UNIT - I

Types of circuit Elements

Basically circuit elements are classified into 4 types

(1) Active & passive elements
(2) Unilateral & Bilateral elements
(3) Linear & Non-linear elements
(4) Lumped & Distributed elements.

(1) **Active** :- Active elements are the elements which are capable of delivering power (or) energy.

Ex:- Voltage source, current source.

Passive :- Passive elements are the elements which are capable of receiving energy.

Ex:- Resistor, Inductor, capacitor.

(2) **Unilateral** :- The elements which has different relationship b/n the voltage & current for two possible directions of current are called unilateral elements. Ex:- Diodes, Metals, Rectifiers

Bilateral :- Bilateral elements are the elements which has voltage-current relation is the same for current flowing in either direction.

Ex:- R, L, C

(3) Linear :- An element is said to be linear, if its voltage current characteristic is at all times a st. line through the origin which satisfies principle of superposition.

Ex:- R, L, C

Non-linear :- The elements which doesn't have its voltage current characteristics is at all times not a st. line through the origin (or) which doesn't satisfies the principle of superposition called non-linear elements.

Ex:- Diodes, Rectifiers.

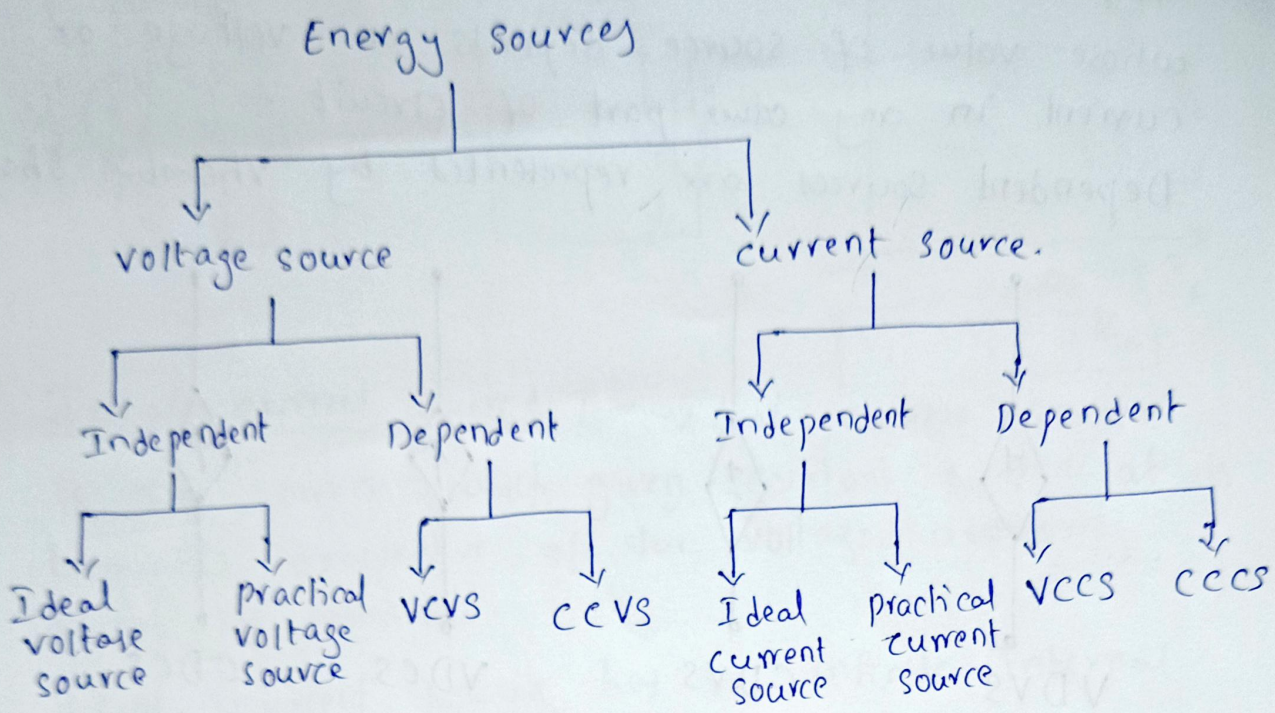
(4) Lumped :- Lumped elements are the elements which are small in size when compared to the wave length of applied signals.

Ex:- R, L, C

Distributed :- The elements which are not electrically seperable for an electrical or analytical purposes.

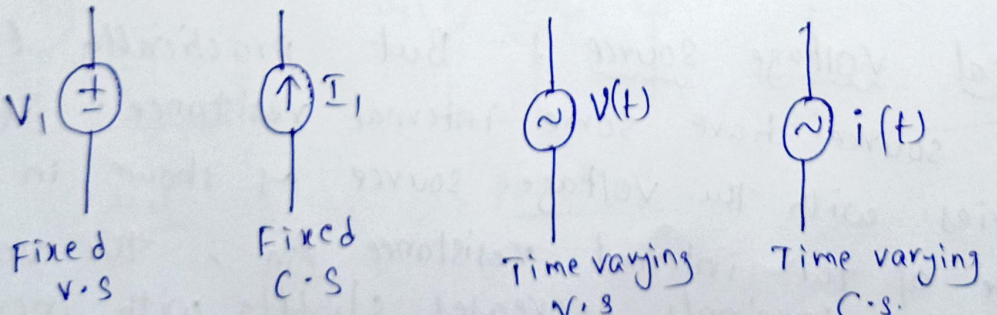
Ex:- Transmission lines.

Energy Sources



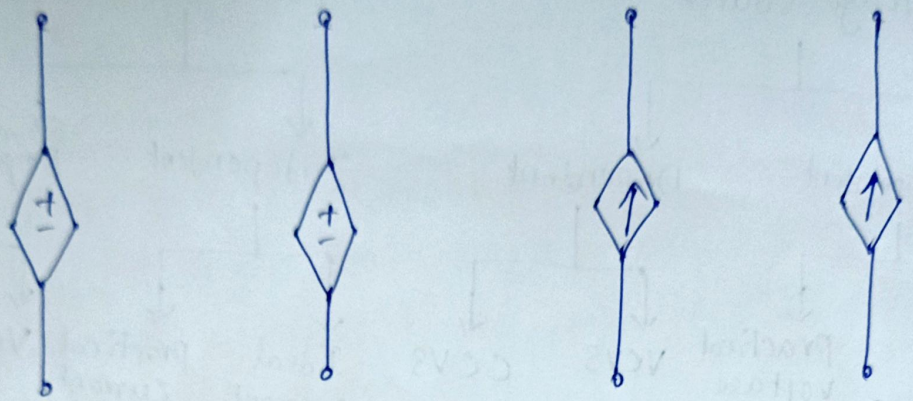
VCVS → voltage controlled voltage source
CCVS → current controlled " "
VCCS → voltage controlled current source
CCCS → current controlled " "

Independent source :- An independent (voltage or current) source maintains a fixed or varying with time (voltage or current) which is not affected by any other quantity.



Dependent Sources :- Dependent sources are those whose value of source depends on voltage or current in any other part of circuit.

Dependent sources are represented by rhombus shape.



VDVS CDVS VDCS CDCS

Ideal voltage source :- Ideal voltage source is defined as the energy source which gives constant voltage across its terminals irrespective of the current drawn through its terminals.

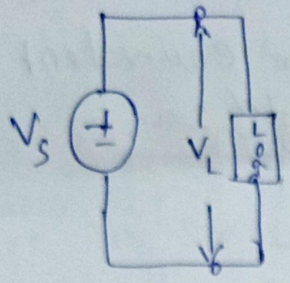
That means ideal v.s have zero internal Resistance. ($R_{in} = 0$)

∴ The o/p voltage always remains constant & equal to the EMF of the source.

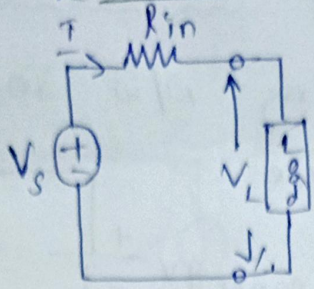
It is because there is no voltage drop inside the source when current drawn from it.

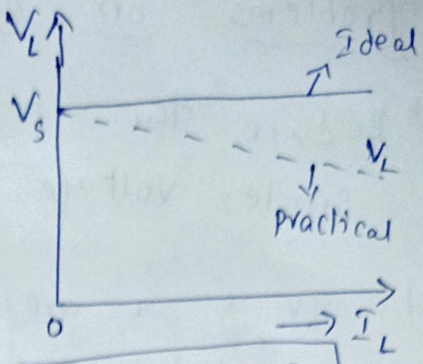
Practical voltage source :- But practically Every voltage source have some internal resistance. It is in series with the voltage source as shown in fig. Because of the internal resistance R_{in}, the voltage across the terminals decreases slightly with increase in current.

Ideal



practical





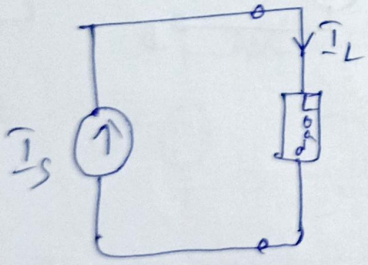
$$V_L = V_S - I R_{in}$$

Ideal current source :- Ideal current source is the source which gives constant current at its terminals irrespective of the voltage appearing across its terminals.

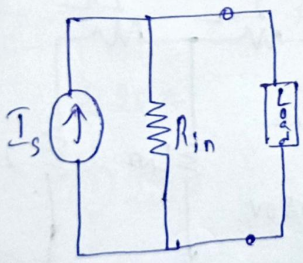
Ideal current source has (∞) infinite internal resistance. $R_{in} = \infty$

Practical current source :- But practically, every current source has high internal resistance, shown in parallel with current source.

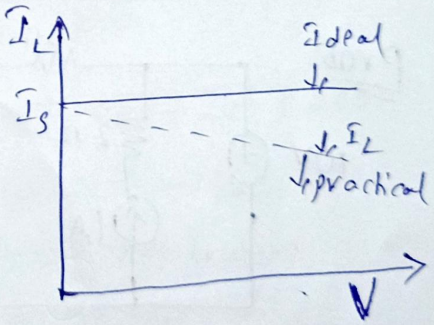
Because of this terminal internal resistance, current through its terminals decreases slightly with increase in voltage across its terminals.



Ideal

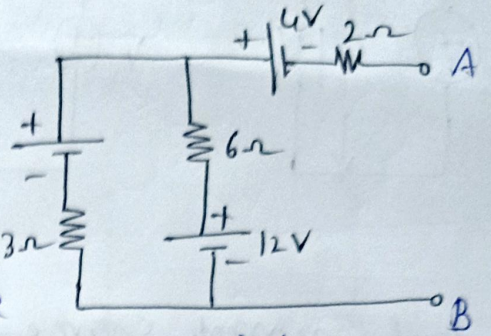


practical



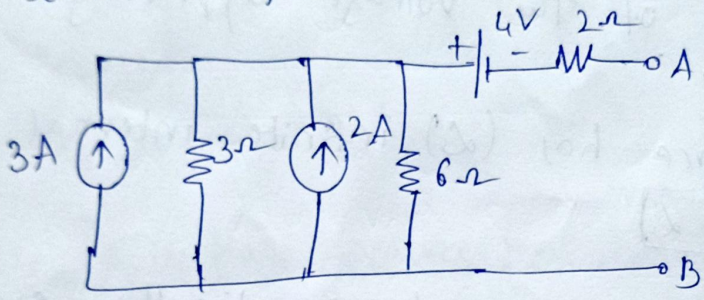
Problems on source Transformation

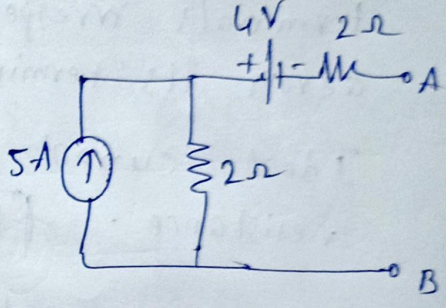
Prob Reduce the following N/w into an equivalent single voltage source.



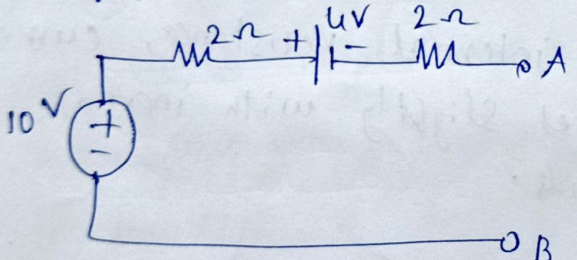
Sol:- 9V & 3Ω are in series convert into a current source in parallel with a resistance

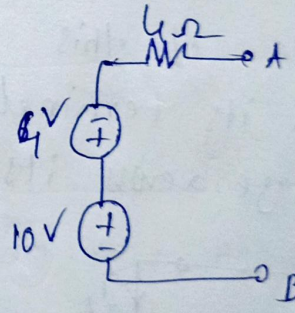
//ly 6Ω & 12V are in series convert into a current source in parallel with a resistance.

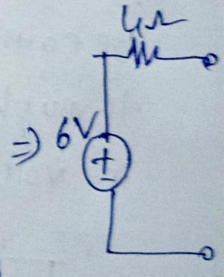




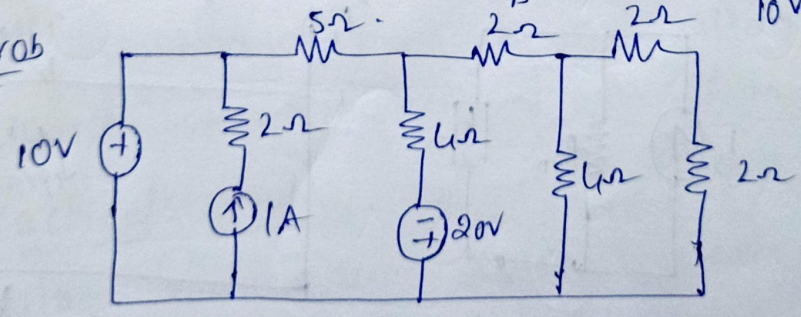
convert 5A & 2Ω are in parallel into a V.S in series with a resistance.







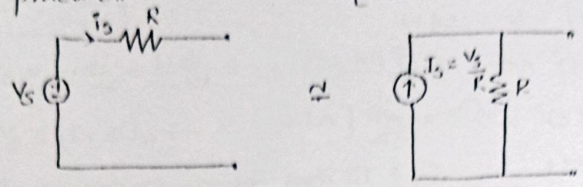
Prob



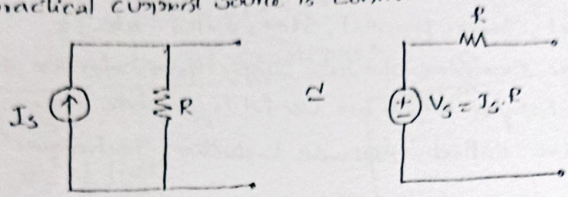
Source Transformation Techniques:

The Source Transformation Techniques are used to convert a complex network into a simple network by that it is very easy to find the response.

A practical voltage source [Voltage source in series with resistance] is converted into a practical current source [current source in parallel with resistance]

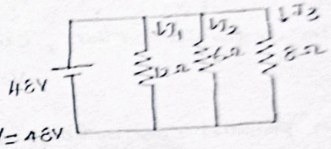


A practical current source is converted into a practical voltage source.



Problems on kcl & kvl:

1) Find the currents I_1, I_2, I_3 and I in the fig. shown



From the fig.

We notice that,

all components are connected in parallel

i.e., voltage across each component is same i.e., $V = 48V$

$V_1 = V_2 = V_3 = V = 48V$

We find the current in each resistor by using ohm's law.

$$I_1 = \frac{V}{R_1} = \frac{48}{12} = 4A$$

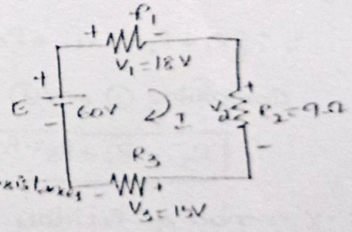
$$I_2 = \frac{V}{R_2} = \frac{48}{6} = 8A$$

$$I_3 = \frac{V}{R_3} = \frac{48}{8} = 6A$$

W.r.t to kcl, $I = I_1 + I_2 + I_3$

$$I = 4 + 8 + 6 = 18A$$

2) From the circuit shown in fig.



i) Determine V_2 by KVL

ii) Determine I, R_1 & R_3

iii) find the power in the 6Ω, 9Ω & 5Ω resistors

By Applying KVL,

$$E - V_1 - V_2 - V_3 = 0$$

(or) $$E = V_1 + V_2 + V_3$$

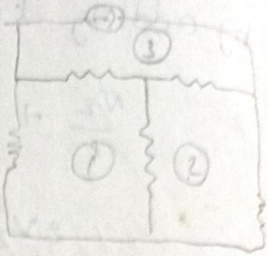
$$V_2 = E - V_1 - V_3 = 60 - 18 - 15 = 27V$$

ii) $$I = \frac{V_2}{R_2} = \frac{27}{9} = 3A$$

Mesh Analysis (or) Loop current method

(1) Identify the no. of meshes for the given ckt.

No of meshes = 3



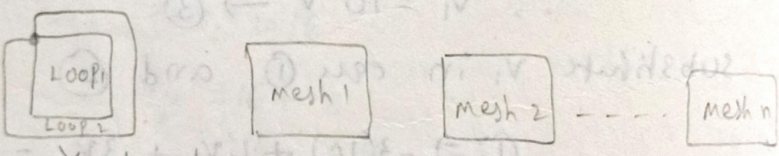
(2) Indicate the meshes and give the number 1, 2, 3 - - - n for each and every loop.

(3) Assume the loop currents $i_1, i_2, \ldots i_n$ for the loops 1, 2, 3 - - - n respectively. in clockwise direction (or) Anticlockwise direction.

(4) Apply the KVL for each and every loop, & assume that the current is in clockwise direction and write the loop equs or KVL equs or mesh equ's (or) voltage equ's and solve the all equ's by using cramer's method (or) general method.

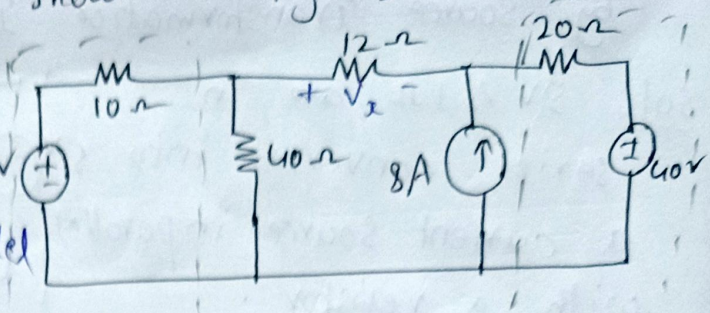
In mesh analysis unknowns are Loop currents.

Mesh:- Individual loops are called the meshes. It doesn't contains any loop with in.

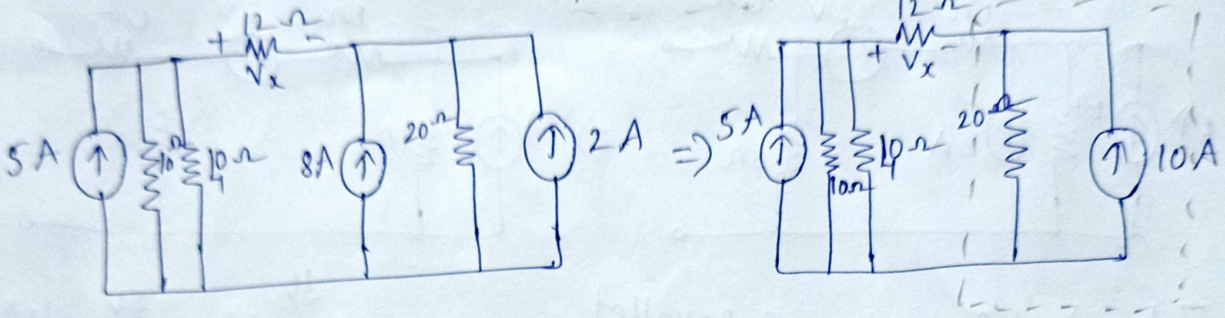


X not mesh.

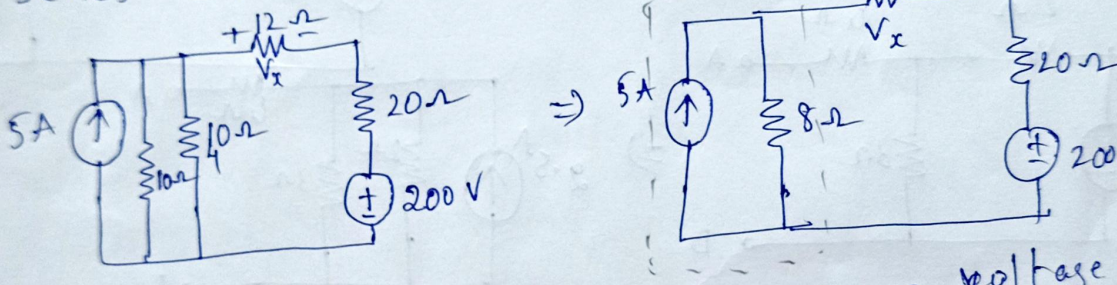
prob: using source Transformation Technique Find the Voltage V_x of shown in fig below.



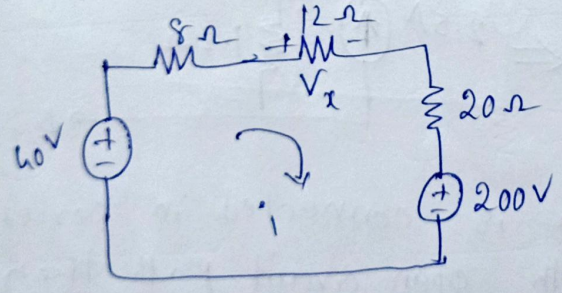
Sol: convert 40V & 20Ω & also 50V & 10Ω are in series into a current source in parallel with a resistance.



10A & 20Ω are in parallel convert into a V.S in series with a resistance.



convert 5A & 8Ω in parallel into a voltage source in series with a resistor.



Apply KVL

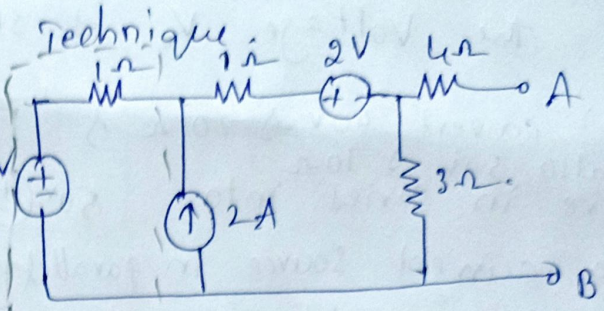
$40 = 8i + 12i + 20i + 200$

$40i = -160$

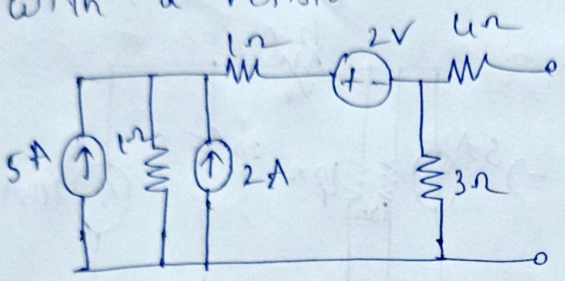
$i = -4A$.

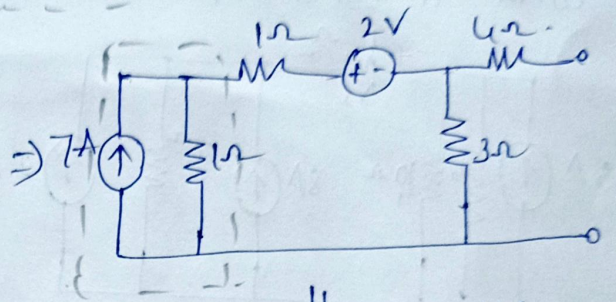
$\therefore V_x = 12i = 12 \times -4 = \underline{-48 \text{ volts}}$.

Prob = Determine the applied voltage across AB terminals by source transformation technique.

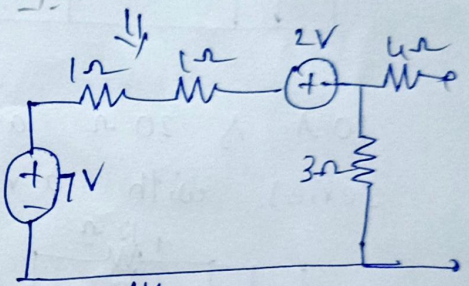


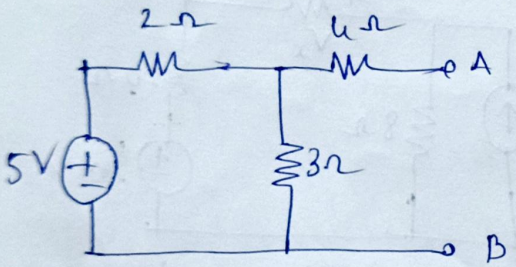
Sol:- 5V & 1Ω are in series converted into 5A current source in parallel with a resistor.

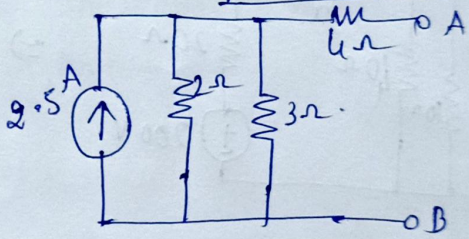


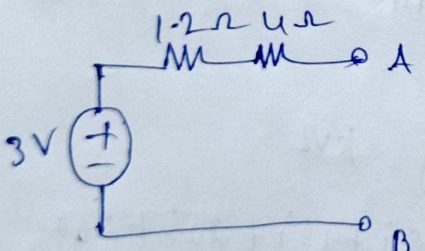


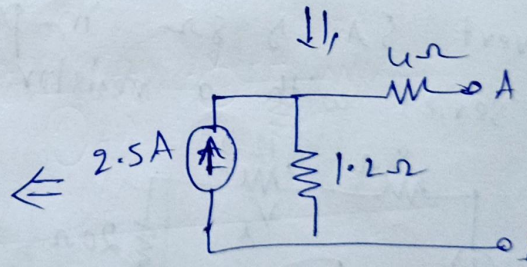
7A & 1Ω are in parallel convert into V.S in series with resistor.

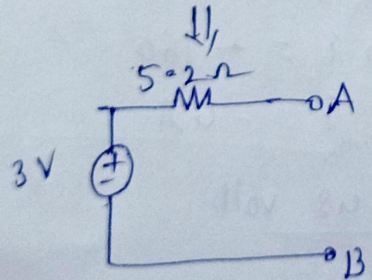








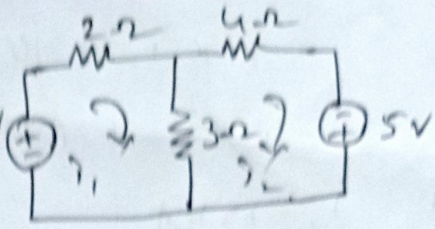




5.2Ω is connected in series with open circuit path then 5.2Ω is neglected.

$\therefore V_{AB} = 3V.$

prob Find mesh currents i_1 & i_2 for the ckt shown in fig.



Sol: Apply KVL for loop ①

$$9 = 2i_1 + 3(i_1 - i_2)$$

$$5i_1 - 3i_2 = 9 \rightarrow ①$$

Apply KVL for loop ②

$$3(i_2 - i_1) + 4i_2 = 5$$

$$-3i_1 + 7i_2 = 5 \rightarrow ②$$

Solve eqn ① & ②

$$3 \times \quad 5i_1 - 3i_2 = 9 \Rightarrow 15i_1 - 9i_2 = 27$$

$$5 \times \quad -3i_1 + 7i_2 = 5 \Rightarrow -15i_1 + 35i_2 = 25$$

$$26 i_2 = 52$$

$$i_2 = \frac{52}{26} = 2$$

$\boxed{i_2 = 2A}$ $\boxed{i_1 = 3A}$.

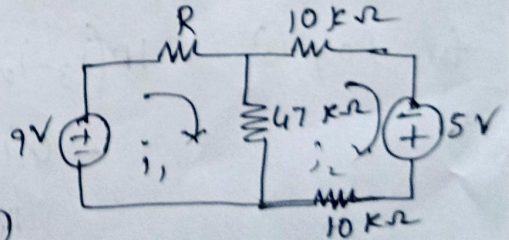
Substitute i_2 in eqn ①

$$5i_1 - 3(2) = 9$$

$$5i_1 = 9 + 6 \Rightarrow i_1 = \frac{15}{5} = 3A$$

prob:- Find the value of R if $i_1 = 1.5$ mA.

Sol:- Apply KVL for loop ①



$$9V = i_1 R + 47K(i_1 - i_2)$$

$$9 = (47K + R)i_1 - 47K i_2 \rightarrow ①$$

Apply KVL for loop ②

$$10K\,i_2 + 10K\,i_2 + 47K(i_2 - i_1) = 5$$

$$-47K\,i_1 + 67K\,i_2 = 5 \rightarrow ②$$

Given $i_1 = 1.5\,mA$.

② $\Rightarrow$ $-47K(1.5m) + 67K\,i_2 = 5$

$$-47 \times 1.5 + 67 \times 10^3\, i_2 = 5$$

$$i_2 = 1.12\,mA.$$ 1.32 mA

① $\Rightarrow$ $(R + 47K)(1.5\,m) - 47K(i_2) = 9$

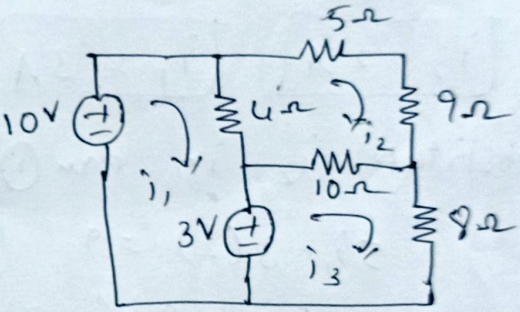
$$(R+47K)(1.5\,m) = 47 \times 1.12 + 9$$

$$= 52.64 + 9 = 61.64$$

$$R + 47K = \frac{61.64}{1.5\,m} = 41.09\,K \Rightarrow R = 41.09 - 47$$

$R = -5.9\,\Omega$ for the ckt

Prob:- Find loop currents i_1, i_2 & i_3 shown in fig.



Sol:- for loop ①

$$10 = 4(i_1 - i_2) + 3$$

$$4i_1 - 4i_2 = 7 \rightarrow ①$$

for loop ②, $4(i_2 - i_1) + 5i_2 + 9i_2 + 10(i_2 - i_3) = 0$

$$-4i_1 + 28\,i_2 - 10i_3 = 0 \rightarrow ②$$

for loop ③, $10(i_3 - i_2) + 8i_3 = 3$

$$-10\,i_2 + 18\,i_3 = 3 \rightarrow ③.$$

$$18\,i_3 = 3 + 10\,i_2.$$

Solve eqn ① & ② $i_3 = \frac{3}{18} + \frac{10}{18} i_2$

& ③ $i_1 = 2.21\,A$ $i_2 = 0.469\,A$, $i_3 = -0.427\,A$

Nodal Analysis

(1) A nodal analysis is more preferable to the circuit existed by the current sources.

(2) A nodal analysis can also preferable to the circuit existed by both volt sources and current sources.

(3) by both independent sources and dependent sources.

(i) Ideal - volt source, current source.

A nodal Analysis is an Analytic method to find the voltages for various branches of the circuit.

These branch voltages can be obtained from the node voltages.

Procedure for obtaining node voltages

(1) Identify the number of nodes of the given circuit.

(2) Give the numbers $1, 2, 3, \ldots, n$

(3) choose one of the node as the reference or ground node.

(4) Take the voltage at the ground node $V_n = 0$

(5) Assume the voltages $V_1, V_2, \ldots, V_{n-1}$

(6) Apply KCL at each and every node except ground node.

(7) which are obtained. These equations are called current equations (or) node equations

(8) Solve the above equation by using cramer's rule or conventional rule.

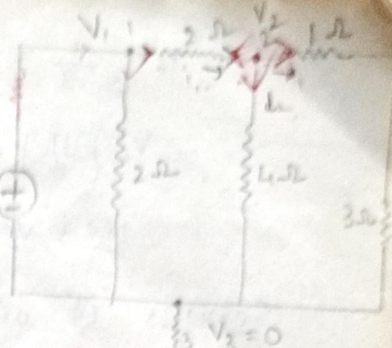
(9) If the circuit contains 'n' nodes there must be a in 'n-1' nodal equations.

Problems

(1) For the given circuit find the branch voltage and node voltages.

By applying the KCL at node V_1

$V_1 = 10\ V \longrightarrow (1)$



By applying the KCL at node 'V_2'

$$\frac{V_2}{4} + \frac{V_2}{1+3} + \frac{V_2 - V_1}{2} = 0$$

$$V_2\left[\frac{1}{4} + \frac{1}{4} + \frac{1}{2}\right] - \frac{V_1}{2} = 0$$

$$V_2[1] - \frac{10}{2} = 0 \quad [\because \text{from equ } (1)\ V_1 = 10]$$

$$V_2 = 5\ V$$

$i = i_1 + i_2$

$\frac{V_1 - V_2}{2} = \frac{V_2}{4} + \frac{V_2}{4} = \frac{V_2 - 0}{4}$

∴ Branch voltages are $V_{2\Omega} = 10$ volts.

$V_{2\Omega} = V_1 - V_2 = 10 - 5 = 5\ V$

$V_{4\Omega} = V_2 = 5\ V$

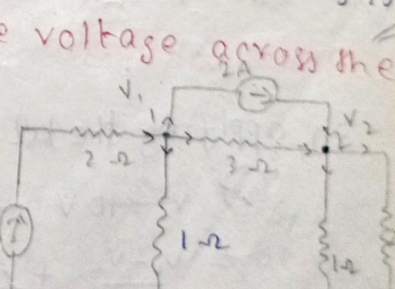
By voltage division principle, $V_{1\Omega} = V_{4\Omega} \times \frac{1}{1+3} = \frac{5 \times 1}{4} = \frac{5}{4}$

$= 1.2\ V$

$V_{3\Omega} = V_{4\Omega} \times \frac{3}{1+3} = \frac{5 \times 3}{4} = \frac{15}{4}$

$= 3.75\ V$

Prob 2: For the given circuit find the voltage across the 3Ω resistor using KCL.



~~By applying~~

1. Assuming the node voltages V_1 and V_2 at the nodes 1 and 2 and the voltage at the reference node is zero ie; $V_3 = 0$

By applying the KCL at node '1'

$$5 = 2 + \frac{V_1}{1} + \frac{V_1 - V_2}{3}$$

$$V_1\left[\frac{1}{1} + \frac{1}{3}\right] - \frac{V_2}{3} = 5 - 2$$

$$1.33\ V_1 - 0.33\ V_2 = 3 \longrightarrow (1)$$

At node '2', $\frac{V_2 - V_1}{3} + \frac{V_2}{1} + \frac{V_2}{2} = 2$

Duality:- in an electrical n/w itself there are pairs of terms which can be interchanged to get a new circuit. Such a pair of dual terms are given below.

Mesh	Node
current	voltage
capacitance	Inductance
Inductance	capacitance
Resistance	conductance
open ckt	short ckt
KCL	KVL
KVL	KCL

Procedure to draw the dual of the n/w.

(1) place a node in each loop of the original n/w. These nodes are non-reference (or) represent nodes in the dual n/w.

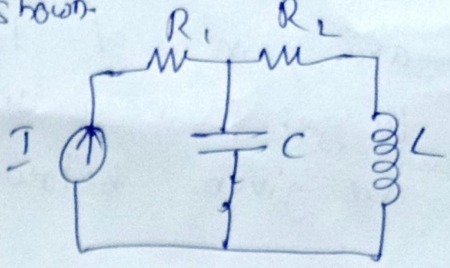
(2) place a node outside of the given n/w. This node is reference node of the dual n/w.

(3) connect all the nodes by dotted line through the elements, travelling only one element at a time.

(4) For each element tranversed in the original n/w replace each element by its dual element.

Ex!- construct the dual for the n/w shown in fig.

Sol:- The given n/w is redrawn as shown.

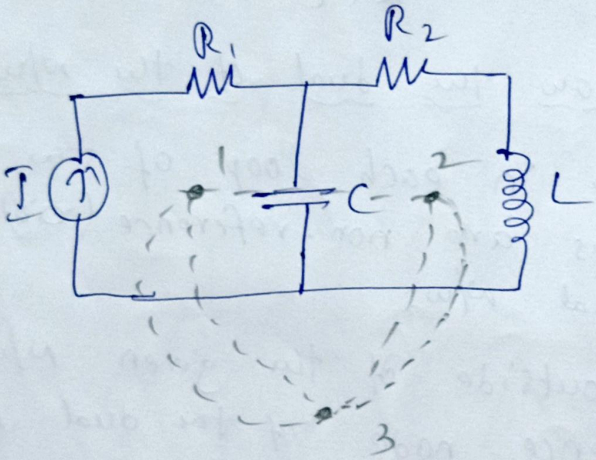


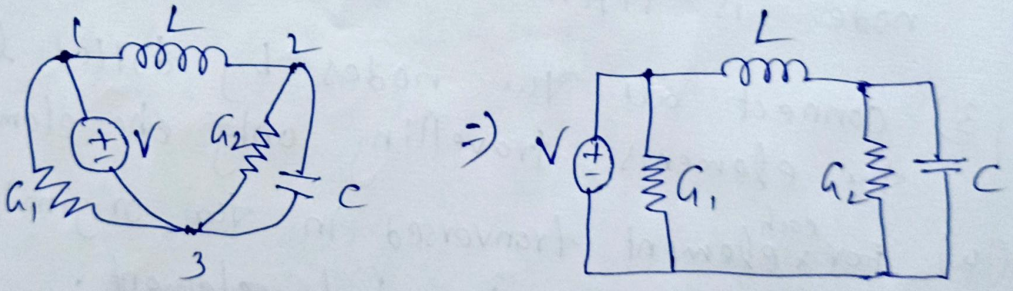
(2) Two nodes (node 1 & node 2) are placed in the two loops which acts as represent nodes.

(3) The node 3 is placed outside the n/w which act as reference node.

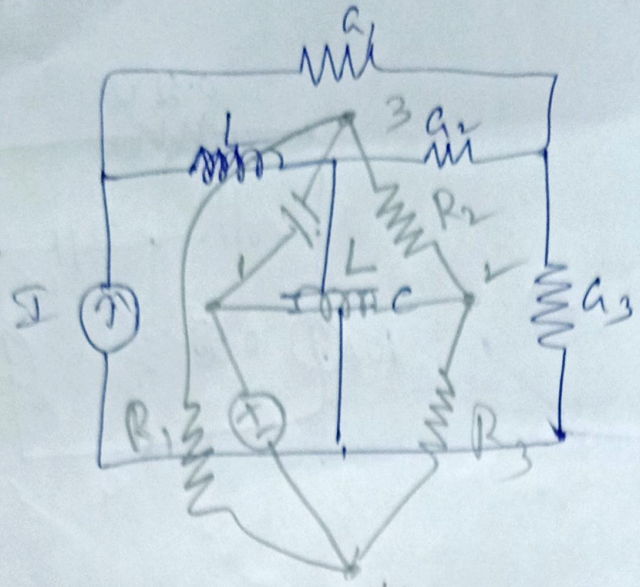
(4) All the nodes are connected by dotted line by traversing through the elements. while traversing each element is replaced by its dual.

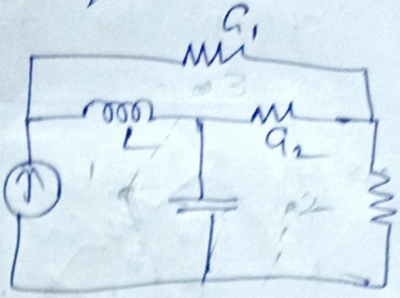
(5) The resultant dual n/w is shown.



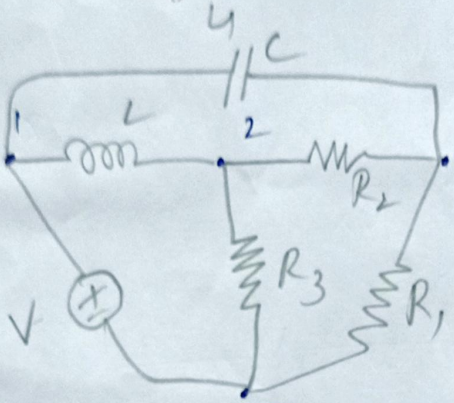


Prob: Construct the dual of the n/w shown

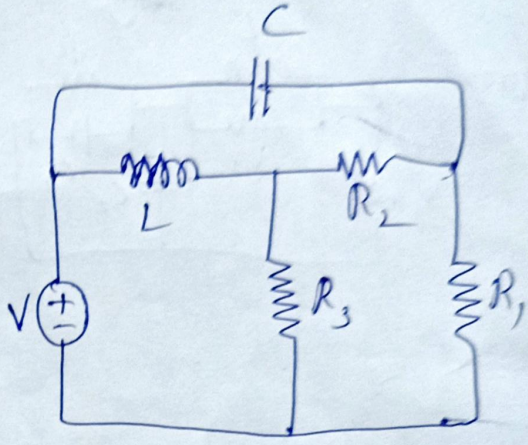




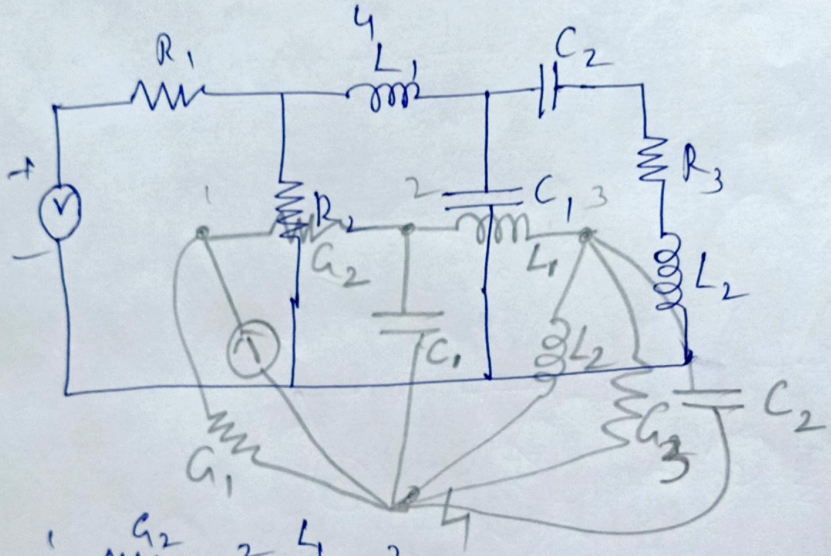
4



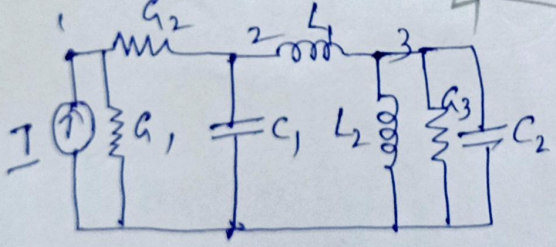
⇒



Prob:-

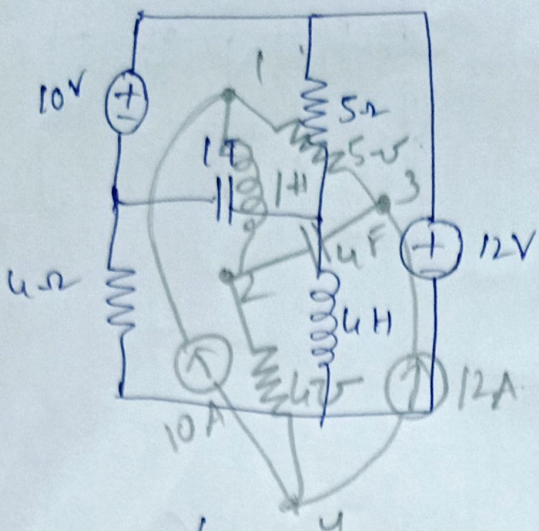


Sol:-

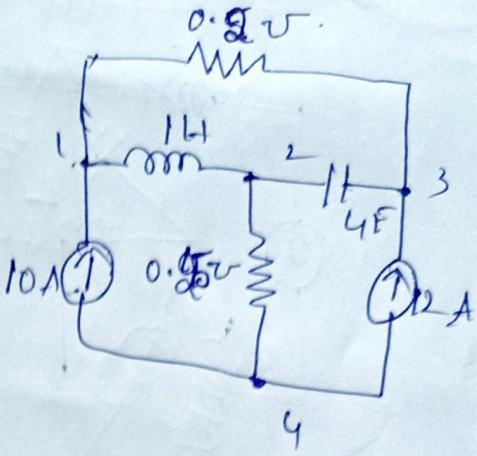


4

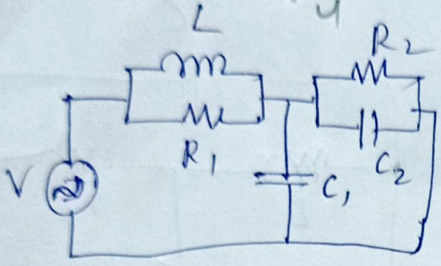
prob: obtain the dual of the circuit.



⇒

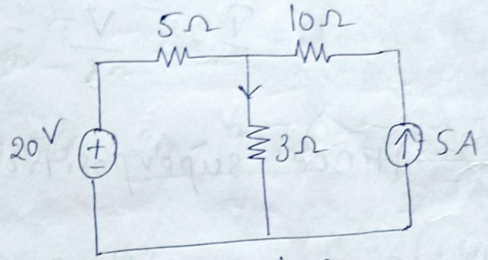


Prob

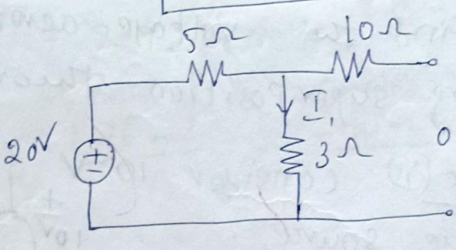


Superposition Theorem :- Superposition theorem states that in any linear network containing two or more sources, the response in any element is equal to the algebraic sum of the responses caused by individual sources acting alone, while the other sources are non operative. that is, while considering the effect of individual sources, other ideal voltage sources & ideal current current sources in the n/w are replaced by short circuit & open circuit across their terminals.

Prob:- using superposition theorem, Determine the current pasing thrayh 3Ω resistor shown in fig:

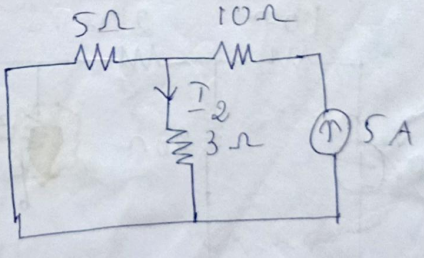


Sol:- **Case (i):-** consider 20 V voltage source & open circuit the 5 A current source.



$$I_1 = \frac{20}{8}$$
$$= 2.5\ A$$

Case (ii) consider 5 A current source & short circuit the 20 v voltage source.



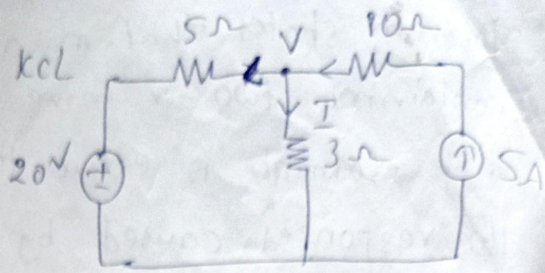
Apply the current divider rule, $I_2 = 5 \times \frac{5}{3+5}$

$$= \frac{25}{8} = 3.125\ A.$$

The total current pasing through the 3Ω resistor is $2.5 + 3.125 = 5.625$ Amps.

Case(iii) Consider both the sources

Apply Nodal Analysis (or) KCL



$$\frac{V-20}{5} + \frac{V}{3} = 5$$

$$3V - 60 + 5V = 75$$

$$8V - 60 = 75$$

$$8V = 135$$

$$V = \frac{135}{8} = 16.875 \text{ volts.}$$

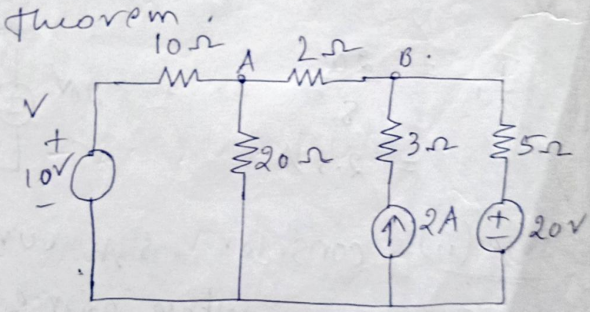
Current passing through 3Ω resistor is

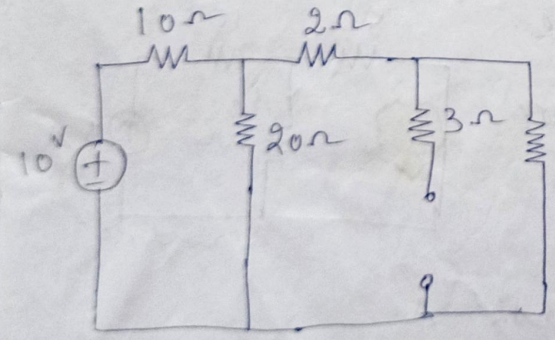
$$I = \frac{V}{3} = \frac{16.875}{3} = 5.625 \text{ Amps}$$

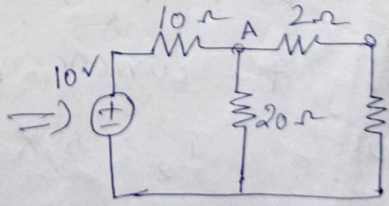
Hence superposition theorem is verified.

Prob: Find the voltage across the 2Ω resistor by using superposition theorem.

Sol: Case (i) Consider 10 V voltage source.







Apply KCL at node 'A' then $\frac{V-10}{10} + \frac{V}{20} + \frac{V}{7}$

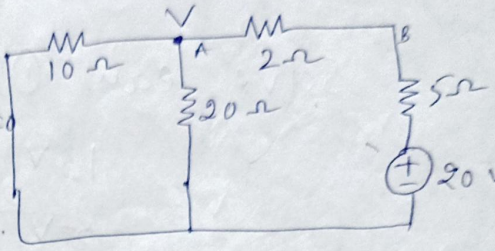
$$V = 3.41 \text{ volts.}$$

By applying voltage divider rule, voltage across 2Ω resistor,

$$V_{2\Omega} = V_{total} \times \frac{2}{5+2}$$

$$V_{2\Omega} = 3.41 \times \frac{2}{7} = 0.97 \text{ volts.}$$

case (ii) consider 20 V voltage source,



Apply KCL, $\frac{V-20}{7} + \frac{V}{10} + \frac{V}{20} = 0$

$$V = \frac{2.86}{0.293} = 9.76 \text{ volts.}$$

The voltage across 2Ω resistor is

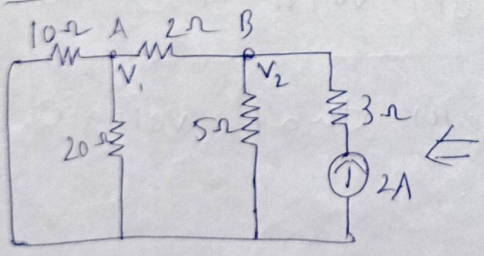
$$V_{2\Omega} = I_{2\Omega} \times 2 = 1.462 \times 2 = -2.924 \text{ volts}$$

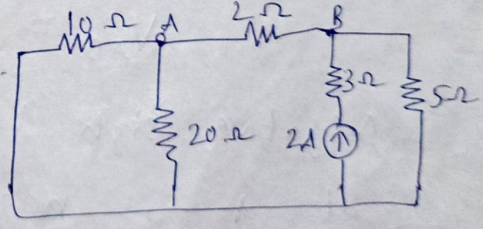
$$I_{2\Omega} = \frac{20-20}{7} = \frac{9.76}{7} = \frac{20-9.76}{7} = \frac{10.24}{7} = 1.462 \text{ A}$$

$$V_{2\Omega} = 1.462 \times 2 = 2.924 \text{ volts.}$$

$$I_{2\Omega} = \frac{V-20}{7} = \frac{9.76-20}{7} = -\frac{10.24}{7} = -1.462 \text{ Amps}$$

case (iii) consider 2 A current source.





Apply current divider rule,

Apply KCL at node A, $\frac{V_1}{20} + \frac{V_1}{10} + \frac{V_1 - V_2}{2} = 0$

$$V_1 + 2V_1 + 10V_1 - 10V_2 = 0$$

$$13V_1 - 10V_2 = 0 \rightarrow (1)$$

Apply KCL at node B,

$$\frac{V_2}{5} + \frac{V_2 - V_1}{2} = 2$$

$$2V_2 + 5V_2 - 5V_1 = 20$$

$$-5V_1 + 7V_2 = 20$$

$$5V_1 - 7V_2 + 20 = 0 \rightarrow (2)$$

Solve eqn (1) & (2).

$$13V_1 - 10V_2 = 0$$

$$5V_1 - 7V_2 + 20 = 0$$

$V_1 = 4.878$ Volt

$V_2 = 6.341$ volt.

Voltage across 2Ω resistor

$$V_1 - V_2 = 4.878 - 6.341$$

$$= -1.463 \text{ volts.}$$

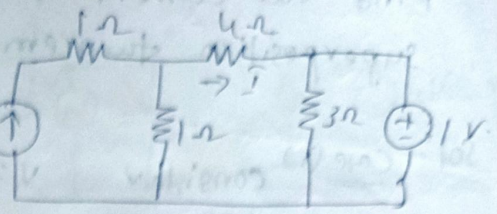
According super position theorem, voltage across 2Ω resistor is

$$V = 0.97 - 2.924 - 1.463$$

$$V_{AB} = -3.417 \text{ volts.}$$

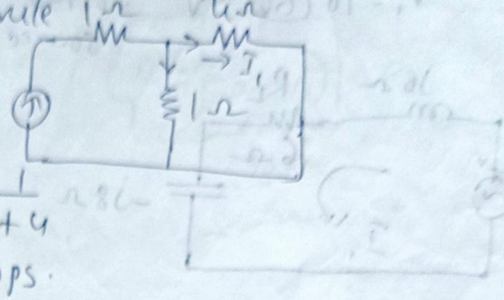
Negative sign indicates that voltage at 'A' is negative.

Prob: Find the current i in the circuit shown using superposition.



Sol: case(i) consider 1A current source.

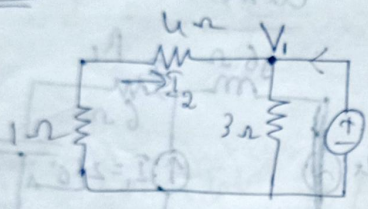
Apply current divider rule



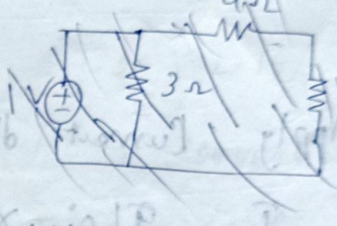
I_1 = Total current $\times \frac{1}{1+4}$

$= \frac{1}{5} = 0.2$ Amps.

case(ii) consider 1V voltage source.



$\Rightarrow$



Apply KCL, then $V_1 = 1V$

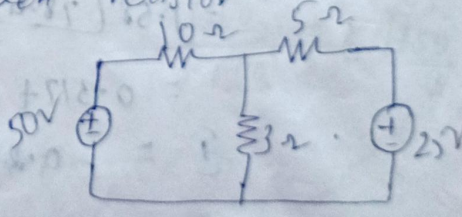
Current flows through $(4+1) = 5\Omega$ is $\frac{V_1}{5} = \frac{1}{5} = 0.2A$

$I_2 = -0.2$ A

According to superposition theorem,

$I_1 + I_2 = 0.2 - 0.2 = 0$ Amps

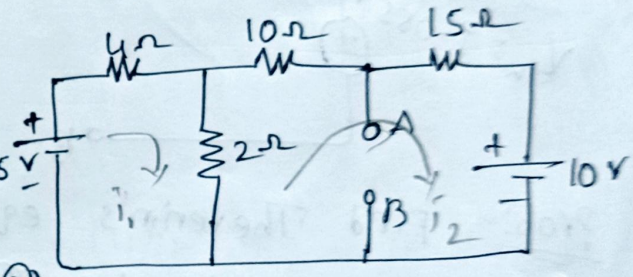
Find the current in each resistor



Thevenin's Theorem :- Thevenin's theorem states that any linear active two terminals N/w containing resistance & voltage sources or current sources can be replaced by a single voltage source V_{th} in series with a single resistance R_{th}.

The Thevenin's equivalent voltage V_{th} is the open circuit voltage at the network terminals, & the Thevenin resistance R_{th} is the resistance b/n the N/w terminals when all the sources are replaced with their internal resistances.

Prob:- obtain Thevenin equivalent circuit at terminals AB for the N/w shown in fig.



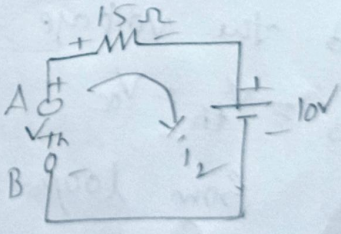
sol:- **step (1)**:- To find V_{th} across AB

loop (1), $25 = 6i_1 - 2i_2 \rightarrow (1)$

loop (2), $-2i_1 + 27i_2 = -10 \rightarrow (2)$.

solve eqn (1) & (2)

$\boxed{i_1 = 4.145 \text{ Amps}}$ $\boxed{i_2 = -0.063 \text{ A}}$



$V_{AB} = V_{th} = 15i_2 + 10$

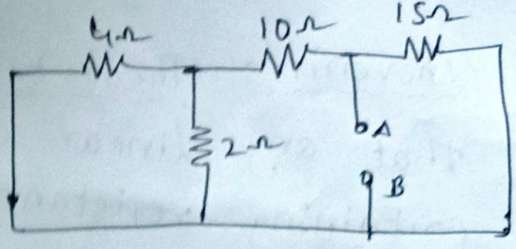
$= 15(-0.063) + 10$

$V_{th} = -0.945 + 10$

$\boxed{V_{th} = 9.055 \text{ volts}}$

Step 2:- To find R_{th}

Short ckt the 25V & 10V voltage sources.



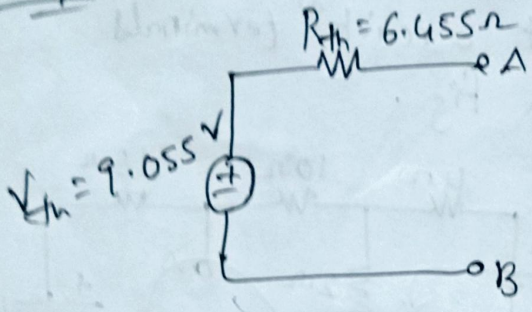
$$R_{eqn} = R_{AB} = R_{th} = \left[(4 // 2) + 10\right] // 15$$

$$= \left[\frac{8}{6} + 10\right] // 15 = \frac{68}{6} // 15$$

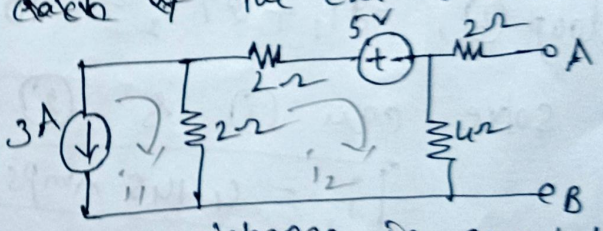
$$= \frac{34}{3} // 15$$

$$R_{th} = \frac{\frac{34}{3} \times 15^{5}}{\frac{34}{3} + 15} = \frac{170}{79/3} = \frac{510}{79} = 6.455\,\Omega.$$

Step 3:- Thevenin's Equivalent ckt



Prob:- Find Thevenin's equivalent resistance & voltage across terminals AB for the ckt shown in fig.



Sol:- Step 1:- To find V_{th}

Neglect 2Ω resistance & find the voltage across 4Ω resistance i.e. equal to

$$V_{AB} = V_{th} = V_{oc}.$$

From loop (1), $i_1 = -3A.$

loop ②, $2i_2 + 5 + 4i_2 + 2(i_2 - i_1) = 0$

$\therefore i_1 = -3$

$8i_2 - 2i_1 + 5 = 0$

$8i_2 + 6 + 5 = 0$

$8i_2 = -11$

$i_2 = \frac{-11}{8}$ Amps.

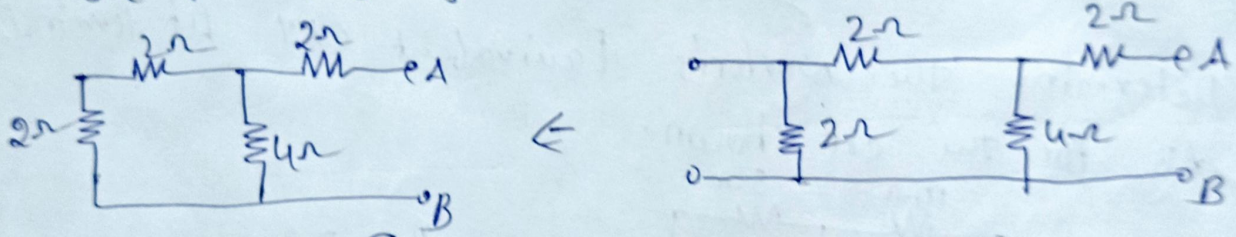
The voltage across 4Ω is equal to the voltage across AB terminals.

$V_{4\Omega} = V_{AB} = V_{oc} = V_{th} = i_2 \times 4 = \frac{-11}{8} \times 4$

$\boxed{V_{th} = -5.5 \text{ VOLTS}}$

Step 2:- To find R_{th}

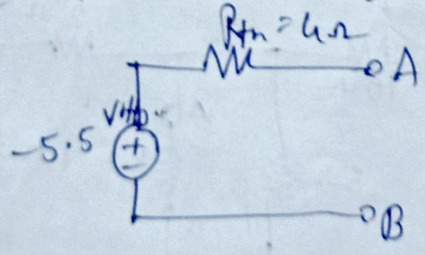
S.C the V.S & O.C the C.S.



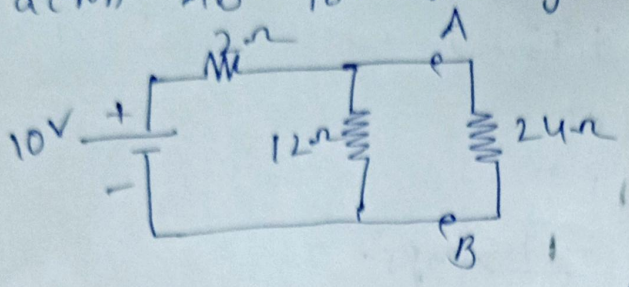
$R_{th} = R_{eqn} = [(2+2) // 4] + 2 = [4 // 4] + 2 = \frac{16}{8} + 2$

$\boxed{R_{th} = 4\Omega}$

∴ Thevenin's Equivalent ckt



Prob: Determine the Thevenin's Equivalent ckt across AB for the given ckt shown in fig.



$V_{th} = 8.52$ V

$R_{th} = 1.71\ \Omega$

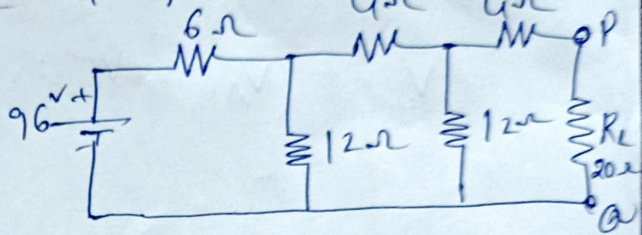
$I_L = 0.33$ Amps

Prob: For the following ckt Determine

(i) The open ckt voltage at the load terminals P@

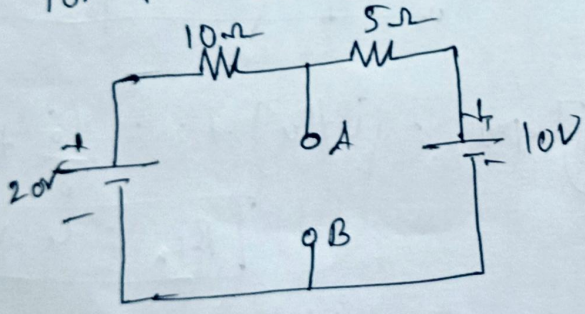
(ii) Thevenin's resistance at P@

(iii) The load current.



Problems on Norton's Theorem

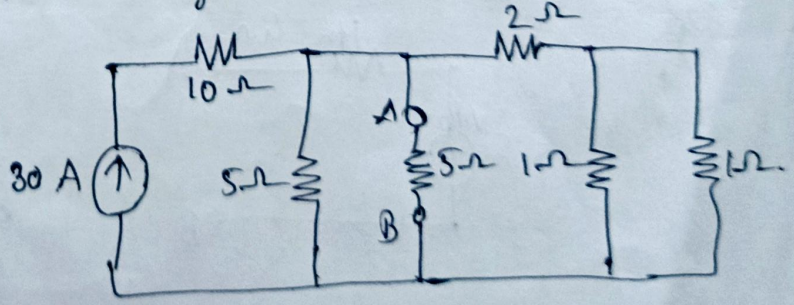
Determine the Norton's Equivalent ckt at terminals AB for the ckt shown.



$R_N = 3.33\ \Omega$

$I_N = 4$ A.

Determine the current through AB terminals using Norton's theorem.

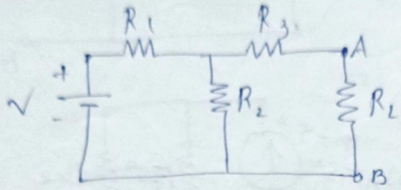


$I_N = 30$ A

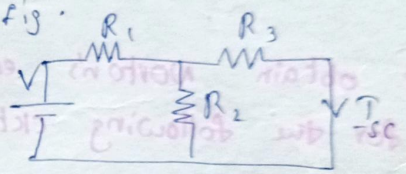
$R_N = 1.67\ \Omega$.

Norton's theorem :- This theorem states that in any two terminal linear network with voltage, current sources & resistances can be replaced by a simple equivalent ckt consisting of a current source in ||el with resistance. The value of current source is equal to the short ckt current b/w two terminals of the n/w & the resistance is equal to the equivalent resistance measured b/n the two terminals with all voltage sources short circted & all current sources open ckted.

Proof :-



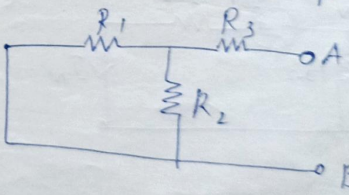
Step 1 :- Disconnect the load resistance R_L & short ckt the terminals as shown in fig.



current $I = \frac{V}{R_1 + [R_2 // R_3]}$

$$I_{sc} = I \times \frac{R_2}{R_2 + R_3} = \frac{V R_2}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

Step 2 :- Calculation of Thevenin's resistance from the terminals AB when the v/t source is replaced by short ckt.

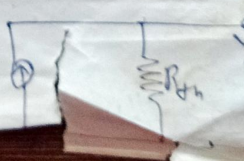


$$R_{th} = R_3 + [R_1 // R_2]$$

$$= R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

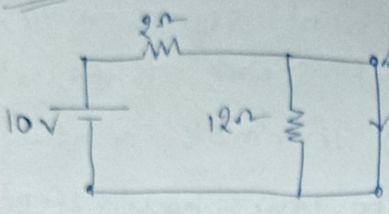
The norton's equivalent ckt with current source equal to I_{sc} & resistance R_N in ||el with it & is shown fig.

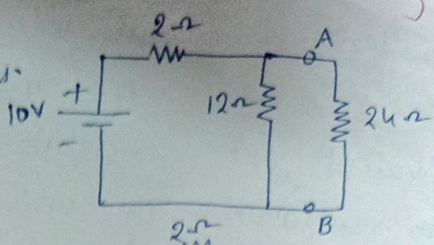
$$I_L = I_{sc} \times \frac{R_{th}}{R_{th} + R_L}$$



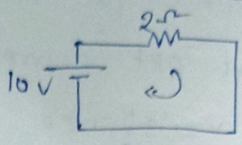
→ Find the current through the 2Ω resistor using Norton's theorem.

Sol: short ckt the AB terminals.



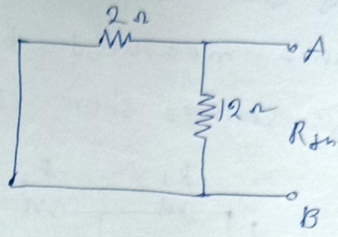


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$I_N = I_{SC} = \frac{10}{2} = 5A$

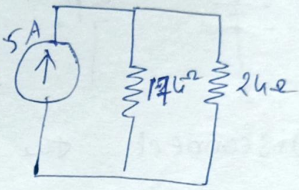
Determination of R_{th} or R_N :-



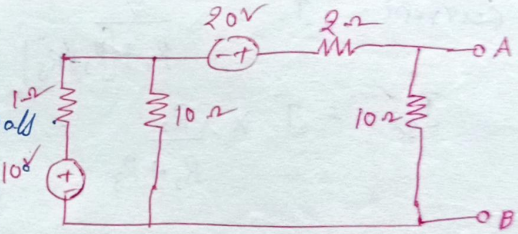
$R_N = 2 // 12 = 1.714\ \Omega$.

Norton's equivalent ckt is

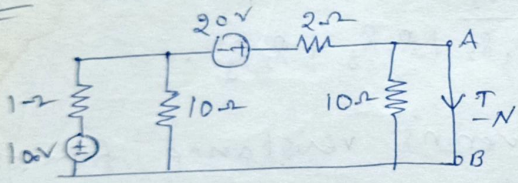
$I_{2\Omega} = 5 \times \frac{1.714}{1.714 + 2\Omega} = 0.733A.$



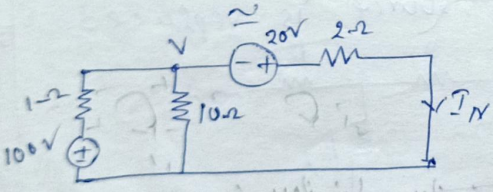
→ obtain Norton's equivalent ckt at AB terminals for the following ckt.



Sol: short ckt the AB terminals



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By applying KCL at node

$\frac{V-100}{1} + \frac{V}{10} + \frac{V+20}{2} = 0$

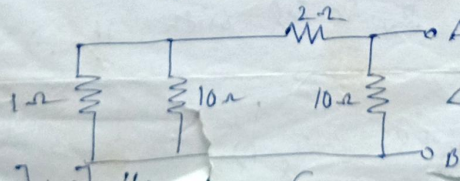
$V - 100 + \frac{5V + 100 + V}{10} = 0$

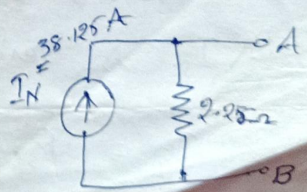
$16V = 900$

$V = 56.25\ V.$

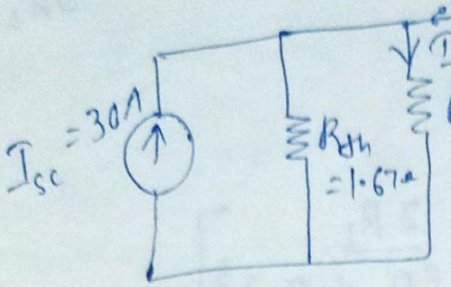
$I_N = \frac{56.25 + 20}{2} = 38.125\ A.$

To find R_N:





$R_N = [1 // 10] + 2 // 10 = \frac{10}{11} + 2 // 10 = 2.25\ \Omega = R_N.$



$$I_L = 30 \times \frac{1.67}{1.67+5}$$

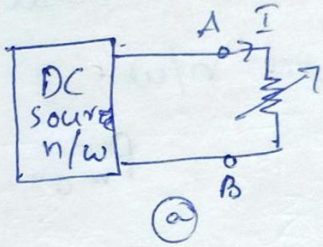
$$= 30 \times \frac{1.67}{6.67} = \frac{50.1}{6.67}$$

$$I_L = 7.51 \text{ A}.$$

Maximum power transfer theorem:-

Statement:- This theorem states that a resistive load connected to a dc n/w receives maximum power when the load resistance is equal to the internal resistance of the source n/w as seen from the load terminals.

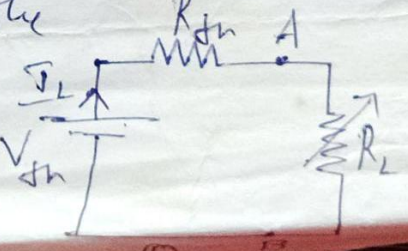
Explanation:- A variable resistance R_L is connected to a source n/w as shown in fig (a) while fig (b) represents the thevenin's voltage & thevenin's resistance R_{th} of the source n/w.



The aim is to determine the value of R_L such that it receives max. power from the dc source.

From fig (b) the current through the load resistance R_L is given by

$$I_L = \frac{V_{th}}{R_{th} + R_L}$$



The power delivered to the resistive load is given by

$$P_{max} \text{ or } P_L = I_L^2 R_L$$

$$P_L = \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 R_L$$

By adjusting the value of R_L, the max. power can be delivered through R_L if & only if $\frac{dP_L}{dR_L} = 0$.

$$\frac{dP_L}{dR_L} = \frac{d}{R_L}\left[\frac{V_{th}^2 R_L}{(R_{th}+R_L)^2}\right]$$

$$= V_{th}^2\left[\frac{1}{(R_{th}+R_L)^2} - \frac{2R_L}{(R_{th}+R_L)^3}\right] \quad \because \frac{du}{dP} = v\,\frac{du}{dP} - u\,\frac{dv}{dP}$$

$$\frac{dP_L}{dR_L} = V_{th}^2\left[\frac{R_{th}-R_L}{(R_{th}+R_L)^3}\right] = 0$$

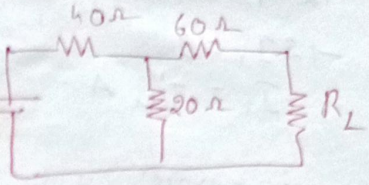
$$R_{th} - R_L = 0 \Rightarrow \boxed{R_{th} = R_L}$$

It is clear that the max. power is transfered to the load resistance when the load resistance R_L is equal to the internal resistance (R_{th}) of the source n/w.

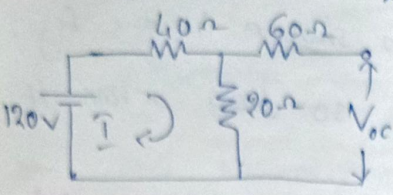
$$P_{max} = \frac{V_{th}^2 R_L}{(R_{th}+R_L)^2}$$

$$P_{max} = \frac{V_{th}^2 R_{th}}{4R_{th}^2} = \frac{V_{th}^2}{4R_{th}}$$

Prob: Find the value of R_L of the ckt shown in fig. for max. power transfer. Also find the max. power transfered.



Sol: Remove the load resistance

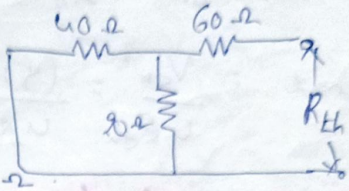


$$I = \frac{120}{40+20} = 2A$$

$$V_{th} = I \times 20 = 20 \times 2 = 40V$$

$$R_{th} = [40 // 20] + 60$$

$$= \frac{40 \times 20}{60} + 60 = 73.34\ \Omega$$



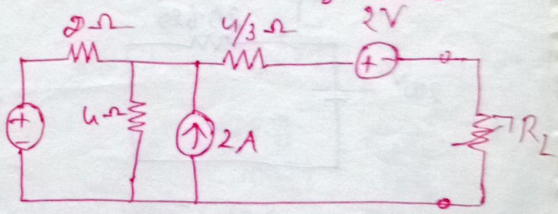
For max. power transfer $R_{th} = R_L$

$$R_L = 73.34\ \Omega.$$

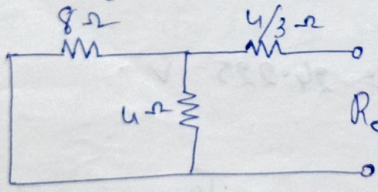
∴ max. power transfered $P = \frac{V_{th}^2}{4R_{th}} = \frac{40 \times 40}{4 \times 73.34}$

$$P_L = 5.45 \text{ watts}.$$

Prob: Find the value of the load resistance when the max. power is transfered across it & also find the max. power.



Sol: The value of $R_{th} = R_L$ for P_{max}. To find R_{Thev} or R_{th} open the current source & short the voltage sources.



$$\Rightarrow R_{th} = [8 // 4] + 4/3$$

$$= 4\ \Omega.$$

$$\therefore R_L = R_{th} = 4\ \Omega \text{ for max. power transfer.}$$

To find P_{max}, find V_{th} across load terminals.

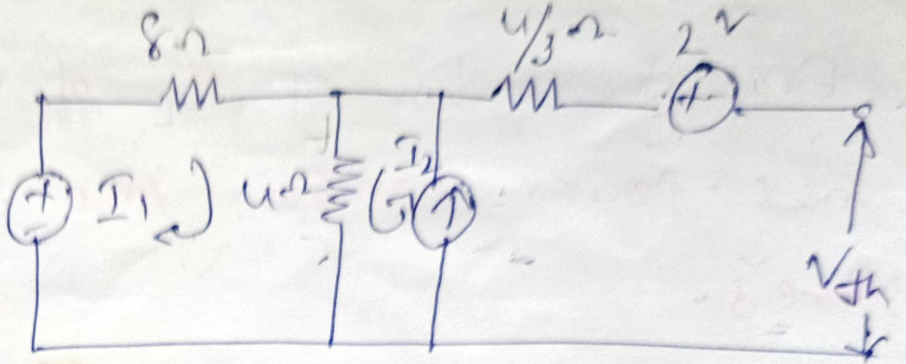
From the branch of current source, $I_2 = 2A$.

$-8I_1 - 4I_1 - 4I_2 + 5 = 0$

$-12I_1 - 4(2) + 5 = 0$

$-12I_1 = 3$

$I_1 = -1/4\ A.$



So drop across $8\Omega = 8I_1 = 2V$ as shown in the fig.

There is no current through $4/3\ \Omega$ so drop across it is zero.

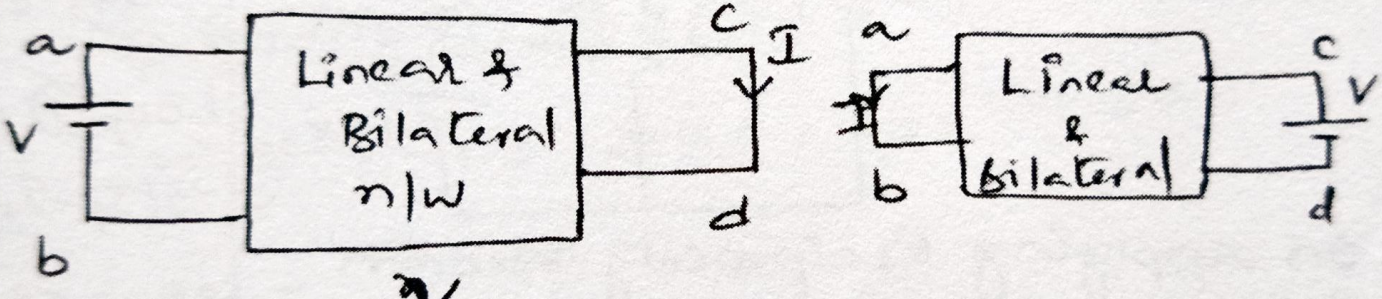
$\therefore V_{th} = V_{AB} = 5 + 2 - 2 = 5V$

$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{(5)^2}{4 \times 4} = \frac{25}{16} = 1.5625$ watts.

$V_{AB} = 1 - 4 - 2 = -5V$

Reciprocity Theorem:

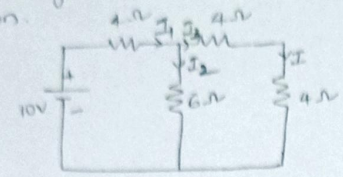
In any linear and bilateral network it consists of only one independent source the ratio of excitation to the response is same while interchanging the excitation and response.



$$\frac{V}{I} = \frac{V}{I}$$

Then Reciprocity theorem is verified

① Verify Reciprocity theorem for the voltage and current in the network shown.



Sol. Case(i)

Apply KCL,

$I_1 = I_2 + I_3$

$\frac{10 - V_1}{4} = \frac{V_1}{6} + \frac{V_1}{8}$

$\frac{10}{4} - V_1\left[\frac{1}{4} + \frac{1}{6} + \frac{1}{8}\right] = 0$

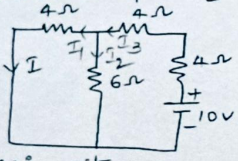
$V_1 = 4.6159\ V$

$I = \frac{V_1}{4+4} = \frac{4.61}{8} = 0.576\ A$

$\frac{V}{I} = \frac{10}{0.576} = 17.54$

Ratio of excitation to response, $\frac{V}{I} = 17.54$

Case ii)



On Applying Reciprocity theorem by changing excitation & response.

KCL at node,

$I_1 + I_2 = I_3$

$\frac{V_1}{4} + \frac{V_1}{6} = \frac{10 - V_1}{8}$

$V_1\left[\frac{1}{4} + \frac{1}{6} + \frac{1}{8}\right] = \frac{10}{8}$

$V_1\,[0.541] = 1.25$

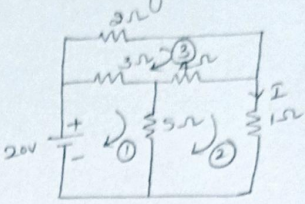
$V_1 = 2.3105\ V$

$I_1 = \frac{V_1}{4} = \frac{2.3105}{4} = 0.5776$

$\frac{V}{I} = \frac{10}{0.5776} = 17.3$

Reciprocity theorem is verified

② Verify the Reciprocity theorem for the circuit shown in the fig.



KVL for loop①

$20 - 3(I_1 - I_3) - 5(I_1 - I_2) = 0$

$8I_1 - 5I_2 - 3I_3 = 20$ —①

Loop② ⇒ $-1I_2 - 5(I_2 - I_1) - 4(I_2 - I_3) = 0$

$5I_1 - 10I_2 + 4I_3 = 0$ —②

Loop③ ⇒ $-4(I_3 - I_2) - 2I_3 - 3(I_3 - I_1) = 0$

$3I_1 + 4I_2 - 9I_3 = 0$ —③

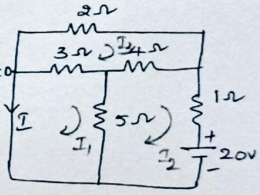
on solving ①, ② & ③ we get,

$I_1 = 9.42\ A$, $I_2 = 7.26A$, $I_3 = 6.36A$

$\therefore \boxed{I = I_2 = 7.26A}$

$\frac{V}{I} = \frac{20}{7.26} = 2.75$

on changing excitation and response



Loop① ⇒

$-3(I_1 - I_3) - 5(I_1 - I_2) = 0$

$8I_1 - 5I_2 - 3I_3 = 0$ —①

Loop② ⇒

$-5(I_2 - I_1) - 4(I_2 - I_3) - I_2 - 20 = 0$

$5I_1 - 10I_2 + 4I_3 = 20$ —②

Loop③ ⇒ $-3(I_3 - I_1) - 2I_3 - 4(I_3 - I_2) = 0$

$3I_1 + 4I_2 - 9I_3 = 0$ —③

on solving, we get

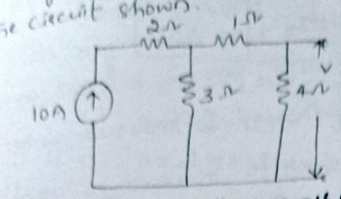
$I_1 = -7.26\ A$, $I_2 = -8.025\ A$

$I_3 = -5.98A$

$\frac{V}{I} = \frac{20}{-I_1} = \frac{20}{7.26} = 2.75$

Hence Reciprocity theorem is verified

③ Verify Reciprocity theorem for the circuit shown.



Sol: Response, V = Voltage across 4Ω

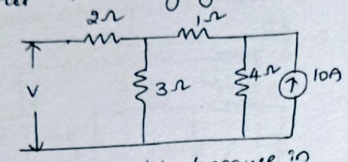
$= 4 \times i_{4\Omega}$

$i_{4\Omega}$ is $= 10 \times \frac{3}{3+4+1}$

$= 10 \times \frac{3}{8} = 3.75$

$\therefore V = 4(3.75) = 15V$

After interchanging,



2Ω is removable because in series with open circuit voltage.

$\therefore V$ = Voltage across 3Ω

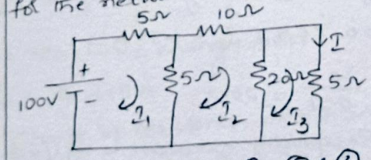
$= 3 \times i_{3\Omega}$

$i_{3\Omega} = 10 \times \frac{4}{3+4+1} = 5A$

$\therefore V_{3\Omega} = 3(5) = 15V$

Reciprocity Theorem is verified.

④ Verify Reciprocity theorem for the voltage(V) and current I for the network shown



Sol: Apply KVL for loop ①, ② & ③

Loop① ⇒ $100 - 5I_1 - 5(I_1 - I_2) = 0$

$10I_1 - 5I_2 = 100 \rightarrow ①$

Loop② ⇒ $-10I_2 - 5(I_2 - I_1) - 20(I_2 - I_3) = 0$

$5I_1 - 35I_2 + 20I_3 = 0 \rightarrow ②$

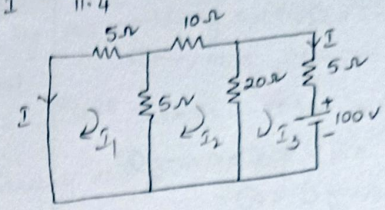
Loop③ ⇒ $-20(I_3 - I_2) - 5I_3 = 0$

$-20I_2 + 15I_3 = 0 \rightarrow ③$

on solving ①, ② & ③ we get

$I_1 = 14.2A$, $I_2 = 8.57A$, $I_3 = 11.4A$

$\frac{V}{I} = \frac{100}{11.4} = 8.771$



on Applying KVL,

Loop① ⇒ $-5I_1 - 5(I_1 - I_2) = 0$

$-10I_1 + 5I_2 = 0$

$10I_1 - 5I_2 = 0 \rightarrow ①$

Loop② ⇒ $-5(I_2 - I_1) - 10I_2 - 20(I_2 - I_3) = 0$

$5I_1 - 35I_2 + 20I_3 = 0 \rightarrow ②$

Loop③ ⇒ $-100 - 5I_3 - 20(I_3 - I_2) = 0$

$20I_2 - 25I_3 = 100 \rightarrow ③$

on solving ①, ② & ③

we get, $I_1 = -2.4A$, $I_2 = -4.84A$

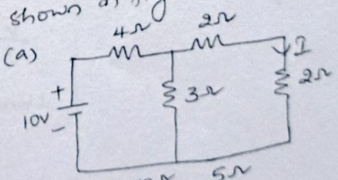
$I_3 = -7.87A$

$\frac{V}{I} = \frac{V}{-I_1} = \frac{20}{-(-2.4)} = 8.33$

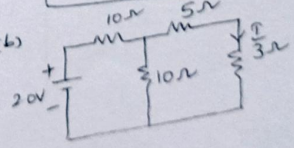
$\therefore$ Reciprocity Theorem is verified.

⑤ Verify the Reciprocity Theorem for the Voltage V and current I for the network shown in fig.

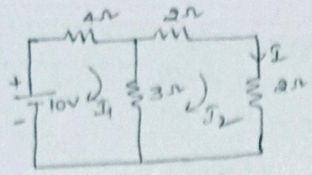
(a)



(b)



a)



Case (i) Apply KVL,

Loop ① ⇒ $10 - 4I_1 - 3(I_1 - I_2) = 0$

$10 - 4I_1 - 3I_1 + 3I_2 = 0$

$7I_1 - 3I_2 = 10 \rightarrow ①$

Loop ② ⇒ $-3(I_2 - I_1) - 2I_2 - 2I_2 = 0$

$3I_1 - 7I_2 = 0 \rightarrow ②$

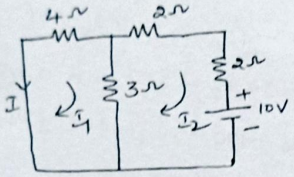
on solving ① & ②

$I_1 = 1.75\,A$, $I_2 = 0.75\,A$

$I = I_2 = 0.75\,A$

$\frac{V}{I} = \frac{10}{0.75} = 13.33$

Case (ii)



on Applying KVL,

Loop ① ⇒ $-4I_1 - 3(I_1 - I_2) = 0$

$7I_1 - 3I_2 = 0 \rightarrow ①$

Loop ② ⇒ $-3(I_2 - I_1) - 2I_2 - 10 = 0$

$3I_1 - 7I_2 = 10 \rightarrow ②$

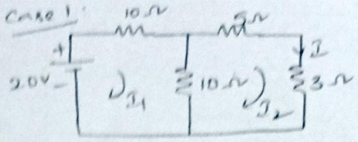
on solving ① & ② we get

$I_1 = -0.75\,A$, $I_2 = -1.75\,A$

$\frac{V}{I} = \frac{10}{0.75} = 13.33$

Hence Reciprocity Theorem is verified.

(b) Case i:



on Applying KVL,

Loop ①

$20 - 10I_1 - 10(I_1 - I_2) = 0$

$20 - 10I_1 - 10I_1 + 10I_2 = 0$

$20I_1 - 10I_2 = 20 \rightarrow ①$

Loop ② ⇒

$-10(I_2 - I_1) - 5I_2 - 3I_2 = 0$

$-10I_2 + 10I_1 - 5I_2 - 3I_2 = 0$

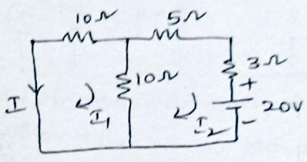
$10I_1 - 18I_2 = 0 \rightarrow ②$

on solving ① & ②, we get

$I_1 = 1.384\,A$, $I_2 = 0.7692\,A$

$\frac{V}{I} = \frac{20}{0.7692} = 26.00$

Case ii:



on Applying KVL,

Loop ①:

$-10I_1 - 10(I_1 - I_2) = 0$

$20I_1 - 10I_2 = 0 \rightarrow ①$

Loop ②:

$-10(I_2 - I_1) - 5I_2 - 3I_2 - 20 = 0$

$-10I_2 + 10I_1 - 5I_2 - 3I_2 = 20$

$10I_1 - 18I_2 = 20 \rightarrow ②$

Solving ① & ②, we get

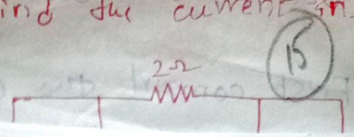
$I_1 = 0.76\,A$, $I_2 = 1.53\,A$

$\frac{V}{I} = \frac{20}{0.76} = 26.3$

Hence, Reciprocity Theorem is verified.

calculate ... Norton's equi... find the current in 2Ω resistor

... first convert the current



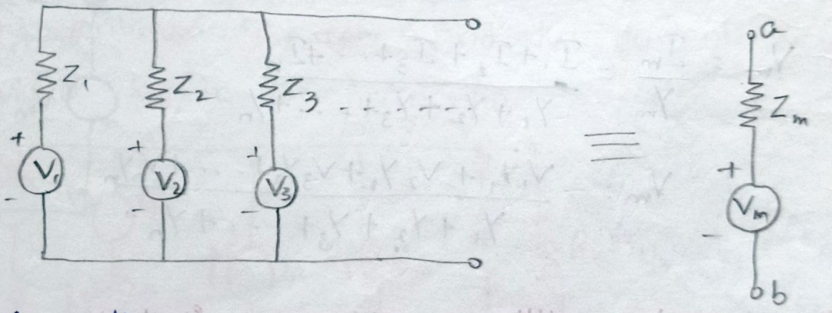
Milliman Theorem :-

Milliman theorem states that if 'n' voltage sources $V_1, V_2, \ldots V_n$ having internal impedances (or series impedances) $Z_1, Z_2, \ldots Z_n$ respectively are connected in ||el as shown in fig. then these sources may be replaced by a single voltage V_m having internal series impedance Z_m where V_m & Z_m are given by the following equations.

$$V_m = \frac{V_1Y_1 + V_2Y_2 + V_3Y_3 + \ldots}{Y_1 + Y_2 + Y_3 + \ldots}$$

$$Z_m = \frac{1}{Y_1 + Y_2 + Y_3 \ldots}$$

where $Y_1, Y_2, \ldots Y_n$ are the admittances corresponding to $Z_1, Z_2, \ldots Z_n$.



Proof A voltage source V_1 with series impedance Z_1 may be replaced by a current source $I_1 = V_1 \cdot Y_1$ with shunt admittance $\left(Y_1 = 1/Z_1\right)$. Accordingly the n/w of fig (a) may be replaced by its equivalent n/w shown in fig (b) where

$$I_1 = V_1 Y_1 \quad \ldots \quad I_n = V_n Y_n$$

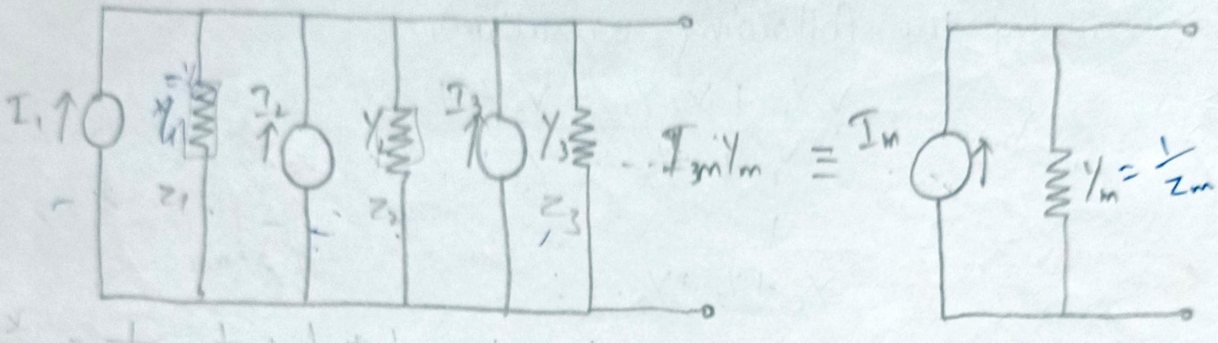
& $Y_1 = \frac{1}{Z_1}$

$Y_2 = \frac{1}{Z_2}$ ... $Y_n = \frac{1}{Z_n}$

Network in fig (b) is equivalent to that of fig(c) whereby

$$I_m = I_1 + I_2 + I_3 - \cdots + I_n$$

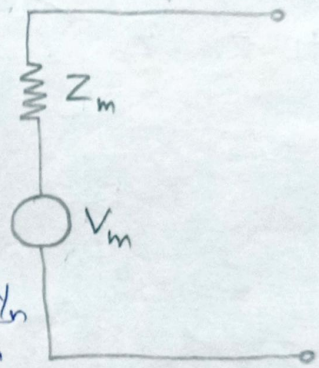
$$Y_m = Y_1 + Y_2 + Y_3 - \cdots + Y_n$$



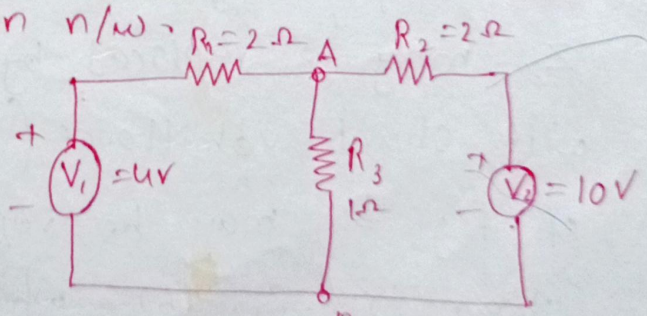
$$Z_m = \frac{1}{Y_m} = \frac{1}{Y_1 + Y_2 + \cdots + Y_n}$$

$$V_m = \frac{I_m}{Y_m} = \frac{I_1 + I_2 + I_3 + \cdots + I_n}{Y_1 + Y_2 + Y_3 + \cdots + Y_n}$$

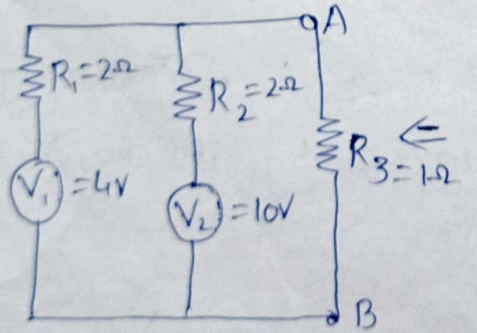
$$V_m = \frac{V_1 Y_1 + V_2 Y_2 + V_3 Y_3 + \cdots + V_n Y_n}{Y_1 + Y_2 + Y_3 + \cdots + Y_n}$$

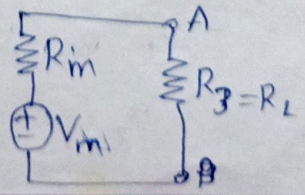


Prob: using Milliman Theorem, find the current in resistor R_3 in the given n/w.



Sol:-





... must convert the current ... 2Ω ... resistor.

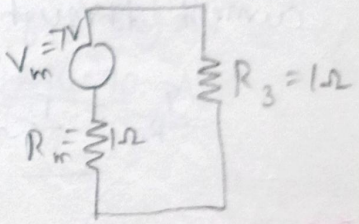
By The Two voltage sources V_1 & V_2 with series resistances R_1 & R_2 may be combined into one. The equivalent n/w is then as shown below.

Then by milliman theorem,

$$V_m = \frac{V_1Y_1 + V_2Y_2}{Y_1 + Y_2} = \frac{(4 \times 0.5) + (10 \times 0.5)}{0.5 + 0.5}$$

$$= 7 \text{ volts.}$$

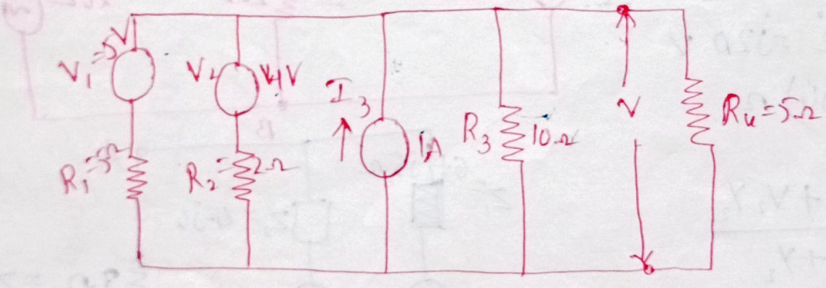
$$R_m = \frac{1}{Y_1 + Y_2} = \frac{1}{0.5 + 0.5} = 1\Omega.$$



Hence current through R_3 is

$$I_{R_3} = I_{1\Omega} = \frac{V_m}{R_m + R_3} = \frac{7}{1+1} = \frac{7}{2} = 3.5 \text{ Amps.}$$

Ex: Using milliman Theorem find the current & voltage is resistor R_4 in the given n/w.

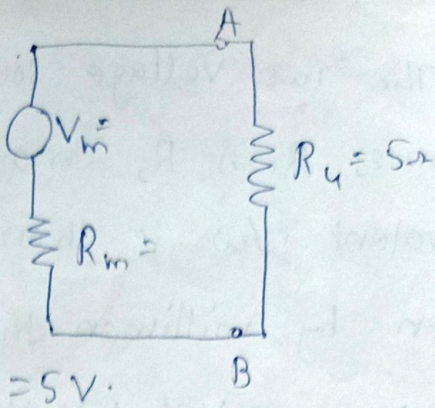


current source I_3 with shunt resistor R_3 may be converted into equivalent voltage source V_3 with series resistor R_3 where $V_3 = I_3 \cdot R_3 = 1 \times 10 = 10V$.

$$V_m = \frac{V_1Y_1 + V_2Y_2 + V_3Y_3}{Y_1 + Y_2 + Y_3}$$

$$= \frac{(5\times 0.2) + (4\times 0.5) + (10\times 0.1)}{0.2 + 0.5 + 0.1}$$

$$V_m = \frac{1+2+1}{0.8} = \frac{4}{0.8} = \frac{40}{8} = 5\,V.$$

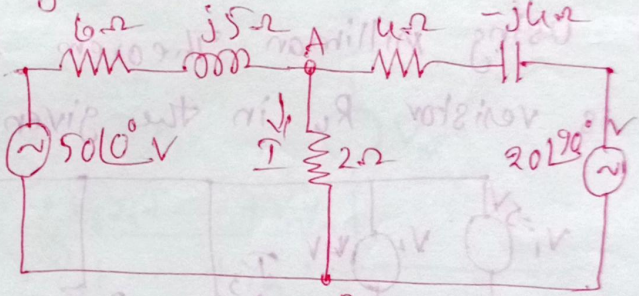


$$R_m = \frac{1}{Y_1 + Y_2 + Y_3} = \frac{1}{0.8} = \frac{10}{8} = 5/4\ \Omega$$

current through R_4 resistor is

$$I_{R_4} = I_{5\Omega} = \frac{V_m}{R_m + R_4} = \frac{5}{\frac{5}{4}+5} = \frac{20}{25} = \frac{4}{5} = 0.8\ \text{Amp}.$$

Prob Using milliman theorem find the current through 2Ω resistance in the fig shown.

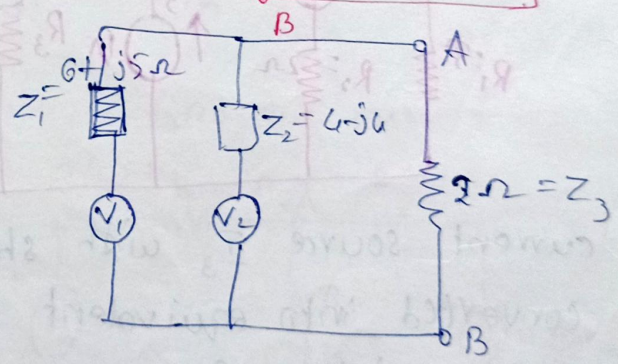


$V_1 = 50\angle 0^\circ = 50 + j0\ V$

$Z_1 = (6 + j5)\ \Omega$

$V_2 = 20\angle 90^\circ = j20\ V$

$Z_2 = (4 - j4)\ \Omega$



$$V_m = \frac{V_1Y_1 + V_2Y_2}{Y_1 + Y_2}$$

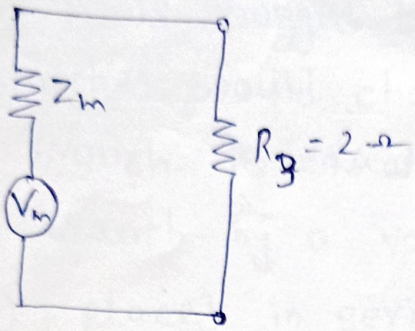
$$= \frac{\frac{50+j0}{6+j5} + \frac{j20}{4-j4}}{\frac{1}{6+j5} + \frac{1}{4-j4}}$$

$$V_m = \frac{2.42 - j1.6}{0.22 - j0.043} = 11.96 - j4.93\ V$$

$12.86\angle -49.13 = 9.69 - j8.46$

$$Z_m = \frac{1}{Y_1 + Y_2} = \frac{1}{\frac{1}{6+j5} + \frac{1}{4-j4}} = \frac{1}{0.22 - j0.043}$$

$$Z_m = (4.32 - j0.83)\,\Omega \qquad 4.34 - j0.83$$



current is 2Ω resistor is

$$I_{2\Omega} = \frac{V_m}{Z_m + Z_3} = \frac{11.96 - j4.93}{4.32 - j0.83 + 2}$$

$$I_{2\Omega} = (1.96 - j0.52)\,A$$

$$I = 2.03 \angle -14.90\ A.$$

$I_{2\Omega} = 1.6 - j1.130\ A.$

$70.7 \angle 42.36$

$Z_{th} = 4.23 - j1.15$

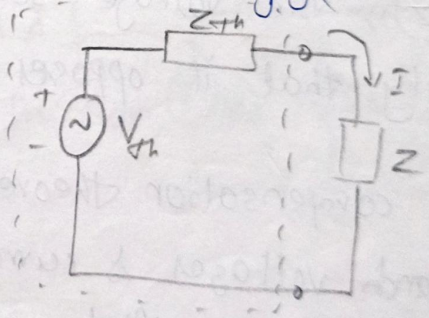
$= 4.38 \angle -15.25$

$P_{max} = 29.17$ watt

Compensation Theorem:

Compensation Theorem states that, in a linear time-invariant network when the impedance 'Z' of a branch, carrying current I is changed by ΔZ, the currents in all other branches would change. The change in current in any branch is equal to the current produced in that branch by a voltage source of magnitude $(I \times \Delta Z)$, placed in series with the altered impedance $(Z + \Delta Z)$. such that it opposes the original current 'I' when all other sources in the n/w are removed ie. voltage sources replaced by short circuit & current sources are replaced by open circuit.

Proof Any linear network can be replaced by a Thevenin's equivalent circuit as shown in fig (a)



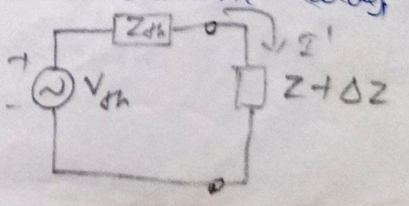
when $I = \dfrac{V_{th}}{Z_{th} + Z}$

When an impedance 'Z' of any branch of the n/w is changed by ΔZ as shown in fig (b).

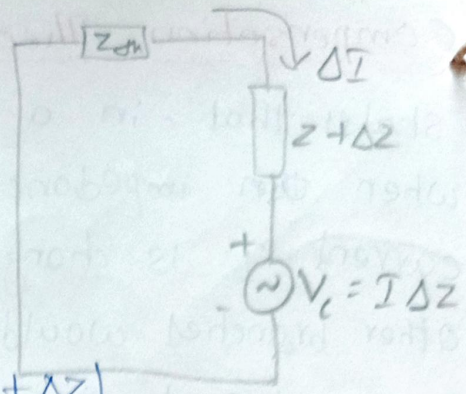
The current through that branch is changed to I'.

Before change in impedance 'Z', the current was

$$I' = \frac{V_{th}}{Z_{th} + Z + \Delta Z}$$



The change in current ΔI is



$$\Delta I = I' - I$$

$$= \frac{V_{th}}{Z_{th}+Z+\Delta Z} - \frac{V_{th}}{Z_{th}+Z}$$

$$= \frac{V_{th}(Z_{th}+Z) - V_{th}(Z_{th}+Z+\Delta Z)}{(Z_{th}+Z+\Delta Z)(Z_{th}+Z)}$$

$$= \frac{-V_{th}\Delta Z}{(Z_{th}+Z+\Delta Z)(Z_{th}+Z)}$$

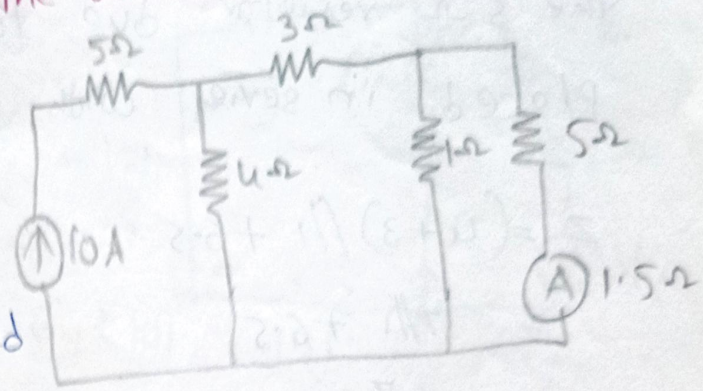
$$\Delta I = \frac{-I\Delta Z}{Z_{th}+Z+\Delta Z} \quad \because \frac{V_{th}}{Z_{th}+Z} = I \text{ from eqn (1)}$$

where $V_c = I\Delta Z$

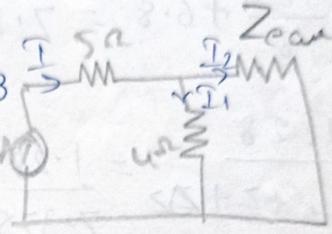
It is to be noted that due to negative sign of term $(I \times \Delta Z)$, the voltage source $(I \times \Delta Z)$ with such a polarity that it opposes the current direction as that of I.

* The compensation theorem is used to find the changes in branch voltages & currents when any of the branches is changed or modified.

Prob Determine the Ammeter reading when connected to 5Ω resistor as shown in fig using compensation theorem, the internal resistance of the ammeter is 1.5Ω.



Sol: The actual current 'I' flowing through 5Ω before connecting ammeter is required to be calculated first.



$Z_{eq} = 3.833\ \Omega$

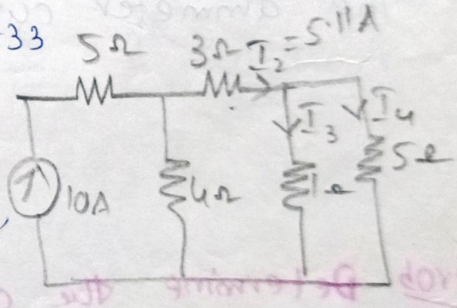
Total Impedance $Z_{eq} = [5 // 1] + 3$

$$Z_{eq} = 3 + \frac{5}{6}$$

$$= 3.833\ \Omega$$

By applying current division principle, the current I_2 is given by, $I_2 = 10 \times \frac{4}{4 + 3.833}$

$$I_2 = 5.11\ A.$$



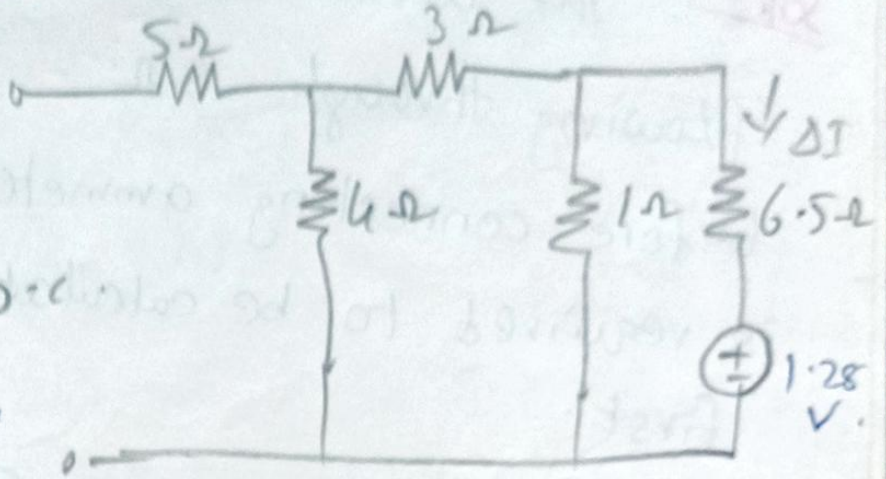
Again by applying current division,

$$I_4 = I_{5\Omega} = 5.11 \times \frac{1}{6}$$

$$I = 0.852\ A.$$

For measurement of this current when ammeter of 1.5Ω is inserted as shown in fig, the circuit condition is disturbed. The internal resistance of ammeter get added into 5Ω & therefore the current through 5Ω is changed from I to I'.

According to compensation theorem, the change in current $\Delta I = I' - I$ is the current flowing through the 5Ω resistor due to a voltage source $(I \times \Delta Z)$. placed in series with 5Ω.



$$Z = (4+3) \,//\, 1 + 6.5$$

$$= 7 \,//\, 1 + 6.5$$

$$= \frac{7}{8} + 6.8 = 7.375\,\Omega.$$

$$-\Delta I = \frac{1.28}{Z_{th} + Z + \Delta Z} = \frac{1.28}{7.375}$$

$$V_c = I\,\Delta Z = 0.852 \times 1.5$$

$$= 1.28\ V.$$

$$-\Delta I = 0.1736$$

$$\Delta I = -0.1736\ A.$$

The ammeter current $I' = I + \Delta I$ $\quad \because \Delta I = I' - I$

$$= 0.852 - 0.1736$$

$$I' = 0.678\ A.$$

Tellegen's Theorem :-

Tellegen's theorem states that in a n/w containing linear/non linear, passive/active, time variant/time invariant circuit elements & independent/controlled sources, the algebraic sum of power delivered to the branches at any given time is zero, provided the kirchoff's laws are satisfied.

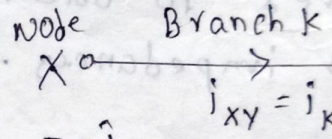
Thus if a n/w has 'b' branches then the tellegen's theorem states that

$$\sum_{k=1}^{b} V_k i_k = 0$$

where V_k is the voltage across k^{th} branch & i_k is the current through k^{th} branch.

Proof :-

Refering to fig. for k^{th} branch of a n/w, Let $i_k = i_{xy} = -i_{yx}$ be the current & $V_k = V_x - V_y = -(V_y - V_x)$ be the voltage across it.



The instantaneous power supplied to the k^{th} branch is given by $V_k \cdot i_k = (V_x - V_y) \cdot i_{xy}$

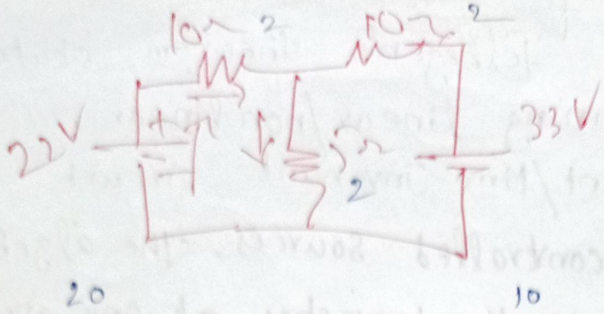
This power can also be written as

or $V_k i_k = -(V_y - V_x)(-i_{yx}) = (V_y - V_x) \cdot i_{yx}$

Summing these two relations, we get.

$$2 V_k i_k = (V_x - V_y) \cdot i_{xy} + (V_y - V_x) \cdot i_{yx}$$

$$V_k \cdot i_k = \frac{1}{2}\left[(V_x - V_y) \cdot i_{xy} + (V_y - V_x) \cdot i_{yx}\right]$$



Reciprocity Theorem: In any linear n/w consisting of linear & bilateral impedances & active sources, the ratio of voltage 'V' introduced in one loop to the current I in other loop is same as the ratio obtained if the positions of V & I are interchanged in the n/w. while calculating the ratio, the sources other than one which is considered to obtain the ratio, must be replaced by their impedances.

The OM Sai Ram

Transient Analysis

The circuit having constant sources is said to be in steady state if the currents & the voltages do not change with time. Thus circuits with currents & voltages having constant amplitude & constant frequency, sinusoidal functions are also considered to be in steady state. That means the amplitude or frequency of a sinusoidal never changes in steady state circuit.

In a network containing energy storage elements with change in excitations, the currents & voltages change from one state to another state. The behaviour of the voltage or current when it is changed from one state to another is called "Transient state".

The time taken for the circuit to change from one steady state to another steady state is called "transient time".

→ Electrical devices are conducted by switches which are closed inorder to supply & are opened to disconnect the supply.

The switching operation will change the current & voltages in the device.

(i) Purely resistive device will allow the instantaneous changes in currents & voltages.

(ii) An inductive device will not allow sudden changes in currents (Delay in the change in the currents)

(iii) capacitive devices will not allow sudden changes in voltages (Delay in the change in the voltages)

Hence when the switching operation is performed in the inductive & capacitive devices, the currents voltages in the devices will take certain time to change from preswitching value ($t=0^-$) to steady value after switching ($t=0^+$).

∴ The study of switching conditions in the circuit is called transient analysis.

→ The state of the circuit from the instant of switching to attain the steady state is called transient state.

→ The transient response is the response of the circuit from the instant of switching to attain the steady state.

In order to study the response w.r.to switching conditions.

(i) The switching instant is taken as a time origin ($t=0$).

ii) The time $t=0^-$ is used to denote the time instant just prior to switching. i.e., just

iii) The time $t=0^+$ is used to denote the time instant immediately after switching.

Apart from switching the transients will also occur due to change in circuit elements.

In electrical engineering the transient analysis is useful tool for analysis of switching conditions in the circuit breakers, relays, various types of loads and also for the analysis of faulty conditions in electrical devices.

→ The transient response of a circuit due to stored energy devices alone (without external source) is called natural response (or) source free response or transient response.

→ The response of a circuit due to an external source is called forced response (or) steady state response

The application of KVL & KCL to circuits containing energy storage elements results in differential equation rather than algebraic equation.

When we consider a differential equation, the complete solution consists of 2 parts.

(i) complimentary function.

(ii) particular function.

R

The solution of the differential equation repres
the response of a circuit ie, complimentary → natural
particular → forced

→ These natural response or transient response eventually dies after some time. :)

→ Forced response that is the response that remains after transients dies out.

Initial conditions:

Element	at $t=0$. Initial state	Steady State (or) Final state
R	R $I_R(0^-) \neq I_R(0^+)$	R $V_R(0^-) \neq V_R(0^+)$
uncharged → $I_0=0$, L	open ckt.	S.C
$V_0=0$, + − C	S.C	O.C
→ I_0, L	I_0	
V_0, + − C	+ − V_0	O.C

In inductor

$i_L(0^-) = i_L(0^+)$

$i_L(0^-) = 0 = I_L(0^+)$

$I_L(0^-) = I_0 = I_L(0^+)$

But $V_L(0^-) \neq V_L(0^+)$

In capacitor

$V_c(0^-) = V_c(0^+)$

$V_c(0^-) = 0 = V_c(0^+)$

$V_c(0^-) = V_0 = V_c(0^+)$

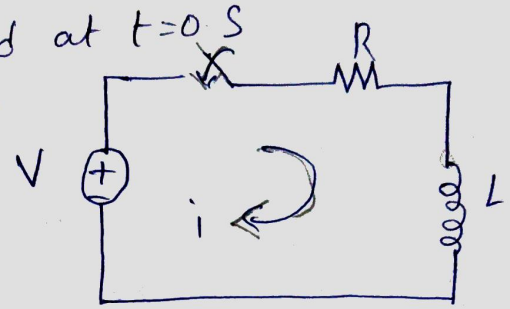
$I_c(0^-) \neq I_c(0^+)$

DC response of RL series ckt by using Differential method:-

Consider a ckt consisting of a resistance & inductance as shown in fig.

The inductor in the ckt is initially uncharged & is in series with the resistor.

when the switch 'S' is closed at $t=0$



Now by applying Mesh analysis,

$$V = iR + L\, di/dt$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L}$$

It is in the form of $\frac{dy}{dt} + Py = Q$ (Non homogeneous eqn)

where $y = i$, $P = \frac{R}{L}$, $Q = \frac{V}{L}$

$$y(I.F) = \int Q(I.F) + C$$

where $I.F = e^{\int P\, dt} = e^{\int \frac{R}{L} dt} = e^{\frac{R}{L}t}$

$$\therefore i\left(e^{R/L\,t}\right) = \int \frac{V}{L} e^{R/L\,t} + C$$

$$i\left(e^{R/L\,t}\right) = \frac{V}{L} \frac{e^{R/L\,t}}{R/L} + C$$

$$i = \frac{\frac{V}{L} \cdot \frac{L}{R} e^{R/L\,t}}{e^{R/L\,t}} + \frac{C}{e^{R/L\,t}}$$

$$\boxed{i = \frac{V}{R} + Ce^{-R/L\,t}} \rightarrow (1)$$

C = arbitary constant.

To find the value of 'C' we go to initial condition

For a uncharged inductor, $i(0^-) = 0 = i(0^+)$

when switch 'S' is closed at $t=0$. At $t=0^-$, ie, just before closing the switch S, the current in the inductor is zero. since the inductor does not allow sudden changes in currents, at $t=0^+$ just after the switch is closed, the current remains zero.

Thus at $t=0$, $i=0$, then eqn (1) becomes

$$0 = \frac{V}{R} + C e^{-R/L(0)}$$

$$\boxed{C = -V/R}$$

(1) $\Rightarrow$

$$i = \frac{V}{R} - \frac{V}{R} e^{-R/L\, t}$$

$$i = \frac{V}{R}\left(1 - e^{-R/L\, t}\right) \rightarrow (2)$$

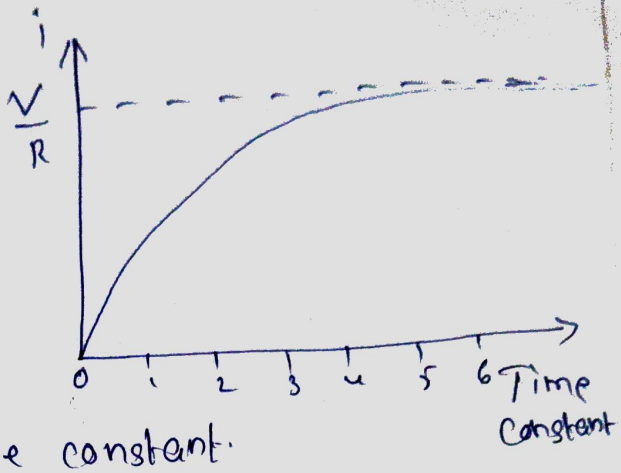
Now eqn (1) consists of 2 parts.

(i) steady state part (V/R)

(ii) Transient part $\left(-\frac{V}{R} e^{-R/L\, t}\right)$

When the switch 'S' is closed, the response reaches the steady state value after a time interval as shown in fig.

Here the transient period is defing as the time taken for the current to reach its final or steady state value from its initial value.



The term "L/R" is a time constant.

$$\tau = L/R .$$

'L/R' is the time required for the current to reach from its initial value of zero to the final value V/R.

The time constant of a function $\frac{V}{R} e^{-(R/L)t}$ is the time at which exponent 'e' is unity.

$$\tau = L/R \text{ sec.}$$

The transient part of the solution is

$$i = -V/R\, e^{-R/L\, t} = -\frac{V}{R} e^{-t/\tau}$$

At one T.C, the transient term reaches 36.8 % of its final value.

$$i(\tau) = -\frac{V}{R} e^{-t/\tau} = -\frac{V}{R} e^{-1} = -0.368\, V/R$$

$$i(2\tau) = -\frac{V}{R} e^{-2} = -0.135\, V/R$$

$$i(3\tau) = -\frac{V}{R} e^{-3} = -0.0498\, V/R$$

$$i(5\tau) = \frac{V}{R} e^{-5} = -0.0067\, V/R .$$

After 5 TC, the transient part reaches more than 99 percent of its final value.

Voltage across the resistor is

$$V_R = iR = R\frac{V}{R}\left[1-\exp\left(\frac{R}{L}t\right)\right]$$

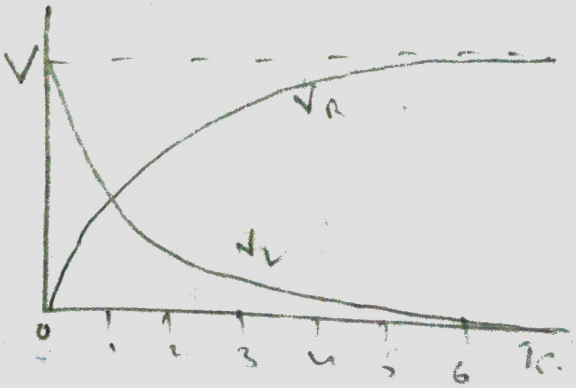
$$V_R = V\left[1-\exp\left(-\frac{R}{L}t\right)\right]$$

Illy voltage across the inductance is

$$V_L = L\,di/dt = L\frac{V}{R}\cdot\frac{R}{L}\,e\left(-\frac{R}{L}t\right)$$

$$V_L = V\exp\left(-\frac{R}{L}t\right)$$

The responses of V_R & V_L are shown in fig.



Power in the resistor is

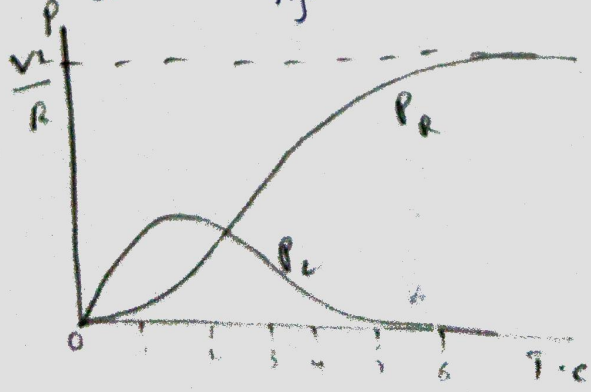
$$P_R = iV_R$$

$$= \frac{V^2}{R}\left[1-2\exp\left(-\frac{R}{L}t\right)+\exp\left(-\frac{2R}{L}t\right)\right]$$

Power in the inductor is

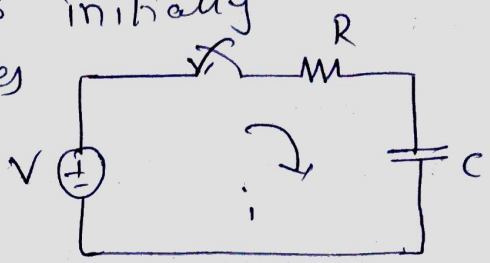
$$P_L = V_L i = \frac{V^2}{R}\left[\exp\left(\frac{R}{L}t\right)-\exp\left(-\frac{2R}{L}t\right)\right]$$

The responses are shown in fig.



DC Response of RC series circuit

consider a ckt consisting of resistance & capacitance as shown in fig.



The capacitor in the ckt is initially uncharged & it is in series with the resistor.

When the switch 'S' is closed at $t=0$. now applying KVL then the eqn becomes

$$V = iR + \frac{1}{C}\int i\,dt$$

Differentiate the above equation

$$0 = R\frac{di}{dt} + \frac{1}{C} i$$

$$di/dt + \frac{1}{RC} i = 0 \quad \text{(Homogeneous eqn)}$$

It is in the form $dy/dt + Py = 0$

The solution is $y(I.F) = C$

$$I.F = e^{\int P\,dt}$$

where $y = i$, $P = \frac{1}{RC}$

$$I.F = e^{\int \frac{1}{RC}\,dt}$$

$$= e^{\frac{1}{RC}t}$$

$$i\, e^{t/RC} = C$$

$$i = \frac{C}{e^{t/RC}} = C e^{-t/RC} \rightarrow (1)$$

where 'c' is the arbitary constant.

To find the value of 'c' we go to initial conditions.

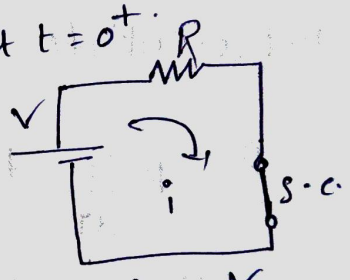
For a uncharged capacitor $v(0^-) = 0 = v(0^+)$ when switch 's' is closed at $t=0$. since the capacitor never allow sudden changes in voltage, it will act as a short circuit at $t=0^+$ so the current in the circuit at $t=0^+$ is V/R.

$\therefore$ at $t=0$, the current $i = V/R$.

① $\Rightarrow$ $\frac{V}{R} = Ce^0$

$\therefore \boxed{C = \frac{V}{R}}$

at $t=0^+$



$i = \frac{V}{R}$

① $\Rightarrow$ $\boxed{i = \frac{V}{R} e^{-t/RC}} \rightarrow ②$

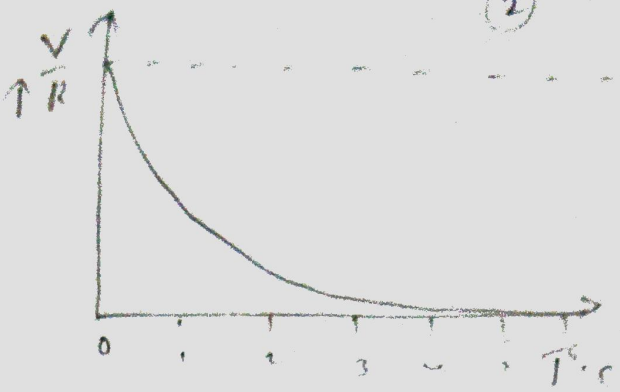
when switch now eqn ② consists only transient part i.e; $\frac{V}{R} e^{-t/RC}$.

Because at steady state, capacitor acts as o.c therefore current in the circuit is zero.

when switch 's' is closed., the response decays with time as shown in fig.

The term RC is the time constant & is denoted by τ

$$\boxed{\tau = RC} \text{ sec.}$$



After 5 T.C the current reaches 99 percent of its final value.

voltage across the resistor is

$$v_R = iR = \frac{V}{R} e^{-t/RC} \cdot R$$

$$\boxed{v_R = Ve^{-t/RC}}$$

Avg voltage across the ~~inductor~~ capacitor is

$$v_c = \frac{1}{C} \int i \, dt$$

$$= \frac{1}{C} \int \frac{V}{R} e^{-t/RC} \, dt$$

$$= -\frac{V}{RC} e^{-t/RC} \cdot RC + c$$

$$v_c = -Ve^{-t/RC} + c$$

at $t = 0$, voltage across the capacitor is zero.

ie., $v_c = 0$.

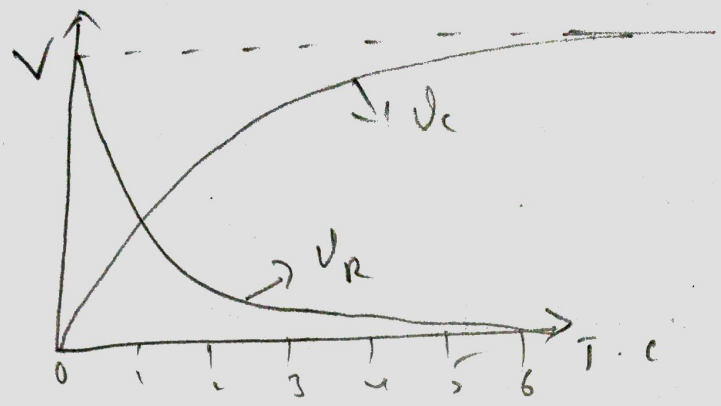
$$0 = -Ve^{-0} + c$$

$$\Rightarrow c = V.$$

$$\therefore v_c = V - Ve^{-t/RC}$$

$$\boxed{v_c = V\left(1 - e^{-t/RC}\right)}$$

The responses as shown in fig.



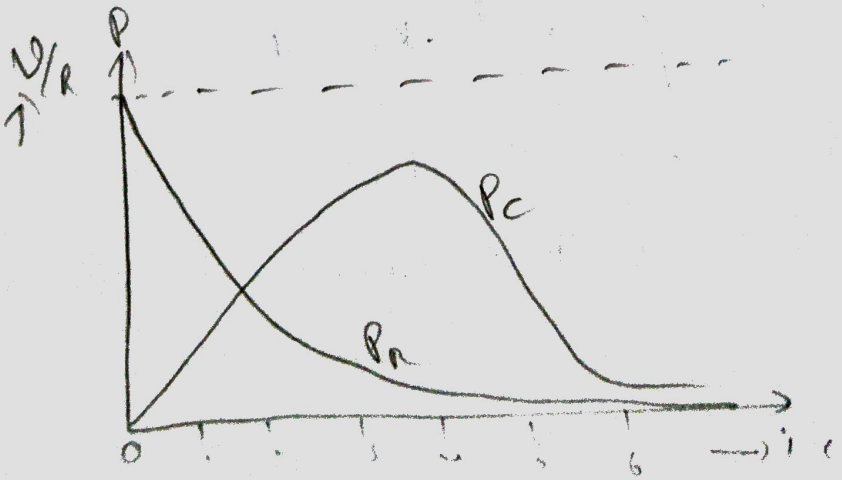
Power in the resistor.

$$P_R = i v_R = \frac{V^2}{R} e^{-2t/RC}$$

Power in the ~~capacitor~~ capacitor.

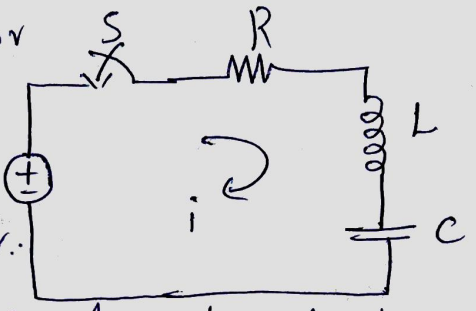
$$P_C = i v_c = \frac{V^2}{R}\left[e^{-t/RC} - e^{-2t/RC}\right]$$

The responses are as shown in fig.



DC Response of R-L-C series circuit using D.M.

Consider a ckt consisting of resistance, inductance & capacitance as shown in fig.



The inductor & capacitor in the ckt are initially uncharged & they are V in series with the resistor.

when the switch 's' is closed at $t=0$

Now applying KVL;

$$V = iR + L\,di/dt + \frac{1}{C}\int i\,dt$$

Differentiate the above equation.

$$0 = R\,di/dt + L\frac{d^2i}{dt^2} + \frac{1}{C}i$$

$$\frac{d^2i}{dt} + \frac{R}{L}di/dt + \frac{1}{LC}i = 0$$

The above equation is second order linear differential equation, with only complementary function. The particular solution for the above equation is zero.

Characteristics equation for the above equation is

$$\left[D^2 + \frac{R}{L}D + \frac{1}{LC}\right]i = 0. \longrightarrow (1)$$

The roots are

$$D_1, D_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

differential equation repre...

Assuming $k_1 = -\frac{R}{2L}$ & $k_2 = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$D_1 = k_1 + k_2$ & $D_2 = k_1 - k_2$

Here k_2 may be +ve, -ve & zero.

Case (i)

k_2 is +ve, when $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

The roots are real & un equal & give the over damped response as shown in fig.

Then the eqn (1) becomes,

$$\left[D - (k_1 + k_2)\right]\left[D - (k_1 - k_2)\right] i = 0$$

The solution for the above equation is

$$i = c_1 e^{(k_1 + k_2)t} + c_2 e^{(k_1 - k_2)t}$$

The current curve for the overdamped case is as shown.

Case (ii) k_2 is -ve, when $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

The roots are complex conjugate, & give the undamped response as shown.

$$\left[D - (k_1 + jk_2)\right]\left[D - (k_1 - jk_2)\right] i = 0$$

The solution for the above equation is

$$i = e^{k_1 t}\left[c_1 \cos k_2 t + c_2 \sin k_2 t\right]$$

The current curve for the undamped case is as shown.

case (iii):- k_2 is zero, when $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

The roots are equal, it gives the critically damped response as shown.

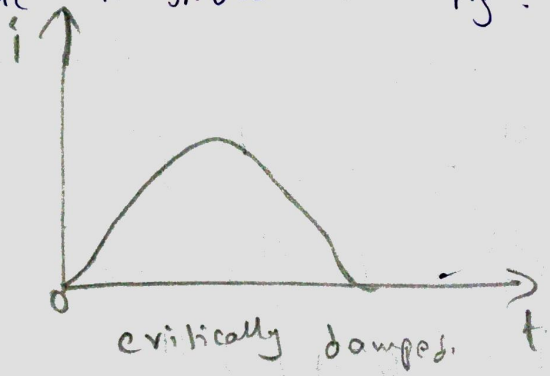
then the eqn (1) becomes,

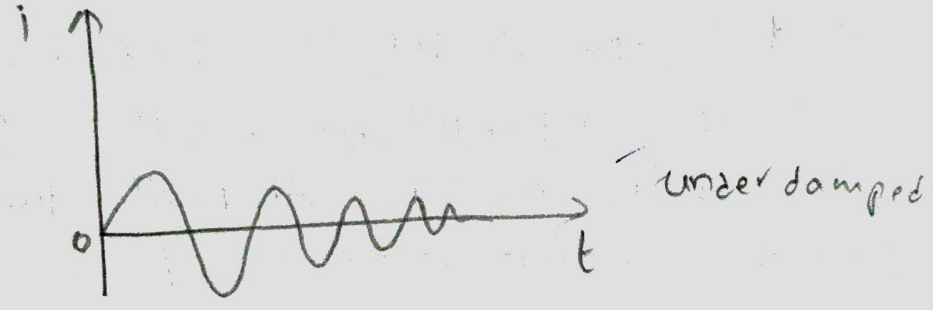
$$\left[(D-k_1)(D-k_1)\right] i = 0$$

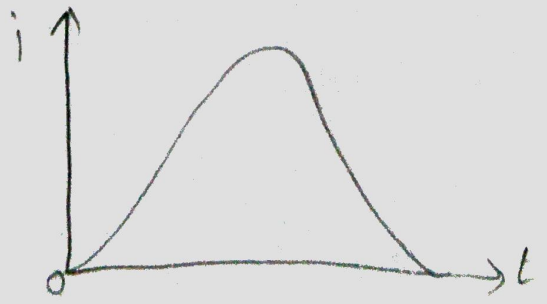
the solution for the above equation is

$$i = e^{k_1 t}(C_1 + C_2 t)$$

The current curve for the critically damped case is shown in fig.

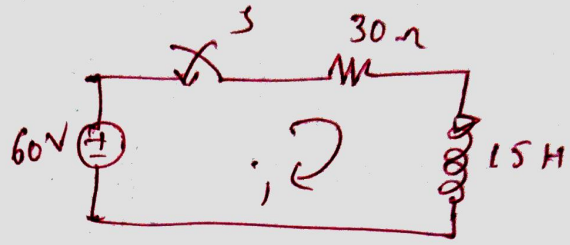






differential equation revel

Prob A series RL circuit with $R = 30\Omega$ & $L = 15H$ has a constant voltage $V = 60V$ applied at $t = 0$ as shown in fig. Determine the current i, the voltage across resistor & inductor.



Sol:- By applying KVL,

$$15\, di/dt + 30i = 60$$

$$di/dt + 2i = 4$$

The general solution for the above equation is

$$i e^{2t} = 4\int e^{2t} + c$$

$$i e^{2t} = 4\frac{e^{2t}}{2} + c$$

$$i = 2 + c e^{-2t}$$

At $t = 0$, the switch 'S' is closed.

since the inductor never allows sudden changes in currents. At $t = 0^-$ the current in the circuit is zero.

$$\therefore \quad t = 0, \quad i = 0$$

$$\therefore \quad 0 = 2 + c$$

$$c = -2$$

$$\therefore \quad i = 2 - 2e^{-2t} \Rightarrow 2(1 - e^{-2t}) \text{ Amps.}$$

voltage across the resistor $V_R = iR$

$= 2(1-e^{-2t})\,30$

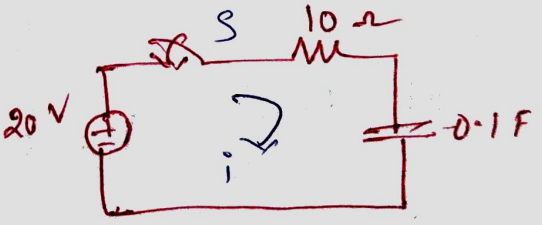
$= 60(1-e^{-2t})$ volts.

Voltage across the inductor, $V_L = L\,di/dt$

$V_L = 15\,\frac{d}{dt}(2-2e^{-2t})$ $\Rightarrow 15[-2(e^{-2t})(-2)]$

$\boxed{V_L = 60\,e^{-2t}}$ volts.

→ A series RC circuit consists of resistor of 10Ω & capacitor of 0.1 F as shown in fig. A constant voltage of 20 V is applied to the circuit at $t=0$. Obtain the current equation. Determine the voltages across the resistor & capacitor.



Sol:- By applying KVL,

$10\,i + \frac{1}{0.1}\int i\,dt = 20$

Differentiate the above equation,

$10\,di/dt + \frac{1}{0.1}\,i = 0$

$\frac{di}{dt} + i = 0.$

The solution for the above equation is

$i\,(I.F) = C$ $\qquad I.F = e^{\int P\,dt}$

$i\,e^{t} = C \Rightarrow i = Ce^{-t}$ $\qquad = e^{t}$

At $t=0$, switch 's' is closed. Since the capacitor does not allow sudden changes in voltage, the current in the circuits is $i = \frac{V}{R} = \frac{20}{10} = 2\,A$

at $t=0$ $\quad i = 2\,A$

$\therefore \boxed{i = 2e^{-t}}$

Voltage across the resistor is $v_R = iR$

$$v_R = 2e^{-t}(10) = 20e^{-t} \text{ volts.}$$

$$v_c = \frac{1}{C}\int i\,dt = \frac{1}{0.1}\int 2e^{-t}\,dt$$

$$= \frac{2}{0.1}\frac{e^{-t}}{-1} + c$$

$$v_c = -20e^{-t} + c$$

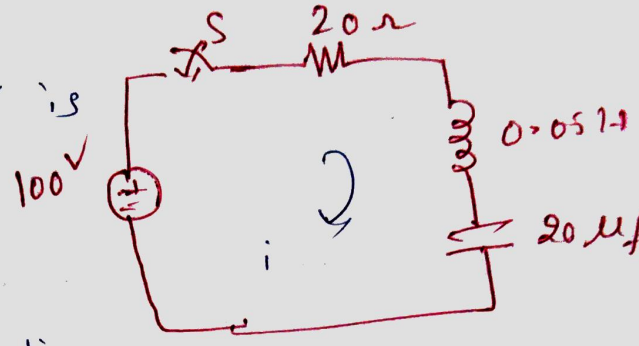
at $t=0$, $v_c = 0$

$$0 = -20e^{0} + c \Rightarrow c = +20.$$

$$v_c = -20e^{-t} + 20$$

$$\boxed{v_c = 20\left[1 - e^{-t}\right] \text{ volts.}}$$

→ The circuit consists of resistance, inductance & capacitance in series with a 100 V constant source when the switch is closed at $t = 0$. Find the current transient.



Sol:- At $t = 0$ switch 'S' is closed. when

Apply KVL

$$100 = 20i + 0.05\, di/dt + \frac{1}{20\mu f}\int i\, dt$$

Differentiate the above eqn

$$0.05\frac{d^2i}{dt^2} + 20\, di/dt + \frac{1}{20\times10^{-6}} i = 0.$$

$$\therefore \frac{d^2i}{dt^2} + 400\, di/dt + 10^6 i = 0$$

$$\therefore \left[D^2 + 400D + 10^6\right] i = 0$$

$$D_1, D_2 = -\frac{400}{2} \pm \sqrt{\left(\frac{400}{2}\right)^2 - 10^6}$$

$$= -200 \pm \sqrt{(200)^2 - 10^6}$$

$$D_1 = -200 + j\,979.8$$

$$D_2 = -200 - j\,979.8.$$

$\therefore$ The current $i = e^{k_1 t}\left[C_1 \cos k_2 t + C_2 \sin k_2 t\right]$

$$i = e^{-200t}\left[C_1 \cos 979.8t + C_2 \sin 979.8t\right]$$

∴ At $t = 0$ the current flowing through the circuit is zero $i = 0$

$$i = 0 = (1)\left(C_1 \cos 0 + C_2 \sin 0\right)$$

$$C_1 = 0$$

$$\boxed{i = e^{-200t} C_2 \sin 979.8\,t \text{ Amps}}$$

Differentiate the above equation.

$$\frac{di}{dt} = C_2\left[e^{-200t}\cos 979.8\,t\,(979.8) + e^{-200t}(-200)\sin 979.8\,t\right]$$

$$\frac{di}{dt} = C_2\left[e^{-200t}(979.8)\cos 979.8\,t - 200\,e^{-200t}\sin 979.8\,t\right]$$

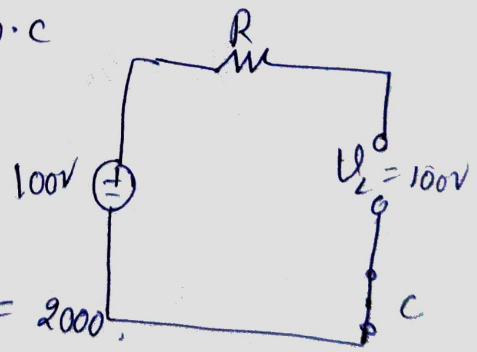
At $t = 0$, the voltage across the inductor is 100 V.

∴ inductor acts as o.c

$$V_L = 100$$

$$L\,di/dt = 100$$

$$di/dt = \frac{100}{L} = \frac{100}{0.05} = 2000$$



∴ At $t = 0$, $di/dt = 2000$

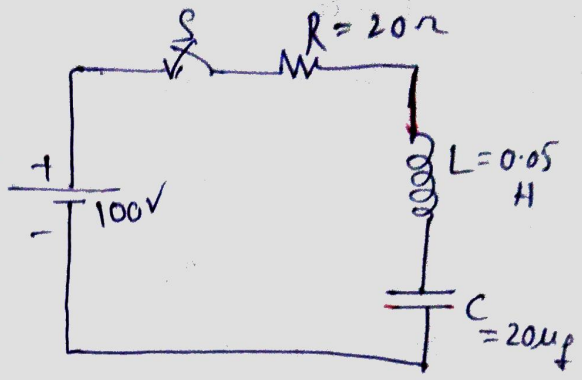
$$2000 = C_2\left[979.8\cos 0 - 200\,e^{0}\sin 0\right]$$

$$2000 = C_2\,979.8 \Rightarrow C_2 = \frac{2000}{979.8} = 2.04$$

The current equation

$$\therefore i = e^{-200t}(2.04\sin 979.8\,t) \text{ Amps.}$$

→ The ckt shown in fig. consists of resistor, inductor, capacitor in series with a 100 v constant source when the switch is closed at t=0. Find the current transient !



Sol:- $V = iR + L\,di/dt + \frac{1}{C}\int i\,dt$

$$100 = 20i + 0.05\frac{di}{dt} + \frac{1}{20\mu}\int i\,dt$$

Diff.

$$20\frac{di}{dt} + 0.05\frac{d^2i}{dt^2} + \frac{1}{20\mu}i = 0$$

$$\frac{d^2i}{dt^2} + \frac{20}{0.05}\frac{di}{dt} + \frac{1}{20\times0.05\mu}i = 0$$

$$\frac{d^2i}{dt^2} + 400\frac{di}{dt} + 10^6 i = 0$$

$$(D^2 + 400D + 10^6)i = 0$$

$$D = \frac{-400 \pm \sqrt{(400)^2 - 4\times10^6}}{2}$$

$$D = -200 \pm 979.8j$$

$D_1 = -200 + j979.8$, $D_2 = -200 - j979.8$

The above roots are complex conjugate then the solution of current is

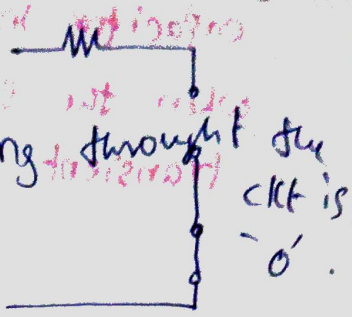
$$i = e^{k_1 t}\left[C_1\cos k_2 t + C_2 \sin k_2 t\right]$$

$k_1 = -200$ $\quad k_2 = 979.8$

$$i = e^{-200t}\left[C_1\cos 979.8\,t + C_2 \sin 979.8\,t\right] \rightarrow (1)$$

Now applying initial conditions.

$i(0^-) = i(0^+)$ at $t = 0^+$



at $t = 0$ $i(0^+) = 0$ i.e, current flowing throught the ckt is '0'.

Then the current equation becomes

$$0 = e^{-0}(C_1 + 0)$$

$$C_1 = 0$$

i.e, at $t=0$, $\therefore i = e^{-200t}(0)\cos 979.8t + C_2 \sin 979.8t$

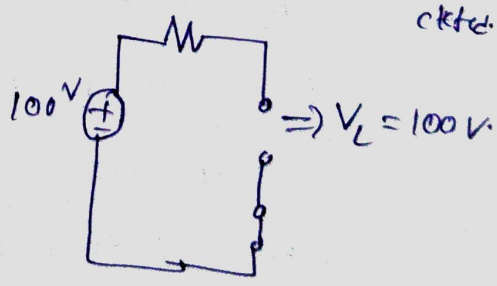
$$i = e^{-200t}[C_2 \sin 979.8t]$$

Diff. $\frac{di}{dt} = e^{-200t}[C_2 \cos 979.8t (979.8)] + C_2 \sin 979.8t \left[e^{-200t}(-200)\right]$

$$\frac{di}{dt} = e^{-200t} C_2 [979.8 \cos 979.8t - 200 \sin 979.8t] \rightarrow (2)$$

Now we should find C_2, so we need to know the value of $\frac{di}{dt}$ at $t=0$.

we know that at initial state L - open & C - short ckted.



ie, $V_L = 100$ V $\Rightarrow V_L = 100$ V.

$$L\frac{di}{dt} = 100$$

$$\frac{di}{dt} = \frac{100}{0.05} = 2000.$$

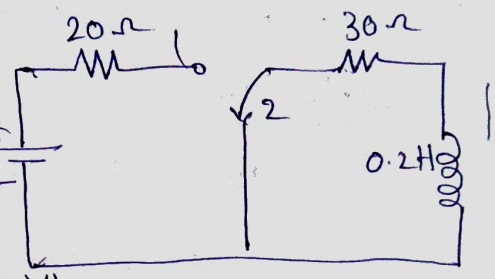
From eqn (1), $2000 = e^{-0}(979.8)C_2$

$$C_2 = \frac{2000}{979.8} = 2.04.$$

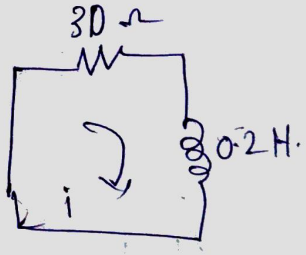
∴ The solution of eqn (1) becomes.

$$i = e^{-200t}\left[2.0\cos 979.8t\right].$$

→ For the ckt shown find the current equation when the switch is changed from position 1 to position 2 at t=0.



Sol:- At t=0, ie, at the instant of switch is open & at the same instant the switch is changed from position 1 to position 2.



Apply KVL,

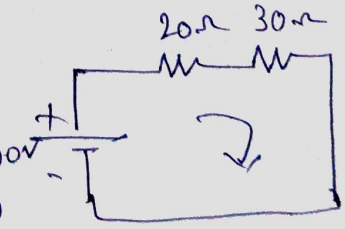
$$0 = 30i + 0.2\frac{di}{dt}$$

$$\frac{di}{dt} + \frac{30}{0.2}i = 0$$

$$\frac{di}{dt} + 150\,i = 0$$

$$i\,e^{150t} = c$$

$$\boxed{i = ce^{-150t}}$$



At $t=0^-$, $i_L(0^-) = i(0^+) = I_0$

Since the switch remains closed at position 1 for a long time. so the circuit might have been attain steady state. At steady state the inductor acts as short ckt.

∴ current flowing through the ckt is I_0.

$$i(0^-) = I_0 = \frac{100}{50} = 2A.$$

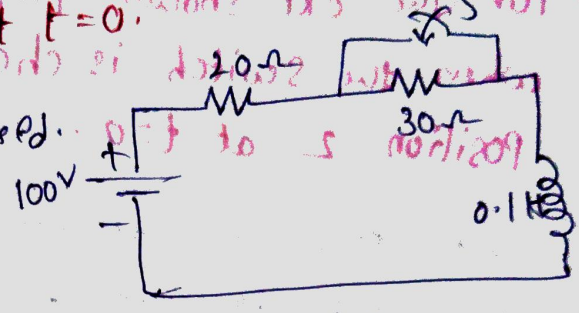
$i(0^-) = i(0^+) = I_0 = 2\ A$.

$2 = c\,e^{0}$

$\boxed{c = 2}$.

$\boxed{i = 2\,e^{-150t}}$ Amps.

→ Find the complete expression for the current when switch is closed at $t = 0$.



Sol:- At $t = 0$, switch is closed.

Apply KVL,

$$100 = 20i + 0.1\frac{di}{dt}$$

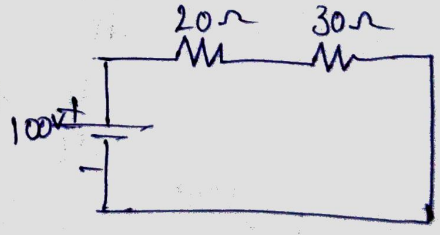
$$\frac{di}{dt} + 200i = 1000$$

$$i\,(e^{200t}) = 1000 \int 5\,e^{200t} + c$$

$$\boxed{i = 5 + c\,e^{-200t}}$$.

At $t = 0^+$ $\quad i(0^-) = i(0^+)$

At $t = 0$, $\quad i(0^+) = I_0 = 2A$.



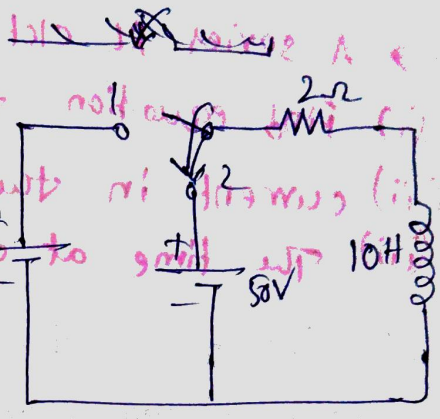
$$i(0^-) = I_0 = \frac{100}{50} = 2A.$$

$2 = 5 + c\,e^{0}$

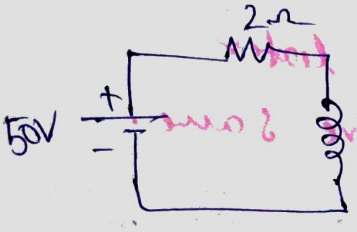
$c = -3$.

$\therefore \boxed{i = 5 - 3\,e^{-200t}}$ Amps.

→ In the R-L ckt as shown in fig the switch is closed at position 1 for long time & then moved to position 2 at time $t=0$. Determine the response of current & voltage across R & L ?



Sol: when it is in position 2 the ckt becomes



$$50 = 2i + 10\frac{di}{dt}$$

$$\frac{di}{dt} + 0.2i = 5$$

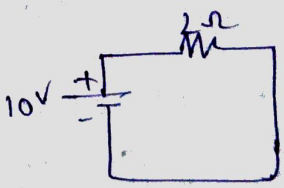
$$i e^{0.2t} = \frac{5}{0.2} e^{0.2t} + c$$

$$\boxed{i = 25 + c e^{-0.2t}}$$

Initial condition when switch is at position 1

$$i(0^+) = i(0^-)$$

$$i(0^-) = \frac{10}{2} = 5A.$$



at $t=0$, $i = 5A$.

$$5 = 25 + c e^{-0}$$

$$\boxed{c = -20}$$

$$\boxed{i = 25 - 20e^{-0.2t}}$$

$$V_R = iR = 2\left[25 - 20e^{-0.2t}\right]$$

$$= 50 - 40e^{-0.2t} \text{ volt.}$$

$$V_L = L\frac{di}{dt} = 10\frac{d}{dt}\left[25 - 20e^{-0.2t}\right]$$

$$= 10\left(-20\,e^{-0.2t}(-0.2)\right)$$

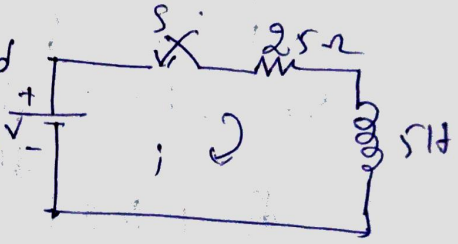
$$\boxed{V_L = 40\,e^{-0.2t}} \text{ Volts}$$

→ A series RL ckt shown in fig.

(i) Find equation for charging current & also V_R & V_L

(ii) current in the circuit 0.5 sec later.

(iii) The time at which V_R & V_L are same.



Sol:- At $t=0$, the switch 's' is closed

Applying KVL, $100 = 25i + 5\frac{di}{dt}$

$$\frac{di}{dt} + 5i = 20.$$

$$ie^{5t} = 4e^{5t} + c$$

$$\boxed{i = 4 + ce^{-5t}}$$

(i) At $t=0$, $i=0$, $c=-4$

$$i = 4 - 4e^{-5t}$$

$$i = 4(1 - e^{-5t}).$$

(ii) at $t = 0.5$ sec,

$$i = 4\left[1 - e^{-5(0.5)}\right]$$

$$i = 3.672 \text{ Amps.}$$

(iii) $V_R = V_L$

$V_R = 50$ V, $V_L = 50$ V.

$$V_L = L\,di/dt$$

$$V_R = iR$$

$$= 4(1 - e^{-5t})25$$

$$\boxed{V_R = 100\,(1 - e^{-5t})}$$

$$V_L = 5\frac{d}{dt}(4 - 4e^{-5t})$$

$$= 5\,(20)\,e^{-5t}$$

$$\boxed{V_L = 100\,e^{-5t}}$$

$\therefore 50 = 5\,20\,(e^{-5t})$

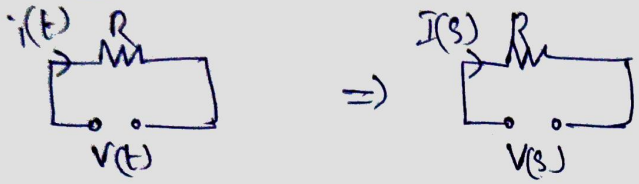
$$= 100\,e^{-5t}$$

$$e^{-5t} = 0.5$$

$$-5t = -0.693$$

$$\boxed{t = 0.13 \text{ sec}}.$$

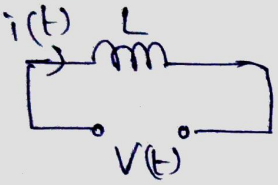
Resistance



$$V(s) = R\, I(s)$$

$$R = \frac{V(s)}{I(s)}$$

Inductance

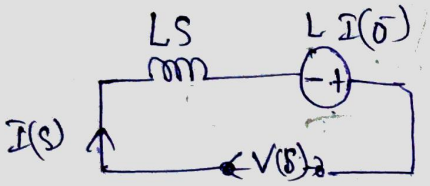


$$V(t) = L\frac{di(t)}{dt}$$

A.L.T

$$V(s) = L[sI(s) - I(0^-)]$$

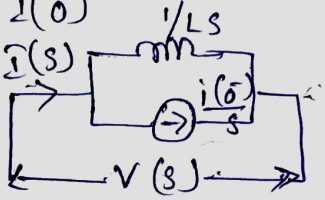
$$V(s) = LsI(s) - LI(0^-)$$



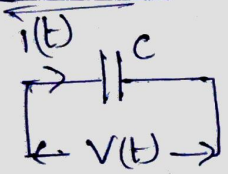
(or)

$$i(t) = \frac{1}{L}\int_0^t V(t) + i(0^-)$$

$$I(s) = \frac{1}{Ls}V(s) + \frac{i(0^-)}{s}$$



capacitance:-



$$i(t) = C\frac{dV(t)}{dt}$$

A.L.T

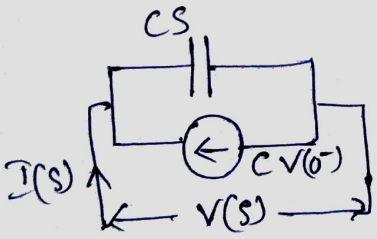
$$I(s) = C[sV(s) - V(0^-)]$$

$$I(s) = CsV(s) - CV(0^-)$$

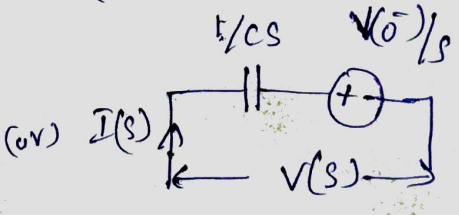
(or)

$$V(t) = \frac{1}{C}\int i\,dt + V(0^-)$$

$$V(s) = \frac{1}{Cs}I(s) + \frac{V(0^-)}{s}$$



(or)



$$I(s) + CV(0^-) = CsV(s)$$

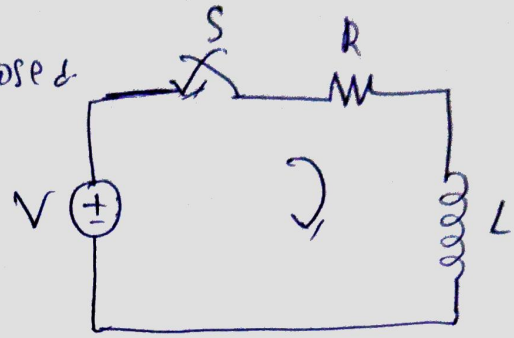
$$V(s) = \frac{1}{Cs}I(s) + \frac{CV(0^-)}{Cs}$$

$$V(s) = \frac{1}{Cs}I(s) + \frac{V(0^-)}{s}$$

$$T_1(s) = \frac{4(s+0.3)}{s^2 + \frac{9}{25}s + \frac{2}{125}}$$

DC Response of RL series ckt, by using L.T.

At $t = 0$ switch 'S' is closed



Apply KVL;

$$V = iR + L\, di/dt$$

Apply Laplace transform,

$$\frac{V}{s} = I(s)R + LsI(s)$$

$$= I(s)(R + Ls)$$

$$I(s) = \frac{\frac{V}{s}}{R + Ls} = \frac{V}{sL\left(s + \frac{R}{L}\right)} = \frac{V/L}{s(s + R/L)}$$

$$I(s) = \frac{V/L}{s(s+R/L)} = \frac{A}{s} + \frac{B}{s+R/L}$$

$$\frac{V/L}{s(s+R/L)} = \frac{A(s+R/L) + Bs}{s(s+R/L)}$$

$$\frac{V}{L} = A(s + R/L) + Bs$$

If $s = 0$, $\frac{V}{L} = A\,R/L \Rightarrow \boxed{A = V/R}$

If $s = -R/L$, $\frac{V}{L} = -B\,R/L \Rightarrow \boxed{B = -V/R}$

$$I(s) = \frac{V/R}{s} + \frac{-V/R}{s + R/L}$$

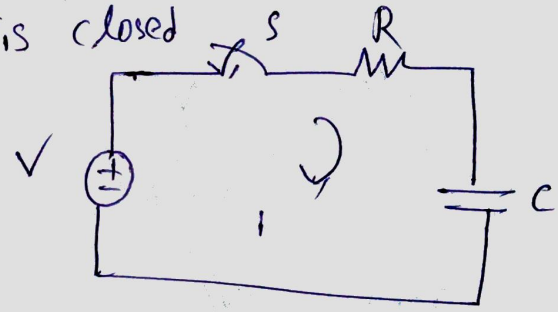
By apply Inverse Laplace transform,

$$i(t) = V/R - V/R\, e^{-R/L\, t}$$

$$\boxed{i(t) = \frac{V}{R}\left[1 - e^{-R/L\, t}\right]}$$

Dc Response of RC series ckt by using L-T.

At $t=0$, switch 's' is closed



Apply KVL,

$$V = iR + \frac{1}{C}\int i\, dt$$

Apply Laplace transform,

$$\frac{V}{s} = I(s)R + \frac{1}{Cs} I(s)$$

$$\frac{V}{s} = I(s)\left(R + \frac{1}{Cs}\right)$$

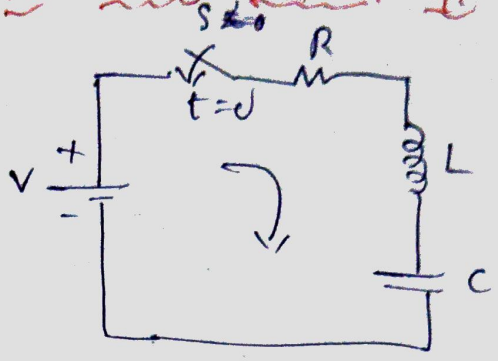
$$I(s) = \frac{V}{s\left(R + \frac{1}{Cs}\right)} = \frac{VCs}{s(RCs+1)}$$

$$= \frac{CV}{RCs+1} = \frac{V/R}{s + 1/RC}$$

By applying I-L-T,

$$\boxed{i(t) = V/R\, e^{-1/RC\, t}}$$

Dc response of R-L-C series ckt using Laplace T/F.



At $t=0$, switch is closed.

Apply KVL,

$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

Apply Laplace transform,

$$\frac{V}{s} = I(s)R + LsI(s) + \frac{1}{Cs}I(s)$$

$$\frac{V}{s} = I(s)\left[R + Ls + \frac{1}{Cs}\right]$$

$$I(s) = \frac{V}{s\left[R + Ls + \frac{1}{Cs}\right]}$$

$$= \frac{V}{s\left(\frac{RCs + LCs^2 + 1}{Cs}\right)}$$

$$= \frac{CV}{LC\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

$$I(s) = \frac{V/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \longrightarrow (1)$$

The roots of the denominator polynomial are

$$s_1, s_2 = \frac{-R/L \pm \sqrt{(R/L)^2 - 4\left(\frac{1}{LC}\right)}}{2}$$

$$s_1, s_2 = -R/2L \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \longrightarrow (2)$$

The roots of the denominator polynomial may be complex or real depending upon the values of R, L, C.

The three cases are

case(i) :- $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$ → real & unequal

case(ii) :- $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$ → complex conjugate

case(iii) :- $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$ → real & equal

Here the response is critically damped.

The critical damped is achieved by choosing the value of R which makes the term $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$ for a given values of L & C.

Let R_c be the value of 'R' for critical damped.

∴ At critical damping.

$$\left(\frac{R_c}{2L}\right)^2 = \frac{1}{LC}$$

$$\frac{R_c}{2L} = \sqrt{\frac{1}{LC}}$$

$$R_c = 2\sqrt{\frac{L}{C}}.$$

The ratio of resistance of circuit & the resistance for critical damping is called damping ratio.

Denoted by δ.

$$\delta = \frac{R}{R_c} = \frac{R}{2\sqrt{\frac{L}{C}}} = \frac{R}{2}\sqrt{\frac{L}{C}}.$$

In the R-L-C circuit, if the resistance is zero then the equation (2) we can say that the roots are purely imaginary $\left(\pm j\sqrt{\frac{1}{LC}}\right)$ & the current will be "completely oscillatory".

$$S_1, S_2 = \pm\sqrt{\frac{-1}{LC}}$$

$$= \pm j\sqrt{\frac{1}{LC}}$$

$$= \pm j\omega_n.$$

where $\omega_n = \sqrt{\frac{1}{LC}}$ is natural frequency of oscillation.

on substituting $R=0$ in eqn (1)

$$I(s) = \frac{V/L}{s^2 + 1/LC}$$

$$I(s) = \frac{V/L}{s^2 + \omega_n^2}$$

$$I(s) = \frac{V}{L\omega_n}\left[\frac{\omega_n}{s^2 + \omega_n^2}\right]$$

Applying inverse Laplace transform.

$$i(t) = \frac{V}{L\omega_n}\sin\omega_n t$$

$$\boxed{i(t) = \frac{V}{\omega_n L}\sin\omega_n t}$$ → completely oscillatory.

The coefficients of the denominator polynomial of eqn (1) can be expressed in terms of δ & ω_n.

$$I(s) = \frac{V/L}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

The roots of denominator polynomial are

$$s_1, s_2 = \frac{-2\delta\omega_n \pm \sqrt{(2\delta\omega_n)^2 - 4\omega_n^2}}{2}$$

$$= \frac{-2\delta\omega_n \pm \sqrt{4\delta^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= -\delta\omega_n \pm \sqrt{(\delta^2 - 1)\omega_n^2}$$

$$s_1, s_2 = -\delta\omega_n \pm \sqrt{\delta^2 - 1}\ \omega_n$$

depending upon the value of 'δ' the roots may be real or complex.

Case (i) If δ lies b/n 0 & 1 then

$0 < \delta < 1$ ie, $\delta < 1$ & $\delta^2 < 1$

then $\sqrt{\delta^2 - 1}$ is imaginary.

The roots are complex conjugate. In this case the current i(t) will be damped oscillatory. Response is underdamped.

Now,

$$I(s) = \frac{V/L}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

The denominator polynomial can be rearranged in the form of $I(s) = \frac{V/L}{(s + \delta\omega_n)^2 - \delta^2\omega_n^2 + \omega_n^2}$

$$I(s) = \frac{V/L}{(s+\delta\omega_n)^2 + \omega_n^2(1-\delta^2)}$$

Let $\omega_n^2(1-\delta^2) = \omega_d^2$

$$\boxed{\omega_d = \omega_n\sqrt{(1-\delta^2)}}$$ where ω_d is the damped frequency of oscillation.

$$I(s) = \frac{V/L}{(s+\delta\omega_n)^2 + \omega_d^2}$$

Multiply & divide by ω_d.

$$I(s) = \frac{V}{\omega_d L} \frac{\omega_d}{(s+\delta\omega_n)^2 + \omega_d^2}$$

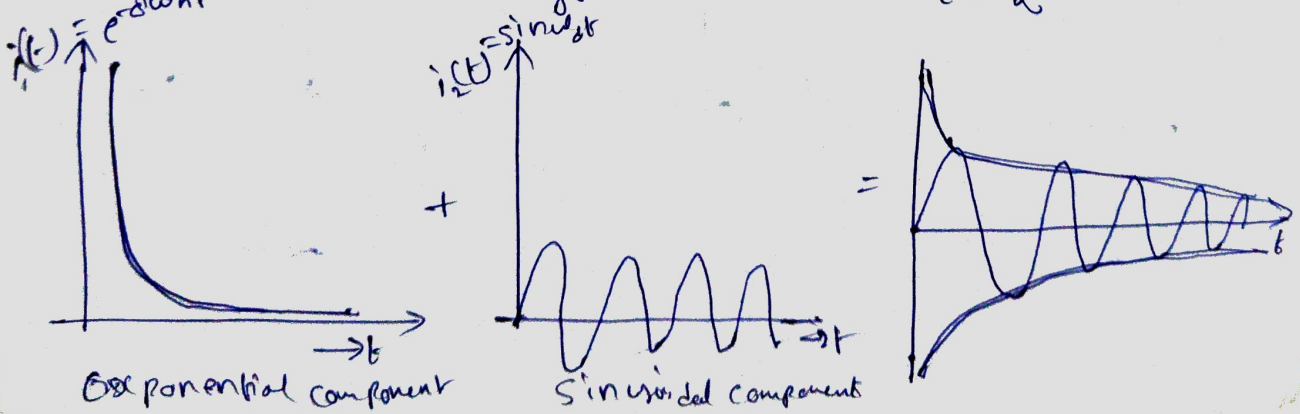
Apply inverse Laplace transform

$$i(t) = \frac{V}{\omega_d L}\left[e^{-\delta\omega_n t} \sin\omega_d t\right].$$

The equation has a sinusoidal component of frequency ω_d.

The sinusoidal oscillations are damped by the exponential term $e^{-\delta\omega_n t}$.

Hence the current $i(t)$ will have damped oscillations whose amplitude decays to zero at $t = \infty$



Case(ii) if $\delta = 1$ then $\sqrt{\delta^2 - 1} = 0$ then the roots are real & equal.

In this case the oscillations of the current are eliminated. Here it is critically damped response.

$$I(s) = \frac{V/L}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

if $\delta = 1$,

$$I(s) = \frac{V/L}{s^2 + 2\omega_n s + \omega_n^2}$$

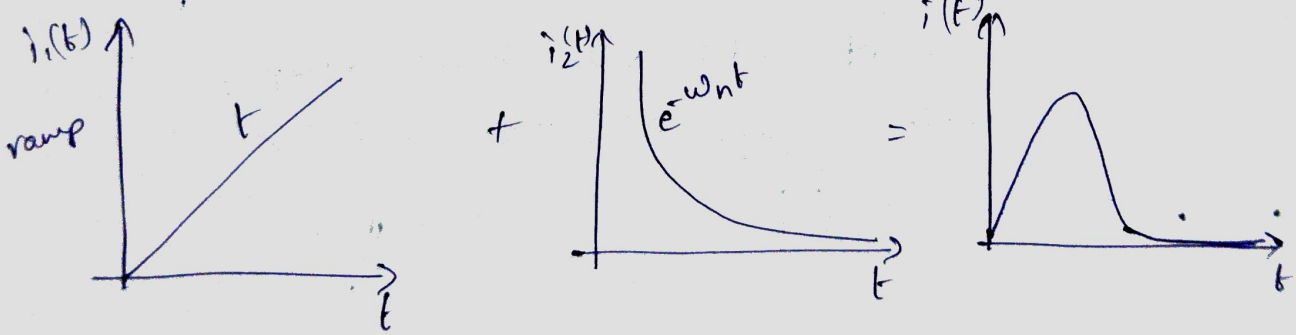
$$I(s) = \frac{V/L}{(s+\omega_n)^2}$$

Apply inverse L-T,

$$\boxed{i(t) = \frac{V}{L} t\, e^{-\omega_n t}}$$

The equation has a ramp component & decaying exponential component.

Initially the current raises due to ramp component & gradually decays to zero.



Case(iii) If $\delta > 1$, $\delta^2 > 1$ then $\sqrt{\delta^2 - 1}$ is real so the roots are real & unequal.

In this case the response of current is overdamped response.

$$I(s) = \frac{V/L}{\omega_n^2 + s^2 + 2s\omega_n\delta}$$

The roots of the denominator polynomial are

$$s_1, s_2 = -\delta\omega_n \pm \omega_n\sqrt{\delta^2 - 1}$$

$$s_1 = -\delta\omega_n + \omega_n\sqrt{\delta^2 - 1}$$

$$s_2 = -\delta\omega_n - \omega_n\sqrt{\delta^2 - 1}$$

$$I(s) = \frac{V/L}{(s - s_1)(s - s_2)}$$

$$I(s) = \frac{V/L}{(s - s_1)(s - s_2)} = \frac{K_1}{s - s_1} + \frac{K_2}{s - s_2}$$

$$I(s) = \frac{V}{2L\omega_n\sqrt{\delta^2 - 1}} \frac{1}{s - s_1} - \frac{V}{2L\omega_n\sqrt{\delta^2 - 1}} \frac{1}{s - s_2}$$

Apply I-L-T

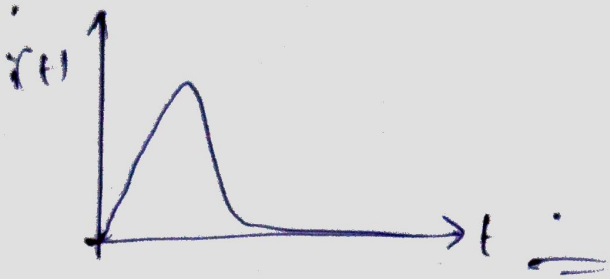
$$i(t) = \frac{V}{2L\omega_n\sqrt{\delta^2 - 1}} \left[e^{s_1 t} - e^{s_2 t}\right]$$

$$= \frac{V}{2L\omega_n\sqrt{\delta^2 - 1}} \left[e^{(-\delta\omega_n + \omega_n\sqrt{\delta^2 - 1})t} - e^{(-\delta\omega_n - \omega_n\sqrt{\delta^2 - 1})t}\right]$$

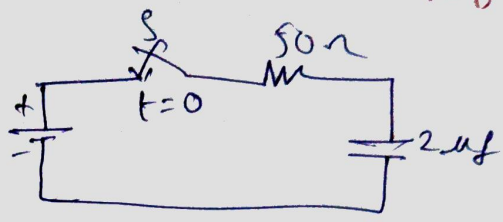
$$= \frac{V}{2L\omega_n\sqrt{\delta^2 - 1}} e^{-\delta\omega_n t}\left[e^{\omega_n\sqrt{\delta^2 - 1}\, t} - e^{-\omega_n\sqrt{\delta^2 - 1}\, t}\right].$$

The equation is response of overdamped RLC ckt.

overdamped response of current decays at a faster rate than the underdamped & critically damped responses.



In the ckt shown in fig below switch 'k' is closed at $t=0$. Find the values of i, di/dt & d^2i/dt^2 at $t=0^+$.



Sol:- Apply KVL,

$$10 = 50i + \frac{1}{2\mu}\int i\,dt$$

$$\frac{1}{2\mu} i + 50\frac{di}{dt} = 0$$

$$\frac{di}{dt} + \frac{1}{2\times10^{-6}\times50} i = 0$$

$$\frac{di}{dt} + \frac{1}{10^{-4}} i = 0 \rightarrow (1)$$

$$\frac{di}{dt} + 10^4 i = 0$$

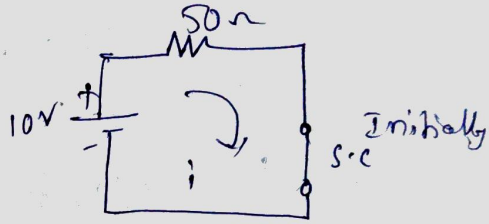
$$\frac{di}{dt} + 10^4(0.2) = 0$$

$$\frac{di}{dt} + 10^4 \times \frac{2}{10} = 0$$

$$\frac{di}{dt} + 2\times10^3 = 0$$

$$di/dt = -2000 \text{ A/sec.}$$

at $t=0^+$



$$i(0^+) = \frac{10}{50}$$

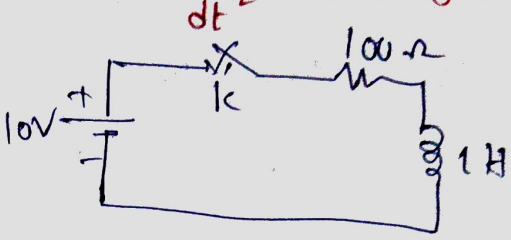
$$= 0.2 \text{ A.}$$

$(1) \Rightarrow$ $\frac{d^2i}{dt^2} + \frac{1}{10^{-4}}\frac{di}{dt} = 0.$

$$\frac{d^2i}{dt^2} + 10^4(-2000) = 0$$

$$\frac{d^2i}{dt^2} = 20\times10^6 \text{ A/sec}^2$$

→ In the ckt shown in fig below, switch 'k' is closed at $t=0$. Find the values of i, di/dt, $\frac{d^2i}{dt^2}$ at $t=0^+$.



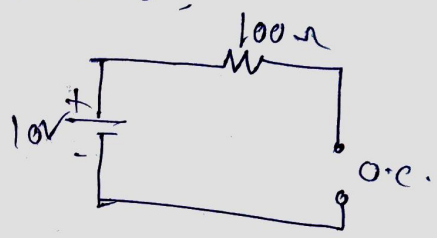
Sol:- Apply KVL,

$$10 = 100i + \frac{di}{dt}$$

$$\frac{di}{dt} + 100i = 10 \quad \Rightarrow (1)$$

$$\boxed{\frac{di}{dt} = 10 \text{ A/sec}}$$

at $t=0^+$,



$i(0^+) = 0$.

(1) $$\frac{d^2i}{dt^2} + 100\frac{di}{dt} = 10$$

$$\frac{d^2i}{dt^2} + 100(10) = 10$$

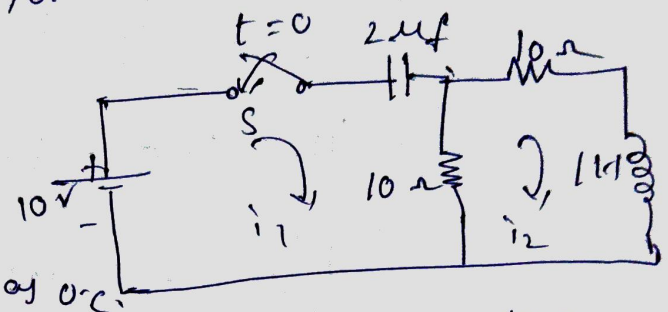
$$\frac{d^2i}{dt^2} = 10 - 1000 = -990 \text{ A/sec}^2.$$

$$V_L = L\frac{di}{dt}$$

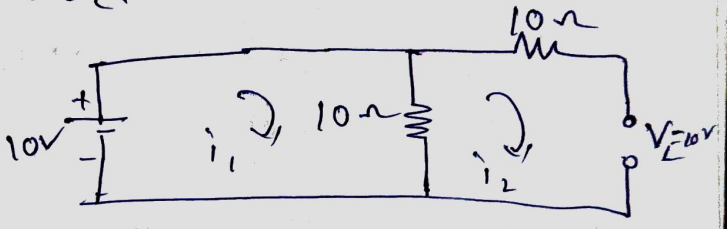
$$10 = 1\frac{di}{dt}$$

$$\boxed{\frac{di}{dt} = 10 \text{ A/sec}}$$

In the ckt shown in the below fig. the switch is closed at $t=0$. Find the values of i_1 & i_2 / di_1/dt $\frac{di_2}{dt}$ * $\frac{d^2 i_1}{dt^2}$ & $d^2 i_2/dt^2$ at $t=0^+$.



Sol:- at $t=0^+$ initially capacitor acts as S.C & inductor acts as O.C.



$i_1(0^+) = \frac{10}{10} = 1$ A.

$i_2(0^+) = 0$

$V_L = L\frac{di_2}{dt}$

$10 = \frac{di_2}{dt} \Rightarrow \boxed{\frac{di_2}{dt} = 10 \text{ A/sec.}}$

Apply KVL for loop ①, for loop ②

$10 = \frac{1}{2\mu}\int i_1 dt + 10(i_1 - i_2) \rightarrow ①$

$10 i_2 + (1)\frac{di_2}{dt} + 10(i_2 - i_1) \rightarrow ② = 0.$

at $t=0^+$ $10\left(\frac{di_1}{dt} - \frac{di_2}{dt}\right) + \frac{1}{2\mu} i_1 = 0 \rightarrow ①$

$10\frac{di_1}{dt} - 10(10) + \frac{1}{2\mu}(1) = 0$

$10\frac{di_1}{dt} = 100 - \frac{1}{2\mu}$

$= 100 - 0.5\times10^6 \Rightarrow \frac{di_1}{dt} = 10 - 5\times10^4$

$= 10 - 50000 \Rightarrow -49990$ A/sec

$\boxed{\frac{di_1(0^+)}{dt} = -49990 \text{ A/sec}^2}$

$\frac{5}{10\times10^6} = 5\times10^5$

(2) $\Rightarrow$ $20i_2 - 10i_1 + \frac{di_2}{dt} = 0$

Differentiate the above equation.

$$20\frac{di_2}{dt} - 10\frac{di_1}{dt} + \frac{d^2i_2}{dt^2} = 0$$

$$20(10) - 10(-49990) + \frac{d^2i_2}{dt^2} = 0$$

$$\frac{d^2i_2}{dt^2} = -499900 - 200$$

$$\boxed{\frac{d^2i_2}{dt^2} = -499700 \text{ A/sec}^2}$$

(1) $\Rightarrow$ $10\frac{di_1}{dt} - 10\frac{di_2}{dt} + \frac{1}{2\mu}\frac{di_1}{dt} = 0$

Differentiate the above equation

$$10\frac{d^2i_1}{dt^2} - 10\frac{d^2i_2}{dt^2} + \frac{1}{2\mu}di_1/dt = 0$$

$$10\frac{d^2i_1}{dt^2} - 10(-499700) + \frac{1}{2\mu}(-49990) = 0$$

$$10\frac{d^2i_1}{dt^2} = \frac{49990}{2\mu} - 4997000$$

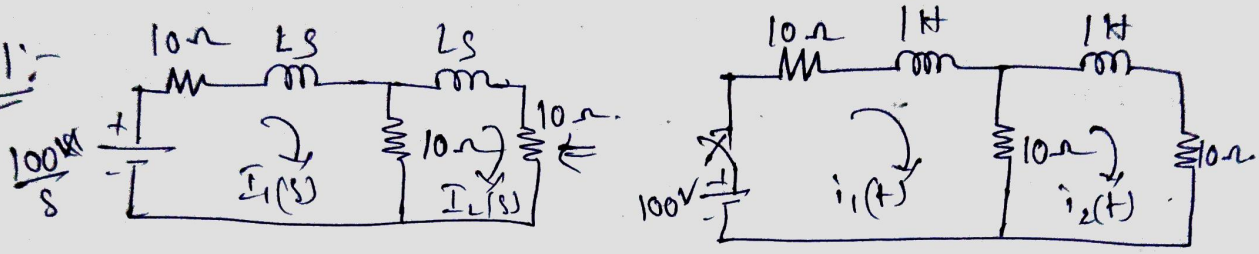
$$\frac{d^2i_1}{dt^2} = \frac{4999}{2\mu} - 499700$$

$$\frac{d^2i_1}{dt^2} = 24995 \times 10^5 - 499700$$

$$\frac{d^2i_1}{dt^2} =$$

2 In the N/w shown in the fig below, the switch 'S' is closed at t=0. Find the values of $i_1(t)$ & $i_2(t)$ assuming zero initial conditions currents through inductors.

Sol:-



Applying KVL,

$$10\,I_1(S) + S\,I_1(S) + 10\left[I_1(S) - I_2(S)\right] = \frac{100}{S}$$

$$\left[20+S\right] I_1(S) - 10\,I_2(S) = \frac{100}{S} \longrightarrow (1)$$

$$S\,I_2(S) + 10\,I_2(S) + 10\left[I_2(S) - I_1(S)\right] = 0$$

$$\left[20+S\right] I_2(S) - 10\,I_1(S) = 0$$

$$I_2(S) = \frac{10}{20+S}\,I_1(S) \longrightarrow (2)$$

(2) in (1)

$$\left[20+S\right] I_1(S) - 10\left(\frac{10}{20+S}\right) I_1(S) = \frac{100}{S}$$

$$(20+S)^2\,I_1(S) - 100\,I_1(S) = \frac{100(20+S)}{S}$$

$$I_1(S)\left(S^2+40S+300\right) = \frac{100(S+20)}{S}$$

$$I_1(S) = \frac{100(S+20)}{S\left(S^2+40S+300\right)}$$

$$I_1(S) = \frac{100(S+20)}{S(S+10)(S+30)}$$

$$I_1(s) = \frac{100(s+20)}{s(s+10)(s+30)} = \frac{A}{s} + \frac{B}{s+10} + \frac{C}{s+30}$$

$$\Rightarrow 100(s+20) = A(s+10)(s+30) + B\,s(s+30) + C\,s(s+10)$$

Put $s=0$, $2000 = 300A$

$$\boxed{A = 6.66}$$

Put $s=-10$, $1000 = -B(200)$

$$\boxed{B = -5}$$

Put $s=-30$, $-1000 = C\,600$

$$\boxed{C = -1.66}$$

$\frac{10}{2} = \frac{5}{3}$

$1.6.6$

$$I_1(s) = \frac{6.66}{s} + - \frac{5}{s+10} - \frac{1.66}{s+30}$$

Apply I.L.T

$$\boxed{i_1(t) = 6.66 - 5e^{-10t} - 1.66e^{-30t}}$$

(2) $\Rightarrow$ $I_2(s) = \frac{10}{s+20} I_1(s)$

$$I_2(s) = \frac{10}{s+20} \cdot \frac{100(s+20)}{s(s+10)(s+30)}$$

$$I_2(s) = \frac{1000}{s(s+10)(s+30)}$$

Apply Partial fractions.

$$I_2(s) = \frac{1000}{s(s+10)(s+30)} = \frac{A}{s} + \frac{B}{s+10} + \frac{C}{s+30}$$

$I_2(s) \Rightarrow \quad 1000 = A(s+10)(s+30) + B\,s(s+30) + C\,s(s+10)$

$s = 0, \quad 1000 = 300A$

$$A = 3.33$$

$s = -10 \quad 1000 = -B\,200$

$$B = -5$$

$s = -30 \quad 1000 = C(600)$

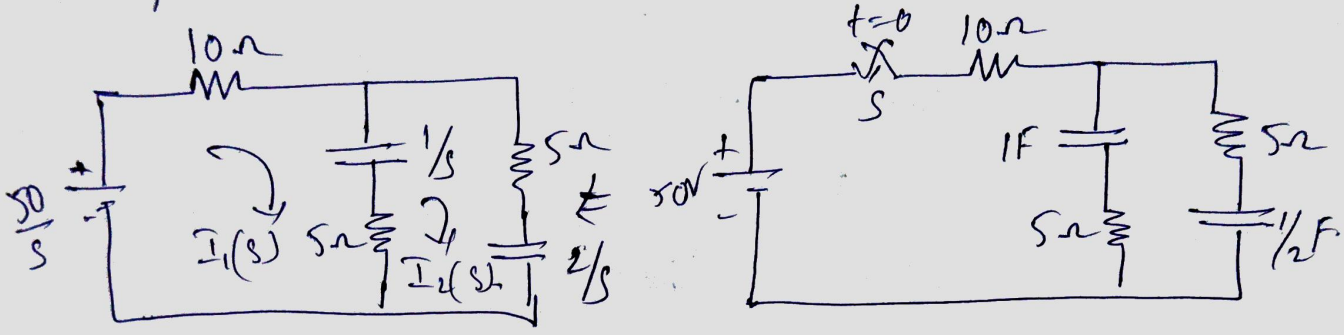
$$C = 1.66$$

$$I_2(s) = \frac{3.33}{s} + \frac{5}{s+10} + \frac{1.66}{s+30}$$

Apply Inverse laplace T/F.

$$i_2(t) = 3.33 - 5e^{-10t} + 1.66e^{-30t}$$

→ In the N/w shown in fig below, the switch is closed at t=0, is there no initial charge on either of the capacitors. Find the current 'i' by Laplace T/F method.



Applying KVL,

loop ① $\frac{50}{s} = 10\,I_1(s) + \frac{1}{s}\left[I_1(s) - I_2(s)\right] + 5\left[I_1(s) - I_2(s)\right]$

$\frac{50}{s} = \left[15 + \frac{1}{s}\right] I_1(s) - \left(\frac{1}{s} + 5\right) I_2(s) \rightarrow ①$

loop ②

$5\,I_2(s) + \frac{2}{s} I_2(s) + 5\left[I_2(s) - I_1(s)\right] + \frac{1}{s}\left[I_2(s) - I_1(s)\right] = 0$

$\left[10 + \frac{3}{s}\right] I_2(s) - \left[5 + \frac{1}{s}\right] I_1(s) = 0$

$I_2(s) = \left[\frac{5s+1}{10s+3}\right] I_1(s) \rightarrow ②$

② in ①

$\frac{50}{s} = \left[15 + \frac{1}{s}\right] I_1(s) - \frac{(5s+1)}{s}\left[\frac{5s+1}{10s+3}\right] I_1(s)$

$50 = (15s+1)\, I_1(s) - \frac{(5s+1)^2}{10s+3} I_1(s)$

$50(10s+3) = \left[(15s+1)(10s+3) - (5s+1)^2\right] I_1(s)$

$= \left[150s^2 + 55s + 3 - 25s^2 - 10s - 1\right] I_1(s)$

$50(10s+3) = \left[125s^2 + 45s + 2\right] I_1(s)$

$$I_1(s) = \frac{50(10s+3)}{125s^2+45s+2} = \frac{500(s+0.3)}{125s^2+45s+2}$$

$$= \frac{500(s+0.3)}{125\left[s^2+\frac{45}{125}s+\frac{2}{125}\right]}$$

$$I_1(s) = \frac{500(s+0.3)}{(s+0.052)(s+0.308)}$$

$$\Rightarrow \frac{500(s+0.3)}{(s+0.052)(s+0.308)} = \frac{A}{s+0.052} + \frac{B}{s+0.308}$$

$$I_1(s) \Rightarrow 500(s+0.3) = A(s+0.308) + B(s+0.052)$$

$s = -0.052$

$$I_1(s) \Rightarrow 500(0.248) = A(0.256)$$

$$\boxed{A = 484.375}$$

$s = -0.308$

$$500(-0.008) = B(-0.256)$$

$$B = \frac{4}{0.256} = 15.625$$

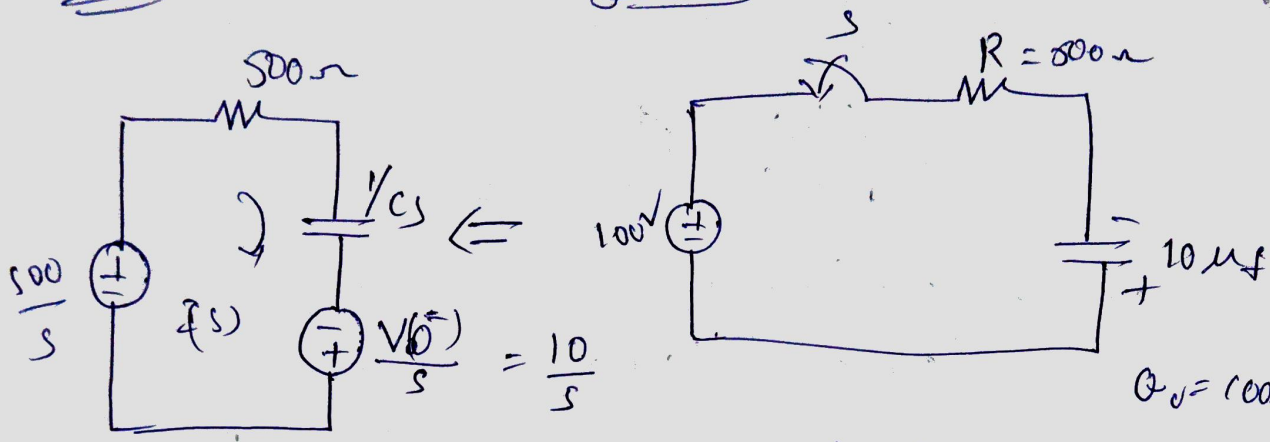
$$\therefore I_1(s) = \frac{484.375}{s+0.052} + \frac{15.625}{s+0.308}$$

$$\boxed{i_1(t) = 484.375\, e^{-0.052t} + 15.625\, e^{-0.308t}}$$

Ans

Prob:

using L-T



$Q_0 = 100\mu C$

$V(0^-) = \frac{Q}{C} = \frac{100}{10} = 10V$

Apply KVL,

$$\frac{100}{s} + \frac{10}{s} = 500\, I(s) + \frac{1}{10\times10^{-6}s} I(s)$$

$$\frac{110}{s} = I(s)\left[500 + \frac{1}{10^{-5}s}\right]$$

$$110 = I(s)\left(500s + 10^5\right)$$

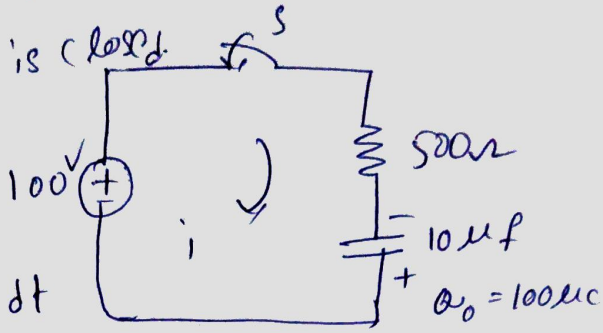
$$I(s) = \frac{110}{500s+10^5} = \frac{110/500}{s+\frac{10^5}{500}} = \frac{11/50}{s+200}$$

$$I(s) = \frac{0.22}{s+200}$$

Apply inverse L.T.

$$\boxed{i(t) = 0.22\, e^{-200t}}$$

prob:- The 10μf capacitor in RC circuit has initial charge of 100μc with polarities as shown in fig. At t=0 the switch being closed, a dc voltage of 100 v is applied. Find the expression for the current.



Sol:- at t=0, switch 's' is closed.

Apply KVL,

$$100 = 500\,i + \frac{1}{10\times10^{-6}}\int i\,dt$$

differentiate the above equation.

$$0 = 500\,di/dt + \frac{1}{10\times10^{-6}}\,i$$

$$\frac{di}{dt} + \frac{1}{5000\times10^{-6}}\,i = 0$$

$$di/dt + 200\,i = 0$$

the solution for the above equation.

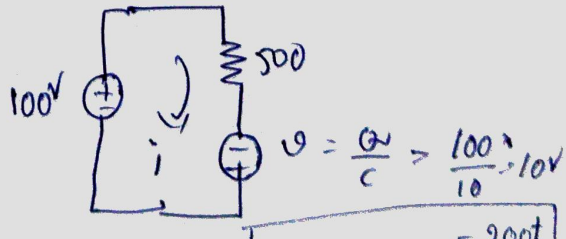
$$\boxed{i = c\,e^{-200t}} \rightarrow (1)$$

To find the value of 'c' we go to initial conditions.

at t=0 i = ?

100 = 10 + 500 i

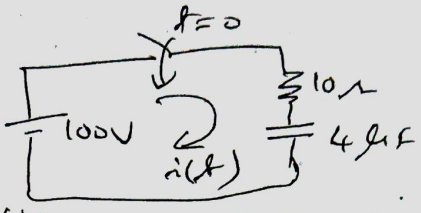
$$i = \frac{110}{500} = 0.22\ A.$$



(1) $\Rightarrow$ $0.22 = c\,e^{0} \Rightarrow c = 0.22$ $\therefore$ $\boxed{i = 0.22\,e^{-200t}}$

① Det. the $i(t)$ in the ckt shown for $t>0$, when the switch is closed at $t=0$.

$v_0 = 800\ \mu V$.



Sol: $10\,I(s) + \frac{1}{4\times10^{-6}s}\,I(s) + \frac{v_c(0)}{s} = \frac{100}{s}$

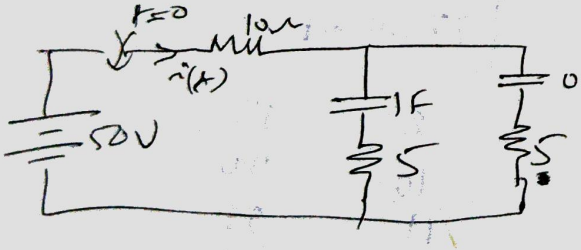
$v_c(0) = \frac{q_0}{C} = 200V$

$I(s)\left(10 + \frac{10^6}{4s}\right) = \frac{100}{s} - \frac{200}{s} = -\frac{100}{s}$

$I(s) = \frac{100}{s\left(10+\frac{10^6}{4s}\right)} = \frac{100\times 4s}{s(40s+10^6)} = \frac{10}{s+\frac{10^6}{40}}$

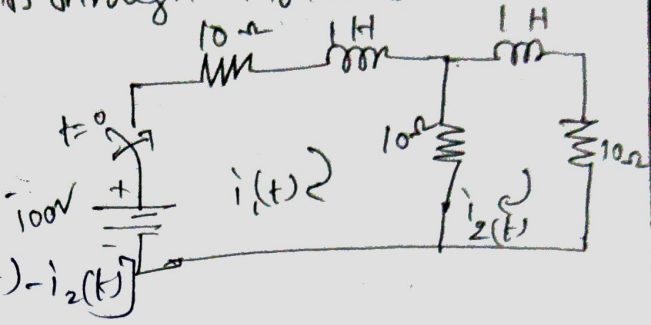
$\therefore\ i(t) = 10\,e^{-25\times10^3 t}$ Amp.

② In the ckt shown, the switch is closed at $t=0$. Assume no initial charge on either of the capacitors. Find i.



$i(t) = 0.125\,e^{-0.3t} + 3.875\,e^{-0.05\frac{2}{t}}$

Prob:- In the N/w shown in fig below, the switch is closed at t=0. Find the values of $i_1(t)$ & $i_2(t)$ assuming zero initial currents through inductors.



Sol:- Loop equations for

Apply KVL for loop ①

$$100 = 10 i_1 + L\frac{di_1}{dt} + 10\left[i_1(t) - i_2(t)\right]$$

Apply L-T

$$\frac{100}{s} = 10 I_1(s) + s I_1(s) + 10\left[I_1(s) - I_2(s)\right]$$

$$\Rightarrow I_1(s)(20+s) - 10 I_2(s) = \frac{100}{s} \rightarrow (1)$$

KVL for loop ②

$$10\left[i_2(t) - i_1(t)\right] + 1\frac{di_2}{dt} + 10 i_2 = 0$$

Apply L-T

$$10\left[I_2(s) - I_1(s)\right] + s I_2(s) + 10 I_2(s) = 0$$

$$(20+s) I_2(s) - 10 I_1(s) = 0$$

$$I_1(s) = I_2(s)\left[\frac{20+s}{10}\right] \rightarrow (2)$$

eqn ② in eqn ①

$$I_2(s)\left(\frac{20+s}{10}\right)(20+s) - 10 I_2(s) = \frac{100}{s}$$

$$I_2(s)\left[\frac{(20)^2 + 40s + s^2}{10} - 10\right] = \frac{100}{s}$$

$$I_2(s)\left[\frac{s^2 + 40s + 400 - 100}{10}\right] = \frac{100}{s}$$

$$I_2(s) = \frac{1000}{s(s^2 + 40s + 300)}$$

$$I_2(s) = \frac{1000}{s(s+30)(s+10)} = \frac{K_1}{s} + \frac{K_2}{s+30} + \frac{K_3}{s+7}$$

$$1000 = K_1(s+30)(s+7) + K_2 s(s+7) + K_3 s(s+30)$$

If $s=0$,

$$K_1(210) = 1000$$

$$K_1 = \frac{1000}{210} = \frac{100}{21} = 4.76$$

If $s=-30$, $\quad 1000 = K_2(-30)(-23)$

$$K_2 = \frac{1000}{690} = \frac{100}{69} = 1.449$$

If $s=-7$, $\quad 1000 = K_3(-7)(23)$

$$K_3 = \frac{-1000}{161} = -6.21$$

$$I_2(s) = \frac{4.76}{s} + \frac{1.449}{s+30} - \frac{6.21}{s+7}$$

Applying I.L.T

$$\boxed{i_2(t) = 4.76 + 1.449\, e^{-30t} - 6.21\, e^{-7t}}$$

From eqn (2) $\quad I_2(s) = I_1(s)\left[\frac{10}{20+s}\right]$

substitute in eqn (1) $\quad I_1(s)(20+s) - 10\, I_1(s)\left[\frac{10}{20+s}\right] = \frac{100}{s}$

$$I_1(s)\left[(20+s) - \frac{100}{20+s}\right] = \frac{100}{s}$$

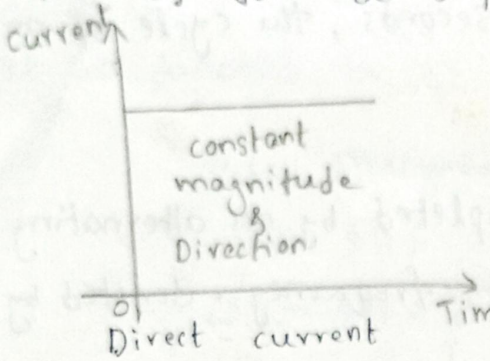
$$I_1(s)\left[\frac{(20)^2 + s^2 + 40s - 100}{20+s}\right] = \frac{100}{s}$$

$$I_1(s) = \frac{100}{s} \times \frac{(20+s)}{s^2+40s+300} = \frac{100(s+20)}{s(s+30)(s+10)}$$

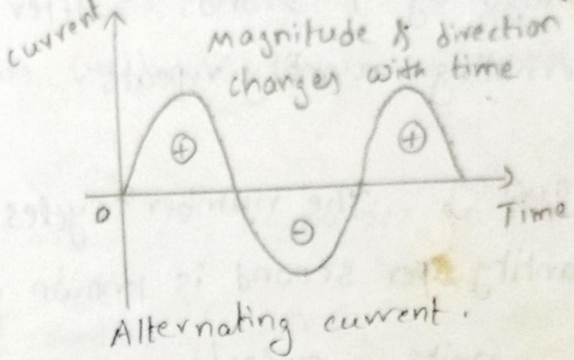
UNIT - III

Single phase AC circuits

An alternating current (a.c) is the current which changes periodically both in magnitude & direction w.r.to time, where as the dc supply has constant magnitude w.r.to time.

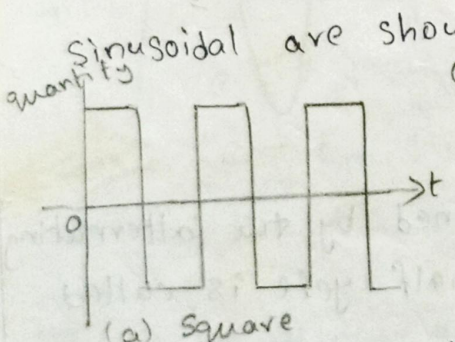


Direct current

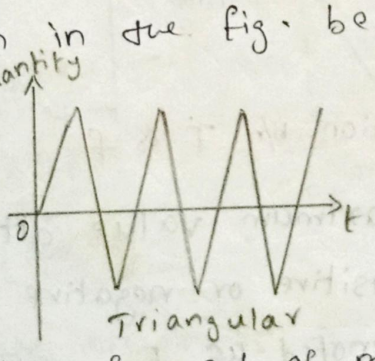


Alternating current.

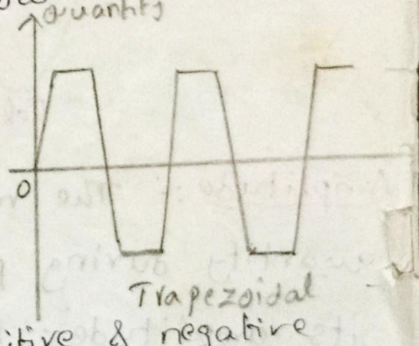
The waveform of alternating voltage or current is shown purely sinusoidal in the fig shown above. But, in practice, a quantity which undergoes variations in its instantaneous values, in magnitude as well as direction w.r.to some zero reference is called alternating quantity. The graph of such quantity against time is called its waveform. various types of alternating waveforms other than sinusoidal are shown in the fig. below



(a) Square



Triangular



Trapezoidal

Cycle :- Each repetition of a set of positive & negative instantaneous values of the alternating quantity is called a cycle.

Such repetition occurs at regular interval of time. Such a waveform which exhibits variations that reoccur after a regular time interval is called Periodic waveform.

** one cycle corresponds to 2π radians or 360°.

Angular

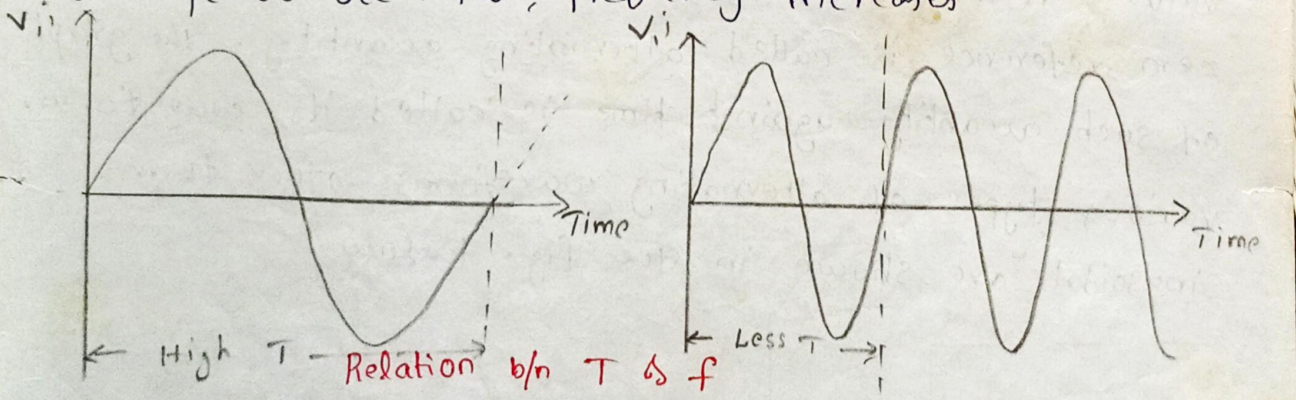
Time period :- The time taken by an alternating quantity in e to complete its one cycle is known as its time period an denoted by T seconds. After every T seconds, the cycle of an the alternating quantity repeats. it

Frequency :- The number of cycles completed by an alternating quantity per second is known as its frequency, denoted by f.

Units = cycles/second or Hertz.

$$f = \frac{1}{T} \text{ Hz}$$

As time Period increases, frequency decreases while as time period decreases, frequency increases.



Relation b/n T & f

Amplitude :- The maximum value attained by the alternating quantity during positive or negative half cycle is called its amplitude. Denoted as E_m or I_m.

Angular frequency (ω):- It is the frequency expressed in electrical radians per second. As one cycle of an alternating quantity corresponds to 2π radians, the angular frequency can be expressed as ($2\pi \times$ cycles/sec). It is denoted by 'ω' & its unit is radians/second.

$$\omega = 2\pi \times \text{cycles/sec} \qquad \because \text{cycles/sec} = f.$$
$$= 2\pi f$$
$$\omega = 2\pi \times \frac{1}{T} = \frac{2\pi}{T} \text{ radians/sec.}$$

Equation of an alternating quantity

According to faraday's laws of electromagnetic induction

$$e = BlV \sin\theta$$

$BlV = E_m$ = maximum value of the induced emf in conductor.

$$\boxed{e = E_m \sin\theta} \text{ volts}$$

$$e = E_m \sin(\omega t)$$
$$e = E_m \sin(2\pi f t)$$
$$e = E_m \sin\left[\frac{2\pi}{T} \cdot t\right]$$

Note: In all the above equations, the angle θ is expressed in radians. Hence while calculating the instantaneous value of the emf, it is necessary to calculate the sine of the angle expressed in radians.

This alternating emf drives a current through the electrical load which also varies in similar manner. Its frequency is the same as the frequency of the generated emf. Hence it can be expressed as

$$\boxed{i = I_m \sin\theta}$$

I_m = Max or peak value of current

This max. value depends on the resistance of the electrical circuit to which an emf is applied. The instantaneous value of this sinusoidal current set up by the emf. can be expressed as,

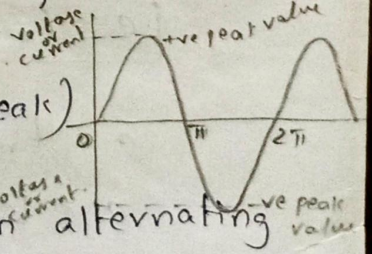
$$i = I_m \sin \omega t$$

$$i = I_m \sin 2\pi f t$$

$$i = I_m \sin\left(\frac{2\pi t}{T}\right).$$

Peak to Peak Value :- The peak to peak value of the sine wave is the magnitude of the +ve peak to the -ve peak.

Peak to peak = (positive peak + negative peak)



Average value :- The average value of an alternating quantity is defined as that value which is obtained by averaging all the instantaneous values over a period of half cycle.

(or)

The average value can be obtained by finding the area under the curve & dividing by the period.

Note:- For a symmetrical a.c, the average value over a complete cycle is zero as both positive & negative half cycles are exactly identical. Hence, the average value is defined for half cycle only.

For an unsymmetrical a.c, the average value must be obtained for one complete cycle.

$$I_{ave} = \frac{\text{Area under curve for half cycle}}{\text{Length of base over half cycle.}}$$

RMS value (or) Effective value:- The Effective or r.m.s value of an alternating current is given by that steady current (D.c) which, when flowing through a given ckt for a given time, produces the same amount of heat as produced by the alternating current which when flowing through the same ckt for the same time.

If i(t) is the a.c, then the heat produced by it for a small interval of time dt seconds is

$$dH = i^2(t)\, R\, dt$$

∴ The heat produced by a over the entire time period 'T' is $H = \int_0^T i^2(t)\, R\, dt$

If this heat is equal to the produced by steady dt of 'i' Amperes passing through 'R' for the time 'T' seconds

$$H = I^2 R T$$

$$\therefore I^2 R T = R \int_0^T i^2(t)\, dt$$

$$I^2 = \frac{1}{T} \int_0^T i^2(t)\, dt$$

$$\boxed{I_{rms} = \sqrt{\frac{1}{T} \int_0^T i^2(t)\, dt}}\ .$$

Form Factor (K_f):- Form factor is ratio of RMS value to the average value of an ac quantity.

$$\text{Form factor} = \frac{0.707\, I_m}{0.632\, I_m} = \frac{I_{RMS}}{I_{avg}}\ .$$

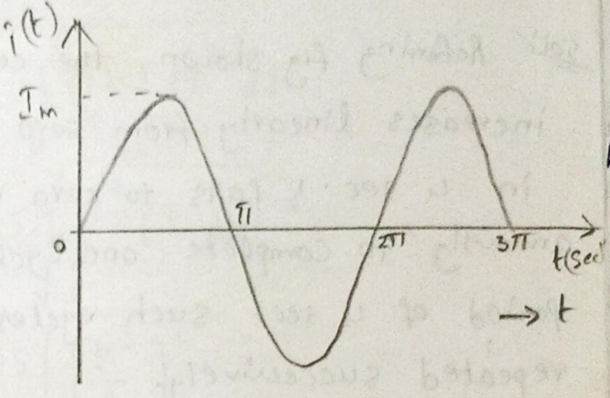
Peak factor (or) Amplitude factor (or) crest factor (K_p)

It is defined as the ratio of max. value to RMS value of an ac quantity.

$$\text{Peak factor } (K_p) = \frac{\text{Peak value}}{\text{RMS }''} = \frac{I_m}{0.707\, I_m}$$

$$= \frac{1}{0.707} = 1.414.$$

rob for the sinusoidal waveform, find out the Rms & Average values.



ol:

$$i(t) = I_m \sin t$$

$$I_{avg} = \frac{1}{T}\int_0^t i(t)\,dt$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin t\,dt$$

$$= \frac{I_m}{\pi}\left[-\cos t\right]_0^{\pi}$$

$$= -\frac{I_m}{\pi}\left[-1-1\right]$$

$$= +2\frac{I_m}{\pi}$$

$$\boxed{I_{avg} = 0.632\, I_m}$$

$$I_{Rms} = \sqrt{\frac{1}{T}\int_0^{\pi} i^2(t)\,dt}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (I_m \sin t)^2\,dt}$$

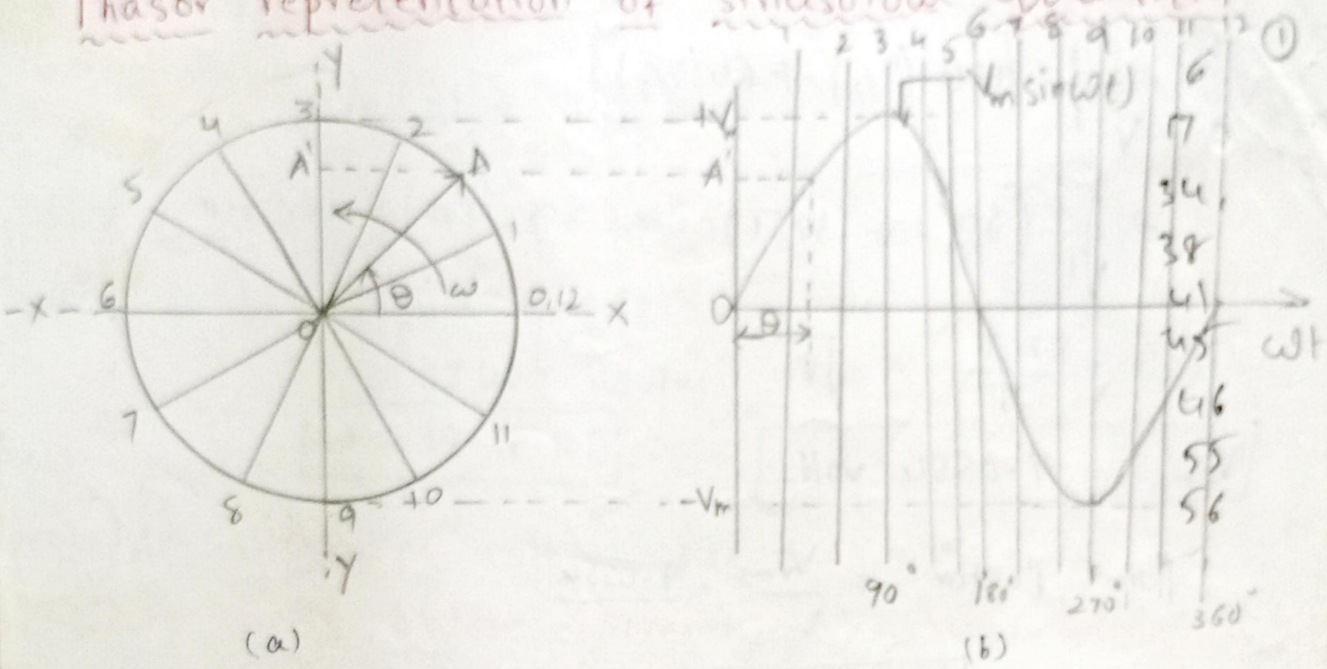
$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{2\pi}\left[\frac{1-\cos 2t}{2}\right]dt}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\left[t\right]_0^{2\pi} - \left[\frac{\sin 2t}{2}\right]_0^{2\pi}\right]}$$

$$= \sqrt{\frac{I_m^2}{4\pi}(2\pi - 0)}$$

$$I_{rms} = \sqrt{\frac{I_m^2}{2}} \Rightarrow \frac{I_m}{\sqrt{2}} = 0.707\, I_m = I_{rms}\ //$$

Phasor representation of sinusoidal quantities



(a) (b)

Fig(a) shows a vector OA rotating at a constant angular velocity ω in anticlockwise direction. Such rotating vector is known as a phasor. When time $t=0$ the angle made by OA with +ve X-axis is $\omega t = 0$ ie, OA lies on +ve X-axis.

When $\omega t = 30^\circ, 60^\circ, 90^\circ, \ldots 360^\circ$ etc, OA occupies positions O1, O2, O3, ... OO respectively as shown.

For any angle $\omega t = \theta$, position of phasor OA is such that its vertical component (ie, projection of OA on Y-axis) is $OA' = OA \sin\theta = OA \sin\omega t$.

If OA' = instantaneous value of sinusoidal quantity (say) V, then $V = OA \sin\omega t$, Therefore the phasor OA must represent the maximum value V_m of that sinusoidal quantity $V = V_m \sin\omega t$. Thus any sinusoidal quantity is represented by a phasor having its length equal to the

maximum value of the sinusoidal quantity.

Conclusion

(1) A sinusoidal quantity can be represented by a phasor

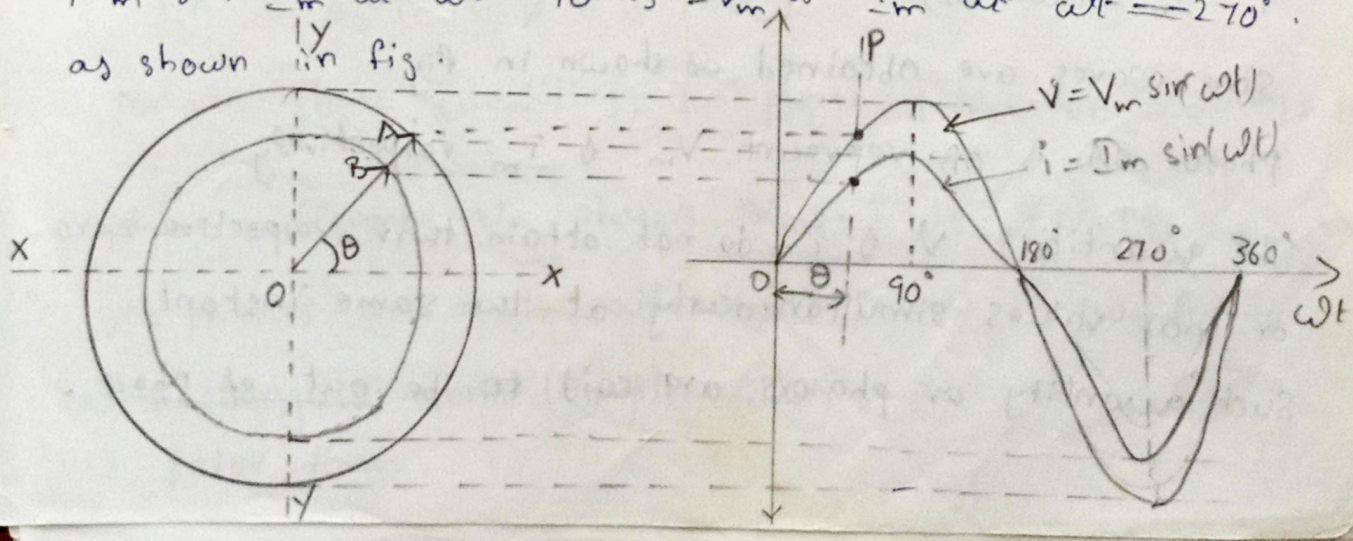
(2) The length of the phasor is equal to the max. value of the sinusoidal quantity. But for the sake of convenience, the rms value is represented by the length of the phasor in practice.

(3) Though the phasor is a rotating vector, its position is fixed in the space while drawing phasor diagram.

In-phase & out of phase sinusoidal quantities

In phase sinusoidal quantities

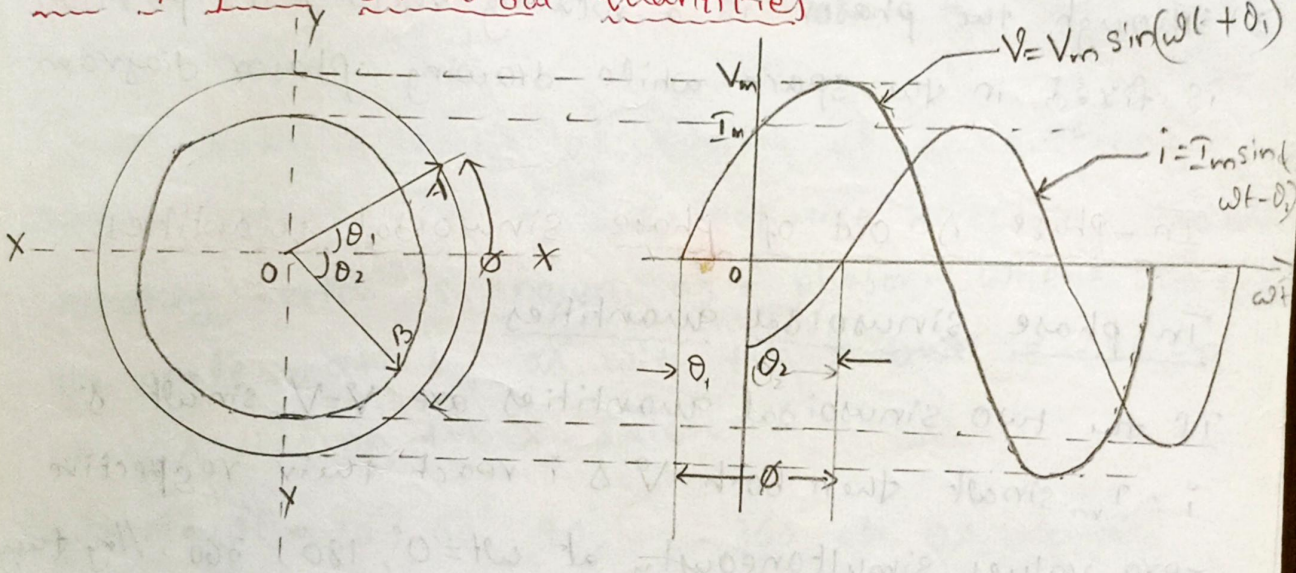
If the two sinusoidal quantities are $V = V_m \sin \omega t$ & $i = I_m \sin \omega t$ then both V & i reach their respective zero values simultaneously at $\omega t = 0°, 180°, 360°$. //ly they reach their respective max. values simultaneously ie $+V_m$ & $+I_m$ at $\omega t = 90°$ & $-V_m$ & $-I_m$ at $\omega t = 270°$. as shown in fig.



These quantities V & i are represented by phasors OA & OB respectively such that $OA = V_m$ & $OB = I_m$ & both OA & OB are in one line or in phase. such phasors or quantities are known as in-phase phasors or quantities.

Thus in-phase quantities reach their respective zero or max. values simultaneously at the same instant of time.

out of phase sinusoidal quantities



If the two sinusoidal quantities $V = V_m \sin(\omega t + \theta_1)$ & $i = I_m \sin(\omega t - \theta_2)$ are plotted against angle 'ωt', the sine waves are obtained as shown in fig.

Phasor OA & OB represent V_m & I_m respectively

The quantities 'V' & 'i' do not attain their respective zero or max values simultaneously at the same instant.

Such quantity or phasors are said to be <u>out of phase</u>.

The quantity 'v' becomes zero when $\omega t = \theta_1$ ie, θ_1 radians earlier than $\omega t = 0$.

The quantity 'i' becomes zero when $\omega t = \theta_2$ ie, θ_2 radians latter than $\omega t = 0$.

Phase or phase angle :- It is defined as the angle ωt covered by the sinusoidal quantity since it passed through its last zero value while increasing in the +ve direction.

Ex:- Fig. shown in In-phase sinu. quantity.

In that sine wave of $v = V_m \sin \omega t$, the phase angle of v at position 'p' is $\omega t = \theta$.

Phase difference (ϕ):- If two sinusoidal quantities v & i are out of phase, the time angle ϕ lapsed b/n their respective zero or maximum values is known as the phase difference b/n them.

For out of phase quantities, the phase difference b/n v & i is $\phi = (\theta_1 + \theta_2)$.

For in-phase quantities, the phase difference is always '0'.

phase difference b/n phasors V & I is $\phi = \theta_1 + \theta_2$

The phasors are mathematically expressed in two forms.

(a) Rectangular form (complex form).

(b) Polar form

Rectangular form

In this form, the phasor is represented by its horizontal component (X-component) & vertical (Y-component)

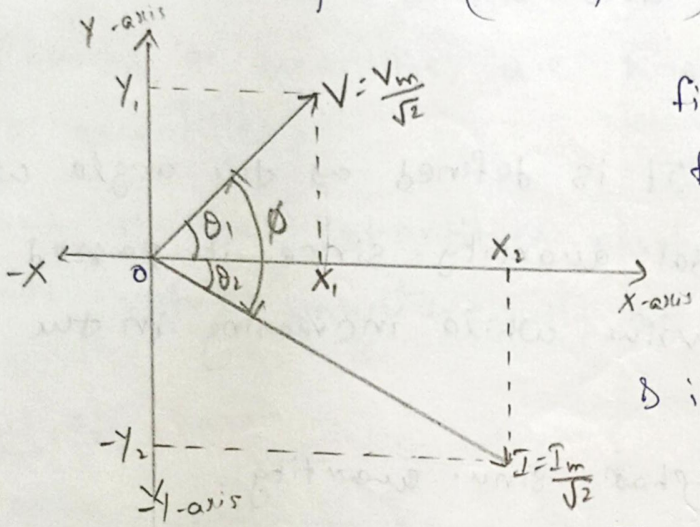


fig. shows a phasor diagram for two sinusoidal quantities.

$v = V_m \sin(\omega t + \theta_1)$ & $i = I_m \sin(\omega t - \theta_2)$

The length of the phasor, $V = V_{rms} = \frac{V_m}{\sqrt{2}}$ & it makes an angle of $+\theta_1$ with +ve X-axis.

The length of the phasor, $I = I_{rms} = \frac{I_m}{\sqrt{2}}$ & it makes an angle of $-\theta_2$ with the +ve X-axis.

The phasors $\bar{V}$ & $\bar{I}$ are expressed as

$$\bar{V} = OX_1 + jOY_1 \quad \& \quad \bar{I} = OX_2 - jOY_2$$

where OX_1 & OX_2 are X-components

& OY_1 & OY_2 are Y- " & j term

The X-component is known as the real part and the Y component (j terms) is known as the imaginary part.

Importance of operator j :- The multiplication of any phasor quantity by operator 'j' gives the rotation of that quantity by 90° in the anticlockwise direction from +ve X-axis.

i.e., ph 'j' is an operator which rotates a phasor by +90°.

'j' is an imaginary number equal to $\sqrt{-1}$.

$j^2 = -1$ gives the rotation of phasor by 180° from +ve X-axis.

$j^3 = j^2 \times j = -1 \times j = -j$

ie; it gives the rotation of phasor by 270° from +ve X-axis ie; phasor on negative Y-axis.

$j^4 = j^2 \times j^2 = -1 \times -1 = 1$, gives the rotation of phasor by 360° from +ve X-axis ie; phasor on +ve X-axis.

Polar form:- In this form, the phasor is represented by its magnitude (length or rms value) & the phase angle made by the phasor with the positive X-axis.

Eg. the phasors $\bar{V}$ & $\bar{I}$ are expressed as

$$\bar{V} = |V| \angle \theta_1 \quad \& \quad \bar{I} = |I| \angle -\theta_2$$

where $|V|$ & $|I|$ are the magnitudes or rms values & θ_1 & $-\theta_2$ are the phase angles made with the +ve X-axis by phasors $\bar{V}$ & $\bar{I}$ respectively.

R to P $\quad \bar{V} = OX_1 + jOY_1$

$$V = |V| \angle \theta_1$$

$$|V| = \sqrt{(OX_1)^2 + (OY_1)^2}$$

$$\theta_1 = \tan^{-1}\left(\frac{OY_1}{OX_1}\right)$$

P to R $\quad V = |V| \angle -\theta_1$ (or) $|V| \angle \theta_1$

to

$V = OX_1 + jOY_1$, where $OX_1 = |V| \cos(-\theta_1)$ (or) $|V| \cos(\theta_1)$

$OY_1 = |V| \sin(-\theta_1)$ (or) $|V| \sin(\theta_1)$

Prob: The max. values of alternating voltage & current are 400 V & 20A respectively in a ckt connected to a 50 Hz supply. The instantaneous values of voltage & current are 283 V & 10 A respectively at $t=0$, both increasing positively.

(i) write down the expression for the voltage & current at time 't'.

(ii) Determine the power consumed in the ckt. Take the voltage & current to be sinusoidal.

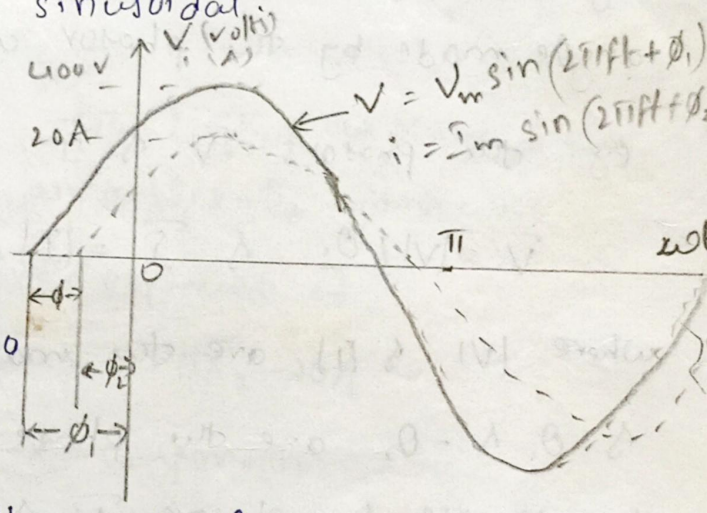
Sol:- Given $V_m = 400\,V$

$I_m = 20A$

$f = 50\,Hz$

$V_0 = 283\,V$

$I_0 = 10A$ at $t=0$



(i) Expression for the voltage is $V = V_m \sin(2\pi ft + \phi_1)$ & for the current is $i = I_m \sin(2\pi ft + \phi_2)$

At $t=0$, $V = 283\,V$ & $i = 10A$

$$V = 283 = 400 \sin(2\pi \times 50 \times 0 + \phi_1)$$

$$283 = 400 \sin\phi_1$$

$$\sin\phi_1 = \frac{283}{400} = 0.7075$$

$$\phi_1 = 0.786 \text{ rad (or) } 45.032^\circ.$$

∴ Expression for the voltage at time 't' is

$$V = 400 \sin(2\pi ft + 0.786)$$

$\therefore v = 400 \sin(314.159t + 0.786)$ V

At $t = 0$, $i = 10 = 20 \sin(2\pi \times 50 \times 0 + \phi_2)$

$$\sin\phi_2 = \frac{10}{20} = 0.5$$

$$\phi_2 = 30^\circ = 0.5236 \text{ rad.}$$

$\therefore$ Expression for the current at time 't' is

$$i = 20 \sin(2\pi \times 50 \times t + 0.5236)$$

$$i = 20 \sin(314.159t + 0.5236) \text{ A.}$$

(ii) phase angle b/n voltage & current $\Rightarrow \phi = \phi_1 - \phi_2$

$\phi = 45^\circ - 30^\circ = 15^\circ$ i.e., ~~voltage lags~~ current by 15°.

$\therefore$ Pf $= \cos\phi = \cos 15^\circ = 0.966$ lagging.

$\therefore$ power consumed in the ckt $= P = VI\cos\phi$

$$P = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi$$

$$= \frac{400 \times 20}{2} \times 0.966 = 3863.7 \text{ W}$$

$$P = 3.8637 \text{ KW.}$$

Prob: An alternating current of frequency 60 Hz has a max. value of 12 A.

(i) write down the equation for instantaneous values.
(ii) Find the value of the current after 1/360 seconds
(iii) Time taken to reach 9.6 A for the first time.

Sol:- given $f = 60$ Hz, $I_m = 12$ A

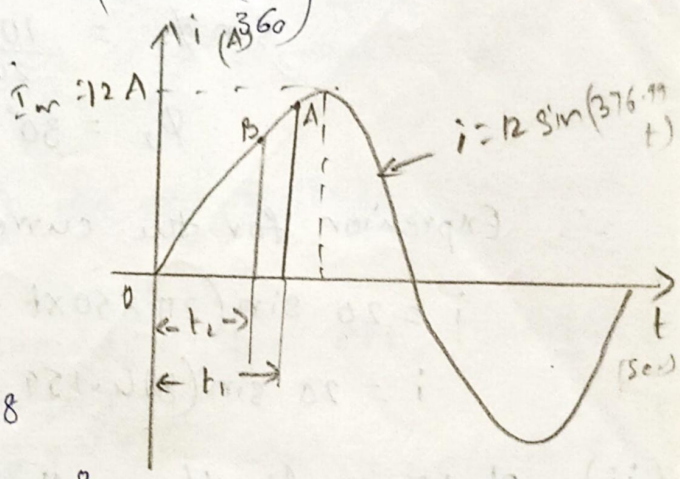
(i) Instantaneous current is given by $i = I_m \sin(\omega t)$

$\therefore i = 12 \sin(2\pi f t)$

$$i = 12 \sin(376.99t)$$

(ii) when $t_1 = 1/360$, $i_1 = 12 \sin\left(376.99 \times \frac{1}{360}\right)$

$$i_1 = 10.3923 \text{ A}.$$



(iii) For $i_2 = 9.6$ A

$$i_2 = 12 \sin(376.99 t_2)$$

$$9.6 = 12 \sin(376.99 t_2)$$

$$\sin(376.99 t_2) = \frac{9.6}{12} = 0.8$$

$$376.99 t_2 = \sin^{-1}(0.8) = 53.13^\circ = 0.9273 \text{ rad}.$$

$$\therefore t_2 = 0.00246 \text{ sec}.$$

Study of single phase AC circuits

Purely Resistive circuit :-

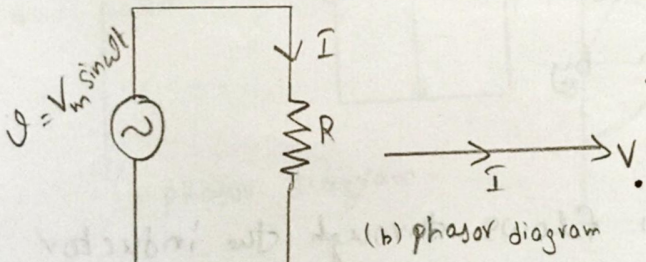
Suppose an alternating voltage $v = V_m \sin \omega t$ is applied to a pure resistance R. The instantaneous current is $i = v/R = \frac{V_m \sin \omega t}{R}$

The value of 'i' will be max. when $\sin \omega t = 1$

$$I_m = \frac{V_m}{R}$$

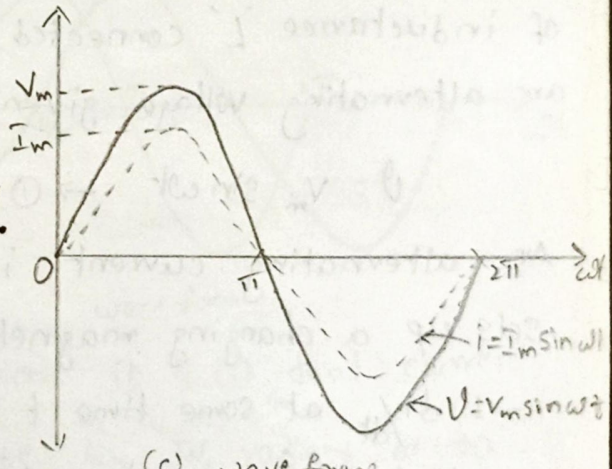
$$\boxed{i = I_m \sin \omega t}$$

It is clear that applied voltage & ckt current are in phase. ie, phase difference $\phi = 0$.



(a) ckt diagram

(b) phasor diagram



(c) wave forms

Power:- Instantaneous power is given by

$$p = v\,i = V_m \sin \omega t \cdot I_m \sin \omega t$$

$$p = V_m I_m \sin^2 \omega t$$

$$p = \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$p = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

This equation shows that the waveform of power 'p' is a constant power $\frac{V_m I_m}{2}$ with a double frequency sinusoidal power $\frac{V_m I_m}{2} \cos 2\omega t$ superimposed on it.

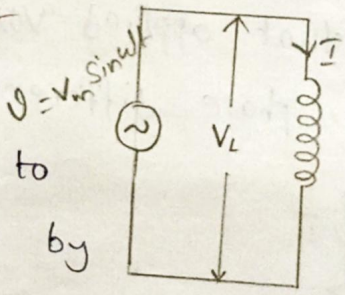
∴ The avg. of second term is zero

∴ The total avg. power $P = \frac{V_m I_m}{2} + 0$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = VI \text{ watts.}$$

Purely Inductive circuit:-

Consider a pure inductor of inductance 'L' connected to an alternating voltage given by



$$v = V_m \sin\omega t \rightarrow (1)$$

An alternating current i flows through the inductor & sets up a changing magnetic field. This induces a back emf $-L\, di/dt$ at some time t. Suppose that the value of applied p.d at time 't' is v ($v = V_m \sin\omega t$).

Applying KVL,

$$v - V_L = 0$$

$$v - L\, di/dt = 0$$

$$V_m \sin\omega t = L\, di/dt$$

$$di = \frac{V_m}{L} \sin\omega t\, dt$$

Integrating on b.s

$$i = \frac{V_m}{L} \int \sin \omega t \, dt$$

$$i = \frac{V_m}{L} \left[\frac{-\cos \omega t}{\omega} \right]$$

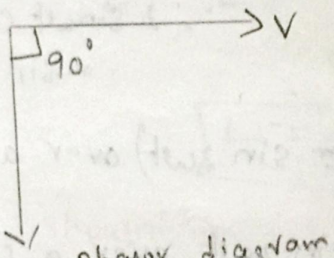
$$i = \frac{V_m}{\omega L} (-\cos \omega t)$$

$$i = \frac{V_m}{\omega L} \sin(\omega t - \pi/2)$$

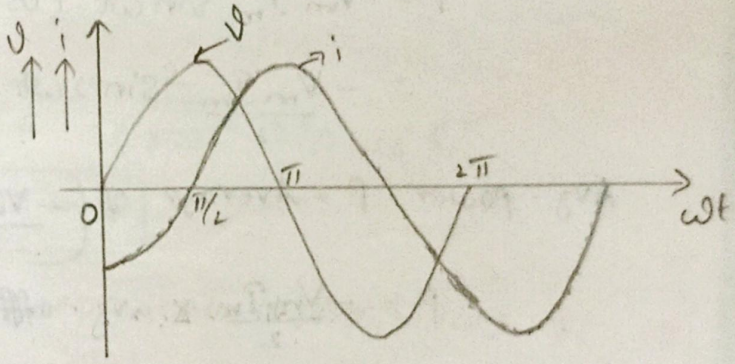
The value of 'i' will be max. when $\sin(\omega t - \pi/2)$ is 1.

$$I_m = \frac{V_m}{\omega L}$$

$$i = I_m \sin(\omega t - \pi/2) \longrightarrow (2)$$



phasor diagram.



wave form.

It is clear that from equations (1) & (2) that current lags behind the applied voltage by $\pi/2$ radians or 90°. This is shown in the phasor diagram as well as wave diagram.

SOL

Inductive Reactance :- Inductance not only causes the current to lag behind the voltage, but it also limits the magnitude of current in the ckt.

$$I_m = \frac{V_m}{\omega L}$$

The quantity ωL is called the inductive reactance X_L of the inductor, therefore measured in Ω.

$$\omega L = X_L = \frac{V_m}{I_m} = \frac{V_m/\sqrt{2}}{I_m/\sqrt{2}} = \frac{V_{rms}}{I_{rms}}$$

$X_L = \omega L = 2\pi f L$ will be in Ω.

L is in Henries & f is in Hz.

Power Instantaneous power, $p = vi$

$$p = (V_m \sin \omega t)\left(I_m \sin(\omega t - \pi/2)\right)$$

$$p = -V_m I_m \sin \omega t \cos \omega t$$

$$= -\frac{V_m I_m}{2} \sin 2\omega t \qquad \because 2 \sin \omega t \cos \omega t = \sin 2\omega t.$$

Avg. power P = Average of $\left(-\frac{V_m I_m}{2} \sin 2\omega t\right)$ over a cycle

$$P = -\frac{V_m I_m}{2} \times \text{avg. of } \sin 2\omega t \text{ over a cycle}$$

$$= -\frac{V_m I_m}{2} \times 0$$

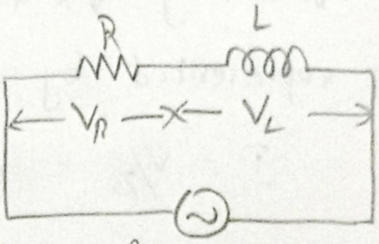
$$\boxed{P = 0}.$$

Reactive power, $Q = VI \sin\phi = VI \sin 90^\circ$

$$Q = VI = \frac{V^2}{X_L} = I^2 X_L \text{ VAR's.}$$

R-L series ckt :-

A pure resistance of 'R' ohms connected in series with a coil of pure inductance 'L' henry shown in fig.



Let V = rms value of applied voltage

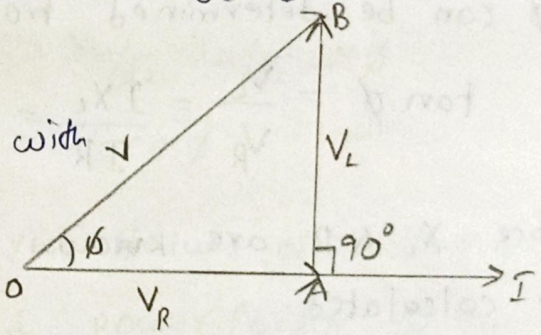
$\bar{I}$ = " " " the ckt current

$V_R = \bar{I}R$, where V_R is in phase with $\bar{I}$

$V_L = \bar{I}X_L$ " V_L leads the $\bar{I}$ by 90°.

$= \bar{I}(j\omega L)$

Taking the current as the reference phasor, the phasor diagram of the ckt can be drawn as shown in fig below.



The voltage drop (V_R) is in phase with current & is represented in magnitude & direction by the phasor OA. The voltage drop V_L ($V_L = \bar{I}X_L$) leads the current by 90° & represented in magnitude & direction by the phasor AB. The applied voltage 'V' is the phasor sum of these two drops. i.e.,

$$V = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{(\bar{I}R)^2 + (\bar{I}X_L)^2}$$

$$V = \bar{I}\sqrt{R^2 + (X_L)^2} = \bar{I}Z$$

$$\therefore \bar{I} = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{Z} \qquad \because Z = \sqrt{R^2 + X_L^2}$$

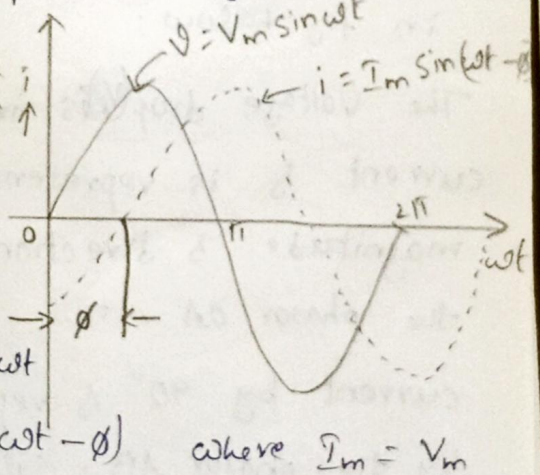
The quantity $\sqrt{R^2+X_L^2}$ is called impedance of the ckt. It is represented by Z & is measured in ohms(Ω).

$$I = V/Z \quad \text{where} \quad Z = \sqrt{R^2+X_L^2} \qquad X_L = 2\pi fL.$$

The magnitude of impedance in R-L series ckt depends upon the values of R, L, & supply frequency 'f'.

phase angle:- The phasor digram that ckt current I lags behind the applied voltage 'V' by ϕ°. The wave diagram shown in fig. The value of phase angle ϕ can be determined from the phasor diagram.

$$\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R}$$



Since X_L & R are known, ϕ can be calculated.

If the applied voltage is $v = V_m \sin\omega t$

current is $i = I_m \sin(\omega t - \phi)$ where $I_m = \frac{V_m}{Z}$

In an inductive ckt, current lags behind the applied voltage. The angle of lag (ie, ϕ) is greater than 0° but less than 90°. It is determined by ratio of inductive reactance to resistance ($\tan\phi = X_L/R$) in the ckt.

Power:- Instantaneous power, $p = vi$

$= V_m \sin\omega t \times I_m \sin(\omega t - \phi)$

$= \frac{1}{2} V_m I_m [2 \sin\omega t \sin(\omega t - \phi)]$

$= \frac{1}{2} V_m I_m [\cos\phi - \cos(2\omega t - \phi)]$

$= \frac{1}{2} V_m I_m \cos\phi - \frac{1}{2} V_m I_m \cos(2\omega t - \phi)$

The second term $\frac{1}{2} V_m I_m \cos(2\omega t - \phi)$, whose avg value over one complete cycle is zero.

$\therefore$ Avg. power, $P = \frac{V_m I_m}{2} \cos\phi$

$P = V_m/\sqrt{2} \; I_m/\sqrt{2} \cos\phi$

$P = VI \cos\phi$.

where V & I are the rms values of voltage & current. The term $\cos\phi$ is called power factor of the ckt & its value is given by (from phasor diagram)

$$p.f = \cos\phi = \frac{IR}{IZ} = \frac{V_R}{V} = R/Z$$

$P = VI \cos\phi$

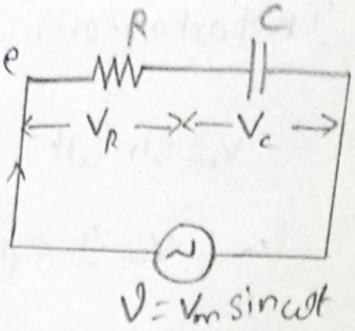
$= IZ \cdot I \cdot \frac{R}{Z}$

$P = I^2 R$.

Power is consumed in resistance only. Inductance does not consume any power.

R-C series ckt :-

A resistance R ohms connected in series with a capacitor of 'C' farads.



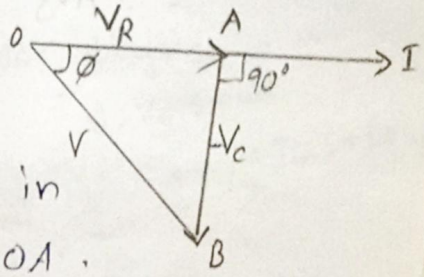
V = rms value of applied voltage

I = " " " the ckt current

$V_R = IR$, where V_R is in phase with 'I'.

$V_c = IX_c$ where V_c lags 'I' by 90°.

Taking current as the reference phasor, the phasor diagram of the ckt shown in fig.



The voltage drop V_R ($V_R = IR$) is in phase with current & is represented in magnitude & direction by the phasor OA.

The voltage drop V_c ($V_c = IX_c$) lags behind the current by 90° & is represented in magnitude & direction by the phasor AB.

The applied voltage 'V' is the phasor sum of these 2 drops.

ie:- $V = \sqrt{V_R^2 + V_c^2}$

$= \sqrt{(IR)^2 + (IX_c)^2}$

$= I\sqrt{R^2 + X_c^2}$

$I = \dfrac{V}{\sqrt{R^2 + X_c^2}}$

$V = V_R - jV_c$

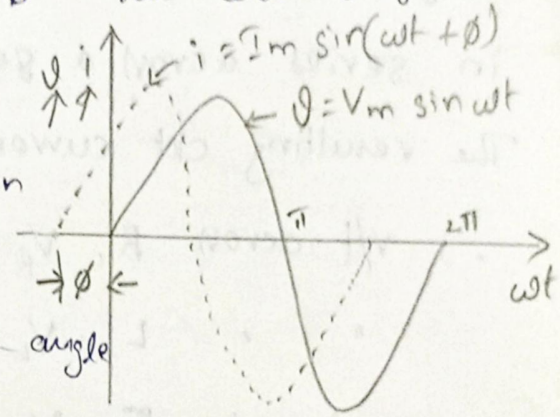
$IZ = IR - jIX_c$

$Z = R - jX_c$

The quantity $\sqrt{R^2 + X_c^2}$ is called impedance of the ckt.

$I = V/Z$ where $Z = \sqrt{R^2 + X_c^2}$.

Phase angle: The phasor diagram that ckt current I leads the applied v/t V by ϕ^0. The wave diagram shown in fig.



The value of phase angle ϕ can be determined from the phasor diagram. The value of the phase angle can be determined as under

$$\tan\phi = \frac{-V_C}{V_R} = \frac{-IX_C}{IR} = \frac{-X_C}{R}$$

$$\phi = \tan^{-1}\left(\frac{-X_C}{R}\right)$$

Power:-

$$v = V_m \sin\omega t$$

$$i = I_m \sin(\omega t + \phi)$$

Avg power P = Avg of $v \times i$

$$P = VI\cos\phi \quad \text{(same as that of RL)}$$

where V & I are the rms values of voltage & current.

R-L-C series circuit :-

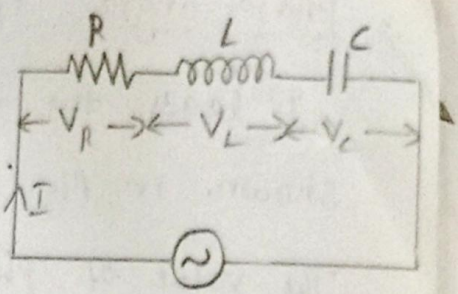
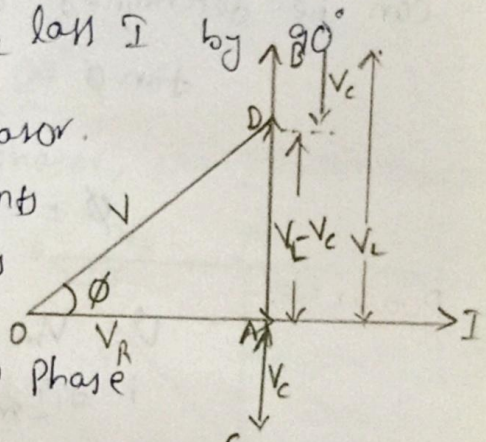


Fig. shows that R, L & C are connected in series across a supply v/t V. (rms). The resulting ckt current is I (rms)

$\therefore$ v/t across R, $V_R = IR$ ---- V_R is in phase with I

" " L, $V_L = IX_L$ ---- V_L leads I by 90°

" " C, $V_C = IX_C$ ---- V_C lags I by 90°



Taking current as the reference phasor. In the phasor diagram OA represents V_R, AB represents V_L & AC represents V_C. It may be seen that V_L is in phase opposition to V_C. For the case $V_L > V_C$ so that net voltage drop across L-C combination is $V_L - V_C$ & is represented by AD. Therefore, the applied ap voltage V is the phasor sum of V_R & $V_L - V_C$ & is represented by OD.

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

The quantity $\sqrt{R^2 + (X_L - X_C)^2}$ is called impedance of the ckt.

ckt power factor, $\cos\phi = R/Z$

$$\cos\phi = \frac{R}{\sqrt{R^2+(X_L-X_C)^2}} = \frac{R}{Z}$$

$$\tan\phi = \frac{V_L - V_C}{V_R}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

Power $P = VI\cos\phi$

$$P = I \cdot Z \cdot I\left(\frac{R}{Z}\right)$$

$$P = I^2 R .$$

Three cases of R-L-C series ckt :

$$Z = \sqrt{R^2+(X_L-X_C)^2}$$

(i) when $X_L - X_C$ is +ve ie, $(X_L > X_C)$ phase angle ϕ is +ve & the ckt will be inductive (current I will lag behind the voltage V by ϕ).

(ii) when $X_L - X_C$ is -ve ie, $(X_C > X_L)$ phase angle ϕ is negative and the ckt is capacitive. (current I leads the applied voltage V by ϕ).

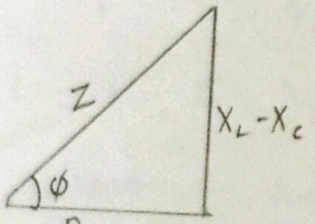
(iii) when $X_L - X_C$ is zero ie, $(X_L = X_C)$ then the ckt is purely resistive. (current I & applied v/t V will be in phase ie, $\phi = 0^\circ$).

If the equation for the applied v/t is

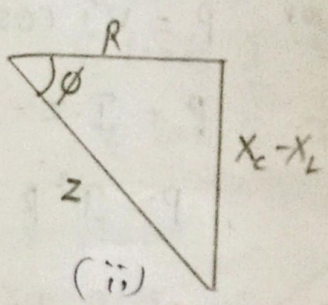
$v = V_m \sin \omega t$, then equ. for the ckt current will be $i = I_m \sin(\omega t \pm \phi)$

where $I_m = V_m / Z$.

The value of ϕ will be +ve or -ve depending upon which reactance (X_L or X_C).



(i)



(ii)

Fig (i) shows the impedance traingle of the ckt for the case when $X_L - X_C$ where as the impedance triangle in fig(ii) for the case when $X_C - X_L$.

Prob A load of 30 kw operates at 0.8 lag pf when connected to a 500 V, 50 Hz source. Find

(a) the current in the load
(b) the pf angle
(c) the impedance
(d) the resistance
(e) the reactance of the load.

Sol:- Given $P = 30\,kw$

$\cos\phi = 0.8$ lag

$V = 500$, $f = 50\,Hz$.

(i) $P = VI\cos\phi$.

$$I = \frac{P}{V\cos\phi} = \frac{30\,k}{500 \times 0.8} = 75\,A$$

(ii) Pf angle, $pf = \cos\phi \Rightarrow \phi = \cos^{-1}(0.8)$

$= 36.8^\circ$.

(iii) $Z = V/I = \frac{500}{75} = 6.67\,\Omega$

(iv) $R = Z\cos\phi \Rightarrow 6.67\cos\phi$

$= 6.67(0.8) = 5.336\,\Omega$

(v) $X = Z\sin\phi = 6.67(0.6) = 4.002\,\Omega$.

$v = V_m \sin\omega t = 500\sqrt{2}\sin 314t$

$i = I_m \sin(\omega t - \phi) = 75\sqrt{2}\sin(314t - 36.8^\circ)$

Prob A series RC load takes the power of 6400 w when connected to a 210 V, 25 Hz main. The v/t drop across the resistor is 160 V. calculate

(a) the current, (b) pf (c) Impedance (d) The resistance (e) capacitance & write the equations for the v/t & current.

Sol:- Given voltage drop across resistor $V_R = 160\ V = IR$

Power $P = I^2R = 6400\ W$

39(a) $P = I(IR) \Rightarrow I = \frac{P}{IR} = \frac{P}{V_R} = \frac{6400}{160} = 40\ A.$

(b) $Pf = \cos\phi = \frac{P}{VI} = \frac{6400}{210 \times 40} = 0.762$

$\phi = \cos^{-1}(0.762) = 40.36^\circ$

(c) $Z = \frac{V}{I} = \frac{210}{40} = 5.25\ \Omega$

$\sin(40.36) = 0.647$

(d) $R = Z\cos\phi = 5.25\,(0.762) = 4.0005\ \Omega$

(e) $X_c = \sqrt{Z^2 - R^2} = \sqrt{(5.25)^2 - (4.0005)^2}$

$\sqrt{27.5625 - 16.004}$

$\sqrt{11.5585}$

$X_c = 3.399\ \Omega$

(or) $X_c = Z\sin\phi$

$= 5.25(0.647)$

$= 3.39675$

$X_c = \frac{1}{2\pi f C} \Rightarrow C = \frac{1}{2\pi \times 50 \times 3.399} =$

$V = 210 \times \sqrt{2} \sin\omega t \Rightarrow 210 \times \sqrt{2} \sin(2\pi \times 25 \times t)$

$= 296.94 \sin 157t\ V.$

$i = I_m \sin(\omega t + \phi)$

$= 40 \times \sqrt{2} \sin(157t + 40.36)$

$i = 56.56 \sin(157t + 40.36^\circ)$ Amps.

30/10

Rectangular forms

Prob Express the following numbers in exp in to polar forms.

Ans

(1) $A_1 = 6+j8$ $\quad A_1 = 10\angle 53.13^\circ$

(2) $A_2 = 6-j8$ $\quad A_2 = 10\angle 306.87^\circ$

(3) $A_3 = -6+j8$ $\quad A_3 = 10\angle 126.87^\circ$

(4) $A_4 = -6-j8$ $\quad A_4 = 10\angle 233.13^\circ$

Prob convert the following polar forms to rectangular forms.

(i) $12\angle 60^\circ$ Ans $6 - j10.39$

(ii) $-50\angle 285^\circ$ $\quad -12.94 + j48.3$.

Prob If $A = 2+j5$ & $B = 4-j6$ then find $A \times B$ & $(A+B)/(A-B)$.

Sol: $A+B = 6-j$, $A-B = -2+j11$

$A \times B \Rightarrow A = 5.385\angle 68.2^\circ$

$B = 7.211\angle -56.31^\circ$

$A \times B = 38.831\angle 11.89^\circ = 38+j8$

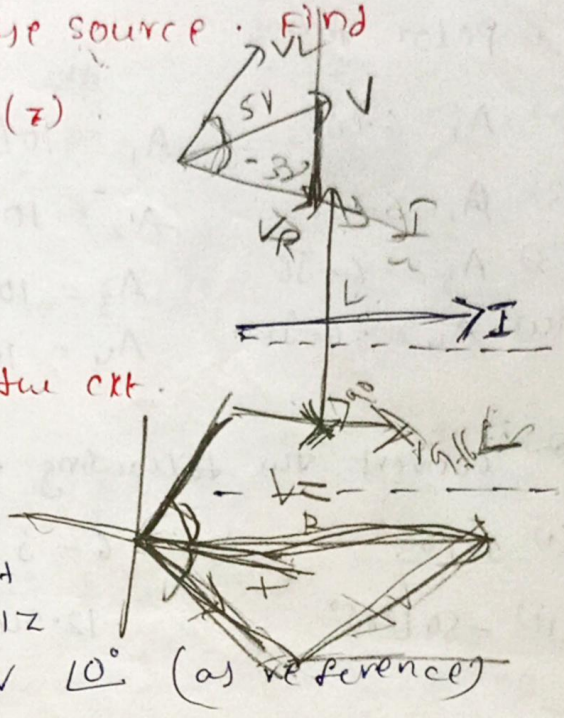
$$\frac{A+B}{A-B} = \frac{6-j}{-2+j11} = \frac{6.083\angle -9.46^\circ}{11.18\angle 100.30^\circ}$$

$$= 0.544\angle -109.76^\circ$$

$$(A+B)/(A-B) = -0.184 - j0.512.$$

Prob A 4Ω ~~inductor~~ Resistor is connected to a 10mH inductor across a 100 V, 50 Hz voltage source. Find

(i) Impedance of the ckt, (Z)
(ii) I/p current
(iii) Drop across R & L
(iv) pf of the ckt.
(v) Real power consumed in the ckt.
(vi) Total power supplied.



Ans given data

$R = 4\Omega$
$L = 10\,mH$
$f = 50\,Hz$
$V = 100\,V \angle 0^\circ$ (as reference)

(i) $Z = R + jX_L$

$X_L = 2\pi fL$
$= 2\pi \times 50 \times 10 \times 10^{-3}$
$= 3.14\,\Omega$

$Z = 4 + j\,3.14\,\Omega$

$Z = 5.085\,\Omega \angle 38.13^\circ$

(ii) $I = V/Z = \dfrac{100\angle 0^\circ}{5.085\angle 38.13} = 19.66\,A \angle -38.13^\circ$

Magnitude of i/p = 19.66 A.

(iii) Drop across R, ie, $V_R = IR = 19.66\angle -38.13^\circ \times 4$

$= 78.64 \angle -38.13^\circ\,V.$

$V_L = jIX_L = j19.66\angle -38.13^\circ \times 3.14$

$= 61.73 \angle 51.87\,V$ $\quad \because j = 1\angle 90^\circ$

(iv) pf, $\cos\phi = \cos(-38.13^\circ)$

$= 0.787$ lag.

(v) Real power $= VI\cos\phi = 100 \times 19.66 \times 0.787 = 1547$ watts.

(vi) Total power, $S = VI = 100 \times 19.66 = 1966\,VA$.

(c) The voltage drop across the inductance is

$$V_L = j I X_L$$

$$V_L = j(27.39 \angle -51.47) \times 2\pi \times 50 \times 0.02)$$

$$= 1\angle 90^\circ \cdot (27.39 \angle -51.47^\circ)(6.28)$$

$$= 172 \angle 90 - 51.47^\circ$$

$$V_L = 172 \angle 38.43^\circ \text{ V}$$

$$V_R = IR = (27.39 \angle -51.47) \cdot 5$$

$$V_R = 136.95 \angle -51.47^\circ \text{ V}.$$

[(check : $V_R + V_L = V$ vectorially)

$V = 136.95 \angle -51.47^\circ + 172 \angle 38.43^\circ = 220 + j0$ V.]

Prob In a series R-L ckt, R = 10 Ω, L = 20 mH. The circuit current being 10 sin 314 t A, find the total voltage drop (v) across the element R & L. Also obtain the phase angle b/n i & v.

Sol: the phase angle θ b/n v & i is given by

$$\theta = \tan^{-1} \frac{\omega L}{R}$$

$$= \tan^{-1} \frac{2\pi \times 50 \times 20 \times 10^{-3}}{10}$$

$$\theta = 32.13^\circ$$

$\omega = 314 = 2\pi f$ i.e., $f = 50$ Hz

$\cos\phi = \frac{R}{Z}$

$\cos\phi = \frac{10}{11.201} = 0.846$

$\phi = \cos^{-1}(0.846)$

given $i = 10 \sin 314t$

$$v = I_m \cdot Z \sin(\omega t + \theta)$$

$$= 10\sqrt{R^2 + (\omega L)^2} \sin(\omega t + \theta)$$

$$= 10\sqrt{10^2 + (314 \times 20 \times 10^{-3})^2} \sin(314t + 32.13^\circ).$$

$v = 118.08 \sin(314t + 32.13^\circ)$ volts. (here i lags v by 32.13°)

Prob In a series R-L ckt, the current & voltage are given as $i = 1\cos(314t - 20^\circ)$, $v = 10\cos(314t + 10^\circ)$. Find the values of R & L.

Sol:- Let I be the reference vector, i lags V by $(20+10) = 30^\circ$.

Also $\frac{V_m}{I_m} = \frac{10}{1} = 10 = Z$

$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$10 = \sqrt{R^2 + (314L)^2} \Rightarrow R^2 + (314L)^2 = 100 \rightarrow (1)$$

& $\tan 30^\circ = 0.577 = \frac{314L}{R} \rightarrow (2)$

solving (1) & (2) we get $L = 15.9$ mH & $R = 8.66\ \Omega$.

i.e., $R = \frac{314L}{0.577}$

Substitute 'R' in eqn (1) we get L & substitute (1) in eqn (2) we get R.

Prob In a series RC ckt, the values of $R = 10\Omega$ & $C = 25$ nf, A sinusoidal voltage of 50 MHz is applied & the maximum voltage across the capacitance is 2.5 V. Find the max. voltage across the series combination.

Sol:- Given $R = 10\Omega$, $f = 50$ MHz, $C = 25$ nf, $V_c = 2.5$ V, $V_{max} = ?$

$$V_{max} = I_{max} Z \Rightarrow I_{max}\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

To find $I_{max} = \omega C V_{max} \Rightarrow 2\pi f \times 25 \times 10^{-9} \times 2.5$

$\Rightarrow 314 \times 10^6 \times 25 \times 10^{-9} \times 2.5$

$I_{max} = 19.625$ A.

$$V_{max} = I_{max} \cdot Z$$

$$= 19.625 \sqrt{(10)^2 + \left(\frac{1}{2\pi \times 50 \times 10^6 \times 25 \times 10^{-9}}\right)^2}$$

$$V_{max} = 196.26 \text{ V}$$

prob In a series RCL circuit, the max. inductor voltage is twice the capacitor voltage maximum. However, the circuit current lags the applied voltage by 30° & the instantaneous drop across the inductance is given by $V_L = 100 \sin 377t$ V. Assuming resistance being 20 Ω, find the values of the inductance & capacitance

Sol:- Given $V_{L\,max} = 2 V_{C\,max}$

$$I_{max} \omega L = 2 \frac{I_{max}}{\omega C}$$

Given $V_L = 100 \sin 377t$ ⇒ $V_{L\,max}$

$$100 = 2 \frac{I_{max}}{\omega C} = I_{max} \omega L$$

$$\frac{I_{max}}{\omega C} = 50 \Rightarrow 50\omega = \frac{I_{max}}{C} \rightarrow (1)$$

$\phi = 30°$, $R = 20\Omega$

$$\Rightarrow \frac{100}{\omega} = I_{max} \cdot L \rightarrow (2)$$

In series RLC, $\tan\theta = \frac{X_L - X_C}{R}$

$$= \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$\tan 30° = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$0.577 = \frac{\omega L - 1/\omega C}{20}$$

$$0.577 = \frac{\omega^2 LC - 1}{20\omega C} \rightarrow (3)$$

Prob A 50 Hz sinusoidal voltage $v = 311 \sin \omega t$ is applied to a RL series circuit. If the magnitude of resistance is 5Ω & that of inductance is 0.02 H,

(a) calculate the rms or effective value of steady state current & relative phase angle.

(b) obtain the expression for the instantaneous current.

(c) compute the effective magnitude & phase of voltage drops appearing across each circuit element.

Sol: Given $v = 311 \sin \omega t = V_m \sin \omega t$

$R = 5\Omega$

$L = 0.02$ H

Calculate $I_{rms} = ?, = \phi$

$i = ?$

$V_R = ?$ & $V_L = ?$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{311}{\sqrt{2}} = 220 V$$

(a) $$\underline{I} = \frac{\underline{V}}{Z} = \frac{220 \angle 0^\circ}{Z}$$

$$= \frac{220 \angle 0^\circ}{8.03 \angle 51.47^\circ}$$

$$\underline{I} = 27.39 \angle -51.47^\circ \text{ A}$$

$Z = R + j\omega L$

$= 5 + j\, 2\pi \times 50 \times 0.02$

$Z = (5 + j\, 6.28)\ \Omega$ rectangular form

Polar form $Z = \sqrt{5^2 + 6.28^2} \angle \tan^{-1} \frac{6.28}{5}$

$Z = 8.03 \angle 51.47^\circ$

Thus the effective value of the circuit current element is 27.39 A while the angle of lag of the current from the voltage is 51.47°.

(b) The instantaneous current is then given by

$i = I_m \sin(\omega t + \theta)$

$= 27.39 \times \sqrt{2} \sin(\omega t - 51.47)$

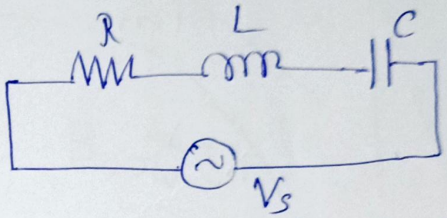
$i = 38.73 \sin(\omega t - 51.47^\circ)$ A.

UNIT-IV

Resonance :- Resonance is a phenomenon in which the applied voltage & resulting current are in phase. In other words AC circuit is said to be in resonance if it exhibits unity pf $\cos\phi = 1$.

Series Resonance: A series circuit is said to be in Resonance if its net reactance is zero. The frequency at which ~~at which it happens~~ resonance occurs is known as resonance frequency (f_r).

Let us consider the RLC series circuit shown in fig.



At resonance condition,

net reactance $X_L - X_C = 0$

(or) $X_L = X_C$

$$\omega_L = \frac{1}{\omega_C}$$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

$$Z = R + j(X_L - X_C)$$

$$Z = R + j(0)$$

$$\boxed{Z = R}$$

At resonance condition, the series R-L-C circuit becomes

(i) pure resistive ckt ie, current is inphase with voltage.

(ii) $X_L - X_C = X = 0$

(iii) $Z = R$ (min)

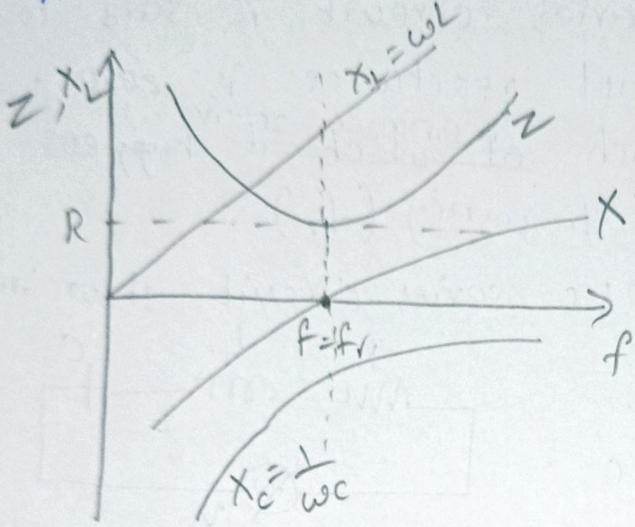
(iv) $\cos\phi = 1$, $\phi = 0$

(v) $I = \frac{V}{Z} = \frac{V}{Z(\min)}$ = max current is max.

$$I_{max} = \frac{V}{Z} = \frac{V}{R}.$$

Graphical Representation of series Resonance.

consider an alternating voltage of constant magnitude with variable frequency is applied to a series RLC ckt.



If $f < f_r$, $X_C > X_L$
Nature of ckt is capacitance → I leads V

If $f = f_r$, $X_C = X_L$
Nature of ckt is resistive
→ I inphase V

If $f > f_r$, $X_C < X_L$,
Nature of ckt is inductive
→ I lags V.

Resistance (R):- Resistance is independent of frequency hence it is represented by a st. line with some magnitude.

Inductive Reactance (X_L):- W.k.T $X_L = \omega L = 2\pi f L$

ie, $X_L \propto f$ ∴ f ↑ X_L also ↑

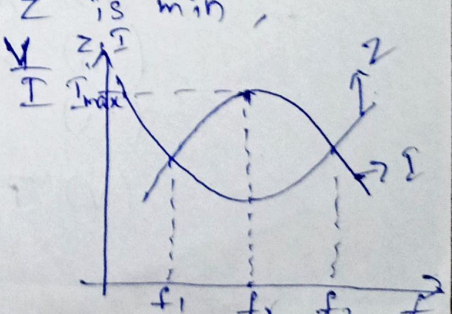
Hence the graph is a st. line passing through origin.

capacitive reactance (X_C):- W.k.T $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

ie, $X_C \propto \frac{1}{f}$ ∴ f ↑, X_C ↓

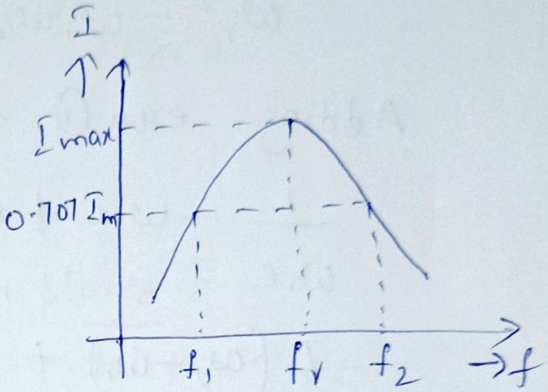
Hence the graph is rectangular hyperbola.

Current (I):- At resonance freq. as Z is min, current will be max, ∴ $Z \propto \frac{V}{I}$



Bandwidth: It is the difference b/n higher cutoff frequency f_2 & lower cutoff frequency f_1 is called as band width.

Derivation of bandwidth



At lower cutoff frequency

$X_C > X_L$ & it is assumed that $X_C - X_L = R$

$$\frac{1}{\omega_1 C} - \omega_1 L = R \rightarrow (1)$$

At upper cutoff frequency, $X_L > X_C$ & it is assumed as $X_L - X_C = R$

$$\omega_2 L - \frac{1}{\omega_2 C} = R \rightarrow (2)$$

Equate eqn (1) & (2).

$$\frac{1}{\omega_1 C} - \omega_1 L = \omega_2 L - \frac{1}{\omega_2 C}$$

$$\frac{1}{\omega_1 C} + \frac{1}{\omega_2 C} = \omega_2 L + \omega_1 L$$

$$\frac{1}{C}\left[\frac{1}{\omega_1} + \frac{1}{\omega_2}\right] = (\omega_1 + \omega_2) L$$

$$\frac{1}{C}\left[\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right] = (\omega_1 + \omega_2) L$$

$$\boxed{\omega_1 \omega_2 = \frac{1}{LC}} \rightarrow (3)$$

w.k.t, $f_r = \frac{1}{2\pi\sqrt{LC}} \Rightarrow 2\pi f_r = \frac{1}{\sqrt{LC}}$

$$\omega_r^2 = \frac{1}{LC} \rightarrow (4)$$

from eqn (3) & eqn (4).

$$\omega_r^2 = \omega_1\omega_2 = \frac{1}{LC} \rightarrow (5)$$

Adding eqn (1) & eqn (2).

$$\frac{1}{\omega_1 C} - \omega_1 L + \omega_2 L - \frac{1}{\omega_2 C} = 2R.$$

$$L(\omega_2 - \omega_1) + \frac{1}{C}\left[\frac{1}{\omega_1} - \frac{1}{\omega_2}\right] = 2R$$

$$L(\omega_2 - \omega_1) + \frac{1}{C}\left[\frac{\omega_2 - \omega_1}{\omega_1\omega_2}\right] = 2R$$

from eqn (5) sub $\frac{1}{C}$ as $\omega^2 L$

$$\Rightarrow \omega_1\omega_2 = \frac{1}{LC} = \omega_r^2$$

$$\therefore L(\omega_2 - \omega_1) + \omega_r^2 L\left[\frac{\omega_2 - \omega_1}{L\omega_r^2}\right] = 2R$$

$$2L(\omega_2 - \omega_1) = 2R$$

$$\omega_2 - \omega_1 = \frac{R}{L}$$

$$2\pi f_2 - 2\pi f_1 = \frac{R}{L}$$

$$2\pi(f_2 - f_1) = \frac{R}{L}$$

Band width, $\boxed{f_2 - f_1 = \frac{R}{2\pi L}}$

Quality factor:- It is defined as the ratio of reactive power of inductor or capacitor to the true power (real power) in the resistance in series with the coil or capacitance.

$$Q = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{2\pi fL}{R} \text{ (or) } \frac{1}{2\pi fCR}.$$

Derivation

Q-factor is also defined as

$$Q = 2\pi \times \frac{\text{Max. Energy Stored}}{\text{Energy dissipated per cycle.}}$$

In case of Inductor

Max. energy stored $= \frac{1}{2} L i^2$

Energy dissipated per cycle = Avg. power in the resistor X time

$$= \left(\frac{I_m}{\sqrt{2}}\right)^2 R \times t$$

$$\therefore Q = 2\pi \frac{\frac{1}{2} L I_m^2}{\left(\frac{I_m}{\sqrt{2}}\right)^2 R \times \frac{1}{f}} = \frac{2\pi fL}{R} = \frac{\omega L}{R}.$$

In case of Capacitor ~~Inductor~~

Max. energy stored $= \frac{1}{2} C V_m^2$

Energy dissipated per cycle = Avg. power in the resistor X time.

$$= \left(\frac{I_m}{\sqrt{2}}\right)^2 R \times t.$$

$$\therefore Q = 2\pi \frac{\frac{1}{2} C V_m^2}{\left(\frac{I_m}{\sqrt{2}}\right)^2 R \cdot t}$$

$V = IX_c$

$V_m = \frac{I_m}{\omega C}$

$$= 2\pi \frac{\frac{1}{2} C \left(\frac{I_m}{\omega C}\right)^2}{\left(\frac{I_m}{\sqrt{2}}\right)^2 R \cdot 1/f}$$

$$= \frac{2\pi f}{R} C \frac{1}{(2\pi f C)^2}$$

$$Q = \frac{1}{2\pi f C} \cdot \frac{1}{R} = \frac{1}{\omega C R}$$

At resonance,

$$Q = \frac{1}{\omega C R} = \frac{\omega L}{R}$$

$$= \frac{2\pi f_r L}{R} = \frac{2\pi \times \frac{1}{2\pi\sqrt{LC}} \times L}{R}$$

$$\frac{1}{\omega C R} = \frac{1}{2\pi f_r C R}$$

$$= \frac{1}{2\pi} \frac{1}{2\pi\sqrt{LC}} \cdot CR$$

$$= \frac{1}{R} \frac{\sqrt{LC}}{C} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Relation b/n quality factor, resonant frequency & band width

$$\boxed{Q = \frac{f_r}{B.W}}$$

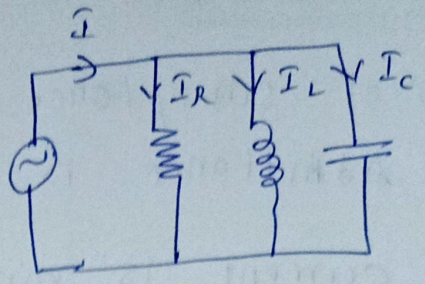
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$BW = \frac{R}{2\pi L}$$

$$Q = \frac{\frac{1}{2\pi\sqrt{LC}}}{\frac{R}{2\pi L}} = \frac{1}{R}\frac{L}{\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Parallel Resonance :-

A Parallel RLC ckt is said to be at resonance when net susceptance is zero



Apply KCL,

$$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C$$

$$\frac{V_s}{Z_{eq}} = \frac{V_s}{R} + \frac{V_s}{X_L} + \frac{V_s}{X_C}$$

$$\frac{1}{Z_{eq}} = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

$$Y = \frac{1}{R} + \frac{1}{j\omega L} + \frac{1}{1/j\omega C}$$

$$= \frac{1}{R} - \frac{j}{\omega L} + j\omega C$$

$$Y = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

At resonance,

$$\omega C - \frac{1}{\omega L} = 0$$

$$2\pi f_r C - \frac{1}{2\pi f_r L} = 0$$

$$2\pi f_r C = \frac{1}{2\pi f_r L}$$

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

At resonance

(1) Net susceptance will be zero

(2) Admittance is min ie, $Y = \frac{1}{R}$ & Impedance is max.

(3) current is min $\because$ Z is max. $I = V/Z$

$$\frac{1}{jX_L} + \frac{1}{-jX_c}$$

$$\frac{-j}{X_L} + \frac{j}{X_c} = -jB_L + jB_c$$

$$j(B_c - B_L) = 0$$

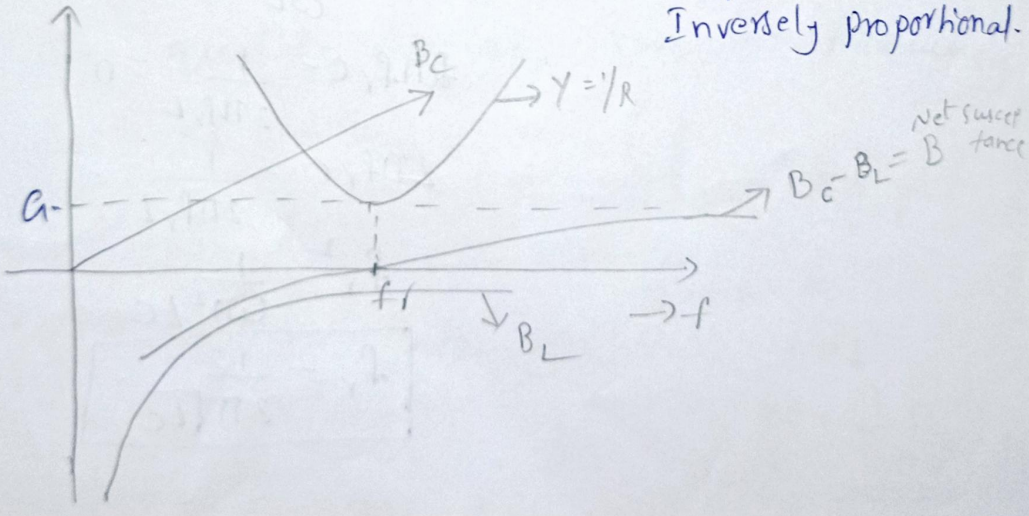
net susceptance $\boxed{B_c - B_L = 0}$

$$\Rightarrow B_c = \frac{1}{X_c} = \frac{1}{1/\omega c} = \frac{1}{1/2\pi fc} = 2\pi fc$$

$f \propto B_c$ (Directly proportional.)

$$\Rightarrow B_L = \frac{1}{X_L} = \frac{1}{2\pi fL} = \frac{1}{2\pi fL}$$ ie, $B_L \propto \frac{1}{f}$

Inversely proportional.



Relationship b/n Quality factor, fr & Bandwidth.

Quality factor :-

It is the ratio of b/n resonant frequency & band width.

$$Q = \frac{f_r}{B.W}$$

$$W.K.T = f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$B.W = \frac{R}{2\pi L}$$

$$Q = \frac{\frac{1}{2\pi\sqrt{LC}}}{R/2\pi L} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\therefore \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

UNIT - IV

Magnetic circuits

The presence of charges in space or in a medium creates an electric field. similarly the flow of current in a conductor sets up a magnetic field.

Electric field is represented by electric flux lines, magnetic flux lines are used to describe the magnetic field.

The path of the magnetic flux lines is called magnetic ckt (or)

The path formed for the passage of flux is known as magnetic circuit.

Just as a flow of current in the electric circuit require the presence of an electromotive force, so the production of magnetic flux requires the presence of magneto-motive force (mmf).

Now we will discuss some properties related to magnetic flux.

(i) **Magneto motive force (MMF):-** A measure of the ability of a coil to produce flux is called the magneto-motive force. It may be considered as a magnetic pressure, just as emf is considered as an electric pressure. A coil with 'N' turns which is carrying a current of I amperes constitutes a magnetic circuit

& produces an mmf of NI ampere turns. The source of flux (ϕ) in the magnetic circuit is the mmf. The flux produced in the circuit depends on mmf & the length of the circuit.

$$MMf = NI \quad \text{ie, Ampere turns (AT)}.$$

(ii) **Flux density (B)** The magnetic flux lines start & end in such a way that they form closed loops. Flux density (B) is the flux per unit area.

units of magnetic flux is webers (Wb).

$$B = \frac{\phi}{A} \quad Wb/m^2 \text{ (or) Tesla.}$$

(iii) **Magnetic field Strength (H)** (or) Magnetic field intensity

The magnetic field strength of a circuit is given by the mmf per unit length.

$$H = \frac{NI}{l} \quad AT/m.$$

where l is the mean length of the magnetic path ie, $2\pi R$

Relation b/n B & H in a medium is given by the equation,

$$B = \mu H$$

where $\mu = \mu_0 \mu_r$ is the permeability of the medium in Henry/metre (H/m).

μ_0 = absolute permeability of free space

$= 4\pi \times 10^{-7}$ H/m

& μ_r = relative permeability of the medium.

$\mu_r = 1$ for air & non magnetic materials.

Reluctance: It is the property of the medium which opposes the passage of magnetic flux. The magnetic reluctance is analogous to resistance in the electric ckt.

units is AT/wb.

Air has a much higher reluctance than does iron or steel. For this reason, magnetic circuits used in electric machines are designed with very small air gaps.

$$\text{reluctance} = \frac{\text{mmf}}{\text{flux}}$$

Reciprocal of reluctance is known as permeance

$$\text{ie> permeance} = \frac{1}{\text{reluctance}} = \frac{\text{flux}}{\text{mmf}} \quad \text{wb/AT}$$

The reluctance of the magnetic circuit is directly proportional to length(l) & inversely proportional to area of cross section.

$$\text{reluctance} \propto l/a$$

$$\text{reluctance} = \frac{l}{\mu a}$$ where μ is permeability of the medium.

$$\text{reluctance} = \frac{\text{mmf}}{\text{flux}} = \frac{l}{\mu a}$$

$$\frac{NI}{\phi} = \frac{l}{\mu a} \Rightarrow \frac{NI}{l} = \frac{\phi}{\mu a}$$

$$H = \frac{1}{\mu} \cdot B \Rightarrow \boxed{B = \mu H}$$

Analogy b/n Magnetic & Electric circuit

Electric ckt	Magnetic circuit
EMf in volts	MMF in AT
current in Amperes $= \frac{V}{R}$	flux in wb $= \frac{MMf}{reluctance}$
voltage drop $= IR$ volts.	MMf drop $= \phi \times$ reluctance AT.
Electric field intensity $= \frac{V}{l}$ volt/m	Magnetic field strength (or) intensity $= \frac{NI}{l}$ AT/m
current density $(J) = \frac{I}{a}$ Amp/m^2	Flux density $(B) = \phi/A$ wb/m^2.
Resistance $R = \frac{\rho l}{a}$ Ω.	Reluctance $= \frac{l}{\mu A}$ AT/Web.
conductance $(G) = 1/R$ $\mho$	permeance $= \frac{1}{\text{reluctance}} = \frac{\mu a}{l}$ web/AT

Faraday's laws of Electromagnetic Induction:-

Faraday's first law states that whenever the no. of Magnetic lines of flux linking with the coil changes an emf is induced in the coil.

Faraday's second law states that magnitude of the induced emf is directly proportional to rate of change of flux linkages.

$$e \propto \frac{d\psi}{dt}$$

$$e = N \frac{d\phi}{dt}.$$

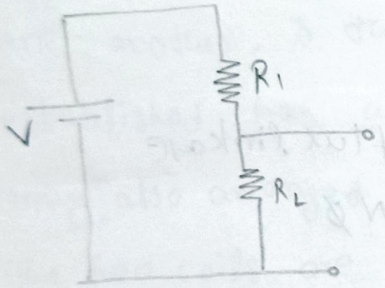
Coupled circuits

Two circuits are said to be coupled when energy transfer takes place from one circuit to the other when of the circuits is energised. There are many types of couplings like conductive coupling as shown by the potential divider in fig (a).

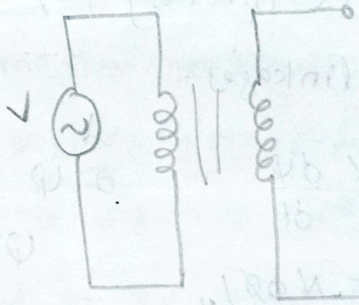
Inductive or magnetic coupling as shown by a two winding transformer in fig(b).

conductive and inductive coupling as shown by an auto transformer in fig (c).

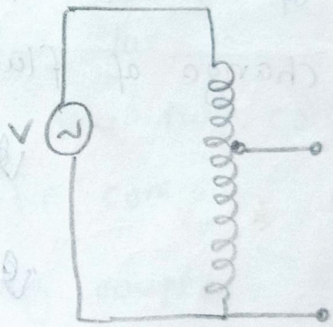
A majority of the electrical circuits in practice are conductively or electromagnetically coupled.



(a)

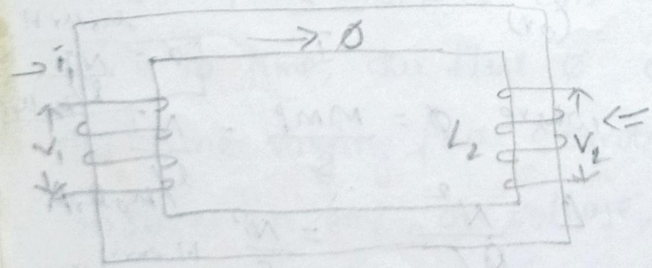


(b)

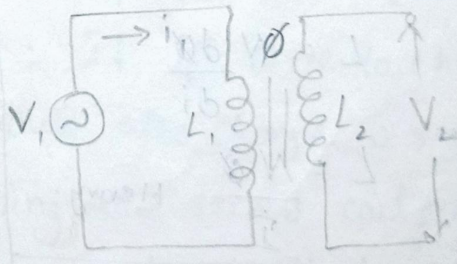


(c)

Self Inductance:



(b)



(a)

As we already discussed in the previous units about the property of inductance of a coil.

A voltage is induced in a coil when there is a time rate of change of current through it.

$\therefore v(t) = L \frac{di(t)}{dt}$ where $v(t)$ is the voltage across the coil.

$i(t)$ is the current through the coil & L is the inductance of the coil.

According to faraday's laws of electromagnetic induction, whenever flux linking with a ckt or conductor or coil changes an emf is induced in it & that magnitude of induced emf is directly proportional to rate of change of flux linkages.

$v \propto \frac{d\psi}{dt}$ & ψ = flux linkage

$\psi = N\phi$

$v = N d\phi/dt$

$\therefore v = L di/dt = N d\phi/dt$

$L = N \frac{d\phi}{di}$

$L = \frac{N\phi}{I}$ Henry.

(or)

$L = \frac{N\phi}{I}$

where $\phi = \frac{MMF}{S} = \frac{NI}{l/\mu_0\mu_r A}$

$L = \frac{N^2}{l/\mu_0\mu_r A} = \frac{N^2}{S}$ Henry.

$H = \frac{NI}{l}$

$B = \phi/A$

$\phi = B \cdot A$

$= \mu_0\mu_r H$

$\phi = \frac{NI}{l/\mu_0\mu_r A}$

$\phi = \frac{MMF}{Reluctance}$

$\phi = NI/S$

(or)

$L = \frac{N\phi}{I} \Rightarrow \frac{N\phi}{I} \cdot \frac{N}{N} = \frac{N^2\phi}{NI} = N^2/S$ $\because S = \frac{NI}{\phi} \Rightarrow 1/S = \phi/NI$

Self Inductance:

An induced EMF due to change of flux produced by relative motion b/n the conductor and flux is called Dynamically induced EMF.

An induced EMF due to a change of flux produced by a time varying current $i(t)$ flowing through the conductor is called statically induced EMF.

The magnitude of statically induced EMF in a coil having N turns & carrying a current $i(t)$, is $\propto$ to the rate of change of flux linkages.

$e \propto$ rate of change of flux linkages.

$$e \propto \frac{d\psi}{dt} \qquad \psi = N\phi$$

$$e = N\frac{d\phi}{dt} \text{ volts.}$$

w.k.t $\quad e = L\, di/dt$

$$N\frac{d\phi}{dt} = L\frac{di}{dt}$$

Self inductance $\boxed{L = N\frac{d\phi}{di}}$

$$L = \frac{N\phi}{I} \Rightarrow \frac{N}{I}\,\frac{NI}{\mathcal{R}} \qquad \mathcal{R} = \frac{l}{\mu_0\mu_r A}$$

$$\boxed{L = \frac{N^2}{\mathcal{R}} = \frac{N^2}{l/\mu_0\mu_r A} = \frac{N^2}{S}}$$

$$L = \frac{N\phi}{I}\cdot\frac{N}{N}$$

$$L = \frac{N^2\phi}{NI} \Rightarrow \boxed{\frac{N^2}{S} = L}$$

$$\phi = \frac{MMF}{reluc} \qquad \phi = \frac{NI}{\mathcal{R}} \Rightarrow \phi = NI/S$$

$$\phi = \frac{NI}{S} = \frac{NI}{\frac{l}{\mu A}}$$

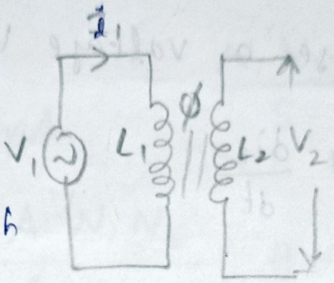
$$\phi = \frac{NI}{\frac{l}{\mu_0\mu_r A}}$$

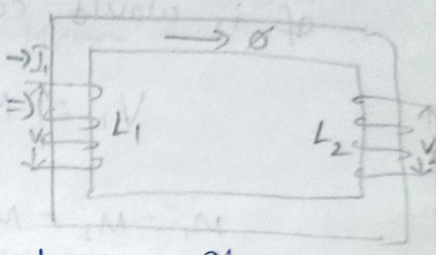
Concept of Mutual Inductance :-

Mutual Inductance is the property associated with two or more coils or inductors which are in close proximity & the presence of common magnetic flux which links the coils.

A T/F is such a device whose operation is based on mutual inductance.

Let us consider two coils L_1 & L_2 are shown in fig. which are sufficiently close together, so that the flux produced by i_1 in coil L_1 also link coil L_2.





We assume that the coils do not move with respect to one another, & the medium in which the flux is established has a constant permeability. The two coils may be also arranged on a common magnetic core.

The two coils are said to be magnetically coupled.

When a voltage V_1 is applied across L_1, a current i_1 will start flowing in this coil & produces the flux ϕ & this flux also links coil L_2. If i_1 were to change w.r.to time, the flux ϕ would also change w.r.to time. The time varying flux surrounding the second coil, L_2 induces an emf, or voltage, across the terminal of L_2; this voltage is proportional to the time rate of change

of current flowing through the first coil L_1.

emf of mutual induction,

$$V_2 = M_2 \frac{di_1}{dt}$$

If current i_2 is made to pass through coil L_2 as shown in fig below with coil L_1 open, a change of i_2 would cause a voltage V_1 in coil L_1 given by

$$V_1 = M_1 \frac{di_2}{dt}$$

$M_1 = M_2 = M$

$V_1 = M\, di_2/dt$ volts, $V_2 = M \frac{di_1}{dt}$ volts

$$e_1 = N_1 \frac{d\phi_1}{dt}, \quad e_2 = N_2 \frac{d\phi_2}{dt}$$

coefficient of coupling

we know that

$$L_1 = \frac{N_1^2}{S} = \frac{N^2}{l/\mu_0 \mu_r A}$$

$$L_2 = \frac{N_2^2}{S} = \frac{N_2^2}{l/\mu_0 \mu_r A}$$

$$\phi_1 = \frac{N_1 I_1}{l/\mu_0 \mu_r A} = \frac{NI}{S}$$

suppose a fraction k_1 of this flux ie, $k_1 \phi_1$ is linked with other coil. B

$$\therefore M = \frac{k_1 \phi_1 \times N_2}{I_1}$$

$$M = N_2 \frac{\phi_{12}}{I_1}$$

$$M = k_1 \frac{N_1 N_2}{l/\mu_0 \mu_r A}$$

$$\phi_2 = \frac{N_2 I_2}{l/\mu_0 \mu_r A}$$

Suppose a fraction k_2 of this ϕ_2 ie, $k_2 \phi_2$ is linked with coil A.

$$M = k_2 \frac{N_1 N_2}{l/\mu_0 \mu_r A}$$

$$\therefore M^2 = K_1 K_2 \frac{N_1^2}{\mathcal{R}_{mol} A} \cdot \frac{N_2^2}{\mathcal{R}_{mol} A}$$

$$= K_1 K_2 L_1 L_2$$

$$M = \sqrt{K_1 K_2} \sqrt{L_1 L_2}$$

$$M = K \sqrt{L_1 L_2} \qquad \because K = \sqrt{K_1 K_2}$$

$$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$$

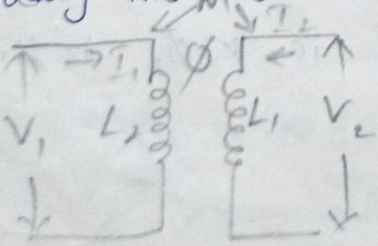
K is called the coefficient of coupling.

If the flux linking due to one coil completely links with the other then value of 'K' is unity.

If the flux of one coil does not at all link with the other, then $K = 0$.

In the first case, when $K = 1$, coils are said to be tightly coupled & when $K = 0$ the coils are magnetically isolated from each other.

Dot convention :- It gives the information that there is a mutual inductance b/n the coils & also that it is used to establish the choice of correct sign for the mutually induced voltages in coupled circuits.



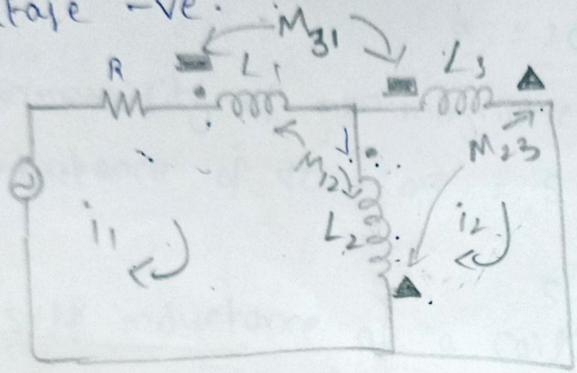
$$V_1 = L_1 \, di_1/dt \pm M \, di_2/dt$$

$$V_2 = L_2 \, di_2/dt \pm M \, di_1/dt$$

If both the currents entering or leaving the dots the sign of the voltage is +ve.

If one of the coils current is entering the dot & another coil current is leaving the dot then the sign of the voltage -ve.

Ex:



Applying KVL for loop ①

$$i_1 R_1 + L_1 \frac{di_1}{dt} + M_{21} \frac{d(i_1 - i_2)}{dt} + L_2 \frac{d}{dt}(i_1 - i_2) + M_{31} \frac{di_2}{dt}$$

$$+ M_{12} \frac{di_1}{dt} + M_{32} \frac{di_2}{dt} = v$$

②

$$L_2 \frac{d(i_2 - i_1)}{dt} + L_3 \frac{di_2}{dt} - M_{32} \frac{di_2}{dt} - M_{12} \frac{di_1}{dt}$$

$$+ M_{13} \frac{di_1}{dt} - M_{23} \frac{d(i_2 - i_1)}{dt}$$

$$L_2 \frac{d(i_2 - i_1)}{dt} + L_3 \frac{di_2}{dt} - M_{21} \frac{di_1}{dt} - M_{23} \frac{di_2}{dt} + M_{31} \frac{di_1}{dt} = 0$$

$$- M_{32} \frac{d(i_2 - i_1)}{dt} = 0$$

Series connection of coupled circuits

Let there be two inductors connected in series with self inductances L_1 & L_2 & mutual inductance of M. Two kinds of series connections are possible; (a) series aiding
(b) series opposing.

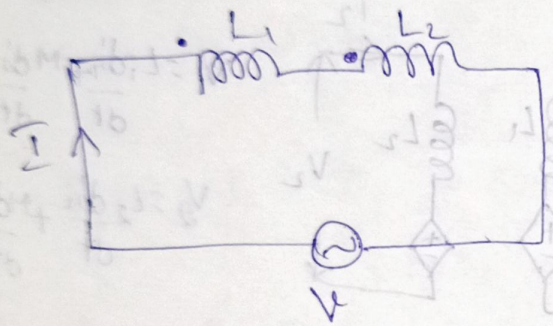


fig:- series aiding

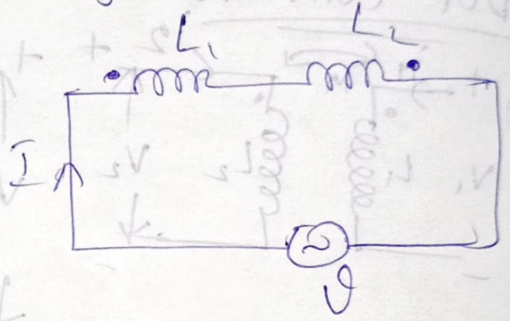


fig:- series opposing

$$v = v_1 + v_2$$

$$V = L_1 \frac{di}{dt} + M \frac{di}{dt} + L_2 \frac{di}{dt} + M \frac{di}{dt}$$

$$v = (L_1 + L_2 + 2M) \frac{di}{dt}$$

$$v = L_{eq} \frac{di}{dt}$$

$$\therefore \boxed{L_{eq} = L_1 + L_2 + 2M}$$

$$v = v_1 + v_2$$

$$v = L_1 \frac{di}{dt} - M \frac{di}{dt} + L_2 \frac{di}{dt} - M \frac{di}{dt}$$

$$v = (L_1 + L_2 - 2M) \frac{di}{dt}$$

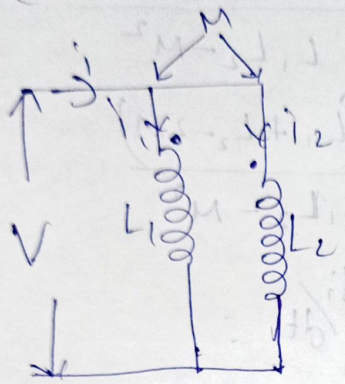
$$v = L_{eq} \frac{di}{dt}$$

$$\therefore \boxed{L_{eq} = L_1 + L_2 - 2M}$$

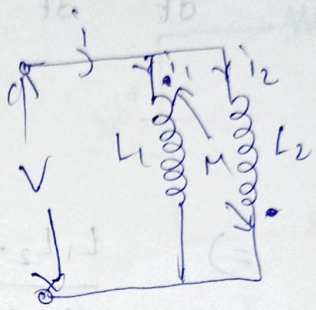
Parallel connection of coupled coils

Consider two inductors with self inductances L_1 & L_2 connected in parallel which are mutually coupled with mutual inductance M. There are two kinds of parallel connection are possible.

(a) Parallel aiding (b) Parallel opposing



Parallel aiding



Parallel opposing.

$$v = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \rightarrow (1)$$

$$v = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \rightarrow (2)$$

$$\begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} di_1/dt \\ di_2/dt \end{bmatrix} = \begin{bmatrix} V \\ V \end{bmatrix}$$

$\because V = L_{eqv} \, di/dt.$

$$\Rightarrow i = i_1 + i_2 = di/dt = di_1/dt + di_2/dt$$

$$\frac{di_1}{dt} = \frac{\begin{bmatrix} V & M \\ V & L_2 \end{bmatrix}}{\begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix}} = \frac{V(L_2 - M)}{L_1 L_2 - M^2} \rightarrow (3)$$

$$\frac{di_2}{dt} = \frac{\begin{vmatrix} L_1 & V \\ M & V \end{vmatrix}}{\begin{vmatrix} L_1 & M \\ M & L_2 \end{vmatrix}} = \frac{V(L_1 - M)}{L_1 L_2 - M^2} \rightarrow (4)$$

So

(a) $\therefore di/dt = \frac{di_1}{dt} + \frac{di_2}{dt} = \frac{V(L_2 - M) + V(L_1 - M)}{L_1 L_2 - M^2}$

$$= \frac{V(L_1 + L_2 - 2M)}{L_1 L_2 - M^2}$$

$\left(V = L_{eqn} \frac{di}{dt} \right) \Rightarrow \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \; di/dt$

$\therefore$ L_{eqn} or $L_{eff} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$

Ans if the voltage induced due to mutual inductance oppose the self induced emf in each coil by the dot convention.

$\therefore$ The equivalent inductance is given by

$$L_{eqn} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

→ A non magnetic core has the core length of 100 cm & its cross sectional area is 20 cm^2. It carries a coil of 500 turns. Find its self inductance.

Sol: Given non magnetic core : $\mu = \mu_0$, $N = 500$

$l = 100\ cm = 1\ m$

$a = 20\ cm^2 = 20 \times 10^{-4}\ m^2$

Permeability of non magnetic core $= \mu = \mu_0 = 4\pi \times 10^{-7}$ H/m

reluctance of the core $= S = \frac{l}{\mu_0 A} = \frac{1}{4\pi \times 10^{-7} \times 20 \times 10^{-4}}$

$S = 397.887 \times 10^6$ AT/Wb.

self inductance of a coil, $L = \frac{N^2}{S}$

$L = \frac{(500)^2}{397.887 \times 10^6}$

$L = 0.628 \times 10^{-3}$ henry $= 0.62$ mH.

→ An iron ring having a mean length circumference of 250 mm & square cross section of 20 mm × 20 mm is wound with a coil of 100 turns. Calculate (i) current in the coil to set up a magnetic flux of 480×10^{-6} wb

(ii) The self inductance of the coil at this current,

(iii) self induced voltage in the coil when the current is switched off in 0.005 sec.

B (wb/m^2)	1.0	1.2	1.4
H (AT/m)	350	600	1250

Sol:- Given iron ring : $l = 250\ mm = 0.25\ m$

$a = 20 \times 20 = 400\ mm^2 = 400 \times 10^{-6}\ m^2$

(i) Flux density in the ring $B = \phi/a = \frac{480 \times 10^{-6}}{400 \times 10^{-6}} = 1.2$ T.

$N = 100$, $\phi = 480 \times 10^{-6}$ wb, $di = I_2 - I_1$

$= 0 - I$ $dt = 0.005$ sec.

From B-H curve data, for $B = 1.2\ T$, $H = 600\ A/m$

$\therefore$ Total mmf $= H \times l = 600 \times 0.25 = 150\ AT$

$\therefore$ coil current $= I = \frac{mmf}{N} = \frac{150}{100} = 1.5\ A$

(ii) self inductance of coil $= L = \frac{N\phi}{I} = \frac{100 \times 1.480 \times 10^{-6}}{1.5}$

$= 0.032\ H = 32\ mH$.

(iii) self induced voltage $= e = -L\, di/dt$

$= -0.032 \times \frac{0-I}{dt}$

$= -0.032 \times \frac{0-15}{0.005} = 9.6$ Volts.

→ Two Identical 1000 turns coils X & Y lie in ||el planes such that 60 percent of the lines of force produced by one coil link the other. A current of 5A in X produces in it a flux of 0.5 m wb. If the current in X changes from +6A to −6 A in 0.1 sec. what will be the magnitude of the electromotive force induced in Y? calculate self inductance of each coil & the mutual inductance. (June 2009) · 6M.

Sol: Given two identical coils X & Y : $N_X = N_Y = 1000$,

coefficient of coupling $= k = 0.6$,

$I_X = 5A$, $\phi_X = 0.5\ m\ wb = 0.5 \times 10^{-3}\ wb$,

$di_X = i_{X2} - i_{X1} = (-6) - 6$, $dt = 0.1$ sec.

As the coils are identical, their self inductances are same.

self inductance of coil X $= L_X = \frac{N_X \phi_X}{I_X} = \frac{1000 \times 0.5 \times 10^{-3}}{5}$

$= 0.1\ H$.

$\therefore$ self inductance of coil Y $= L_Y = L_X = 0.1\ H$.

Mutual inductance : $M = k\sqrt{L_x L_y} = 0.6\sqrt{0.1 \times 0.1}$

$= 0.06$ H.

emf induced in a coil Y : $e_y = -M\frac{di_x}{dt} = -0.06 \times \frac{-6-6}{0.1}$

$= 7.2$ Volts.

→ The no. of turns in a coil is 250 when a current of 2A flows in this coil the flux in the coil is 0.3 mwb. when this current is reduced to zero in 2ms, the voltage induced in a coil lying in the vicinity of coil is 63.75 V. If the coefficient of coupling b/n the coils is 0.75, find self inductances of two coils, mutual inductance & no. of turns in the second coil. (June 2008) COM

Sol:- Given $N_1 = 250$, $I_1 = 2A$, $\phi_1 = 0.3$ m wb $= 0.3 \times 10^{-3}$ wb

$dI_1 = (0-2)$, $dt = 2$ ms $= 2 \times 10^{-3}$ sec

$E_{21} = 63.75$ V, $k = 0.75$.

self inductance of first coil $= L_1 = \frac{N_1 \phi_1}{I_1} = \frac{250 \times 0.3 \times 10^{-3}}{2}$

$L_1 = 37.5 \times 10^{-3}$ H $= \boxed{37.5 \text{ mH}}$.

Emf induced in second coil, $E_{21} = -M\frac{dI_1}{dt} = 63.75$

$63.75 = -M\frac{(0-2)}{2 \times 10^{-3}}$ $\Rightarrow M = 63.75 \times 10^{-3}$ H

$\boxed{M = 63.75 \text{ mH}}$.

But $M = k\sqrt{L_1 L_2}$ (or) $M^2 = k^2 L_1 L_2$

$M^2 = (0.75)^2 (37.5 \times 10^{-3}) L_2$

$(63.75 \times 10^{-3})^2 = (0.75)^2 (37.5 \times 10^{-3}) L_2$

$L_2 = 192.67$ mH.

$$L_1 = \frac{N_1^2}{S} \quad \& \quad L_2 = \frac{N_2^2}{S}$$

$$\frac{L_2}{L_1} = \left(\frac{N_2}{N_1}\right)^2 \Rightarrow \frac{N_2}{N_1} = \sqrt{\frac{L_2}{L_1}}$$

$$N_2 = N_1\sqrt{\frac{L_2}{L_1}} = 250\sqrt{\frac{192.67}{37.5}}$$

$$\boxed{N_2 = 566.67}$$

→ Two inductors have self inductances of 0.1 mH & 0.4 mH & a mutual inductance of 0.15 mH. What is the value of coefficient of coupling b/n them? If a current of $I(t) = 3\sin t + 1.5\sin 2t$ is passed through the first inductor, what is the expression for the voltage induced in the second coil. (June 2009) 8 M.

Sol: Given $L_1 = 0.1$ mH, $L_2 = 0.4$ mH, $M = 0.15$ mH,

$$I_1(t) = 3\sin t + 1.5\sin 2t$$

$M = k\sqrt{L_1 L_2}$ ⇒ coefficient of coupling $k = \frac{M}{\sqrt{L_1 L_2}}$

$$k = \frac{0.15}{\sqrt{0.1 \times 0.4}} = 0.75$$

Voltage induced in second coil is given by

$$V_2(t) = -M\frac{dI_1}{dt} = -0.15\times10^{-3}\frac{d}{dt}(3\sin t + 1.5\sin 2t)$$

$$= -0.15\times10^{-3}(3\cos t + 1.5\times 2\cos 2t)$$

$$V_2(t) = 3\cos t + 3\cos 2t \text{ volts}.$$

$$V_2(t) = -0.45\times10^{-3}\cos t - 0.45\times10^{-3}\cos 2t$$

$$= -0.45\times10^{-3}(\cos t + \cos 2t)$$

→ A non magnetic ring having a mean diameter of 30 cm & a cross sectional area of 4 cm^2 is uniformly wound with two coils A & B one over other. A has 100 turns & B has 250 turns. Calculate the mutual inductance b/n the coils. Also calculate the emf induced in B when a current of 6 A in A is reversed in 0.02 sec. Derive the formulae used. (June 2008) 16 M

Sol:- Given non magnetic ring, $\mu_r = 1$ ∴ $d = 30\ cm = 0.3\ m$

$a = 4\ cm^2 = 4 \times 10^{-4}\ m^2$.

$N_A = 100$ $N_B = 250$ $I_1 = 6\ A$, $I_2 = -6\ A$, $dt = 0.02\ sec$

Reluctance $S = \frac{l}{\mu_0 \mu_r a} = \frac{\pi d}{4\pi \times 10^{-7} \times 1 \times 4 \times 10^{-4}} = \frac{\pi \times 0.3}{4\pi \times 10^{-7} \times 1 \times 4 \times 10^{-4}}$

$= 1875 \times 10^3\ AT/wb.$

Mutual inductance, $M = \frac{N_A N_B}{S} = \frac{100 \times 250}{1875 \times 10^3} = 0.013\ H$

EMF induced in coil B $= e_B = -M \frac{dI_A}{dt} = -0.013 \times \frac{(-6)-6}{0.02}$

$\boxed{e_B = 7.8\ V}$.

→ The mean diameter of a steel ring is 40 cm & flux density of 0.9 wb/m^2 is produced by 3500 Ampere turns per metre. If the cross section of the ring be 15 cm^2 & the number of turns 400. Calculate.

(1) The exciting current

(2) The self inductance

(3) The exciting current & the inductance when an air gap of 2mm is cut in the ring; the flux density being the same.

Ignore leakage & Ringing. (June 2008) 16 M.

Sol:- Given: Steel ring: $d = 40\ cm = 0.4\ m$

$B = 0.9\ T$, $H = 3500\ AT/m$

$a = 15\ cm^2 = 15 \times 10^{-4}\ m^2$, $N = 440$, $l_g = 2\ mm = 2 \times 10^{-3}\ m$.

(a) Steel ring without air gap.

$$mmf_s = H \cdot l = H \times \pi d = 3500 \times \pi \times 0.4 = 4398\ AT.$$

$\therefore$ Exciting current, $I = \frac{mmf}{N} = \frac{4398}{440} = 10\ A$.

(b) Flux $\phi = B \cdot a = 0.9 \times 15 \times 10^{-4} = 13.5 \times 10^{-4}\ Wb$.

$\therefore$ self inductance, $L = \frac{N\phi}{I} = \frac{440 \times 13.5 \times 10^{-4}}{10} = 0.0594\ H$.

(c) Steel ring with air gap.

$$mmf_g = H_g \cdot l_g = \frac{B_g \cdot l_g}{\mu_0} = \frac{0.9 \times 2 \times 10^{-3}}{4\pi \times 10^{-7}} = 1432\ AT.$$

$\therefore$ Total $mmf = mmf_s + mmf_g$

$= 4398 + 1432 = 5830\ AT$.

$\therefore$ Exciting current, $I = \frac{mmf}{N} = \frac{5830}{440} = 13.25\ A$.

$\therefore$ self inductance, $L = \frac{N\phi}{I} = \frac{440 \times 13.5 \times 10^{-4}}{13.25}$

$L = 0.0448\ H$.

Two port Networks

Z-parameters - For the two port n/w the i/p & o/p voltages V_1 & V_2 can be expressed interms of i/p & o/p currents I_1 & I_2 respectively as

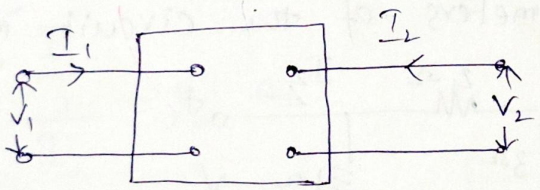
$$[V] = [Z][I]$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

In a two-port n/w representation, the n/w is assumed to be a rectangular box & the directions of i/p port & o/p port currents & voltages have been shown in fig.



$I_1, I_2 \rightarrow$ i/p, o/p currents
$V_1, V_2 \rightarrow$ i/p & o/p voltages

Case(i) Assuming the o/p of the two port n/w to be open circuited

$I_2 = 0$ then

$$V_1 = Z_{11} I_1 + Z_{12}(0) \Rightarrow Z_{11} = \frac{V_1}{I_1}$$

$$V_2 = Z_{21} I_1 + Z_{22}(0) \Rightarrow Z_{21} = \frac{V_2}{I_1}$$

Case(ii) Assuming the i/p port of the same two port n/w to be o/c

ie, $I_1 = 0$ then $V_1 = Z_{11}(0) + Z_{12} I_2 \Rightarrow Z_{12} = V_1/I_2$

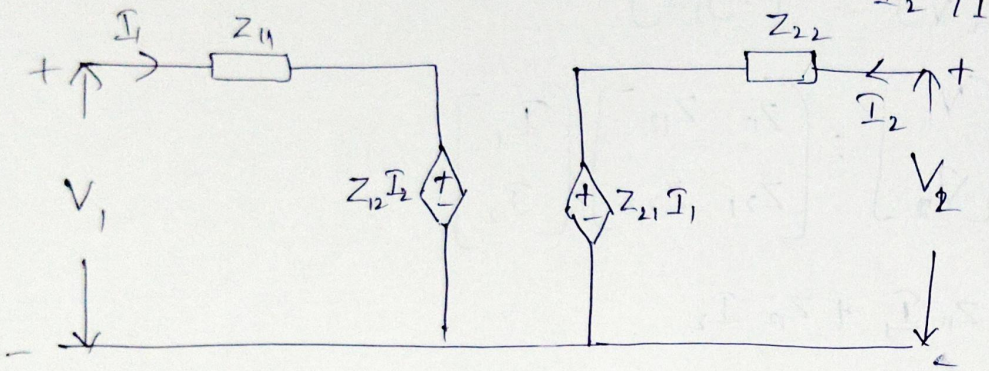
$V_2 = Z_{21}(0) + Z_{22} I_2 \Rightarrow Z_{22} = V_2/I_2$

Input driving point impedance $Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$

Reverse transfer impedance $Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$

Forward transfer impedance $Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$

Output Driving point impedance $Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$

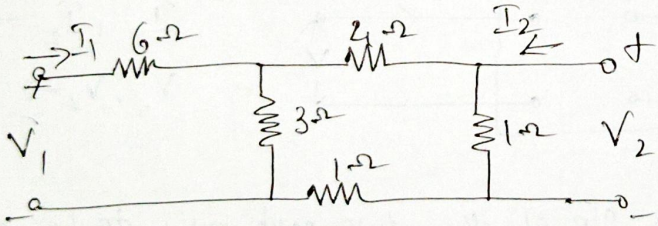


Basic z-parameters equivalent ckt.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{22} I_2 + Z_{21} I_1$$

Ex: Find the z-parameters of the circuit in following fig.



Sol: case (i) when $I_2 = 0$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$R_{eqn} = [(4+1+1)\,//\,3\Omega] + 6$$

$$= [6\,//\,3] + 6$$

$$= 2 + 6$$

$$= 8$$

$$I_1 = \frac{V_1}{R_{eqn}}$$

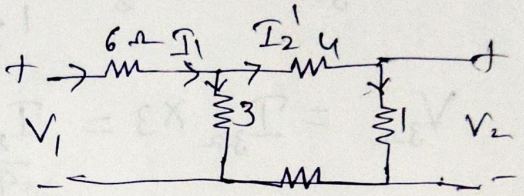
$$= \frac{V_1}{8}$$

$$Z_{11} = \frac{V_1}{I_1} = \frac{V_1}{\frac{V_1}{8}} = 8\ \Omega$$

$$Z_{21} = \frac{V_2}{I_1}$$

V_2 is voltage across 1Ω resistor.

By applying current division principle.



$$I_2' = I_1 \times \frac{3}{3+6}$$

$$= I_1 \frac{3}{9}$$

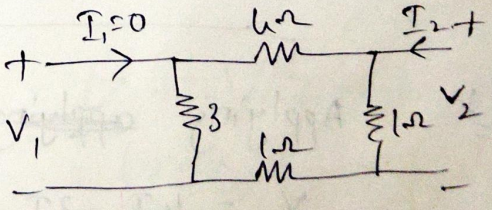
$$I_2' = \frac{I_1}{3}$$

$$V_2 = I_2' \times 1 = \frac{I_1}{3}$$

$$\therefore Z_{21} = \frac{\frac{I_1}{3}}{I_1} = 1/3\ \Omega$$

Case(ii) Now i/p port is o.c $\Rightarrow I_1 = 0$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$



$$R_{eqn} = (3+4+1)//1$$

$$= 8//1$$

$$= \frac{8}{9}$$

$$I_2 = \frac{V_2}{R_{eq}} = \frac{V_2}{8/9}$$

$$Z_{22} = \frac{V_2}{I_2} = 8/9\ \Omega$$

$$Z_{12} = \frac{V_1}{I_2}$$ V_1 is voltage across 3Ω resistor

By applying current division principle.

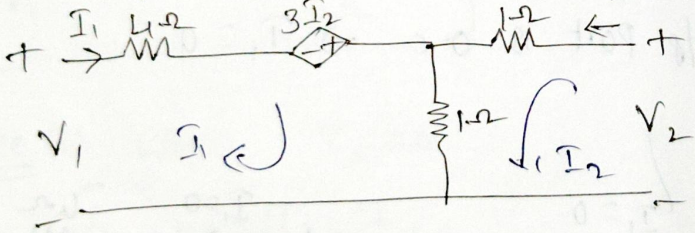
$$I_{3\Omega} = I_2 \times \frac{1}{1+8} = \frac{I_2}{9}$$

$$V_{3\Omega} = I_{3\Omega} \times 3 = \frac{I_2}{9} \times 3 = \frac{I_2}{3} = V_1$$

$$Z_{12} = \frac{\frac{I_2}{3}}{I_2} = 1/3\ \Omega.$$

$$\therefore \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 8 & 1/3 \\ 1/3 & 8/9 \end{bmatrix}.$$

→ Find the Z-parameters of the N/W shown.



Sol:- Applying ~~applying~~ KVL for loop ①

$$V_1 = 4I_1 - 3I_2 + (I_1 + I_2)1$$

$$V_1 = 5I_1 - 2I_2 \qquad V_1 = Z_{11}I_1 + Z_{12}I_2$$

KVL for loop ①

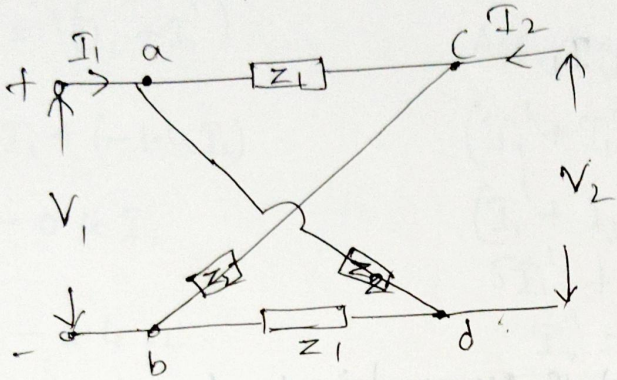
$$V_2 = 1\Omega I_2 + (I_2 + I_1)1$$

$$V_2 = I_1 + 2I_2$$

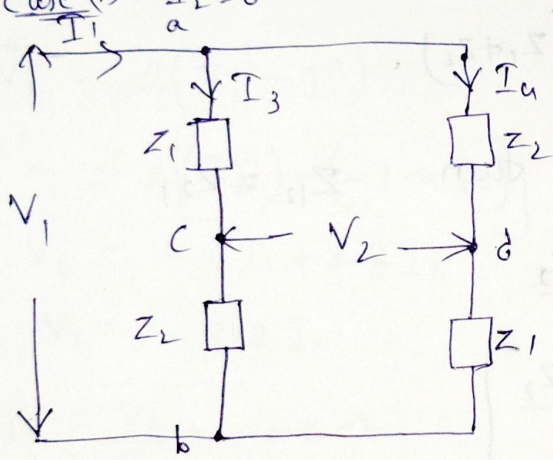
$$\boxed{V_2 = Z_{21} I_1 + Z_{22} I_1}$$

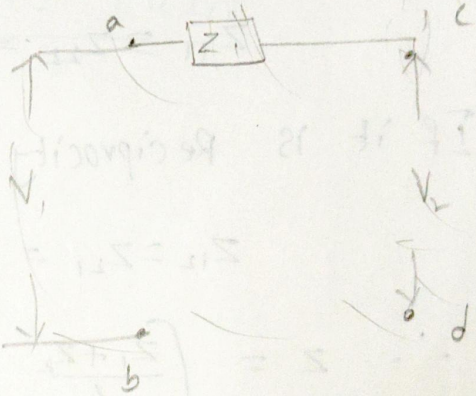
$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_2 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ 1 & 2 \end{bmatrix}$$

Prob:- Find the Z-parameters for the lattice n/w shown.



Sol:- Case (i) $I_2 = 0$





$$V_1 = (Z_{eqv})\, I_1$$

$$= I_1 \left(\frac{Z_1 + Z_2}{2}\right)$$

$$\frac{V_1}{I_1} = \frac{Z_1 + Z_2}{2} = Z_{11}$$

$$Z_{eqv} = (Z_1 + Z_2) /\!/ (Z_1 + Z_2)$$

$$= \frac{(Z_1 + Z_2)(Z_1 + Z_2)}{(2Z_1 + 2Z_2)}$$

$$= \frac{Z_1 + Z_2}{2}$$

$$I_3 = I_1 \times \frac{(Z_1+Z_2)}{(Z_1+Z_2)+(Z_1+Z_2)} = I_1 \times \frac{1}{2}$$

$$I_3 = I_1/2$$

$$\text{W.K.T } I_4 = I_1/2$$

$$V_2 = (V_a - V_b)$$

$$= I_3 \times Z_1 - I_4 \times Z_2$$

$$= \frac{I_1}{2} \times Z_1 - \frac{I_1}{2} \times Z_2$$

$$V_2 = \left(\frac{Z_1 - Z_2}{2}\right) I_1$$

$$\frac{V_2}{I_1} = \frac{Z_1 - Z_2}{2}$$

$$Z_{21} = \frac{Z_1 - Z_2}{2}$$

As the given n/w is symmetrical n/w then

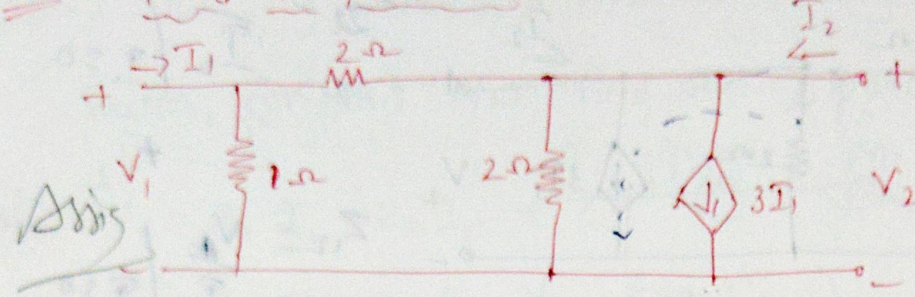
$$Z_{11} = Z_{22} = \tfrac{1}{2}(Z_1+Z_2)$$

If it is Reciprocity n/w then $Z_{12} = Z_{21}$

$$Z_{12} = Z_{21} = \frac{Z_1 - Z_1}{2}$$

$$\therefore Z = \begin{bmatrix} \frac{Z_1+Z_2}{2} & \frac{Z_1-Z_2}{2} \\ \frac{Z_1-Z_2}{2} & \frac{Z_1+Z_2}{2} \end{bmatrix}$$

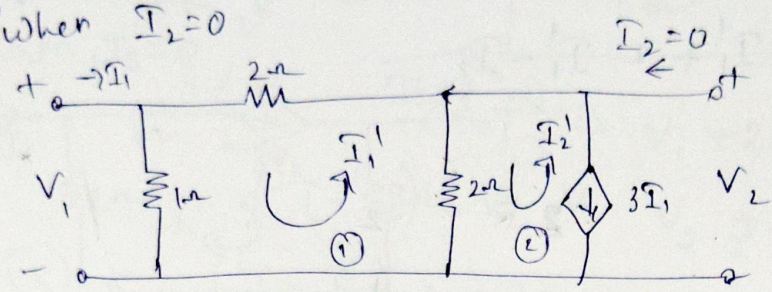
1.Ob Find Z-Parameters



$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

case(i) when $I_2 = 0$



$V_1 = 1(I_1 + I_1')$

$V_1 = I_1 + (-1.4 I_1)$

$V_1 = -0.4 I_1$

$Z_{11} = \frac{V_1}{I_1} = -0.4\ \Omega.$

Applying KVL for loop ①

$(I_1' + I_1) + 2 I_1' + 2(I_1' - I_2') = 0$

$(I_1' + I_1) + 2 I_1' + 2(I_1' + 3I_1) = 0$

$5 I_1' + 7 I_1 = 0$

$I_1' = -\frac{7}{5} I_1$

$\boxed{I_1' = -1.4 I_1}$

$V_2 = 2(I_2' - I_1')$

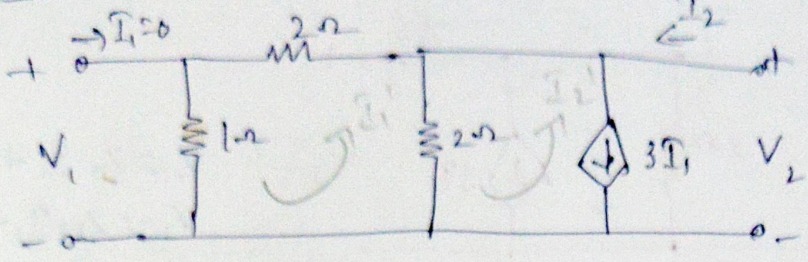
$= 2[(-3I_1) - (-1.4 I_1)]$

$V_2 = -6 I_1 + 2.8 I_1$

$V_2 = -3.2 I_1$

$Z_{21} = \frac{V_2}{I_1} = -3.2\ \Omega.$

case (ii) when $I_1 = 0$



$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

$I_1' = +I_2$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

$V_1 = I_1'$

$V_2 =$

$I_1' \cdot 1 + 2I_1' + 2I_1' - I_2$

$V_2 = 2(I_2 - I_1')$

$V_2 = 2(+I_2 - I_1') \rightarrow (2)$

Apply KVL for loop (1)

$I_1' + 2I_1' + 2(I_1' - I_2') = 0$

$3I_1' + 2(I_1' - I_2) = 0$

$5I_1' - 2I_2 = 0$

$$I_1' = +\frac{2}{5} I_2$$

eqn (2) $\Rightarrow V_2 = 2(+I_2 - \frac{2}{5} I_2)$

$= +2I_2 - \frac{4}{5} I_2$

$V_2 = +\frac{6}{5} I_2$

$$\frac{V_2}{I_2} = Z_{22} = +1.2\,\Omega$$

$2(+I_2 - I_1') = V_2$

$2(+I_2 + \frac{2}{5} I_2) = V_2$

$+2I_2 + \frac{4}{5} I_2 = V_2$

$+10I_2 + 4I_2 = V_2$

$+6I_2 = V_2$

$\frac{6}{5} = +1.2$

$Z_{22} = +1.2\,\Omega$

$\frac{V_2}{I_2}$

$V_1 = I_1'$

$+\frac{2}{5}$

$+0.4 = Z_{12}$

$+\frac{2}{5}$

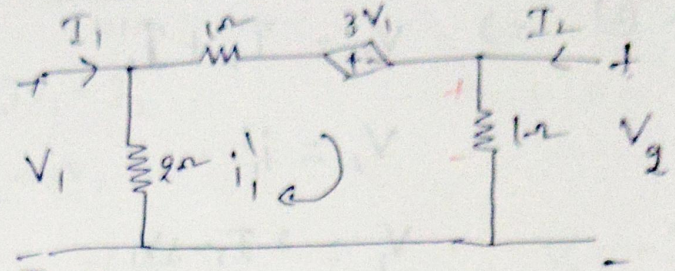
$\frac{V_1}{I_2}$

$Z_{12} = \frac{V_1}{I_2}$

$V_1 = I_1'$

$= +\frac{2}{5} I_2 \Rightarrow \frac{V_1}{I_2} = +\frac{2}{5} = +0.4\,\Omega = Z_{12}$

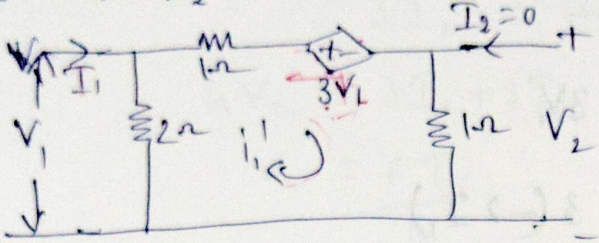
Prob:- The fig. shows a n/w containing a single controlled source, for this n/w, find the Z parameters.



Sol:- $V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

Case(i) when $I_2 = 0$



$V_1 = 2(I_1 - i_1') \rightarrow (1)$

$V_2 = (I_2 + i_1') \rightarrow (2)$

Applying KVL for the loop

$V = IR$

$I_1 = 2A$

$3I_1$

$3 \times 2 = 6V$

$$2(i_1' - I_1) + i_1' + 3V_1 + i_1' = 0$$

$$2i_1' - 2I_1 + i_1' + 3V_1 + i_1' = 0$$

$$4i_1' - 2I_1 + 3V_1 = 0$$

$$4i_1' = 2I_1 - 3V_1$$

$$i_1' = \frac{2I_1 - 3V_1}{4}$$

Substitute 'i_1'' in eqn (1)

$$V_1 = 2\left[I_1 - \left(\frac{2I_1 - 3V_1}{4}\right)\right]$$

$$= 2I_1 - \frac{2I_1 + 3V_1}{2}$$

$$= \frac{2I_1}{2} + \frac{3V_1}{2}$$

$$V_1 = I_1 + \frac{3V_1}{2} \Rightarrow I_1 = V_1 - \frac{3V_1}{2} \Rightarrow \frac{+2V_1 - 3V_1}{2} = -V_1/2$$

$$\frac{V_1}{I_1} = \boxed{2 = Z_{11}}$$

eqn (2) $\Rightarrow$ $V_2 = I_2 + i_1'$

$V_2 = i_1'$ $\quad I_2 = 0$ $\therefore$ port 2 is O.C.

$$V_2 = \frac{2I_1 - 3V_1}{4}$$

$$4V_2 = 2I_1 - 3V_1$$

$$= 2I_1 - 3(-2I_1)$$

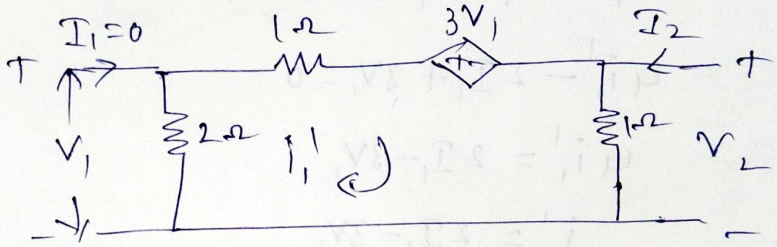
$$= 2I_1 + 6I_1$$

$$4V_2 = 8I_1$$

$$\frac{V_2}{I_1} = \boxed{2 = Z_{21}}$$

case(ii)

When $I_1 = 0$



Applying KVL for the loop,

$$2.\ i_1' + i_1' + 3V_1 + (i_1' + I_2) = 0$$

$$4i_1' + 3V_1 + I_2 = 0$$

$$i_1' = \frac{-3V_1 - I_2}{4}$$

$$V_1 = -2\left(\frac{-3V_1 - I_2}{4}\right)$$

$$= \frac{3V_1}{2} + \frac{I_2}{2} \Rightarrow -\frac{V_1}{2} = \frac{I_2}{2} = \frac{V_1}{I_2} = \boxed{-1 = Z_{12}}$$

$V_1 = 2(I_1 - i_1')$ $\rightarrow$ (3)

$V_1 = -2i_1'$ $\rightarrow$

$V_2 = (I_2 + i_1')$ $\rightarrow$ (4)

$V_2 = I_2 + i_1$

$= I_2 + \frac{-3V_1 - I_2}{4}$

$4V_2 = 4I_2 - 3V_1 - I_2$

$4V_2 = 3I_2 - 3V_1$

$= 3I_2 - 3(-I_2)$ $\quad \because V_1 = -I_2$

$4V_2 = 3I_2 + 3I_2$

$V_2 = \frac{6I_2}{4}$

$\frac{V_2}{I_2} = \boxed{3/2 = Z_{22}}$.

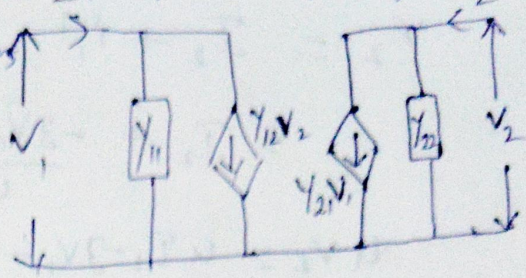
$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 2 & 3/2 \end{bmatrix}$.

Y-Parameters

The Y-parameters of a two port network can be obtained by expressing I_1 & I_2 interms of V_1 & V_2.

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$



when $V_2 = 0$

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$$ = short ckt input driving point admittance

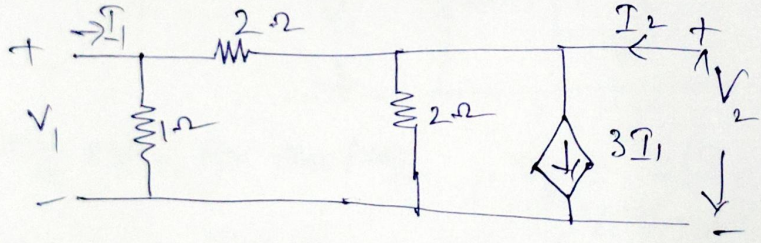
$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$ = Forward transfer admittance

when $V_1 = 0$

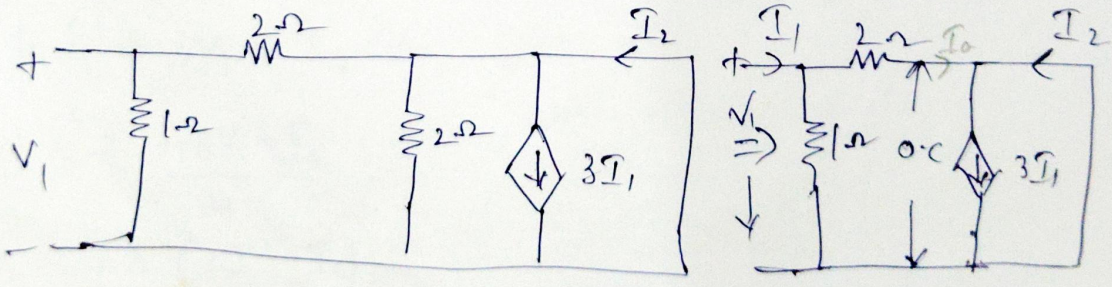
$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$ - reverse transfer admittance

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$ - short ckt output driving admittance

Prob Find the Y-parameters for the given n/w shown in below



case (i) when $V_2 = 0$



$$I_a = 3I_1 - I_2 \rightarrow (A)$$

$$V_1 = (I_1 - I_a)1 \rightarrow (1)$$

Apply KVL,

$$(I_a - I_1) + 2I_a = 0$$

$$3I_a = I_1$$

$$\boxed{I_a = \frac{I_1}{3}}$$

$(A) \Rightarrow$ $$\frac{I_1}{3} = 3I_1 - I_2$$

$$I_1 = 9I_1 - 3I_2$$

$$3I_2 = 8I_1$$

$$\boxed{I_2 = \frac{8}{3}I_1}$$

$(1) \Rightarrow$ $$V_1 = I_1 - I_a$$

$$= I_1 - \frac{I_1}{3}$$

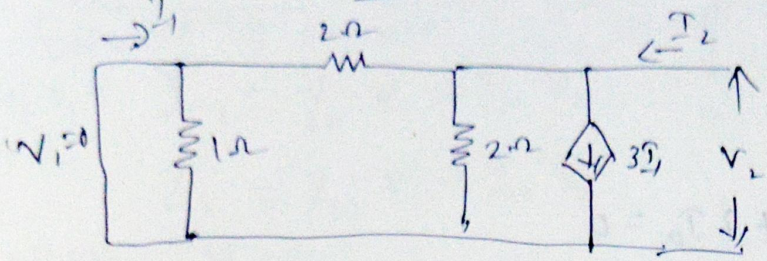
$$= \frac{2I_1}{3}$$

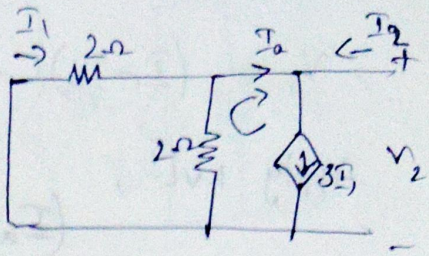
$$\frac{I_1}{V_1} = \frac{3}{2} = \boxed{1.5 = Y_{11}}$$

$$Y_{21} = \frac{I_2}{V_1} = \frac{\frac{8}{3}I_1}{\frac{2}{3}I_1} = \frac{8}{2} = 4 \quad \therefore \boxed{Y_{21} = 4}$$

~~case~~ ii

Case (ii) when $V_1 = 0$





$$2I_1 + (I_1 - I_a)2 = 0$$

$$4I_1 - 2I_a = 0$$

$$2I_a = 4I_1$$

$$I_a = 2I_1 \rightarrow (3)$$

$$V_2 = -2(I_a - I_1) \rightarrow (4)$$

$$V_2 = 2(2I_1 - I_1)$$

$$V_2 = -2I_1$$

$$I_a + I_2 = 3I_1$$

$(1) \Rightarrow 2I_1 + I_2 = 3I_1$

$$\boxed{I_2 = I_1}$$

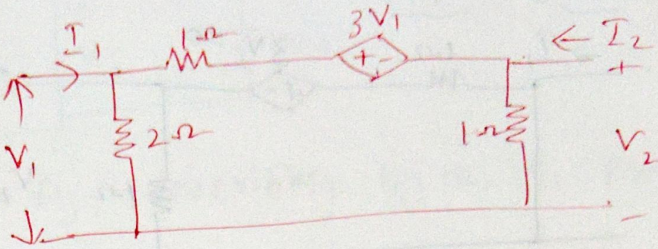
$$Y_{12} = \frac{I_1}{V_2} = \frac{1}{2}$$

$$(2) \Rightarrow Y_{22} = \frac{I_2}{V_2} = \frac{I_1}{2I_1} = \frac{1}{2}$$

$$\therefore \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ 4 & -\frac{1}{2} \end{bmatrix}.$$

Prob

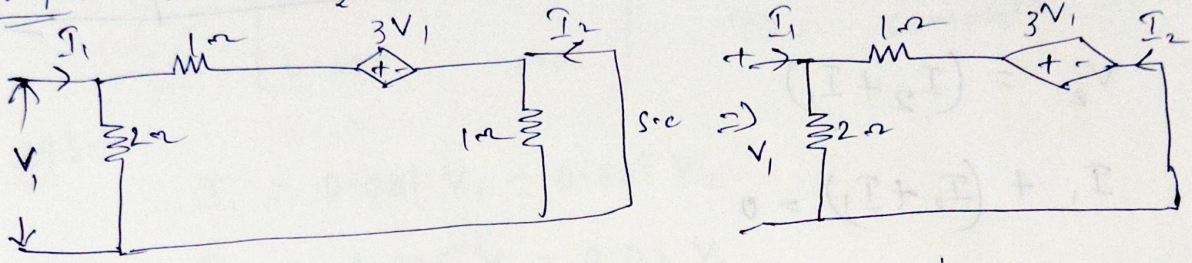
Find the Y-parameters for the n/w shown. (7)



$I_1 = Y_{11} V_1 + Y_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$

step ① when $V_2 = 0$



$Y_{11} = \frac{I_1}{V_1}$

$Y_{21} = \frac{I_2}{V_1}$

$V_1 = 2(I_1 + I_2)$

$V_1 = 2\left[I_1 + \frac{3V_1 - 2I_1}{3}\right]$

$= \frac{2}{3}\left[3I_1 + 3V_1 - 2I_1\right]$

$= \frac{2}{3}(3V_1 + I_1)$

$3V_1 = 6V_1 + 2I_1$

$2I_1 = -3V_1$

$\frac{I_1}{V_1} = -3/2 = Y_{11}$

$2(I_2 + I_1) + I_2 = 3V_1$

$3I_2 + 2I_1 = 3V_1$

$I_2 = \frac{3V_1 - 2I_1}{3}$

$I_1 = -\frac{3V_1}{2}$

$I_2 = \frac{3V_1 - 2\left(-\frac{3V_1}{2}\right)}{3}$

$= \frac{3V_1 + 3V_1}{3}$

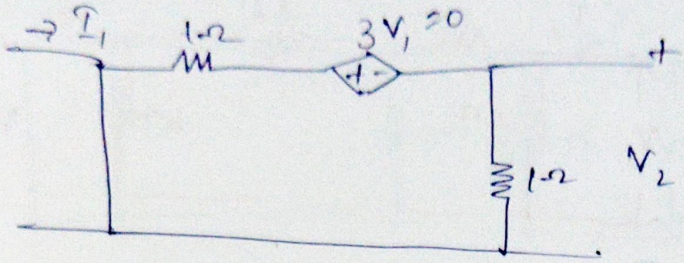
$= \frac{6V_1}{3}$

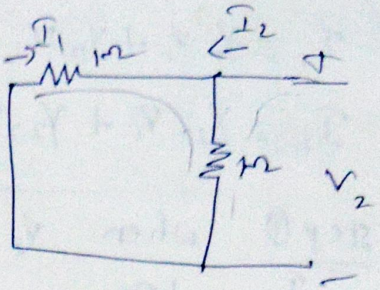
$I_2 = 2V_1 \Rightarrow \frac{I_2}{V_1} = 2 = Y_{21}$

$Y_{11} = -3/2$

$Y_{21} = 2$

Step ② $V_1 = 0$





$$V_2 = (I_2 + I_1)$$

$$I_1 + (I_2 + I_1) = 0$$

$$2I_1 + I_2 = 0$$

$$I_1 = -\frac{I_2}{2}$$

Substitute I_1 in V_2

$I_2 = 2V_2 \leftarrow V_2 = \frac{I_2}{2} \Rightarrow \frac{I_2}{V_2} = 2 = Y_{22}$

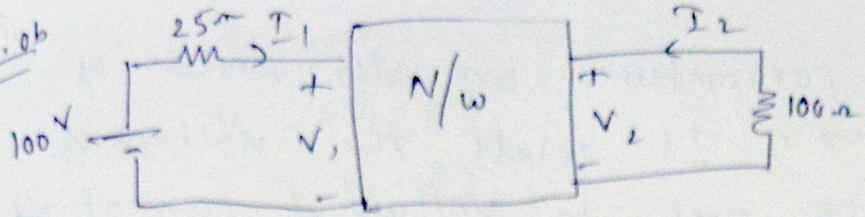
$$I_1 = -\frac{I_2}{2} = -\left(\frac{2V_2}{2}\right)$$

$$I_1 = -V_2$$

$$\frac{I_1}{V_2} = -1 = Y_{12} \qquad \therefore Y_{22} = 2, \quad Y_{12} = -1$$

$$\therefore \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} -3/2 & -1 \\ 2 & 2 \end{bmatrix}$$

1.6b



In the ckt shown in fig, describing by the s.c parameters

$$Y = \begin{bmatrix} 0.01 & -0.005 \\ 0.005 & 0.02 \end{bmatrix}$$

Then compute V_1, V_2 ?

Sol:-

$$I_1 = 0.01 V_1 - 0.005 V_2$$

$$I_2 = 0.005 V_1 - 0.02 V_2$$

From the N/W $\quad I_1 = \dfrac{100 - V_1}{25} \Rightarrow 4 - 0.04 V_1$

$$I_2 = \frac{-V_2}{100} = -0.01 V_2$$

$$4 - 0.04 V_1 = 0.01 V_1 - 0.005 V_2$$

$$-0.01 V_2 = 0.005 V_1 - 0.02 V_2$$

$\therefore V_1 = 68.57$ V $\qquad V_1 = 78.6$ V

$V_2 = -114.285$ V. $\qquad V_2 = -13.114$ V.

Transmision parameters are also known as ABCD parameters. It relate the voltage & current at one port to voltage & current at the other port.

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Matrix form.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

In power tr. lines ~~different~~ the conventional current direction of I_2 is opposite to that of the designed direction assigned to I_2.

∴ A minus sign include for I_2.

ABCD parameters are also known as transmision parameters because they provide a link b/n supply & receiving end voltages & currents considering the circuit elements to be linear.

ABCD Parameters

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

when o/p port of two port n/w is to be o.c ie, $I_2 = 0$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \Rightarrow \frac{1}{A} = \text{open ckt voltage gain}$$

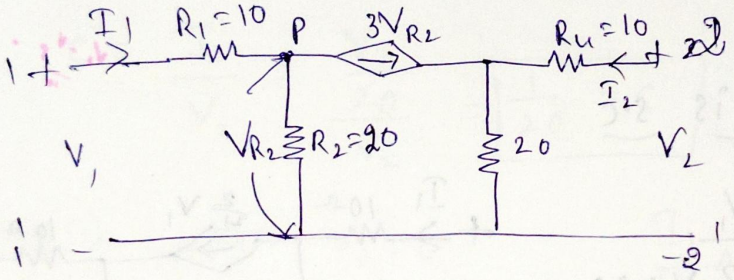
$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \Rightarrow \frac{1}{C} = \text{open ckt transfer impedance}$$

when o/p port of two port n/w is to be s.c ie; $V_2 = 0$

$$-B = \left.\frac{V_1}{I_2}\right|_{V_2=0} \Rightarrow \frac{-1}{B} = \text{short ckt transfer admittance}$$

$$-D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = -\frac{1}{D} = \text{short ckt current gain}$$

Prob calculate transmission parameters for the following ckt.



Sol:- At node P,

$$V_p = V_{R_2}$$

$$\frac{V_p - V_1}{10} + \frac{V_p}{20} + 3V_p = 0$$

$$2V_p - 2V_1 + V_p + 60V_p = 0$$

$$V_p = \frac{2}{63}V_1$$

$10\left(\frac{10.20}{1,2}\right)$

$$\frac{2V_p - 2V_1 + V_p + 60V_p}{20}$$

$$63V_p = 2V_1 \qquad \frac{63V_p - 2V_1}{20}$$

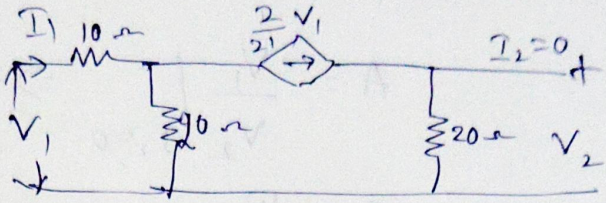
$$V_p = \frac{2}{63}V_1$$

Current source $I_{cs} = 3 \times \frac{2}{63} V_1 = \frac{2}{21} V_1$

case(i) when $2-2'$ is o.c

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$



$V_2 = 20 \times \frac{2}{21} V_1$

$V_2 = \frac{40}{21} V_1$

$A = \frac{V_1}{V_2} = \boxed{\frac{21}{40} = A}$

$C = \frac{I_1}{V_2}\Big|_{I_2=0} = \frac{\frac{V_1 - V_p}{10}}{V_2} = \frac{V_1 - \frac{2}{63} V_1}{\frac{40}{21} V_1} = \frac{V_1 - \frac{2}{63} V_1}{10} \times \frac{21}{40 V_1}$

$= \left(\frac{V_1}{10} - \frac{2}{630} V_1\right) \times \frac{21}{40 V_1}$

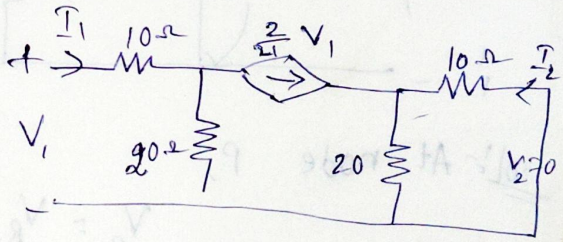
$C = \frac{61 V_1}{630} \times \frac{21}{40 V_1}$

$C = \boxed{\frac{61}{1200}}$

case(ii) when $2-2'$ is s.c

$B = -\frac{V_1}{I_2} = \frac{-V_1}{-\frac{2}{21} V_1 \times \frac{20}{30}}$

$\boxed{B = \frac{63}{4}}$



$I_2 = \frac{2}{21} V_1 \times \frac{20}{20+10}$

$I_2 = \frac{2}{21} V_1 \times \frac{20}{30}$

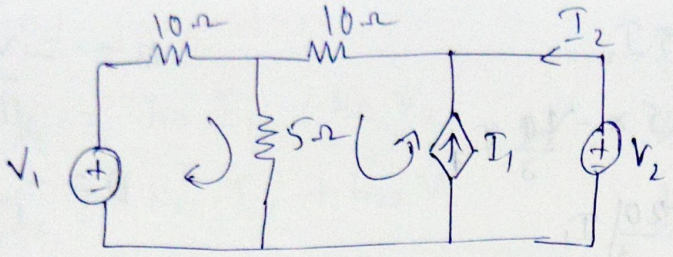
direction is opposite.

$D = -\frac{I_1}{I_2} = \frac{\frac{61}{630} V_1}{\frac{20}{30} \times \frac{2}{21} V_1}$

$\frac{61}{630} \times \frac{63}{4} = \boxed{\frac{61}{40} = D}$

$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{21}{40} & \frac{63}{4} \\ \frac{61}{1200} & \frac{61}{40} \end{bmatrix}$

Prob:

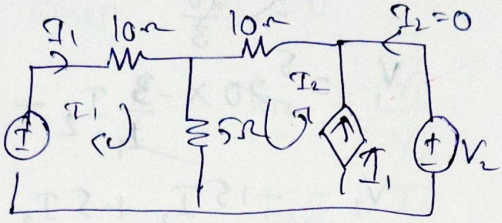


Find ABCD Parameters

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

Soln: Case (i) When $I_2 = 0$



$I_2 = I_1$

$V_1 = 10I_1 + 5(I_1 + I_1)$

$V_1 = 20I_1 \rightarrow (1)$

$5(2I_1) + 10I_1 = V_2$

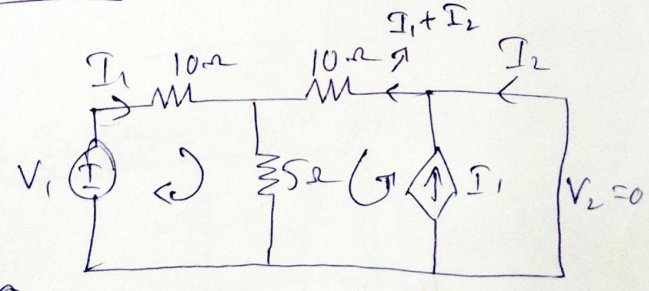
$20I_1 = V_2 \rightarrow (2)$

$I_1 = \frac{V_2}{20}$

(1) $\Rightarrow V_1 = 20 \times \frac{V_2}{20} \Rightarrow \frac{V_1}{V_2} = \boxed{A = 1}$

$C = \frac{I_1}{V_2} = \frac{\frac{V_2}{20}}{V_2} = \boxed{\frac{1}{20} = C}$

Case (ii) when $V_2 = 0$



$V_1 = 10I_1 + 5(I_1 + I_1 + I_2)$

$V_1 = 20I_1 + 5I_2 \rightarrow (1)$

$5(2I_1 + I_2) + 10(I_1 + I_2) = 0 \rightarrow (2)$

$20I_1 + 15I_2 = 0$

$I_2 = -\frac{20}{15}I_1 = -\frac{4}{3}I_1$

(1) ⇒ $V_1 = 20 I_1 + 5 I_2$

$= 20 I_1 + 5 \times -\frac{20}{3} I_1$

$= 20 I_1 - \frac{20}{3} I_1$

$= \frac{40}{3}$

$V_1 = 20 \times -\frac{3}{4} I_2 + 5 I_2$

$V_1 = -15 I_2 + 5 I_2$

$V_1 = -10 I_2$

$-\frac{V_1}{I_2} = B = 10$

$D = -\frac{I_1}{I_2} = -\left(\frac{-3}{4}\right)\frac{I_2}{I_2}$

$\boxed{D = \frac{3}{4}}$

$B = -\frac{V_1}{I_2}$

$D = -\frac{I_1}{I_2}$

$I_1 = -\frac{3}{4} I_2$

$1 = A \frac{1}{20} = C$

$\frac{3}{4} - \frac{10}{20}$

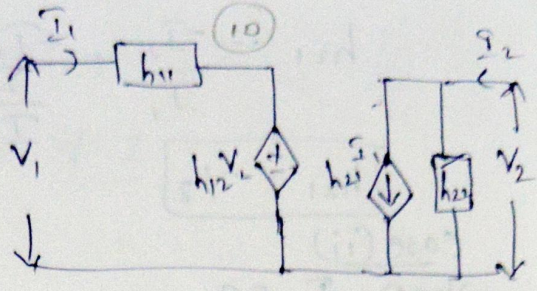
$\frac{3}{4} - \frac{1}{2}$

$\Rightarrow \frac{3-2}{4} = 1/4$

Hybrid Parameters

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$



When $V_2 = 0$

$$h_{11} = \frac{V_1}{I_1}$$

$$h_{21} = \frac{I_2}{I_1}$$

When $I_1 = 0$

$$h_{12} = \frac{V_1}{V_2}$$

$$h_{22} = \frac{I_2}{V_2}$$

h_{11} = Input impedance

h_{21} = forward current gain

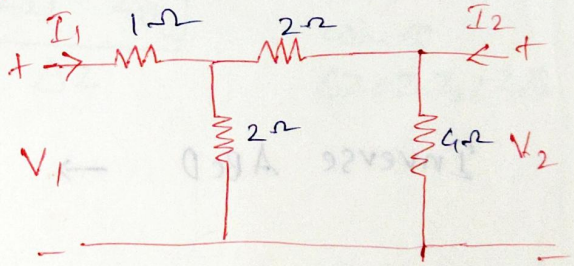
h_{12} = Reverse voltage gain

h_{22} = output admittance.

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 =$

Prob Find the h-parameters of the N/w shown in fig.

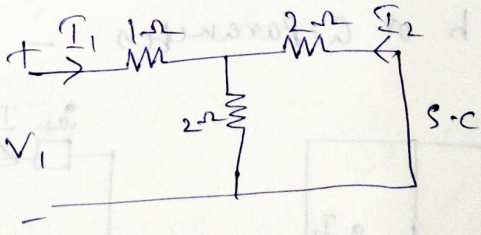


Sol:- The equations of hybrid parameters are

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

case (i) when $V_2 = 0$



$$h_{11} = \frac{V_1}{I_1}$$

$$V_1 = I_1 \times (2 // 2) + 1$$

$$= \frac{2 I_1}{I_1}$$

$$= I_1 2$$

$$h_{11} = 2\ \Omega.$$

$$h_{21} = \frac{-I_2}{I_1} = \frac{-\frac{I_1}{2}}{I_1}$$

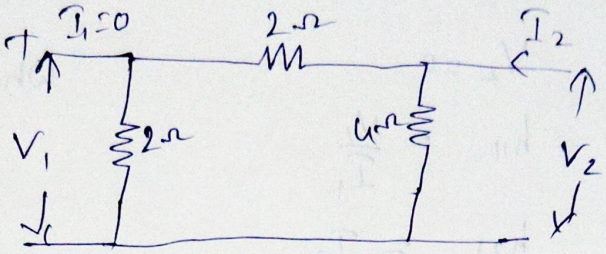
$$\boxed{h_{21} = -1/2}$$

$$I_2 = I_1 \times \frac{2}{2+2} = \frac{2I_1}{4}$$

$$I_2 = \frac{I_1}{2}$$

case (ii)

when $I_1 = 0$



$$h_{12} = \frac{V_1}{V_2}$$

$$= \frac{2I_2}{2I_2}$$

$$\boxed{h_{12} = \frac{1}{2}}$$

$$V_2 = I_2 (2+2) // 4$$

$$V_2 = I_2\, 2$$

V_1 = voltage drop across 2Ω.

$$V_1 = 2 \times I_2 \times \frac{4}{4+4}$$

$$= 2I_2 \frac{4}{8}$$

$$V_1 = I_2$$

$$h_{22} = \frac{I_2}{V_2} = \frac{I_2}{2I_2}$$

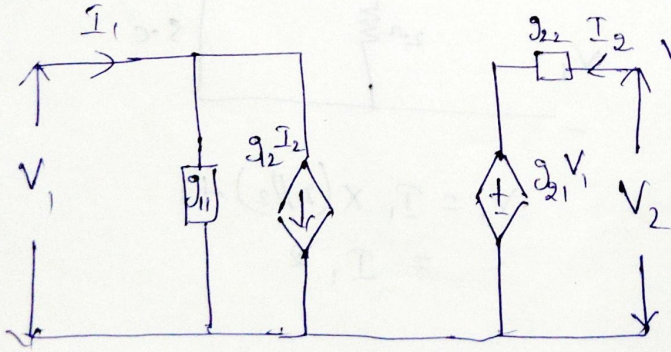
$$\boxed{h_{22} = 1/2}$$

Inverse ABCD $\rightarrow$ $V_2 = A' V_1 - B' I_1$

$I_2 = C' V_1 - D' I_1$

Inverse h or G-Parameters $-$ $I_1 = g_{11} V_1 + g_{12} I_2$

$V_2 = g_{21} V_1 + g_{22} I_2$



Equivalent ckt of g-parameters

Relation b/n Z & Y Parameters

To find Y-Parameters in terms of Z.

$$V = IZ$$

$$Z^{-1}V = IZZ^{-1}$$

$$Z^{-1}V = I$$

$$Z^{-1} = \frac{I}{V}$$

$$Z^{-1} = Y$$

$$Y = Z^{-1}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1}$$

$$Z^{-1} = \frac{\text{Adjoint}}{\det \text{ of } Z} = \frac{\begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}}{\Delta Z}$$

where $\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$

$$Y = Z^{-1} = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

$$\therefore Y_{11} = \frac{Z_{22}}{\Delta Z}, \quad Y_{12} = \frac{-Z_{12}}{\Delta Z}, \quad Y_{21} = \frac{-Z_{21}}{\Delta Z}, \quad Y_{22} = \frac{Z_{11}}{\Delta Z}$$

To find Z-parameters in terms of Y.

$$Y = Z^{-1}$$

$$Z = Y^{-1}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

$$Y^{-1} = \frac{\text{Adjoint}}{\det of\ Y} = \frac{\begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}}{\Delta Y}$$

where

$\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21}$

$$Z = Y^{-1} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$$

$$Z_{11} = \frac{Y_{22}}{\Delta Y} \quad Z_{12} = \frac{-Y_{12}}{\Delta Y}, \quad Z_{21} = \frac{-Y_{21}}{\Delta Y}, \quad Z_{22} = \frac{Y_{11}}{\Delta Y}.$$

Relation b/n Z & H parameters

To find Z-parameters in terms of H paramters.

$$V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (1)$$
$$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (2)$$
$$V_1 = Z_{11} I_1 + Z_{12} I_2 \rightarrow (3)$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \rightarrow (4)$$

solving V_2 from equ (2)

$$V_2 = \frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} I_1 \rightarrow (5)$$

Substituting equ (5) in equ (1)

$$V_1 = h_{11} I_1 + h_{12} \left[\frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} I_1 \right]$$

$$V_1 = h_{11} I_1 + \frac{h_{12}}{h_{22}} I_2 - \frac{h_{12} h_{21}}{h_{22}} I_1$$

$$V_1 = \left[h_{11} - \frac{h_{12} h_{21}}{h_{22}}\right] I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$V_1 = \left[\frac{h_{11} h_{22} - h_{21} h_{12}}{h_{22}}\right] I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$V_1 = \frac{\Delta h}{h_{22}} + \frac{h_{12}}{h_{22}} I_2 \rightarrow (6)$$

compare eqn (3) & eqn (6)

$$\boxed{Z_{11} = \frac{\Delta h}{h_{22}}, \quad Z_{12} = \frac{h_{12}}{h_{22}}}$$

compare eqn (5) & eqn (4)

$$\boxed{Z_{22} = \frac{1}{h_{22}} \qquad Z_{21} = -\frac{h_{21}}{h_{22}}}.$$

To find h-parameters interms of z-parameters.

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \rightarrow (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \rightarrow (2)$$

$$V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (3)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (4)$$

solving I_2 from eqn (2)

$$I_2 = \frac{1}{Z_{22}} V_2 - \frac{Z_{21}}{Z_{22}} I_1 \rightarrow (5)$$

substitute eqn (5) in eqn (1)

$$V_1 = Z_{11} I_1 + Z_{12}\left[\frac{1}{Z_{22}} V_2 - \frac{Z_{21}}{Z_{22}} I_1\right]$$

$$V_1 = \left[Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22}} \right] I_1 + \frac{Z_{12}}{Z_{22}} V_2$$

$$V_1 = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{22}} I_1 + \frac{Z_{12}}{Z_{22}} V_2$$

$$V_1 = \frac{\Delta Z}{Z_{22}} I_1 + \frac{Z_{12}}{Z_{22}} V_2 \rightarrow (6)$$

comparing eqn (3) & (6)

$$h_{11} = \frac{\Delta Z}{Z_{22}}, \quad h_{12} = \frac{Z_{12}}{Z_{22}}$$

From eqn (4) & (5)

$$h_{21} = -\frac{Z_{21}}{Z_{22}} \qquad h_{22} = \frac{1}{Z_{22}}$$

Relation b/n Z & ABCD Parameters

To find Z parameters in terms of ABCD parameters.

$$V_1 = A V_2 - B I_2 \rightarrow (1)$$

$$I_1 = C V_2 - D I_2 \rightarrow (2)$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \rightarrow (3)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \rightarrow (4)$$

solving V_2 from eqn (2)

$$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2 \rightarrow (5)$$

substituting V_2 in eqn (1)

$$V_1 = A \left(\frac{1}{C} I_1 + \frac{D}{C} I_2 \right) - B I_2$$

$$= \frac{A}{C} I_1 + \frac{AD}{C} I_2 - B I_2$$

$$V_1 = \frac{A}{C} I_1 + \left(\frac{AD - BC}{C}\right) I_2 \rightarrow (6)$$

Compare eqn (3) & eqn (6)

$$\boxed{Z_{11} = \frac{A}{C} \qquad Z_{12} = \frac{AD-BC}{C}}$$

Compare eqn (4) & (5)

$$\boxed{Z_{21} = \frac{1}{C} \qquad Z_{22} = \frac{D}{C}}$$

To find ABCD parameters Parameters interms Z

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \rightarrow (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \rightarrow (2)$$

$$V_1 = A V_2 - B I_2 \rightarrow (3)$$

$$I_1 = C V_2 - D I_2 \rightarrow (4)$$

solving I_1 from eqn (2)

$$I_1 = \frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2 \rightarrow (5)$$

Substitute eqn (5) in eqn (1)

$$V_1 = Z_{11}\left[\frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2\right] + Z_{12} I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \frac{Z_{11} Z_{22}}{Z_{21}} I_2 + Z_{12} I_2$$

$$= \frac{Z_{11}}{Z_{21}} V_2 - \left[\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}}\right] I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \frac{\Delta Z}{Z_{21}} I_2 \rightarrow (6)$$

Compare eqn (3) & eqn (6)

$$\boxed{A = \frac{Z_{11}}{Z_{21}}, \quad -B = -\frac{\Delta Z}{Z_{21}}}$$

compare eqn (4) & (5)

$$\boxed{C = \frac{1}{Z_{21}}, \quad -D = -\frac{Z_{22}}{Z_{21}}}$$

Show that $AD - BC = 1$

$$\frac{Z_{11} Z_{22}}{Z_{21}^2} - \frac{\Delta Z}{Z_{21}^2} = \frac{Z_{11} Z_{22}}{Z_{21}^2} - \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}^2}$$

$$\Rightarrow \frac{Z_{11} Z_{22} - Z_{11} Z_{22} + Z_{12} Z_{21}}{Z_{21}^2}$$

$$\Rightarrow \frac{Z_{12} Z_{21}}{Z_{21}^2} \Rightarrow \frac{Z_{12}}{Z_{21}} = 1$$

$$\Rightarrow \boxed{Z_{12} = Z_{21}}$$

Relation b/n Y & ABCD parameters

To find Y parameters interms of ABCD parameters

$$V_1 = AV_2 - BI_2 \rightarrow (1)$$

$$I_1 = CV_2 - DI_2 \rightarrow (2)$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \rightarrow (3)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \rightarrow (4)$$

solving I_2 from eqn (1)

$$I_2 = \frac{A}{B}V_2 - \frac{1}{B}V_1 \rightarrow (5)$$

Substitute eqn (5) in eqn (2)

$$I_1 = CV_2 - D\left(\frac{A}{B}V_2 - \frac{1}{B}V_1\right)$$

$$= CV_2 - \frac{AD}{B}V_2 + \frac{D}{B}V_1$$

$$= BC\left(-\frac{AD+BC}{B}\right)V_2 + \frac{D}{B}V_1$$

$$I_1 = \frac{D}{B}V_1 + \frac{-(AD-BC)}{B}V_2 \rightarrow (6)$$

Compare eqn (3) & eqn (6)

$$\boxed{Y_{11} = \frac{D}{B}, \quad Y_{12} = \frac{-(AD-BC)}{B}}$$

Compare eqn (4) & eqn (5)

$$\boxed{Y_{21} = -\frac{1}{B} \quad Y_{22} = \frac{A}{B}}.$$

To find ABCD Parameters in terms of Y parameters.

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \rightarrow (1)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \rightarrow (2)$$

$$V_1 = A V_2 - B I_2 \rightarrow (3)$$

$$I_1 = C V_2 - D I_2 \rightarrow (4)$$

Solving V_1 from eqn (2)

$$V_1 = \frac{1}{Y_{21}} I_2 - \frac{Y_{22}}{Y_{21}} V_2 \rightarrow (5)$$

substitute eqn (5) in eqn (1)

$$I_1 = Y_{11}\left[\frac{1}{Y_{21}} I_2 - \frac{Y_{22}}{Y_{21}} V_2\right] + Y_{12} V_2$$

$$I_1 = \frac{Y_{11}}{Y_{21}} I_2 - \frac{Y_{11} Y_{22}}{Y_{21}} V_2 + Y_{12} V_2$$

$$= \frac{Y_{11}}{Y_{21}} I_2 - \left(\frac{Y_{11} Y_{22} - Y_{12} Y_{21}}{Y_{21}}\right) V_2$$

$$I_1 = \frac{Y_{11}}{Y_{21}} I_2 - \frac{\Delta Y}{Y_{21}} V_2 \rightarrow (6)$$

compare eqn (4) & (6)

$$\boxed{C = \frac{-\Delta Y}{Y_{21}}, \quad -D = \frac{Y_{11}}{Y_{21}}}$$

Compare eqn (3) & (5)

$$\boxed{A = \frac{-Y_{22}}{Y_{21}} \quad -B = \frac{1}{Y_{21}}}$$

Relation b/n Y & H parameters

To find Y-parameters interms of h parameters.

$$V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (2)$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \rightarrow (3)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \rightarrow (4)$$

solving I_1 from eqn (1)

$$I_1 = \frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2 \rightarrow (5)$$

substitute eqn (5) in eqn (2)

$$I_2 = h_{21}\left[\frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2\right] + h_{22} V_2$$

$$= \frac{h_{21}}{h_{11}} V_1 - \frac{h_{12} h_{21}}{h_{11}} V_2 + h_{22} V_2$$

$$= \frac{h_{21}}{h_{11}} V_1 + \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{11}} V_2$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \frac{\Delta h}{h_{11}} V_2 \rightarrow (6)$$

compare eqn (4) & (6)

$$\boxed{Y_{21} = \frac{h_{21}}{h_{11}} \quad Y_{22} = \frac{\Delta h}{h_{11}}}$$

compare eqn (3) & (5)

$$\boxed{Y_{11} = +\frac{1}{h_{11}} \; ; \; Y_{12} = -\frac{h_{12}}{h_{11}}}$$

I_1

To find h-parameters in terms of Y-parameters:

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \rightarrow (1)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \rightarrow (2)$$

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \rightarrow (3)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \rightarrow (4)$$

Solving V_1 from eqn (1)

$$V_1 = \frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2 \quad \rightarrow (5)$$

substitute eqn (5) in eqn (2)

$$I_2 = Y_{21}\left[\frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2\right] + Y_{22} V_2$$

$$= \frac{Y_{21}}{Y_{11}} I_1 - \frac{Y_{21} Y_{12}}{Y_{11}} V_2 + Y_{22} V_2$$

$$= \frac{Y_{21}}{Y_{11}} I_1 + \frac{Y_{11} Y_{22} - Y_{21} Y_{12}}{Y_{11}} V_2$$

$$I_2 = \frac{Y_{21}}{Y_{11}} I_1 + \frac{\Delta Y}{Y_{11}} V_2 \quad \rightarrow (6)$$

Compare eqn (4) & (6)

$$\boxed{h_{21} = \frac{Y_{21}}{Y_{11}}, \quad h_{22} = \frac{\Delta Y}{Y_{11}}}$$

Compare eqn (3) & (5)

$$\boxed{h_{11} = \frac{1}{Y_{11}}, \quad h_{12} = \frac{-Y_{12}}{Y_{11}}}$$

Relation b/n ABCD & h Parameters

To find ABCD parameters interms of h parameters

$$V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (2)$$

$$V_1 = A V_2 - B I_2 \rightarrow (3)$$

$$I_1 = C V_2 - D I_2 \rightarrow (4)$$

solving I_1 from eqn (2)

$$I_1 = \frac{1}{h_{21}} I_2 - \frac{h_{22}}{h_{21}} V_2 \rightarrow (5)$$

substitute eqn (5) in eqn (1)

$$V_1 = h_{11}\left[\frac{1}{h_{21}} I_2 - \frac{h_{22}}{h_{21}} V_2\right] + h_{12} V_2$$

$$= \frac{h_{11}}{h_{21}} I_2 - \frac{h_{11} h_{22}}{h_{21}} V_2 + h_{12} V_2$$

$$= \frac{h_{11}}{h_{21}} I_2 - \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{21}} V_2$$

$$V_1 = \frac{h_{11}}{h_{21}} I_2 - \frac{\Delta h}{h_{21}} V_2 \rightarrow (6)$$

compare eqn (3) & (6)

$$\boxed{A = -\frac{\Delta h}{h_{21}}, \quad -B = \frac{h_{11}}{h_{21}}}$$

compare eqn (4) & (5)

$$\boxed{C = -\frac{h_{22}}{h_{21}}, \quad -D = \frac{1}{h_{21}}}$$

To find h-parameters in terms of ABCD Parameters

$$V_1 = AV_2 - BI_2 \rightarrow (1)$$

$$I_1 = CV_2 - DI_2 \rightarrow (2)$$

$$V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (3)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (4)$$

solving I_2 from Eqn (2)

$$DI_2 = CV_2 - I_1$$

$$I_2 = \frac{C}{D} V_2 - \frac{1}{D} I_1 \rightarrow (5)$$

Substitute I_2 in eqn (1)

$$V_1 = AV_2 - B\left(\frac{C}{D} V_2 - \frac{1}{D} I_1\right)$$

$$= AV_2 - \frac{BC}{D} V_2 + \frac{B}{D} I_1$$

$$V_1 = \frac{AD - BC}{D} V_2 + \frac{B}{D} I_1 \rightarrow (6)$$

Compare eqn (3) & (6)

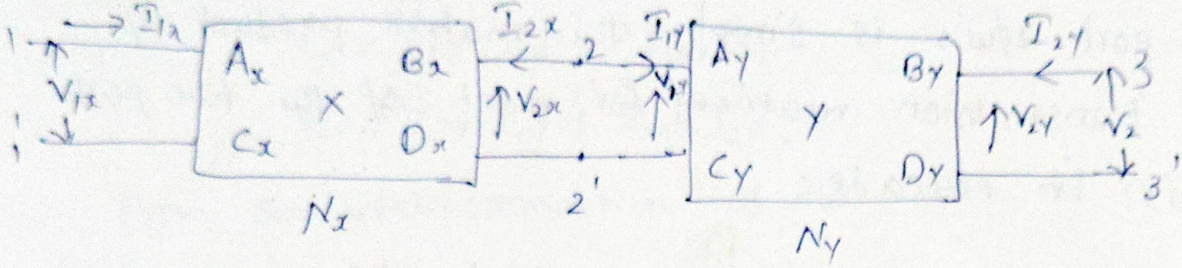
$$\boxed{h_{11} = \frac{B}{D}, \quad h_{12} = \frac{AD - BC}{D}}$$

Compare eqn (4) & (5)

$$\boxed{h_{21} = \frac{-1}{D}, \quad h_{22} = \frac{C}{D}}$$

UNIT - VI

Cascade connection



Let us consider two two-port n/w's N_x & N_y connected in cascade with port voltages & currents as indicated in fig above.

The matrix representation of ABCD parameters for the n/w X is as under.

$$\begin{bmatrix} V_{1x} \\ I_{1x} \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} V_{2x} \\ -I_{2x} \end{bmatrix}$$

And for the network 'y', the matrix representation is

$$\begin{bmatrix} V_{1y} \\ I_{1y} \end{bmatrix} = \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_{2y} \\ -I_{2y} \end{bmatrix}$$

It is observed that $V_{1x} = V_1$, $I_{1x} = I_1$,

$V_{2x} = V_{1y}$, $I_{2x} = -I_{1y}$, $V_{2y} = V_2$, $I_{2y} = I_2$

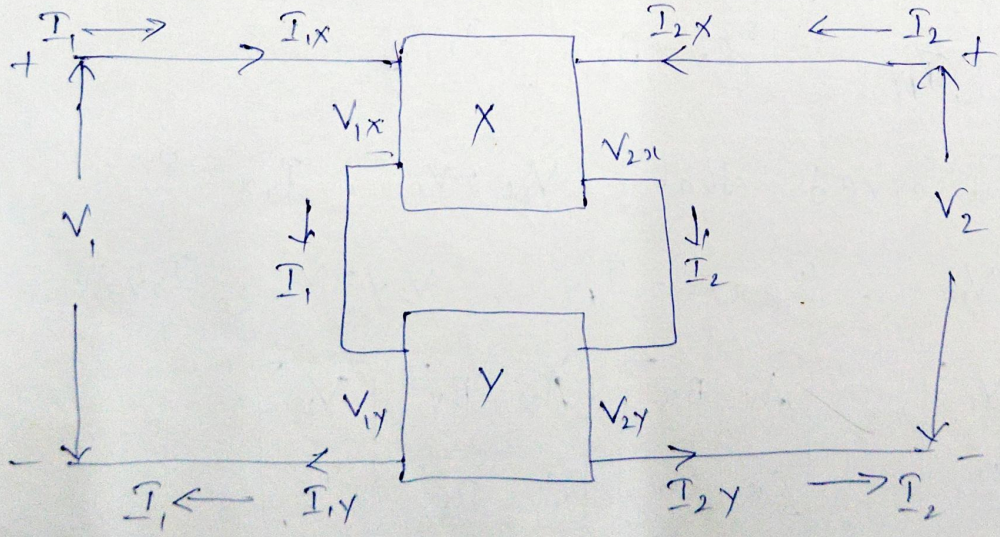
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

This result may be generalised for any no. of two-ports connected in cascade, & we see that the overall transmission parameter matrix for cascaded two-port N/w's is simply the matrix product of the transmission matrices for each of the two port N/w's in cascade.

Interconnection of two-port Networks

Series connection of two port N/w

When two port N/w's are connected in cascade, the parameters of the interconnected N/w can be conveniently expressed with the help of ABCD Parameters. In a similar way, the Z-Parameters can be used to describe the parameters of series connected two port N/w's & Y-parameters can be used to describe parameters of paralled connected two port N/w's. A series connection of two port N/w's is shown in fig.



$$V_{1x} = Z_{11x} I_{1x} + Z_{12x} I_{2x}$$

$$V_{2x} = Z_{21x} I_{1x} + Z_{22x} I_{2x}$$

$$V_{1y} = Z_{11y} I_{1y} + Z_{12y} I_{2y}$$

$$V_{2y} = Z_{21y} I_{1y} + Z_{22y} I_{2y}$$

From the inter-connection of the n/w's, it is clear that

$$I_1 = I_{1x} = I_{1y} \quad , \quad I_2 = I_{2x} = I_{2y}$$

$$V_1 = V_{1x} + V_{1y} \quad ; \quad V_2 = V_{2x} + V_{2y}$$

$$V_1 = Z_{11x} I_1 + Z_{12x} I_2 + Z_{11y} I_1 + Z_{12y} I_2$$

$$V_1 = \left[Z_{11x} + Z_{11y}\right] I_1 + \left[Z_{12x} + Z_{12y}\right] I_2$$

$$V_2 = Z_{21x} I_1 + Z_{22x} I_2 + Z_{21y} I_1 + Z_{22y} I_2$$

$$= \left[Z_{21x} + Z_{21y}\right] I_1 + \left[Z_{22x} + Z_{22y}\right] I_2$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

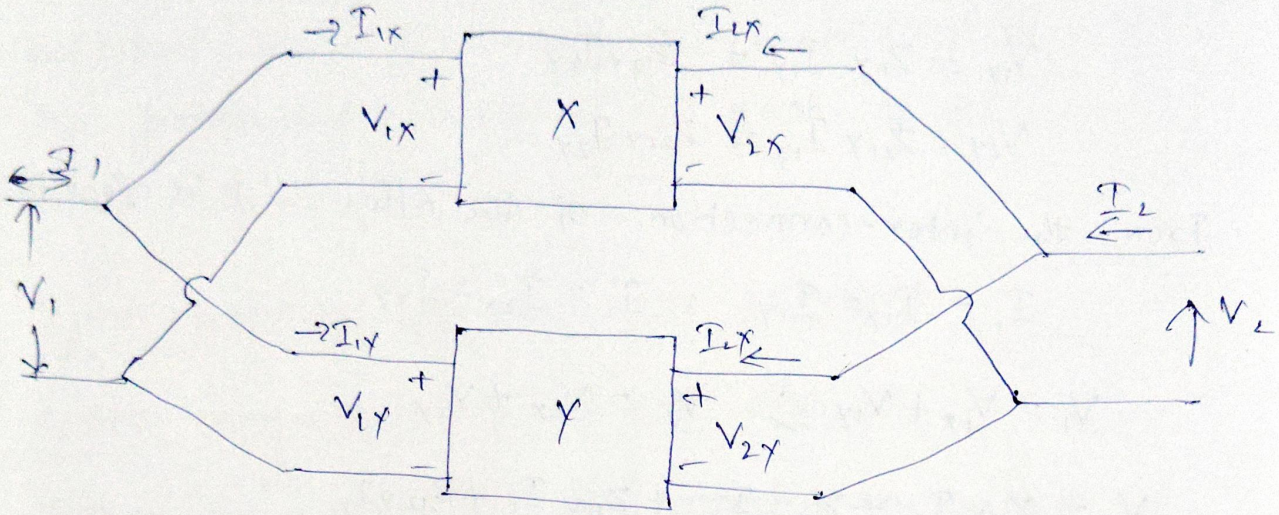
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$Z_{11} = Z_{11x} + Z_{11y} \qquad Z_{21} = Z_{21x} + Z_{21y}$$

$$Z_{12} = Z_{12x} + Z_{12y} \qquad Z_{22} = Z_{22x} + Z_{22y}$$

So in series connection of two port n/w's, the z-parameters are sum of the individual corresponding parameters of individual two port n/w's.

Parallel connection of two-port N/w



$$V_1 = V_{1x} = V_{1y} \qquad V_2 = V_{2x} = V_{2y}$$

$$I_1 = I_{1x} + I_{1y} \qquad I_2 = I_{2x} + I_{2y}$$

for N/w X, the equations of Y-parameters are

$$I_{1x} = Y_{11x} V_{1x} + Y_{12x} V_{2x}$$

$$I_{2x} = Y_{21x} V_{1x} + Y_{22x} V_{2x}$$

for N/w Y, $I_{1y} = Y_{11y} V_{1y} + Y_{12y} V_{2y}$

$$I_{2y} = Y_{21y} V_{1y} + Y_{22y} V_{2y}$$

$$I_1 = I_{1x} + I_{1y} = Y_{11x} V_1 + Y_{12x} V_2 + Y_{11y} V_1 + Y_{12y} V_2$$

$$= [Y_{11x} + Y_{11y}] V_1 + [Y_{12x} + Y_{12y}] V_2$$

$$I_2 = I_{2x} + I_{2y} = Y_{21x} V_1 + Y_{22x} V_2 + Y_{21y} V_1 + Y_{22y} V_2$$

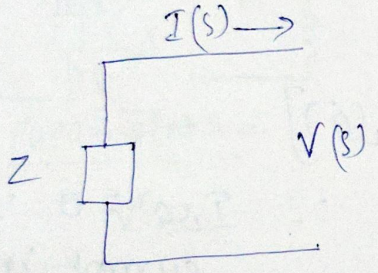
$$= [Y_{21x} + Y_{21y}] V_1 + [Y_{22x} + Y_{22y}] V_2$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$\therefore Y_{11} = Y_{11x} + Y_{11y} \qquad Y_{21} = Y_{21x} + Y_{21y}$$
$$Y_{12} = Y_{12x} + Y_{12y} \qquad Y_{22} = Y_{22x} + Y_{22y}$$

So in ||el connection of two port N/w's, the Y-parameters are sum of the corresponding parameters of individual two port N/w's.

Concept of transformed N/w



Transformed N/w.

$$V(t) = I(t) \cdot Z$$
$$V(s) = I(s) \cdot Z(s)$$
$$\frac{V(s)}{I(s)} = Z(s)$$

$\therefore$ Transformed impedance $Z(s) = \frac{V(s)}{I(s)}$

Transformed admittance $Y(s) = \frac{1}{Z(s)} = \frac{I(s)}{V(s)}$

Ex: **Resistance** :- The time domain expression relating voltage & current for the resistor is

$$V(t) = R\, i(t)$$

Applying Laplace transform to the above equ.

$$V(s) = R \cdot I(s).$$

