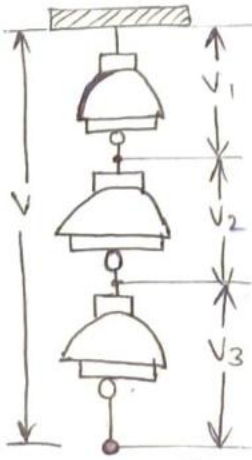


fig ⓐ

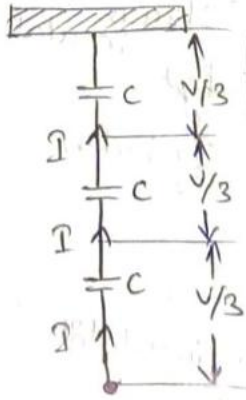


fig ⓑ

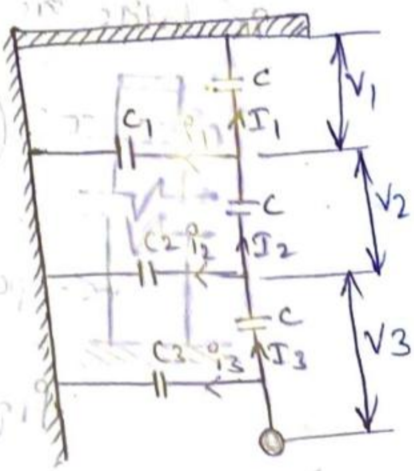


fig ⓒ

If there were mutual capacitance alone, then charging current would have been the same through all the discs and consequently voltage across each unit would have been the same i.e., V/3 are shown in fig (b)

However, in actual practice, capacitance also exists b/w metal fitting of each disc and tower or earth. This is known as shunt capacitance C_1.

Due to shunt capacitance, charging current is not the same through all the discs of the string as shown in fig (c). therefore, voltage across each disc will be different. obviously the disc nearest to the line conductor will have the maximum voltage i.e., ref to fig (c), V_3 will be much more than V_2 or V_1.

Important points:

⇒ The voltage on a string of suspension insulators doesn't distribute it's self uniformly across the individual discs due to the presence of shunt capacitance.

⇒ The disc nearest to the conductor has maximum voltage across it. As we move towards the cross-arm the voltage across each disc goes on decreasing.

⇒ The unit nearest to the conductor is under maximum electrical stress & is likely to be punctured. therefore, means must be provided to equalise the potential across each unit.

⇒ If the voltage impressed across the string were dc then voltage across each unit would be the same. It is because insulator capacitances are ineffective for dc.

⇒ String Efficiency: The voltage across the string of suspension insulators is not uniformly distributed across various units or discs. The disc nearest to the conductor has much higher potential than the other discs. This unequal potential distribution is undesirable and is usually expressed in terms of string efficiency.

" The ratio of voltage across the whole string to the product of no: of discs and the voltage across the ~~string~~ disc nearest to the conductor is known as

String efficiency %:

$$\text{String Efficiency} = \frac{\text{Voltage across the string}}{n \times \text{Voltage across disc nearest to the conductor}}$$

where,

n = no: of discs in the string.

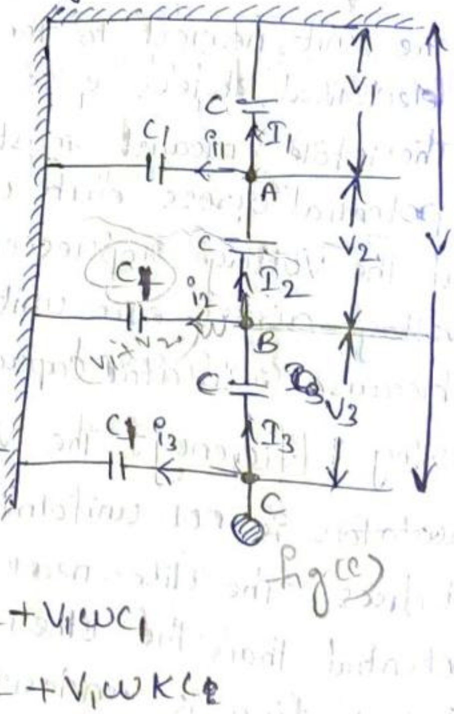
* String Efficiency decides the potential distribution along the string.
* The greater the string efficiency, the more uniform is the voltage distribution

Mathematical Expression: fig© shows the equivalent ckt for the self capacitance of a 3-disc string. let us assume the self capacitance each disc is c. let us assume that shunt capacitance C_1 is some fraction K of self capacitance

$\boxed{\text{i.e., } C_1 = KC}$

starting from the cross arm or tower, the voltage across each unit is V_1, V_2 & V_3 respectively as shown.

Applying KCL at node A, we get



fig(c)

$I_2 = i_1 + I_1$

$V_2 \omega c = V_1 \omega c + V_1 \omega C_1$

$V_2 \omega c = V_1 \omega c + V_1 \omega K c$

$V_2 = V_1 + V_1 K$

$\boxed{V_2 = V_1(1+K)} \longrightarrow ①$

Applying KCL at node B, we get

$I_3 = I_2 + i_2$

$V_3 \omega c = V_2 \omega c + V_2 \omega C_1 + V_1 \omega C_1$

$= V_2 \omega c + V_2 \omega K c + V_1 \omega K C$

$V_3 = V_2 + V_2 K + V_1 K$

$\boxed{V_3 = V_2(2+K)} \longrightarrow ②$

$$I_3 = I_2 + i_2$$

$$V_3 \omega C = V_2 \omega C + (V_1 + V_2)\omega C_1 \qquad [C_1 = kC$$

$$= V_2 \omega C + V_1 \omega C k + V_2 \omega k C$$

$$V_3 = V_2 + V_1 k + V_2 k \qquad V_2 = V_1(1+k)$$

$$= V_1(1+k) + V_1 k + \cancel{V_2}\, V_1(1+k)k$$

$$= V_1 + V_1 k + V_1 k + V_1 k + V_1 k^2$$

$$* \quad = V_1(3k + k^2 + 1)$$

$$\boxed{V_3 = V_1(k^2 + 3k + 1)} \longrightarrow (2)$$

Voltage between conductor and earth

$$V = V_1 + V_2 + V_3$$

$$V = V_1 + V_1(1+k) + V_1(k^2 + 3k + 1)$$

$$= V_1 + V_1 + V_1 k + V_1 k^2 + 3V_1 k + V_1$$

$$= V_1(2 + k + k^2 + 3k + 1) = V_1(3 + k^2 + 4k)$$

$$V = V_1(k^2 + 4k + 3)$$

$$* \quad \boxed{V = V_1(1+k)(3+k)} \longrightarrow (3)$$

from eq (1), (2) & (3), we get

$$V_1 = \frac{V_2}{1+k} = \frac{V_3}{k^2+3k+1} = \frac{V}{(1+k)(3+k)} \longrightarrow (4)$$

Voltage across top unit

$$V_1 = \frac{V}{(1+k)(3+k)}$$

Voltage across second unit, third unit

$$V_2 = V_1(1+k) \qquad V_3 = V_1(k^2 + 3k + 1)$$

% string efficiency

$$= \frac{\text{Voltage across string}}{n \times \text{Voltage across disc nearest to the conductor}} \times 100$$

$$= \frac{V}{3V_3} \times 100$$

$$* \quad \boxed{\begin{aligned} V_4 &= V_1[1 + 6k + 5k^2 + k^3] \\ V_5 &= V_1[1 + 10k + 15k^2 + 7k^3 + k^4] \end{aligned}}$$

Important points:

⇒ If $K = 0.2$ (say), then from equation (4), we get $V_2 = 1.2V_1$ & $V_3 = 1.64V_1$. This clearly shows that disc nearest to the Conductor has maximum Voltage across it.

⇒ The greater line voltage of $(K = \frac{C_1}{C})$ the more non-uniform is the potential across the discs and lesser is the string efficiency.

⇒ The inequality in voltage distribution increases with the increase of number of disc in the string.

∴ shorter string has more efficiency than the Longer one.

Methods of improving string Efficiency:

If the insulation of the highest stressed insulator (i.e., nearest to the Conductor) breaks down or flash over takes place, the breakdown of other units will take place in succession. This necessitates to equalise the potential across the Various units of the string i.e., to improve the string efficiency.

The Various methods are:

1) By using longer Crossarm: The value of string efficiency depends upon the value of K i.e., ratio of shunt Capacitance to mutual Capacitance.

⇒ The lesser the Value of k, the greater is the string efficiency and more uniform is the voltage distribution. The Value of k can be decreased by reducing the shunt Capacitance. In order to reduce shunt capacitance the distance of Conductor from tower must be increased i.e., lower cross-arm should be used.

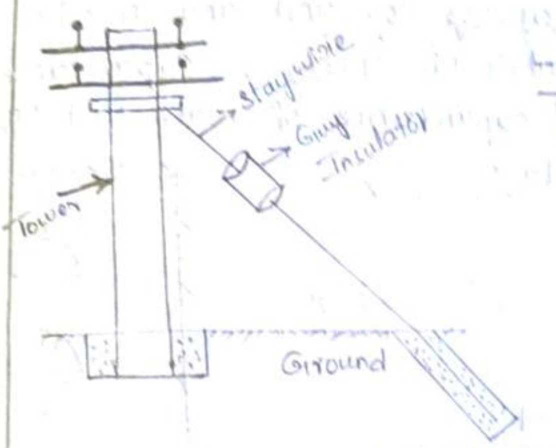
However, the limitations of cost & strength of tower don't allow the use of very long cross-arm.

In practice, $k = 0.1$ is the limit that can be achieved by this method.

2) By grading the insulators: In this method, insulators of different dimensions are so chosen that each has a different Capacitance. The insulators are capacitance graded i.e., they are assembled in the string in such a way that the top unit has the minimum capacitance, increasing progressively as the bottom unit is reached. Since the Voltage is inversly proportional to the Capacitance, this method tends to equalise the potential distribution

→ continuation on next

Guy Insulator:



→ It used at low voltage lines, Transformers.

→ It is made up of porcelain material

→ Galvanization → zink coating on Iron

→ In order to prevent leakage current from tower to ground without guy insulator it may harmful to living organisms.

problems

1) A 33kV overhead transmission line having 3 units in the string of insulator. The capacitance b/w each insulator pin and earth is 11% of self capacitance of each insulator. find the distribution of voltages among three insulators and string efficiency

sol $V = 33kV = \frac{33}{\sqrt{3}} = 19.05kV$

$n = 3$ units

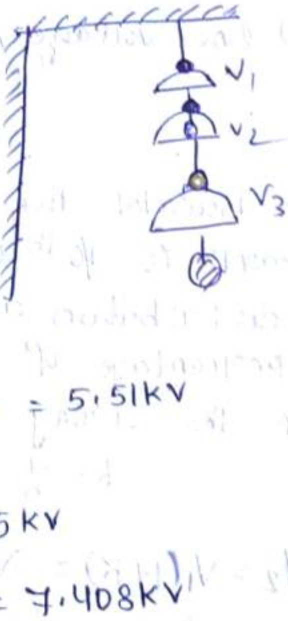
k = Capacitance b/w each insulator & earth is 11% of self capacitance

k = 11% of self capacitance

$k = \frac{11}{100} = 0.11$

$V_2 = V_1(1+k)$

$V_3 = V_1(k^2+3k+1)$



$V = V_1(1+k)(3+k)$

$19.05\times10^3 = V_1(1+0.11)(3+0.11)$

$= V_1(1.11)(3.11)$

$= 3.4521V_1$

$V_1 = \frac{19.05\times10^3}{3.4521} = 5518.38V = 5.51kV$

$V_2 = 5518.38(1+0.11) = 6.125kV$

$V_3 = 5518[(0.11)^2+3(0.11)+1] = 7.408kV$

$\text{String } \eta = \frac{V_1+V_2+V_3}{n\times V_3}\times100 = \frac{5518.38+6125+7408}{3\times7408}\times10$

$= \frac{19.05\times10^3}{3\times7408}\times100 = 85.7$

2) A 3-ϕ overhead transmission line is ... middle unit insulators. The potential across top unit and middle unit are 8kV and 11kV. Calculate (i) ratio of capacitance b/w pin and earth to self capacitance of each unit (ii) string efficiency (iii) line voltage.

$n=3$

$V_2 = V_1(1+k)$

$11\times10^3 = 8\times10^3(1+k)$

$k = \frac{11\times10^3}{8\times10^3} - 1$

$k = 0.375$

$V_3 = V_1(1+3k+k^2) = 8\times10^3[1+3\times0.375+(0.375)^2]$

$= 18.125kV$

(i) $V = V_1+V_2+V_3 = 11+8+18.125 = 37.125$

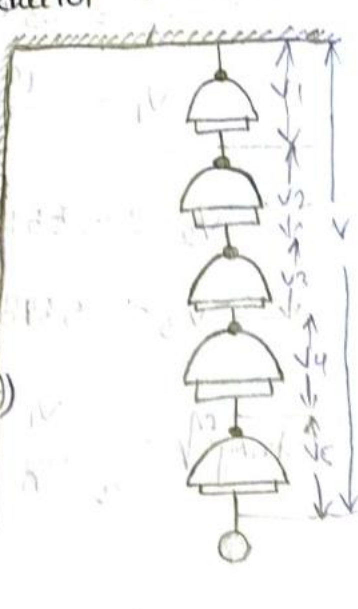
(ii) String Efficiency $\eta_{string} = \frac{\text{Voltage of the string}}{n\times\text{voltage of the bottom disc}}\times100$

$= \frac{37.125}{3\times18.125}\times100 = 68.27\%$

(iii) line Voltage, $V_L = \sqrt{3}V_{ph}$

$= \sqrt{3}\times37.125 = 64.29kV$

3) 5 Insulator disc string capacitance b/w each unit and earth is 1/6th of the mutual capacitance. Find the voltage distribution across each insulator in the string as percentage of voltage of the conductor to earth. Determine the string efficiency?



Sol: $k = \frac{1}{6} = 0.167$

$V_2 = V_1(1+k) = V_1(0.167+1)$

$V_2 = 1.167V_1 \longrightarrow (1)$

$V_3 = V_1(1+3k+k^2) = V_1(1+3\times0.167+0.167^2)$

$V_3 = 1.529V_1 \longrightarrow (2)$

$V_4 = V_1(1+6k+5k^2+k^3)$

$= V_1(1+6\times0.167+5\times0.167^2+0.167^3)$

$$V_4 = 2.146 V_1 \longrightarrow (3)$$

$$V_5 = V_1(1+10K+15K^2+7K^3+K^4) = V_1(1+10\times0.167+15\times0.167^2+7\times0.167^3+0.167^4)$$

$$V_5 = 3.122 V_1 \longrightarrow (4)$$

$$V_1 = \frac{V_2}{1+K} = \frac{V_3}{1+3K+K^2} = \frac{V_4}{1+6K+5K^2+K^3} = \frac{V_5}{1+10K+15K^2+7K^3+K^4}$$

$$V = \frac{V_1}{(1+K)(3+K)} = \frac{V_1}{(1+0.167)(3+0.167)}$$

$$V = 0.2706 V_1 \longrightarrow (5)$$

$$V_2 = \frac{1.167 \cancel{V_1}}{0.2706 \cancel{V_1}} = 4.286$$

$$V_3 = \frac{1.529 \cancel{V_1}}{0.2706 \cancel{V_1}} = 5.65$$

$$V_4 = \frac{2.146 \cancel{V_1}}{0.2706 \cancel{V_1}} = 7.93$$

$$V_5 = \frac{3.122 V_1}{0.206 V_1} = 15.155$$

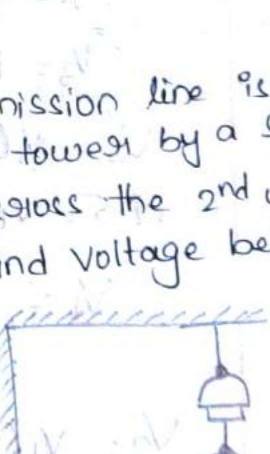
$$V_1 = \frac{1}{0.206} = 4.854$$

$$\eta_{string} = \frac{\text{Voltage of the string}}{n \times \text{Voltage across the bottom disc}} \times 100$$

$$= \frac{8.95 V_1}{5 \times 3.122 V_1} \times 100 = \frac{8.95}{15.61} \times 100$$

$$\eta_{string} = 57.38\%$$

4) Each conductor of a 3-φ Overhead transmission line is suspended from a cross arm of a steel tower by a string of 4 suspential insulators. The voltage across the 2nd unit is 15KV and across 3rd unit is 21KV. find voltage between disk/conductors and string efficiency.



$V_2 = 15\,KV$

$V_3 = 21\,KV$

$$\frac{V_2}{1+K} = \frac{V_3}{K^2+3K+1}$$

$$V_2(1+K^2+3K) = V_3(1+K)$$

$$15(K^2+3K+1) = 21(1+K)$$

$\frac{15}{21}(k^2+3k+1) = (1+k) \Rightarrow 0.714k^2+2.143k+0.714 = 1+k$

$0.714k^2+1.143k-0.286 = 0$

$k = 0.22$

$V_2 = V_1(1+k)$

$V_1 = \frac{V_2}{1+k} = \frac{15}{1+0.22} = 12.29\,kV$

$V_4 = V_1(1+6k+5k^2+k^3) = 31.62\,kV$

$V = V_1+V_2+V_3+V_4$

$= 12.29+15+21+31.62$

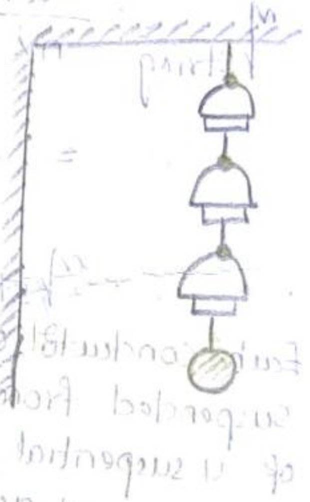
$V = 79.91\,kV$

$\eta_{string} = \frac{79.91 \times 10^3}{4 \times 31.62 \times 10^3} \times 100$

$= 63.18\%$

$= 63.18\%$

5) Each line of a 3-φ system is suspended by a string of 3 similar insulators. If the voltage across the line unit disc near the conductor is 17.5KV. Calculate the line to neutral voltage. Assume that the shunt capacitance between each insulator and earth is $\frac{1}{8}$th of the capacitance of the insulator it's self find the string efficiency.



$k = \frac{1}{8} = 0.125$

$V_3 = 17.5\,KV$

$V_3 = V_1(k^2+3k+1)$

$17.5 = V_1(0.125^2+3\times0.125+1)$

$V_1 = \frac{17.5}{0.125^2+3\times0.125+1} = 12.58\,KV$

$V_2 = V_1(1+k) = 12.58(1+0.125) = 14.157\,KV$

$V = V_1+V_2+V_3 = 44.23\,KV$

$\eta_{string} = \frac{44.23}{3 \times 17.55} \times 100 = 84.26\%$

⇒ Method's to improve string efficiency:

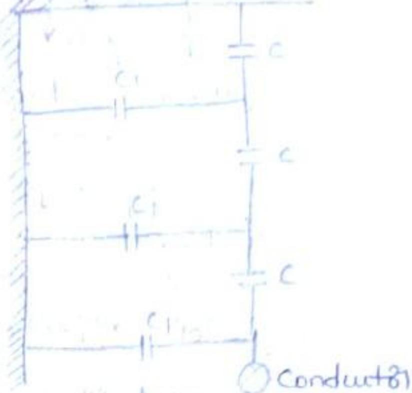
method-1

1) By Using longer cross-arm: string efficiency depends on the value of K.

Cross arm length ... longer ... mechanically weak

where K is ratio of shunt capacitance to the mutual capacitance.

* Lesser the value of K, greater the string efficiency and more uniform voltage distribution



$\eta \uparrow \rightarrow K \downarrow \rightarrow C_1 \downarrow \rightarrow$ long cross arm

method-2

3) Grading of Insulator: Capacitance Grading

Bottom disc near the conductor has ... capacitance and goes on to down to top the capacitance decreases.

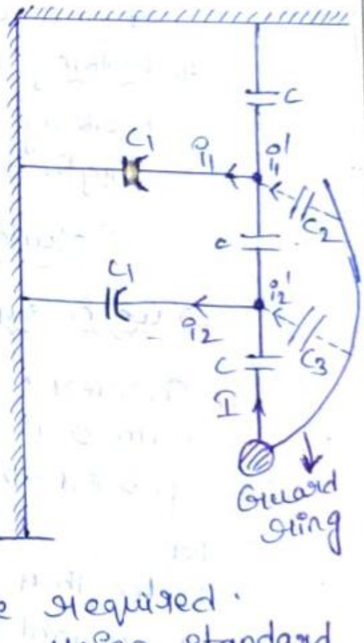
Top unit has the minimum capacitance, increasing progressively as the bottom unit (i.e., nearest to conductor)

different materials are used in the disc i.e., different grading so the cost increases.

Capacitances are used

method-3

4) By Using Guard Ring: Guard Ring which is a metal ring electrically connected to the conductor and surround the bottom insulator. This method also called as static shielding.



* The design of the ring should be such that shunt capacitance currents i_1, i_2 etc, are equal to metal fitting line capacitance currents i_1', i_2' etc, consequently the same charging current I flows through each unit of a string, leading to uniform potential distribution across the units.

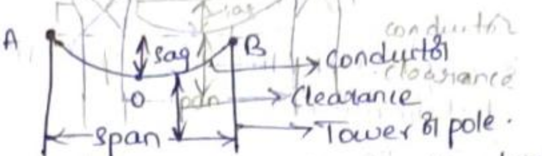
⇒ continuation

across the units in the string. This method has the disadvantage that a large number of different sized insulators are required. However good results can be obtained by using standard insulator for most of the string and larger units for that near to the line conductor.

~~3) By Using guard Ring:~~

IInd Mid

Sag:- The difference in level between points of supports of poles and lowest point of the conductor is called sag.



66KV → clearance: 1m, → span: 300m

Sag > span

Span:- the distance between the two adjacent transmission line tower or pole.

- The sag is greater than the span.
- In order to avoid the breakages of conductor in the transmission line the conductor - the conductor is streched called sag.

while connecting dampers we decrease the sag at the installation

⇒ Factor's effecting sag:

1) weight of the conductor: If the weight of conductor material used is more then the sag in transmission line is also increased. Height of the tower ↑

2) Length of the span: If the length of the span is longer then the sag in the transmisson lines is increased by square times.

3) Temperature: If the temperature increases then the sag in the transmission line is increaced.

4) Working Tensile strength: The sag is inversely proportional to the working tensile strength of the conductor at temperature length of the span etc., remains Constant.

Calculation of sag

⇒ when supports are at equal levels:

Consider a conductor between two equilevel supports A and B with O as the lowest point as shown in fig. It can be proved that lowest point will be at the mid-span

let

l = length of span

w = weight per unit length of conductor

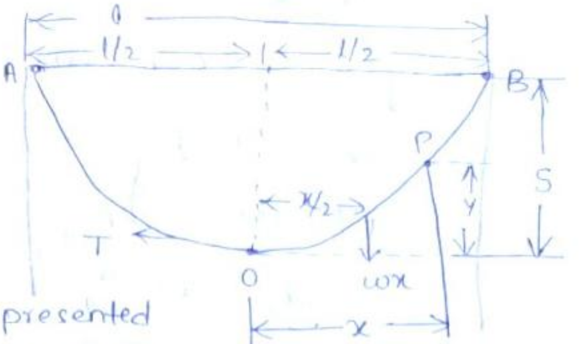
T = Tension in the conductor.

Consider a point P on the conductor. Taking the lowest point O as the origin, let the co-ordinates of point P be x and y. Assuming that the curvature is so small that curved length is equal to its horizantal projection the two forces acting on the portion OP of the conductor are:

a) weight 'wx' of the conductor acting at a distance $\frac{x}{2}$ from O.

b) Tension T acting at O.

Equating the moments of above two forces about point O, we get



$$Ty = wx \times \frac{x}{2}$$

$$y = \frac{wx^2}{2T}$$

The maximum sag is represented by the value of y at either of the supports A and B.

At support A $x = \frac{l}{2}$, $y = S$

$$\text{Sag}, S = \frac{w(l/2)^2}{2T} = \frac{wl^2}{8T}$$

$$\boxed{\therefore\ S = \frac{wl^2}{8T}}$$

⇒ when supports are at unequal levels:

In hilly areas, we generally come across conductors suspended between supports at unequal levels.

A conductor suspended between two supports A and B which are at different levels. The lowest point on the conductor is O.

let

l = span length

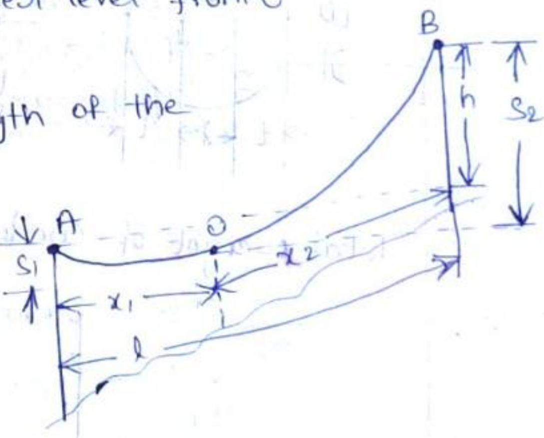
h = Difference in levels between two supports

x_1 = Distance of support at lower level from O.

x_2 = Distance of support at higher level from O.

T = Tension in the conductor.

If w is the weight per unit length of the conductor.



$$\text{Sag}, S_1 = \frac{wx_1^2}{2T}$$

$$S_2 = \frac{wx_2^2}{2T}$$

$$x_1 + x_2 = l \longrightarrow ①$$

Now $S_2 - S_1 = \frac{w}{2T}\left[x_2^2 - x_1^2\right] = \frac{w}{2T}\left[(x_2+x_1)(x_2-x_1)\right]$

$$= \frac{wl}{2T}\left[x_2 - x_1\right] \qquad \boxed{x_1 + x_2 = l}$$

$$S_2 - S_1 = h$$

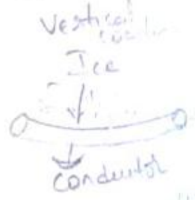
$$h = \frac{wl}{2T}(x_2 - x_1)$$

$$x_2 - x_1 = \frac{2Th}{wl} \longrightarrow (2)$$

Solving equation (1) and (2), we get

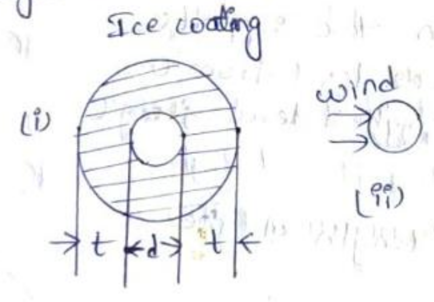
$$x_2 + \cancel{x_1} = l$$
$$x_2 - \cancel{x_1} = \frac{2Th}{wl}$$
$$2x_2 = l + \frac{2Th}{wl}$$
$$\boxed{x_2 = \frac{l}{2} + \frac{Th}{wl}}$$

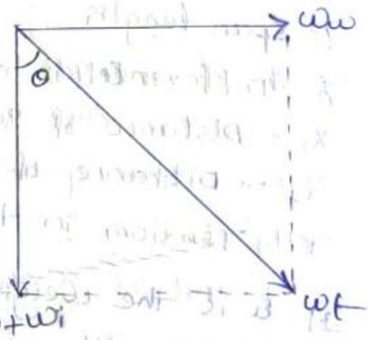
$$\rightarrow x_1 = l - x_2 = l - \frac{l}{2} + \frac{Th}{wl}$$
$$\boxed{x_1 = \frac{l}{2} - \frac{Th}{wl}}$$



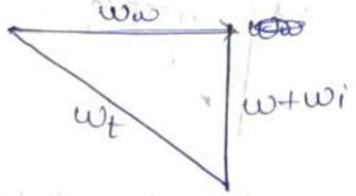
(iii) Effect of wind and ice loading:

If the transmission lines are laid in hilly areas cause the transmission line conductor are effected sometime with heavy wind pressure and ice falling.
The weight of the conductor (w) and ice (w_i) acts horizantally.
The force due to the wind (w_w) acts horizantally i.e., right angle to the surface of the conductor.





Total weight of conductor per unit length



Accolding to phythogolous theolem

$$w_t = \sqrt{(w_w)^2 + (w + w_i)^2}$$

where, w = weight of the conductor per unit length
= Conductor material density × Volume per unit length.

w_i = weight of ice per unit length
= density of ice × Volume of ice per unit length.
$$= \text{density of ice} \times \frac{\pi}{4}\left[(d+2t)^2 - d^2\right] \times 1$$

$= \text{density of ice} \times \frac{\pi}{4}\left[d^2 + 4t^2 + 4dt - d^2 \right]$

$= \text{density of ice} \times \frac{\pi}{4} \times 4\left[t^2 + dt \right]$

$= \text{density of ice} \times \pi t(d+t)$

w_w = wind force per unit length

= wind force per unit area × projected area

= wind pressure × [(d+2t) × 1]

Volume of ice per unit length

$$= \frac{\pi}{4}\left[(d+t)^2 - d^2 \right] \times 1 = \frac{\pi}{4}\left[4dt + 4t^2 \right] = \pi t(d+t)$$

Consider the wind and Ice loading on the conductor:

i) The conductor sets itself in a plane at an angle θ to the vertical where

$$\tan\theta = \frac{w_w}{w + w_i}$$

ii) the sag in the conductor is given by

$$\boxed{S = \frac{w_t l^2}{2T}}$$

iii) Vertical sag = $S\cos\theta$

~~working tensile strength~~ =

factor of safety = $\frac{\text{Maximum stress}}{\text{permissible stress}}$

Corona:- The whole phenomenon of hising noise, the violet glow and production of Ozone gas is known as corona.

p.63 called corona

or

Ionization of air surrounding the power conductors

⇒ Dielectric strength of air equal to 30kv/cm peak at NTP

Critical disruptive voltage: The minimum phase to neutral voltage at which corona occurs.

$$V_c = m_0 g_0 \delta r \log \frac{d}{r} \text{ kv/ph}$$

$m_0 = 1$ for polished conductor, 0.98 to 0.92 for dirty conductor,

0.87 to 0.8 for standard conductor

δ = air density factor = $\frac{3.92b}{273+t}$

g_0 = breakdown strength of the air at 76cm of mercury &

25°c = 30kv/cm or 21.2 kv/cm (r.m.s)

Visual Corona Critical Voltage: It is the minimum phase-neutral voltage at which corona glow appears all along the line conductors.

$$V_v = m_v g_0 \delta r \left(1 + \frac{0.3}{\sqrt{\delta r}}\right) \log_e \left(\frac{d}{r}\right) \text{ kV/ph}$$

m_v = irregularity factor
= 1 for polished , 0.72 to 0.82 for rough Conductor

Emperical formula for power loss due to corona

formatation of corona is always accompanied by energy loss which dissipated in the form of light , heat , sound and chemical action . when disruptive voltage is exceeded, the power loss due to corona is given by

$$P = 242.2 \left(\frac{f+25}{\delta}\right) \sqrt{\frac{r}{d}} (V - V_c)^2 \times 10^{-5} \text{ kW/km/ph}$$

f = frequency
V = phase to neutral voltage (r.m.s)
V_c = Disruptive Critical Voltage.

Methods of reducing corona effect

* By increasing Conductor size.
* Using Bundled Conductors.
* Using Hollow conductors.
* Using Corona rings.
* By increasing conductor spacing.

factors affecting corona

1) Atmosphere
2) field around the conductor.
3) Conductor size.
4) Spacing b/w Conductor.
5) No: of Conductor /phases.
6) profile of conductor.
7) Surface conditions of conductor.

The curves of Tension & Sag Versus Temperature are called Stringing charts.

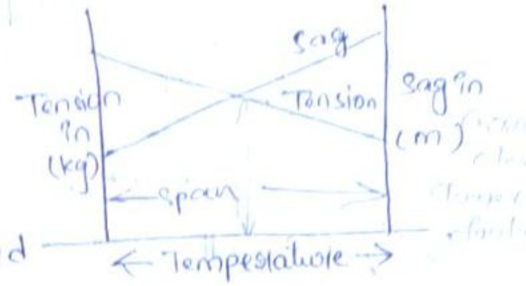
String chart

* stringing chart is useful in knowing the sag and tension at any temperature. stringing chart gives the data per sag to be allowed and the tension to be allowed for a particular temperature.
* stringing chart prepared by calculating the sag and tension on the Conductor under worst Conditions

such as maximum wind pressure and minimum tempe-rature by assuming a suitable factor safety

Graph

* Now the graph of tension verses temperature and sag verses temperature can be plotted as shown in fig.



* This graph is plotted for a fixed span and is called as "stringing chart".

Sag problem's

1) A transmission line has a span of 275m between level supports. The conductor has an effective diameter of 1.96cm. And weights 0.865kg/m. Its Ultimate strength is 8060kg's. If the conductor has ice coating of radial thickness 1.27cm and is subjected to a wind pressure of 3.9 gram/cm² of projected area. calculate sag for safety factor '2' weight of 1cc of ice 0.91.

Sol- Span, $l = 275m$

Conductor diameter, $d = 1.96cm$

weight of the conductor/m = 0.865 kg/m

wind pressure = 3.9 g/cm²

Ultimate strength = 8060 kg

Safety factor = 2

$$\text{Tension}(T) = \frac{\text{Ultimate strength}}{\text{f.o. Safety}} = \frac{8060}{2} = 4030$$

$$w_t = \sqrt{(w+w_i)^2 + w_w^2}$$

weight of ice per metre length (w_i) = density of ice x Volume of ice/m

Volume of ice per metre (100cm) = $\pi + (d+t) \times 100 cm^2$

$$w_i = 0.91 \times 1288 = 1172 = 1.172 \, kg$$

wind force/m length of conductor

$$(w_w) = \text{pressure} \left[(d+2T) \times 100\right] = 3.9\left[(1.96 + 2 \times 1.27) \times 100\right]$$

$$= 1755$$

$$w_w = 1.755 kg$$

Total weight of conductor

$$w_t = \sqrt{(w+w_i)^2 + (w_w)^2}$$

$$= \sqrt{(0.865+1.172)^2 + (1.755)^2}$$

$$w_t = 2.6887 \text{ kg}$$

Now, $Sag = \dfrac{w_t l^2}{8T} = \dfrac{(2.6887)(275)^2}{8 \times 4030}$

$$Sag = 6.3\text{m}$$

Un Equal Support

2) An overload transmission line conductor having a parabolic configuration weights 1.925 kg/m of length. The area of cross section of the conductor is 2.2cm^2 and the ultimate strength 8000kg/cm^2. The supports are 600m apart having 15m difference of levels. Calculate the sag from the taller of two supports which must be allowed. so that the factor of safety is '5' assume the ice load is 1kg/m and there is no wind pressure.

Sol- weight of conductor, $w = 1.925$ kg/m

Area of cross section of conductor, $A = 2.2\text{cm}^2$

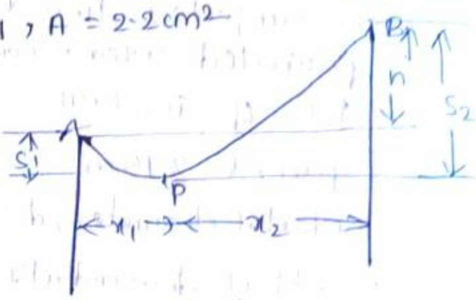
Ultimate strength $= 8000 \text{kg/cm}^2$

Span $l = 600\text{m}$

Factor of safety $= 5$

levels difference, $h = 15\text{m} = S_2 - S_1$

Ice load, $w_i = 1\text{kg/m}$



$$\text{Tension}, T = \frac{\text{Ultimate strength} \times \text{Area of cross section}}{\text{factor of safety}}$$

$$= \frac{8000 \times 2.2}{5}$$

$$= 3520\text{W}$$

$$w_t = \sqrt{(w + w_i)^2} = w + w_i = 1.925 + 1 = 2.925\text{kg}$$

from question

$$l = x_1 + x_2 = 600 \longrightarrow (1)$$

we know that

$$h = S_2 - S_1$$

$$= \frac{w_t x_2^2}{2T} - \frac{w_t x_1^2}{2T} = \frac{w_t}{2T}\left[x_2^2 - x_1^2\right]$$

$$15 = \frac{w_t}{2T}\left[(x_2 - x_1)(x_2 + x_1)\right]$$

$$x_2 - x_1 = \frac{15 \times 2T}{w_t(x_1 + x_2)} = \frac{15 \times 2(3520)}{2.925(600)}$$

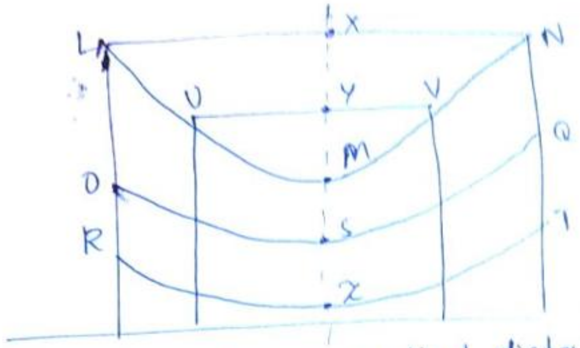
$x_2 - x_1 = 60 \longrightarrow (2)$

Solve (1) & (2) we get

$x_1 = 270\,m$

$x_2 = 330\,m$

Sag Templete :- At the time of planning sag template is necessary to decide the location of towers along the route of Transmission line.



(2)

1) The horizantal and vertical distances represent span and sag

2) The Upper Curve LMN represents Conductor length.

4 - Transients in power system

Transient phenomenon last in power system for a very short period of time ranging from few "µsec to 1 sec".

⇒ During transient system is subjected to greatest stress from excessive over currents or voltages which depends on severity it can cause extensive damage.

⇒ In some cases there is a complete shutdown of plant or black out of whole city or area.

⇒ Types of system transient's :-

1) Lightning 2) Switching 3) Resonance 4) Short circuit.

$X_L = X_C$

⇒ Lightning and switching are most common and usually most severe.

⇒ In E.H.V system the voltage transients or surges caused by switching i.e., opening and closing of circuit breaker are becoming important.

⇒ On cable system lightning transients rarely occur, other causes become more important.

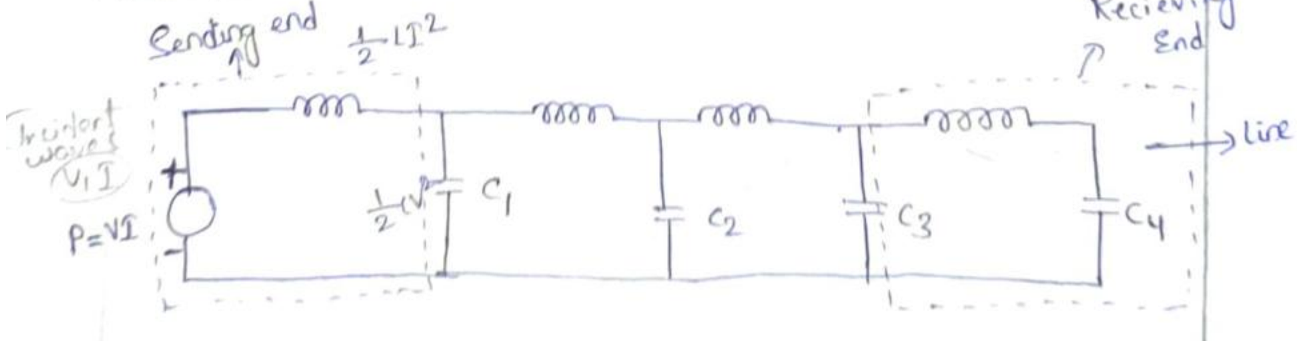
⇒ Depending on speed of transients can be classified as

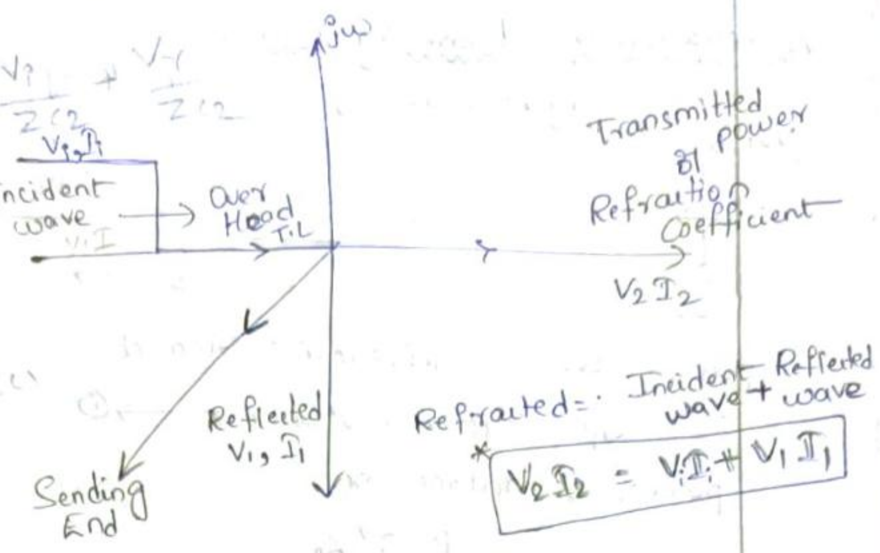
(i) surge phenomenon [extremely fast transient]

(ii) short circuit phenomenon [medium fast transient]

(iii) Transient stability [slow transient]

Travelling wave Analysis





Refracted = Incident wave + Reflected wave

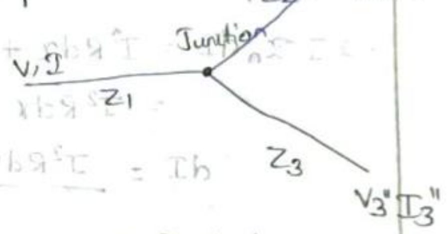
$$\boxed{V_2 I_2 = V_i I_i + V_1 I_1}$$

Important **

Reflection & Refraction Coefficients of T-Junction

A Voltage wave 'V' is travelling over the line with surge impedances Z_1 as shown in figure. when it reaches the junction it looks a change in impedance and therefore suffers reflection and Refraction. let V_2'', I_2'' and V_3'', I_3'' be the voltages and currents in the lines having surge impedances Z_2 and Z_3 respectively.

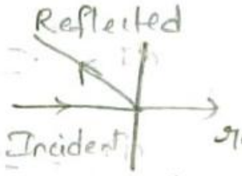
$$V_2'' = V_3'' = V''$$



$$V + V' = V''$$

$$\left.\begin{aligned} I = \frac{V}{Z_1} ; \quad I' = \frac{-V'}{Z_1} \end{aligned}\right\} \longrightarrow (1)$$

$$I_2'' = \frac{V''}{Z_2}, \quad I_3'' = \frac{V''}{Z_3}$$



$$I + I' = I_2'' + I_3'' \longrightarrow (2)$$

$$\frac{V}{Z_1} - \frac{V'}{Z_1} = \frac{V''}{Z_2} + \frac{V''}{Z_3}$$

Refracted = Incident + Reflected

(1) in (2)

Sub $V' = V'' - V$

$$\frac{V}{Z_1} - \left[\frac{V'' - V}{Z_1}\right] = \frac{V''}{Z_2} + \frac{V''}{Z_3}$$

$$\frac{2V}{Z_1} = V''\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right]$$

Refraction Coefficient

$$\boxed{V'' = \frac{\frac{2V}{Z_1}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}}$$

Attenuation of travelling wave:-

→ loss less

fig a: Travelling wave on lossy line.

power loss in differential element

$$dp = I^2 R\,dx + V^2 G\,dx \longrightarrow ①$$

power at distance x,

$$P = I^2 Z_n$$

Differential power $dp = -2 I Z_n dI \longrightarrow ②$

Z_n = Natural impedance of line. The "-ve" sign indicates there is a reduction power as the wave travels with the time.

Equate $Eq①$ & $Eq②$

$$P = I^2 Z_n$$

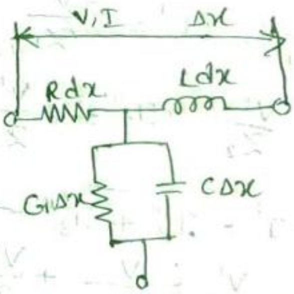
$$\boxed{\because \frac{d}{dx} x^n = n x^{n-1}}$$

$$dP = -2 I Z_n dI$$

$$-2 I Z_n dI = I^2 R dx + V^2 G dx$$

$$= I^2 R dx + (I^2 Z_n^2) G dx$$

$$dI = \frac{I^2 R dx + (I^2 Z_n^2) G dx}{-2 I Z_n}$$



$$dI = \frac{-I(R + Z_n^2 G)}{2 Z_n} dx$$

$$\frac{dI}{I} = -\left[\frac{R + Z_n^2 G}{2 Z_n}\right] dx \;\cancel{+A}$$

$$\ln I = -\left[\frac{R + Z_n^2 G}{2 Z_n}\right] x + A$$

At $x = 0$, $I = I_0$.

$$\therefore A = \ln I_0$$

$$\ln I \;\cancel{- \ln I_0} = -\left[\frac{R + Z_n^2 G}{2 Z_n}\right] x \;\cancel{+ \ln I_0} + \ln I_0$$

$$\ln \frac{I}{I_0} = -ax \qquad \ln I - \ln I_0 = -\left[\frac{R + Z_n^2 G}{2 Z_n}\right] x$$

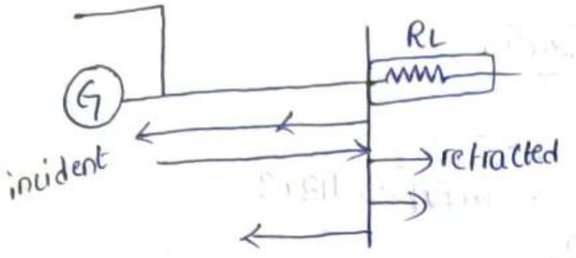
$$\boxed{I = I_0 e^{-ax}} \qquad \boxed{a = \frac{R + Z_n^2 G}{2 Z_n}}$$

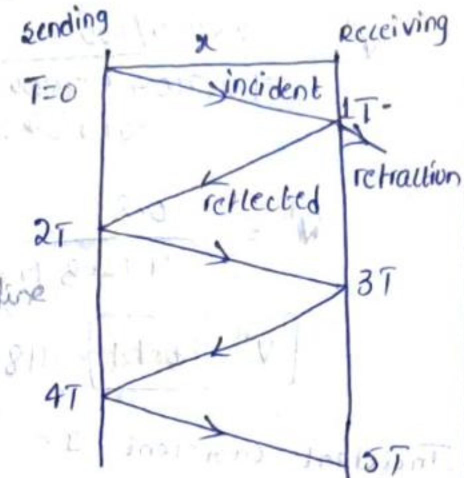
Similarly

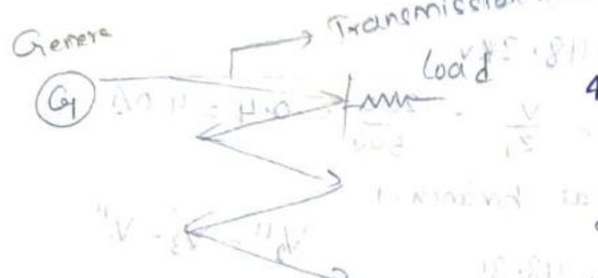
$$\boxed{V = V_0 e^{-ax}}$$

Bewley Lattice diagram:- Also Called as Zig - Zag

It is a time distance graph with the help of lattice diagram we can find the motion & direction of travelling waves. (reflection, refraction, incident) at any instant of time.

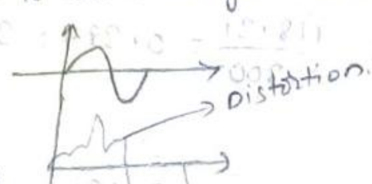


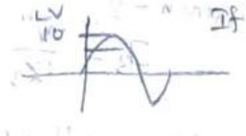




at any instant of time we find the direction & motion of the transmission line

LV → due to line R we get distortion's.
HV → due to corona we get distortion's





If the magnitude is decreases after some instant of time it is called Attenuation

Surge Impedance

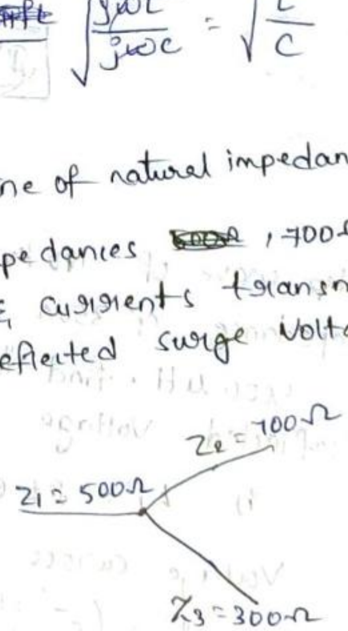
Surge impedance of a line is defined as the square root of the ratio of Series impedance & shunt admittance

$$\sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+jX_L}{G+jX_C}} \quad \text{if } R=0,\ G=0$$

$$\sqrt{\frac{jX_L}{jX_C}} = \sqrt{\frac{X_L}{X_C}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}$$

Surge High current's & Voltages are flow with in a short time

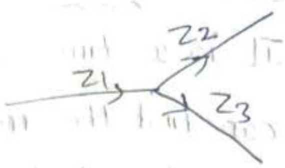
**
(**) 1) A surge of 200kV travelling on a line of natural impedance of 500Ω arrives at a junction with two lines of impedances 700Ω & 300Ω respectively. find the surge voltage & currents transmitted into each branch line also find the reflected surge voltage & current's.



Sol

$$V'' = \frac{\dfrac{2V}{Z_1}}{\dfrac{Z_2Z_3 + Z_1Z_3 + Z_1Z_2}{Z_1Z_2Z_3}}$$

$$V'' = \frac{2V}{Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}} \times \frac{Z_2 Z_3}{Z_2 Z_3}$$



$$V'' = \frac{2 \times 200/500}{\frac{700 \times 300 + 500 \times 300 + 500 \times 700}{300 + 700 + 500}}$$

$$V'' = \frac{0.8}{\cancel{4.1333} \; \cancel{7.888}} = \cancel{0.40Pk} \; 118.3$$
0.007

$\boxed{V'' = \cancel{0.0} \; 118.3 \text{KV}}$

Incident current $I = \frac{V}{Z_1} = \frac{200}{500} = 0.4 = 400A$

Transmitted current at branch-1

$V_2'' = V_3'' - V''$

$$I_2'' = \frac{V_2''}{Z_2} = \frac{118.31}{700} = 0.169A = 169A$$

Branch-2

$$I_3 = \frac{V''}{Z_3} = \frac{118.31}{300} = 0.394 = 394A$$

$$V_R = V'' - V$$

$$V_R = 118.31 - 200 = -81.69V$$

Reflected current

$$I' = I_2'' + I_3'' - I$$

$$= 0.169 + 0.394 - 0.4$$

$V = 110KV$
$Z_1 = 500\Omega$
$Z_2 = 455\Omega$
$Z_3 = 55\Omega$

$\boxed{I' = 0.625}$ $\boxed{I' = 163A}$

$I' = \cancel{625}A$

A step wave of 110 kv travells through a line having a surge impedance of 350Ω. The line is terminated by an inductance 5000 μH. find the voltage across the inductance and reflected voltage wave.

$E = 110kV$
$Z_0 = 350\Omega$
$L = 5000\mu H$

i) $E_L = 2E e^{-\frac{Z_0 t}{L}}$ kv

Voltage across inductance, $E_L = 2E e^{-\frac{Z_0 t}{L}}$ kV

$V'' = V\left(2e^{-\frac{Z_0 t}{L}} - 1\right)$

$$E_L = 2 \times 110 \times e^{-\frac{350t}{5000 \times 10^{-6}}}$$

$$\boxed{E_L = 220 e^{-0.07t}}$$

3) A 3-φ transmission line has Conductor $\overset{2.5cm}{\times m}$ diameter spaced 1.5m apart in Equilateral formation. The resistance and leakage are negligible find natural impedance of a line.

sol

diameter = 2.5 cm

for lossless line : $G = 0$; $R = 0$

$$\text{so, } Z = \sqrt{\frac{L}{C}} \qquad r = \frac{D}{2} = \frac{2.5}{2} = 1.25cm$$

Spacing = 1.5 m = 150 cm

$$L = 2\times10^{-7} \ln\left(\frac{d}{r}\right)$$

$$= 2\times10^{-7} \ln\left(\frac{150}{1.25}\right) = 0.957 \mu H$$

$$C = \frac{2\pi\varepsilon_0}{\ln \frac{d}{r}} = \frac{2\pi\times 8.854\times10^{-12}}{\ln\left(\frac{150}{1.25}\right)} = 11.62\times10^{-12} F$$

$$Z = \sqrt{\frac{0.957\times10^{-6}}{11.62\times10^{-12}}} = 286.98 \approx 287 \Omega$$

Reflection and Refraction Coefficient

Consider a lossless transmission line which has a surge impedance of Z_0 terminated through a resistance R. when the wave travelling along the line and absorbs any change, then it is partly or totally reflected.

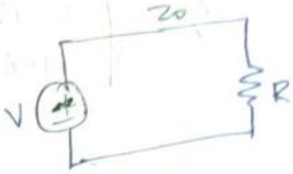
The Expression for reflected Current is:

$$i'' = \frac{-V''}{Z_0}$$

where V'' & i'' are reflected Voltage and current.

let V & i be the transmitted Voltage

V' & i' be the incident waves.



Incident Current $i' = \frac{V'}{Z_0}$

Reflected current $i'' = -\frac{V''}{Z_0}$

transmitted Current, $i = \frac{V}{R}$.

$i = i' + i''$ & $V = V' + V''$

$$\frac{V}{R} = \frac{V'}{Z_0} - \frac{V''}{Z_0} \longrightarrow (1)$$

$[V'' = V - V']$

$$= \frac{V'}{Z_0} - \frac{V - V'}{Z_0}$$

$$= \frac{2V'}{Z_0} - \frac{V}{Z_0}$$

Transmitted Voltage, $V = \frac{2R}{Z_0 + R} V' \longrightarrow (2)$

Transmitted Current, $i = \frac{V}{R} = \frac{2}{Z_0 + R} V'$

$$= \frac{V'}{Z_0} \times \frac{2Z_0}{Z_0 + R}$$

$$i = i' \cdot \frac{2Z_0}{Z_0 + R} \longrightarrow (3)$$

The Coefficient of transmitted or refraction Current waves is $\frac{2Z_0}{Z_0 + R}$ and transmitted coefficient for Voltage waves $= \frac{2R}{Z_0 + R}$

Substituting v as v' + v'' in equation in (1)

$$\frac{v' + v''}{R} = \frac{v'}{Z_0} - \frac{v''}{Z_0}$$

$$v'' = v' \times \frac{R - Z_0}{R + Z_0}$$

$$i'' = -\frac{v''}{Z_0} = -\frac{v'}{Z_0} \times \frac{R - Z_0}{R + Z_0} = -i' \times \frac{R - Z_0}{R + Z_0}$$

Coefficient of reflection for current waves $= -\frac{R - Z_0}{R + Z_0}$

& reflected coefficient for voltage waves $= +\frac{R - Z_0}{R + Z_0}$

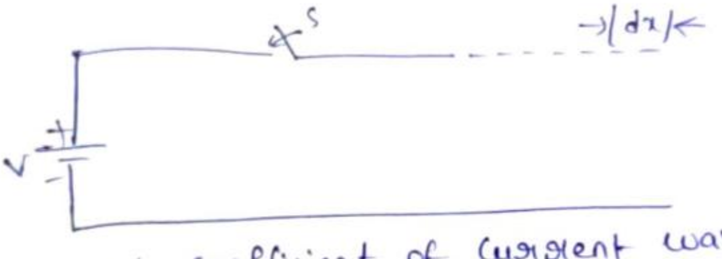
Line open ckt at receiving end

when the receiving end is open circuited i.e., $R = \alpha$, the equivalent circuit is show in below fig

Consider the transmitted coefficient of voltage wave $= \frac{2R}{Z_0 + R}$

when $R = \alpha$,

Transmitted coefficient of voltage wave $= \frac{2}{1 + \frac{Z_0}{R}} = \frac{2}{1 + \frac{Z_0}{\alpha}} = 2$



transmitted coefficient of current wave $= \frac{2Z_0}{\alpha + R} = 0$

& Reflection coefficient of voltage wave $= \frac{R - Z_0}{R + Z_0} = 1$

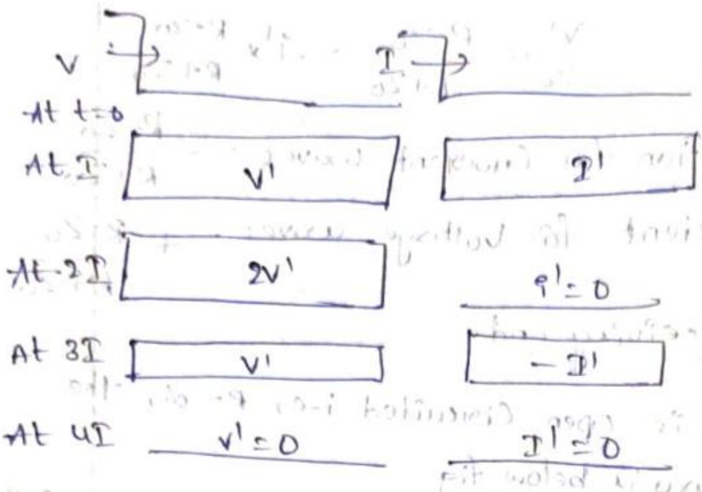
Reflection coefficient of current wave $= -\frac{R - Z_0}{R + Z_0} = -\frac{1 - Z_0/R}{1 + Z_0/R}$

$= -1$

The transmitted coefficient is two i.e., the voltage at the open ended line is $2v'$: This means that the voltage of open ended line is raised by v' due to reflection. the open ended line is raised by v' due to reflection.

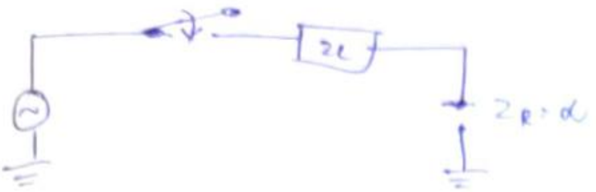
Transmitted wave = Incident wave + reflected wave.

for an open-ended line, a travelling voltage wave is reflected back with a positive sign and the coefficient of reflection is Unity. The above cycle is repeated for voltage and Current waves. This cycle occupies the time taken for a wave to travel four times the length of line and is explained through



Values of coefficients for open ckt line :-

line terminated open ckt



$$\frac{V}{V^+} = \frac{2Z_R}{Z_R+Z_C} \quad \text{Eq (2)}$$

$Z_R = \alpha$

$$\frac{V}{V^+} = \frac{\cancel{Z_R}\,2}{\cancel{Z_R}\left(1+\frac{Z_c}{Z_R}\right)} = \frac{2}{1+\frac{Z_c}{\alpha}} = \frac{2}{1+0} = 2$$

$$\frac{V^-}{V^+} = \frac{Z_R-Z_c}{Z_R+Z_c} = \frac{\cancel{Z_R}\left(1-\frac{Z_c}{Z_R}\right)}{\cancel{Z_R}\left(1+\frac{Z_c}{Z_R}\right)} = \frac{1-\frac{Z_c}{Z_R}}{1+\frac{Z_c}{Z_R}}$$

$$\frac{V^-}{V^+} = \frac{1-0}{1+0} = 1$$

$$\frac{V^-}{V^+} = 1$$

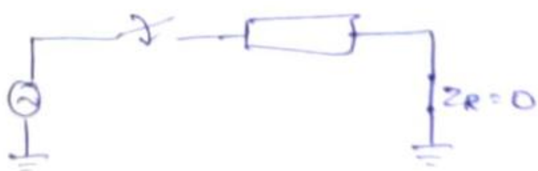
$$\frac{I}{I^+} = \frac{2Z_c}{Z_R+Z_c} \qquad \frac{I^-}{I^+} = -\frac{V^-}{V^+}$$

$Z_R = \alpha$

$$\frac{I^-}{I^+} = -1$$

$$\frac{I}{I^+} = \frac{2Z_c}{\alpha+Z_c} = 0$$

Coefficient for short ckt line



$$\frac{V}{V^+} = \frac{2Z_R}{Z_R+Z_c} \qquad \frac{V^-}{V^+} = \frac{Z_R-Z_c}{Z_R+Z_c}$$

$Z_R = 0$ $\qquad$ $Z_R = 0$

$$\frac{V}{V^+} = 0 \qquad \frac{V^-}{V^+} = -\frac{Z_c}{Z_c} = -1$$

$$\frac{I}{I^+} = \frac{2Z_c}{Z_R+Z_c}$$

$Z_R = 0$

$$\frac{I}{I^+} = 2$$

$$\frac{I^-}{I^+} = -\frac{V^-}{V^+} = -(-1)$$

$$\frac{I^-}{I^+} = 1$$

Travelling Wave

1) what is travelling wave: In general when an object vibrates, a disturbance in the form of mechanical wave is created. this mechanical wave travels through the medium from one point to another carrying energy as it moves.

2) what is surge phenomena?

Every ckt will undergo two states i.e., transient state and steady state. The transient state will be for a very short period of time but it is more significant than the steady state. During the transient state, transient's are setup in the system. The transients are also called as surges which are defined as the movement of charges along the conductor. whenever there is a sudden change in the ckt parameter's there will be a transient state which gives rise to surges.

3) Attenuation

Attenuation is the reduction in magnitude of crest or peak of a travelling wave as it travels along the length of the line. Attenuation mainly occurs due to corona and the effect is more one positive waves than on negative waves. The shorter waves are vulnerable than longer waves.

Attenuation of a line can be found using foust and Menger empirical formulae, which is given by.

$$V = \frac{V}{1+KxV}$$

where, v = surge voltage at distance x km from origin kv

k = Attenuation constant.

= 0.00037 for chopped waves (2μs)

= 0.00019 for short waves (5μs)

= 0.0001 for long waves (20μs)

V = surge kv at point of origin and

x = Distance travelled. km.

Attenuation constant

Attenuation constant is Real part of propagation constant that determines change in magnitude per unit wave length.

Distortion

Distortion is the change in the wave shape due to losses in the transmission line. As the wave travels along the length of the line, the losses cause the wave to elongate to become smooth and the voltage and current waves become similar.

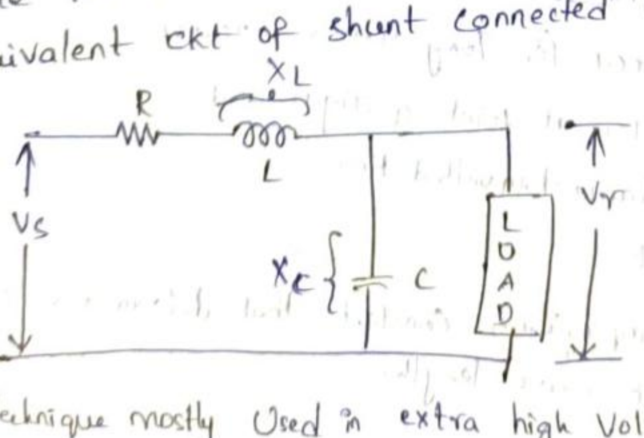
At low voltages, losses due to line resistance causes distortion, whereas at high voltages, corona causes distortion of the waves.

line termination types

1) short ckt line
2) Open ckt line
3) Line terminated with an impedance equal to surge impedance.
4) line connected to cable.
5) Bifurcated line.
6) line terminated through a resistance
7) Reactive termination.

Shunt compensation

Shunt compensation is a technique used to control by voltage and power factor by limitting the losses to the minimum value. this is the most widely used technique in which the reactive power of current is supplied to the inductive load with the help of shunt capacitance to reduce the out of phase component of current. the equivalent ckt of shunt connected capacitor is shown in fig



This technique mostly used in extra high voltage transmission systems to control the steady state over voltages during light load conditions. The usage of shunt capacitance ↑ the transfer reactance of the system further reducing the power transfer capability in the transmission line. The capability of power transfer can be ↑ by removing the shunt reactors under heavy load conditions.

In practice the shunt reactors are permanently kept connected to avoid the increase of voltage which is due to sudden fall of load.

Advantages

1) The energy can be considerably be saved due to low or less losses.
2) The span of equipments connected to it is increased.
3) The transformers and switch gears can be prevented from over loading.
4) The voltage at the receiving end can be improved.

Disadvantages

1) The response of voltage dip's or fluctuations is not so rapid when compared to series compensation.

Advantages of Bewley's lattice diagram

1) Bewley's lattice diagram technique is used to study travelling wave problem's.
2) It help's in solving the transient problem directly in time domain
3) It can be drawn for voltage as well as for current.
4) It helps in observing the position and direction of all successive reflections of voltage and current waveform's.
5) Good accuracy can be achieved by lumping resistance at one or more points along the time.
6) History of any wave can be determined easily.

UNIT V

CABLES

Cable's :- The Combination of conductor and its insulation is called Cable.

=> Underground cable: It is the combination of one or more conductors Convered with suitable insulation and surrounded by a protecting cover. $\boxed{\text{Upto 66 kV}}$

OH upto 765 kV

xx

* Comparison between Overhead line and Underground Cable:

-> Initial Cost : Underground cable initial cost is more than Overhead line
-> public safety : public safety is more in Underground cable
-> Maintanance Cost :- Maintenance Cost is more in Overhead lines than Underground
-> frequency of fault :- faults mostly occurs in Overhead line
-> frequency of accident :- OH↑, UG↓
-> fault location or detection :- OH Easy identification
-> Joints :- OH -> defecult, UG -> easy.
-> Interferance with Communication line -> OH ↑, UG↓
-> Working voltage :- upto 66K UG, OH upto 765
-> lighting :- OH↑ UG↓
-> charging current :- OH↑, UG↓
-> Surge effect :- Voltage drop's less in Underground and more in Overhead because there are no joints at Underground

xx -> The conductor used in cable should be tinned standard copper or alluminium of high conductor. (different conductors in cable)

Classification of cable:

=> According to no: of conductors :-

1) 1-core
2) 2-core
3) 3-core
4) 4-core

=> Rating of Tran. Voltage :-

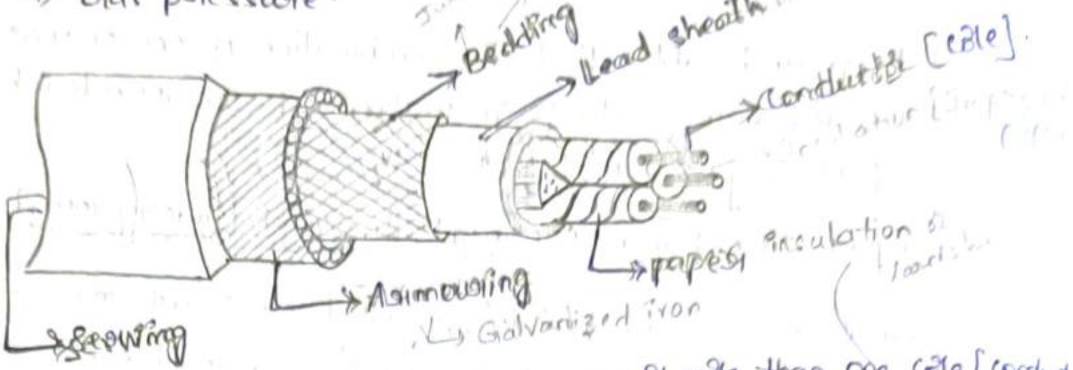
1) Low Voltage Cable for operating voltage upto 1000V
2) High Voltage cable for operating voltage upto 11,000V
3) Super Tension Cable for operating voltage upto 33,000V
3) Super Tension Cable for operating voltage upto 66,000V
4) Extra high Tension cables for operating voltage upto 132000V
5) Extra super Voltage cables for operating voltage upto

=> According to nature of application of insulation and lead sheathing :-

1) Belted type (lead sheath given to all the conductors) (ii) 2) H-type or Screened cables (seperate lead sheath for each conductor) 3) (iii) S.L. Type 4) HSL Type pressure Cable.

→ According to method of improving the dielectric stress.

1) solid type
2) Oil filled
3) External oil pressure
4) Gas pressure.



1) Core or Conductor: A cable may have one or more than one core [conductor] depending upon the type of service for which it is used. Conductor made of thinned copper or aluminium and are usually stranded in order to provide flexibility to the cable

2) Metallic sheath: Metallic sheath of lead or alloy or aluminium is provided around the insulation to protect it against moisture

3) Bedding: for protection of sheath against corrosion and from mechanical injury.

4) Armouring & Serving: Over the layer of Bedding "armouring" consist of one or two layers of galvanized steel wire is provided to save the cable from mechanical injury.

** Insulation Resistance of Single Core Cable

The cable is provided with a suitable thickness of insulation material in order to prevent leakage current. The opposition offered by insulation to leakage current is known as "Insulation Resistance" of a cable.

Consider a single-core cable of conductor radius r_1 and internal sheath radius r_2

let L = be the length of the cable
ρ = be the resistivity of the insulation.

Let us consider a very small layer of insulation of thickness 'dx' at a radius x.

The length through which leakage current tends to flow is 'dx' and the area of cross section offered to this flow is $2\pi x L$.

Insulation resistance of considered layer

$$R = \rho \frac{dx}{2\pi x l} \rightarrow \frac{\rho l}{a}$$

Insulation resistance of whole cable

area of circle — $2\pi x l$

length = dx

$$R = \int_{r_1}^{r_2} \rho \frac{dx}{2\pi x L}$$

$$= \frac{\rho}{2\pi l}\int_{r_1}^{r_2} \frac{1}{x}\,dx = \frac{\rho}{2\pi l}\left[\log x\right]_{r_1}^{r_2}$$

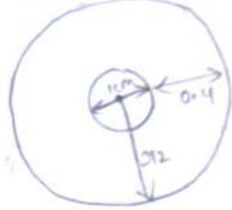
$$= \frac{\rho}{2\pi l}\left[\log r_2 - \log r_1\right] = \frac{\rho}{2\pi l}\left[\log\left(\frac{r_2}{r_1}\right)\right]$$

$$\boxed{\uparrow R = \frac{\rho}{2\pi l \downarrow}\left[\log\left(\frac{r_2}{r_1}\right)\right]}$$

when the insulation resistance 'R' increases the length of the conductor decreases. Hence insulation resistance is inversely proportional to legth of the conductor.

problem

1) A single core cable has a conductor diameter of 1cm and insulation thickness of 0.4cm. If the specific resistance of insulation is $5\times10^{14}\,\Omega/cm$. Calculate the insulation resistance for a 2km length of the cable.



Sol

diameter, $d = 1cm$

$\rho = 5\times10^{14}\,\Omega cm = 5\times10^{12}\,\Omega\text{-}m$

length, $l = 2km = 2000m \times$ ~~20cm~~

$r_1 = \frac{d}{2} = \frac{1cm}{2} = 0.5cm \rightarrow 0.5\times10^{3}\,m$

$1m = 100cm$

$r_2 = 0.5 + 0.4 = r_1 + \text{thickness}$

$= 0.9cm \rightarrow 0.9\times10^{3}\,m$

$$R = \frac{\rho}{2\pi l}\left[\log\left(\frac{r_2}{r_1}\right)\right] = \frac{5\times10^{12}}{2\pi\times2000}\times\left[\log\left(\frac{0.9\times10^{3}}{0.5\times10^{3}}\right)\right]$$

$R = 101.56\times10^{6}\,\Omega(m)$

2) The insulation Resistance of a single core cable is $495\,M\Omega$ if the core diameter is 2.5cm and resistivity of insulation is $4.5\times10^{14}\,\Omega\text{-}cm$. find the insulation thickness.

$r_1 = 1.25cm$

$R = 495\times10^{6}\,\Omega$ $\quad l = 1km = 1000m$

$d = 2.5cm$

Antilog

$r_1 = \frac{d}{2} = \frac{2.5}{2} = 1.25cm$

$\rho = 4.5\times10^{14}\,\Omega\text{-}cm = 4.5\times10^{12}\,\Omega\text{-}m$

$$\log\frac{r_2}{r_1} = \frac{2\pi R l}{\rho} = \frac{2\pi\times495\times10^{6}\times1000}{4.5\times10^{12}}$$

$\log\left(\frac{r_2}{r_1}\right) = \cancel{0.0069}\ 0.69$

$\frac{r_2}{r_1} = e^{0.69} = 1.996$

$r_2 = 1.996 \times 1.25$

$r_2 = 2.495\,\Omega\cdot cm$

Thickness of insulation $= r_2 - r_1$

$= 2.495 - 1.25 = 1.23$

3) A single core cable 5km long has an insulation resistance of 0.4MΩ. The core diameter is 20mm and the diameter of the core over the insulation is 50mm. Calculate the resistivity of the insulating material.

Sol: length = 5km = 5000m

$R = 0.4 \times 10^6\,\Omega$

$d_1 = 20mm \quad d_2 = 50mm$

$r_1 = 10mm \quad r_2 = 25mm$

$\rho = ?$

$$R = \frac{\rho}{2\pi l}\left[\log_e\left(\frac{r_2}{r_1}\right)\right]$$

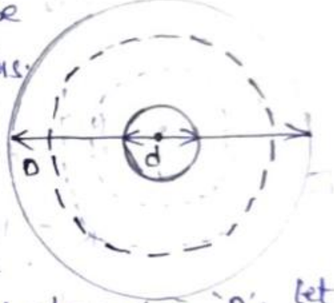
$$\rho = \frac{2\pi l R}{\log_e\left(\frac{r_2}{r_1}\right)} = \frac{2\pi \times 5000 \times 0.4 \times 10^6}{\log_e\left(\frac{25}{10}\right)} = \frac{2\pi \times 5000 \times 0.4 \times 10^6}{\ln\left[\frac{25}{10}\right]}$$

$$\rho = 13.72 \times 10^9\,\Omega\text{-}m$$

Coaxial → two circles are at same axis

Concentric → two circles having the common centre point

Capacitance of a single core cable:



A single core cable can be considered to be equivalent to two long co-axial cylinders. The conductor of the cable is the inner cylinder while the outer cylinder is represented by lead sheath which is at earth potential. Consider a single core cable with conductor diameter d and inner sheath diameter D. Let the charge per metre axial length of the cable be Q coulombs and ε be the permittivity of the insulation material between core and lead sheath. Obviously $\varepsilon = \varepsilon_0 \varepsilon_r$ where ε_r is the relative permittivity of the insulation.

Consider a cylinder of radius x metres and axial length 1 metre. The surface area of this cylinder is $= 2\pi x \times 1$

$= 2\pi x\,(m^2)$

∴ Electric flux density at any point P on the

$$E_x = \frac{Q}{2\pi x \varepsilon} = \frac{Q}{2\pi x \varepsilon_0 \varepsilon_r}$$

considered cylinder is

$$D_x = \frac{Q}{2\pi x} \quad C/m^2$$

Electric intensity at point P, $E_x = \frac{D_x}{\varepsilon} = \frac{Q}{2\pi x \varepsilon} = \frac{Q}{2\pi x \varepsilon_0 \varepsilon_r}$ Volt's/m

The workdone in moving a unit positive charge from point 'p' through a distance dx in the direction of electric field is $E_x dx$. Hence, the workdone in moving a unit positive charge from conductor to sheath, which is the potential difference V between conductor and sheath, is given by:

$$V = \int_{d/2}^{D/2} E_x dx = \int_{d/2}^{D/2} \frac{Q}{2\pi x \varepsilon_0 \varepsilon_r} dx = \frac{Q}{2\pi \varepsilon_0 \varepsilon_r} \log_e \frac{D}{d}$$

Capacitance of the Cable is

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{2\pi \varepsilon_0 \varepsilon_r} \log_e \frac{D}{d}} \quad F/m$$

$$= \frac{2\pi \varepsilon_0 \varepsilon_r}{\log_e \frac{D}{d}} \quad F/m$$

$$= \frac{2\pi \times 8.854 \times 10^{-12} \times \varepsilon_r}{2.303 \log_{10}(D/d)} \quad F/m$$

$$= \frac{\varepsilon_r}{41.4 \log_{10}(D/d)} \times 10^{-9} \quad F/m$$

If the cable has a length of 'l' metre's, then capacitance of the cable is

$$C = \frac{\varepsilon_r l}{41.4 \log_{10} \frac{D}{d}} \times 10^{-9} \; F$$

1) A single core cable has a conductor diameter of 1cm and internal sheath diameter of 1.8cm. If impregnated paper of relative permittivity 4 is used as the insulation. Calculate the capacitance for 1km length of the cable.

Sol:

~~length~~ diameter $d = 1cm$ $\qquad \varepsilon_r = 4$

$r = \frac{1}{2} cm = 0.5cm$

length = 1km = 1000m $\qquad D = 1.8cm$

$R = \frac{1.8}{2} =$

$$C = \frac{\varepsilon_r l}{41.4 \log_{10}(\frac{D}{d})} \times 10^{-9} = \frac{4 \times 1000}{41.4 \times \log_{10}(\frac{1.8}{1})} \times 10^{-9} = 0.378 \times 10^{-6}$$

$$= 0.378 \mu F$$

Calculate the capacitance and charging current of a single core cable used on a 3-phase, 66KV system. The cable is 1km long having a core diameter of 10cm and an impregnated paper insulation of thickness 7cm. The relative permitivity of the insulation may be taken as 4 and the supply at 50Hz.

$\varepsilon_r = 4$, $l = 1000m$

$d = 10cm$, $D = 10 + 2\times7 = 24cm$

$$C = \frac{\varepsilon_r l}{41.4 \log_{10}\left(\frac{D}{d}\right)} \times 10^{-9} = \frac{4\times1000}{41.4\log\left(\frac{24}{10}\right)} \times 10^{-9} \; 0.254\times10^{-6} = 0.254\mu F$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{66\times10^3}{\sqrt{3}} = 38.11kV$$

$$\text{Charging Current} = \frac{V_{ph}}{X_c} = \frac{V_{ph}}{\frac{1}{2\pi fc}} = 2\pi fc\, V_{ph}$$

$$= 2\pi\times50\times38.11\times10^3 \times 0.25\times10^{-6}$$

Charging Current = 2.993A

Insulation Resistance

The conductor of single-core cable has a diameter of 6mm, the diameter over the insulation is 24mm. If the insulation Resistance of the cable is 1600Ω/km. Calculate the specific resistance of the dielectric is used

Sol

$d_1 = 6mm$ $\quad l = 1km = 1000m$

$r_1 = \frac{6}{2} = 3mm$ $\quad R = 1600\Omega/km = 16\times10^6\ \Omega/m$

$d_2 = 24mm$

$r_2 = \frac{24}{2} = 12mm$

$\log_e = \ln()$

$\log_{10} = \log()$

$$R = \frac{\rho}{2\pi l}\left[\log\left(\frac{r_2}{r_1}\right)\right] \Rightarrow 16\times10^6 = \frac{\rho}{2\pi\times1000}\left[\log_e\left(\frac{12^4}{3}\right)\right]$$

$$\rho = \frac{16\times10^6\times2\pi\times1000}{\ln(4)} = 72.52\times10^9\ \Omega\text{-m}$$

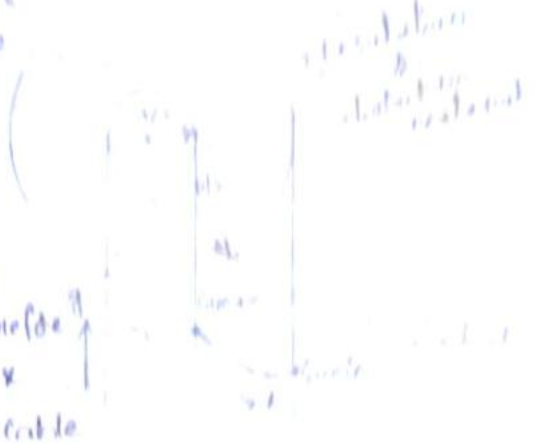
$$\boxed{\rho = 72.52\times10^9\ \Omega\text{-m}}$$

Dielectric stress in a Single-core Cable:

Under operating conditions, the insulation of the cable is subjected to electrostatic forces. This is known as electric stress. This is k.a. dielectric stress at any point in a cable is infact the potential gradient.

Consider a single core cable with core diameter d and internal sheath diameter D. As provided in Art, the electric intensity at a point x meters from the centre of the cable

$$E_x = \frac{Q}{2\pi\varepsilon_0\varepsilon_r x} \text{ Volt's/m}$$



By definition electric intensity is equal to potential gradient. therefore potential gradient (g) at a point x meters from the centre of a cable is

$$g = E_x = \frac{Q}{2\pi\varepsilon_0\varepsilon_r x} \text{ V/m} \longrightarrow (1)$$

we know,

$$V = \frac{Q}{2\pi\varepsilon_0\varepsilon_r} \log_e \frac{D}{d} \text{ V}$$

$$Q = \frac{2\pi\varepsilon_0\varepsilon_r V}{\log_e \frac{D}{d}} \longrightarrow (2)$$

substitute (2) in (1) we get

$$g = \frac{\frac{2\pi\varepsilon_0\varepsilon_r V}{\log_e D/d}}{2\pi\varepsilon_0\varepsilon_r x} = \frac{V}{x\log_e\left(\frac{D}{d}\right)} \text{ V/m} \longrightarrow (3)$$

$\therefore$ Maximum potential gradient is

$$\boxed{g_{max} = \frac{2V}{d\log_e\left(\frac{D}{d}\right)} \text{ V/m}}$$

put $x = \frac{d}{2}$ → maximum potential gradient occurs surface the cable

$\therefore$ minimum potential gradient is

$$\boxed{g_{min} = \frac{2V}{D\log_e\left(\frac{D}{d}\right)} \text{ V/m}}$$

$$\frac{g_{max}}{g_{min}} = \frac{\frac{2V}{d\log_e\left(\frac{D}{d}\right)}}{\frac{2V}{D\log_e\left(\frac{D}{d}\right)}} = \frac{D}{d}$$

$$\boxed{\frac{g_{max}}{g_{min}} = \frac{D}{d}}$$

from the figure it is observed that the dielectric stress gradually decrease as we go from centre of conductor to sheath. Hence the dielectric stress is not uniform.

Due to this non-uniform distribution of dielectric stress, it is difficult to design size of the cable. In order to have uniform distribution of dielectric stress, the grading of cable is used.

1) A 33kV single core cable has a conductor diameter of 1cm and a sheath of inside diameter 4cm. find the maximum and minimum stress in the insulation.

Sol-

$V = 33kV = 33\times10^3 V$

$d = 1cm = 1\times10^{-2} m$ $\qquad g_{max} = ?$

$D = 4cm = 4\times10^{-2} m$ $\qquad g_{min} = ?$

$$g_{max} = \frac{2V}{d\log_e(\frac{D}{d})} = \frac{2\times33\times10^3}{10^{-2}\times\log_e(\frac{4}{1})} = 4.7\times10^6 V/m$$

$$g_{min} = \frac{2V}{D[\log_e(\frac{D}{d})]} = \frac{2\times33\times10^3}{10^{-2}\times4\times\log_e(\frac{4}{1})} = 1.19\times10^6 V/m$$

2) The maximum and minimum stresses in the dielectric of a single core cable are 40kV/cm (rms) and 10 kV/cm (rms) respectively. If the conductor diameter is 2cm, find (i) thickness of insulation (ii) operating voltage.

Sol-

$G_{max} = 40kV/cm$ $\qquad d = 2cm = 2\times10^{-2}m$

$G_{min} = 10kV/cm$

$$\frac{G_{max}}{G_{min}} = \frac{D}{d} \Rightarrow \frac{40}{10} = \frac{D}{2}$$

$D = 8cm = 8\times10^{-2}m$

Thickness of Insulation $= \frac{D-d}{2} = \frac{8cm-2cm}{2} = \frac{6cm}{2} = 3cm.$

$$G_{max} = \frac{2V}{d[\log_e(\frac{D}{d})]} = \frac{2\times V}{2\times\log_e(\frac{8}{2})\times10^{-2}}$$

$$40\times10^3\times10^{-2}\times2\times\log_e(4) = V$$

$$\boxed{V = 55.45\,kV}$$

surface of conductor has more gradient when distance ↑ it decreases at the surface of conductor we have higher insulation and goes on the insulation ↓ i.e., different dielectrics are used. using different insulations

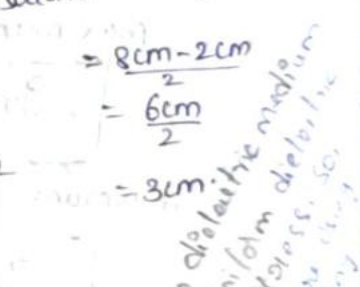
Uniform dielectric medium
Non-Uniform dielectric stress, so we are doing grading

Grading of cables :- Two types of grading → Capacitance, → Inter sheath

The process of achieving uniform electrostatic stress in the dielectric of cables is known as "Grading of cables".

Capacitance grading :- the process of achieving uniformity in the dielectric stress by using of different dielectrics known as capacitance grading.

It has already been shown that electrostatic stress in a single core cable has a maximum value (g_{max}) at the conductor



surface and goes on decreasing as we move towards the sheath.

=) The maximum voltage that can be safely applied to a cable depends upon g_{max} i.e, electrostatic stress at the conductor surface. For safe working of a cable having homogeneous dielectric, the strength of dielectric must be more than g_{max}

Capacitance Grading

In Capacitance grading the homogeneous dielectric is replaced by a ~~cap~~ composite dielectric. The composite dielectric consists of various layers of different dielectrics in such a manner that relative permittivity ε_r of an layer ~~pr~~ is inversely proportional to its distance from the centre. Under such conditions, the value of potential gradient at any point in the dielectric is "constant and is independent of its distance from the centre. In other words, the dielectric stress in the cable is same everywhere and the grading is ideal on. However, ideal grading requires use of an infinite number of dielectrics which is an impossible task. In practice, two or three dielectrics are used in the decreasing order of permittivity -the dielectric of highest permitivity being used near the core.

The capacitance grading can be explained ~~the~~ beautifully by referring to fig(a). There are three dielectrics of other diameter d_1, d_2 & D and of relative permitivity ε_1, ε_2 and ε_3. If the permitivities are such that $\varepsilon_1 > \varepsilon_2 > \varepsilon_3$ and the three dielectrics are worked at the same maximum stress then.

$$\frac{1}{\varepsilon_1 d} = \frac{1}{\varepsilon_2 d_1} = \frac{1}{\varepsilon_3 d_2}$$

$$\varepsilon_1 d = \varepsilon_2 d_1 = \varepsilon_3 d_2$$

~~potential difference across the linear layer is~~

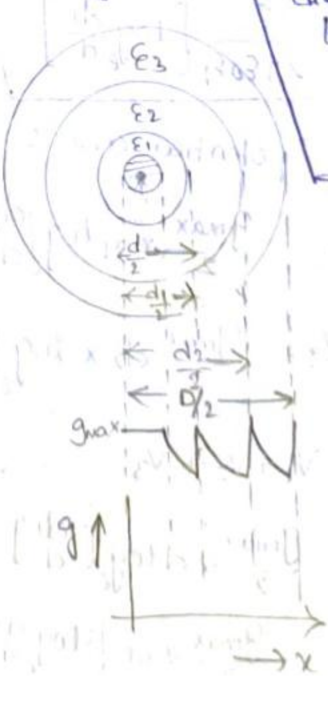
The process of achieving uniformity in the dielectric stress by using layer of different dielectrics is known as capacitance grading

Capacitance grading

$\varepsilon_r \propto \frac{1}{x}$ =) $\varepsilon_r = \frac{K}{x}$

where 'K' is constant

potential gradient at a distance 'x' from the centre.



$$E = \frac{Q}{2\pi\varepsilon_0\varepsilon_r x}$$

$$= \frac{Q}{2\pi\varepsilon_0\left(\frac{K}{x}\right)x}$$

$$= \frac{Q}{2\pi\varepsilon_0 K}$$

$$g = \frac{Q}{2\pi\varepsilon_0\varepsilon_{r1} x}$$

$\boxed{\varepsilon_{r1} = \varepsilon_1 \quad x = \frac{d}{2}}$ → distance

$$g_{1max} = \frac{Q}{2\pi\varepsilon_0\varepsilon_{r1}\left(\frac{d}{2}\right)}$$

$$g_{1max} = \frac{Q}{\pi\varepsilon_0\varepsilon_1 d} \qquad \varepsilon_{r} = \varepsilon_2 \quad x = \frac{d_1}{2}$$

$$g_{2max} = \frac{Q}{2\pi\varepsilon_0\varepsilon_2\frac{d_1}{2}} = \frac{Q}{\pi\varepsilon_0\varepsilon_2 d_1}$$

$$g_{3max} = \frac{Q}{2\pi\varepsilon_0\varepsilon_3\frac{d_2}{2}} = \frac{Q}{\pi\varepsilon_0\varepsilon_3 d_2}$$

if $g_{1max} = g_{2max} = g_{3max} = g_{max}$

$$\frac{\cancel{Q}}{\cancel{\pi\varepsilon_0}\varepsilon_1 d} = \frac{\cancel{Q}}{\cancel{\pi\varepsilon_0}\varepsilon_2 d_1} = \frac{\cancel{Q}}{\cancel{\pi\varepsilon_0}\varepsilon_3 d_2}$$

$$\frac{1}{\varepsilon_1 d} = \frac{1}{\varepsilon_2 d_1} = \frac{1}{\varepsilon_3 d_2}$$

$$\boxed{\varepsilon_1 d = \varepsilon_2 d_1 = \varepsilon_3 d_2}$$

Potential difference across the linear layer:- $\varepsilon_{r1} = 1$

$$V_1 = \int_{d/2}^{d_1/2} g\,dx = \int_{d/2}^{d_1/2} \frac{Q}{2\pi\varepsilon_0\varepsilon_{r1} x}\,dx = \frac{Q}{2\pi\varepsilon_0\varepsilon_1}\left[\log_e x\right]_{d/2}^{d_1/2}$$

$$\frac{Q}{2\pi\varepsilon_0\varepsilon_1} = \frac{g_{max}}{2} \times d$$

$$= \frac{Q}{2\pi\varepsilon_0\varepsilon_1}\left[\log_e\frac{d_1}{2} - \log_e\frac{d}{2}\right] = \frac{Q}{2\pi\varepsilon_0\varepsilon_1}\left[\log_e\frac{d_1}{d}\right]$$

$$\boxed{V_1 = \frac{Q}{2\pi\varepsilon_0\varepsilon_1}\left[\log_e\frac{d_1}{d}\right]} \Rightarrow V_1 = \frac{g_{max}}{2} \times d \log_e\frac{d_1}{d}$$

Similary, potential across second and third layer.

$$V_2 = \frac{g_{max}}{2} \times d_1 \log_e\left(\frac{d_2}{d_1}\right)$$

$$V_3 = \frac{g_{max}}{2} d_2 \times \log_e\frac{D}{d_2}$$

$$V_T = V_1 + V_2 + V_3$$

$$= \frac{g_{max}}{2} \times d\log_e\left(\frac{d_1}{d}\right) + \frac{g_{max}}{2} \times d_1\log_e\left(\frac{d_2}{d_1}\right) + \frac{g_{max}}{2} \times d_2\log_e\left(\frac{D}{d_2}\right)$$

$$V = \frac{g_{max}}{2} \times \left[d\log_e\frac{d_1}{d} + d_1\log_e\frac{d_2}{d_1} + d_2\log_e\frac{D}{d_2}\right]$$

11) A single core lead sheath cable is graded by using three dielectric's of relative permittivity 5, 4 & 3 respectively. the conductor diameter is 2cm and overall diameter = 8cm If the three dielectrics are worked at the same maximum stress of 40kv/cm, find the safe working voltage of the cable. what will be the value of safe working voltage for an ungraded cable, assuming the same conductor and overall diameter and the maximum dielectric stress?

Sol- $\varepsilon_1 = 5$, $\varepsilon_2 = 4$, $\varepsilon_3 = 3$, $g_{max} = 40 kv/cm$.

$$d = 2cm \quad D = 8cm$$

Graded cable, as the maximum die estress in the three dielectric is same.

$$\varepsilon_1 d = \varepsilon_2 d_1 = \varepsilon_3 d_2$$

$$\varepsilon_1 d = \varepsilon_2 d_1 \qquad \varepsilon_2 d_1 = \varepsilon_3 d_2$$

$$5 \times 2 = 4 \times d_1 \qquad 4 \times 2.5 = 3 \times d_2$$

$$d_1 = \frac{10}{4} = 2.5 \qquad d_2 = \frac{4 \times 2.5}{3} = 3.34$$

$$V = \frac{g_{max}}{2}\left[d \log_e \frac{d_1}{d} + d_1 \log_e \frac{d_2}{d_1} + d_2 \log_e \frac{D}{d_2}\right]$$

$$= \frac{40 \times 10^3}{2}\left[2 \times \log_e \frac{2.5}{2} + 2.5 \times \log_e \frac{3.34}{2.5} + 3.34 \times \log_e \frac{8}{3.34}\right] = 81.808 kv$$

Safe working voltage for cable $= \frac{81.808}{\sqrt{2}} = 57.84 kv$.

Ungraded cable :- permisible peak voltage for the cable

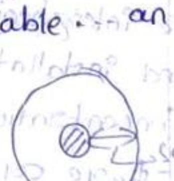
$$= \frac{g_{max}}{2} \times d \log_e \frac{D}{d}$$

$$= \frac{40 \times 10^3}{2} \times 2 \times \log_e\left(\frac{8}{2}\right)$$

$$= 55.45 kV$$

Safe working voltage (r.m.s) for the cable $= \frac{55.44}{\sqrt{2}} = 39.2 kV$

this for the this example shows the utility of grading the cable. the same overall dimension(D); same conductor diameter (d) and the grading cable can be operated at a voltage $(57.84 - 39.20) =$ 18.64kv (rms) higher than the homogeneous cable. an increase of about 47%.



Same insulation is used that is called homogeneous dielectric

Inter sheath grading :- In this method of cable grading, a homogeneous dielectric is used, but it is divided into various type layers by replacing metallic intersheaths b/w the core and lead sheath

The intersheaths are held at suitable potentials which are inbetween the core potential and earth potential.

* This arrangement improves voltage distribution in the dielectric of the cable and consequently more uniform potential gradient is obtained.

⇒ Consider a cable of core diameter 'd' and outer diameter of lead sheath is D. Suppose that two intersheaths of diameter d_1 and d_2 are inserted into the homogeneous dielectric and maintained at some fixed potentials. Let V_1, V_2 and V_3 respectively be the voltage between core and intersheath 1, between intersheath 1 and 2 and between intersheath 2 and outer lead sheath. As there is a define potential difference between the inner and outer layer of each intersheath, therefore each sheath can be treated like a homogeneous single core cable.

As proved in art., 11.9.

Maximum stress between core and intersheath 1 is

$$g_{max} = \frac{V_1}{\frac{d}{2}\log_e\frac{d_1}{d}}$$

$$g_{2max} = \frac{V_2}{\frac{d_1}{2}\log_e\frac{d_2}{d_1}}$$

$$g_{3max} = \frac{V_3}{\frac{d_2}{2}\log_e\frac{D}{d_2}}$$

Since, the dielectric is homogeneous, the maximum stress in each layer is the same

i.e., $g_{1max} = g_{2max} = g_{3max} = g_{max}$

$$\frac{V_1}{\frac{d}{2}\log_e\frac{d_1}{d}} = \frac{V_2}{\frac{d_1}{2}\log_e\frac{d_2}{d_1}} = \frac{V_3}{\frac{d_2}{2}\log_e\frac{D}{d_2}}$$

As the cable behaves like three capacitors in series, therefore all the potentials are in phase i.e., voltage between conductor and earthed lead sheath is

$$V = V_1 + V_2 + V_3.$$

+ Intersheath grading has three principle disadvantages.

(i) there are complications in fixing the sheath potentials.

(ii) the intersheaths are likely to be damaged during transportation and installation which might result ~~transportation~~ in local concentrations of potential gradient.

(iii) there are considerable losses in the intersheaths due to charging currents. for these reasons, intersheath grading is rarely used.

1) A single core cable of conductor diameter 2cm and lead sheath of diameter 5.3cm is to be ~~used to be~~ used on a 66kV, 3-ϕ system. Two intersheaths of diameter 3.1cm and 4.2cm are introduced between the core and lead sheath. If the maximum stress in the layer is the same, find the voltages on the intersheaths.

Sol: $d = 2cm$ $D = 5.3cm$

$V_{rms} = 66 \times 10^3 V = \frac{66 \times \sqrt{2}}{\sqrt{3}}$ $V_{ph} = 53.9kV$

$d_1 = 3.1cm$, $d_2 = 4.2cm$

$$g_{max} = \frac{V_1}{\frac{d}{2}\log\frac{d_1}{d}} = \frac{V_1}{\frac{2}{2}\log(\frac{3.1}{2})} = 2.8V_1$$

$$g_{2max} = \frac{V_2}{\frac{d_1}{2}\log\frac{d_2}{d_1}} = \frac{V_2}{\frac{3.1}{2}\log(\frac{4.2}{3.1})} = 2.12\,V_2$$

$$g_{3max} = \frac{V_3}{\frac{d_2}{2}\log(\frac{D}{d_2})} = \frac{V_3}{\frac{4.2}{2}\log(\frac{5.3}{4.2})} = 2.04V_3$$

As the maximum stress in the layer same

$$g_{1max} = g_{2max} = g_{3max}$$

$$2.28V_1 = 2.12V_2 = 2.04V_3$$

$$2.28V_1 = 2.12V_2$$

$$V_2 = \frac{2.28}{2.12}V_1 = 1.075V_1$$

$$2.28V_1 = 2.04V_3$$

$$V_3 = \frac{2.28}{2.04}V_1$$

$$V_3 = 1.117V_1$$

Now, $V_1 + V_2 + V_3 = V$

$$V = V_1 + 1.075V_1 + 1.117V_1 = V_1(1+1.075+1.11)$$

$$53.9 = 3.912V_1$$

$$V_1 = \frac{53.9}{3.912} = 16.88kV$$

$$V_2 = 1.075 \times 16.88 = 18.14kV$$

∴ Voltage on first intersheath $= V - V_1 = 53.9 - 16.88$
$= 37.02kV$

Voltage on second intersheath $= V - V_1 - V_2$
$= 53.9 - 16.88 - 18.14$
$= 18.88kV$

Properties of Insulating Materials for cables

1) Insulation Resistance is ↑ (conductor నుండి వచ్చే leakage current ఆపగలిగే ఉండాలి)
2) High Dielectric strength
3) Low Thermal coefficient → Heat తగిలినా కూడా కరిగి melt కాకూడదు
4) Low water absorption → నీటిని పీల్చితే Insulation R ↓ leakage current ↑
5) Non-flammable
6) Chemical stability
7) High mechanical strength.

No one insulating material pocess all the properties.

Various types of ~~Rubber~~ Insulating materials

1) **Rubber:-** Natural rubber is obtained from the milky sap of topical trees.

* Synthetic Rubber is provided from alcohol or oil products.
* It's relative permitivity is between 2 and 3.
* It's dielectric strength is 30kV/mm
* Although pure rubber has reasonably high insulating properties, it suffers from some major drawbacks viz, readily absorbs moisture, maximum safe temperature is low (about 38°c)

2) **Valcanised Indian Rubber (V.I.R)**

* It is prepared by mixing pure rubber with mineral matter such as zinc oxide, red lead etc. and 3 to 5% of sulphur.
* The compound so formed is rolled into thin sheets and cut into strips. The rubber compound is then applied to the conductor and is heated to temperature of about 150°c.
* The whole process is called Vulcanisation and the product obtained is known as Vulcanised India rubber.

⇒ It's main drawback is that sulphur reacts very quickly with copper and for this reason, cables using VIR insulation have tinned copper conductor.

⇒ The VIR insulation is generally used for low and moderate voltage cables.

3) **Impregnated paper:-**

It consists of a chemical pulped paper made from wood chippings and impregnated with some compound such as paraffinic or napthenic material.

⇒ It is because it has the advantages of low cost, low capacitance, high dielectric strength and high insulation resistance.

⇒ The only disadvantage is that paper is hygroscopic and even if it is impregnated with suitable compound, it absorbs moisture and thus lowers the insulation resistance of the cable.

⇒ For this reason, paper insulated cables are always underground cable underground provided with some protective covering and never left unsealed.

4) Varnished Cambric :- It is cotton cloth impregnated and coated with varnish.

* This type of insulation is also known as empire tape.
* Cambric is lapped on to the conductor in the form of a tape and its surfaces are coated with petroleum jelly compound to allow for the sliding of one turnover another as the cable is bent.
* As the varnished cambric is hygroscopic, therefore such cables are always provided with metallic sheath.
* It's dielectric strength is about 4kV/mm and permittivity is 2.5 to 3.8.

5) polyvinyl chloride (PVC)

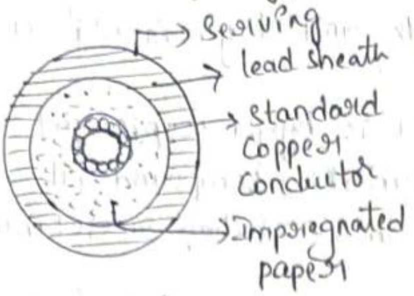
* This material is a synthetic compound. It is obtained from the polymerisation of acetylene and is in the form of white powder.
* For obtaining this material as a cable insulation, it is compounded with certain material known as plasticizers which are liquids with high boiling point.
* The plasticizer forms a gell and renders the material plastic over the desired range of temperature.
* polyvinyl chloride has high insulation resistance, good dielectric strength and mechanical toughness over a wide range of temperature.
* As the machanical properties of PVC are not good as those of rubber, therefore PVC insulation cables are generally used for low and medium domestic lights and power installations.

⇒ Constructional of a single core cable :-

Constructional details of a single-core low tension cable. The cable has ordinary construction because the stresses develope in the ~~caple~~ cable for low voltages (upto 66kV) are generally small.

* It consists of one circular core of tinned standard copper

Insulated by layers of impregnated paper.



In order of an ~~conductor cast~~ to protect the lead sheath from corrosion an overall serving of compounded fibrous material is provide (Jute)

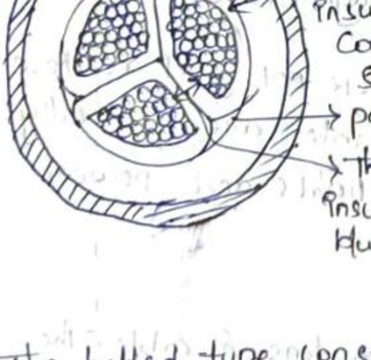
⇒ Cables for 3 phase service : In practice Underground Cables are generally required to deliver ~~upto 66kv, 3-core cable (i.e., multi core construction) is~~ p 3-φ power. For the purpose either 3-core cable or 3- single core cables are ~~be~~ used

* for voltages upto 66kv, 3-core Cable (multi-core) is preferred due to economic reasons.
* However for voltages beyond 66kv, 3-core cables became too large and unwidely and therefore single-core cables are used.
* The following types of cables are generally used for 3-φ service

1) Belted cables

These cables are used for low voltages upto 11kv but in extraordinary cases, their use may be extended upto 22kv.

* The core are insulated from each other by layers of impregnated paper. Another ~~tape~~ layer of impregnated paper tape called paper belt. is wound round the grouped insulated cores.
* The gap b/w the insulated cores is filled with fibrous insulating material. so as to give circular cross-section to the cable.



The cores are generally standard and may be of non circular shape to make better use of available space.

The belt is covered with lead sheath to protect the cable against ingress of moisture and mechanical injury.

The lead sheath is covered with one or two more layers of armouring with outer serving.

The belted type construction is suitable only for low and medium voltages as the electrostatic stresses developed in the cables for these voltages are more or less radial i.e., across the insulation.

However for high voltages (beyound 22kv) the tangential stresses also become important. These stresses etc along the layers of paper insulation.

⇒ In order to overcome this difficulty, screened cables are used where leakage currents are conducted to earth through metallic ~~sh~~ screens.

2) <u>Screened Cables</u>

* These cables are meant for use upto 33kv, but in particular cases their use may be extended to operating voltages upto 66kv. Two principle types of screened cables are
1) H-type cables 2) S.L type cables.

<u>H-type cables</u> :- This type of cable was first designed by H. Hochstadter and hence the name. Each core insulated by layers of impregnated paper. The insulation on each ~~a~~ core is covered with a metallic screen which usually consists of a perforated aluminium coil.

- The cores are laid in such a way that metallic screens make contact with one another. An additional conducting belt is ~~wound~~ wrapped round the three cores.

- The cable has no insulating belt but lead sheath, bedding armouring and serving follow as usual.

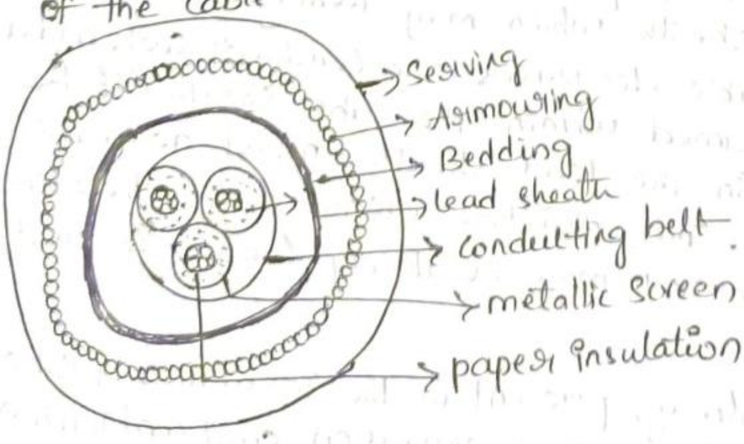
- As the four screens and the lead sheath are at earth potential, therefore, the electric stresses are purely radial and consequently dielectric losses are reduced.

⇒ Two principle advantages are claimed for H-type cables

* The perforations in the metallic screens assist in the complete impregnation of the cable with the compound and thus the possibility of air pockets or voids in the dielectric is eliminated.

* To voids if present tend to reduce the breakdown strength of the cable and may cause considerable damage to the paper insulation

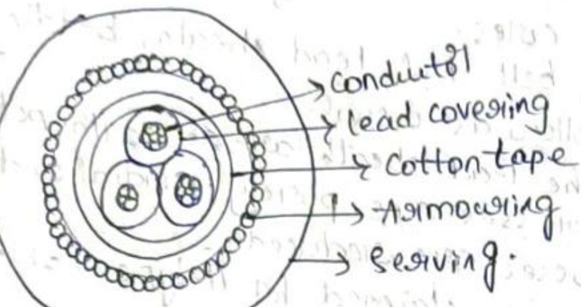
* Secondly the metallic screens increase the heat dissipating of the cable.



2) S.L type cables :- SL the name tells it is a separate lead type cable.

* It is basically H-type cable, but the screen round each core insulation is covered by its own lead sheath.
* there is no overall lead sheath but only armouring and serving are provided.
* the SL type cables have two main advantages over H-type cables.
* (i) The separate sheaths minimise the possibility of core-to core breakdown.
  (ii) Bending of cables becomes easy due to the elimination of overall lead sheath.

=> However the disadvantage is that the three lead sheaths of S.L cable are much thinner than the single sheath of H-cable.



Limitations of Solid type Cables

All the cables of above construction are referred to as solid type cables because solid insulation is used and no gas or oil circulates in the cable sheath.

The voltage limit for solid type cables is 66kV due to the following reason's.

i) As a solid cable carries the load, its conductor temperature increases and the cable compound, expands. This action stretches the lead sheath which may damaged.

ii) when the load on the cable decreases, the conductor cools and a practical vacuum is formed within the cable sheath. If the pinholes are present in the lead sheath, moist air may be drawn into the cable. The moisture reduces the dielectric strength of insulation and may eventually cause the break down of the cable.

iii). In practice, voids are always present in the insulation of cable. formed as a result of differential expansion and contraction of the sheath impregnated compound. The voids nearest to the conductor are the first to break down, the chemical and thermal effects of ionisation causing permanent damage to the paper insulation.

3) pressure Cables

for Voltages beyond 66KV, solid type cables are Unreliable because there is a danger of breakdown of insulation due to the presence of Voids.

when the operating voltages are greater than 66KV, pressure cables are Used. In such cables, Voids are eliminated by increasing the pressure of compound and for this reason they are Called pressure Cables.

Two types of pressure cables:

(i) Oil filled Cables : In such types of cables, channels are provided in the Cable for oil circulation.

⇒ The oil under pressure (it is the same oil used for impregnation) is kept constantly supplied to the channel by means of external reservoirs placed at suitable distances along the route of the Cable.

⇒ Oil Under pressure compresses the layers of paper insulation and is forced into any voids that may have formed between the layers.

⇒ Due to the elimination of Voids, Oil-filled cables can be used for higher voltages, the range being from 66KV upto 230KV.

Oil filled cables are of three types :

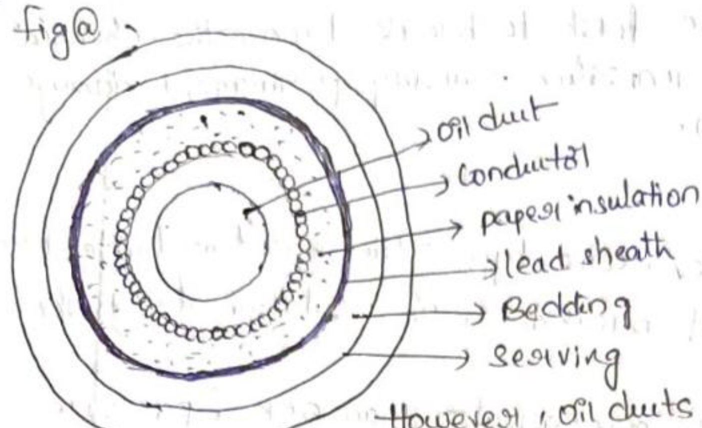
a) Single core Conductor Channel
b) Single core sheath channel
c) Three core filter-space channel.

⇒ The oil Under pressure is supplied to the channel by means of external reservoir. As the channel is made of spiral steel tape, it allows the oil to percolate between copper strands to the wrapped insulation.

⇒ The oil pressure Compress the layers of paper insulation and prevents the possibility of void formation.

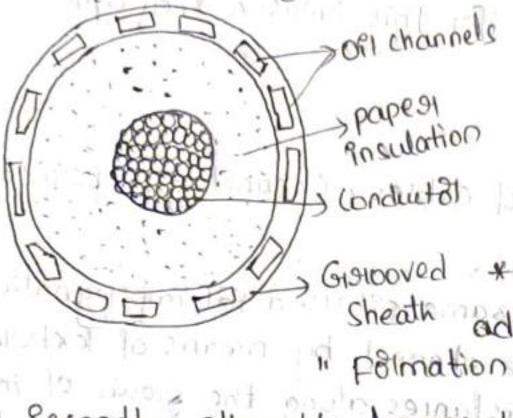
⇒ The system is so designed that when the oil gets expanded due to increase in cable temperature, the extra oil collects in the reservoir.

⇒ However, when the cable temperature falls during light load Conditions, the oil from the reservoir flows to the channel fig(a) :-



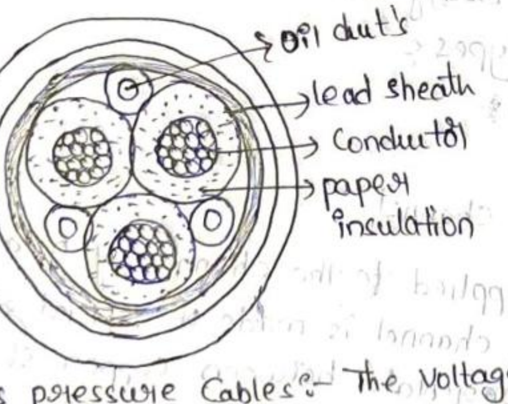
The Constructional details of a single core sheath channel oil filled cable. In this type of cable, the Conductor is solid similar to that of solid cable and is paper insulation.

However, oil ducts are provided in the metallic sheath as shown. In the 3-core oil filter cable shown. fig(b)



* the oil ducts are located in the filler spaces, these channels are Composed of perforated ~~mate~~ metal-ribbon tubing and are at earth potential
* The oil filled Cables have three principle advantages.
1. Formation of voids and ionisation are avoided.
2. Secondly, allowable temperature range and dielectric strength are increased.
3. there is leakage, the defect in the lead sheath is at once indicated and the possibility of earth faults is decreased.

However, their major disadvantages are the high initial Cost and complicated system of laying.



ii) Gas pressure Cables :- The voltage required to set up ionisation inside a void increaseds as the pressure is increased.

* Therefore, if ordinary Cable is subjected to a sufficiently high pressure, the ionisation can be altogether eliminated the section of external pressure Cable designed by Hochstadter, Nogal and Bowden.
* The triangular section reduces the weight and gives low thermal resistance but the main reason for triangular shape is that the lead sheath acts as a pressure membrane.

* The sheath is protected by a thin metal tape. The cable is laid in a gas - tight steel Pipe.
* The pipe is filled with dry nitrogen gas at 12 to 15 atmospheres.
* Moreover, maintanance cost is small and the nitrogen gas help's in quenching any flame. However, it has the disadvantage that the overall cost is very high.