

Measuring Instruments

Fundamentals :-

Instrument :- A device which is used to determine the present value of the quantity under measurement.

Measurement :- The process of determining the amount or capacity of an unknown quantity by comparing with the known or with standard units.

A knowledge of the performance characteristics of an instrument is essential for selecting the most suitable instrument for specific measuring quantities.

There are two basic characteristics

- (i) Static
- (ii) Dynamic.

Static characteristics :- Static characteristics are the criteria used to describe the performance of a measuring instrument. Static characteristics

- include (i) Accuracy
- (ii) precision
- (iii) sensitivity
- (iv) ~~linearity~~ Resolution
- (v) Threshold

They are essential for understanding how closely an instrument's output corresponds to the true value of a measurement.

(i) Accuracy :- The degree of exactness (closeness) of a measurement compared to the ~~scope~~ True value of the quantity.

(True value :- The true value is the actual value of a physical quantity which is ideally obtained from a perfect measurement without any errors).
→ Many repeated measurements & calculate the average value → TV.

(ii) Precision :- Precision is the degree of agreement among a series of measurements of the same quantity under the same conditions. It indicates how close repeated measurements are to each other.

Note :- precision value may or may not be accurate value.

Any instrument should have high precision.

(iii) Sensitivity :- Sensitivity in measurement is the ratio of a change in the instrument's o/p to the corresponding change in the i/p quantity being measured.

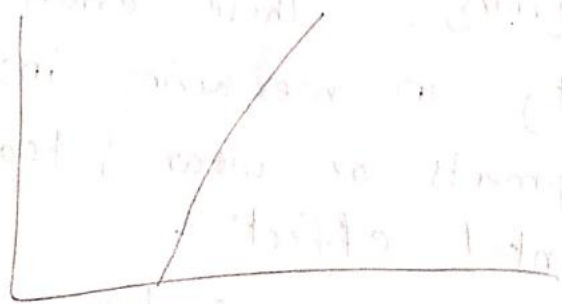
[A higher sensitivity means the instrument is more responsive to small variations in the i/p, while a lower sensitivity means it responds to large changes.]

(iv) Resolution :- If the i/p is slowly increased from some i/p value, it will again be found that o/p doesnot change at all until a certain increment is exceeded. 2 cm.

This increment is called Resolution. The smallest change in a physical quantity that an instrument can respond.

(v) Threshold :- The min. value of the i/p that can be detected & measured by an instrument is called as Threshold.

Threshold is the minimum i/p value to cause a detectable change in the o/p. Est. :- if i/p = 0 o/p = 0. That means the minimum i/p value where there is a change in its o/p is called Threshold.



Types of static error :- The static error of a measuring instrument is the numerical d/f b/n the true value of a quantity & its value as obtained by measurement. $E_s = V_m - V_t$ static error can be +ve or -ve.

Static errors are categorised as

(i) Gross errors (or) human errors

(ii) systematic errors

(iii) Random errors.

(i) Gross errors :- Gross errors occur due to human mistakes & errors in using measuring instruments or taking wrong readings or mistakes in recording observations.

→ These errors can't be completely eliminated. But we can try to minimise them by taking proper care while taking readings or recording observations.

(ii) systematic errors :- These errors occur due to lack of quality in measuring instruments such as defective equipments or wear & tear or aging effects or environmental effects.

They are also called as bias.

There are basically 3-types of systematic errors.

(a) Instrumental (b) Environmental (c) Observational

Instrumental Errors:- occur due to design & constructional features or the mechanical structure of the instruments.

Environmental Errors:- occur due to environmental effects such as temperature, pressure, humidity etc.

Observational Errors:- occur due to errors in observation. The most common error is the parallax error.

(iii) Random Errors:- These errors occur due to unknown causes & reasons which are very difficult to determine.

These errors are very small in magnitude & follow laws of probability & can be treated mathematically.

Sources of Errors: - The sources of error, other than the inability of a piece of hardware to provide a true measurement, are as follows.

- (1) Insufficient knowledge of process parameters
 - a) design conditions
- (2) Poor design
- (3) change in process parameters, irregularities, etc
- (4) poor maintenance,
- (5) Errors caused by person operating the instrument or equipment
- (6) certain design limitations,

(4)

Statistical Analysis of Data :- In measurement system there are various type of errors like gross errors, systematic errors, observational errors, Instrument errors, random errors & to calculate these errors, There are some errors which are to be determined at the time of measurement but there are some errors which can't be determined directly but we have to do some mathematical calculations to determine them. Examples of such type of errors are Random errors.

Random errors can't be determined in the process of measurement. We have to do some mathematical analysis.

The mathematical analysis of various measurements is called statistical analysis of data.

Suppose if the same reading is taken no. of times using different observers, different instruments & by different ways of measurement then what is the effect on that reading by using different ways, different observers & different instruments.

Statistical analysis helps to determine analytically the uncertainty of the final test result.

How we are doing statistically analysis on the mathematical analysis over the measurement.

There are different terms which have to be calculated.

- They are
- (i) Arithmetic mean
 - (ii) Median
 - (iii) Deviation from mean
 - (iv) Average Deviation
 - (v) Standard Deviation
 - (vi) Variance.

(i) Arithmetic Mean :- When the no. of readings of the same measurement are taken, the most likely value from the set of measured variable values is called the "Arithmetic Mean" of the no. of readings taken.

The arithmetic mean of 'n' measurements ^{is given} ~~at~~ by the expression

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

$$\bar{x} = \frac{\sum_{n=1}^n x_n}{n}$$

where \bar{x} = Arithmetic Mean

x_n = n^{th} reading

n = total no. of readings

Ex :- If the true value of a resistance is 100 Ω .

If it is measured by different observers

$$\boxed{100.1 + 100.01 + 99.9 + 98.9} / 4 = \text{mean}$$

The mean is very close to true value if no. of readings is very large.

But taking that no. of readings is so complicated. so, in such a case instead of measuring large no. of readings i.e. instead of measuring Arithmetic mean we calculated the ^{another term} ~~called~~ Median.

(ii) Median: For a set of 'n' measurements x_1, x_2, \dots, x_n written down in ascending order of magnitude, the median is given by

$$x_{\text{median}} = x_{(n+1)/2}$$

Ex: If the no. of measurements are 11 i.e.,

$$x_1, x_2, \dots, x_{11}$$

$$x_{\text{median}} = x_{(11+1)/2} = x_{12/2} = x_6$$

(or) If the no. of measurements is even. i.e., 12.

$$x_{\text{median}} = \frac{x_6 + x_7}{2}$$

$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}$

then the median is average of middle 2 readings

$$\text{Median} = \frac{x_6 + x_7}{2}$$

(iii) Deviation from Mean: The deviation tells us about the departure of a given reading from the arithmetic mean of the data set.

$$d_i = x_i - \bar{x}$$

d_i - deviation of i^{th} reading

x_i - value of i^{th} reading

\bar{x} - Arithmetic mean.

The deviation can be a +ve or -ve value i.e.,

If $x_i > \bar{x} \rightarrow +ve$

If $x_i < \bar{x} \rightarrow -ve$

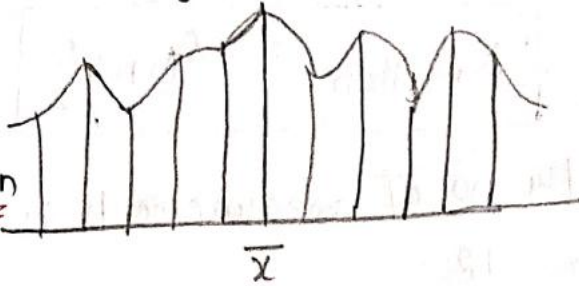
(iv) Average Deviation: It is defined as the sum of the absolute values of the deviations divided by the no. of readings.

$$\bar{D} = \frac{\sum |d_i|}{n}$$

Consider only magnitude deviation, neglect whether it may be +ve or -ve deviation.

(v) Standard Deviation :- The amount by which the 'n' measurement values are spread about its mean is expressed as standard deviation.

The standard deviation is also known Root Mean Square deviation.



$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + \dots + d_n^2}{n}} = \sqrt{\frac{\sum d_i^2}{n}}$$

Standard deviation is square root of sum of individual deviations squared divided by the no. of readings.

Suppose if the no. of readings taken is < 20 then

the standard deviation $\sigma = \sqrt{\frac{d_1^2 + d_2^2 + \dots + d_n^2}{n-1}}$

* Less the value of standard deviation, more accurate is the measurement.

(vi) Variance :- Variance means square deviation.

↓
Square of standard deviation.

$$V = \sigma^2 = \left(\frac{d_1^2 + d_2^2 + d_3^2 + \dots + d_n^2}{n} \right)^2$$

$$V = \sigma^2 = \frac{d_1^2 + d_2^2 + d_3^2 + \dots + d_n^2}{n}$$

If $n < 20$, then

$$V = \sigma^2 = \frac{d_1^2 + d_2^2 + d_3^2 + \dots + d_n^2}{n-1}$$

Prob:- In a survey of 15 food the following given data, calculate

- (i) Arithmetic Mean (ii) Median (iii) Deviation of each value (iv) Average deviation (v) Standard Deviation.

Sol:- Given $x_1 = 49.7$ $x_2 = 50.1$ $x_3 = 50.2$ $x_4 = 49.6$
 $x_5 = 49.7$.

(i) Arithmetic Mean $\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$
 $= 49.86$.

(ii) Median $= x_{(n+1)/2}$ No. of readings taken = 5 i.e. odd.
 $= x_{(5+1)/2} = x_3$

∴ Median is $x_3 = 50.2$.

(iii) Deviation of each value is ^{Calculated} given by

$$d_i = x_i - \bar{x}$$

$$d_1 = x_1 - \bar{x} = 49.7 - 49.86 = -0.16$$

$$d_2 = x_2 - \bar{x} = 50.1 - 49.86 = 0.24$$

$$d_3 = x_3 - \bar{x} = 50.2 - 49.86 = 0.34$$

$$d_4 = x_4 - \bar{x} = 49.6 - 49.86 = -0.26$$

$$d_5 = x_5 - \bar{x} = 49.7 - 49.86 = -0.16$$

(iv) Average deviation $\bar{D} = \frac{\sum |d_i|}{n}$
 $= \frac{0.16 + 0.24 + 0.34 + 0.26 + 0.16}{5}$
 $\bar{D} = \frac{1.16}{5} = 0.232$

$$\text{Standard deviation } \sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2}{n-1}}$$

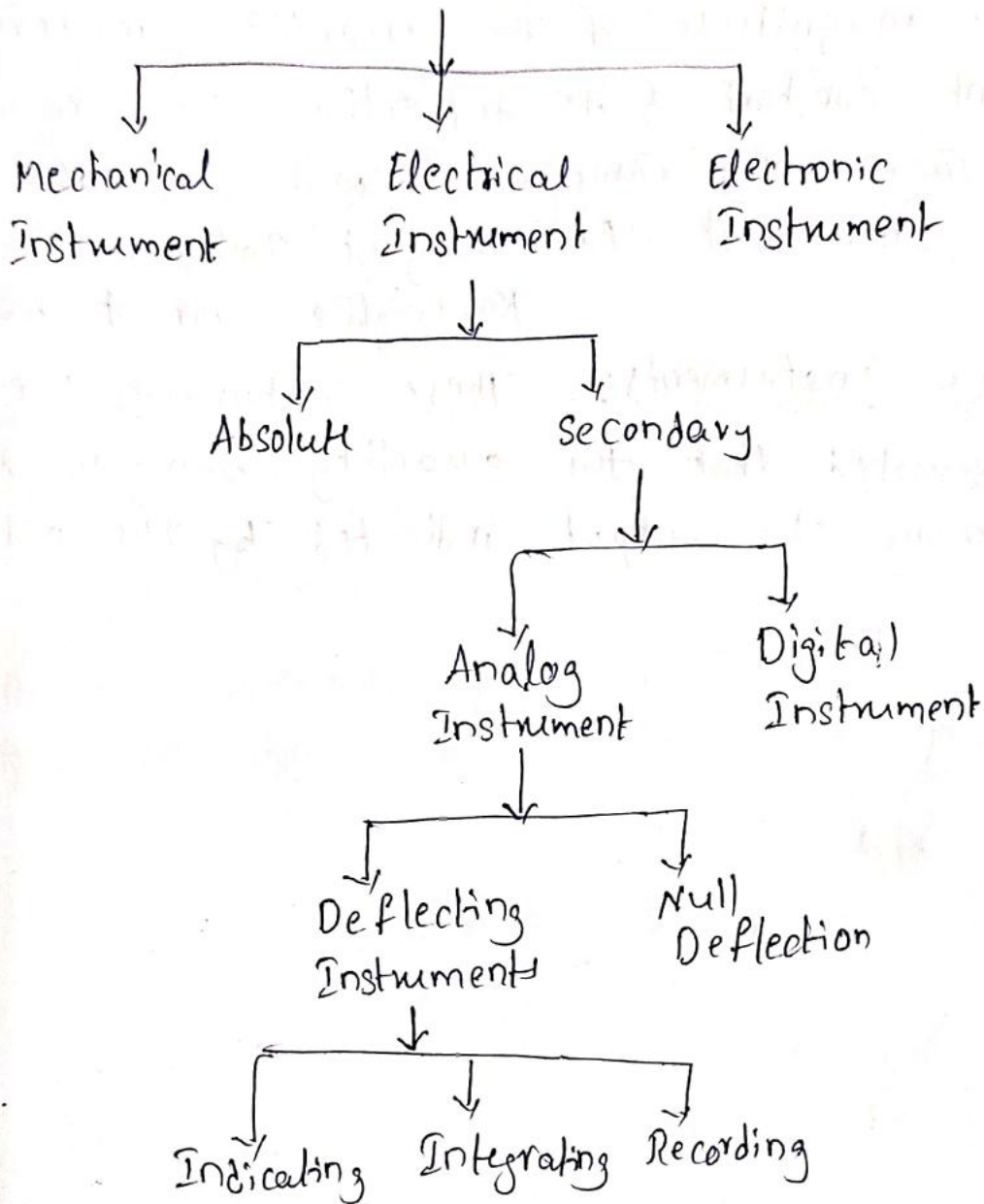
$$\sigma = \sqrt{\frac{(0.16)^2 + (0.24)^2 + (0.34)^2 + (0.26)^2 + (0.16)^2}{4}}$$

$$= \sqrt{\frac{0.292}{4}} = \sqrt{0.073}$$

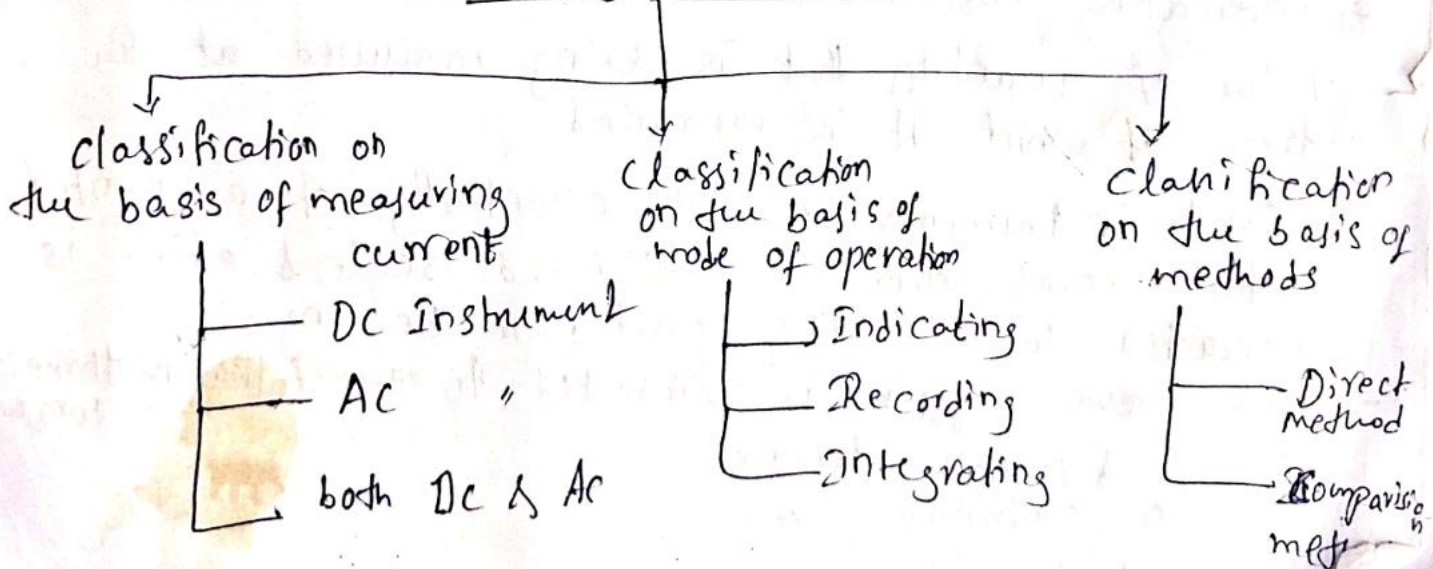
$$\sigma = 0.27$$

UNIT - I

Instrument



Analog Instrument



which measures quantity to be measured in terms of physical parameter.

Absolute Instruments :- The instruments of this type gives the magnitude of the quantity in term of instrument constant & its deflection. Such instruments do not require any comparison with any other standard instrument. Ex: - Tangent Galvanometer, Rayleigh's current balance

Secondary Instruments :- These instruments are so constructed that the quantity being measured by observing the output indicated by the instrument

Essentials of Indicating instruments

- Indicating instruments are those which indicate the value of quantity that is being measured at the time at which it is measured.
- Such instruments consist essentially of a pointer which moves over a calibrated scale & which is attached to a moving pivoted in bearing.
- The moving system is subjected to the following three torque
 - (i) A deflecting torque
 - (ii) A controlling "
 - (iii) A damping "

Measuring Instruments

The measurement of a given quantity is the result of comparison b/n the quantity to be measured & a definite standard. The instruments which are used for such measurements are called measuring instruments.

Necessary requirements:- for any measuring instrument

- (i) with the introduction, the ckt conditions should not be altered.
- (ii) power consumed by the instruments for their operation should be as small as possible.

UNIT- I

Measuring Instruments

Electrical Measurements :- classification

Electrical measurements can be classified
(1) Analog instruments
(2) Digital instruments.

An Analog device is one in which the output is a continuous function of time & maintains a constant relation to its ~~input~~ input.

Analog instruments :- classification

Analog instruments can be classified based on quantity they can be measured.

Ex:- Ammeter, Wattmeter, voltmeter, pf meter, frequency meters etc

Analog instruments can be classified according to the kind of current that can be measured by them.

- (i) Direct current (d.c)
- (ii) Alternating current (a.c)
- (iii) both dc & ac

Analog instruments may be classified based on mode of operation according to which of the effects is used for their working.

- (i) Indicating
- (ii) Recording
- (iii) Integrating

Indicating instruments:- This type of instruments are used to indicate the magnitude of a quantity being measured. They generally use a pointer for this purpose. Ex:- A, V, W.

The analog indicating instruments may be divided into two groups.

- (a) Electromechanical instruments
- (b) Electronic instruments

Recording instruments:- This type of instruments give a continuous record of the quantity being measured over a specified period. (used in generative station).

Integrating instruments:- Integrating instruments totalize events over a specific period of time.

The summation, which they give is the product of time & an electrical quantity. Ex:- Energy meters & Ampere-hour meters.

The analog instruments may also be classified on the basis of method used for comparing the unknown quantity (measured) with the unit of measurement as.

- (i) Direct measuring instruments
- (ii) Comparison instruments

Direct measuring instrument:- These instruments convert the energy of ^{which is to be} the measured directly into the energy that actuates the instrument & the value of unknown quantity is measured or displayed directly. Ex:- Energy meter, A, V, W.

used in MC & MI
Magnetic effect: - Most of the A, V, W utilize the magnetic effect for the production of deflecting force or torque.

→ Magnetic effect is used in Moving coil & MI Instruments

→ when a current carrying coil or conductor is placed in a magnetic field it experiences a mechanical force. (which in turn ^{the coil} produces a torque)

→ Then the coil tends to move & deflects the pointer attached to it.

Heating effect or Thermal effects: -

→ when a current to be measured is passed through a coil ~~same~~ which heats it, causes a rise in the temperature of the coil.

Due to this temp. rise whatever the amount of heat produced can be converted to emf with the help of thermocouple attached to the coil.

→ This effect can be used to measure both ac & dc quantities:

→ Hot wire instrument is also based on the principle of heating effect for producing deflecting torque.

Electrostatic effect - This effect is used only for measuring the voltages on both ac & dc.

- This effect is well suited for measuring very high voltages.
- The principle of operation of electrostatic instruments is the force of attraction or repulsion b/n two charged bodies, which leads to the production of deflecting force.
- When two plates are charged & potential d/f is applied b/n them there exists a force b/n them. It may be a force of attraction or force of repulsion. (This force will cause to deflect the pointer).

Electromagnetic induction effect - This phenomenon used for measuring ac quantities only & mostly used in energy meters.

- The deflecting ~~torque~~ ^{force} required is produced by the ~~eddy~~ currents induced in the ~~rotating~~ disc due to the flux caused by the electromagnet.
 - When a non-magnetic conductor formed as disc is placed in a magnetic field produced by electromagnets excited by alternating currents, an emf is induced in the disc according to F.L.E.M.I. If closed path is provided the emf forces a current to flow in the disc. The force produced by the interaction of induced currents & alternating magnetic fields makes the ~~pointer~~ ^{disc} move.
- unknown marking.

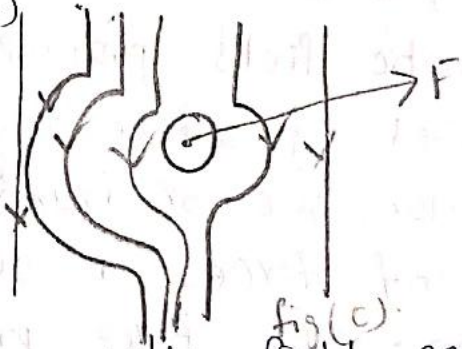
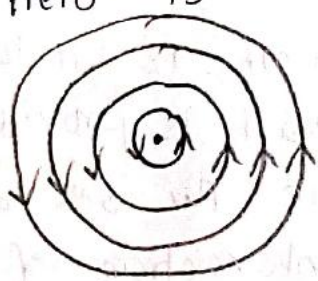
Comparison Instruments :- These instruments convert the energy measure the unknown quantity with by comparison with a standard.

Ex - DC & AC bridges.

As said earlier analog instruments may be classified according to the principle of operation they utilize. The effects they utilize are

- (i) Magnetic effect
- (ii) heating " (or) thermal
- (iii) electrostatic "
- (iv) electromagnetic " (or) induction effect.
- (v) Hall effect.

Magnetic effect :- Torque is a measurement of force which causes an object to rotate about an axis. Consider a current carrying conductor is as shown in fig (a). A uniform magnetic field in fig (b). Let the current carrying conductor be placed in the uniform magnetic field, the resultant field is as shown in fig (c).



This results in distortion of magnetic field causing a force 'F' to act from left to right. The reversal of current direction will cause a force 'F' in opposite direction i.e., from right to left.

Thermal or Heating effect :- The current to be measured is passed through a small element which heats it. The temp. rise is converted into an emf by a thermo couple attached to the element.

A thermo couple consists of two dissimilar electric conductors joined at ends to form a closed loop. If the junctions of two dissimilar metals are maintained at different temperatures, a current flows through the closed loop. This current can be measured & is indicative of the rms value of current flowing through the heater elements.

Electrostatic effect :- When two plates are charged, there is a force exerted b/n them. This force is used to move one of the plates. The instruments working on this principle are called electrostatic instruments & they are usually voltmeters.

Induction effect :- When a non-magnetic conductor is formed as disc or drum which is placed in a magnetic field produced by a system of electromagnets excited by alternating currents, an emf is induced in the disc or drum ^{according to F.L.E.M.I.}. If a closed path is provided, the emf forces a current to flow in the disc or drum. The force produced by the interaction of induced currents & the alternating magnetic fields makes the disc move.

The induction effect is mainly utilized in AC energy meters.

Hall effect :- If a strip of conducting material carries current in the presence of a transverse magnetic field as shown in fig. an emf is produced b/n two edges of conductor. The magnitude of voltage depends upon the current, flux density & a property of conductor called Hall effect coefficient

The emf may be measured after amplification. Hall effect elements are extensively used in magnetic measurements. They can also be used for sensing of current.

Another instrument that uses Hall effect is Poynting vector wattmeter. This wattmeter used for measuring the power loss density at the surface of a magnetic material.

Effect

Instruments

Magnetic effect	Ammeters, voltmeters, wattmeters
Heating effect	Integrating meters.
Electrostatic effect	Ammeters & voltmeters, wattmeters
Induction effect	Voltmeters
Hall effect	AC ammeters, voltmeters, wattmeters, energy meters
	Flux meters, ammeters & Poynting vector wattmeter.

Operating forces :- Three types of forces are needed for the satisfactory operation of any indicating instrument. These are

- (i) Deflecting force
- (ii) Controlling force and
- (iii) Damping force

Deflecting force :- The deflecting or operating force is required for moving the pointer from its zero position. The system producing the deflecting force is called "Deflecting system or Moving system". (The deflecting force can be produced by utilizing any of the effects mentioned earlier). Thus the deflecting system of an instrument converts the electric current into a mechanical force called deflecting force. The deflecting system thus acts as the prime mover responsible for deflection of the pointer.

controlling force :- This force is required in an indicating instrument in order that the current produces deflection of the pointer proportional to its magnitude. The system producing a controlling force is called a "controlling system". The functions are

- (i) To produce a force equal & opposite to the deflecting force at the final steady position of pointer. (or) producing output must be directly proportional to i/p.
- (ii) To bring the moving system back to zero position when the current is to be removed.

Rolling torque is generally provided by springs.
Sometimes gravity control is also used.
↳ spring control.

Damping force :- The deflecting force provides some deflection & ~~controlling force~~ ~~at~~ ~~the~~ ~~opposite~~ ~~dir~~ & it should come to rest at a position when the deflecting force is balanced by the controlling force.

So before coming to the rest, pointer always oscillates due to inertia, about the equilibrium position. Unless pointer rests, final reading cannot be obtained. So to bring the pointer to rest within short time, damping system is required.

The system should provide a damping force only when the moving system is in motion.

↳ Damping torque \propto velocity of the moving system
doesn't affect controlling torque. ↓
does not depend on operating current.

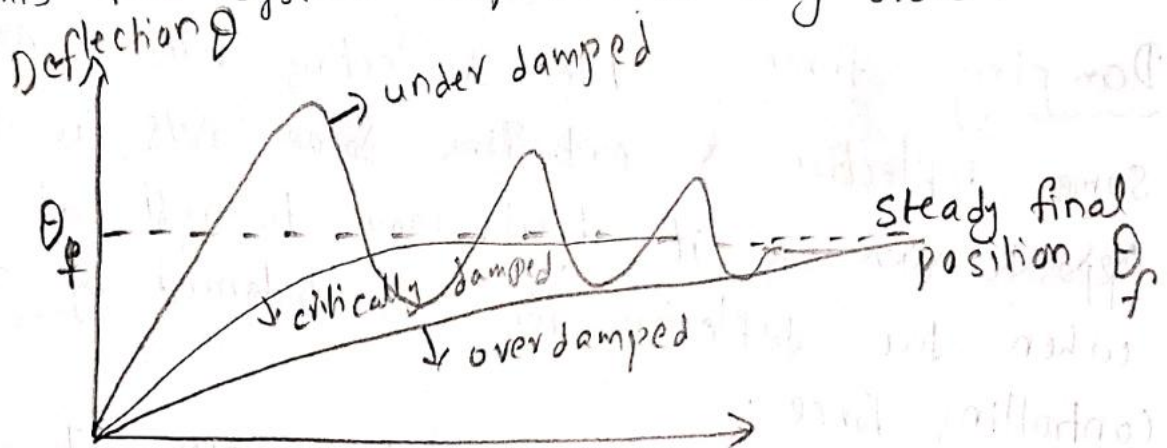
The quickness with which the moving system settles to the final steady position depends on relative damping.

→ If the moving system reaches to its final position rapidly but smoothly without oscillations, the instrument is said to be critically damped.

→ If the instrument is under damped, the moving system will oscillate about the final steady position with decreasing amplitude & will take certain time to come to rest.

In practice this under damped systems are preferred.

If the instrument is said to be overdamped,
 the ^{moving} system moves slowly to its final steady position.
 In this the system response is very slow.



The time response of damping system for various types of damping conditions is shown in fig above.

Methods ^{used} to produce damping torque are

- (i) Air friction damping
- (ii) Fluid friction damping
- (iii) Eddy current damping

Torque equation: - The various parameters involved in torque equation are -

- l = Length of coil in meters
- r = width of coil in "
- N = Number of turns of coil.
- B = Flux density in Wb/m^2 or Tesla.
- I = current through coil in Amperes.
- K = spring (or) Restoring constant in Nm/rad .
- α = Angle b/n plane of coil & direction of magnetic field.
- A = Area of coil in $m^2 = l \times r$

~~θ~~ θ

F = Force on each side of a coil = $NBIl \sin \alpha$ Newtons

T_d = Deflecting torque = $F \times d = NBIl \sin \alpha \times r$

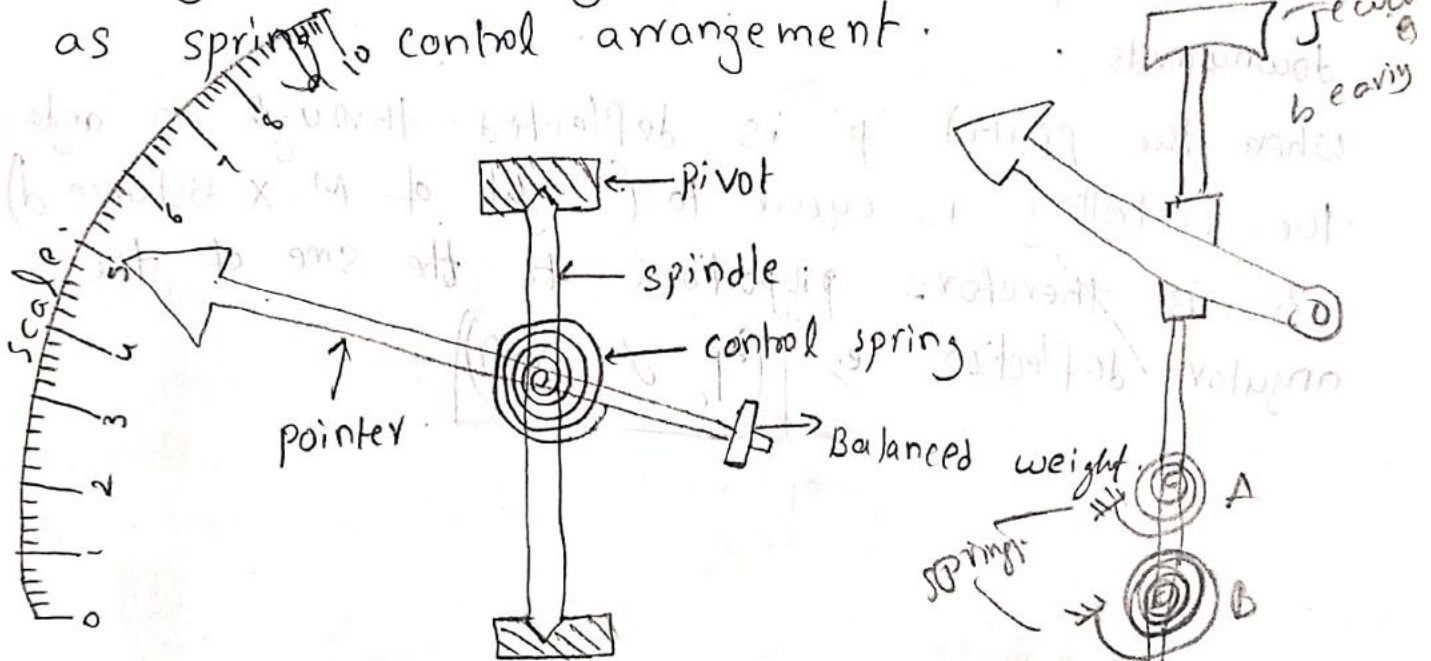
$T_d = NBIl \sin \alpha \Rightarrow T_d = NBIAr \Rightarrow T_d = GI \Rightarrow T_d = GI$ $\because \alpha = 90^\circ$
 $\sin \alpha = 1$

Controlling torque

Controlling torque can be produced by

- (i) spring control
- (ii) Gravity control.

(i) spring control :- Fig. Shows a commonly used as spring control arrangement.



It utilizes a hair spring of large number of turns which is attached to the moving system or spindle. When the pointer is deflected by deflecting torque the spring gets twisted in opposite direction which produces controlling torque. It comes to an equilibrium position when magnitude of deflecting torque is equal to controlling torque. Further, the torque of a spiral spring is proportional to the angle of twist, the controlling torque (T_c) is directly proportional to the angular deflection of the pointer (θ) i.e., $T_c \propto \theta$

The spring material should have the following properties.

- (i) It should be non-magnetic.
- (ii) It must be of low temp. coefficient
- (iii) It should not be subjected to fatigue.

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3 Spring control :- A hair spring attached to the moving system, which produces controlling torque.

The essential requirements for instruments springs are (i) They should be non-magnetic.
(ii) They should not be subjected to fatigue.
(iii) They should have low temp. coefficient.

For most of the instruments, phosphor bronze spiral springs are provided. The inner end of the spring is attached to the spindle while the outer end is attached to the arm which is actuated by a set of screw mounted at the front of the instrument. So zero setting can be easily done.

The controlling ^{torque} provided by the instrument is directly proportional to the angular deflection of the pointer.

The controlling torque produced by the spiral spring is given by

$$T_c = \frac{Ebt^3}{12L} \theta = K_s \theta$$

where E = Young's modulus of spring material in N/m^2

t = thickness in metres

b = Depth in "

L = Length in "

K_s = Spring constant = $\frac{Ebt^3}{12L}$

$$\boxed{T_c \propto \theta}$$

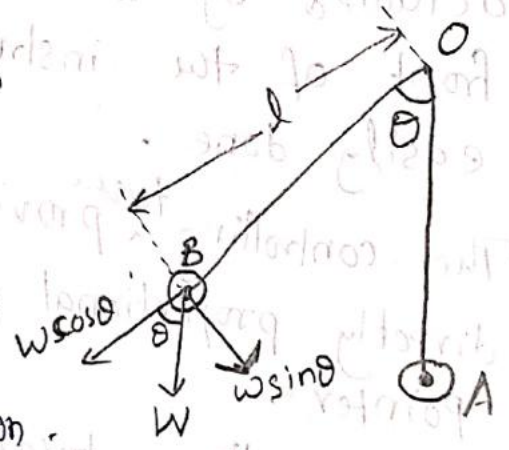
Gravity control: - This type of control consists of a small weight attached to the moving system whose position is adjustable. This weight produces a controlling torque due to gravity. This weight is called control weight.

Fig shows the gravity control system.

At zero position of the pointer, the controlling torque is zero. This position is shown as position 'A' of the weight in the fig shown in below.

If the system deflects, the weight position also changes as shown in fig below.

The system deflects through an angle θ . The control weight acts at a distance l from the center. The component $w \sin \theta$ of this weight tries to restore the pointer back to zero position.



This is nothing but the controlling torque T_c .

Thus $T_c = w \sin \theta \times l$

$T_c = k \sin \theta$

where $k = wl$

$T_c \propto \sin \theta$

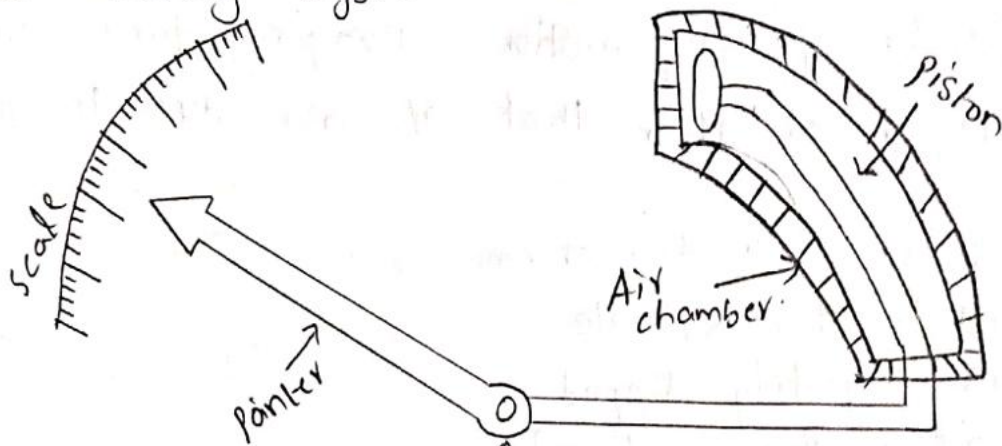
Gravity constant

Thus controlling torque is proportional to sine of angular deflection (or) sine of angle of deflection of moving system.

→ The controlling torque can be varied by simply adjusting the position of control weight upon the arm which carries it.

(ii) To bring the moving system

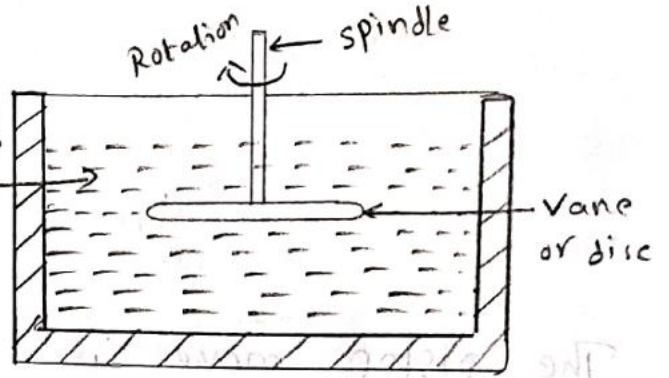
1) Air friction damping:- This arrangement consists of a light aluminium piston which is attached to the moving system, show in fig.



The piston moves in a ^{spindle} fixed air chamber. It is close to one end. The clearance b/n piston & wall chambers is uniform & small. When there are oscillations the piston moves into & out of an air chamber. When piston moves into the chamber, air inside is compressed & pressure of air developed due to friction opposes the motion of pointer. There is also opposition to motion of moving system when piston moves out of the chamber. Thus the oscillations & the overshoot gets reduced due to to & fro motion of the piston in the chamber, providing necessary damping torque. This helps in settling down the pointer to its final steady position very quickly.

Fluid friction damping :- This method is similar to air friction damping, only air is replaced by working fluid. The friction b/n the disc & fluid is used for opposing motion. Damping force due to fluid is greater than that of air due to more viscosity.

It consists of a disc or vane attached to the spindle which is completely dipped in the oil. The frictional force b/n oil & vane is used to produce the damping torque, which opposes the oscillating behaviour of the pointer.



when the moving system moves, the disc moves in oil & frictional force is produced.

- Advantages :-
- Due to more viscosity of fluid, more damping is provided.
 - The oil can also be used for insulation purposes.

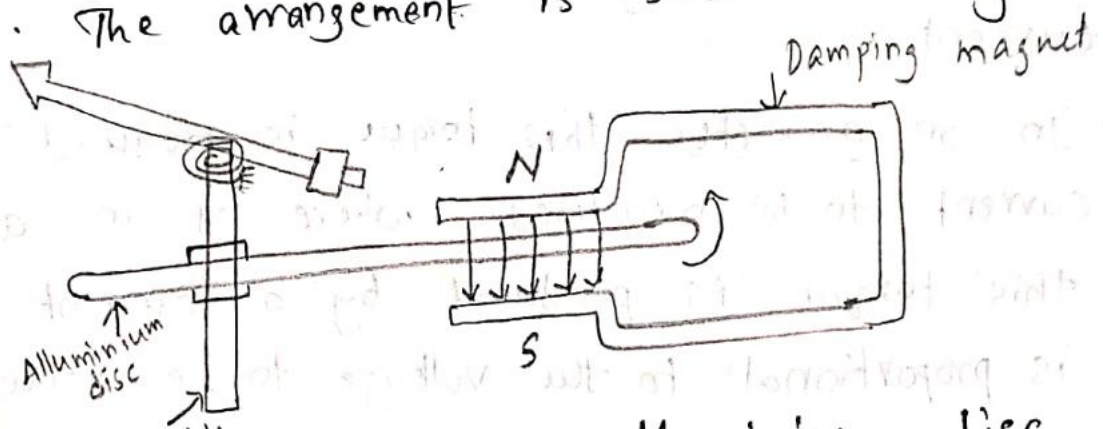
Disadvantages :-

- This can be only used for the instruments which are in vertical position.
- Due to oil leakage, the instruments can't be kept clean.

Eddy current damping :- This is the most effective way to providing damping. It is based on the Faraday's law & Lenz's law. When a conductor moves in a magnetic field it cuts the magnetic flux of lines an emf is induced in it.

And direction of this emf is so as to oppose the cause producing it.

In this method, an aluminium disc is connected to the spindle. The arrangement of disc is such that when it rotates, it cuts the magnetic flux lines of a permanent magnet. The arrangement is shown in fig.



When the pointer oscillates, aluminium disc rotates under the influence of magnetic field of damping magnet. So disc cuts the flux which causes an induced emf in the disc. The disc is a closed path hence induced emf circulates the current through the disc called eddy current.

The direction of such eddy current is so as to oppose the cause producing it. The cause is relative motion b/n disc & field. Thus it produces an opposing torque so as to reduce the oscillations of pointer.

This brings pointer to rest quickly. This is most effective & efficient method of damping.

Analog Ammeters & Voltmeters :-

Ty

Analog Ammeters & voltmeters are classed together as there are no fundamental differences in their operating principles.

The action of all ammeters & voltmeters depends upon a deflecting torque produced by an electric current.

- In an ammeter this torque is produced by a current to be measured where as in a voltmeter this torque is produced by a current which is proportional to the voltage to be measured.
- Ammeters are connected in series in the ckt whose current is to be measured. The power loss in an ammeter is $I^2 R_a$. Ammeters should have low electrical resistance so that they cause a small voltage drop & absorb small powers.
- Voltmeters are connected in parallel with the ckt whose voltage is to be measured. The power loss in ~~an~~ ~~amm~~ voltmeters is V^2/R_v . Voltmeters should have a high electrical resistance, so that the current drawn by them is small & power consumed is small.

Types of Instruments :-

The main types of instruments used as ammeters & voltmeters are

- (i) Permanent magnet moving coil (PMMC)
- (ii) Moving iron
- (iii) Electro dynamometer
- (iv) Hot-wire
- (v) Thermo couple
- (vi) Induction
- (vii) Electrostatic
- (viii) Rectifier

PMMC is used for DC measurements only &

Induction type " Ac measurements only .

The other type of instruments can be used either DC or AC measurements.

Permanent Magnet Moving coil instruments: (PMMC)

The PMMC instruments are most accurate type for DC measurements. PMMC works on the motoring principle. When a current carrying coil is placed in the magnetic field produced by permanent magnet, the coil experiences a force & moves.

Construction:- The general constructional features of PMMC are moving coil, magnetic system, pointer & scale.

The moving coil is either rectangular or circular ^{vant} in shape. It has number of turns of copper wire. The coil is mounted on a rectangular aluminium former which is pivoted on jewelled bearings. The coils move freely in the field of a permanent magnet.

→ The pointer is carried by the spindle & moves over a graduated scale. The pointer is of light weight & by careful alignment of the pointer the parallax errors will be reduced.

working! - When the current carrying coil is placed in a magnetic field it will experience a force. When this instrument is connected to the electrical circuit, the current to be measured will flow through the coil & this current is the responsible for the deflecting torque, such that the pointer moves from its zero position.

If the current in the coil is reversed, the torque will also be reversed, consequently the pointer tries to deflect below zero.

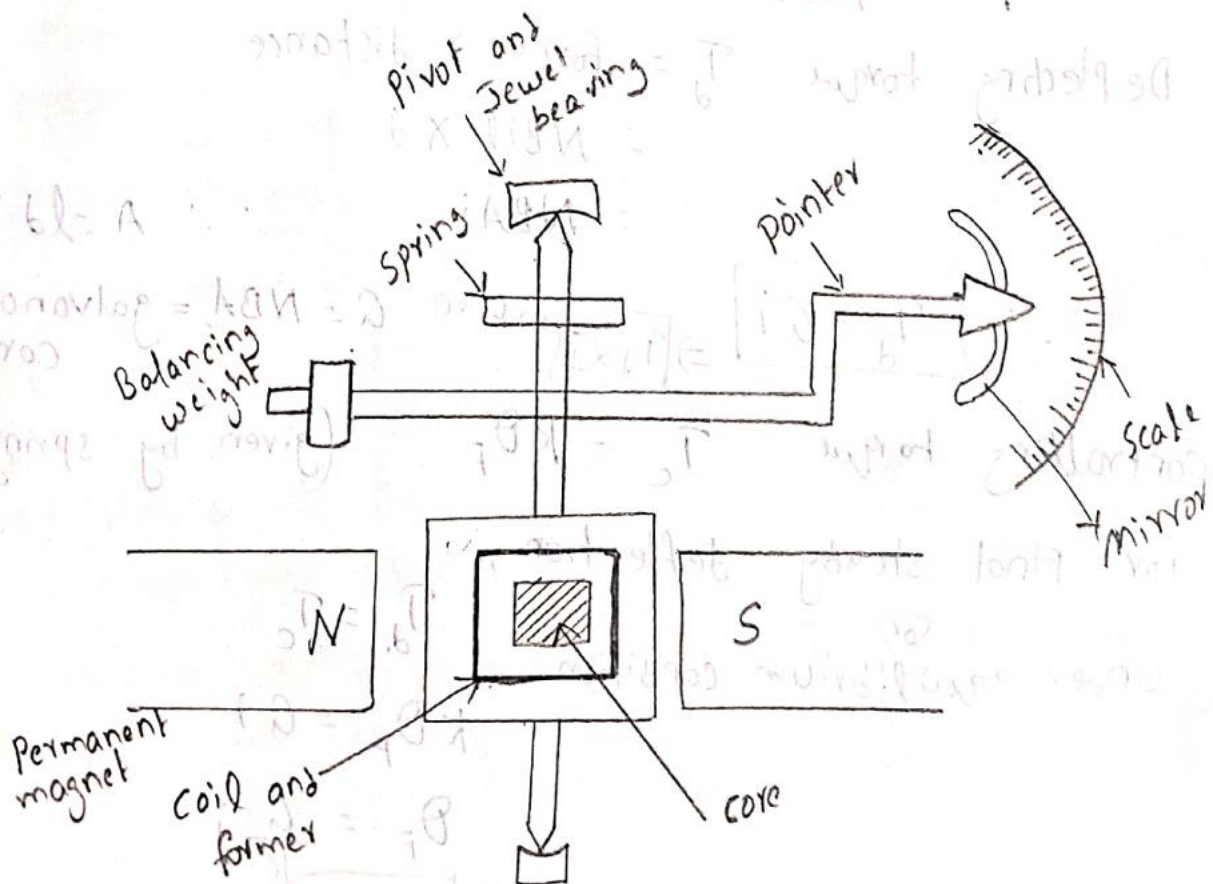
The controlling torque is provided by two phosphor bronze springs. Damping torque is produced by movement of aluminium former moving in the magnetic field.

Advantages:-

- (1) It has uniform scale
- (2) The torque to weight ratio is very high.
- (3) The sensitivity is high.
- (4) It consumes low power
- (5) It has high accuracy
- (6) Instrument is free from hysteresis error
- (7) Extension of instrument range is possible.

Disadvantages:-

- (1) suitable for dc measurements only.
- (2) Ageing of permanent magnet & the control springs introduces the errors.
- (3) The cost is high
- (4) The friction due to jewel pivot suspension.



Construction of PMMC Instrument

Torque equation:

Let l, d = length of vertical & horizontal side^{rs} of coil respectively, in meters. E

N = number of turns ~~of~~ in the coil

B = flux density, wb/m^2

i = current through moving coil, in Amperes

k = spring constant, Nm/rad

θ_F = final steady state deflection of moving coil in radians

Force on each side of coil = $N B i l \sin \alpha$

where α = angle b/n direction of magnetic field & the conductor

~~IF~~ the field is radial, $\therefore \alpha = 90^\circ$

$$F = N B i l$$

Deflecting torque T_d = force \times distance

$$= N B i l \times d$$

$$= N B A i \quad \because A = l d \text{ in } \text{m}^2.$$

$$\therefore \boxed{T_d = G i} \Rightarrow \boxed{T_d \propto i} \text{ where } G = N B A = \text{galvanometer constants.}$$

Now

Controlling torque $T_c = k \theta_F$ (given by springs)

For final steady deflection,

(or)

under equilibrium condition

$$T_d = T_c$$

$$k \theta_F = G i$$

$$\theta_F = \frac{G}{k} i$$

$$\boxed{\theta_F \propto i}$$

Errors in PMMC instruments:- The main sources of errors in moving coil instrument are due to

- (i) weakening of permanent magnets due to aging.
- (ii) weakening of springs due to ageing.
- (iii) change of resistance of moving coil with temperature.
- (iv) Frictional Error
- (v) Temperature Error.

weakening of permanent magnets & springs

The weakening of magnets causes or tends to decrease the deflection, while weakening of springs causes to ~~large~~ increase the deflection for a particular value of current.

The proper use of material & preageing during manufacturing can reduce the errors due to weakening of ~~x~~ springs.

Frictional Errors:- The Error which occurs between the moving coil & the core is known as Frictional Error. The frictional error will be reduced by maintaining high torque - weight ratio. This can be even reduced when the weight of the moving system is made as small as possible.

Temperature error:- Apart from the room temp. it occurs mainly due to the heating of working coil connected internally in the instrument.

This can be reduced by providing the proper ventilation & cooling system to the instrument. Att
c
Some times this error is minimised by winding a series resistance which has a low temp-coefficient called as swamping resistance.

Moving iron instruments! - The most common Ammeters & voltmeters for laboratory use at power frequencies are the moving iron instruments. These instruments are constructed to measure voltage & current to an accuracy needed in most engineering works & still be cheap as compared to any other AC instrument.

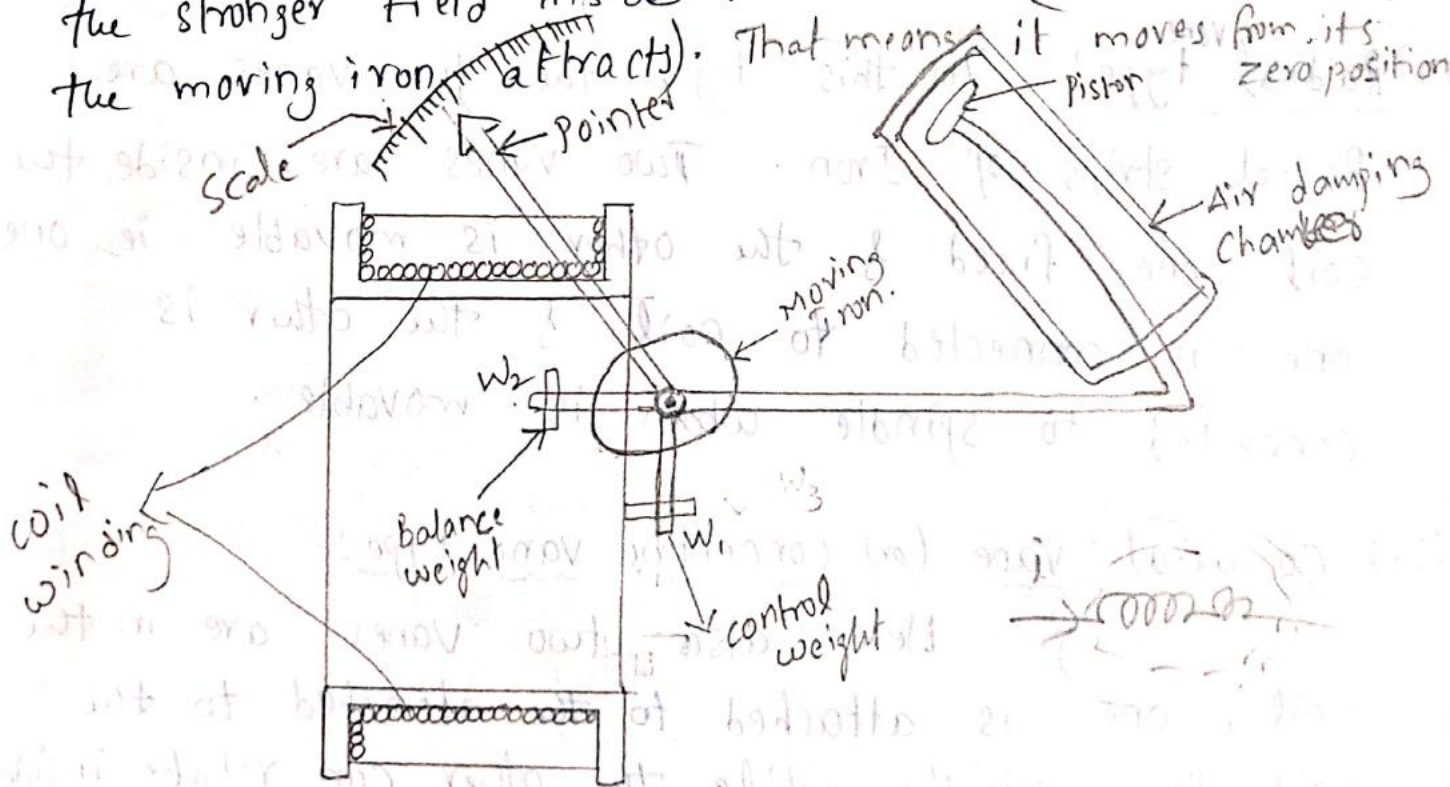
The basic principle of MI instruments is, It consists of a soft iron which is made to move & placed in a magnetic field. When the Electromagnets are excited the soft iron piece experiences a force & it moves from its initial position. [The working principle of these instruments is that a soft iron piece if brought near the magnet gets attracted by the magnet.]
Classification of Moving Iron Instruments:-

There are two types of Moving Iron Instruments. They are namely (i) Attraction type of MI Instrument (ii) Repulsion type of " "

i) Attraction type :- The below figure shows the constructional details of Moving Iron Instruments. It consists of a flat coil & has the windings in it. The number of turns of the fixed coil are dependent on the range of the instrument. For passing the large current through the coil only few turns are required.

→ The moving iron is a flat disc which is mounted on the spindle. This spindle is supported by ^{Jewel} bearings & carries a pointer which moves over a graduated scale.

When the current flows through the coil, ^{becomes an electromagnet} magnetic field is produced & the moving iron moves from weaker field outside the coil to the stronger field inside the coil. (In other words the moving iron attracts). That means it moves from its zero position.



The controlling torque is provided by the springs ^{not} but gravity control may also be used for vertically mounted panel type instruments.

The damping torque is provided by the air friction damping.

Repulsion type:- These instruments have two vanes inside the coil, the one is fixed & other is movable. When the current flows through the coil, both the vanes are magnetised with like polarities. ~~induced on the same~~ Therefore due to like polarities the repulsion force will take place, which causes the movement ^{of} moving vane.

There are two types of Repulsion instruments.

(i) Radial vane type

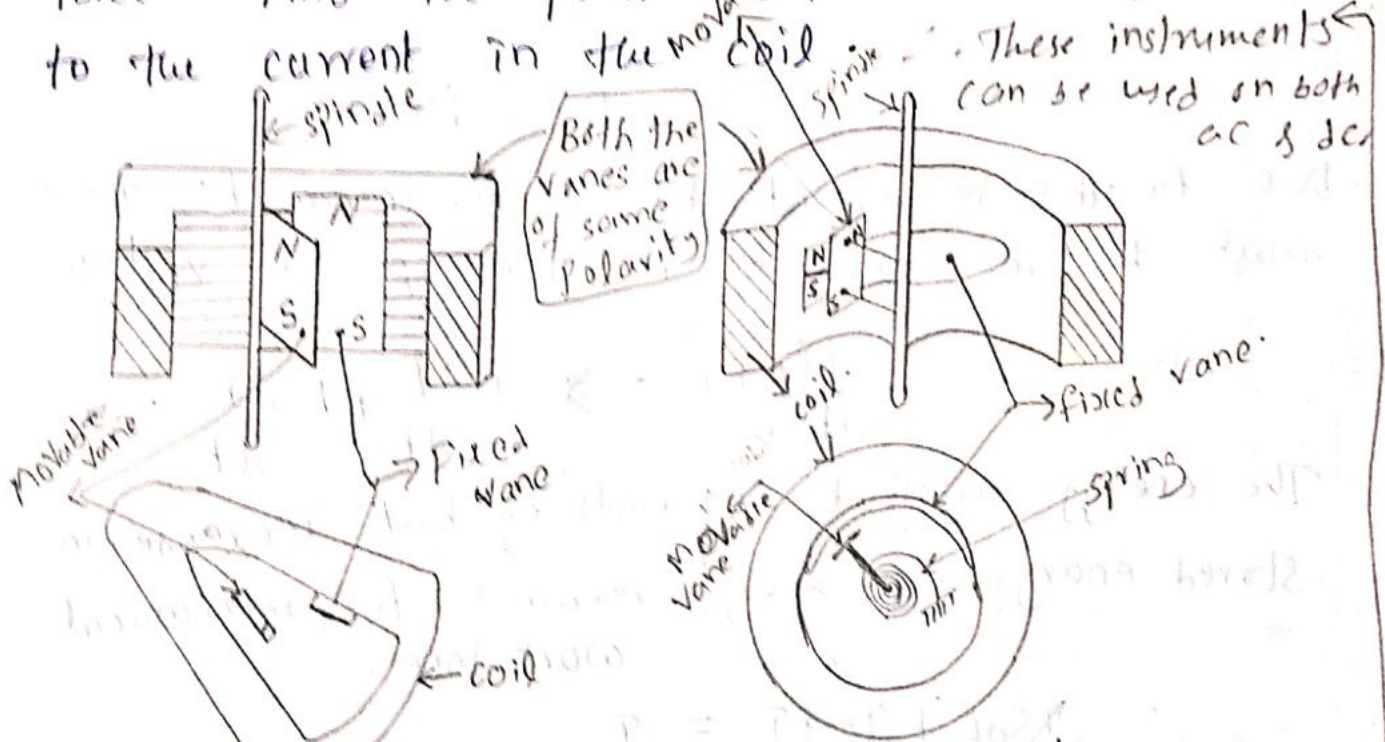
(ii) co-axial vane type.

(i) Radial ^{vane} type:- In this type, the two vanes are radial strips of Iron. Two vanes are inside the coil one fixed & the other is movable. i.e., one is connected to coil & the other is connected to spindle which is movable.

(ii) Co-axial vane (or) concentric vane type:-

Here also two vanes are in the coil, one is attached ~~to the~~ attached to the coil frame rigidly while the other can rotate inside.

Both the vanes are magnetised with same polarity. Thus the movable vane rotates under the repulsive force. And the pointer deflection is proportional to the current in the coil. These instruments can be used on both AC & DC.



It is clear that whatever may be direction of the current in the coil of the instrument, the iron vanes are so magnetised that there is always a force of attraction in the attraction type & repulsion in the repulsion type of inst. Thus $M I$ instruments are unpolarised instruments i.e. they are independent of the direction in which the current passes. Torque equation of Moving Iron Instruments.

An expression for the torque of a moving iron instrument may be derived by considering the energy relations when there is a small increment in current supplied to the instrument. When this happens there will be a small change in deflection $d\theta$ & some mechanical work will be done.

$$\text{Mechanical work done} = T_d d\theta$$

There will be a change in the energy stored in the magnetic field due to change in inductance.

Suppose initially there is the initial current is I , inductance L & the deflection θ , if the current increased by dI , then $I \rightarrow dI$

$$L \rightarrow dL$$

$$\theta \rightarrow d\theta$$

Due to this increment dI in the current, there must be an increase in the applied voltage.

$$\text{i.e., } e = \frac{d(LI)}{dt} \Rightarrow I \frac{dL}{dt} + L \frac{dI}{dt}$$

The energy supplied is nothing but increase in stored energy + energy required for mechanical work done.

$$eI dt = I^2 dL + IL dI$$

Energy supplied $eI dt = I^2 dL + IL dI$

Due to change in current
The stored energy changes from $\frac{1}{2} LI^2$ to $\frac{1}{2} (L+dL)(I+dI)^2$

$$\text{i.e., } \frac{1}{2} (I^2 + 2I dI + dI^2)(L+dL) - \frac{1}{2} LI^2$$

Neglecting higher order terms, this becomes

$$IL dI + \frac{1}{2} I^2 dL$$

From the principle of conservation of energy,

Electrical energy supplied = increase in stored energy + mechanical work done

$$I^2 dL + IL dI = LI dI + \frac{1}{2} I^2 dL + T_d d\theta$$

$$\frac{1}{2} I^2 dL = T_d d\theta$$

$$\text{Dividing by } d\theta \quad T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

The Deflecting torque will be controlled by controlling

torque T_c . $\Rightarrow T_c = k\theta$

(since the moving system is provided with control springs)

At equilibrium position,

$$T_d = T_c$$

$$k\theta = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

The deflection in the MI instrument

is given by - Deflection $\theta = \frac{1}{2} \frac{I^2}{k} \frac{dL}{d\theta}$

Hence the angular deflection is proportional to the square of the operating current, & the instrument has a square law response.

Errors in MI Instruments.

(i) Hysteresis Errors :- This error occurs as the value of flux density is different for same current of ascending & descending values. The value of flux density is higher for descending currents than the meter reads higher values for descending currents & vice versa.

To reduce this errors the iron parts of low flux density are used.

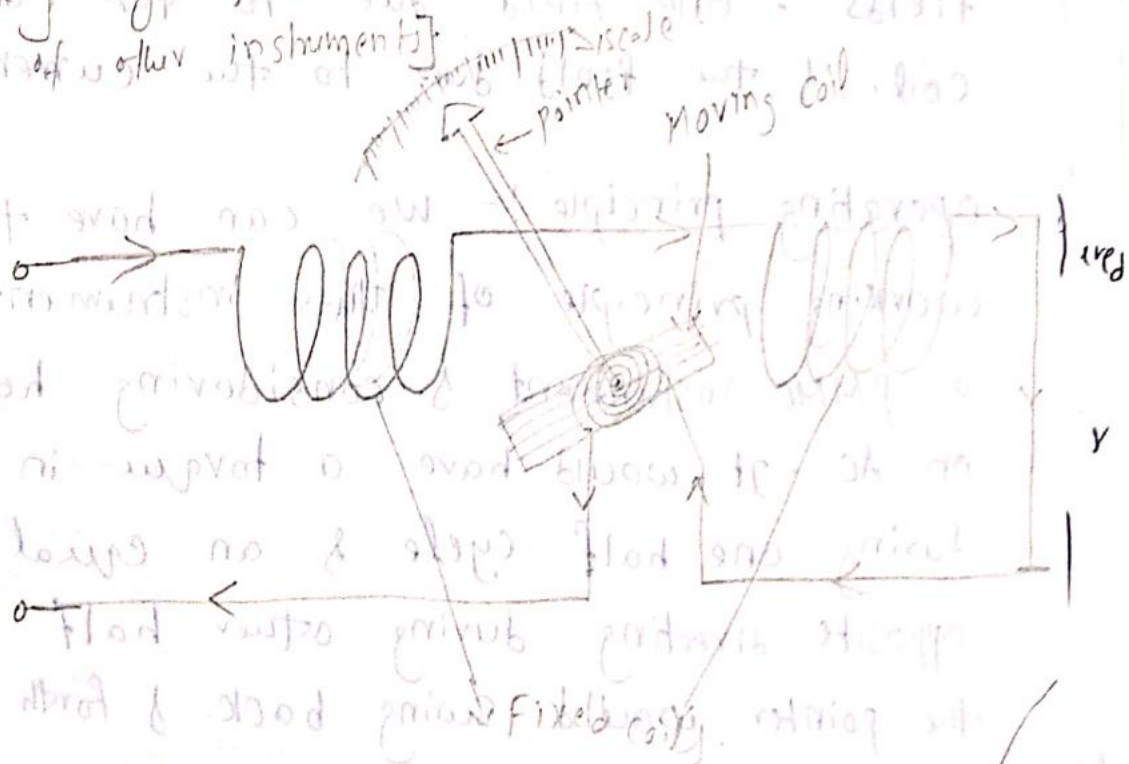
Electrodynamometer type Instruments:- The necessity for the AC calibration of Moving Iron instruments as well as other types of instruments which can't be correctly calibrated & requires the use of transfer instruments.

A Transfer instrument is one that may be calibrated with on DC & then used without modification to measure AC quantities. This requires the transfer type instrument to have same accuracy for both dc & ac, which the electrodynamicometers have.

→ can be used for measurement of current, voltage & power for both dc & ac.

→ Electrodynamicometer type & other transfer type instruments have same accuracy for dc & ac upto 125 Hz.

→ Accuracy is very high & hence can be used in laboratory for calibration of other instruments.



Construction & operation:- Dynamometer type instruments are similar to the PMMC instruments except that permanent magnets are replaced by coil.

It consists of 2 coils

- Fixed coil → to produce operating field.
→ divided into 2 parts to have uniform distribution of flux around the moving coil.
- Moving coil → to produce deflecting torque T_d .

[In PMMC, PM produces the operating field & moving coil is placed inside the coil. When the current passes through the MC, flux will be produced. So the interaction between flux produced by MC & flux produced by PM develops the required torque i.e., T_d].

As like other instruments, the deflection in the instrument develops by the interaction of magnetic fields - one field due to the current in moving coil & the field due to the current in the fixed coil.

Operating principle:- We can have the idea of working principle of this instrument by taking up a PMMC instrument & considering how it behaves on AC - It would have a torque in one direction during one half cycle & an equal effect in the opposite direction during other half cycle. Therefore, the pointer would swing back & forth around zero point.

However, if we want to reverse the direction of flux each time the current through the moving coil reverses, a unidirectional torque would be produced for both negative & positive half cycles.

In electrodynamometer instruments, the field can be made to reverse simultaneously with the current in the movable coil, if the field coil is connected in series with the moving coil as shown in above.

Construction:-

- (i) Fixed Coils :- The ^{operating} field is produced by fixed coils. The coil is divided into two sections to give a uniform field near the centre. The wire wound for fixed coils is stranded to reduce eddy current losses.
- (ii) Moving coil :- The moving coil is wound either as a self sustaining coil or on a non-metallic former. A metallic former cannot be used as eddy currents would be induced in it by alternating field.
- (iii) Moving system :- The moving coil is mounted on an aluminium spindle. The moving system also carries a pointer.

(iv) controlling system:- The controlling torque is provided by two control springs.

(v) Damping system:- Air friction damping is used & is provided by a pair of aluminium vanes, attached to the spindle at the bottom.

(vi) Shielding:- proper shielding is provided against the stray magnetic fields using laminated steel shield.

(The field produced by fixed coils is weaker also due to some external magnetic field & the interference may come that may disturb the reading. To avoid that problem we have to provide the proper shielding such that external magnetic field may not affect the reading).

Torque Equation

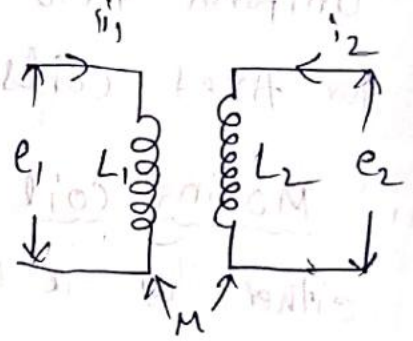
i_1 = current in fixed coil

i_2 = current in moving coil

L_1 = self inductance of fixed coil

L_2 = Mutual inductance of moving coil

M = Mutual Inductance b/n fixed & moving coils



Flux linkages of coil 1, $\Phi_1 = L_1 i_1 + M i_2$

" " " " coil 2, $\Phi_2 = L_2 i_2 + M i_1$

Now $e_1 = \frac{d\Phi_1}{dt}$ & $e_2 = \frac{d\Phi_2}{dt}$

Electrical input energy = $e_1 i_1 dt + e_2 i_2 dt$

$\therefore e = i_1 d\psi_1 + i_2 d\psi_2$ $\therefore e_1 = \frac{d\psi_1}{dt}$

$\Rightarrow e = i_1 d(L_1 i_1 + M i_2) + i_2 d(L_2 i_2 + M i_1)$ $e_1 dt = d\psi_1$
 $\qquad\qquad\qquad e_2 dt = d\psi_2$
 $= i_1 L_1 di_1 + i_1^2 dL_1 + i_1 i_2 dM + i_1 M di_2 + i_2 L_2 di_2 +$
 $\qquad\qquad\qquad \frac{1}{2} i_1^2 dL_1 + M i_2^2 dL_2 + i_1 i_2 dM + i_2 M di_1 \rightarrow (1)$

Energy stored in the magnetic field,

$\frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + i_1 i_2 M$

change in stored energy = $d \left[\frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + i_1 i_2 M \right]$

$\Rightarrow \frac{1}{2} L_1 \cdot 2 i_1 di_1 + \frac{1}{2} i_1^2 dL_1 + \frac{1}{2} L_2 \cdot 2 i_2 di_2 + \frac{1}{2} i_2^2 dL_2 +$
 $i_1 i_2 dM + i_1 M di_2 + i_2 M di_1$

$\Rightarrow L_1 i_1 di_1 + \frac{1}{2} i_1^2 dL_1 + L_2 i_2 di_2 + \frac{1}{2} i_2^2 dL_2 + i_1 i_2 dM$
 $+ i_1 M di_2 + i_2 M di_1 \rightarrow (2)$

\therefore Electrical i/p Energy = change in energy stored + Mechanical work done.

\therefore Mechanical work done = Electrical i/p energy - change in energy stored

Mechanical work done = $\frac{1}{2} i_1^2 dL_1 + \frac{1}{2} i_2^2 dL_2 + i_1 i_2 dM$

Since L_1, L_2 are constants therefore dL_1 & dL_2 are both equal to zero.

∴ Mechanical work done = $i_1 i_2 dM$

If T_d is the instantaneous deflecting torque, $d\theta$ is change in deflection

then ∴ Mechanical work done = $T_d d\theta$

$$\Rightarrow T_d d\theta = i_1 i_2 dM$$

$$\therefore T_d = i_1 i_2 \frac{dM}{d\theta} \rightarrow (3)$$

when operation with DC :-

From equ (3), deflecting torque, $T_d = i_1 i_2 \frac{dM}{d\theta}$

Controlling torque $T_c = k\theta$

Under equilibrium, $T_d = T_c$

$$i_1 i_2 \frac{dM}{d\theta} = k\theta$$

deflection :-

$$\theta = \frac{i_1 i_2}{k} \frac{dM}{d\theta}$$

→ rate of change of mutual inductance

when operating with AC :- In ac operation, the total deflecting torque over a cycle must be obtained by integrating

From equ (3), $T_d = i_1 i_2 \frac{dM}{d\theta}$ over one period.

The average deflection torque for a complete cycle

$$T_d = \frac{1}{T} \int_0^T T_d dt$$

$$\Rightarrow T_d = \frac{1}{T} \int_0^T i_1 i_2 \frac{dM}{d\theta} dt \Rightarrow \frac{dM}{d\theta} \frac{1}{T} \int_0^T i_1 i_2 dt$$

T = time period for one complete cycle

If i_1 & i_2 are sinusoidal currents & are displaced by an angle ϕ , i.e.

$$i_1 = I_{m1} \sin \omega t \quad \& \quad i_2 = I_{m2} \sin(\omega t - \phi)$$

The avg- deflecting torque,

$$T_d = \frac{dM}{d\theta} \frac{1}{T} \int_0^T I_{m1} \sin \omega t \cdot I_{m2} \sin(\omega t - \phi) d(\omega t)$$

$$T_d = \frac{dM}{d\theta} \frac{1}{2\pi} \int_0^{2\pi} I_{m1} \sin \omega t \cdot I_{m2} \sin(\omega t - \phi) d(\omega t)$$

$$= \frac{I_{m1} I_{m2}}{2\pi} \frac{dM}{d\theta} \int_0^{2\pi} \sin \omega t \cdot \sin(\omega t - \phi) d(\omega t)$$

$$= \frac{I_{m1} I_{m2}}{2} \frac{dM}{d\theta} \frac{1}{\pi} \int_0^{2\pi} \sin \omega t \sin(\omega t - \phi) d\omega t$$

$$= \frac{I_{m1} I_{m2}}{2} \frac{dM}{d\theta} \frac{1}{2\pi} \int_0^{2\pi} \frac{2 \sin \omega t \sin(\omega t - \phi)}{2} d\omega t$$

$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$

$$= \frac{I_{m1} I_{m2}}{2} \frac{dM}{d\theta} \frac{1}{2\pi} \int_0^{2\pi} [\cos \phi - \cos(2\omega t - \phi)] d\omega t$$

$$= \frac{I_{m1} I_{m2}}{2} \frac{dM}{d\theta} \frac{1}{2\pi} \left[\int_0^{2\pi} \cos \phi d\omega t - \int_0^{2\pi} \cos(2\omega t - \phi) d\omega t \right]$$

$$= \frac{I_{m1} I_{m2}}{2} \frac{dM}{d\theta} \frac{1}{2\pi} \left[\cos \phi \cdot \omega t \Big|_0^{2\pi} - \left[\frac{\sin(2\omega t - \phi)}{2} \right]_0^{2\pi} \right]$$

$$= \frac{I_{m1} I_{m2}}{2} \frac{dM}{d\theta} \frac{1}{2\pi} \left[\cos \phi \cdot 2\pi - \frac{\sin(4\pi - \phi)}{2} + \frac{\sin(-\phi)}{2} \right]$$

$$= \frac{I_{m1} I_{m2}}{2} \frac{dM}{d\theta} \frac{1}{2\pi} \left[\cos \phi \cdot 2\pi + \frac{\sin \phi}{2} - \frac{\sin \phi}{2} \right]$$

$$T_d = \frac{I_{m1} I_{m2}}{2} \frac{dM}{d\theta} \cos \phi \Rightarrow \boxed{T_d = I_1 I_2 \frac{dM}{d\theta} \cos \phi}$$

$I_{m1} = I_{m2} = \frac{I_m}{\sqrt{2}}$

where I_1 & I_2 are rms values of current flowing in the coils.

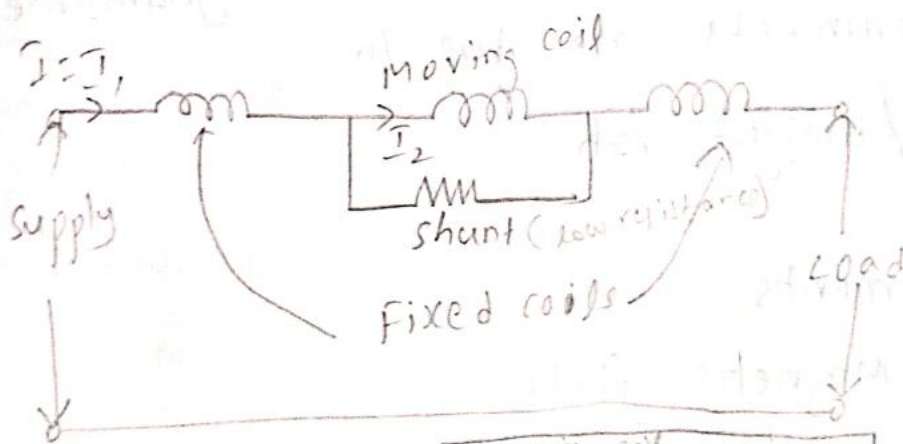
At equilibrium, $T_d = T_c$

$$I_1 I_2 \cos \phi \frac{dM}{d\theta} = k\theta$$

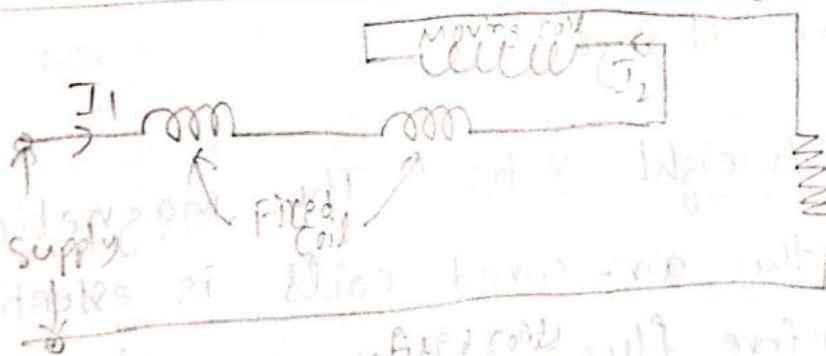
∴ Deflection $\theta = \frac{I_1 I_2 \cos \phi}{k} \frac{dM}{d\theta}$

Deflection is proportional to product of the currents, cosine of the phase angle & rate of change of mutual inductance.

- (1) Low Torque to weight ratio
- (2) Frictionary errors
- (3) Eddy current "
- (4) stray magnetic field errors
- (5) temperature error

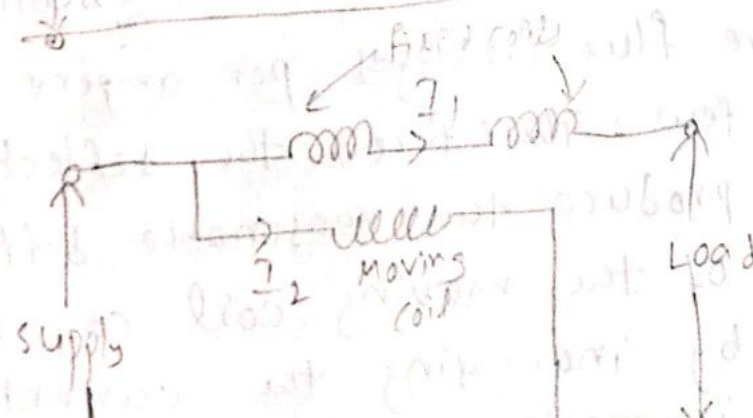


Electrodynamometer
ammeter calibre 100 A



Voltmeter

High non inductive resistance



Comparison of various types of instruments.

<u>Meter type</u>	<u>Control</u>	<u>Damping</u>	<u>Suitability</u>
PMMC	Spring	Eddy current	DC
Electro dynamometer	Spring or gravity	Air friction damping	DC & AC
Electrodynamometer	Spring	"	DC & AC.

Errors in Electro dynamometer instruments.

The main sources of error in a dynamometer type of instruments are due to

- (i) Low torque/weight ratio
- (ii) Frequency
- (iii) Eddy currents
- (iv) External magnetic fields
- (v) Temperature change.

(i) Low Torque/weight ratio:- The magnetic field produced by the air-cored coils is essentially small & therefore flux linkages per ampere in the moving coil are few. Therefore, the deflecting torque is low. Thus to produce the reasonable deflecting torque, the mmf of the moving coil can be increased either by increasing the current through the coil or by increasing the no. of turns.

The current cannot be increased beyond 200 mA otherwise the springs would be heated too much. On other hand an increase in the number of turns of the moving coil, it will give rise to increased weight of the moving parts.

Resulting low torque/weight ratio error. This error also increases the friction losses in EDM type instruments.

Compensation:- This error can be minimized by careful design of the coils.

$$\text{Flux linkages } \psi = \phi N$$

$$T = F \times d \quad \psi = BAN$$

$$T = NBil \times d = NBAI$$

Frequency error:- The change in the frequency causes to change self inductances of moving coil & fixed coil. This causes the error in the reading.

The frequency error can be reduced by having equal time constant for both fixed & moving coil circuits.

Eddy current errors:- In metal parts of the instrument the eddy currents get produced. The eddy currents interact with the instrument current, to cause change in the deflecting torque, to cause error. Hence metal parts should be kept as minimum as possible. Also the resistivity of the metal parts used must be high, to reduce the eddy currents.

Stray magnetic fields error :- ~~sim~~ The operating field in electro-dynamometer instrument is very weak. Hence external magnetic field can interact with the operating field to cause change in the deflection, that causes the error. To reduce the effect of stray magnetic field, ~~the shields must be used for~~ proper shielding must be provided.

Temperature errors :- The temperature errors are caused due to the self heating of the coil, which causes change in the resistance of the coil. Thus temperature compensating resistors can be used to eliminate the temperature errors.

Extension of Range :- There are 4 common devices for extending the range of instruments. Those are namely, shunts, multipliers, potential transformers & current transformers.

Shunts & ~~multipliers~~ current transformers are used to extend the range of ammeters, while the multipliers & potential transformers are used ~~to~~ extend the range of voltmeters.

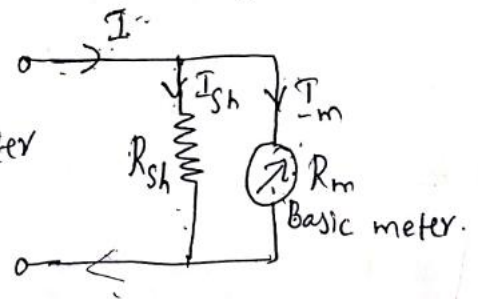
Generally, the instrument's springs serves as current leads to moving coil. Their (springs) current carrying capacity limits the current which can be safely carried to about 20 mA. For measuring higher currents in p.m.c. the overall current should be bypassed through a resistor called shunt.

Extension of range of Ammeters: (upto 50 A). Ammeter shunts:-

- The amount of current that can be passed to moving coil is limited (20 mA) due to weight constraint & current carrying capacity of springs.
(∵ weight of moving part should be minimum → conductor size ↓ → I_m ↓)
- So to increase the range of ammeters, a low value of resistance that ~~can~~ ^{must} be connect in parallel with the meter called "shunt".

Whenever large values of higher currents are to be measured, the major part of current is bypassed through a low resistance known as "shunt".

Fig:- shows the basic meter & its shunt to produce an ammeter



Let R_m = internal resistance of meter i.e. coil.

R_{sh} = Resistance of shunt

$I_{fs} = I_m$ = full scale deflection current

I_{sh} = shunt current, I = current to be measured

Since, shunt resistance is in parallel with the meter, so the voltage drop across shunt & meter is same.

$$\therefore I_{sh} R_{sh} = I_m R_m$$

$$R_{sh} = \frac{I_m R_m}{I_{sh}}$$

But from the ckt, $I_{sh} = I - I_m$

$$\therefore R_{sh} = \frac{I_m R_m}{I - I_m}$$

$$R_{sh} = \frac{I_m R_m}{I_m \left(\frac{I}{I_m} - 1 \right)} = \frac{R_m}{\left(\frac{I}{I_m} - 1 \right)}$$

$$\therefore \boxed{R_{sh} = \frac{R_m}{m-1}}$$

ie., the ratio of total current to meter current is known as multiplying power

Construction of shunt

The general requirements are

- (i) The resistance of shunt should not vary with time
- (ii) The temp. coefficient of shunt & instrument should be low.
- (iii) They should have low thermal Electromotive force.

"Manganin" is usually used for shunts of DC instrument & the "constantan" for AC circuits.

Multi-range Ammeters :- The current range can be further extended by a number of shunts, selected by a range switch. Such meter is called Multirange Ammeter.

The circuit consists of 4 shunts R_{sh1} , R_{sh2} , R_{sh3} & R_{sh4} which are placed in parallel with the meter to give four different current ranges

$$I_1, I_2, I_3 \& I_4.$$

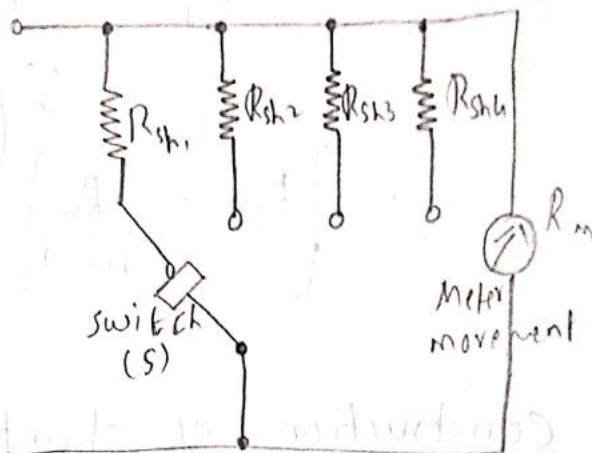
Let $m_1, m_2, m_3, \text{ \& } m_4$ are the multiplying powers for currents $I_1, I_2, I_3 \text{ \& } I_4$.

From the formula,

$$R_{sh} = \frac{R_m}{m-1}$$

$$R_{sh1} = \frac{R_m}{m_1-1}, \quad R_{sh2} = \frac{R_m}{m_2-1}$$

$$R_{sh3} = \frac{R_m}{m_3-1}, \quad R_{sh4} = \frac{R_m}{m_4-1} \text{ --- etc.,}$$

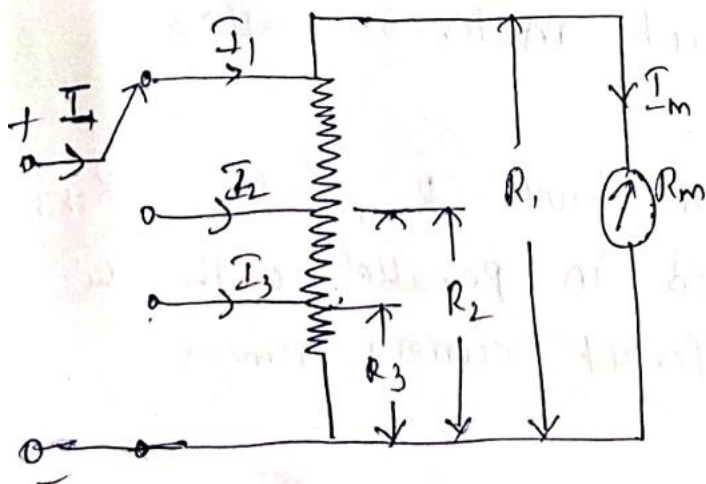


A "make-before-break" switch is provided so that the meter movement can't be damaged when changing the current range from one to another.

When an ordinary switch is provided, the meter may get damaged ^{when changing} from one to another. (Current range)

Wattmeter multipliers :- Universal shunt; for Aryton shunt:-

The advantage of universal shunt is it eliminates the possibility of ~~not~~ having a meter in the circuit without shunt.



For switch at position 1,

$$(I_1 - I_m) R_1 = I_m R_m$$

$$R_1 = \frac{I_m R_m}{I_1 - I_m} = \frac{R_m}{\frac{I_1}{I_m} - 1}$$

$$R_1 = \frac{R_m}{m_1 - 1} \quad \text{where } m_1 = \frac{I_1}{I_m}$$

when switch at position 2,

$$(\bar{I}_2 - \bar{I}_m) R_2 = \bar{I}_m (R_1 - R_2 + R_m)$$

$$R_2 = \frac{\bar{I}_m (R_1 - R_2 + R_m)}{\bar{I}_2 - \bar{I}_m} = \frac{R_1 - R_2 + R_m}{\frac{\bar{I}_2}{\bar{I}_m} - 1}$$

$$\bar{I}_2 R_2 - \bar{I}_m R_2 = \bar{I}_m (R_1 - R_2 + R_m)$$

$$R_2 = \frac{R_1 - R_2 + R_m}{m_2 + 1} \quad \bar{I}_2 R_2 = \bar{I}_m (R_1 + R_m)$$

$$R_2 = \frac{(R_1 + R_m)}{(\bar{I}_2 / \bar{I}_m)}$$

on simplification

$$R_2 = \frac{R_m + R_1}{m_2}$$

when switch is at position 3,

$$(\bar{I}_3 - \bar{I}_m) R_3 = \bar{I}_m (R_1 - R_3 + R_m)$$

$$\bar{I}_3 R_3 - \bar{I}_m R_3 = \bar{I}_m (R_1 - R_3 + R_m)$$

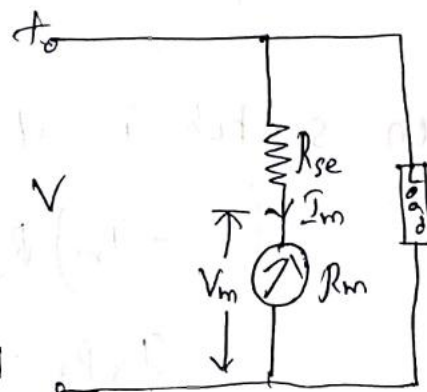
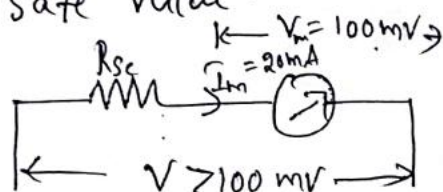
$$\bar{I}_3 R_3 = \bar{I}_m (R_1 + R_m)$$

$$R_3 = \frac{R_1 + R_m}{(\bar{I}_3 / \bar{I}_m)}$$

$$R_3 = \frac{R_1 + R_m}{m_3}$$

Voltmeter multipliers! - (upto 500 V): - The amount of current that can be passed to moving coil is limited (20 mA) due to weight constraint & current carrying capacity of spiral springs. [multiplier limits the current through the meter so that it doesn't exceed the value for full scale deflection]

→ so it can measure only upto 100 mV. To extend the range of voltmeter a series resistance of magnanin or constantan is used to limit the current to a safe value.



voltmeter multiplier is a high non-inductive resistance connected in series with the voltmeter coil

↳ is used for increasing the range of voltmeter.

Let R_{se} = Resistance of multiplier (Ω)

R_m = resistance of meter (Ω)

$I_m = I_{fs}$ = full scale deflection of current (A)

V = voltage across the load (V)

V_m = voltage drop across the meter (V) = $I_m R_m$

By voltage division rule, $V_m = V \frac{R_m}{R_{se} + R_m}$ $R_m + R_{se} = \frac{V}{I_m}$

$V = V_m + V_{se}$
 $= I_m (R_m + R_{se})$

$\Rightarrow \frac{R_{se} + R_m}{R_m} = \frac{V}{V_m} = m_v$

where m_v = multiplication factor of voltmeter.

$\Rightarrow R_{se} + R_m = \frac{V}{V_m} R_m \Rightarrow R_{se} = (m_v - 1) R_m$

Properties of multiplier resistance (R_{se}): -

- (1) Resistance should not change with time.
- (2) The temp. coefficient of resistance must be very low.
- (3) Should provide cooling to dissipate the heat produced.

made up of ~~Manganin~~ Manganin \rightarrow dc meter.

" " constantan \rightarrow ac meter.

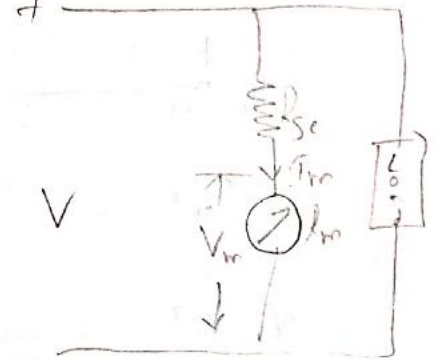
Voltmeter sensitivity:-

$$S_v = \frac{1}{I_{fs}} = \frac{1}{(V_m/R_m)} = \frac{R_m}{V_m} = \Omega/V \text{ (or) } A^{-1}$$

$$V = I_m R_{se} + I_m R_m$$

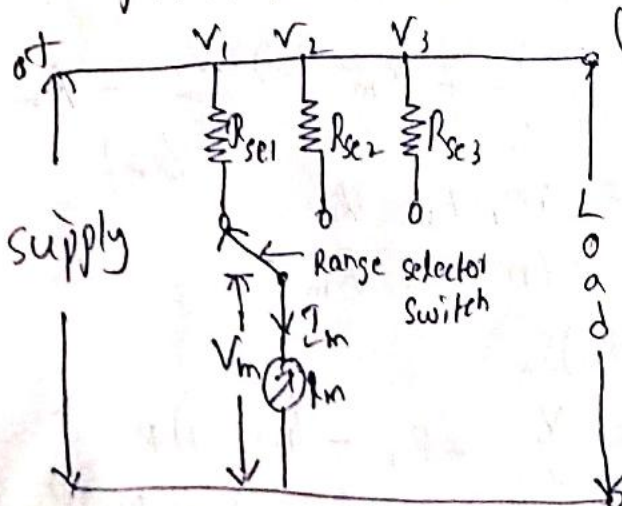
$$R_{se} = \frac{V}{I_m} - R_m \quad \therefore I_m = I_{fs}$$

$$R_{se} = V S_v - R_m$$



$$R_{se} = \frac{V}{I_m} - R_m$$

Multirange voltmeter:- In a multirange voltmeters different ~~full~~ voltage ranges may be obtained by the use of individual multiplier resistors or by a potential divider arrangement.



(i) Individual Multipliers:-

we can obtain different voltage ranges by connecting different values of series resistors in series with the meter.

The number of these resistors is equal to the number of ranges required.

$$R_{se1} = R_m(m_1 - 1) \quad \text{where } m_1 = \frac{V_1}{V_m}$$

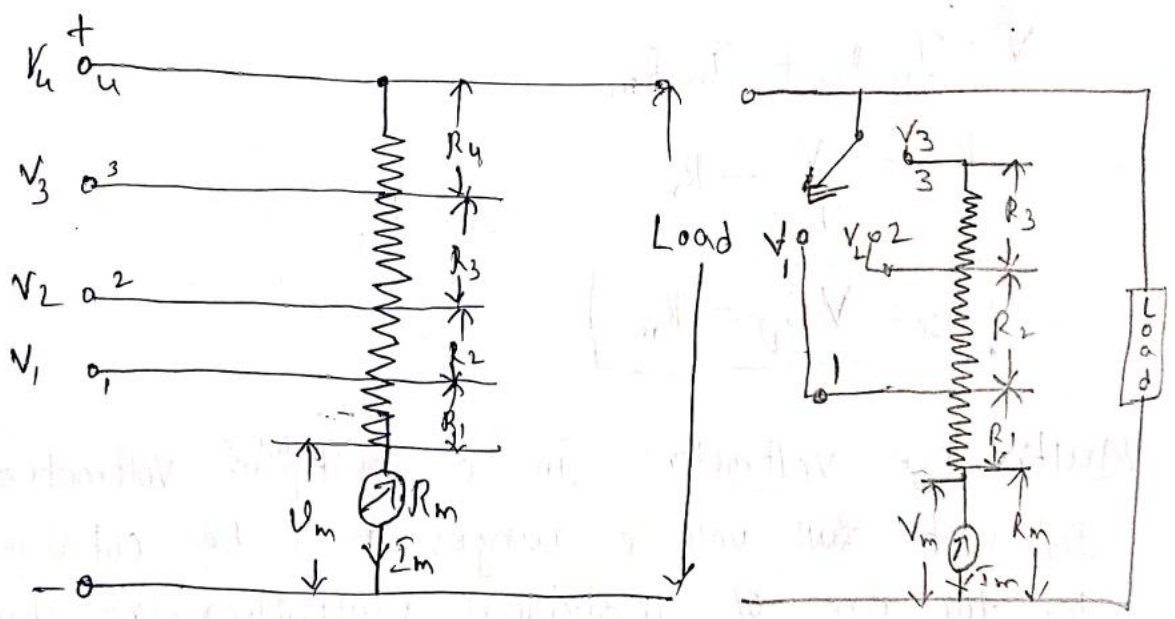
$$R_{se2} = R_m(m_2 - 1)$$

$$R_{se3} = R_m(m_3 - 1)$$

$$m_2 = \frac{V_2}{V_m} \quad \text{and} \quad m_3 = \frac{V_3}{V_m}$$

where m_1, m_2, m_3 are the multiplying factors for respective resistors R_1, R_2 & R_3

(ii) Potential divider arrangement: - In this arrangement the connections are made at the junctions of resistances R_1, R_2, R_3, R_4 in series to obtain the V_1, V_2, V_3, V_4 voltage ranges.



At position 1, $V_1 = (R_1 + R_m) I_m$

$$R_1 = \frac{V_1}{I_m} - R_m \Rightarrow \frac{V_1}{(V_m/R_m)} - R_m$$

$$R_1 = m_1 R_m - R_m$$

$$R_1 = (m_1 - 1) R_m$$

at position 2, $V_2 = (R_1 + R_2 + R_m) I_m$

$$R_2 = \frac{V_2}{I_m} - R_1 - R_m \Rightarrow \frac{V_2}{(V_m/R_m)} - R_m - (m_1 - 1) R_m$$

$$R_2 = m_2 R_m - R_m - (m_1 - 1) R_m \Rightarrow R_2 = (m_2 - m_1) R_m$$

at position 3, $V_3 = I_m (R_m + R_1 + R_2 + R_3)$

$$R_3 = \frac{V_3}{I_m} - R_m - R_1 - R_2$$

$$= \frac{V_3}{\left(\frac{V_m}{R_m}\right)} - R_m - (m_1 - 1)R_m - (m_2 - m_1)R_m$$

$$R_3 = (m_3 - 1 - m_1 + 1 - m_2 + m_1) R_m$$

$$\boxed{R_3 = (m_3 - m_2) R_m}$$

Problems

→ A PMMC instrument has a coil of dimensions 10mm x 8mm. The flux density in the airgap is 0.15 wb/m². If the coil is wound for 100 turns carrying a current of 5mA then calculate the deflecting torque. Calculate the deflection if the spring constant is 0.2 x 10⁻⁶ Nm/degree.

Sol: - Given data Area = l x d = 10 mm x 8 mm = 80 mm²
 $B = 0.15 \text{ wb/m}^2$ $= 80 \times 10^{-6} \text{ m}^2$

$$I = 5 \text{ mA}, N = 100$$

spring constant, $K = 0.2 \times 10^{-6} \text{ Nm/degree}$

$$T_d = NBA I = 100 \times 0.15 \times 80 \times 10^{-6} \times 5 \times 10^{-3} \text{ Nm}$$

$$T_d = 6 \times 10^{-6} \text{ Nm}$$

Now $T_d = T_c \Rightarrow 6 \times 10^{-6} \neq K \theta$

$$\theta = \frac{6 \times 10^{-6}}{0.2 \times 10^{-6}} = 30 \text{ degree}$$

→ A moving coil instrument gives a full scale deflection for a current of 20 mA with a p.d of 200 mV across it. Calculate

(i) shunt required to use it as an ammeter to get a range of 0-200 A

(ii) multiplier required to use it as a voltmeter of range (0-500) V

Ans:-

Given $I_m = 20 \text{ mA}$

$V_m = 200 \text{ mV}$

$\therefore R_m = \frac{V_m}{I_m} = \frac{200}{20} = 10 \Omega$

For using it as an Ammeter

(i) $R_{sh} = ?$ $R_{sh} = \frac{R_m}{m}$ where $m = \frac{I}{I_m} = \frac{200}{20} = 10$

$I = 200 \text{ A}$

$R_{sh} = \frac{10}{20 \times 10^3 - 1} = 0.0005 \Omega$

(ii) For using it as a voltmeter: $V = 500 \text{ V}$

$R_{se} = R_m(m-1)$

$= 10(2.5 \times 10^3 - 1)$

$R_{se} = 24.99 \text{ k}\Omega$

$m = \frac{V}{V_m} = \frac{500}{200 \times 10^{-3}} = 2.5 \times 10^3$

→ A moving coil instrument has a resistance of 5 Ω b/w terminals & full scale deflection is obtained with a current of 0.02 A. This instrument is to be used with manganin shunt to measure 100 A. Calculate the error caused by a 15°C rise in temperature.

- (i) when internal resistance of 5Ω due to copper only is used.
 (ii) when a 4Ω manganin swamping resistor in series with 1Ω copper resistance is used in the coil.

Take $\alpha_c = 0.4\% \text{ per } ^\circ\text{C}$ & $\alpha_m = 0.015\% \text{ per } ^\circ\text{C}$
 $0.004 / ^\circ\text{C}$ $0.00015 / ^\circ\text{C}$

Ans Given data $R_m = 5\Omega$, $I_m = 0.02\text{A}$, $I = 100\text{A}$

$$m = \frac{I}{I_m} = \frac{100}{0.02} = 5000$$

$$R_{sh} = R_m / (m-1) = \frac{5}{5000-1} = \frac{5}{4999} = 0.001\Omega$$

Shunt resistance after a rise of 15°C is

$$R_{sh} = R_1 (1 + \alpha t) = 0.001 \left(1 + 15 \times \frac{0.015}{100} \right) = 0.00100225$$

(i) Instrument with only copper.

$$R_m = 5 \left(1 + 15 \times \frac{0.4}{100} \right) = 5.3\Omega$$

$$\therefore \text{current through instrument} = \frac{I}{m} \times \frac{0.00100225}{5.3 + 0.00100225} = 0.018907\text{A}$$

$$\therefore \text{error in reading} = \frac{0.018907 - 0.02}{0.02} \times 100 = -5.46\%$$

$$\therefore \text{meter reads} = 100 - \frac{5.46}{100} \times 100 = 94.534\text{A}$$

(ii) Instrument swamping resistor

$$= 1 \left(1 + 15 \times 0.004 \right) + 4 \left(1 + 15 \times 0.00015 \right) = 1.06 + 4.009 = 5.069\Omega$$

$$I_m = 100 \times \frac{0.00100225}{5.069 + 0.00100225} = 0.019768 \text{ A}$$

$$\text{Error in reading} = \frac{0.019768 - 0.02}{0.02} \times 100$$

$$= -1.1588 \%$$

$$\text{Meter reading} = 100 - \frac{1.1588}{100} \times 100$$

$$= \underline{\underline{98.84 \text{ A}}}$$

→ A basic d'Arsonval meter movement with an internal resistance $R_m = 100 \Omega$ & full scale current of $I_m = 1 \text{ mA}$ is to be converted into a multirange dc voltmeter with ranges $(0-1) \text{ V}$, $0-10 \text{ V}$ & $0-50 \text{ V}$. Find the values of various resistances in potential divider arrangement.

Sol:

$$\text{Given } R_m = 100 \Omega, I_m = 1 \text{ mA}, V_m = I_m R_m = 100 \times 1 = 100 \text{ mV}$$

$$m_1 = \frac{V_1}{V_m} = \frac{1}{100 \text{ m}} = 10$$

$$m_2 = \frac{V_2}{V_m} = \frac{10}{100 \text{ m}} = 100$$

$$m_3 = \frac{50}{100 \text{ m}} = 500$$

$$R_1 = R_m (m_1 - 1) = 100 (10 - 1) = 900 \Omega$$

$$R_2 = R_m (m_2 - m_1) = (100 - 10) 100 =$$

$$R_3 = R_m (m_3 - m_2) = (500 - 100) 100 =$$

Prob:- A moving coil instrument whose resistance is 25Ω gives a full-scale deflection with a current of 1 mA . This instrument is to be used with a manganin shunt to extend its range to 100 mA . Calculate the error caused by 10°C rise in temp. when

- (i) copper moving coil is connected directly across the manganin shunt.
 (ii) A 75Ω manganin resistance is used in series with the instrument moving coil.

Sol:- $m = \frac{I}{I_m} = 100$ Given $I_m = 1\text{ mA}$
 $I = 100\text{ mA}$
 $R_m = 25\Omega$
 $R_{sh} = R_m / (m-1) = \frac{25}{100-1} = 0.2525\Omega$

Instrument resistance for 10°C rise in temp

$$R_{mt} = 25(1 + 10 \times 0.004) = 26\Omega$$

shunt resistant for 10°C rise in temp.

$$R_{sh_t} = 0.2525(1 + 10 \times 0.0015) = 0.2529\Omega$$

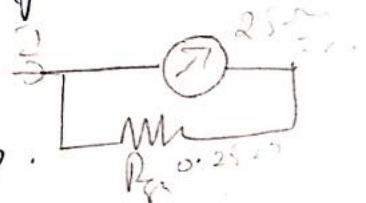
current I_{mt} through the meter for 100 mA in the main circuit for 10°C rise in temperature.

$$I_{mt} = 100 \frac{0.2529}{26 + 0.2529} = 0.963\text{ mA}$$

Normal meter current 1 mA

\therefore Error due to rise in temperature.

$$= (0.963 - 1) \times 100 = 3.7\%$$



Total resistance in the meter circuit.

(b) $\Rightarrow R_m + R_{sw} = 100\Omega$

shunt resistance $R_{sh} = \frac{R_m}{m-1} = \frac{100}{100-1} = 1.01\Omega$.

Resistance of the instrument ckt for 10°C rise in temp.
 $R_{mt} = 25(1+10 \times 10^{-5}) + 75(1+10 \times 10^{-5}) = 101.11\Omega$

shunt resistance for 10°C rise in temp.

$R_{sh_t} = 1.01(1+10 \times 10^{-5}) = 1.0115\Omega$.

Instrument current for 100 mA in the main circuit for

$I_{mt} = 100 \times \frac{1.0115}{101.11 + 1.0115} = 0.9905\text{mA}$ 10°C rise in temp.

$\therefore \text{Error} = (0.9905 - 1) \times 100 = -0.95\%$

$$V = V_{se} + V_m$$

$$V_{se} = V - V_m$$

$$I_{se} = \frac{V_{se}}{R_{se}} = \frac{V - V_m}{R_{se}} = R_{se}$$

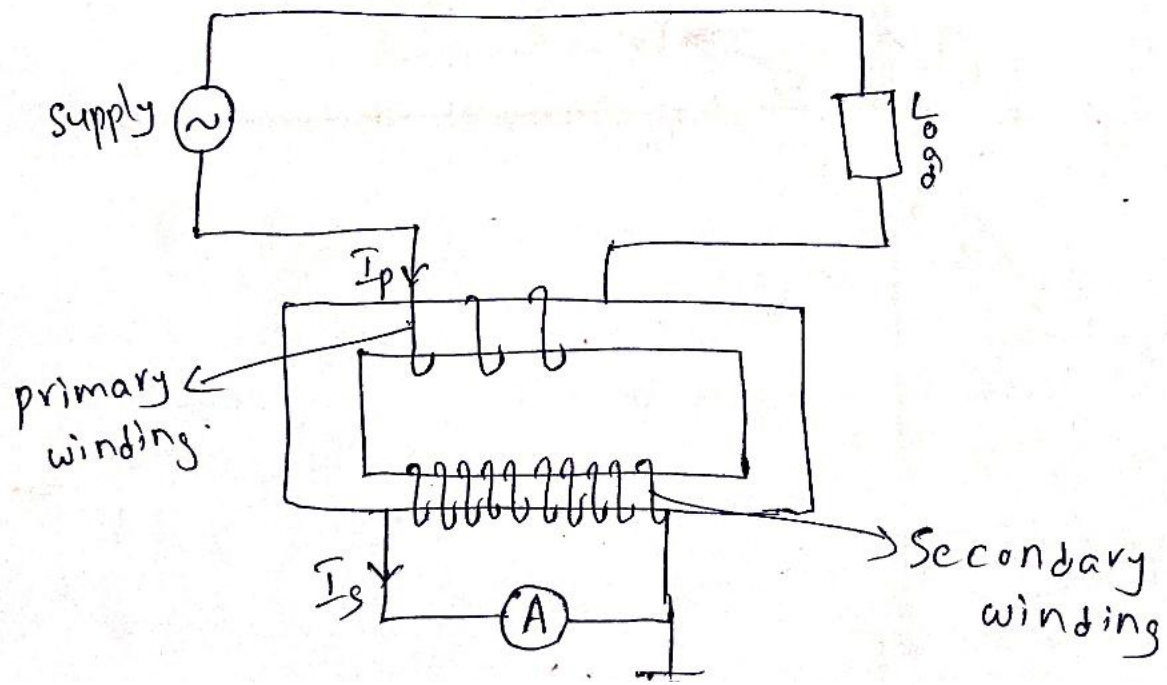
$$\Rightarrow \frac{V - V_m}{R_{se}} = R_{se} \Rightarrow \frac{V - V_m}{(m-1)R_m} = R_{se}$$

UNIT-VI

Instrument Transformers & Potentiometers

Current Transformers :- The current transformer is used with its primary winding connected in series with the line carrying the current to be measured & therefore the primary current is dependent on load connected to system but not depends upon the load (burden) connected on the secondary winding of the C.T.

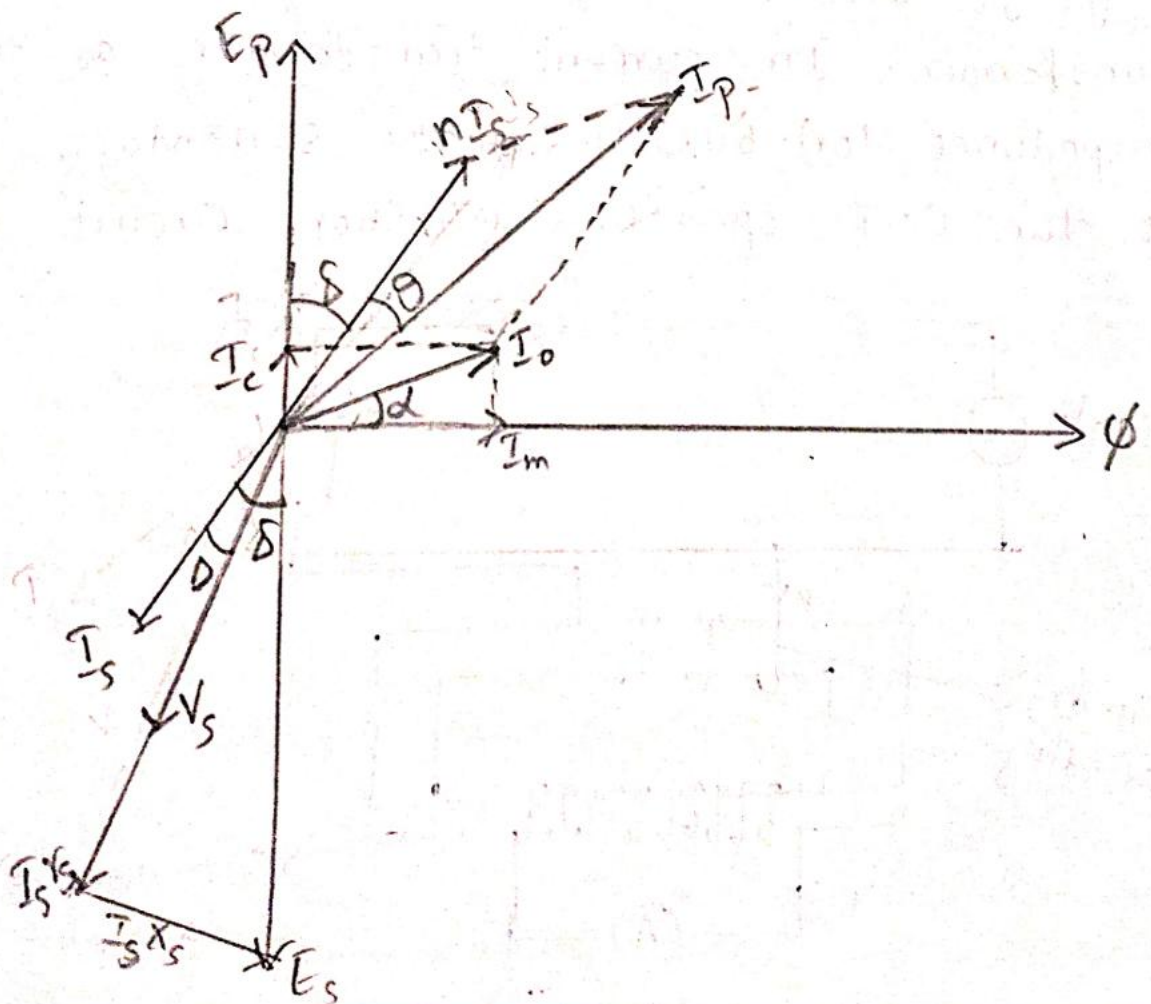
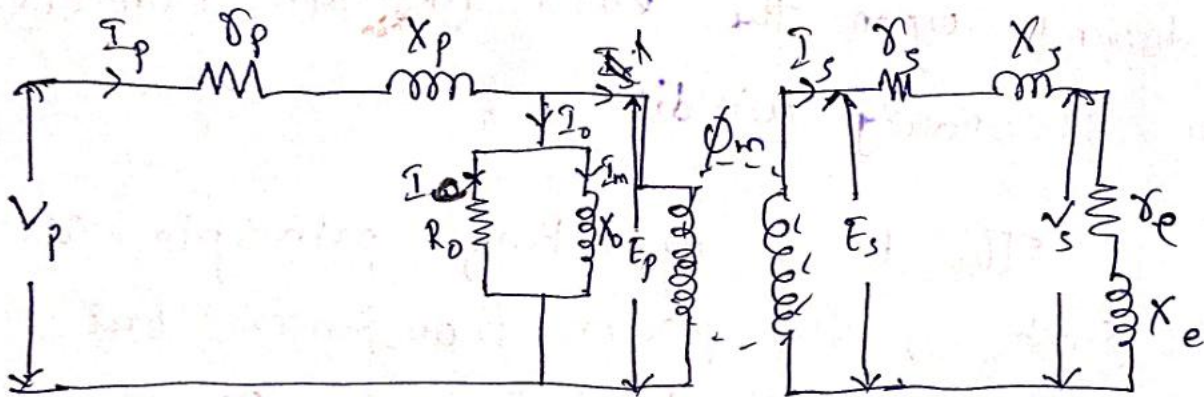
The basic operating principle of C.T is same as the power transformer but slightly differs in operation from the power transformer. In current transformer as the load impedance (or) burden on the secondary is very small, so the C.T operates on short circuit conditions.



From the above diagram, one of the terminals of secondary winding is earthed so as to protect equipment in the event of an insulation breakdown in the C.T.

Expression for Actual Ratio & Phase angle in C.T:

The below figure represent the equivalent ckt & the phasor diagram of current transformer.



Let $V_p =$ primary supply voltage

(2)

$E_p =$ primary winding induced voltage

$I_p =$ primary current

$E_s =$ secondary induced voltage

$V_s =$ secondary terminal "

$I_s =$ secondary current

$I_0 =$ NO-load current

$I_c =$ core loss component

$I_m =$ magnetising

$n =$ Turns ratio $= \frac{\text{No. of secondary w/g turns}}{\text{No. of primary " "}}$

$\phi =$ flux in core. $n = N_s/N_p$

$$\delta = \angle E_s, I_s = \tan^{-1} \left[\frac{X_s + X_e}{r_s + r_e} \right]$$

$$\Delta \phi = \angle V_s, I_s = \tan^{-1} \left[\frac{X_e}{r_e} \right]$$

$$\alpha = \angle I_0, \phi$$

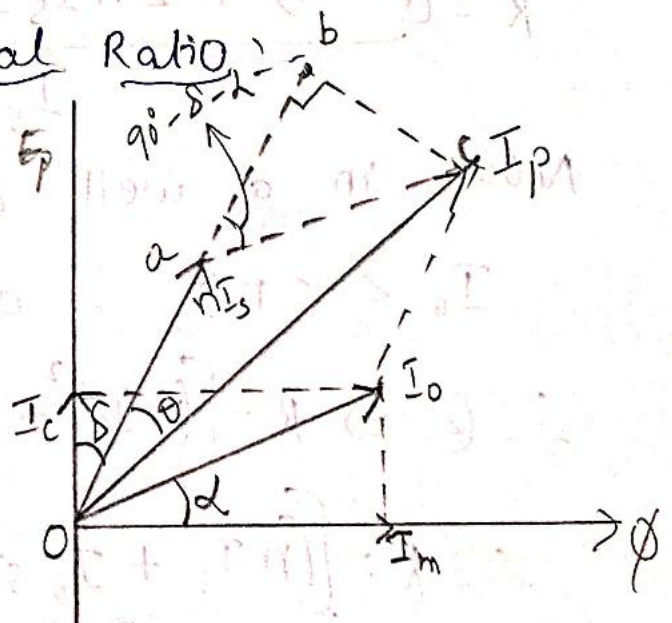
Transformation Ratio (or) Actual Ratio

$$R = \frac{I_p}{I_s}$$

Considering a section of phasor diagram as shown in fig.

$$\angle bac = 90^\circ - \alpha - \delta$$

$$c = I_p, ac = I_0, Oa = nI_s$$



$$\begin{aligned} bc &= ac \sin(90^\circ - (\delta + \alpha)) = I_0 \times \cos(\delta + \alpha) \\ ab &= ac \cos[90^\circ - (\delta + \alpha)] = I_0 \sin(\delta + \alpha) \end{aligned} \quad \left. \begin{array}{l} \therefore \sin(90^\circ - \delta - \alpha) = \frac{bc}{ac} \\ \cos(90^\circ - \delta - \alpha) = \frac{ab}{ac} \end{array} \right\}$$

Now by considering Right angled triangle obc ,

we have $(oc)^2 = (ob)^2 + (bc)^2$

$$= (oa + ab)^2 + (bc)^2$$

$$(oc)^2 = [n I_s + I_0 \sin(\delta + \alpha)]^2 + [I_0 \cos(\delta + \alpha)]^2$$

$$= (n I_s)^2 + I_0^2 \sin^2(\delta + \alpha) + 2n I_s I_0 \sin(\delta + \alpha) + I_0^2 \cos^2(\delta + \alpha)$$

$$= (n I_s)^2 + 2n I_s I_0 \sin(\delta + \alpha) + I_0^2 [\sin^2(\delta + \alpha) + \cos^2(\delta + \alpha)]$$

$$(I_p)^2 = (n I_s)^2 + 2n I_s I_0 \sin(\delta + \alpha) + I_0^2$$

$$I_p = [(n I_s)^2 + 2n I_s I_0 \sin(\delta + \alpha) + I_0^2]^{1/2} \rightarrow (1)$$

\therefore Transformation Ratio $R = \frac{I_p}{I_s}$

$$R = \frac{[(n I_s)^2 + 2n I_s I_0 \sin(\delta + \alpha) + I_0^2]^{1/2}}{I_s} \rightarrow (2)$$

Now in a well designed current transformer

$I_0 \ll n I_s$, adjusting I_0^2 to $I_0^2 \sin^2(\delta + \alpha)$

$$\therefore (2) \Rightarrow R = \frac{[(n I_s)^2 + 2n I_s I_0 \sin(\delta + \alpha) + I_0^2 \sin^2(\delta + \alpha)]^{1/2}}{I_s}$$

$$\therefore R = \frac{[(n I_s + I_0 \sin(\delta + \alpha))]^2}{I_s} = \frac{n I_s + I_0 \sin(\delta + \alpha)}{I_s}$$

$$R = n + \frac{I_0}{I_s} \sin(\delta + \alpha) \rightarrow (3)$$

The above equation is sufficient accurate for practically all purposes.

The above equation is further expanded as:

$$R = n + \frac{I_0}{I_s} [\sin \delta \cos \alpha + \cos \delta \sin \alpha]$$

$$R = n + \frac{I_m \sin \delta + I_c \cos \delta}{I_s} \rightarrow (A)$$

where $I_m = I_0 \cos \alpha$ & $I_c = I_0 \sin \alpha$

Phase angle :- The angle b/n the counter balancing current ($n I_s$) & the primary current (I_p) is known as phase angle which is represented by θ .

From the above diagram, $\tan \theta = \frac{bc}{ob} = \frac{bc}{oatab}$

$$\tan \theta = \frac{I_0 \cos(\delta + \alpha)}{n I_s + I_0 \sin(\delta + \alpha)}$$

as θ is very small, $\tan \theta = \theta$

$$\Rightarrow \theta = \frac{I_0 \cos(\delta + \alpha)}{n I_s + I_0 \sin(\delta + \alpha)}$$

Now I_0 is very small when compared to $n I_s$, therefore the term $I_0 \sin(\delta + \alpha)$ will be neglected.

$$\theta = \frac{I_0 \cos(\delta + \alpha)}{n I_s} \rightarrow (4)$$

$$\theta = \frac{I_0 (\cos \delta \cos \alpha - \sin \delta \sin \alpha)}{n I_s}$$

$$\theta = \frac{I_m \cos \delta - I_c \sin \delta}{n I_s} \quad \left[\begin{array}{l} I_m = I_0 \cos \alpha \\ I_c = I_0 \sin \alpha \end{array} \right]$$

$$\Rightarrow \theta = - \frac{180}{\pi} \left[\frac{I_m \cos \delta - I_c \sin \delta}{n I_s} \right] \text{ degrees.} \rightarrow \textcircled{B}$$

Errors in current Transformers : In general,

The transformation Ratio or Actual ratio (R)

must be equal to the turns Ratio i.e. $\frac{I_2}{I_1} = \frac{N_1}{N_2}$

But actually the transformation Ratio is not equal to the turns ratio because from the

eqn (A) we can say that, the transformation Ratio is depends on magnetizing and core loss

Components of current & also the secondary winding current & the power factor. Due to

this fact, large error is introduced in the measurements. Such an error is known as

Ratio error.

(i) Ratio error : It is defined as

$$\% \text{ Ratio error} = \frac{\text{Nominal Ratio} - \text{Actual Ratio}}{\text{Actual Ratio}}$$

$$\therefore \% \text{ Ratio error} = \frac{Kn - R}{R} \times 100 \%$$

(ii) Phase angle error :- In power measurements, it is must & should that the secondary phasor (current or voltage) must be displaced by exactly 180° from that of the primary phasor (current or voltage). But actually it is not so. The error due to this reason is known as "phase angle error". & is represented by θ

Phase angle error

$$\theta = \frac{180}{\pi} \left[\frac{I_m \cos \delta - I_c \sin \delta}{n I_s} \right] \text{ deg}$$

Approximate Results :- In practice, the loads are largely resistive with some inductive in nature & so δ is true & very small.
 $\therefore \sin \delta = 0$ & $\cos \delta = 1$ [approximately $\delta = 0$]

Now the equations (A) & (B) will be

$$(A) \Rightarrow R = n + \frac{I_m \sin \delta + I_c \cos \delta}{I_s}$$

$$R = n + \frac{I_c}{I_s} \rightarrow (C)$$

$$\text{Imp (B)} \Rightarrow \theta = \frac{180}{\pi} \left[\frac{I_m \cos \delta - I_c \sin \delta}{n I_s} \right] \text{ degrees.}$$

$$\therefore \theta = \frac{180}{\pi} \left[\frac{I_m}{n I_s} \right] \text{ degrees.} \rightarrow (D)$$

In terms of $I_p \Rightarrow n = \frac{I_p}{I_s}$

$$I_s = \frac{I_p}{n}$$

$$(C) \Rightarrow R = n + \frac{I_c}{I_s} \Rightarrow n + \frac{I_c}{I_p/n}$$

$$R = n + \frac{n I_c}{I_p}$$

$$\text{Imp (D)} \Rightarrow \theta = \frac{180}{\pi} \frac{I_m}{n I_s} \Rightarrow \frac{180}{\pi} \frac{I_m}{I_p}$$

$$\theta = \frac{180}{\pi} \frac{I_m}{I_p} \text{ degrees.}$$

It is clear that from eqn (C) & (D) the ratio error is caused by core loss component of current & phase angle error caused by magnetising component of current.

Interface is a collection of method declaration and constructors that one or more

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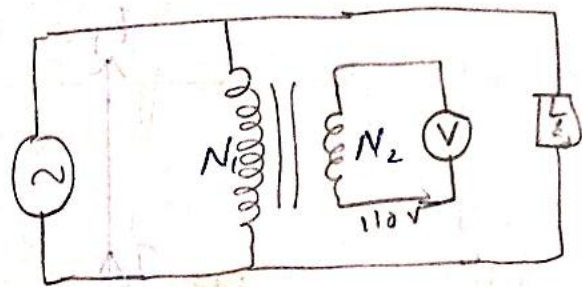
Potential Transformers! - Potential Transformers ⁽¹⁾ Need of Instrument Transformers

are used to operate the voltmeters, the potential coils of wattmeters, & relays from high voltage line.

The primary w/g of potential transformer is connected across the line carrying the voltage to be measured. And the voltage instrument is connected to the secondary.

The secondary winding is designed so that a voltage of 100 V to 120 V is delivered to the instrument load. The normal secondary voltage rating is 110 V.

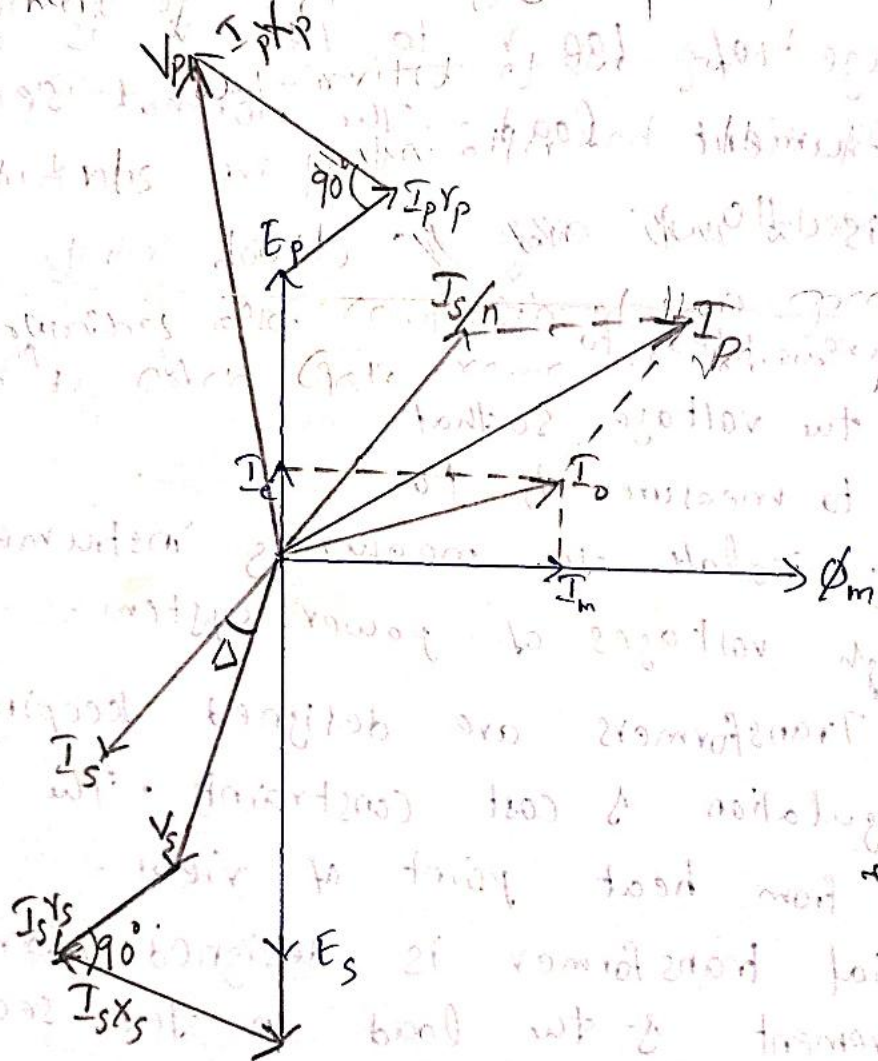
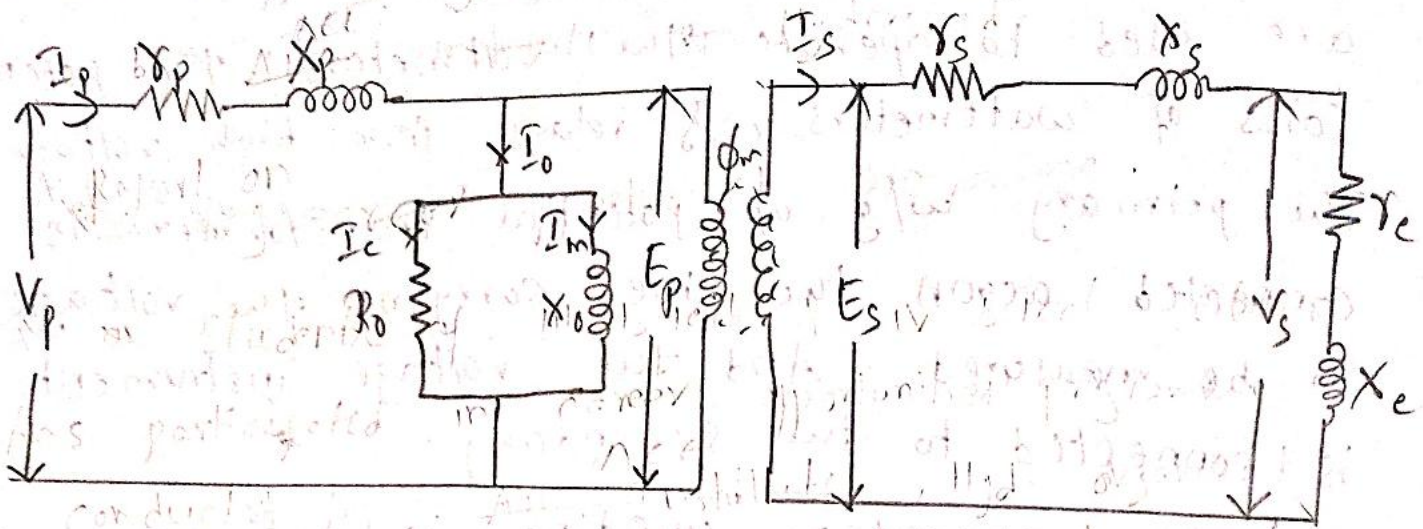
→ They are used to step down the voltage so that it is easy to measure & to physically isolate the measuring instruments/relays from high voltages of power system.



→ power Transformers are designed keeping in view of μ , regulation & cost constraints. The rating is decided from heat point of view.

→ potential transformer is designed for voltage measurement & the load on the secondary is very small as only voltage coil is connected to secondary. The rating is decided based on accuracy requirements (transformation ratio error & phase angle error).

Relationships of potential transformer



The above figures shows the equivalent circuit diagram & the phase diagram of potential T/F. The Theory of potential transformer is same as that of the power transformer.

The main difference is that power loading of Potential transformer is very small & consequently the exciting current is of same order as secondary winding load current.

Let ϕ = working flux.

I_m, I_c = ~~exc~~ magnetizing & iron loss components of current.

I_0 = NO load current.

E_s, E_p = induced emf's on secondary & primary

N_p, N_s = NO. of turns on primary & secondary.

r_s, x_s = secondary winding resistance & reactance.

r_e, x_e = resistance & reactance of load circuit.

Δ = phase angle of secondary load circuit.

Derivation of Ratio error:-

from the phase diagram,

$oa = I_0$, $oc = I_p$, $of = I_s/n$, $og = nV_s$, $ob = V_p$

θ = phase angle of transformer, $V_p = nV_s$

Δ = phase angle of secondary load = nV_s & I_s/n

β = phase angle b/n I_p & nV_s

oa is nV_s extended & ba is perpendicular drawn from b on to nV_s extended.

$$\therefore oa = ob \cos \theta$$

$$oa = V_p \cos \theta \rightarrow (1)$$

\therefore Consider $\triangle oab$

$$\cos \theta = oa/ob$$

$$oa = ob \cos \theta$$

Now consider a Δocd

$$\cos \beta = \frac{cd}{I_p}$$

$$cd = I_p \cos \beta$$

$$I_p \cos \beta = ch + hd$$

$$I_p \cos \beta = \frac{I_s}{n} \cos \Delta + I_c \rightarrow (4)$$

$$\text{||ry } \sin \beta = \frac{od}{I_p}$$

$$I_p \sin \beta = od$$

$$= I_m + eh$$

$$I_p \sin \beta = I_m + \frac{I_s}{n} \sin \Delta \rightarrow (5)$$

Now when θ is very small approximately $\theta = 0$.

$$\cos \theta = \cos 0 = 1$$

$$V_p \cos \theta = V_p \rightarrow (6)$$

Now substitute eqn (4), (5), (6) in eqn (3)

$$\text{we set } V_p = nV_s + n I_s r_s \cos \Delta + n I_s x_s \sin \Delta +$$

$$r_p \left[\frac{I_s}{n} \cos \Delta + I_c \right] + X_p \left[\frac{I_s}{n} \sin \Delta + I_m \right]$$

$$V_p = nV_s + n I_s (r_s \cos \Delta + X_s \sin \Delta) + r_p \left[\frac{I_s}{n} \cos \Delta + I_c \right]$$

$$+ X_p \left(I_m + \frac{I_s}{n} \sin \Delta \right)$$

$$V_p = nV_s + I_s \cos \Delta \left[nr_s + \frac{r_p}{n} \right] + I_s \sin \Delta \left[nX_s + \frac{X_p}{n} \right] + I_c r_p + I_m X_p$$

Consider a Δech (3)

$$\cos \Delta = \frac{ch}{\frac{I_s}{n}}$$

$$ch = \frac{I_s}{n} \cos \Delta$$

$$\text{||ry } \sin \Delta = \frac{eh}{\frac{I_s}{n}}$$

$$eh = \frac{I_s}{n} \sin \Delta$$

$$\therefore V_p = nV_s + \frac{I_s}{n} \cos \Delta (n^2 r_s + r_p) + \frac{I_s}{n} \sin \Delta (n^2 x_s + x_p) + I_c r_p + I_m x_p$$

Now $n^2 r_s + r_p = R_{1e} =$ equivalent resistance referred to primary.

$n^2 x_s + x_p = X_{1e} =$ equivalent reactance referred to primary.

$$\therefore V_p = nV_s + \frac{I_s}{n} [R_{1e} \cos \Delta + X_{1e} \sin \Delta] + I_c r_p + I_m x_p$$

$$I_c r_p + I_m x_p \rightarrow (7)$$

Now, the Transformation Ratio,

$$R = \frac{V_p}{V_s}$$

$$\therefore R = \frac{nV_s + \frac{I_s}{n} [R_{1e} \cos \Delta + X_{1e} \sin \Delta] + I_c r_p + I_m x_p}{V_s}$$

$$\therefore R = n + \frac{I_s/n (R_{1e} \cos \Delta + X_{1e} \sin \Delta) + I_c r_p + I_m x_p}{V_s}$$

The above equation gives the transformation ratio of a potential T/F when referred to primary. $\rightarrow (8)$

Try The Transformation Ratio of a potential T/F when referred to secondary is

$$R = n + \frac{n I_s (R_{2e} \cos \Delta + X_{2e} \sin \Delta) + I_c r_p + I_m x_p}{V_s}$$

$$\rightarrow (9)$$

Derivation of phase angle :- (4)

From the phasor diagram,

$$\tan \theta = \frac{ba}{oa}$$

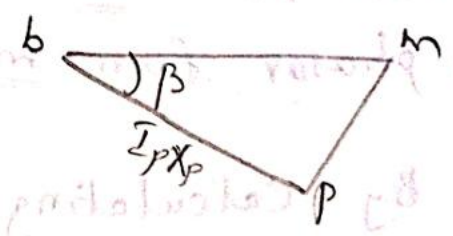
$$\tan \theta = \frac{ba}{nV_s + nI_s r_s \cos \Delta + nI_s X_s \sin \Delta + I_p r_p \cos \beta + I_p X_p \sin \beta}$$

Again $ba = bm + ma$

considering the $\Delta^{le} bmp$,

$$\cos \beta = \frac{bm}{I_p X_p}$$

$$bm = I_p X_p \cos \beta$$

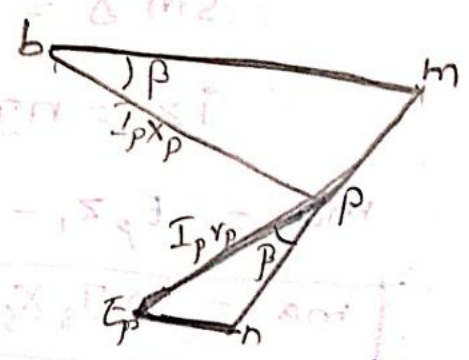


Now if we observe the phasor diagram the term $I_p X_p$ has been included in the region bm . So it is necessary to subtract the region from E_p to n to get exact "bm".

∴ consider $\Delta^{le} E_p n p$.

$$\sin \beta = \frac{E_p \cdot n}{I_p r_p}$$

$$E_p \cdot n = I_p r_p \sin \beta$$



$bm - E_p n \rightarrow$ to get exact bm .

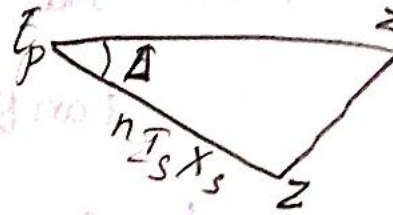
∴ Exact $bm = bm - E_p n$

$$bm = I_p X_p \cos \beta - I_p r_p \sin \beta$$

Now to calculate the ma consider a $\Delta^{le} E_p$

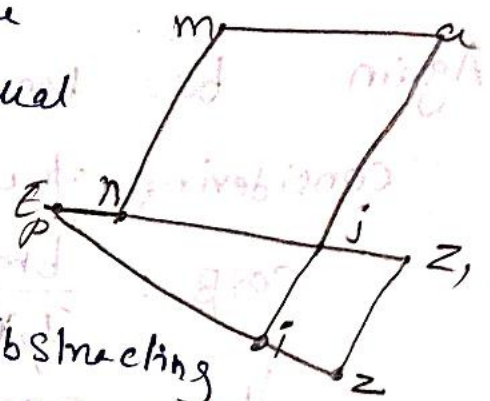
$$\cos \Delta = \frac{E_p Z_1}{n I_s X_s}$$

$$E_p Z_1 = n I_s X_s \cos \Delta$$



Now we have to calculate only ma but we got $E_p Z_1$.

Whatever may be the phasor from j to n will be equal to phasor from m to a.



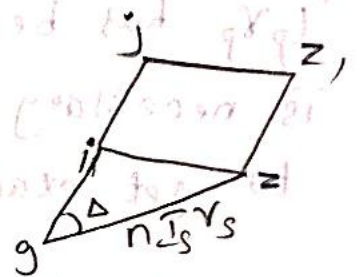
By calculating 'jz' & subtracting it from $E_p Z_1$, we get actual value of ma.

From the phasor diagram $jz = iz$

Considering $\Delta^{le} iz$

$$\sin \Delta = \frac{iz}{n I_s r_s}$$

$$iz = n I_s r_s \sin \Delta$$



$$ma = E_p Z_1 - iz$$

$$ma = n I_s X_s \cos \Delta - n I_s r_s \sin \Delta$$

$$ba = bm + ma$$

$$= (I_p X_p \cos \beta - I_p r_p \sin \beta) + (n I_s X_s \cos \Delta - n I_s r_s \sin \Delta)$$

$$\therefore \tan \theta = \frac{I_p X_p \cos \beta - I_p r_p \sin \beta + n I_s X_s \cos \Delta - n I_s r_s \sin \Delta}{n V_s + n I_s X_s \cos \Delta + n I_s X_s \sin \Delta + I_p r_p \cos \beta + I_p X_p \sin \beta}$$

In the denominator expression, the terms other than nV_s are very small & so neglected.

As θ is very small, $\tan \theta = \theta$

$$\therefore \theta = \frac{I_p X_p \cos \beta - I_p r_p \sin \beta + n I_s X_s \cos \Delta - n I_s r_s \sin \Delta}{n V_s} \rightarrow \text{A}$$

Similar to that of the procedure followed for the Derivation of Ratio error

Consider a $\Delta^{le} ocd$

$$\cos \beta = \frac{cd}{I_p}$$

$$I_p \cos \beta = ch + hd$$

$$I_p \cos \beta = \frac{I_s}{n} \cos \Delta + I_c \rightarrow \text{B}$$

Asy $\sin \beta = \frac{od}{I_p}$

$$I_p \sin \beta = od = I_m + eh$$

$$I_p \sin \beta = I_m + \frac{I_s}{n} \sin \Delta \rightarrow \text{C}$$

Consider a $\Delta^{le} ech$

$$\cos \Delta = \frac{ch}{I_s/n}$$

$$ch = \frac{I_s}{n} \cos \Delta$$

Consider a $\Delta^{le} ech$

$$\sin \Delta = \frac{eh}{I_s/n}$$

$$eh = \frac{I_s}{n} \sin \Delta$$

Substitute (B) & (C) in (A).

$$\theta = \frac{X_p \left(\frac{I_s}{n} \cos \Delta + I_c \right) - r_p \left(I_m + \frac{I_s}{n} \sin \Delta \right) + n I_s X_s \cos \Delta - n I_s r_s \sin \Delta}{n V_s}$$

$$= \frac{I_s \cos \Delta \left[\frac{X_p}{n} + n X_s \right] - I_s \sin \Delta \left(\frac{r_p}{n} + n r_s \right) + I_c X_p - I_m r_p}{n V_s}$$

$$\theta = \frac{\frac{I_s}{n} \cos \Delta (X_p + n^2 X_s) - \frac{I_s}{n} \sin \Delta (r_p + n^2 r_s) + I_c X_p - I_m r_p}{n V_s}$$

$$= \frac{\frac{I_s}{n} \left[\cos \Delta (X_p + n^2 X_s) - \sin \Delta (r_p + n^2 r_s) \right] + I_c X_p - I_m r_p}{n V_s}$$

Now $X_p + n^2 X_s = X_{le}$ & $r_p + n^2 r_s = R_{le}$

$$\theta = \frac{\frac{I_s}{n} (X_{le} \cos \Delta - R_{le} \sin \Delta) + I_c X_p - I_m r_p}{n V_s} \text{ radians} \rightarrow \textcircled{D}$$

Try the above expression can also derived in terms of referred to secondary shown below:

$$\theta = \frac{n I_s \left[X_{2e} \cos \Delta - R_{2e} \sin \Delta \right] + I_c X_p - I_m r_p}{n V_s} \text{ radians} \rightarrow \textcircled{E}$$

Errors in potential Transformers

(6)

(i) Ratio Error :- The actual Ratio of Transformation varies with operating conditions & the error in secondary voltage may be defined as

$$\text{Ratio Error} = \frac{K_n - R}{R} \times 100\%$$

(ii) Phase angle Error :- In an ideal potential transformer there should not be any phase difference b/n primary voltage & secondary voltage. But actually there exists a phase difference b/n V_p & V_s .

Phase angle error,
$$\theta = \frac{I_s}{n} \frac{[X_{le} \cos \Delta - R_{le} \sin \Delta] + I_e X_p - I_m Y_p}{n V_s}$$

Reduction of Errors in potential transformers

(i) Reduction of magnetizing & core loss components.

$$R = n + \frac{I_s}{n} (R_p \cos \Delta + X_p \sin \Delta) + I_e Y_p + I_m X_p$$

$$R - n = \frac{I_s}{n} \frac{V_s}{V_s} (R_p \cos \Delta + X_p \sin \Delta) + I_e Y_p + I_m X_p$$

From the above equation, the difference of Actual Ratio (R) & the turns ratio (n) is made up of two parts.

- (i) one is dependent of I_s
- (ii) other is " on two components of I_0 .

Hence a considerable ~~amprovement~~ improvement in performance can be made by ~~margin~~ reducing I_m & such a reduction requires short magnetic paths, good quality of core material & low flux density in core.

(ii) Reduction of Resistance & Leakage Reactance

Leakage reactance tends to increase in Ratio Error. To minimize the leakage reactance the two w/g's primary & secondary should keep as close as possible.

Winding resistance can be minimized by using thick conductors & by adopting smallest length of mean turn.

(iii) Turns compensation :- At no load condition ($I_s = 0$) the actual ratio exceeds the turns ratio by an amount of $\frac{I_e X_p + I_m X_p}{V_s}$ with an inductive (or) capacitive load there is further increase of ratio error because of voltage drops in resistance & reactances of windings.

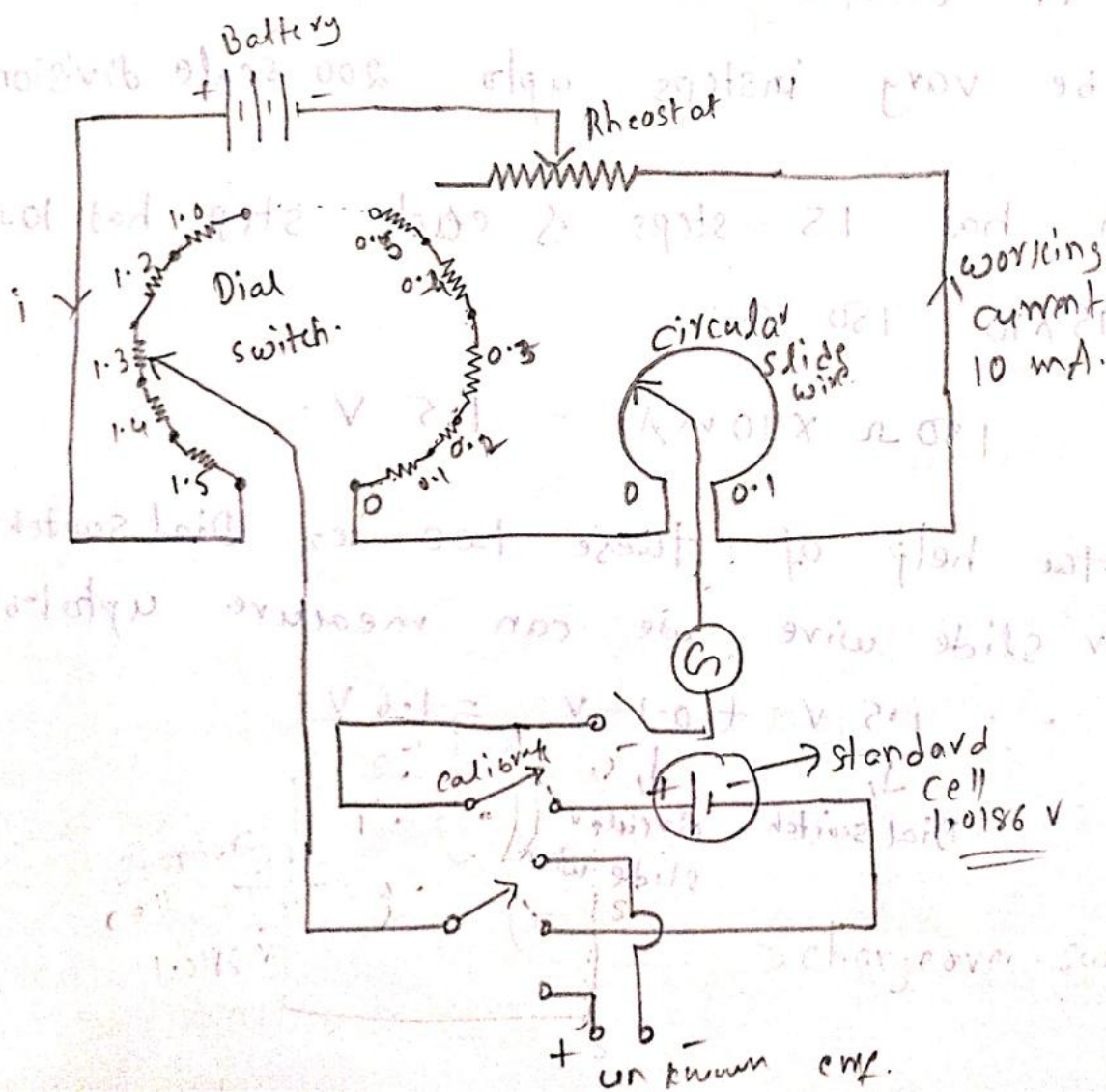
The solution is making the turns ratio less than the nominal ratio. This can be done by either reducing the number of primary w/g turns or increasing the number of secondary w/g turns.

DC Crompton's Potentiometer: (Laboratory type)

Basic slide wire dc potentiometer discussed in the last class has the following disadvantages.

- (i) The slide wire is linear & hence occupies more space which is not possible practically.
- (ii) If the length is decreased then ^{precision} resolution of potentiometer decreases as voltage per unit length increases.

So we use Crompton's potentiometer in practice to decrease size without compromising the resolution. (Smallest measurable voltage).



The battery & Rheostat structure are same as of before.
but slide wire has divided into two sections

→ Dial switch & circular slide wire | generally the working current is 10 mA.

This circular slide wire will have 10Ω resistance & it will be circular & smoothly we can vary the range from 0 to 10Ω .

To total resistance is $10 \Omega \times 10 \text{ mA} = 0.1 \text{ V}$.

we can measure from 0 to 0.1 V in circular slide wire

This can be vary in steps upto 200 scale divisions.

Dial switch has 15 steps & each step has 10Ω

∴ Total resistance is $15 \times 10 = 150 \Omega$

∴ $150 \Omega \times 10 \text{ mA} = 1.5 \text{ V}$.

∴ With the help of these two i.e. Dial switch & circular slide wire we can measure upto 1.6 V

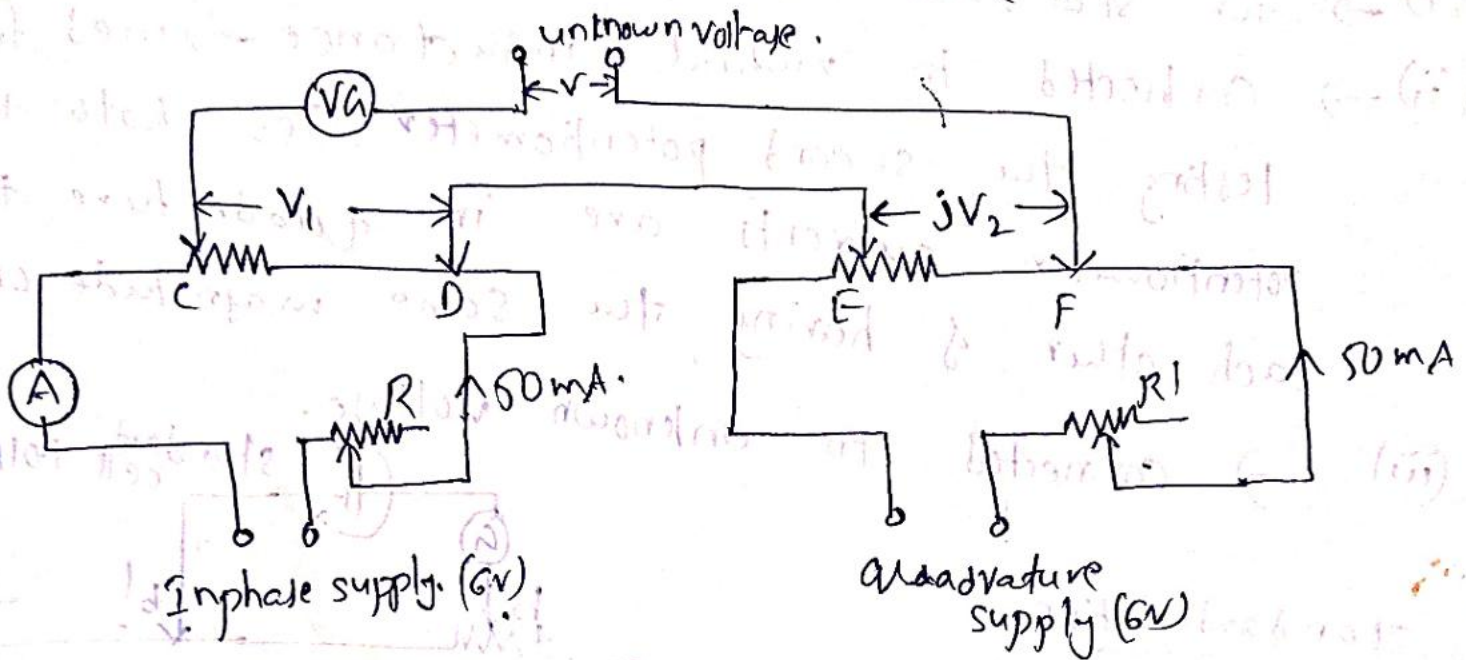
∴ $1.5 \text{ V} + 0.1 \text{ V} = 1.6 \text{ V}$.

↓ Dial switch ↓ circular slide wire

Gall Tinsley (co-ordinate type) AC potentiometer

Unknown voltage is measured in complex form is $\bar{V} = V_1 + jV_2$ can be +ve or -ve,

$$|V| = \sqrt{V_1^2 + V_2^2} \quad \& \quad \theta = \tan^{-1}\left(\frac{V_2}{V_1}\right)$$



- It consists of 2 separate potentiometer ckt enclosed in a common case.
- CD & EF are sliding contacts of inphase & quadrature potentiometers.
- R, R¹ are slide wire current controlling rheostats.
- slide wires are either supplied from 2- ϕ alternator or phase splitting device to convert 1- ϕ to 2- ϕ .
- T₁, T₂ are step down T/F's to step down the supply voltage to 6V & also to isolate the potentiometer circuit from the ac source.

→ S_1, S_2 are signchanging switches

→ S_3 is selector switch.

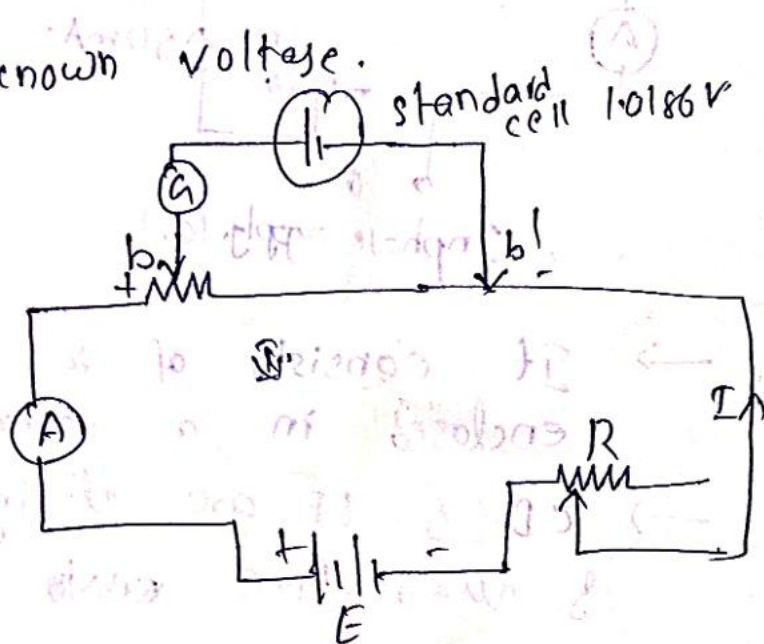
This can be kept in ~~four~~ ^{three} different positions.

(i) → for standardization connected to DC supply

(ii) → connected to mutual inductance → used for testing the second potentiometer i.e., both the potentiometer currents are in quadrature to each other & having the same magnitude or not.

(iii) → connected to unknown voltage.

standardization



$$e = 2\pi f M i \quad i \text{ lags } i' \text{ by } 90^\circ$$

UNIT-II

①

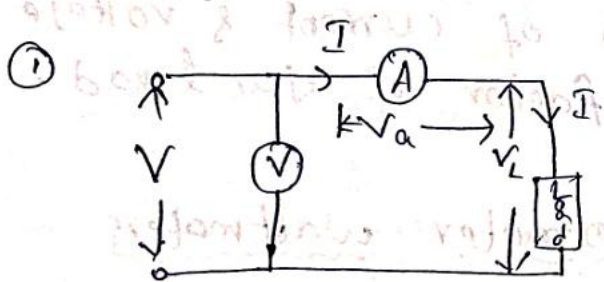
Single phase Dynamometer wattmeter:-

Power in DC circuit

The power taken by a load from a dc supply is given by $P = VI$ watts.

ie, two meters (voltage & Ammeter) are required to measure power.

There are two possible connections of Ammeter & voltmeters.

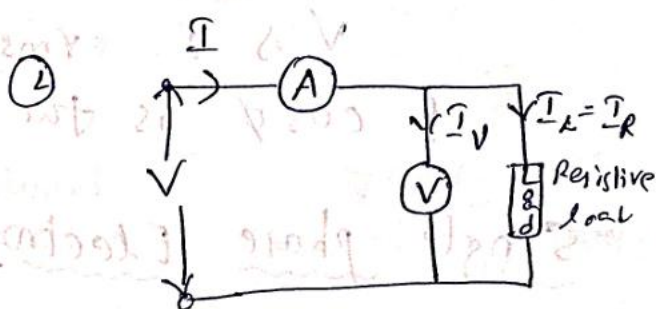


$$V = V_a + V_L \quad \text{and} \quad V_a = IR_a$$

Power consumed by load, $P_L = V_L I_L = V_L I$

$$\begin{aligned} P_L &= (V - V_a) I = VI - V_a I \\ &= VI - (IR_a) I \\ &= VI - I^2 R_a \end{aligned}$$

Power consumed by Load = Power indicated by instruments - Power loss in Ammeter.



$$I = I_L + I_V, \quad I_V = \frac{V}{R_v}$$

Power consumed by load, $P_L = V_L I_L = V(I - I_V)$

$$\begin{aligned} P_L &= VI - V I_V \\ &= VI - \frac{V^2}{R_v} \end{aligned}$$

Power consumed by load = Power indicated by instruments - Power loss in voltmeter.

Power indicated by the instruments is equal to the power consumed by load & power loss by the instrument nearer to the load terminals.

So when power measurements are required, it is an advantage to install a wattmeter in place of voltmeter & Ammeter.

Wattmeter gives direct indication of power & there is no need of multiplying two readings.

The labour involved is reduced & the accuracy is also increased.

Power in AC circuit:-

$$P = V I \cos \phi$$

V & I rms values of current & voltage
& $\cos \phi$ is the power factor of the load.

Single phase Electrodynamometer wattmeters:-

These instruments are similar in design & construction to electrodynamometer type Ammeters & voltmeters.

For measurement of power, the two coils are connected in different circuits.

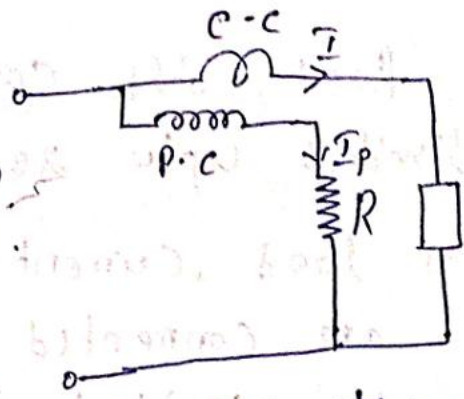
→ The fixed coils (or field coils) are connected in series with the load & so carry the current in the circuit. Simply the fixed coils are current coils of the wattmeter.

→ The moving coil is connected across the voltage & therefore it carries a current proportional to the voltage. So moving coil is the pressure coil or voltage coil of the wattmeter.

A high non-inductive resistance is connected in series with the moving coil.

This serves two purposes

- (i) to limit the current passing through the M.C (I_p)
- (ii) Because of this high resistance, it is dominating the reactance of coil so I_p will be nearly in phase with the voltage

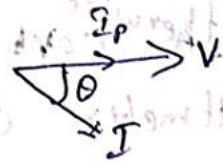


$$P = V I \cos \phi$$

$$I_p = \frac{V}{R} I$$

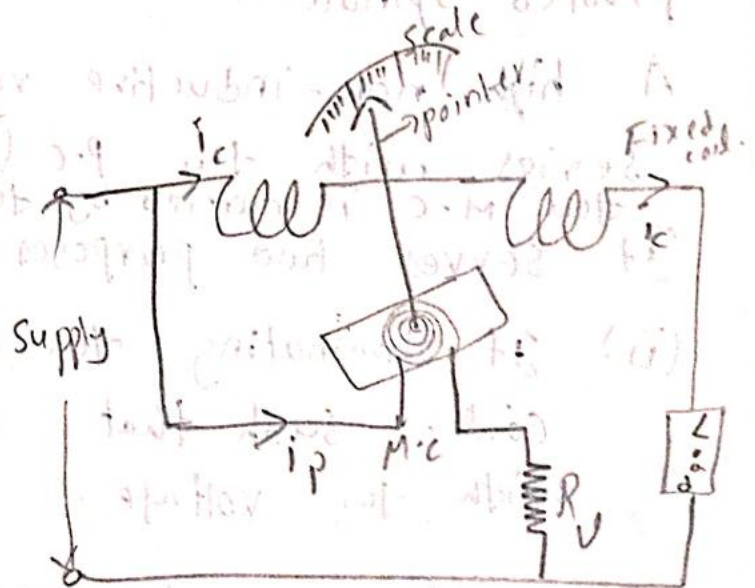
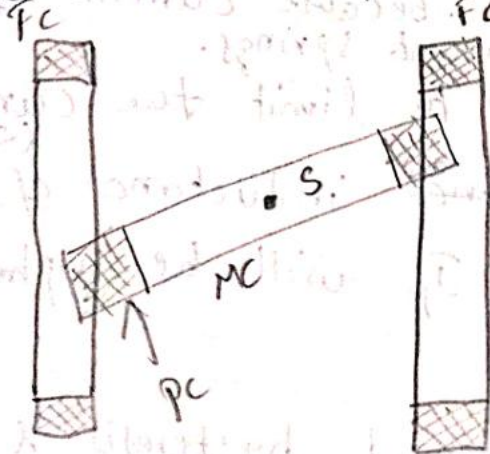
This produces one flux

This produces another flux



\cos of angle b/n those two currents

Construction & operation:-



Fixed coils:- Fixed coils are connected in series with the load and carry the load current \therefore hence called current coil (C.C).

\rightarrow It is divided into two sections to provide more uniform magnetic field near the centre where moving coil is placed.

→ The fixed coils carry the load current they are limited upto 20A. If ~~the~~ ^{we} want load,

If the load current is $> 20A$ i.e. $> 20A$ then the fixed coils are connected in parallel so the current is equally divided into two sections.

Moving coil:- The two halves of the fixed coil, which are connected in series for a basic measuring range, otherwise ~~can~~ ^{can} be connected in parallel to increase the wattmeter current range to twice its original value.

Moving coil:- ^{connected across the supply voltage or pressure coil (P.C.)} The moving coil is mounted on a pivoted spindle. Hence called

A high non-inductive resistance is connected in series with the P.C (or) M.C. because current of the M.C is carried by the instrument springs.

It serves two purposes :- (i) to limit the current ($< 100mA$)

(ii) It dominating the effect ~~and~~ ^{of} inductance of the coil. such that current I_p will be in phase with the voltage.

→ The coils are air cored to avoid hysteresis & eddy current errors.

→ Control system:- spiral hair springs are used to supply current to M.C & also to produce controlling torque T_c .

Damping system: Air friction damping is used. (3)

∴ the operating field is very weak, eddy current damping can not be used.

→ proper shielding is provided against stray magnetic fields using laminated steel shield.

Torque Equation:- The instantaneous torque of an electro-dynamometer instruments is given by

$$T_i = i_1 i_2 \frac{dM}{d\theta}$$

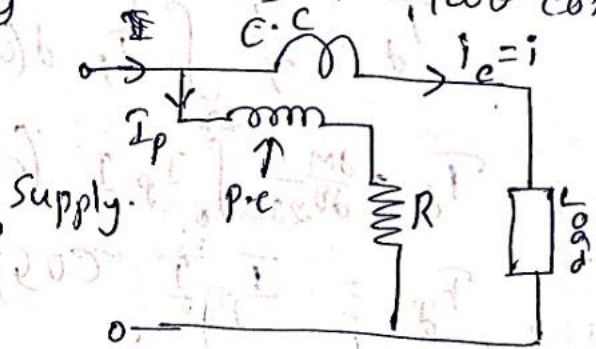
where i_1 & i_2 are instantaneous values of currents in two coils

Let

V & I be the rms value of voltage & current being

Let $V = V_m \sin \omega t$, $i = I_m \sin(\omega t - \phi)$ measured.

Instantaneous value of voltage across the pressure coil ckt



$$V_m = V_m \sin \omega t$$

$$= \sqrt{2} V \sin \omega t$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$i_p = \frac{V_m}{R_p} = \frac{\sqrt{2} V}{R_p} \sin \omega t = \sqrt{2} I_p \sin \omega t \quad \left[\because I_p = \frac{V_p}{R_p} \right]$$

I_p = rms value of current in p.c ckt.

If the current in the current coil lags the voltage by a phase angle ϕ ,

Instantaneous value of current through current coil is

$$i_c = I_m \sin(\omega t - \phi) = \sqrt{2} I \sin(\omega t - \phi)$$

If the current in the current coil lags the voltage by an angle ϕ ,

$$i = I_m \sin(\omega t - \phi)$$

$$i_c = i = \sqrt{2} I \sin(\omega t - \phi)$$

Instantaneous torque,

$$T_i = \sqrt{2} I_p \sin \omega t \times \sqrt{2} I \sin(\omega t - \phi) \frac{dM}{d\theta}$$

$$= 2 I_p I \sin \omega t \sin(\omega t - \phi) \frac{dM}{d\theta}$$

$$= I_p I 2 \sin \omega t \sin(\omega t - \phi) \frac{dM}{d\theta}$$

$$T_i = I_p I (\cos \phi - \cos(2\omega t - \phi)) \frac{dM}{d\theta}$$

Average deflecting torque,

$$T_d = \frac{1}{T} \int_0^T T_i d(\omega t)$$

$$T_d = \frac{dM}{d\theta} \frac{1}{2\pi} \int_0^{2\pi} I_p I (\cos \phi - \cos(2\omega t - \phi)) d(\omega t)$$

$$T_d = I_p I \cos \phi \frac{dM}{d\theta}$$

$$= \left(\frac{VI}{R_p} \right) \cos \phi \frac{dM}{d\theta}$$

$$T_c = k\theta$$

At balance position,

$$k\theta = I_p I \cos \phi \frac{dM}{d\theta}$$

$$\theta = I_p I \cos \phi \left(\frac{dM}{d\theta} \right) / k$$

$$= \frac{VI \cos \phi}{R_p k} \frac{dM}{d\theta}$$

$$\theta = \left(k_1 \frac{dM}{d\theta} \right) p$$

Controlling torque i_c

$$\theta = K_1 \frac{VI \cos \phi}{R_p k} \frac{dM}{d\theta}$$

Low power factor wattmeter (LPF Wattmeter) (4)

Wattmeter $\left\{ \begin{array}{l} \text{UPF - is used to measure power in Resistive ckt.} \\ \text{LPF - is used to measure " in highly inductive ckt.} \end{array} \right.$

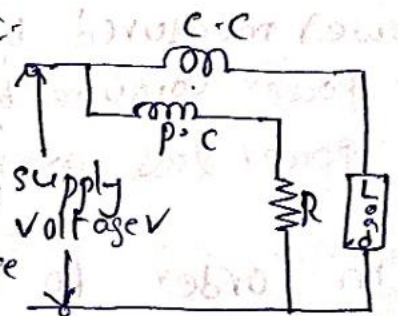
What is the need of LPF Wattmeter:-

- (1) Magnitude of deflecting torque (T_d) is small. ($T_d \propto P \propto V I \cos \phi$) even after the full excitation of the pressure & current coil.
- (2) The error occurs in the reading because of the pressure coil inductance.

So some additional features are added to the ordinary wattmeters so that the meter can measure the power of the Low power factor circuits.

Modifications in ordinary wattmeter:-

- (1) Reduction of pressure coil resistance
- (2) Compensation of Error due to p-c connection
- (3) " " " " Inductance



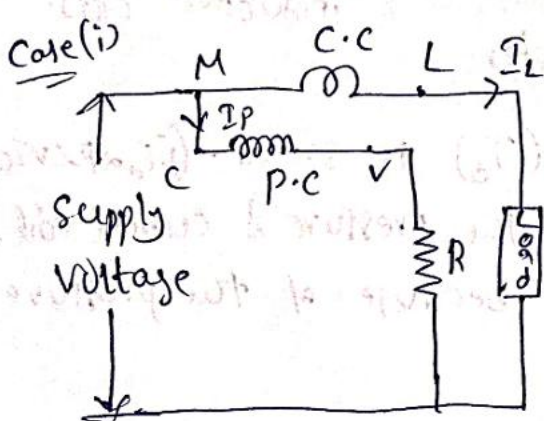
① \rightarrow In an ordinary wattmeter a high value of Resistance 'R' is connected in series with the p.c. in order to limit the current through the coil.

But in case of LPF wattmeter p.c. is designed for low value of resistance so that a high value of current passes through the coil. This current produces the high value of T_d .

To the current in

② compensation of error due to pressure coil connections :-

They are two methods of P.C connections in the wattmeter

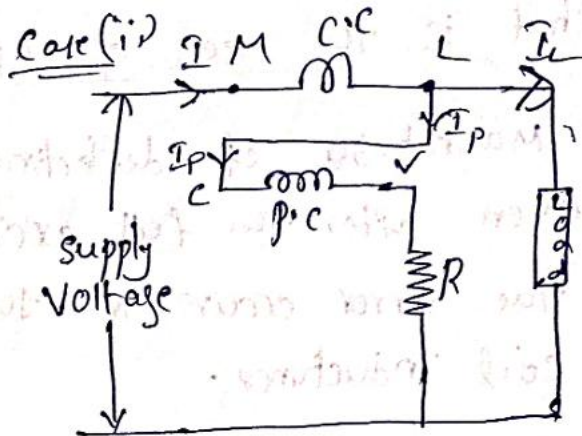


P.C is connected across the supply side.

$$V = V_L + V_{P.C}$$

$$P_w = P_L + I_L^2 R_{P.C}$$

Power measured by wattmeter = Power consumed by load + Power loss across in the C.C



$$I = I_L + I_P$$

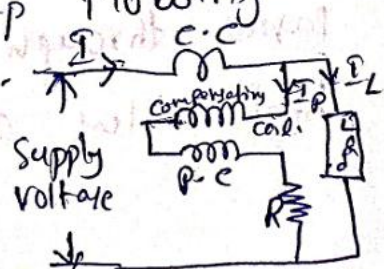
$$P_w = P_L + \frac{V^2}{R_{P.C}}$$

Power measured by wattmeter = Power consumed by load + Power loss in the P.C.

In order to compensate the error due to P.C connection

a compensating coil is connected in series with the P.C. so pressure coil current I_P flows through the compensating coil then magnetic field is produced in it. This magnetic field of compensating coil opposes the magnetic field of C.C.

Thus the error caused by the I_P flowing through the C.C is neutralized.



3) Compensation of error due to p.c inductance = $V I \sin \phi \tan \beta$

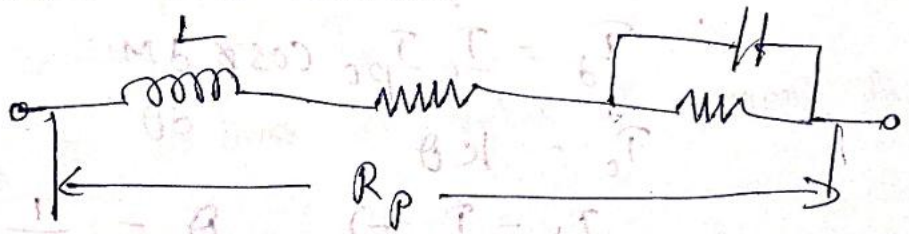
A small amount of inductance is present in the p.c of the wattmeter. This inductance causes error in the reading. This type of error can be reduced by connecting a capacitor in parallel with the p.c resistance. So that the capacitive reactance cancelled out the Inductive reactance of the p.c. Then p.c becomes purely resistive.

The error caused by p.c inductance is given by $V I \sin \phi \tan \beta$.

For low PF $\cos \phi \downarrow$, $\phi \uparrow$

\therefore Error is corresponding large.

So we must compensate the error caused by inductance of p.c - This is done by connecting a capacitor across a part of series resistance in the pressure coil circuit.



Torque Equation! - Instantaneous torque, $T_i = i_1 i_2 \frac{dM}{d\theta}$

$V = \text{Instantaneous voltage} = V_m \sin \omega t = \sqrt{2} V \sin \omega t$

i_{pc} Due to high series resistance, p-c is treated to be purely resistive.

$\therefore i_{pc} = \text{Instantaneous value} = \frac{V}{R_p} = \frac{\sqrt{2} V \sin \omega t}{R_p}$

$\therefore i_{pc} = \sqrt{2} I_{pc} \sin \omega t$

If c-c current lags the voltage by angle ϕ , then its instantaneous value is given by.

$i_c = \sqrt{2} I_c \sin(\omega t - \phi)$ $i_c = I_m \sin(\omega t - \phi)$

Now $i_1 = i_c$ & $i_2 = i_{pc}$

$T_i = \sqrt{2} I_c \sin(\omega t - \phi) (\sqrt{2} I_{pc} \sin \omega t) \frac{dM}{d\theta}$

$= 2 I_c I_{pc} \sin \omega t \sin(\omega t - \phi) \frac{dM}{d\theta}$

2 sara sinus
Average deflection torque
 $\cos(A-B) - \cos(A+B)$

$= I_c I_{pc} [\cos \phi - \cos(2\omega t - \phi)] \frac{dM}{d\theta}$

$T_d = \frac{1}{T} \int_0^T T_i d\omega t$

on simplification

$T_d = I_c I_{pc} \cos \phi \frac{dM}{d\theta}$

$T_c = k\theta$

$T_d = T_c \Rightarrow \therefore \theta = \frac{1}{k} I_c I_{pc} \cos \phi \frac{dM}{d\theta}$

$\theta = \frac{1}{k_1} I_c I_{pc} \cos \phi$

$\theta = k_1 \frac{I_c V}{R_p} \cos \phi$

$\theta = k_2 V \frac{I_c}{R_p} \cos \phi$

where $P = V I_c \cos \phi$

$k_2 = \frac{k_1}{R_p}$

$k_1 = \frac{1}{k} \frac{dM}{d\theta}$

$\therefore \theta \propto P$

$\theta = k_2 P$

Low power factor wattmeters :- Measure of (6)
power in ckt. having low pf by ordinary
electrodynamometer wattmeters is difficult &
inaccurate because

(i) the deflecting torque (T_d) on the moving system
is small even when the current & pressure
coils are fully excited.

(ii) errors occurred because of inductance of p.c.
So some special features are added to the
ordinary wattmeters so that the meter can
measure the power of the low pf circuits.
The features are discussed below.

(1) Pressure coil current :- (Reduction of p.c resistance)

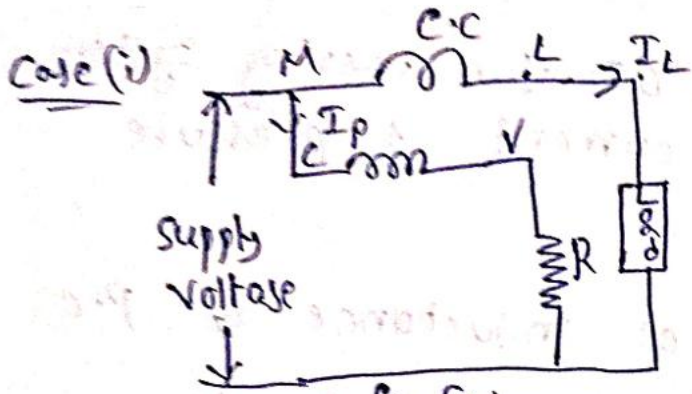
The pressure coil circuit is designed to have a
low value of resistance, so that high value of
current passes through the coil. This current
produces high value of T_d .

In Lpf I_p is 10 times the value of current for high pf
wattmeters.

① Compensation for pressure coil current :-

That means compensation of error due to p.c connections.

They are two methods of p.c connections in the wattmeter.

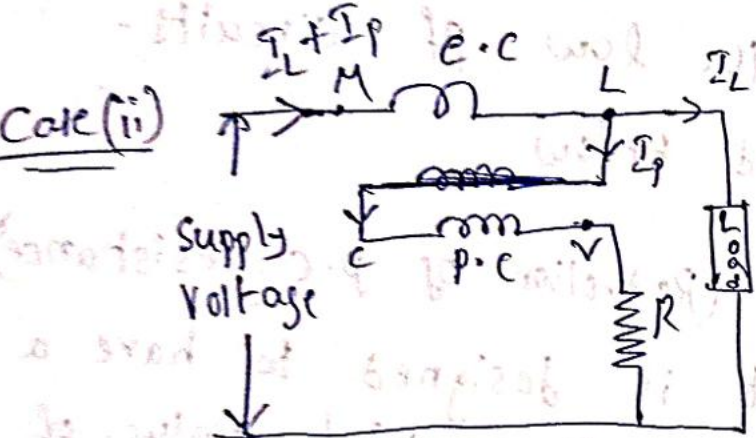


p.c. is connected across the supply side.

$$\therefore V = V_L + V_{D \text{ is C.C.}}$$

$$P_W = P_L + I_L^2 R_{c.c.}$$

For small values of current
 Power measured by wattmeter = power consumed by load + power loss in the C.C. ($I_L^2 R_c$)



C.C. is connected on the supply side.

\therefore It carries p.c. current & load current.

$$I = I_L + I_p$$

$$P_W = P_L + \frac{V^2}{R_p}$$

For high values of current
 power consumed by wattmeter = power consumed by load + power loss in the p.c.

* If the load current is small, the voltage drop in the C.C. is small ^{iey power loss there} then Fig (a) ckt's are preferable.

They introduces very small error. (because $I_L^2 R_c$ is small)

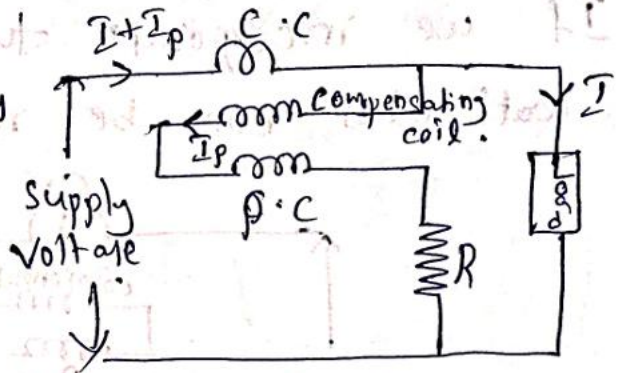
→ If load current is large, the value of p.c. current is very small as compared to load current & hence power loss in pressure coil will be very small. \therefore Fig (b) is preferable.

For small current fig (a) & for large current fig (b) are preferable if the power loss in the instrument is to be neglected.

But However if accuracy requires that the power loss in the instrument has to be considered then fig (b) is preferable.

So to compensate the error caused by power loss in p.c circuit, a compensating coil is connected in series with the p.c ckt as shown in fig.

In this connection, c.c carries a current of $(I + I_p)$ & produces a field corresponding to this current.



The compensating coil carries a current I_p & produces a field corresponding to this current.

→ This field acts in opposition to the current coil field. Thus the resultant field is due to current I only. Hence the error caused by the p.c current flowing in the c.c is neutralized.

③ Compensation of error due to pressure coil inductance

The error caused by P.C inductance is

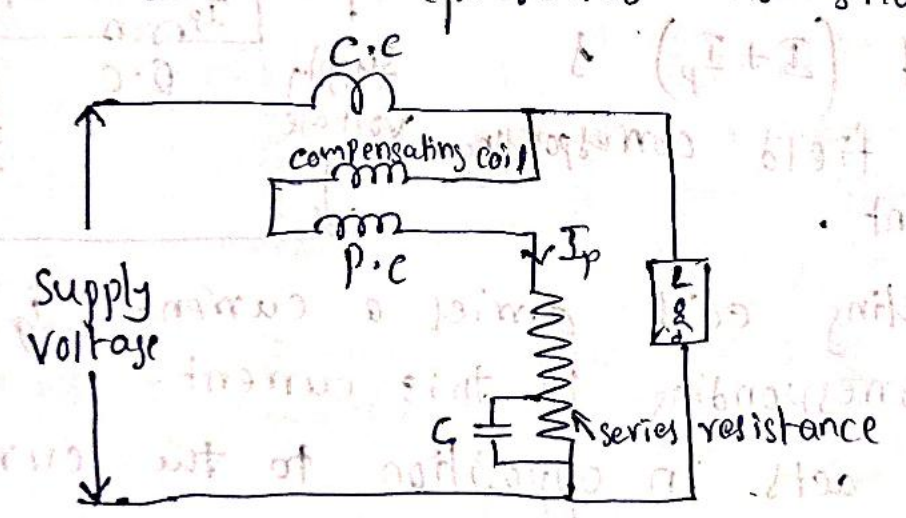
$$VI \sin \phi \tan \beta$$

For low pf, the value of β is large

\therefore the error due P.C inductance is large.

This error can be reduced by connecting a capacitor across a part of series resistance in the pressure coil ckt as shown in fig.

\therefore If we incorporate the features discussed, the Lpf wattmeter can be represented as shown in fig.



Torque equation: - The instantaneous torque of an electrodynamometer instrument is given by

$$T_i = i_1 i_2 \frac{dM}{d\theta}$$

where i_1, i_2 are instantaneous values of currents in two coils.

Let V & I be the rms values of voltage & current being measured.

Let instantaneous value of voltage across the pressure coil circuit $v = V_m \sin \omega t$

$$v = \sqrt{2} V \sin \omega t \quad \because V_m = \frac{V_m}{\sqrt{2}}$$

Instantaneous Value $i_{pc} = \frac{v}{R_p} = \frac{\sqrt{2} V \sin \omega t}{R_p}$

$$i_{pc} = \sqrt{2} I_{pc} \sin \omega t \rightarrow (1)$$

Rms value of current in p.c. circuit

$$I_{pc} = \frac{V}{R_p}$$

If c.c. current lags the voltage by an angle ϕ then its instantaneous value is given by

$$i_c = I_m \sin(\omega t - \phi)$$

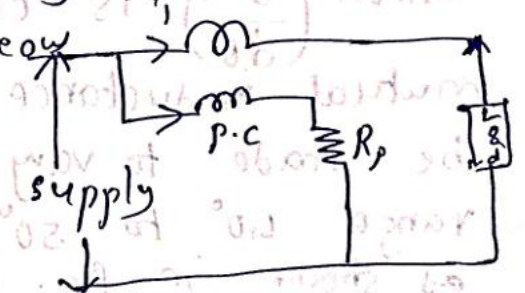
$$i_c = \sqrt{2} I_c \sin(\omega t - \phi) \rightarrow (2)$$

Instantaneous torque $T_i = \sqrt{2} I_c \sin(\omega t - \phi) \sqrt{2} I_{pc} \sin \omega t \frac{dM}{d\theta}$

$$= 2 I_c I_p \sin \omega t \sin(\omega t - \phi) \frac{dM}{d\theta}$$

$$T_i = I_c I_p [\cos \phi - \cos(2\omega t - \phi)] \frac{dM}{d\theta}$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$



Average Deflecting torque $T_d = \frac{1}{T} \int_0^T T_i d(\omega t)$

$$= \frac{1}{2\pi} \int_0^{2\pi} T_i d(\omega t)$$

$$T_d = \frac{1}{2\pi} \int_0^{2\pi} I_c I_{pc} [\cos \phi - \cos(2\omega t - \phi)] d(\omega t) \frac{dM}{d\theta}$$

$$T_d = \frac{1}{2\pi} I_c I_{pc} \cos \phi \frac{dM}{d\theta} \quad \text{[on simplification]}$$

controlling torque $T_c = k\theta$

Under equilibrium condition,

$$T_d = T_c$$

$$k\theta = I_c I_{pc} \cos \phi \frac{dM}{d\theta}$$

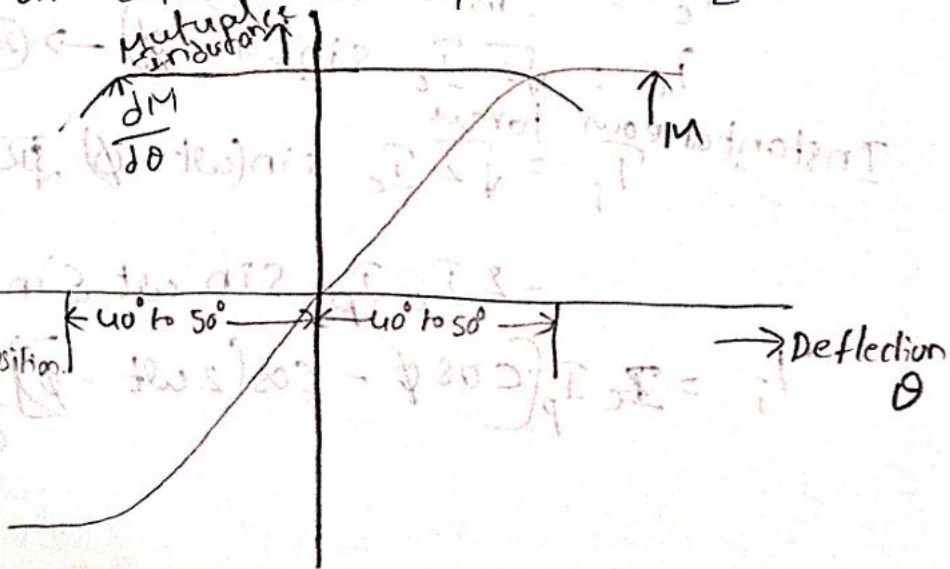
$$\theta = \frac{1}{k} I_c \frac{V}{R_p} \cos \phi \frac{dM}{d\theta}$$

$\therefore \theta \propto P$

$$\theta = K_1 \sqrt{I_c} \cos \phi \Rightarrow \boxed{\theta = K_1 P}$$

It is clear that the deflection is directly proportional to the power being measured & the scale is essentially uniform over the range in which $\left(\frac{dM}{d\theta}\right)$ is constant. By suitable design, the mutual inductance b/n fixed & moving coil can be made to vary linearly with angle over a range 40° to 50° on either side of zero M-I position as shown in fig.

The wattmeters are designed such that $\frac{dM}{d\theta}$ remains almost constant over a range of 40° to 50° on either side of zero M-I position.



In order to read the 3- ϕ wattmeter read correctly, there should not be any mutual interference b/n the two elements.

So Resistance 'R' may be adjusted to compensate for errors caused by mutual interference.

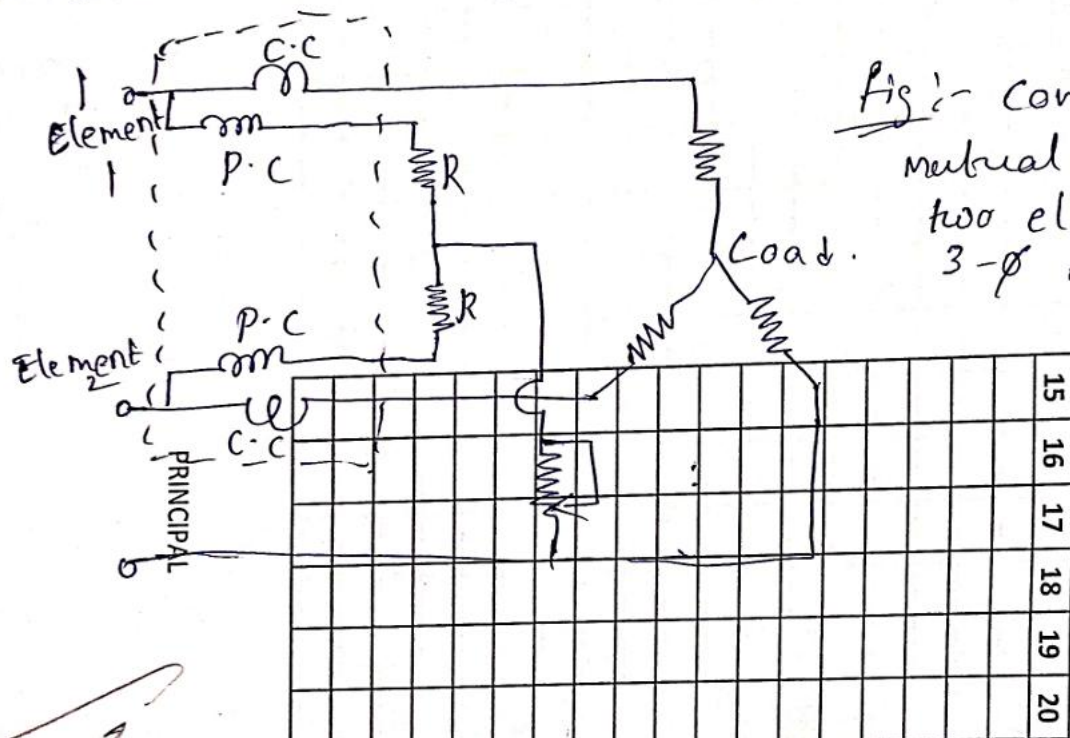
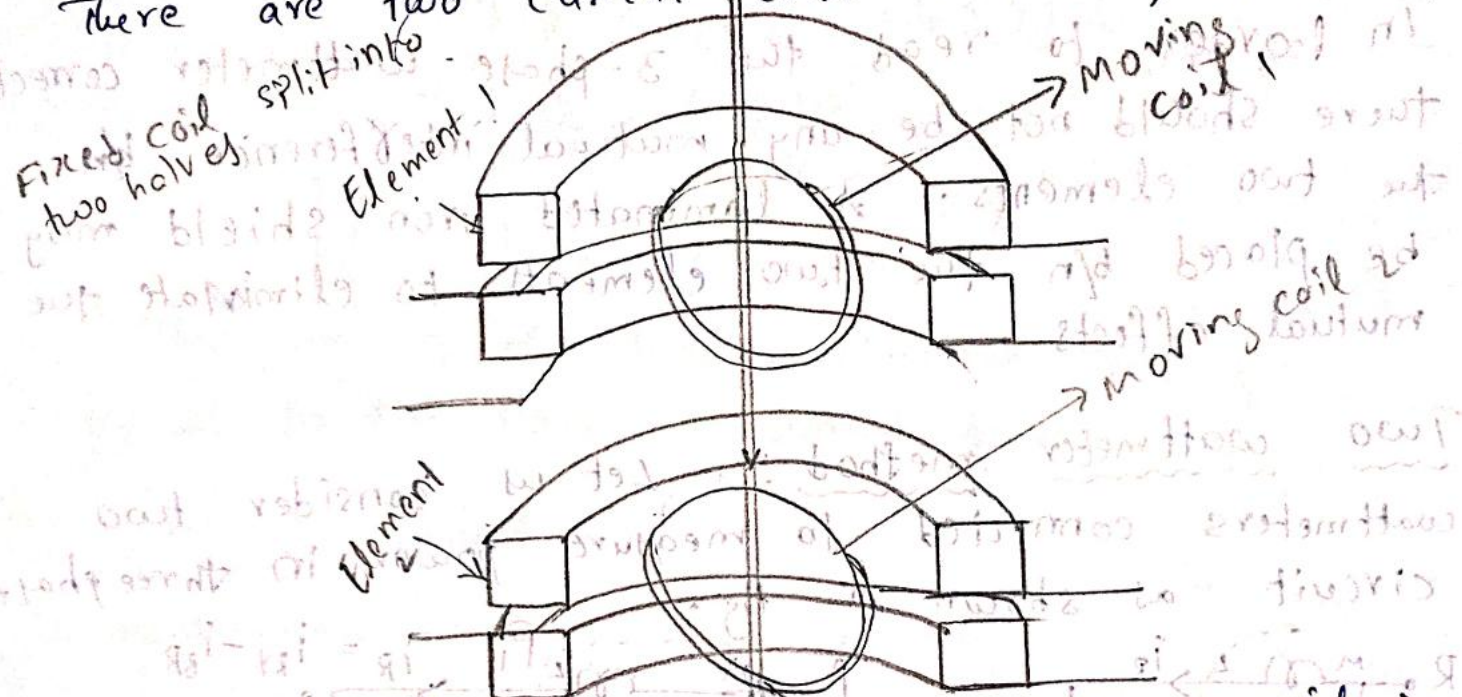


Fig:- Compensation for mutual effects b/n two elements of a 3- ϕ wattmeter.

Three phase wattmeters - - A dynamometer type

three phase wattmeter consists of two separate wattmeter movements mounted together in one case with the two moving coils mounted on the same spindle. The arrangement is shown in fig.

There are two current coils & two pressure coils,



A current coil together with its pressure coil is known as an element. Therefore a three phase wattmeter has two elements.

The connections of two elements of a 3-phase wattmeter are the same as that of two wattmeter method using two single phase wattmeters.

The torque on each element is proportional to the power being measured by it. The total torque deflecting the moving system is the sum of the deflecting torque of the two systems elements.

Sum of the instantaneous readings of two wattmeters

$$\begin{aligned}
 P &= P_1 + P_2 \\
 &= i_R (V_R - V_Y) + i_B (V_B - V_Y) \\
 &= i_R V_R - i_R V_Y + i_B V_B - i_B V_Y \\
 &= i_R V_R + i_B V_B - (i_R + i_B) V_Y \\
 &= i_R V_R - (-i_Y) V_Y + i_B V_B \quad \because i_R + i_Y + i_B = 0 \\
 &\quad \therefore i_R + i_B = -i_Y \\
 &\boxed{P = i_R V_R + i_Y V_Y + i_B V_B}
 \end{aligned}$$

The sum of the two wattmeters reading is equal to the power consumed by the load.

//ry in Δ connection :-

Instantaneous reading of wattmeter 1 is

$$P_1 = i_R V_{RY} = (i_{RY} - i_{BR}) V_{RY}$$

Instantaneous reading of wattmeter 2 is

$$P_2 = i_B V_{BY} = (i_{BR} - i_{YB}) V_{BY}$$

Sum of the instantaneous readings of two wattmeters

$$\begin{aligned}
 P &= P_1 + P_2 \\
 &= i_R V_{RY} + i_B V_{BY} \\
 &= (i_{RY} - i_{BR}) V_{RY} + (i_{BR} - i_{YB}) V_{BY} \\
 &= i_{RY} V_{RY} - i_{BR} V_{RY} + i_{BR} V_{BY} - i_{YB} V_{BY} \\
 &= i_{RY} V_{RY} + i_{BR} V_{BY} + i_{YB} V_{YB} - i_{BR} V_{RY} \\
 &= i_{RY} V_{RY} + i_{YB} V_{YB} + (V_{BY} - V_{RY}) i_{BR} \Rightarrow i_{RY} V_{RY} + i_{YB} V_{YB} + V_{BR} i_{BR} = P
 \end{aligned}$$

∴ The sum of the two wattmeter readings is equal to the power consumed by the load. This is irrespective of whether the load is balanced or unbalanced.

For balanced load! - Measurement of Power

Let V_R, V_Y, V_B be the rms values of phase voltage
 & I_R, I_Y, I_B be the " " " " " " currents.

Line voltages $V_{RY} = V_{YB} = V_{BR} = \sqrt{3} V_{ph}$

Line currents = phase currents (For star connected balanced load)

The power reads by wattmeter 1 is given by.

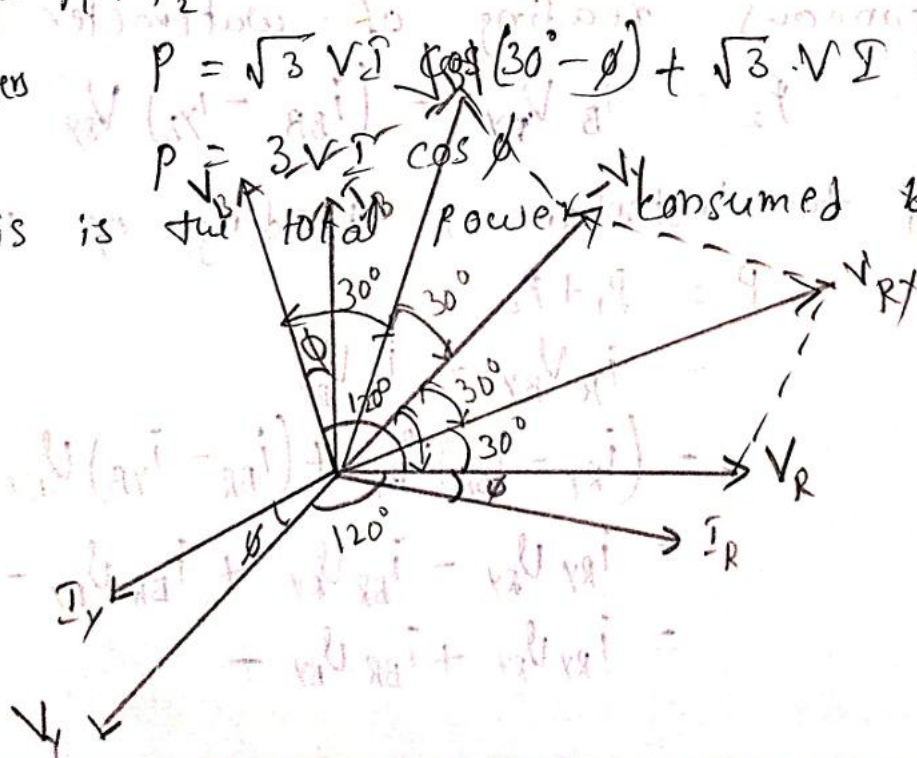
$$P_1 = V_{RY} I_R \cos(\quad) = \sqrt{3} V_{ph} I_{ph} \cos(30^\circ - \phi)$$

By wattmeter 2 $P_2 = I_B V_{BY} \cos(\quad) = \sqrt{3} V_{ph} I_{ph} \cos(30^\circ + \phi)$

Sum of reading of two wattmeters $P = P_1 + P_2$

$$P = \sqrt{3} V I \cos(30^\circ - \phi) + \sqrt{3} V I \cos(30^\circ + \phi)$$

This is the total power consumed by load.



(11)

Difference of two wattmeter readings

$$P_1 - P_2 = \sqrt{3} V I (\cos(30^\circ + \phi) - \cos(30^\circ - \phi))$$
$$= \sqrt{3} V I \sin \phi$$

$$\therefore \frac{P_1 - P_2}{P_1 + P_2} = \frac{\sqrt{3} V I \sin \phi}{3 V I \cos \phi} = \frac{\tan \phi}{\sqrt{3}}$$

$$\phi = \tan^{-1} \sqrt{3} \left(\frac{P_1 - P_2}{P_1 + P_2} \right)$$

Power factor $\cos \phi = \cos \left[\tan^{-1} \sqrt{3} \left(\frac{P_1 - P_2}{P_1 + P_2} \right) \right]$

Effect of PF on the readings of wattmeters

(a) when $\cos \phi = 1$, $\phi = 0$

$$W_1 = \sqrt{3} V I \cos(30^\circ + \phi) = \frac{3}{2} V I$$

$$W_2 = \sqrt{3} V I \cos(30^\circ - \phi) = \frac{3}{2} V I$$

$$\boxed{W_1 = W_2}$$

Each wattmeter reads half of the total power.

(b) when $\cos \phi = 0.5$, $\phi = 60^\circ$

$$W_1 = \sqrt{3} V I \cos(30^\circ + \phi) = 0$$

$$W_2 = \sqrt{3} V I \cos(30^\circ - \phi) = \frac{3}{2} V I$$

$$\therefore \text{Total power } P = \frac{3}{2} V I$$

ie., one of the wattmeter reads zero & the other wattmeter reads the total power.

© when $\text{pf} \cos \phi = 0$, $\phi = 90^\circ$

$$W_1 = \sqrt{3} V I \cos(30^\circ + 90^\circ) = -\frac{\sqrt{3}}{2} V I$$

$$W_2 = \sqrt{3} V I \cos(30^\circ - 90^\circ) = \frac{\sqrt{3}}{2} V I$$

$$\therefore \boxed{P_1 + P_2 = 0}$$

Measurement of Power by three Elements:-

As we already discussed that A c.c. together with its p.c. is known as Element.

The connections of 3 Elements - of a 3- ϕ wattmeter are the same as that of three wattmeter method.

The torque developed on each element is proportional to the power being measured by it. The total torque deflecting the moving system is the sum of the T_d of three elements.

Deflecting torque of element 1 $\propto P_1$

" " " " " " 2 $\propto P_2$

" " " " " " 3 $\propto P_3$

\therefore Total deflecting torque $\propto (P_1 + P_2 + P_3) \propto P$.

Instantaneous power read by element 1, $P_1 = I_1 U_1$

" " " " " " 2, $P_2 = U_2 I_2$

" " " " " " 3, $P_3 = U_3 I_3$

Sum of the instantaneous readings of the wattmeter

$$P = P_1 + P_2 + P_3 = U_1 I_1 + U_2 I_2 + U_3 I_3$$

Power factor Meters :- on measuring the voltage & current, & power in AC circuit, its pf can be calculated from the relationship $\cos \phi = \frac{P}{\sqrt{I}}$. (12)

This method of determining the pf of an electric circuit, is however, of low accuracy & has a number of disadvantages & is rarely used in practice.

It is obviously desirable to have an instantaneous indication of the pf of an AC circuit, especially where this is varying continuously without having recourse to mathematical calculations.

∴ power factor meters indicate directly, by a single reading, the power factor of the circuit to which they are connected.

There are two types of power factor meters.

- (i) Electrodynamometer type
- (ii) Moving Iron type.

1- ϕ Electrodynamometer power factor meter :-

The construction of a 1- ϕ Electrodynamometer type pf meter is shown in fig. It consists of a fixed coil which acts as the current coil. This coil is split into two parts & carries the current of the ckt. Therefore, the magnetic field produced by this coil is proportional to the main current.

Two identical pressure coils A & B pivoted on a spindle constitute the moving system. Pressure coil A has a non-inductive resistance R connected in series with it & pressure coil B has a highly inductive choke coil L connected in series with it. The two coils A & B are connected across the applied voltage of the circuit. The values of R & L are so adjusted that the two coils carry the same value of current at normal frequency.

The current through coil A is in phase with the applied voltage while the current through coil B is lags the voltage by an angle 90° .

For that, the angle b/n the planes of coils is also made equal to Δ . ($\Delta = 90^\circ$)

There is no controlling torque.

In order to simplify, we assume that the current through coil B lags the voltage by exactly 90° . Also that the angle b/n the planes of coils is exactly by 90° .

Now there will be two deflecting torques, one acting on coil A & the other on coil B. The coil windings are so arranged that the torques due to the two coils are opposite in direction.

∴ Therefore the deflection of the instrument is a measure of phase angle of the circuit.

The scale of the instrument can be calibrated in directly interms of powerfactor.

$\frac{dM}{d\theta}$ is constant for radial field but if the field produced fixed coils is not radial then $\frac{dM}{d\theta} \neq \text{constant}$

In this case the field produced by F.C is parallel $\left[\frac{dM}{d\theta} \right]$

The Mutual inductance 'M' b/n F.C's & M.C depends upon the position of M.C. The max. value of M.I is obtained at $\theta = 180^\circ$ & minimum at $\theta = 0^\circ$.

$$M = M_{\max}$$

$$M = -M_{\max}$$

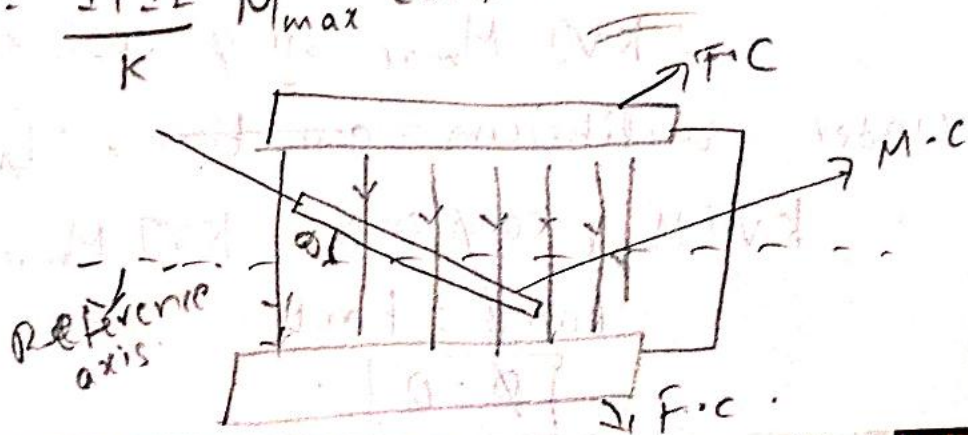
$$M = -M_{\max} \cos \theta$$

$$\frac{dM}{d\theta} = M_{\max} \sin \theta$$

$$\therefore T_d = I_1 I_2 \cos \phi \frac{dM}{d\theta}$$

$$T_d = I_1 I_2 M_{\max} \cos \phi \sin \theta$$

$$\theta = \frac{I_1 I_2 M_{\max} \cos \phi \sin \theta}{K}$$

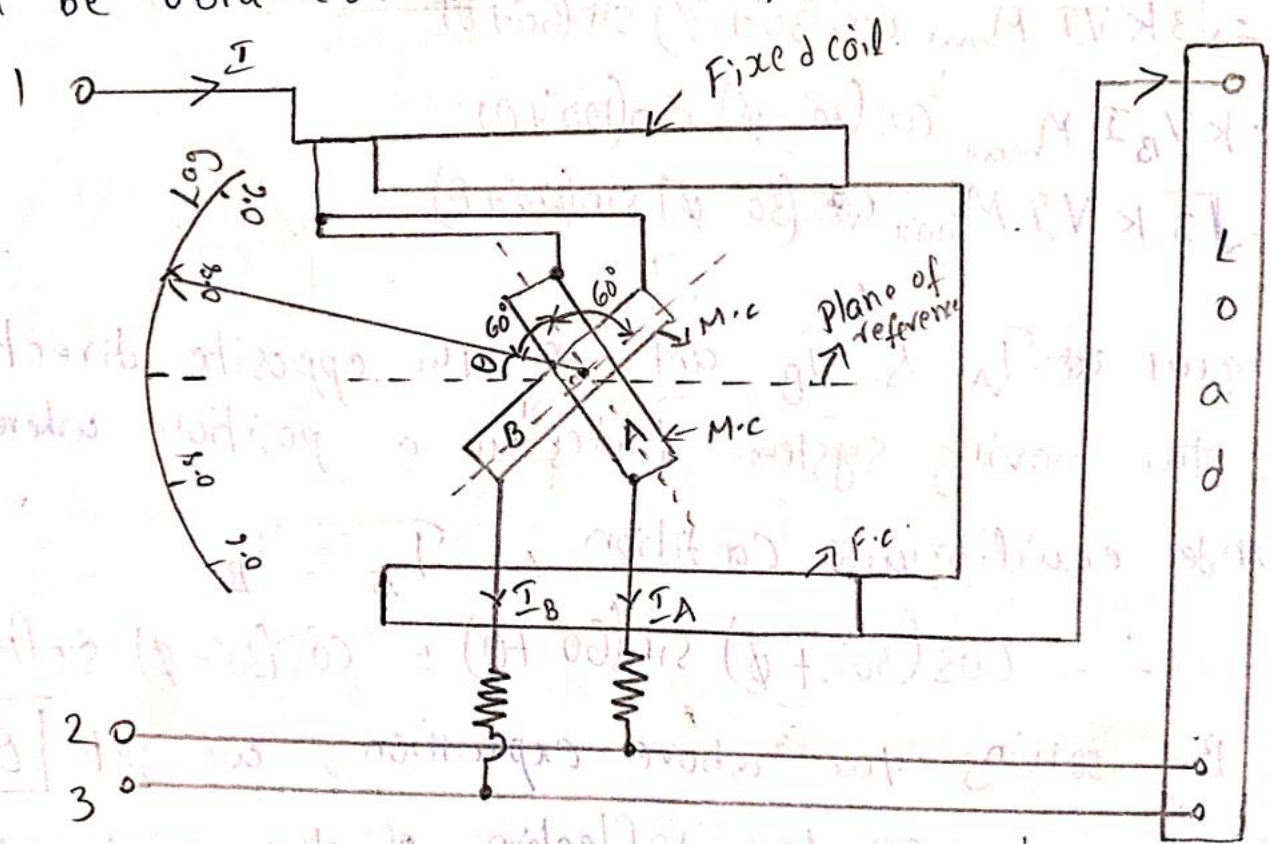


Three phase Electrodynamometer Power factor meter:

The fig. shows the 3- ϕ Electrodynamometer pf meter.

This meter is useful for only balanced loads.

The two moving coils are so placed that the angle b/n their planes is 120° . They are connected across two different phases of the supply ckt. Each coil has a series resistance. There is no necessity for phase splitting by artificial means, since the required phase displacement b/n currents \vec{I}_A & \vec{I}_B in the two moving coils can be obtained from the supply itself.



This is only for balanced loads.

For unbalanced loads two element pf meter has to be used. (where two sets of fixed coils & two sets of M.c.'s mounted on the spindle.)

voltage applied across coil A is V_{12} & as its circuit is resistive, current \underline{I}_A is in phase with V_{12} .
 Any voltage applied across coil B is V_{13} & current \underline{I}_B is in phase with V_{13} as the circuit of coil is resistive.

Let ϕ = phase angle of the circuit

θ = angular deflection from the plane of reference.

Now $V_1 = V_2 = V_3 = V$

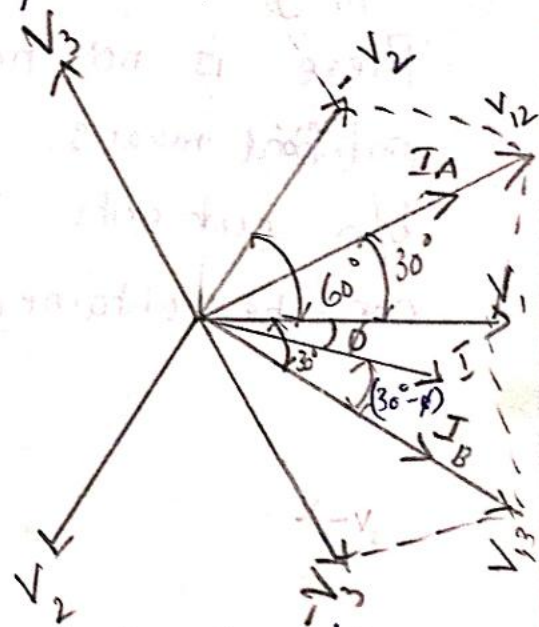
Torque acting on coil A is

$$T_A = k V_{12} I M_{\max} \cos(30^\circ + \phi) \sin(60^\circ + \theta)$$

$$T_A = \sqrt{3} k V I M_{\max} \cos(30^\circ + \phi) \sin(60^\circ + \theta)$$

$$\text{Any } T_B = k V_{13} I M_{\max} \cos(30^\circ - \phi) \sin(120^\circ + \theta)$$

$$= \sqrt{3} k V I M_{\max} \cos(30^\circ - \phi) \sin(120^\circ + \theta)$$



Torques T_A & T_B act in the opposite directions & the moving system takes up a position where $T_A = T_B$.

ie, under equilibrium condition, $T_A = T_B$

$$\therefore \cos(30^\circ + \phi) \sin(60^\circ + \theta) = \cos(30^\circ - \phi) \sin(120^\circ + \theta)$$

By solving the above expression, we get $\theta = \phi$

\therefore Thus the angular deflection of the pointer from the plane of reference is equal to the phase angle of the ckt.

3-4 Construction Moving Iron type Power factor Meter

The following are the essential features of a rotating magnetic field type of instrument -

The pf meter has 3 fixed coils & their axes are 120° displaced from each other. The axes are intersecting each other on the centre line of the instrument.

→ The coils are connected to the three phase supply with the help of the current transformers.

→ 'p' is the fixed coil connected in series with a high resistance across one pair of lines say 2 & 3. There is an iron cylinder 'c' inside coil 'p'.

→ Two sector shaped iron vanes 'v' are fixed to this cylinder. The two vanes are 180° apart in space.

→ The spindle also carries damping vanes & a pointer.

working:- Coil p & the iron system produce an alternating flux, which interacts with the fluxes produced by coils A_1 , A_2 & A_3 . This causes the moving system to take up the angular position determined by the phase angle of the current. (The angular deflection of the iron vane ~~from~~ is equal to the phase angle ϕ .)

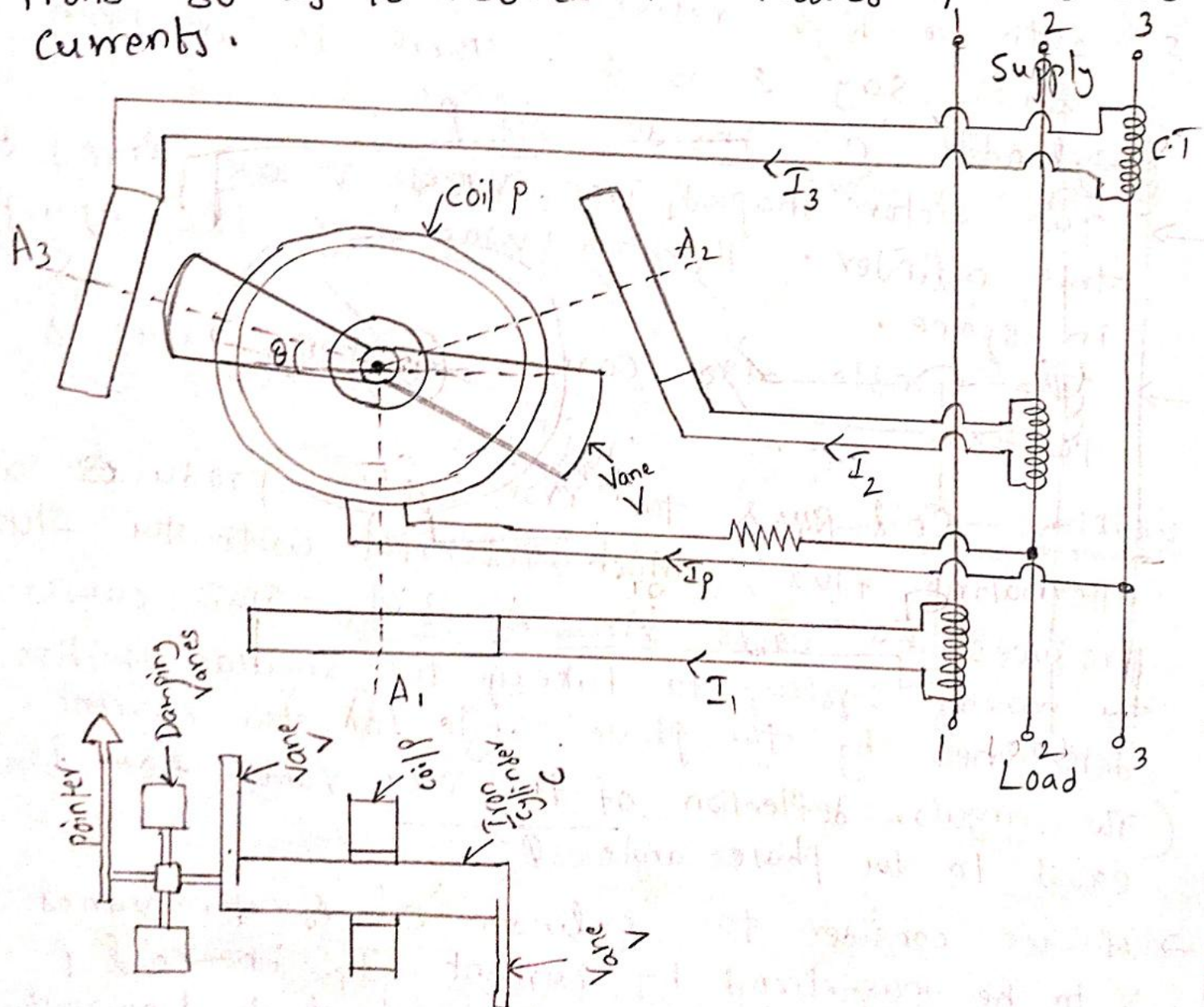
→ If we consider the cylinder 'c' & the vanes 'v', v to be magnetized by current I_p in coil p which is in phase with & proportional to the line voltage

of the system.

U-E

→ It may be noted that the three fixed coils A_1, A_2, A_3 produce a rotating magnetic field & therefore owing to this there will be an induction motor action tending to keep the moving system continuously in the direction of the rotating magnetic field.

This effect can be made negligibly small by using high resistivity metal for the moving irons so as to reduce the values of induced currents.



Now the torque acting on the moving system due to currents in coils A_1, A_2 & A_3 .

The total deflecting torque

$$T_d \propto \left[I_1 I_p \cos(90^\circ - \phi) \sin(90^\circ + \theta) + I_2 I_p \cos(330^\circ - \phi) \sin(210^\circ + \theta) + I_3 I_p \cos(210^\circ - \phi) \sin(330^\circ + \theta) \right]$$

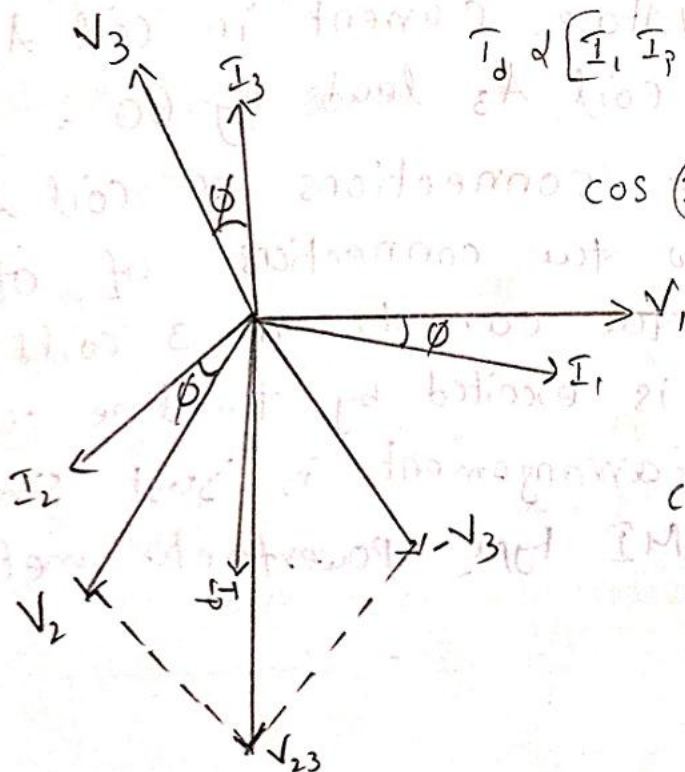
For a steady deflection, the total torque must be zero. Also considering the system to be balanced

ie $I_1 = I_2 = I_3$, we have

$$\cos(90^\circ - \phi) \sin(90^\circ + \theta) + \cos(330^\circ - \phi) \sin(210^\circ + \theta) + \cos(210^\circ - \phi) \sin(330^\circ + \theta) = 0$$

Solving the above expression, we have $\theta = \phi$.

\therefore The deflection of Iron Vane from the reference axis is a direct measure of the phase angle b/w each line current & the corresponding phase voltage.



$$T_d \propto \left[I_1 I_p \cos(I_1 \wedge I_p) + I_2 I_p \cos(I_2 \wedge I_p) + I_3 I_p \cos(I_3 \wedge I_p) \right]$$

$$\cos(I_1 \wedge I_p) = (90^\circ - \phi)$$

$$\cos(I_2 \wedge I_p) = (330^\circ - \phi)$$

$$I_2 \rightarrow I_3 = 120^\circ$$

$$I_3 \rightarrow I_1 = 120^\circ + 120^\circ = 330^\circ - \phi$$

$$I_1 \rightarrow I_p = 90^\circ - \phi$$

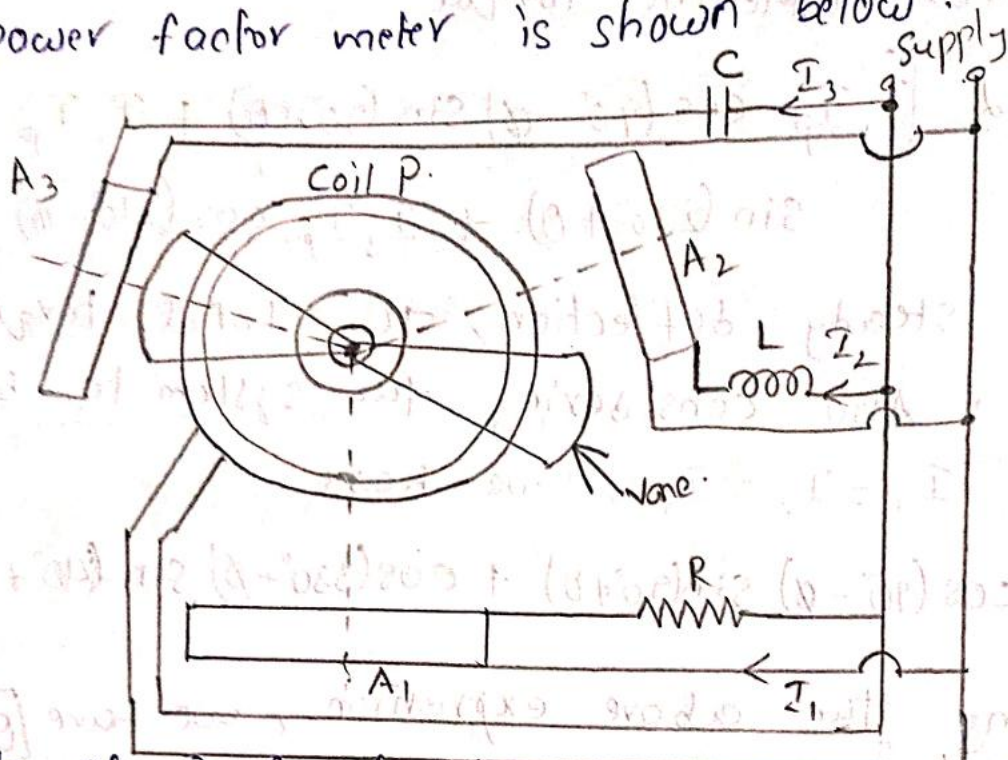
$$\cos(I_3 \wedge I_p) = (210^\circ - \phi)$$

$$I_3 \rightarrow I_1 = 120^\circ + 90^\circ = 210^\circ - \phi$$

$$I_1 \rightarrow I_p = 90^\circ - \phi$$

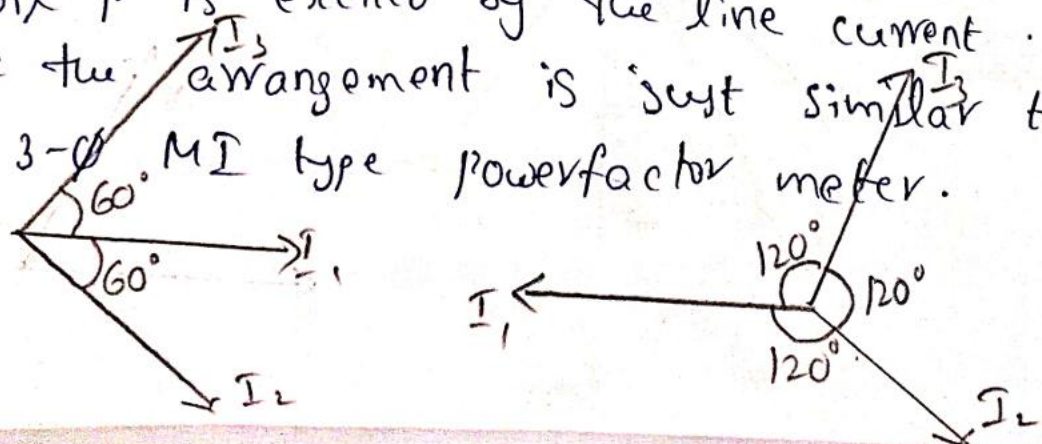
Single phase Moving Iron Power factor meter.

The schematic diagram of 1- ϕ M.I type power factor meter is shown below.



It consists of 3 fixed coils displaced by 120° in space. These coils are connected through impedances to the lines as shown in fig. The values of R, L, C are chosen so that the current in coil A_1 is in phase with the line voltage, current in coil A_2 lags by 60° & current in coil A_3 leads by 60° .

The connections of coil A_1 are reversed w.r. to the connections of other coils so as to make the currents in 3 coils displaced by 120° . Coil P is excited by the line current. The rest of the arrangement is just similar to that of 3- ϕ M.I type power factor meter.



Measurement of Power & Energy:- Energy is the total power delivered or consumed over a time interval, that is

$$\boxed{\text{Energy} = \text{Power} \times \text{time}}$$

The unit of Energy is Joule or watt second which means 1 watt hour over a time interval of one second. If the unit of time is taken as hour, energy is then expressed as watt hours. Thus one kilo watt hour energy means the consumption of 1 kW power over a time interval of one hour.

→ The meter which is used to measure the Energy is known as Energy meters.

Energy meters are classified into 3 types.

1. Electrolytic meters 2. Motor Meters 3. Clock meters.

Out of the above 3-types of Energy meters, the motor meters are commonly used meters. The motor meters are used for the measurement of energy in both DC & AC circuits. In motor meters, the moving system revolves continuously. Thus the speed of rotation is proportional to power consumed.

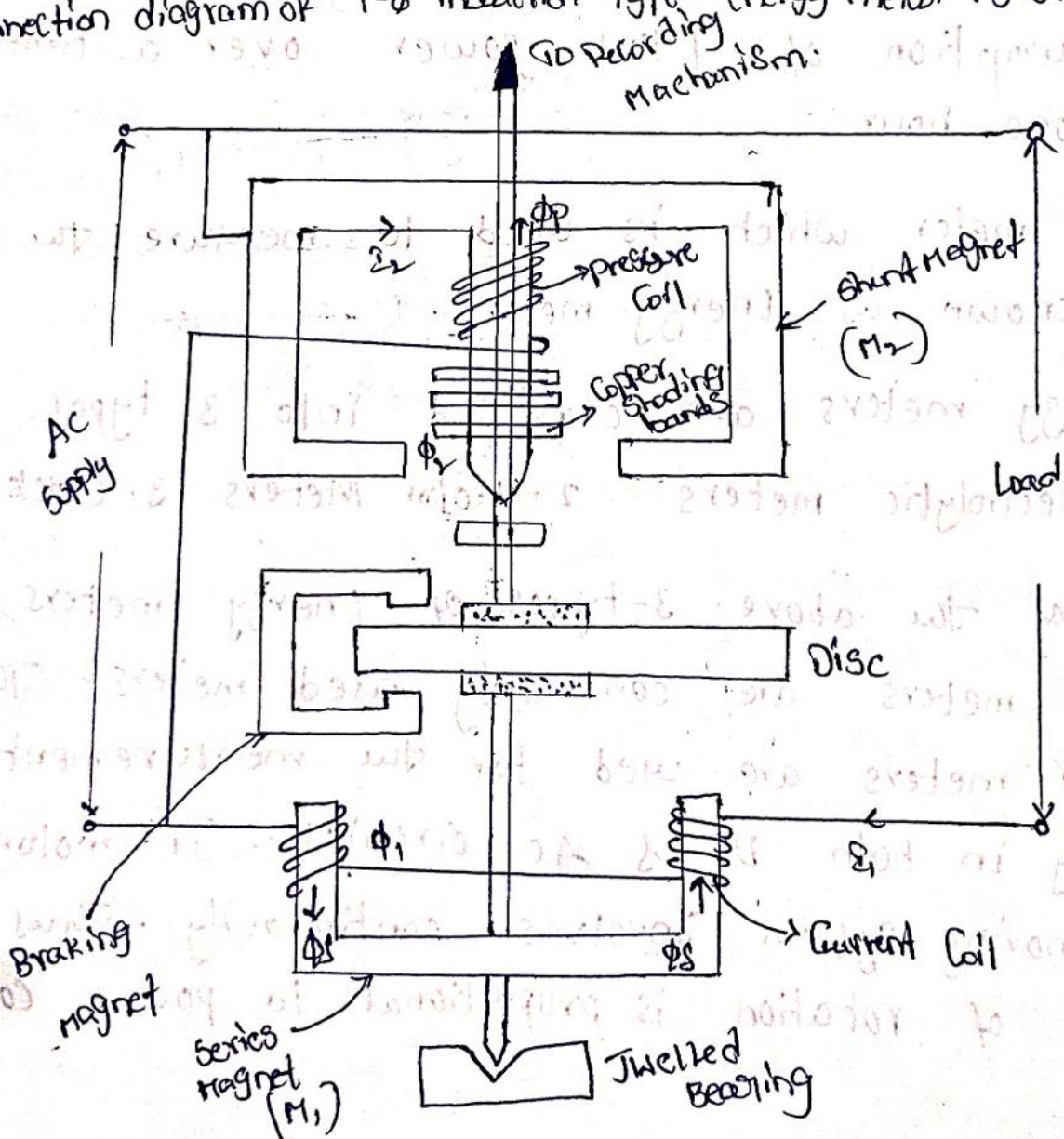
Thus the total number of revolutions made by watt-hour meter in a given interval of time is proportional to the energy supplied.

Regarding to the revolutions a term called "Meter constant" is used.

Meter constant is defined as the no. of revolutions made per kilowatt hour. The value of meter constant is usually marked on the meter.

Single phase Induction type Energy Meters

The connection diagram of 1- ϕ induction type Energy meter is shown below



There are 4 main parts of operating system.

1. Driving system
2. Moving system
3. Braking system
4. Registering system.

(1) Driving system :- The driving system of the meter consists of two electromagnets. The core of these electromagnets is made up of silicon steel laminations. The coil of one of the electromagnets is excited by the load current. This coil is called the current coil.

The coil of second electromagnet is connected across the supply & therefore, carries a current proportional to the supply voltage. This coil is called the pressure coil.

Consequently the two electromagnets are known as series & shunt magnets respectively.

To bring the flux produced by the shunt magnet exactly in quadrature (90°) with the supply voltage, copper shading bands are used on the centre limb.

(2) Moving system :- This consists of an aluminium disc mounted on a light alloy shaft. This disc is positioned in the air gap b/w series & shunt magnets. The upper

Here the rotating shaft has a small magnet at each end, where the upper magnet of the shaft

is attracted to a magnet in the upper bearing & the lower magnet of the shaft is attracted to a magnet in the lower bearing. The moving system thus floats without touching either bearing surface, thus the friction is drastically reduced.

(3) Braking system:- A permanent magnet is placed near the aluminium disc for braking mechanism. This magnet reproduces its own field. The aluminium disc moves in the field of this magnet & thus provides a braking torque.

The braking torque is adjustable, by adjusting the position of the permanent magnet.

(4) Registering system:- The function of Registering (or) counting mechanism is to record continuously a number which is proportional to revolutions made by moving system.

Imp:- The basic working of 1- ϕ Induction type Energy meter is works on two mechanisms.

(i) Mechanism of rotation of an aluminium disc which is made to rotate at a speed proportional to the power.

(ii) Mechanism of counting & displaying the amount of energy transferred.

Torque Equation for 1- ϕ Energy meter :-

One torque is produced by interaction of I_{es} & ϕ_p & another torque is produced by interaction of I_{ep} & ϕ_s .

These two torque are in opposite directions.

Let V = applied voltage.

I = Load current

ϕ = phase angle of load

I_p = pressure coil current

E_{ep} = Eddy emf induced by flux ϕ_p

I_{ep} = Eddy current " by flux ϕ_p

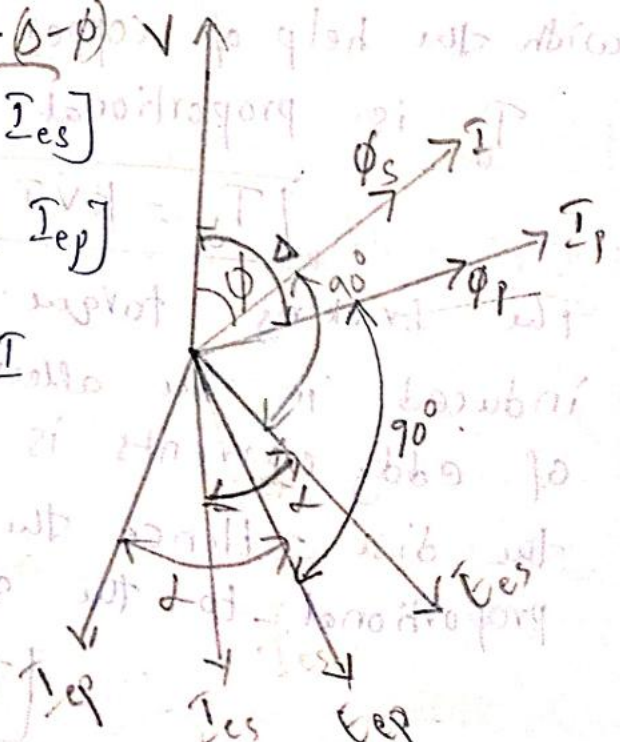
E_{es} = Eddy emf induced by flux ϕ_s

I_{es} = " current "

$$T_{d1} \propto \phi_p I_{es} \cos [\phi_p \text{ \& } I_{es}]$$

$$T_{d2} \propto \phi_s I_{ep} \cos [\phi_s \text{ \& } I_{ep}]$$

$$\phi_p \propto V \text{ \& } \phi_s \propto I$$



Δ - phase angle b/n V & I_p
 α - phase angle of eddy currents.

∴ Net driving torque $T_d \propto T_{d1} - T_{d2}$

$$T_d \propto [\phi_p I_{es} \cos(\phi_p \delta I_{es}) - \phi_s I_{ep} \cos(\phi_s \delta I_{ep})]$$

$$T_d \propto V I \cos[(90^\circ + \alpha) - (\Delta - \phi)] - V I \cos(90^\circ + \alpha + \Delta - \phi)$$

$$T_d \propto V I \left\{ \begin{array}{l} \cos(A - B) \\ \cos[(90^\circ + \alpha) - (\Delta - \phi)] - \cos[(90^\circ + \alpha) + (\Delta - \phi)] \end{array} \right\}$$

$$T_d \propto V I [\sin A \sin B + \sin A \sin B]$$

$$T_d \propto V I [\sin(90^\circ + \alpha) \sin(\Delta - \phi) + \sin(90^\circ + \alpha) \sin(\Delta - \phi)]$$

$$T_d \propto 2 V I \sin(90^\circ + \alpha) \sin(\Delta - \phi)$$

$$T_d \propto 2 V I \cos \alpha \sin(\Delta - \phi)$$

$$T_d = K V I \sin(\Delta - \phi)$$

∴ α is constant

But practically Δ is achieved to be exactly 90° with the help of copper shading bands. so that $\cos \alpha$ is also constant.

T_d is proportional to power in the circuit.

$$\boxed{T_d = k V I \cos \phi}$$

The braking torque is due to eddy currents induced in the aluminium disc. The magnitude of eddy currents is proportional to the speed N of the disc. Hence the braking torque T_b is also proportional to the speed N .

$$\therefore \boxed{T_b \propto N}$$

For the steady speed of rotation, $T_d = T_b$

$$K_1 V I \cos \phi = K_2 N$$

$$N = K V I \cos \phi = K [\text{power}]$$

Speed $\propto N \propto P \Rightarrow$ From this equation we can say that the speed of rotation is proportional to Power & this is possible at $\Delta = 90^\circ$.

$$\begin{aligned} \therefore \text{Total No. of revolutions} &= \int_0^t N dt = \int_0^t K (\text{power}) dt \\ &= K \int_0^t P dt = K \text{ Energy} \end{aligned}$$

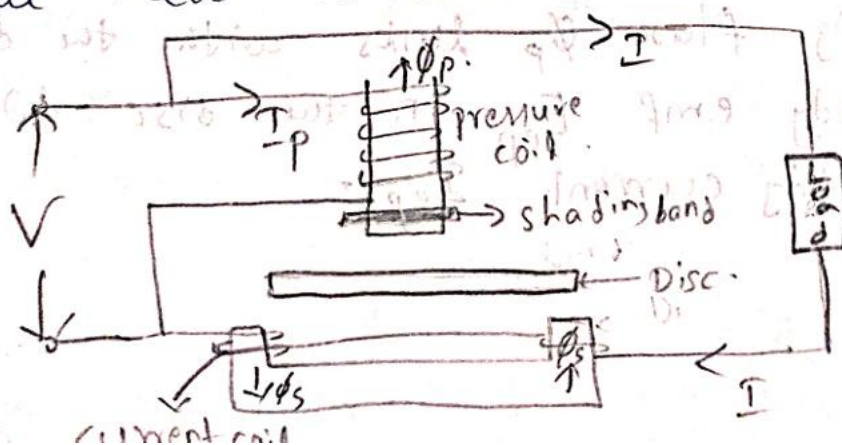
Thus the no. of revolutions of the disc in a given time is the energy consumption by the circuit in that time.

$$K = \text{Meter constant} = \frac{N}{\text{energy}} = \frac{\text{No. of revolutions}}{\text{kWh}}$$

Thus the no. of revolutions of the disc per kWh of energy consumption is called the meter constant.

Theory & operation of single phase Energy meters.

A simple functional diagram of the driving system of the meter is shown below.



The supply voltage is applied across the pressure coil. The pressure coil winding is highly inductive as it has very large no. of turns. Thus the current I_p through the pressure coil is proportional to the supply voltage & lags it by a few degrees less than 90° . This is because the winding has a small resistance.

Current I_p produces a flux ϕ_{pt} . This flux divides itself into two parts ϕ_g & ϕ_p . The major portion ϕ_g flows across the side gaps. Reluctance of this path is small.

The reluctance ~~of~~ the path of flux ϕ_p is large & hence its magnitude is small.

This flux ϕ_p goes across aluminium disc & hence is responsible for production of driving torque.

$\phi_p \rightarrow I_p \rightarrow \phi_p \propto I_p \rightarrow \phi_p \propto V$
 ϕ_p lags V by an angle $\text{slightly} < 90^\circ$.

The alternating flux ϕ_p links with the disc & induces eddy emf E_{ep} in the disc which in turn produces eddy current I_{ep} .

The load current I flows through the current coil & produces a flux ϕ_s . This flux is proportional to the load current & is in phase with it. This flux ϕ_s produces lines with the disc & induces eddy emf E_{es} in the disc which in turn produces eddy current I_{es} .

Now the eddy current I_{es} interacts with flux ϕ_p to produce a torque of eddy current T_{ep} interacts with ϕ_s to produce another torque. These two torques are in the opposite direction & the net torque is the difference b/n these two torque.

Net driving torque $T_d \propto (T_{d1} - T_{d2})$.

Due to this torque the disc will rotate.

The speed of the disc is controlled by the

C shaped magnet called braking magnet. When disc rotates in the field produced by permanent magnet, so it cuts the magnetic flux of lines, an emf is induced which in turn induces eddy current in the disc.

According to Len's law, these eddy currents oppose the cause producing them. Here the cause is

relative motion of the disc with respect to magnet.

Hence, braking torque T_b is produced.

This is proportional to speed N of disc.

$$T_b \propto N$$

By adjusting the position of this magnet, desired speed of disc is obtained.

Errors & their compensation in 1- ϕ Energy meter:

(iv) Speed error: By adjusting the braking magnet which is nearer to the aluminium disc the speed can be adjustable to a desired value.

(v) Error due to voltage variation:

$$\text{Voltage } \uparrow \rightarrow I_p \uparrow \rightarrow \phi_p \uparrow \rightarrow E_{ep} \rightarrow T_{ep}$$

Here ϕ_p interacts with I_{ep} to produce a braking torque.

This is nothing but self braking action.

Due to this self braking torque, the disc will rotate very slowly. i.e., self braking torque $>$ driving torque.

$$\text{If voltage } \downarrow \rightarrow I_p \downarrow \rightarrow \phi_p \downarrow \rightarrow E_{ep} \rightarrow T_{ep}$$

In this case self braking torque $<$ driving torque.

Due to this the disc will rotate very fastly.

Errors in single phase Energy Meter

- (1) creeping
- (2) frequency error $\pm 5\%$
- (3) Frictional error \rightarrow Cu. wire in the air gaps of shunt magnet.
- (4) speed error
- (5) Error due to voltage variation \rightarrow compensated by proper design of a
- (6) phase error
- (7) Temp. error. compensated by providing a magnetic circuit of shunt magnet.

(i) creeping Adjustment: - In some meters, the disc rotates slowly & continuously when there is no load. The rotation of disc without any current through current coil is only due to excitation of pressure coil is called creeping.

To prevent creeping, two holes are drilled in the disc 180° opposite to each other. When a hole comes under shunt magnet, the circular eddy current path gets distorted. This distortion is responsible to produce the torque in opposition to rotation of disc. This stops creeping.

[In some cases, a small piece of Iron is attached to the edge of the disc. The force of attraction of braking magnet on iron piece is responsible to prevent rotation of disc at no loads

Under no-load condition, the disc rotates slowly called creeping effect.

→ creeping effect due to excess supply voltage[!]
or due to stray magnetic fields.

(ii) frequency error - Standard frequency normally
at 50 Hz.

If frequency $\uparrow \downarrow$, reactance $\uparrow \downarrow$, current

To reduce this error, we have to maintain
the frequency within the limits ($\pm 5\%$) -

(iii) frictional error! - This error occurs due to (i)
the rapidity of different parts, or rapidity
of moving mechanism in case of Energy meters.
ie., b/n the spindle & supported jewel bearings.

To reduce this error, Cu. wire in air gaps
of shunt magnet, is placed

we can also reduce this error by connecting
a small magnet at each end of rotating shaft,
where the upper magnet of the shaft is attracted
to a magnet in the upper bearing & the lower
magnet of the shaft is attracted to a magnet
in the lower magnet bearing.

The moving thus floats without touching
either surfaces thus, the friction is drastically
reduced.

(iv) Speed error! - ~~Due to the~~ rotation the registering
or counter mechanism system will read the
wrong reading of electrical energy due to the
rotation of disc \leftarrow ~~at~~

To compensate this error we need to be a proper design of a magnetic ckt of shunt magnet.

(vi) Temperature error :- Due to heating of coils in series & shunt magnet, temperature changes. - Resistance changes

If Resistance increases the angle b/w V & ϕ_p is $\neq 90^\circ$ i.e. $\Delta \neq 90^\circ$

If $\Delta \neq 90^\circ$ then $T_d \neq V I \cos \phi$

i.e. Driving torque $T_d \neq P$

so that the speed of the disc $N \neq P$.

To compensate this error we have to provide a Temperature shunt on the brake magnet.

UNIT - IV
DC Bridges

Resistance Measurements :- The measurement of resistance is an important as the measurement of any other electrical parameter. The basic knowledge of resistance measurement is necessary to understand the working of other instruments used for the measurement of other electrical quantities.

Resistances are classified as low, medium & high resistances based on the values of the resistances.

Low resistances — of the order $\leq 1 \Omega$

Medium resistances — from 1Ω to $0.1 M\Omega$.

High resistances — $> 0.1 M\Omega$

Measurement
Low resistances :-

(1) Ammeter - voltmeter method

(2) Kelvin's double bridge method

(3) Potentiometric method.

High resistance

→ Direct deflection method

→ Loss of charge method

→ Megohm bridge

→ Meggar.

Medium resistances :-

(1) Ammeter - voltmeter method

(2) substitution method

(3) Wheatstone bridge "

(4) ohmmeter method.

Ammeter - voltmeter method ; - The

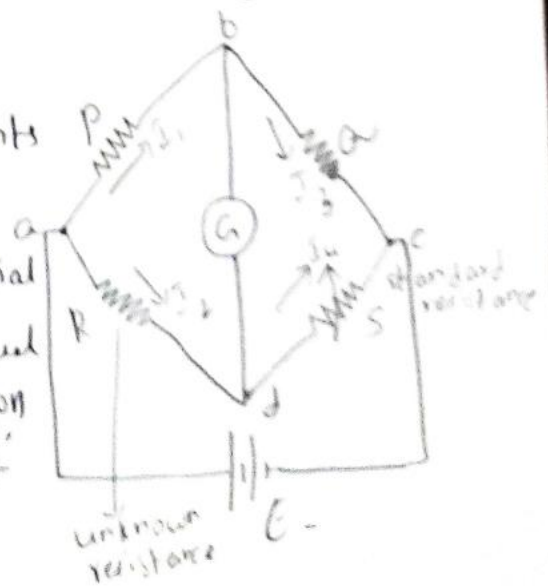
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①
Wheatstone Bridge : A very important device used in the measurement of medium resistances is the wheatstone Bridge. This has been in use longer than almost any electrical measuring instrument. It is still accurate & reliable instrument & is extensively used in the laboratory.

The basic circuit ~~diagram~~ consists of 4 resistive arms consisting of resistances P, Q, R, S together with a source of Battery & a Null Detector usually a Galvanometer.

It works on the principle of Null Deflection. That means, for a bridge is said to be balanced when there is no current passing through the Galvanometer (or) the Galvanometer carries zero current & it does not show any deflection.

Now to have zero current through Galvanometer the points "b" & "d" must be at the same potential. Thus the potential difference across "ab" must equal to the potential difference across "ad".



∴ For the bridge balance, we can write

$$I_1 P = I_2 R \rightarrow (1)$$

When the Galvanometer carries zero current, then

$$I_1 = I_3 \quad \& \quad I_2 = I_4 \rightarrow (2)$$

That means P & Q are in series & R & S are in series.

$$\therefore I_1 = I_3 = \frac{E}{P+Q} \rightarrow (3)$$

$$I_2 = I_4 = \frac{E}{R+S} \rightarrow (4)$$

substitute (3) & (4) in eqn (1) we get.

$$(1) \Rightarrow \frac{E P}{P+Q} = \frac{E R}{R+S}$$

$$\therefore \frac{P}{P+Q} = \frac{R}{R+S} \rightarrow (5)$$

$$\Rightarrow PR + PS = RP + RQ$$

$$PS = RQ$$

$$\therefore \text{The unknown resistance } \boxed{R = \frac{P}{Q} S}$$

where R = unknown resistance

S = standard resistance

P, Q are known of the Ratio arms.

Sensitivity of wheatstone bridge :- (2)

When the bridge is balanced, the current through the galvanometer is zero. But when the bridge is not balanced, the current flows through the galvanometer & causing the deflection. Now, the amount of deflection depends on the sensitivity of galvanometer.

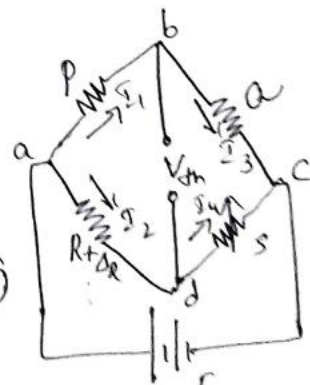
The sensitivity to unbalance can be computed by solving the bridge circuit for a small unbalance.

The unbalance in the circuit is created by changing the resistor "R" to "R + ΔR".

This unbalance leads to cause an emf 'e' to appear across the galvanometer branch. with galvanometer branch open,
Now the voltage drop across ab is,

$$E_{ab} = I_1 P = \frac{E \cdot P}{P + \alpha} \rightarrow (6)$$

$$\text{Hwy } E_{ad} = I_2 (R + \Delta R) = \frac{E (R + \Delta R)}{(R + \Delta R) + s} \rightarrow (7)$$



∴ The d/f b/n the potentials b & d is

$$e = E_{ad} - E_{ab}$$

$$= E \left[\frac{R + \Delta R}{(R + \Delta R) + s} - \frac{P}{P + \alpha} \right] \rightarrow (8)$$

Since from Eqn (3), we know that

$$\frac{P}{P+Q} = \frac{R}{R+S}$$

$$\begin{aligned} \therefore e &= E \left[\frac{R+\Delta R}{R+\Delta R+S} - \frac{R}{R+S} \right] \\ &= E \left[\frac{(R+\Delta R)(R+S) - R(R+\Delta R+S)}{(R+\Delta R+S)(R+S)} \right] \\ &= E \left[\frac{R^2 + RS + R\Delta R + \Delta RS - R^2 - R\Delta R - RS}{R^2 + RS + R\Delta R + S\Delta R + RS + S^2} \right] \\ &= E \left[\frac{S\Delta R}{R^2 + 2SR + S^2 + \Delta R(R+S)} \right] \end{aligned}$$

$$e \approx E \left[\frac{S\Delta R}{(R+S)^2 + \Delta R(R+S)} \right]$$

as $\Delta R(R+S) \ll (R+S)^2$

$$\therefore \boxed{e = \frac{ES\Delta R}{(R+S)^2}} \rightarrow \textcircled{1} \rightarrow \text{voltage across the Galvanometer during unbalance}$$

Now the sensitivity of Galvanometer can be expressed as, the amount of deflection per unit voltage across the Galvanometer.

Mathematically $\boxed{S_v = \frac{\theta}{e}} \rightarrow \textcircled{10}$

(3)

where $S_V =$ Galvanometer sensitivity (or) voltage sensitivity

from

$$S_V = \frac{\theta}{e}$$

$$\theta = S_V e = S_V \cdot \frac{ES \Delta R}{(R+S)^2} \rightarrow (11)$$

Now the overall bridge sensitivity be S_B is defined as the deflection of Galvanometer per unit fractional change in the unknown resistance.

$$\therefore S_B = \frac{\theta}{\Delta R/R}$$

$$\therefore S_B = \frac{S_V \frac{ES \Delta R}{(R+S)^2}}{\Delta R/R} \quad \therefore \text{from eqn (11)}$$

$$S_B = \frac{S_V \cdot ESR}{(R+S)^2} \rightarrow (12)$$

\therefore From the above equation it is clear that the overall bridge sensitivity depends on voltage sensitivity of galvanometer, bridge voltage & bridge parameters.

Now rearranging the above equation,

$$S_B = \frac{S_V E S R}{(R+S)^2}$$

$$= \frac{S_V E}{(R+S)^2} = \frac{S_V E}{\frac{R^2 + S^2 + 2RS}{SR}}$$

$$\therefore S_B = \frac{S_V E \cdot \frac{SR}{R+S+2}}{SR} \rightarrow \textcircled{13}$$

It is clear that Maximum ^{bridge} sensitivity occurs

when $\frac{R}{S} = 1$.

As the ratio becomes either larger (or) smaller than the overall bridge sensitivity decreases.

Note:- For a bridge with equal arms,

$P = Q = R = S$
then the sensitivity

$$S_B = \frac{S_V E}{4}$$

Galvanometer current:- The overall bridge sensitivity is not only depends upon the voltage sensitivity of galvanometer, but also on the current sensitivity of galvanometer.

Now the current passing through galvanometer under unbalanced bridge condition, $I_g = \frac{E_0}{R_0 + G} \rightarrow \textcircled{14}$.

can
(ie) The
b/h

where $E_0 =$ Thevenin's voltage of the bridge. (6)

$R_0 =$ Thevenin's resistance

$G =$ Galvanometer resistance.

~~Apply~~ The current through the galvanometer can be found by finding Thevenin's equivalent ckt.

(ie) Thevenin's voltage & Thevenin's equivalent resistance.

The Thevenin's or open ckt voltage appearing b/w terminals b & d with is,

$$\begin{aligned} E_0 &= E_{ad} - E_{ab} \\ &= I_2(R + \Delta R) - I_1 P \quad \therefore \text{from eqn (6) \& (7)} \\ &= \frac{E(R + \Delta R)}{R + \Delta R + S} - \frac{E P}{P + \omega} \\ &= E \left[\frac{R + \Delta R}{R + \Delta R + S} - \frac{P}{P + \omega} \right] \end{aligned}$$

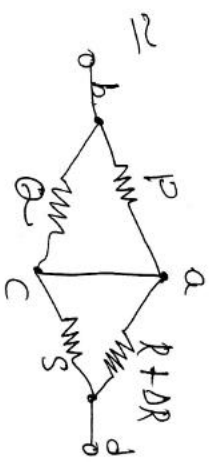
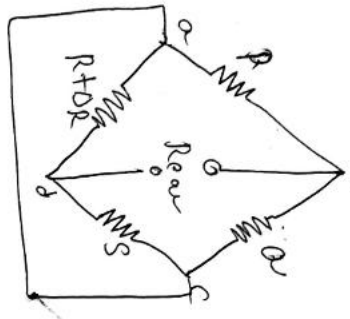
For a bridge with equal arms, $P = \omega = R = S$

$$\begin{aligned} \therefore E_0 &= E \left[\frac{R + \Delta R}{2R + \Delta R} - \frac{R}{R + R} \right] \\ &= E \left[\frac{R + \Delta R}{2R + \Delta R} - \frac{1}{2} \right] \\ &= E \left[\frac{2R + 2\Delta R - 2R - \Delta R}{2(2R + \Delta R)} \right] \end{aligned}$$

$$E_0 = E \left[\frac{\Delta R}{4R + 2\Delta R} \right] \quad \text{when } \Delta R \ll R$$

$$\therefore \boxed{E_0 = E \left(\frac{\Delta R}{4R} \right)} \quad \rightarrow (14)$$

Try the Thevenin's equivalent resistance can be calculated b/h the terminals b & d. The circuit will be



$$\therefore R_0 = P/\omega + (R+DR)/\omega$$

$$= \frac{P\omega}{P+\omega} + \frac{(R+DR)S}{R+DR+S}$$

$$R_0 = \frac{P\omega}{P+\omega} + \frac{RS}{R+S}$$

$\therefore DR \ll R$ & S so neglects.

For bridge with equal arms, $P=Q=R=S$

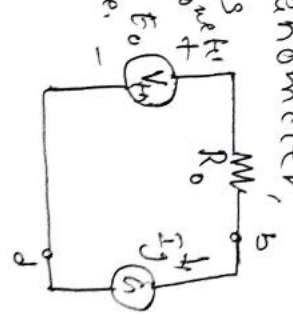
$$\therefore \boxed{R_0 = R} \rightarrow (15)$$

\therefore current through the Galvanometer,

$$I_g = \frac{E_0}{R_0 + G}$$

where G is the galvanometer resistance.

$$I_g = \frac{E \left(\frac{DR}{R} \right)}{R + G}$$



Now, the deflection of Galvanometer is given by

$$\theta = S_{vx} \frac{E S DR}{(R+S)^2}$$

But $S_V = \frac{S_i}{R+G} \rightarrow$ (16) where $S_i =$ Current sensitivity of galvanometer (3)

$$\therefore \theta = \frac{S_i E S DR}{(R+G)^2 (R_0+G)}$$

\therefore Overall bridge sensitivity : $S_B = \frac{\theta}{dR/R}$

$$S_B = \frac{S_i E S DR}{(R_0+G)(R+G)^2} \cdot \frac{R}{dR/R}$$

$$\therefore S_B = \frac{S_i E S R}{(R_0+G)(R+G)^2}$$

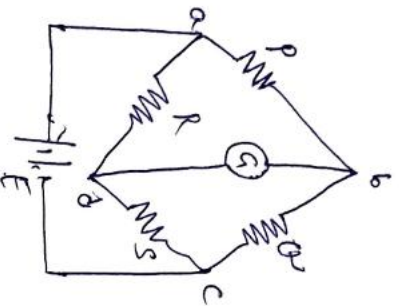
Advantages of wheatstone bridge:-

- (1) The results are not dependent on calibration & characteristics of galvanometer, as it works on null deflection method.
- (2) Due to null deflection method used, the accuracy & sensitivity is higher than the direct deflection meters.
- (3) The source emf & source fluctuations don't affect the bridge balance.

Applications of wheatstone bridge :-

- (1) used basically for measurement of medium resistances ranging 1Ω to $0.1 M\Omega$.
- (2) used to measure the resistance of motor winding relay coils etc.
- (3) used by the telephone companies to locate the cable fault.

Kelvin's double bridge method :-
 It is the modified bridge of wheatstone bridge.



$$\frac{P}{Q} = \frac{R}{S}$$

$$R = \frac{P}{Q} \cdot S$$

at point 'm' \rightarrow s + x
 \rightarrow $S \cdot R = \frac{P}{Q} (S + x)$
 (low resist)

at point 'n' \rightarrow R + y
 $R + y = \frac{P}{Q} \cdot S$
 (high resist)

$$R + y = \frac{P}{Q} (S + x_2)$$

$$\frac{y_1}{y_1 + x_2} = \frac{P}{P + Q}$$

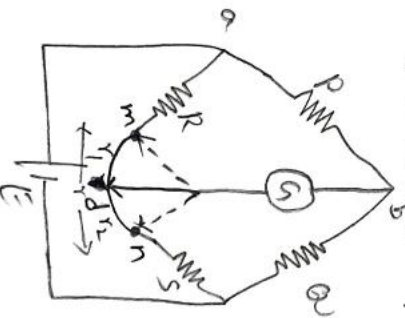
$$y_1 = \frac{P}{P + Q} (y_1 + x_2)$$

$$\frac{y_2}{y_1 + x_2} = \frac{Q}{P + Q}$$

$$y_2 = \frac{Q}{P + Q} (y_1 + x_2)$$

$$\therefore \left[R = \frac{P}{Q} \cdot S \right]$$

All the resistors are connected to each other for the connecting leads. Here the unknown resistance R is connected to S & that connecting leads also have some resistance. And if we want to measure the resistance of that connecting leads we can use kelvin's double bridge because the resistance of connecting leads is very small.



Here connecting lead of resistance 'r' is used to connect the unknown resistance R to the S.

r is divided into y_1 & y_2 such that

$$\frac{y_1}{y_2} = \frac{P}{Q}$$

If we call 'r' as $\frac{r}{2}$

$$\frac{y_1 + y_2}{y_2} = \frac{P + Q}{Q} \quad \text{by } \frac{r}{2} \cdot \frac{y_1}{y_2} = \frac{P}{Q}$$

$$\frac{y_1 + y_2}{y_1} = \frac{P + Q}{P}$$

$$R + \frac{P}{Q} \cdot x = \frac{P}{Q} (S + Q \cdot x)$$

$$R + \frac{P}{Q} \cdot y = \frac{P}{Q} \cdot S + \frac{P}{Q} \cdot Q \cdot y$$

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It is clear that $R = \frac{P}{Q} S$.

If we connect the connecting lead resistor in such a way that there will be no effect of 'r' in the circuit parameters.

But it is impossible to identify that intermediate point 'd' to divide the 'r' into r_1 & r_2 & the ratio of $\frac{r_1}{r_2} = \frac{P}{Q}$.

So instead of searching that intermediate point 'd' we go for another modified bridge wheatstone that is kelvin's double bridge.

ie, Instead of connecting this connecting lead 'r' we use for another set of arms.

Kelvin's Double Bridge

It has a second set of ratio arms, hence the name kelvin double bridge.

First set of ratio arms P & Q
Second " " " " P & Q

galvanometer is connected b/w the two arms.

principle: ratio of $\frac{P}{Q}$ is made equal to $\frac{P}{Q}$ under balance conditions there is no current through the galvanometer which means $V.D$ b/w $E_{ab} = E_{amd}$
ie. $E_{ab} = E_{amd}$

Ans V-D-R $E_{ab} = E_{amd}$

$$E_{ab} = \frac{P}{P+Q} E_{ac}$$

$$E_{ac} = I \left[R + S + \frac{(P+Q)Y}{P+Q+Y} \right]$$

$$E_{ab} = \frac{P}{P+Q} \cdot I \left[\frac{R+S+(P+Q)Y}{P+Q+Y} \right]$$

$$E_{amd} = I \left\{ R + \frac{P}{P+Q} \frac{(P+Q) \cdot Y}{P+Q+Y} \right\}$$

$$E_{amd} = I \left(R + \frac{PY}{P+Q+Y} \right)$$

$$E_{ab} = E_{amd}$$

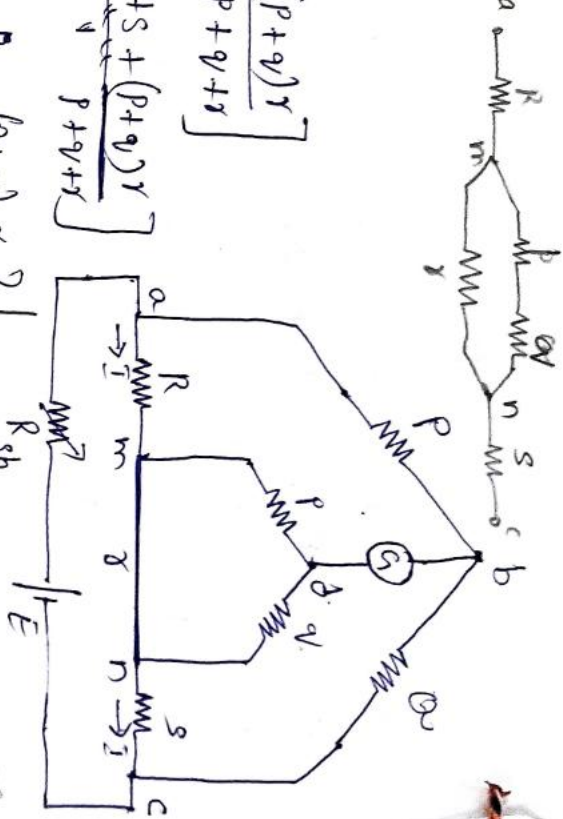
$$\frac{P}{P+Q} \cdot I \left[\frac{R+S+(P+Q)Y}{P+Q+Y} \right] = I \left(R + \frac{PY}{P+Q+Y} \right)$$

$$\frac{R}{P+Q} \left[R+S + \frac{(P+Q)Y}{P+Q+Y} \right] = \left(R + \frac{PY}{P+Q+Y} \right)$$

$$R \left[\frac{1-(P+Q)}{P} \right] = \frac{PY}{P+Q+Y} \times \frac{P+Q}{P} - S - \frac{(P+Q)Y}{P+Q+Y}$$

$$R \left[\frac{P-Q}{P} \right] = \frac{PY}{P+Q+Y} \left[\frac{P+Q}{P} - 1 \right] - S - \frac{QY}{P+Q+Y}$$

$$R = \frac{P}{Q} \cdot S + \frac{QY}{P+Q+Y} \left[\frac{P}{Q} - \frac{P}{Q} \right]$$



$E_{amd} = E_{am} + E_{md}$
 $E_{md} = V_{mn} \times \frac{P}{(P+Q)}$
 $V_{mn} = I \cdot \frac{(P+Q)Y}{P+Q+Y}$
 $E_{md} = \frac{P}{P+Q} \times I \cdot \frac{(P+Q)Y}{P+Q+Y}$

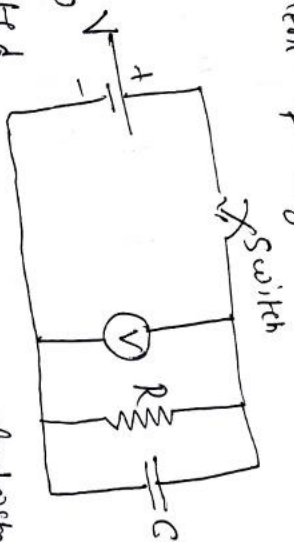
Measurement of High Resistances :-

Loss of charge Method :- This method used to measure insulation resistance of cables or leakage resistance of capacitors or cables.

Fig shows the measurement of high resistance by

loss of charge method.

In this method, the unknown resistance R' to be measured is connected in parallel with a capacitor C' and an electrostatic voltmeter.



From the figure when the switch is closed, then the capacitor starts charging to some suitable voltage & when the switch is in open condition the capacitor starts discharging through the resistance R' .

The voltage across the capacitor at any instant of time is given by

$$V_c = V e^{(-t/Rc)}$$
$$\frac{V_c}{V} = e^{-t/Rc}$$
$$\frac{V_c}{V} = e^{-t/Rc}$$

or
V

$$\ln e\left(\frac{V}{V_c}\right) = \frac{t}{RC}$$

$$\frac{t}{RC} = \ln e \frac{V}{V_c}$$

$$RC = \frac{t}{\ln e\left(\frac{V}{V_c}\right)}$$

$$R = \frac{t}{C \ln e\left(\frac{V}{V_c}\right)}$$

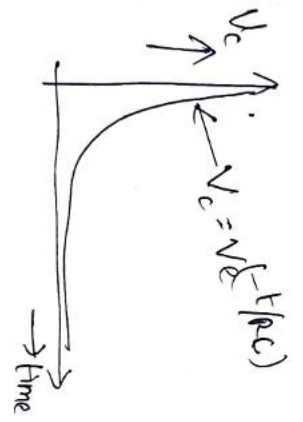
$$R = \frac{t (0.4343)}{C \log_{10}\left(\frac{V}{V_c}\right)}$$

$$\begin{aligned} \therefore \ln e^{10} &= 2.3025 \\ \frac{\ln e^{10}}{2.3025} &= 1 \\ 0.4343 \ln e^{10} &= 1 \\ \ln e^{10} &= \frac{1}{0.4343} \end{aligned}$$

The b when the resistance of high This met a high



The variation of voltage (V_c) with respect to time is given below.



If the resistance 'R' is large than the time taken by the capacitor for discharging is more. In such cases, the process becomes time consuming.

From eqn (1) the calculated value of 'R' is not the actual value because there the leakage resistance of capacitor is not considered. To know the exact value, practically leakage resistance have been considered.

Now
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3025

$$10 = 1$$

$$= \log_{10} 10$$

$$\frac{10}{10} = 1$$

$$= 1343$$

time

R' is

faster

discharging

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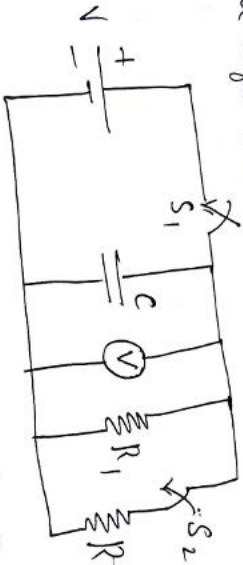
me of

factor

resistor

The below fig (b) shows the law of charge method when the effects of leakage resistance & voltmeter resistance are considered.

This method can be effectively used for measurement of high resistances when a capacitor provided with a high leakage resistance R_1 .



Here R_1 = leakage resistance of capacitor
 R = unknown resistance value.

Now, initially the switch S_2 is in open & switch S_1 is closed then a capacitor charges to

some suitable voltage, when S_1 is closed

Then again switch S_1 is open & S_2 is closed then the capacitor starts discharging through the

resistors R & R_1 , which have been in parallel combination.

Let R' be the equivalent resistance of R & R_1 then

$$R' = \frac{R R_1}{R + R_1} \quad \rightarrow \textcircled{2}$$

Whenever the equivalent resistance is calculated then the discharge equation of capacitor will be

$$R' = \frac{0.368 V}{C \log_{10} \left(\frac{V}{V_c} \right)} \quad \rightarrow \textcircled{3}$$

Now to find the unknown resistance ' R ', the resistance " R " is disconnected from the ckt & the test is repeated. Now the capacitor discharges through only R_1 .

\therefore The value of R_1 will be obtained from the second test. When this value of R_1 is substituted in eqn ②, the unknown resistance will be calculated.

$$\textcircled{2} \Rightarrow R' = \frac{RR_1}{R+R_1}$$

$$RR' + R'R_1 = RR_1$$

$$R'R_1 = R(R_1 - R')$$

$$R = \frac{R'R_1}{R_1 - R'}$$

Measurement of Inductance

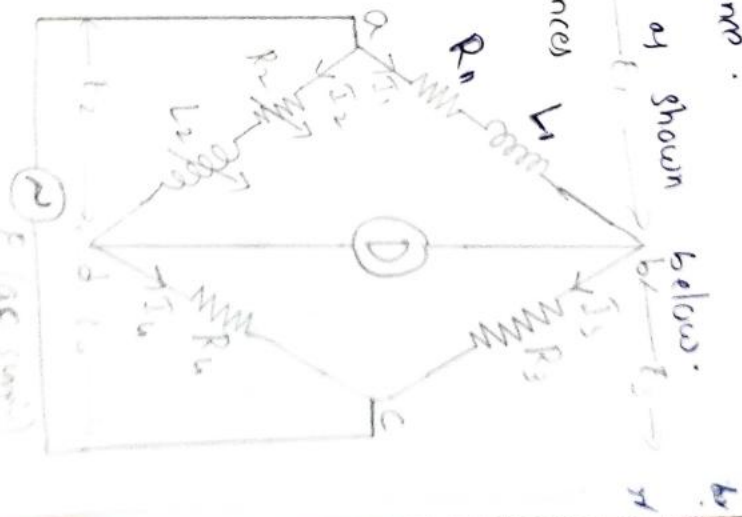
(i) Maxwell Inductance Bridge: This bridge circuit measures an inductance by comparing it with a variable standard value of self inductance.

The circuit & phasor diagrams are shown below.

- It has four Arms → four Impedances
- a Detector
- gives AC power supply

Three known
one unknown

where $L_1 =$ unknown Inductance
 $L_2 =$ Variable " standard Inductance
 $R_2 =$ " " Resistance
 $R_3, R_4 =$ Non inductive Resistances.



For a bridge to be balanced, the voltage drop across ab is equal to V-D across ad.

Now from the concept of General equation of AC Bridge.

For the balanced condition,

$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_1 + j\omega L_1) R_4 = [(R_2 + j\omega L_2) + j\omega L_1] R_3$$

$$R_1 R_4 + j\omega L_1 R_4 = (R_2 + j\omega L_2) R_3 + j\omega L_1 R_3$$

Now by equating Real parts & imaginary parts on b.s, the unknown parameters will be calculated.

$$R_1 R_4 = (R_2 + \omega^2 L_2 L_1) R_3$$

$$j\omega L_1 R_4 = j\omega L_2 R_3$$

$$L_1 = L_2 \frac{R_3}{R_4}$$

Phasor diagram :-

$Z_1 = Z_3, Z_2 = Z_4$

$E_1 = E_2$

$E_3 = E_4$

$E_1 = I_1 R_1 + j I_1 \omega L_1$

$E_2 = I_2 R_2 + j I_2 \omega L_2 +$

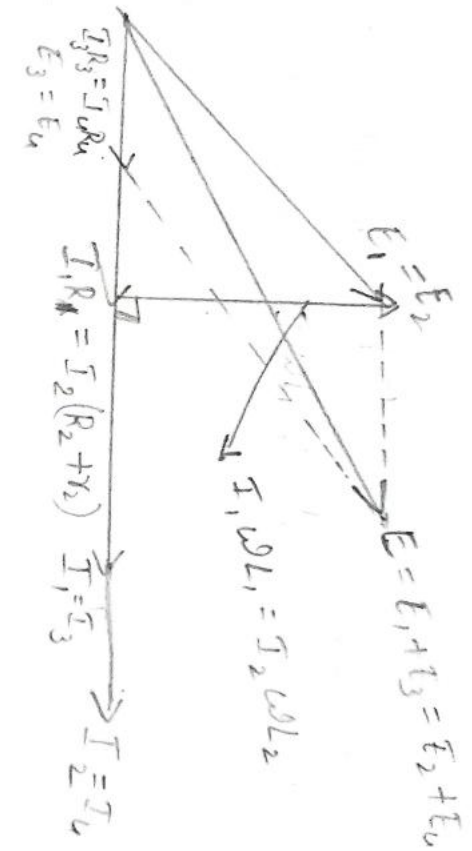
$I_2 \omega L_1$

$E_2 = I_2 (R_2 + j I_2 \omega L_2 +$

$E_1 = I_1 R_1 + j I_1 \omega L_1$

$E = E_1 + E_3$
 $= E_2 + E_4$

Now



Resistors R_3 & R_4 are normally a selection of values from 10, 100, 1000 etc, R_2 is decade resistor

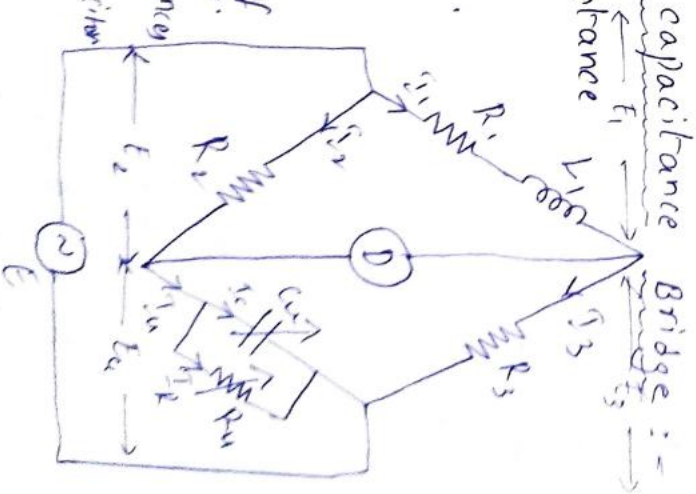
(ii) Maxwell's Inductance - capacitance Bridge :-

In this bridge the Inductance is measured by comparison with a standard capacitance.

The ckt diagram is shown.

where L_1 = unknown inductance
 R_1 = effective resistance of inductor.
 R_2, R_3, R_4 = Non-inductive resistances
 C_u = Variable standard capacitance

$Z_1 = R_1 + j \omega L_1$, $Z_u = R_u // C_u$
 $Z_2 = R_2$
 $Z_3 = R_3$, $Z_4 = \frac{R_u}{1 + j \omega C_u R_u}$



Box.

E_u

Now from the bridge balance Equation

$$Z_1 Z_u = Z_2 Z_3$$

$$(R_1 + j\omega L_1) \left(\frac{R_u}{1 + j\omega C_u R_u} \right) = R_2 R_3$$

$$R_1 R_u + j\omega L_1 R_u = R_2 R_3 [1 + j\omega C_u R_u]$$

$$R_1 R_u + j\omega L_1 R_u = R_2 R_3 + j\omega C_u R_2 R_3 R_u$$

Separating the real terms & imaginary terms,

$$R_1 R_u = R_2 R_3$$

$$j\omega L_1 R_u = j\omega C_u R_2 R_3 R_u$$

$$R_1 = \frac{R_2 R_3}{R_u}$$

$$L_1 = R_2 R_3 C_u$$

Unknown Impedance, $Z_1 = R_1 + j\omega L_1$,

$$Z_1 = \frac{R_2 R_3}{R_u} + j\omega R_2 R_3 C_u$$

Quality factor of the coil (Q) = $\frac{\omega L}{R}$

∴ The ratio of unknown inductive reactance to unknown resistance is known as Quality Factor (or) 'Q' factor.

The expression for Q factor = $\frac{\omega L_1}{R_1}$

$$Q = \frac{\omega R_2 R_3 C_u}{\frac{R_2 R_3}{R_u}} = \omega C_u R_u$$

I_u

are
x.
→



Under balanced condition,

$$I_1 = I_3 \quad \& \quad I_2 = I_u + I_c$$

$$\therefore I_r = I_c$$

$$E_1 = E_2 \quad \& \quad \text{also}$$

when the voltage drop across bc equals to voltage drop across ce, then bridge will be balanced,

$$\therefore E_3 = I_3 R_3 = I_c \left[\frac{1}{j\omega C} \right]$$

$$I_1 R_3 = I_c \left[\frac{1}{j\omega C} \right]$$

$$I_c = I_1 R_3 j\omega C \rightarrow (1)$$

Other balance equations are

voltage drop across 'ab' is equal to the voltage drop across ade.

$$\text{Voltage drop across ab is } E_1 = I_1 (R_1 + r_1 + j\omega L_1)$$

$$\text{" " " ade is } E_2 = I_2 R_2 + I_c r$$

$$\Rightarrow I_1 (R_1 + r_1 + j\omega L_1) = I_2 R_2 + I_c r \rightarrow (2)$$

Also at balanced condition, the arms, de & ce are in series & the sum of voltage drops across de & ce is equal to V.D across cd.

$$I_c r + I_c \frac{1}{j\omega C} = I_u R_u$$

$$I_c \left[r + \frac{1}{j\omega C} \right] = I_u R_u$$

$$I_c \{$$

Now subst

$$(2) \Rightarrow I_1$$

$$I_1 \left[\frac{1}{j\omega C} \right]$$

||ry subst

$$(3) \Rightarrow I_c$$

$$I_1$$

$$I_1 \{$$

$$I_1 \left[\frac{1}{j\omega C} \right]$$

$$\left[\frac{1}{j\omega C} \right]$$

Now

$$(4) \Rightarrow I_1$$

$$\Rightarrow R_1 + r_1 +$$

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(1) 7

v

I_c

$$I_c \left[r + \frac{1}{j\omega c} \right] = (I_2 - I_c) R_u \rightarrow (3) \quad \begin{aligned} \therefore I_2 &= I_u + I_c \\ I_u &= I_2 - I_c \end{aligned}$$

Now substitute eqn (1) in (2).

oltage
ced,

$$(2) \Rightarrow I_1 (R_1 + r_1 + j\omega L_1) = I_2 R_2 + I_c r$$

$$I_1 (R_1 + r_1 + j\omega L_1) = I_2 R_2 + I_1 R_3 j\omega c r$$

$$I_1 [R_1 + r_1 + j\omega L_1 - R_3 j\omega c r] = I_2 R_2 \rightarrow (4)$$

Now substitute eqn (1) in (3).

$$(3) \Rightarrow I_c \left[r + \frac{1}{j\omega c} \right] = (I_2 - I_c) R_u$$

age drop

$$I_1 R_3 j\omega c \left[r + \frac{1}{j\omega c} \right] = [I_2 - j\omega c R_3 I_1] R_u$$

$$I_1 R_3 j\omega c r + I_1 R_3 = I_2 R_u - j\omega c R_3 R_u I_1$$

$$I_1 (R_3 j\omega c r + R_3 + j\omega c R_3 R_u) = I_2 R_u$$

$$I_2 = \frac{I_1}{R_u} [R_3 j\omega c r + R_3 + j\omega c R_3 R_u] \rightarrow (5)$$

Now eqn (5) in eqn (4).

$$(4) \Rightarrow I_1 (R_1 + r_1 + j\omega L_1 - R_3 j\omega c r) = \frac{I_1}{R_u} [R_3 j\omega c r + R_3 + j\omega c R_3 R_u] R_2$$

$$\Rightarrow R_1 + r_1 + j\omega L_1 - R_3 j\omega c r = \frac{R_2 R_3 j\omega c r}{R_u} + \frac{R_2 R_3}{R_u} + j\omega c R_2 R_3$$

s ce
s
ed.

Now equating Real & Imaginary terms we get, Advantage

$$R_1 + r_1 = \frac{R_2 R_3}{R_4}$$

$$R_1 = \frac{R_2 R_3}{R_4} - r_1$$

$$\therefore \boxed{R_1 = \frac{R_2 R_3}{R_4} - r_1}$$

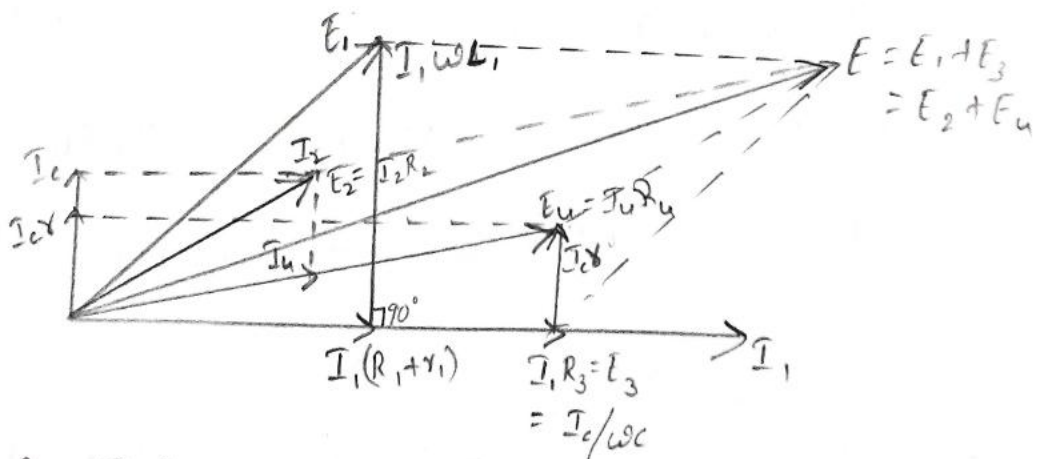
$$L_1 - CR_3 = CR_2 R_3 + \frac{CR_2 R_3 r_1}{R_4}$$

$$L_1 = CR_3 + CR_2 R_3 + \frac{CR_2 R_3 r_1}{R_4}$$

$$L_1 = CR_3 \left[R_2 + r_1 + \frac{R_2 r_1}{R_4} \right]$$

$$\boxed{L_1 = \frac{CR_3}{R_4} \left[R_2 R_4 + r_1 (R_2 + R_4) \right]}$$

Phasor diagram :



$$E_3 = I_3 R_3 = I_2 R_2$$

$$= \frac{I_c}{j\omega C}$$

$$E_u = I_u R_u = I_c r_1 + \frac{I_c}{\omega C}$$

$$I_2 = I_c + I_u$$

Advantages:-

- (1) Other bridges require variable capacitor but a fixed capacitor is used for Anderson's bridge.
- (2) The bridge is easy to balance when compared to the Maxwell's bridge.
- (3) This bridge may be used for determination of capacitance in terms of inductance.

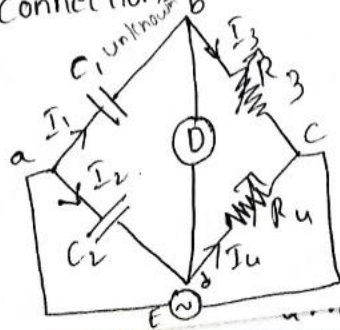
Disadvantages:-

- (1) It is more complex when compared to other bridges.
- (2) Balance Equations are also more complicated to derive.
- (3) Uses more number of components.

Measurement of capacitance :-

Desauty's Bridge:- This bridge is used to determine the unknown capacitance. It is the simplest method of comparing two capacitances.

The connections for circuit diagram & the phasor diagram are shown below.



C_1, C_2 are the loss less capacitors i.e., which are free from dielectric losses.
 R_3, R_4 are non inductive resistance standard variable resistances.
 bc & dc.

Now equating Real & Imag
 $R_1 \perp R_4$

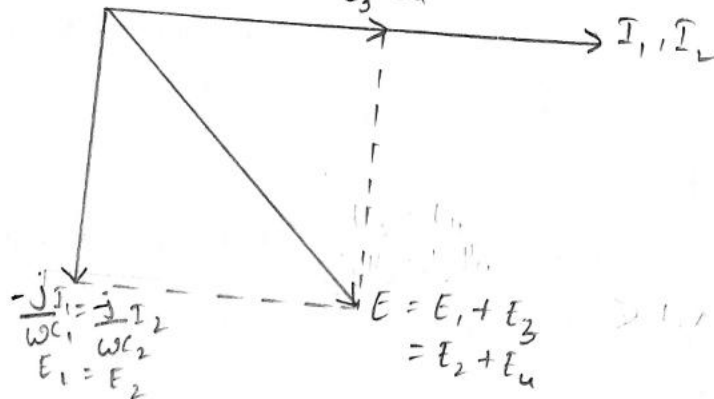
Under balanced condition, $I_1 = I_3$ & $I_u = I_2$

$$E_1 = E_2 \quad \& \quad E_3 = E_4$$

$$\frac{-jI_1}{\omega C_1} = \frac{-jI_2}{\omega C_2} \quad \& \quad I_1 R_3 = I_2 R_4$$

$$I_1 R_3 = I_2 R_4$$

$$E_3 = E_4$$



The bridge is said to be balanced when there is no deflection in the detector.
 Now the balancing equation,

$$Z_1 Z_4 = Z_2 Z_3$$

$$\Rightarrow \frac{1}{j\omega C_1} R_4 = \frac{1}{j\omega C_2} R_3$$

$$\frac{R_4}{C_1} = \frac{R_3}{C_2} \Rightarrow \boxed{C_1 = C_2 \left(\frac{R_4}{R_3} \right)}$$

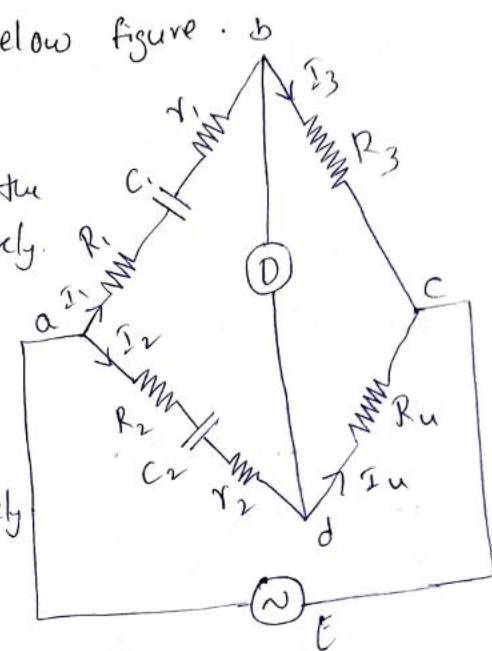
Here the unknown capacitance is obtained in terms of known capacitance (C_2) by varying R_3 & R_4 .

Here it is impossible to obtain balance if both capacitors are not free from dielectric loss.

This bridge can be used only for lossless capacitors.

In order to ^{make} measurements on imperfect capacitors (ie, capacitors having dielectric loss), the bridge is modified as shown in the below figure.

The resistors R_1 & R_2 are connected in series with the capacitors C_1, C_2 respectively. r_1, r_2 are resistances representing the loss components of the two capacitors C_1, C_2 respectively.



At balance,

$$Z_1 Z_u = Z_2 Z_3$$

$$\left(R_1 + r_1 + \frac{1}{j\omega C_1} \right) R_u = \left(R_2 + r_2 + \frac{1}{j\omega C_2} \right) R_3$$

$$\left(R_1 + r_1 \right) R_u + \left(\frac{1}{j\omega C_1} \right) R_u = \left(R_2 + r_2 \right) R_3 + \left(\frac{1}{j\omega C_2} \right) R_3$$

Now equating the real & imaginary parts,

we get

$$\left(R_1 + r_1 \right) R_u = \left(R_2 + r_2 \right) R_3$$

$$\boxed{\frac{R_u}{R_3} = \frac{R_2 + r_2}{R_1 + r_1}} \rightarrow (1)$$

$$\frac{R_u}{j\omega C_1} = \frac{R_3}{j\omega C_2}$$

$$\boxed{\frac{R_u}{R_3} = \frac{C_1}{C_2}} \rightarrow (2)$$

From the above ^{two} equations, the balance may we can say

$$\boxed{\frac{C_1}{C_2} = \frac{R_u}{R_3} = \frac{R_2 + r_2}{R_1 + r_1}} \rightarrow (A)$$

arms bc & dc - gap across

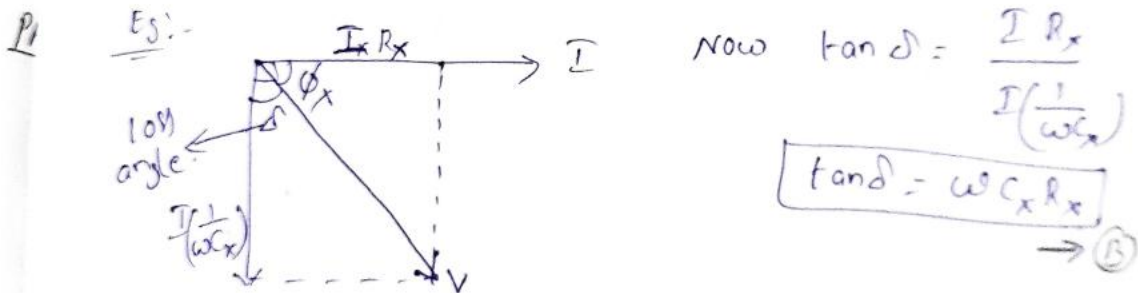
ifications
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 of

Now equating ...

From the above equation, the balance may be obtained by varying the resistors, R_1, R_2, R_3 & R_4 .

Loss Angle:- For a series combination of R_x & C_x , the angle between the voltage across the series combination & the voltage across the capacitor C_x is known as loss angle (δ).

The phasor diagram is shown below.



The dissipation factor for series $R_x - C_x$ circuit is defined as the cot of the phase angle.

From the above phasor diagram,

$$D = \cot \phi = \frac{1}{\tan \phi} = \frac{1}{\frac{I \left(\frac{1}{\omega C_x} \right)}{I R_x}} = \omega C_x R_x$$

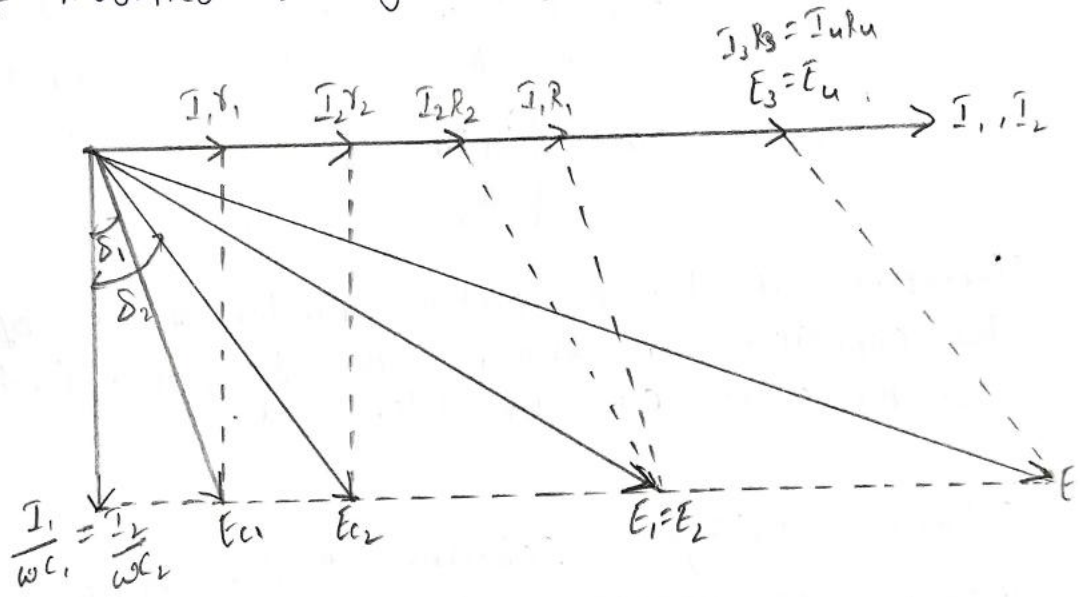
$D = \omega C_x R_x \rightarrow \textcircled{C}$

From the equation B & C we can say

$$D = \tan \delta$$

This bridge can be used ...

Now from the modified circuit diagram of Desauty's Bridge, we have two R-C series combinations. Therefore two loss angles will be obtained. This can be evaluated with the help of phasor diagram of modified Desauty's Bridge as shown in below.



From the above phasor diagram of a bridge under the balance conditions, the angles δ_1 & δ_2 are loss angles of capacitors C_1 & C_2 respectively.

Dissipation factors for the capacitors are

$$\left. \begin{aligned} D_1 &= \tan \delta_1 = \omega C_1 R_1 \\ D_2 &= \tan \delta_2 = \omega C_2 R_2 \end{aligned} \right\} \rightarrow \text{---} \textcircled{A}$$

From eqn (A) $\Rightarrow \frac{C_1}{C_2} = \frac{R_2 + R_1}{R_1 + R_2}$

$$\begin{aligned} C_1 R_1 + C_1 R_2 &= C_2 R_2 + C_2 R_1 \\ C_1 R_1 - C_2 R_2 &= C_2 R_1 - C_1 R_2 \end{aligned}$$

δR_1
 δC_1
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r

)
B)

from
of Detector
capacitor
voltage
drop
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dc

Now

$$\omega C_1 R_1 - \omega C_2 R_2 = \omega C_2 r_2 - \omega C_1 r_1$$
$$= D_2 - D_1 \quad \therefore \text{from eqn (1)}$$

$$D_2 - D_1 = \omega C_1 R_1 - \omega C_2 R_2$$
$$= \omega [C_1 R_1 - C_2 R_2]$$
$$= \omega \left[C_2 \frac{R_1 R_4}{R_3} - C_2 R_2 \right]$$

$$\therefore (2) \Rightarrow \frac{C_1}{C_2} = \frac{R_4}{R_3}$$
$$C_1 = C_2 \frac{R_4}{R_3}$$

$$\therefore D_2 - D_1 = \omega C_2 \left[\frac{R_1 R_4}{R_3} - R_2 \right]$$

Therefore if the dissipation factor of one of the capacitors is known, the dissipation factor for the other can be determined.

Schering Bridge:- Schering Bridge is widely used for capacitance & dissipation factor measurements. In fact, Schering bridge is one of the most important of the AC bridges. It is extensively used in the measurement of capacitance in general & in particular in the measurement of ~~the properties~~ ^{dissipation factor} of insulators, capacitor bushings, insulating oil & other insulating materials ^{which has very small capacitance} (100 pF to 1 μF) with $\pm 2\%$ accuracy. This bridge is particularly suitable for small capacitances & is usually supplied from high frequency (or) a high voltage source.

This bridge can be used

The connections & phasor diagram of Schering bridge is shown below.

where C_1 = unknown capacitor

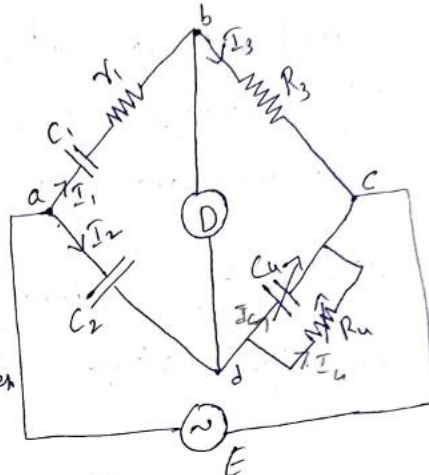
r_1 = a series resistance

representing loss component of C_1

C_2 = Standard capacitor

C_u = Variable capacitor

R_3, R_u = Non-Inductive Resistances



At balanced condition, $Z_1 Z_u = Z_2 Z_3$

$$\left(r_1 + \frac{1}{j\omega C_1}\right) \left(\frac{R_u}{1 + j\omega R_u C_u}\right) = R_3 \left(\frac{1}{j\omega C_2}\right)$$

$$\left(r_1 + \frac{1}{j\omega C_1}\right) R_u = \frac{R_3}{j\omega C_2} (1 + j\omega C_u R_u)$$

$$R_u r_1 + \frac{R_u}{j\omega C_1} = \frac{R_3}{j\omega C_2} + \frac{R_3 R_u C_u}{C_2}$$

$$Z_u = R_u \parallel C_u$$

$$\frac{1}{Z_u} = \frac{1}{R_u} + \frac{1}{X_u}$$

$$= \frac{1}{R_u} + j\omega C_u$$

$$\frac{1}{Z_u} = \frac{1 + j\omega R_u C_u}{R_u}$$

Now by equating real & imaginary parts, we get

$$R_u r_1 = \frac{R_3 R_u C_u}{C_2}$$

$$r_1 = \frac{R_3 C_u}{C_2}$$

$$\frac{R_u}{j\omega C_1} = \frac{R_3}{j\omega C_2}$$

$$C_1 = \frac{R_u}{R_3} C_2$$

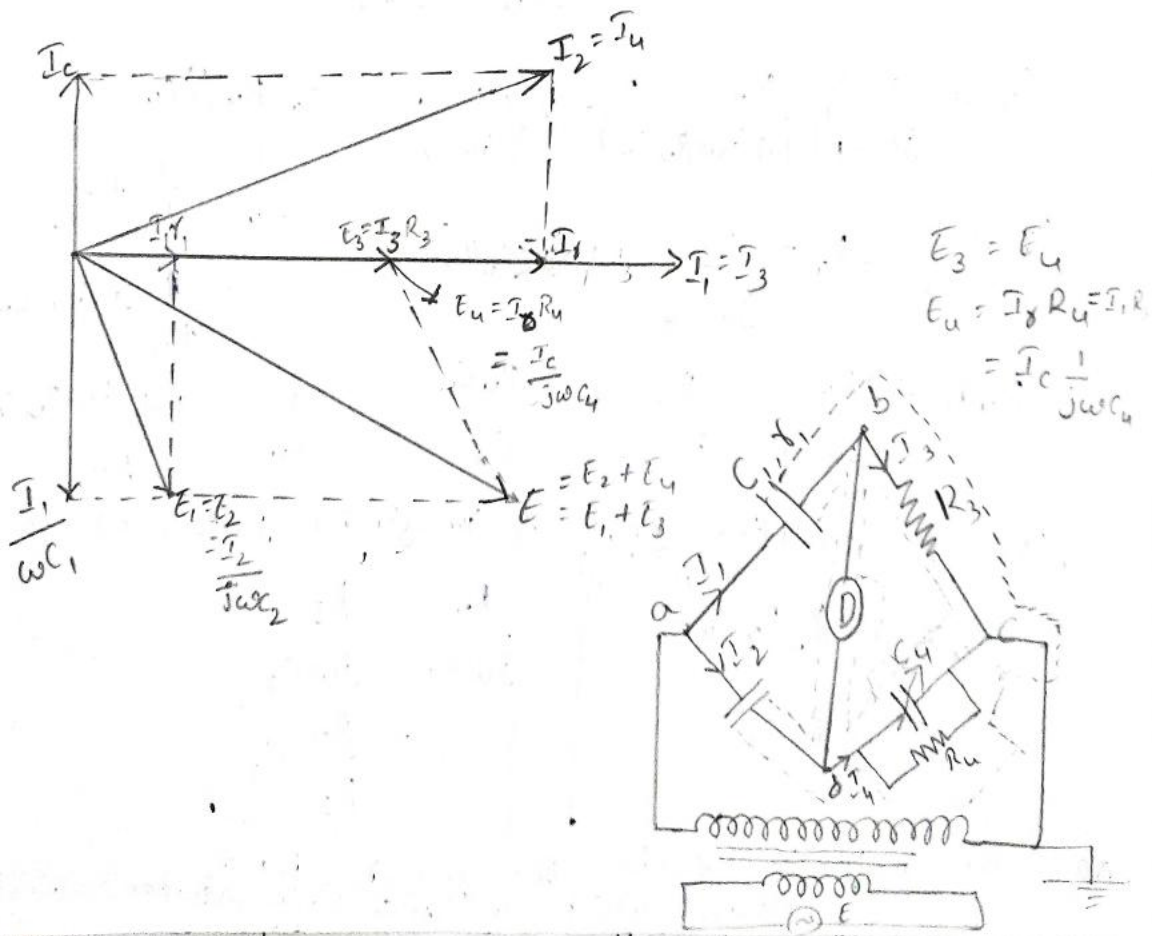
Now the dissipation factor $D = \omega C_1 r_1 = \tan \delta$
 $\Rightarrow \tan \delta = \omega C_1 r_1 = \omega \left(\frac{R_u}{R_3}\right) C_2 \left(\frac{R_3 C_u}{C_2}\right) \Rightarrow D = \tan \delta = \omega C_u R_u$

Now equating

Note! The measurement of small capacitances using this bridge suffers from many disadvantages if carried out at low voltages.

High voltage bridge is certainly preferable for such measurements.

High voltage Schering Bridge :- The circuit diagram for high voltage Schering bridge is shown below.



High voltage Schering Bridge:- Following modifications are made to operate at high voltages.

- (1) High voltage is obtained by step up T/F from 50 Hz supply. Vibration galvanometer is used as detector.
- (2) Arms ab & ad each contain only a capacitor & these capacitors are designed for a high voltage work. Thus the major portion of potential drop occurs in the arms ab & ad & very little voltage drop is there across the arms bc & dc .
- (3) The point 'c' is earthed even if a voltage as high as 100 kv is applied to the bridge. The voltage across arms bc & dc will be of few volts.

This is certainly a great advantage as the controls are located in the arms bc & dc & for the safety of the operator, these controls should be at low potential w.r. to earth.

(4) It is necessary to provide a prevention of dangerous high voltages appearing across arms bc & cd in the case of breakdown of either of high voltage capacitors.

This is done by creating a spark-gap across each of the arms bc & dc .

Branch:- A branch is an element of the n/w having only two ~~element~~ terminals.

Node:- The junction point of two or more branches is known as a node.

Loop or mesh:- A closed path

(5) Earthed screens are provided in order to avoid errors caused due to inter-capacitance between high & low arms of a bridge.

Instead of earthing one point on the circuit, the earth capacitance effect on galvanometer is eliminated by means of "Wagner Earth device".

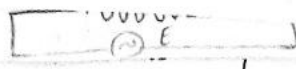
we
→ Step up T/F for high voltage screening bridge.

→ The capacitance is very low. Resing is constant

$\uparrow Z = \frac{1}{j\omega C}$ ab & ad has large Impedance
currents are less so to detect that much amount of ^{small} current we use vibration gal.

→ Impedances are high so voltage drops across ab & ad are high & V.D. across bc & cd are less.

→ In case of Break down of capacitors the entire high voltage applied across the arms bc & cd. So we have to provide a Spark gap. This spark gap produce spark w/h the contact & entire voltage is bypassed to ground.



Wein's Bridge :- Some bridges have the balance equations which involve frequency directly. The wein's bridge is primarily known as a "frequency" determining bridge & is described here not only for measurement of frequency but also for its application in various other useful circuits.

The main circuit diagram of wein's Bridge is shown below.

At balance, $Z_1 Z_u = Z_2 Z_3$

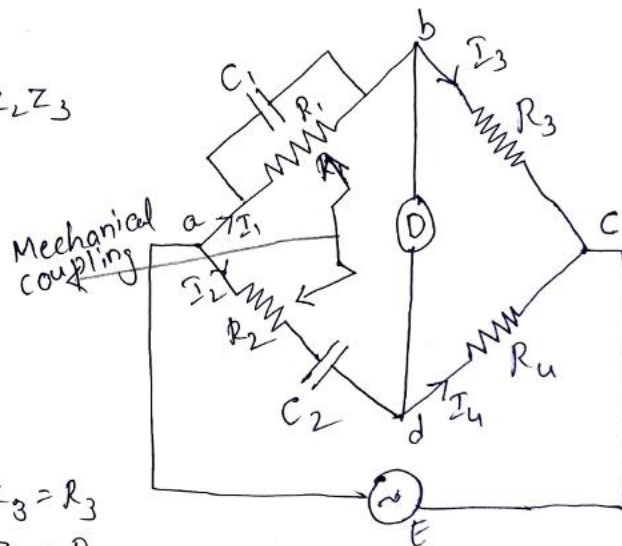
$$\frac{1}{Z_1} = \frac{1}{R_1} + \frac{1}{X_{C_1}}$$

$$= \frac{1}{R_1} + j\omega C_1$$

$$Z_1 = \frac{R_1}{1 + j\omega C_1 R_1}$$

$$Z_2 = R_2 + \frac{1}{j\omega C_2}, \quad Z_3 = R_3$$

$$Z_u = R_4$$



$$\therefore \left(\frac{R_1}{1 + j\omega C_1 R_1} \right) R_u = \left(R_2 + \frac{1}{j\omega C_2} \right) R_3$$

$$R_1 R_u = \left(R_2 R_3 + \frac{R_3}{j\omega C_2} \right) (1 + j\omega C_1 R_1)$$

$$R_1 R_u = R_2 R_3 + j\omega C_1 R_1 R_2 R_3 + \frac{R_3}{j\omega C_2} + \frac{R_3 R_1 C_1}{C_2}$$

Now Equating Real & Imaginary terms on b.s.

$$R_1 R_u = R_2 R_3 + \frac{R_1 R_3 C_1}{C_2} \quad \left| \quad j\omega C_1 R_1 R_2 R_3 + \frac{R_3}{j\omega C_2} = 0 \right.$$

$$R_1 R_4 = R_3 \left[R_2 + \frac{R_1 C_1}{C_2} \right]$$

$$\frac{R_1 R_4}{R_3} = R_1 \left[\frac{R_2}{R_1} + \frac{C_1}{C_2} \right]$$

$$\boxed{\frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_2}} \rightarrow \textcircled{1}$$

In most Wein Bridges,
the components are
chosen that $R_1 = R_2 = R$
 $C_1 = C_2 = C$

$$\therefore \textcircled{1} \Rightarrow \boxed{\frac{R_4}{R_3} = 2} \quad \uparrow$$

$$\textcircled{2} \Rightarrow \boxed{f = \frac{1}{2\pi RC}}$$

Wein's Bridge is suitable for the measurement of frequencies from 100 Hz to 100 kHz. It is possible to obtain an accuracy of 0.1% to 0.5%.

* If the operating frequency is known then the Bridge can be used for the measurement of capacitance.

is bypassed to ground

$$j\omega R_1 C_1 R_2 R_3 + \frac{R_3}{j\omega C_2} = 0$$

$$j\omega R_1 R_2 R_3 C_1 - \frac{j R_3}{\omega C_2} = 0$$

$$\omega R_1 R_2 R_3 C_1 - \frac{R_3}{\omega C_2} = 0$$

$$\omega R_1 R_2 R_3 C_1 = \frac{R_3}{\omega C_2}$$

$$\omega R_1 R_2 C_1 - \frac{1}{\omega C_2} = 0$$

$$\omega^2 R_1 R_2 C_1 C_2 - 1 = 0$$

$$\omega^2 = \frac{1}{R_1 R_2 C_1 C_2}$$

$$\omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$2\pi f = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$\boxed{f = \frac{1}{2\pi \sqrt{R_1 R_2 C_1 C_2}}} \rightarrow \textcircled{2}$$

Prob: Four arms of a bridge are

arm ab = Imperfect capacitor C_1 with series resistance r_1

arm bc = non-inductive resistance R_3

arm cd = " " " " R_4

arm da = imperfect capacitor C_2 with series resistance of r_2 & R_2

A supply of 450 Hz is given to terminals a & c.

At balance $R_2 = 4.8 \Omega$, $R_3 = 2000 \Omega$, $R_4 = 2850 \Omega$, $C_2 = 0.5 \mu\text{F}$
 $r_2 = 0.4 \Omega$.

Calculate C_1 & r_1 also the dissipation factor.

Sol: From the given data when we draw a bridge it is Desauty's bridge.

At balance

$$\left(r_1 + \frac{1}{j\omega C_1}\right) R_4 = R_3 \left(R_2 + r_2 + \frac{1}{j\omega C_2}\right)$$

separating real & Imaginary terms

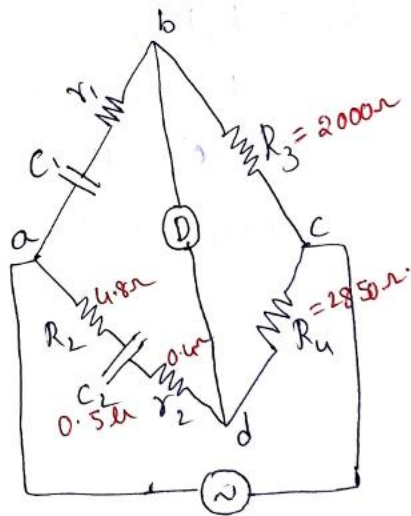
$$r_1 R_4 = R_3 (R_2 + r_2)$$

$$r_1 = \frac{(R_2 + r_2) R_3}{R_4}$$

$$r_1 = \frac{(4.8 + 0.4) 2000}{2850} = 3.65 \Omega$$

$$C_1 = \frac{R_4}{R_3} C_2 = \frac{2850}{2000} \times 0.5 \times 10^{-6} = 0.712 \mu\text{F}$$

Dissipation factor $D = \tan \delta_1 = \omega C_1 r_1 = 2\pi f C_1 r_1$
 $= 2\pi \times 450 \times 0.712 \times 10^{-6} \times 3.65$
 $D = 0.00734$



Prob:- For Maxwell's Inductance-capacitance bridge shown below, The various values of balance is $R_2 = 400 \Omega$, $R_3 = 600 \Omega$, $R_u = 1000 \Omega$, $C_u = 0.5 \mu\text{F}$. Calculate the values of R_1 & L_1 . Also calculate storage factor of the coil if the frequency is 100 Hz.

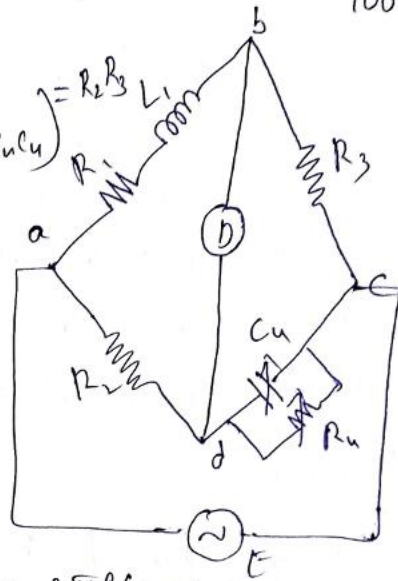
Sol:- At balance, $(R_1 + j\omega L_1) \left(\frac{R_u}{1 + j\omega R_u C_u} \right) = R_2 R_3$

$$\therefore R_1 = \frac{R_2 R_3}{R_u} = \frac{400 \times 600}{1000} = 240 \Omega$$

$$L_1 = R_2 R_3 C_u = 400 \times 600 \times 0.5 \times 10^{-6} = 0.12 \text{ H}$$

$$\therefore \text{Storage factor } Q = \frac{\omega L_1}{R_1} = \frac{2\pi f (0.12)}{240} = \frac{2\pi \times 100 \times 0.12}{240}$$

$$Q = 0.314$$



prob:

The arms of a five node bridge are as follows.

arm ab: an unknown impedance (R_1, L_1) in series with a non-inductive variable resistor γ_1 .

arm bc: a non-inductive resistor $R_3 = 100 \Omega$

arm cd: a non-inductive " $R_4 = 200 \Omega$.

arm da: " " $R_2 = 250 \Omega$.

arm de: a non-inductive variable resistor γ

arm ec: a loss less capacitor $C = 1 \mu\text{F}$

arm be: a detector.

An ac supply is connected b/n a & c.

Calculate the resistance & inductance R_1, L_1

when under balance conditions $\gamma_1 = 43.1 \Omega$ & $\gamma = 229.7 \Omega$.

Sol:

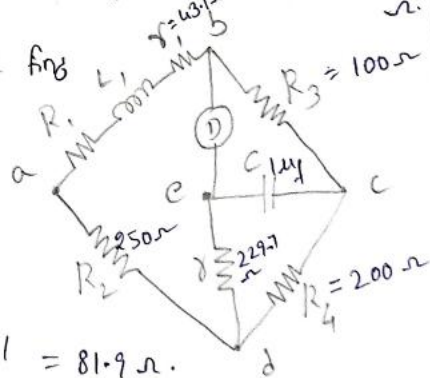
After drawing the N/w, we find that it is the Anderson's bridge.

Under balance conditions,

$$R_1 = \frac{R_2 R_3}{R_4} - \gamma_1 = \frac{250 \times 100}{200} - 43.1 = 81.9 \Omega.$$

$$L_1 = \frac{C R_3}{R_4} [\gamma (R_4 + R_2) + R_2 R_4] = \frac{1 \times 10^{-6} \times 100}{200} [229.7 (200 + 250) + (200 \times 250)]$$

$$L_1 = 0.0766 \text{ H.}$$



Prob: The impedance of the basic ac bridge are
 $Z_1 = 50 \angle 180^\circ$, $Z_2 = 250 \angle 0^\circ$, $Z_3 = 200 \angle 30^\circ$.
calculate the constants of unknown impedance.

Sol: The balancing equation of an AC bridge is

$$Z_1 Z_u = Z_2 Z_3$$

$$Z_u = \frac{Z_2 Z_3}{Z_1} = \frac{(250 \angle 0^\circ)(200 \angle 30^\circ)}{50 \angle 180^\circ}$$
$$= \frac{1000 \angle 30^\circ}{80 \angle 180^\circ}$$

$$\boxed{Z_u = 1000 \angle -150^\circ} \Omega.$$

The constants are $R = Z \cos \phi = 1000 \cos(-150)$
 $X = Z \sin \phi = 1000 \sin(-150)$

Digital Voltmeter

- Voltmeter is an electrical measuring instrument which is used to measure the p.d b/w two points.
- voltage measured can be A.C or D.C.

Two types of voltmeters.

- (i) Analog Voltmeter
- (ii) Digital Voltmeter

→ It is an instrument which measures AC/DC voltage & displays the value directly in numeric form.

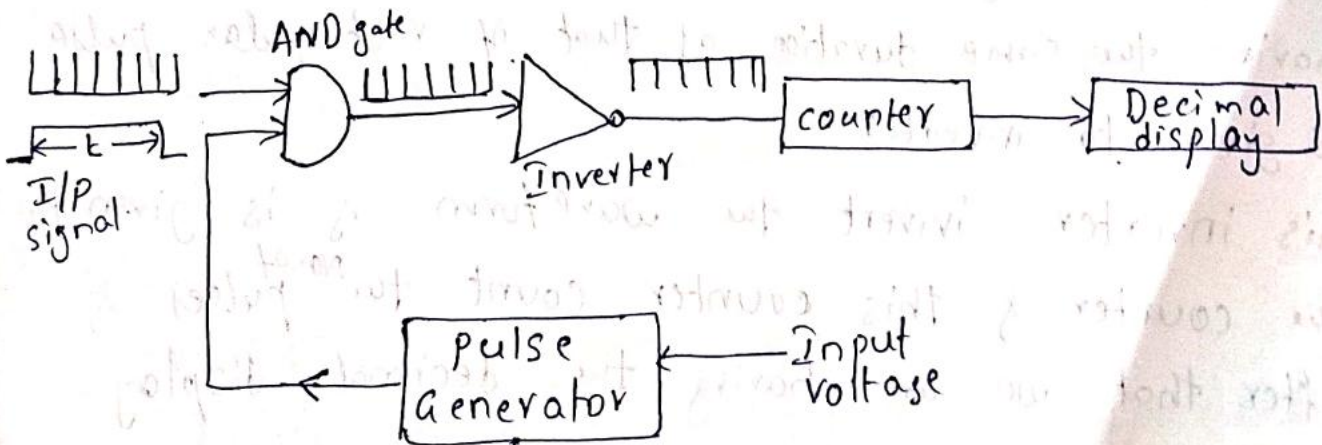
It has a pointer deflected over a calibrated scale. It gives the value of voltage which is to be measured

where as Digital voltmeter measure voltage directly by giving the discrete numerical output.

Adv :- (1) versatile & accurate instrument

(2) reduced size, less power consumption & less cost

Working :-



The input voltage which we want to measure is applied to the pulse generator. This pulse generator's output is given as one of the inputs of the AND gate. The other input of this AND gate will be a rectangular pulse. The output of the AND gate is given to the inverter. The function of the inverter is to invert the waveform. The output of the inverter is given to the counter & this output is given to the decimal display.

(b) Total resistance in the meter circuit.
 $25 + 75$

Construction:-

- (1) Input ^{signal} voltage:- It is the signal whose voltage to be measured.
- (2) pulse generator:- It is a voltage source. It used digital, analog or both techniques to generate rectangular pulse.

The width & frequency — controlled by digital circuit
Amplitude rise & fall time → controlled by Analog circuit

- (3) AND gate:- It gives high output ^{only} when both the inputs are high.

When a train of pulses are fed to it along with rectangular pulse, it provides an o/p having train of pulses with same duration as that of rectangular pulses from the pulse generator.

- (4) O/p of AND gate that is train of pulses which is having the same duration as that of rectangular pulse is given to inverter.

This inverter invert the waveform & is given to the counter & this counter count the ^{no. of} pulses & after that we are having the decimal display.

- (4) Decimal display:- It counts the number of impulses & hence the duration & display the voltage on LED.

The no. of impulses equal to the i/p voltage.

unknown i/p voltage signal

↓
pulse generator

↓ pulse (proportional to the i/p signal)

↓ rectangular pulse

one leg of AND gate ← train of pulses

↓

o/p of AND gate is positive triggered train of duration same as the width of the rectangular pulse.

↓

Inverter → (It converts into negative triggered train).

↓

counter (count the no. of triggers in the pulse).

↓

No. of triggers = i/p signal voltage.

→ working of digital voltmeter is nothing but an analog to digital conversion.

Advantages:-

- ~~reduces~~ eliminates observational errors ↓
- Errors on account of parallax & approximation are eliminated. ↓
- output can be fed to memory device for storage & future computations. speed ↑
- versatile, accurate ↑ cheap ↓ size ↓
- Lower power requirements ↓
- stability increased

Types of Digital Voltmeters

- (1) Ramp type digital voltmeter
- (2) Integrating type
- (3) Potentiometric type
- (4) Successive approximation type DVM
- (5) Continuous balance type DVM

DVM :- It is an instrument which measures the AC/DC voltage & displays the value directly in numeric form instead of pointer deflections.

Adv :-

Accuracy ↑

Speed ↑

Sensitivity ↑

Observational Error ↓

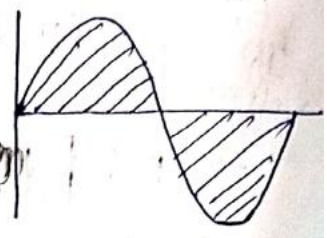
Power requirement ↓

Cost ↓

Size ↓

Integrating type Digital Voltmeter:

→ This measure the true average value of the i/p voltage over a fixed measuring period.



That means here we are using the term integration. i.e. Integrating the given curve or waveform over a period of time gives the avg. value. i.e. we are measuring the area covered under that curve.

→ In this type DVM we use the integration technique which uses the voltage to frequency conversion & then frequency will be measured & that frequency is \propto to magnitude of i/p voltage.

→ This voltmeter employs an integration technique which uses a voltage to frequency conversion.

→ The voltage to frequency converter functions as a feedback control system which governs the rate of pulse generation in proportion to the magnitude of i/p voltage.

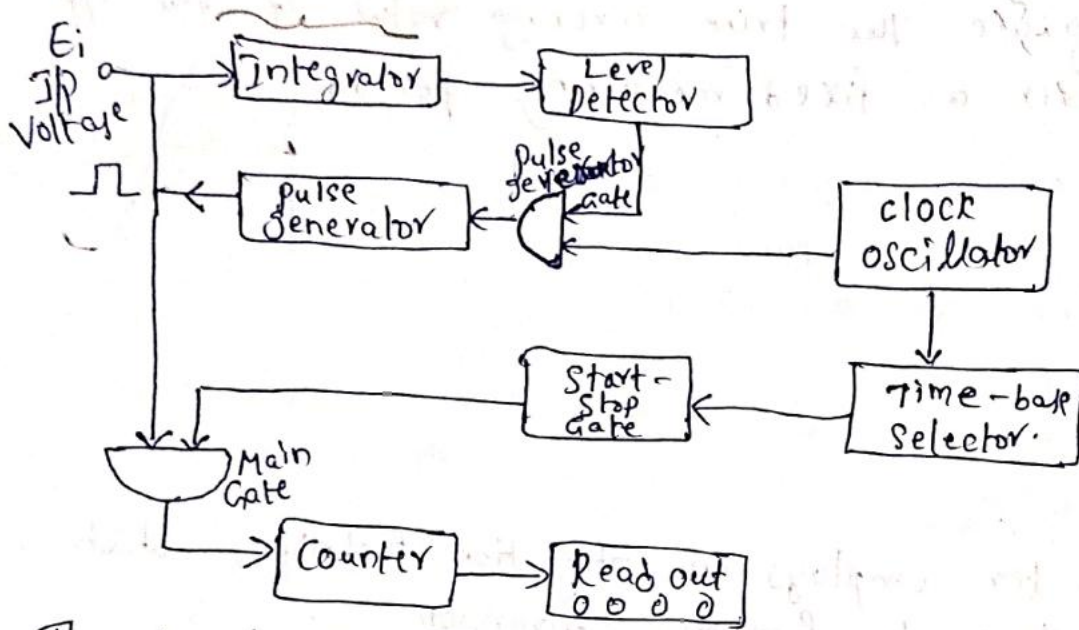
i.e. the converter which acts as a feedback control system & it controls the rate of pulse generation. The pulses in generation is proportion to the magnitude of i/p voltage.

→ A train of pulses is generated.
↓
rate at which pulses are generated is equal to the i/p voltage.
no. of pulses appearing in a definite interval is counted.
i.e. How many no. of pulses are appearing in a fixed period of time that will be counted.

No. of pulses which are generated in a fixed period of time nothing but frequency of the signal. & by calculating the frequency we are measuring the i/p voltage because the rate at which the pulses are generated is proportional to magnitude of i/p voltage.

↓
Frequency is a function of unknown voltage.

Block diagram :-

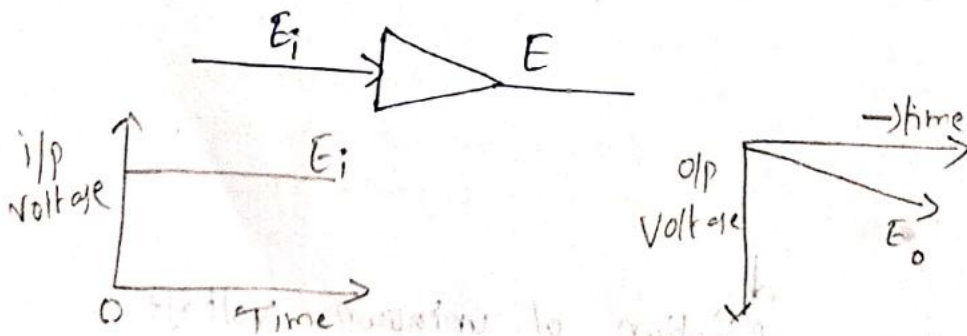


The heart of this technique is operational amplifier which act as an integrator. So this integrator is the main part of DVM.

The i/p of the integrator is the i/p voltage that we want to measure & the o/p of the integrator will be

The o/p of the integrator, $E = -E_i \frac{1}{Rc} \times t$

→ If a constant i/p voltage applied, an o/p voltage is produced at a uniform rate & polarity is opposite.
 Constants of integrator



If the i/p voltage E_i ↑ then the deflection in the o/p voltage is also ↑

→ constant i/p voltage — ramp o/p voltage of opposite polarity.

→ The slope of o/p voltage is determined by the value of the i/p voltage. The o/p voltage is changing in proportion to

Coming back to the block diagram. The o/p of Integrator is given level detector & it detects the magnitude of o/p voltage. that means when the o/p voltage reaches a fixed value then it produces an o/p which triggers the pulse generator gate signal. & it triggers the pulse generator that means it produces no. of pulses proportional to i/p voltage.

→ Level detector → When voltage reaches a certain reference level, the detector sends a pulse to the pulse generator gate.

ie, Level detector acts as a comparator it compares the o/p value with a reference level & when these values are equal it generates a pulse to the pulse generator gate.

→ Greater the values of i/p voltage sharper will be the slope of o/p voltage → quicker the o/p voltage will reach the reference level. ie, quickly pulse will be generated which will trigger the pulse generator.

→ o/p of level detector opens the pulse level gate, permitting pulses from a fixed frequency clock oscillator to pass through pulse generator.

→ pulse generator → produces an o/p pulse of fixed amplitude & width.

ie, o/p pulse is fed back to the integrator

o/p pulse is fed back to the i/p of the integrator.

↓ ie, o/p value of integrator is equal to the reference value
No more pulses from the clock oscillator will pass through the pulse generator

→ when the o/p voltage pulse from the pulse generator has passed, is restored to its original value & starts its rise again.

↓
The pulse generator is trigger by a pulse from the clock generator & entire cycle is repeated.

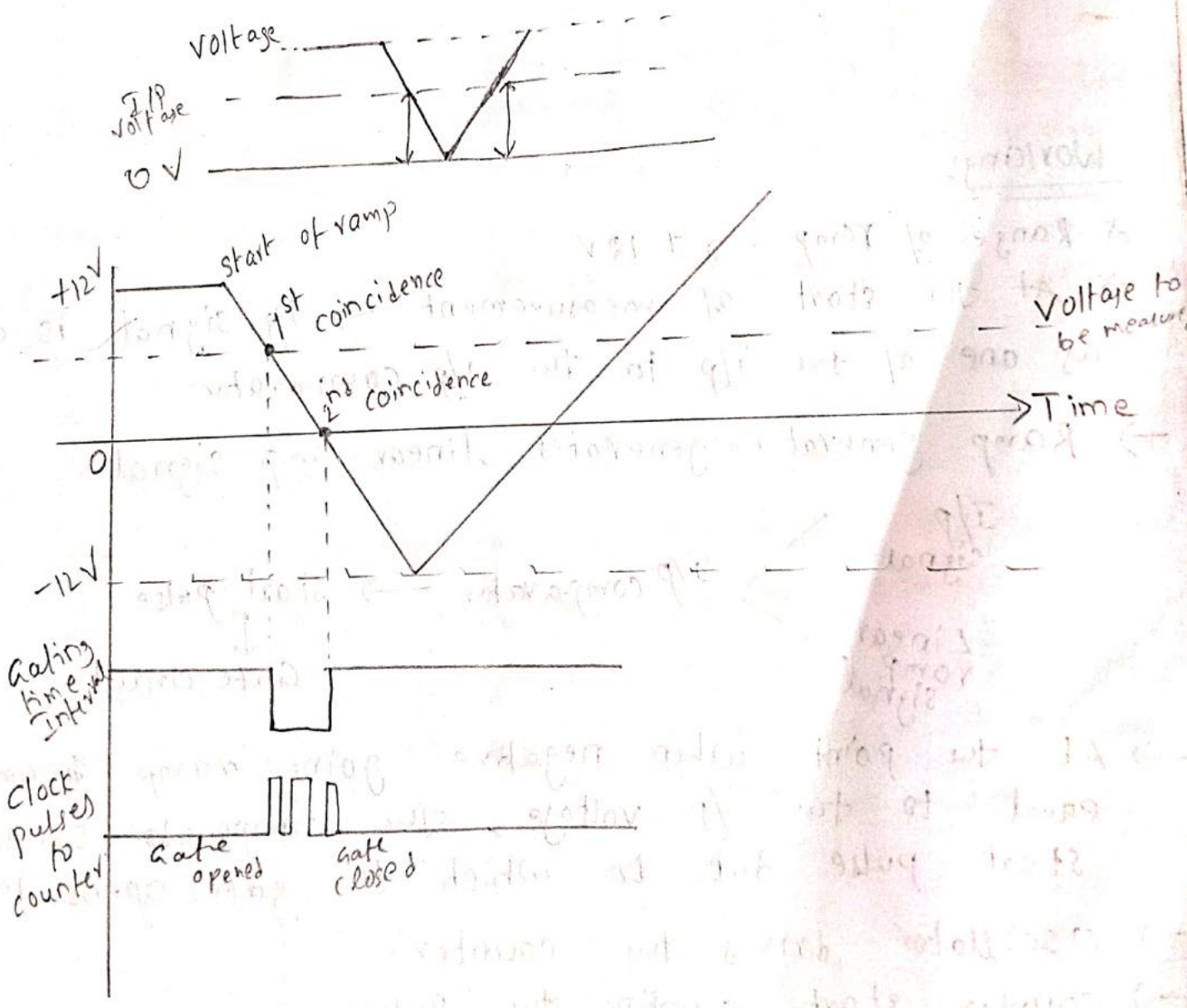
→ The frequency of the wave is a function of the voltage being measured.

→ one pulse from the pulse generator is produced for each cycle of the wave ; the number of pulses produced in a given period of time interval & hence the frequency is an indication by the voltage being measured.

Ramp type DVM :-

operating principle: The operating principle of a ramp type digital voltmeter is to measure the time of linear ramp voltage takes to change from "level of i/p voltage to zero voltage" (or) zero voltage to level of i/p voltage".

Based on the measurement of the time taken by a linear ramp to rise from zero voltage to level of i/p voltage or decrease from level of i/p voltage to zero voltage.

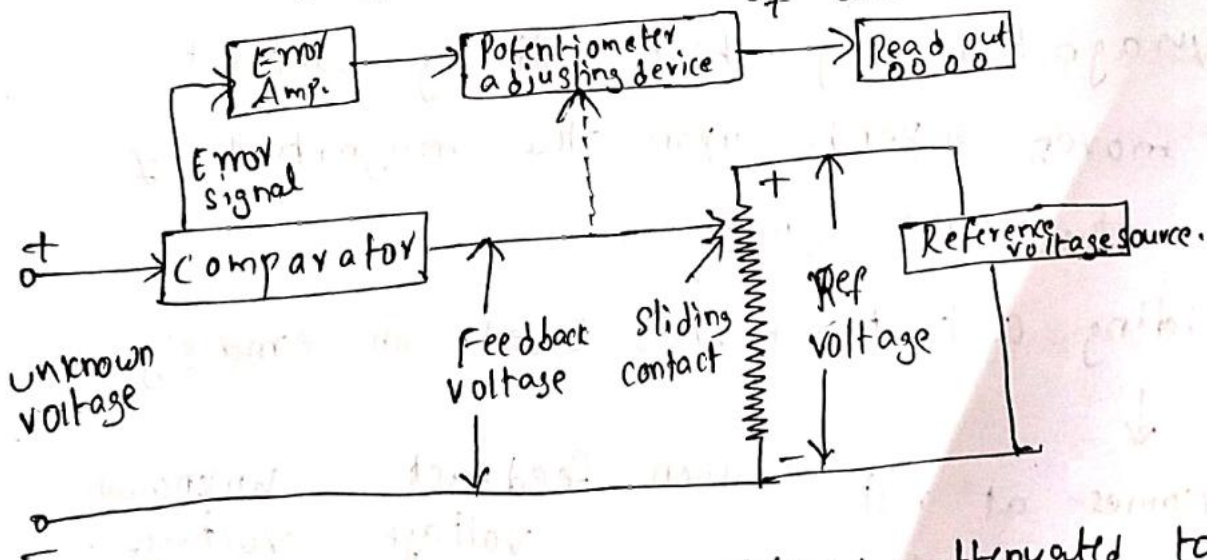


Potentiometric Type Digital Voltmeter

It is a type DVM which measures the potential difference by comparing the unknown voltage with a reference voltage.

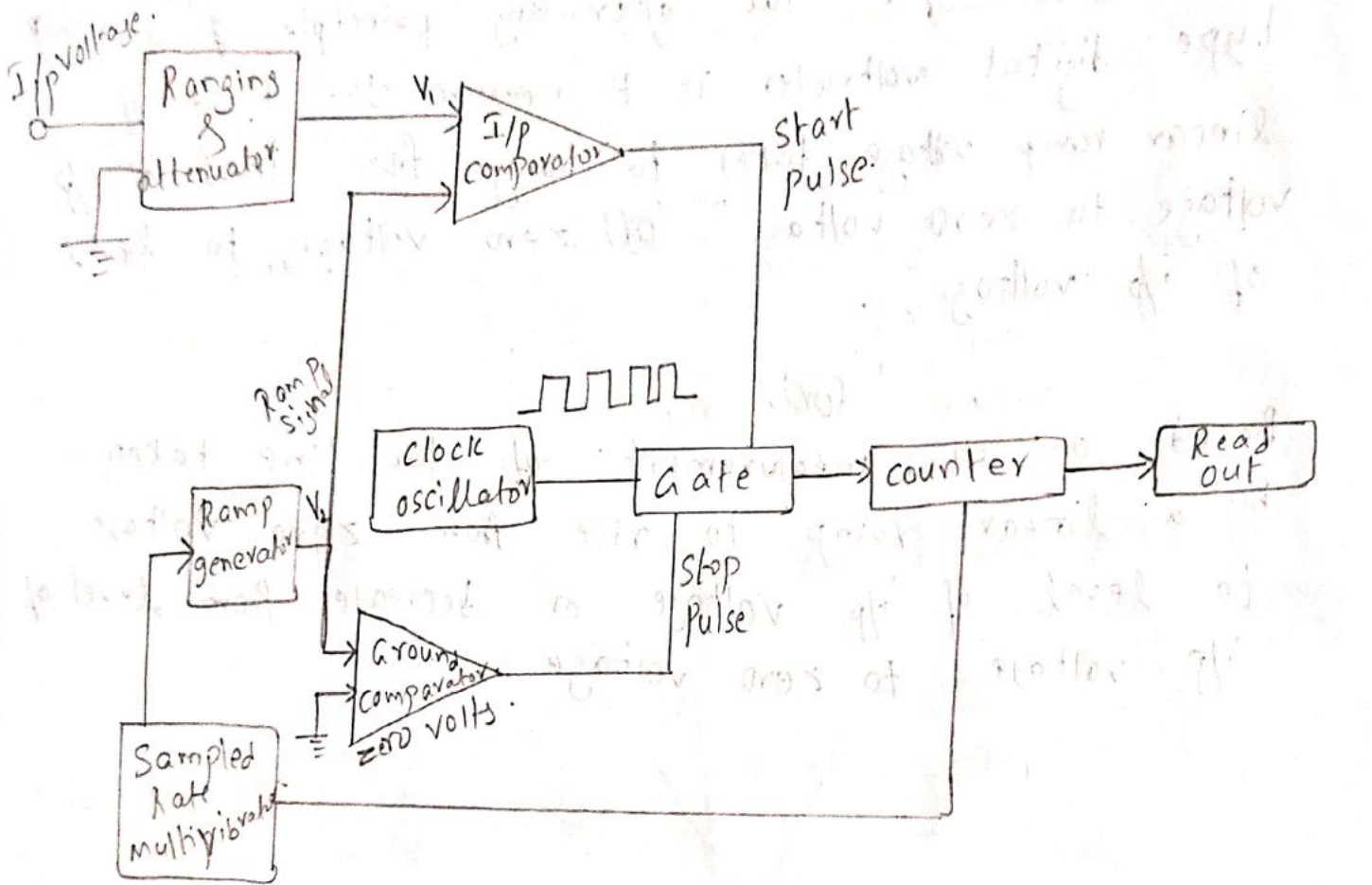
whose value is fixed by the setting of the potentiometer.

- potentiometer setting is changed to obtain null condition.
- At null condition, value of unknown voltage is equal to the reference voltage.
- self balancing potentiometer → Automatically balances.
- Display unit → directly read the value of unknown voltage.



- Unknown voltage → Filtered, attenuated to suitable level.
- comparator (Error detector)
- Reference voltage ← Fixed voltage source.
- Potentiometer that are compared to the i/p voltage.

Block diagram! -

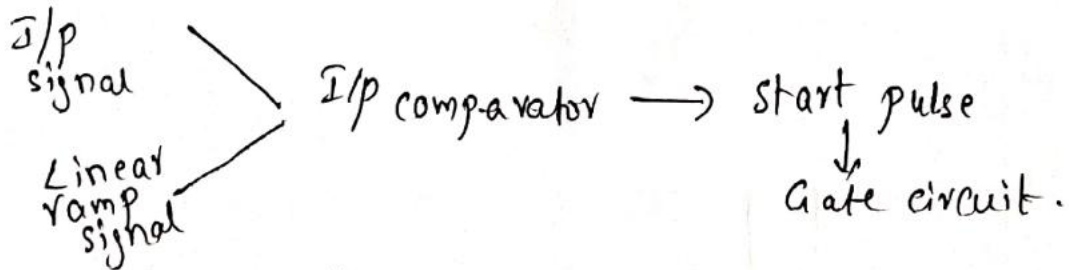


Working! -

* Range of ramp $\rightarrow \pm 12V$

* At the start of measurement \rightarrow i/p signal is applied as one of the i/p to the i/p comparator.

\rightarrow Ramp generator generates linear ramp signal.



\rightarrow At the point when negative going ramp becomes equal to the i/p voltage, the comparator sends start pulse due to which the gate opens. i.e., $V_1 = V_2$

\rightarrow Oscillator drives the counter

\rightarrow counter starts counting the pulses.

Again next start pulse signal activates the control ~~circuit~~ ^{register}, the o/p of the SAR is converted into analog in the DAC & the V_{out} is again compared with V_{in} in the comparator.

If $V_{in} > V_{out}$ then SAR sets '1' in the next MSB i.e. D_6 (or)

If $V_{in} < V_{out}$ the SAR resets '0' in the D_7 & set '1' in the D_6 .

The ring counter then advances one count shifting a '1' in the second MSB of the control register & its reading becomes 11000000

This causes the D/A ~~ca~~ & then converted into analog o/p in DAC & compared with unknown i/p in the comparator.

Again for the next start pulse signal activates the control register & also the ring counter then advances one count shifting a '1' in the third MSB of & its reading

- } when ramp = ground voltage (0V) i.e. $V_2 = 0$
Ground comparator → sends stop pulse

→ A definite number of pulses will be counted by the counter during the start & stop pulses which is a measure of the i/p voltage.
Gate closes. ↓

→ Sample Rate Multivibrator → determines the rate at which measurement cycles are initiated. (done)
↓ digital read out -

→ Oscillation of multivibrator ↓ adjusted

front panel control

↓
RATE

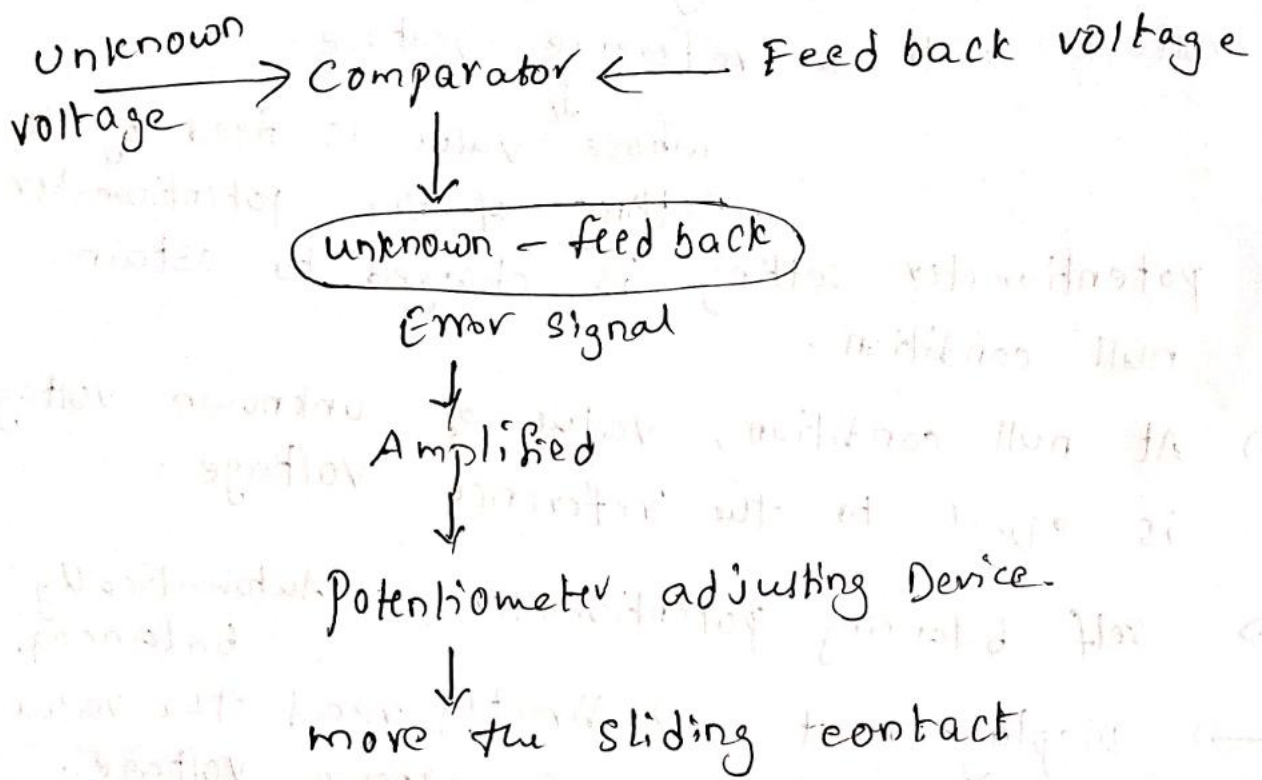
→ The sample rate MV provides an initiating pulse to the ramp generator to start its next ramp voltage.

→ At the same time, a reset pulse is also generated which resets the counter to zero.

Successive Approximation of DVM :-

Principle :- The successive approximation principle can be easily understood using the simple example, the determination of weight of an object. By using balance & placing the object on one side & approximate weight on other side, the weight of object is determined.

→ The value of the feedback voltage depends on the position of the sliding contact.



→ Magnitude by which sliding contact moves depends upon the magnitude of the error signal.

→ sliding contact moves based on error signal.

↓
comes at rest when feedback voltage = unknown voltage.

→ No error voltage → no i/p to potentiometer.

→ The unknown voltage equals the feedback voltage, the reading of the readout device indicates the value of unknown voltage.

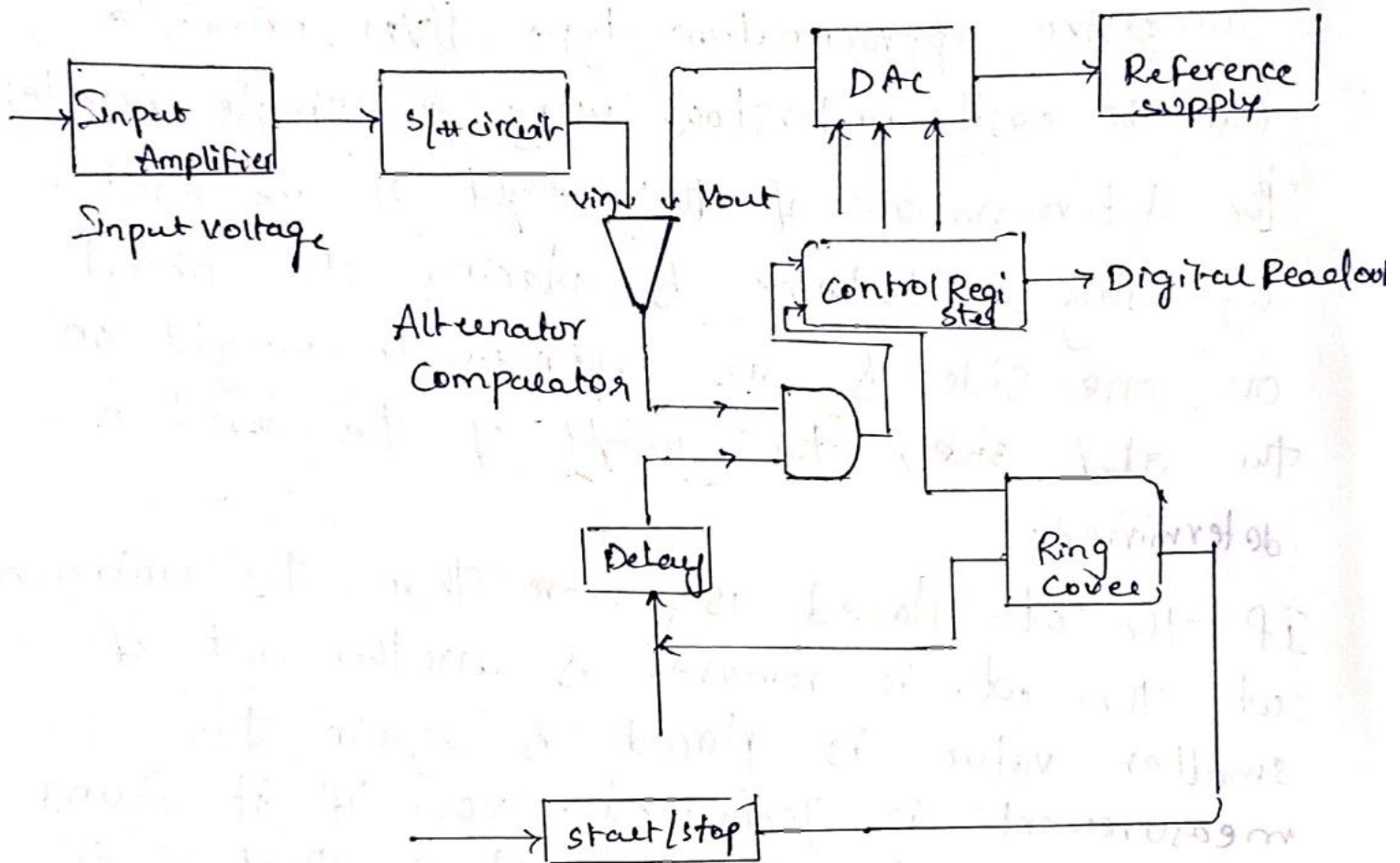
Successive Approximation Type DVM :- The

successive approximation type DVM principle can be easily understood using a simple example; The determination of the weight of an object. By using a balance $\&$ placing the object on one side $\&$ an approximate weight on the other side, the weight of the object is determined.

If the wt. placed is more than the unknown wt, the wt. is removed $\&$ another wt of smaller value is placed $\&$ again the measurement is performed. Now if it found that the wt. placed is less than that of the object, another wt of smaller value is added to the wt already present $\&$ the measurement is performed. If it is found to be greater than the unknown wt. the added wt. is removed $\&$ another wt. of smaller value is added. In this manner by adding $\&$ removing the appropriate wt, the wt. of the unknown object is determined.

Principle :- The successive approximation type DVM uses a digital - to - analog converter (DAC) to generate a series of analog voltages that are compared to the i/p voltage.

Block Diagram



The block diagram of Successive approximation type DVM is shown above.

The i/p attenuator amplifies the i/p voltage & then given to comparator through sample & hold circuit.

When the start pulse signal activates the control circuit, the SAR is cleared. The o/p of the SAR is 0000000 & it is converted into analog o/p in the DAC & the V_{out} is 0.

Block Diagram

Now the i/p unknown voltage V_{in} compares with V_{out} in the comparator.

If $V_{in} > V_{out}$ then the comparator o/p is +ve

The SAR sets the most significant bit (MSB) to 1 ^{remainings are zeros} generates the digital code.

Assume it is 8-bit control

∴ the digital o/p 1 0 0 0 0 0 0 0.

$V_{in} = 1V$	operation	D_7	D_6	D_5	D_4	D_3	D_2	D_1	D_0	compare.
0 0 1 1 0 0 1 1	D_7 set	1	0	0	0	0	0	0	0	$V_{in} < V_{out}$
"	D_6 set	0	1	0	0	0	0	0	0	$V_{in} < V_{out}$
"	D_5 set	0	0	1	0	0	0	0	0	$V_{in} > V_{out}$
"	D_4 set	0	0	1	1	0	0	0	0	$V_{in} > V_{out}$
"	D_3 set	0	0	1	1	1	0	0	0	$V_{in} < V_{out}$
"	D_2 set	0	0	1	1	0	1	0	0	$V_{in} < V_{out}$
"	D_1 set	0	0	1	1	0	0	1	0	$V_{in} > V_{out}$
"	D_0 set	0	0	1	1	0	0	1	1	$V_{in} = V_{out}$

If $V_{in} < V_{out}$ then the comparator o/p is negative (-ve) - then the control circuit resets D_7 .

05/01/17

Unit - 4

EEMG by A.K. Sahoo

Transducers

Introduction :-

As explained earlier, a generalized measuring system consists of 3 major components.

1. measurand
2. Signal Conditioning circuit
3. output.

The input device receives the measurand (or) quantity under measurement and delivers the analogous electrical signal to the signal conditioning circuit. Here, the signal is modified, (or) amplified (or) attenuated into the format acceptable to the output device.

The input quantity for most instrumentation systems is of "non-electrical quantity". In order to use electrical methods and techniques for measurement, the non-electrical quantity is to be converted to electrical quantity by a device known as "Transducer".

* Definition of Transducer :-

A device which is used to convert one form of energy to another form of energy (or)

A Transducer is a device which is used to convert any physical quantity (or) a non-electrical quantity to an electrical quantity.

* Classification of Transducers :-

The Transducers are mainly classified into,

- i. Primary & Secondary Transducers.
- ii. Passive & Active Transducers.
- iii. Analog and Digital Transducers. and
- (iv) Electrical Transducers.

i. Primary & Secondary Transducers :-

Let us consider, a Bourdon's tube which senses (or) takes the pressure as input and, converts the pressure into a displacement. This displacement moves the 'Core' of 'LVDT' which produces the Electrical output.

Here, there are two stages, firstly the pressure is converted to displacement using Bourdon's tube (Primary Transducer) and again the displacement is converted to Electrical output by using an LVDT (Secondary Transducer).

ii. Passive & Active Transducers :-

→ Passive transducers derive the power required for the transduction from an auxiliary power source. They are also known as 'externally powered transducers'.

In the absence of external power, the transducer cannot work and hence it is called as passive Transducer.

→ Active Transducers are those which do not require any auxiliary power source to produce their output. They are also known as 'self generating' type of Transducers, since they can generate their own voltage (or) current output.

The Potentiometer is the best example of Passive Transducer and the Piezo-Electric Crystal is an best example for an Active Transducer.

iii, Analog and Digital Transducer:-

The Analog Transducers Convert the input quantity into an analog output which is a Continuous function of time.
→ Strain gauge, LVDT are the best examples of Analog Transducers.

The Digital Transducers that Convert the input quantity into an electrical output which is in the form of pulses.

→ Digital Tachometer is an example of Digital Transducers.

(iv) Electrical Transducers:-

While measuring non-electrical quantities, a detector is used. This detector converts the physical quantity into displacement. This displacement is given as an input to an electrical transducer which gives an electrical output.

The electrical signal may be a voltage (or) current or a frequency and production of these signals is based on electrical effects which may be resistive, capacitive, inductive etc.,. The electrical quantity so produced is measured by standard methods used for electrical measurements. The result gives the magnitude of the physical quantity (or) condition being measured.

* Advantages of Electrical Transducers:

The advantages of converting physical quantities into analog electrical quantities are:

1. Electrical amplification and attenuation can be done easily.
2. The effects of friction in a transducer will be minimized.
3. The Electrical (or) Electronic Systems can be controlled with a very small power level.
4. The electrical output can be easily analyzed, transmitted and processed for the purpose of measurement.
5. Telemetry is used in almost all communication systems. The entire aerospace research and development is totally based on the Telemetry. For better communication using Telemetry, the Electrical and Electronic principles have to be employed.

* Characteristics and Choice of Transducers:

When choosing a transducer for any application the 1. Input 2. Transfer and 3. output characteristics have to be considered.

1. Input characteristics:

i. Type of Input and Operating Range:

The foremost consideration for the choice of transducer is the input quantity it is going to measure and its operating range. A physical quantity may be measured through the number of transducers. However, the choice of particular transducer that is selected for the purpose, depends on useful range of i/p quantity over which a transducer can be used.

The useful operating range of a transducer may be a decisive factor in selection of a transducer for a particular application.

ii, Loading Effects :-

Ideally, a transducer should have no loading effects. The magnitude of loading effects can be expressed in terms of force, power (or) energy extracted from the quantity under the measurement for proper working of a transducer.

Therefore, a transducer that is selected for particular application should not extract force, power (or) energy from the input quantity.

2. Transfer characteristics :-

These characteristics have been divided into

- i, Transfer function
- ii, Error
- iii, Response of transducer

i, Transfer function :-

The transfer function of a transducer defines the relation between the input and output quantity and is expressed as,

$$Q_o = f(Q_i)$$

where, Q_o = output quantity
 Q_i = input quantity

The sensitivity of a transducer is defined as the differential quotient,

$$S = \frac{dQ_o}{dQ_i}$$

Some manufacturers specify the scale factor and is still called as sensitivity.

$$\text{Scale factor} = \frac{1}{S} = \frac{dQ_i}{dQ_o}$$

ii, Error :- For example, the output on account of input Q_i has to be Q_o but in practice an output obtained as Q_o' . Then the difference between Q_o & Q_o' is Error.

mathematically, $\text{Error} (E) = q_0' - q_0$.

Again the Error has been divided into 3 categories.

(a) Scale Error (b) Dynamic Error (c) Error due to Noise

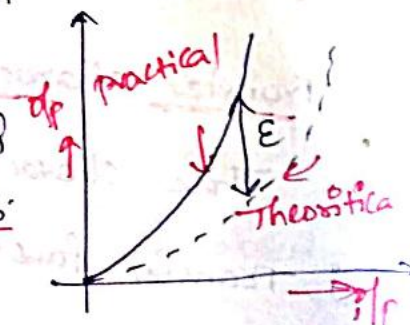
→ Zero Error :-

In this case, the output deviates from the correct value by a constant factor over range of transducer.



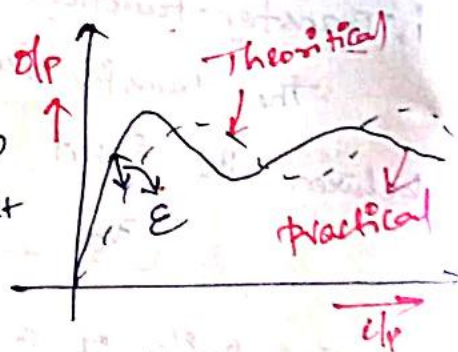
→ Sensitivity Error :-

This error occurs when the observed dp deviates from the correct value by a constant value. Suppose the theoretical dp is q_0 , but in practice it is kq_0 , where k is constant.



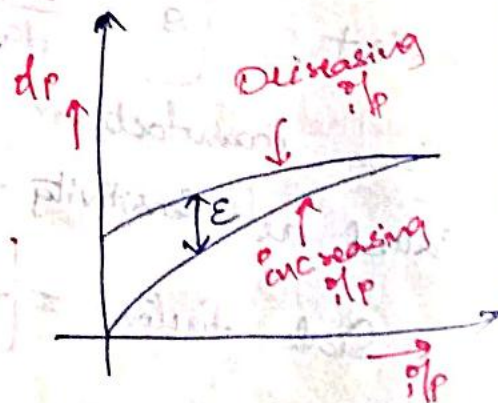
→ Non-Conformity :-

This is the case in which the experimentally obtained transfer function deviates from theoretical T.F for almost every input.



→ Hysteresis :-

All transducers subject to the effects of hysteresis. The dp of a transducer not only depends on ip but also upon input quantity previously applied to it.



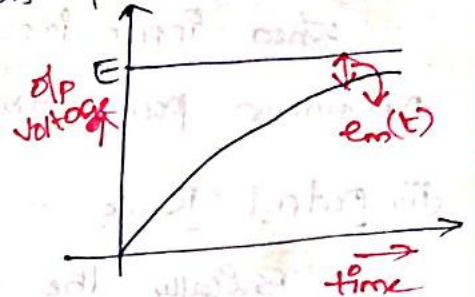
For decreasing values, a greater dp is obtained than the increasing values of same value of input.

(b) Dynamic Error :-

Dynamic Error occurs only when the input quantity is varying with the time. Due to presence of storage elements the output cannot follow exactly.

for e.g., an R-C series ckt to which an step ip of magnitude E is applied. The voltage across capacitor after a time t is, $e_c = E(1 - e^{-t/\tau})$

$$\text{Now, Dynamic Error } e_m(t) = E - e_c \\ = E e^{-t/\tau}$$



(c) Error due to Noise :-

Noise ~~from~~ originating from the transducer which vary with the time are superimposed on the output signal. Due to this the output may not get exact according to input.

(ii) Transducer Response to Environmental Influences :-

The response of transducer to environmental influences is of a great importance. Generally, the performance of transducer fully depends on T.F and Errors provided that transducer is in constant environment. If the transducers are subjected to environmental disturbances, precautions are to be taken, so that, changes in T.F and Errors do not occur.

3. Output Characteristics :-

These are classified into i) type of o/p, ii) o/p impedance and iii) useful range

i) Type of output :-

Generally, the output of any transducer is electrical in nature. The electrical output may be voltage, current (or) impedance. These output quantities may (or) may not be acceptable to the later stages. So, they have to be manipulated by signal conditioning circuit.

(i) Output Impedance :-

The loading effect of a transducer decreases with decreasing the output impedance.

A transducer acts as a constant voltage source if its output impedance is lower than the input impedance.

When input impedance is equal to the output impedance then maximum power transfer will take place.

(ii) Output Range :-

Basically, the output range of a transducer are very low, hence these are not allowed to drive the systems due to low signal and noise distortion.

To overcome such problems, the transducer is inbuilt with amplifier at output which strengthens the signal level.

* Principle of Operation :-

(a) Resistive Transducers :-

In general, the resistance of a metallic conductor is expressed by a simple equation, $R = \frac{\rho L}{A}$.

Any method of varying one of the quantities involved in the above relation can be the design basis of electrical resistive transducer. There are number of ways in which resistance can be changed by a physical phenomenon. The Potentiometer works on the principle of change in the value of resistance with change in length of the conductor. Strain Gauge works on the principle that resistance of a conductor changes when strained. This property can be used for the measurement of displacement, force and the pressure.

The Resistivity of the material changes with the change of temperature thus causing a change in Resistance. This property may be used for the measurement of temperature.

Thus, an Electrical Resistive transducers have a wide variety of applications.

⑤ Inductive Transducers :-

The Inductive Transducers is generally works on the following principles (i) Self Inductance.
(ii) Mutual Inductance.

Inductive Transducer is a simple and most popular type of displacement transducer in which the variation of inductance as a function of displacement is achieved by self inductance

(or) mutual inductance.

In general, the value of self inductance is,

$$L = \frac{N^2}{S}$$

But $S = \text{Reluctance of coil} = \frac{l}{\mu a}$

$$\therefore L = \frac{N^2 \mu a}{l}$$

where, N = Number of turns of a coil

μ = permeability of core

a = Area of magnetic circuit

l = length of the magnetic circuit.

Thus, the variation of self inductance may be achieved

- due to,
- (i) change in number of turns.
 - (ii) change in Reluctance
 - (iii) change in permeability.

Similarly, the mutual inductance between the two coils is,

$$M = k \cdot \sqrt{L_1 \cdot L_2}$$

where, k = Coefficient of Coupling

Thus, the mutual inductance can be varied by varying any of the self inductances or by varying coefficient of coupling.

In general, inductive transducers are used for measurement of displacement, force, pressure, velocity etc.

© Capacitive Transducers :-

The Capacitive Transducers are basically works on the principle of change in capacitance

In general, the capacitances used in capacitive transducers are

i) Parallel Plate Capacitor :-

The capacitance of parallel plate capacitor is given by,

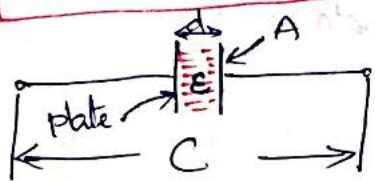
$$C = \frac{\epsilon A}{d} \text{ F}$$

Where, $\epsilon = \epsilon_0 \epsilon_r \Rightarrow \epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$

ϵ_r = Relative permittivity

A = cross sectional area of plates

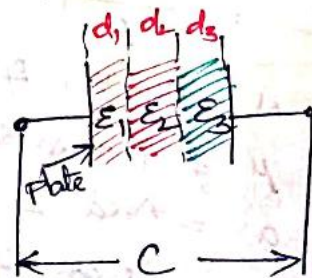
d = plate separation.



ii) Composite Capacitor :-

This capacitance consists of more than one dielectric medium in between the plates.

It consists of 3 layers of dielectrics having relative permeabilities of ϵ_1, ϵ_2 & ϵ_3



∴ Capacitance

$$C = \frac{\epsilon_0 A}{\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2} + \frac{d_3}{\epsilon_3}} \text{ F}$$

iii) Cylindrical Capacitor :-

In this system, the plates are cylindrical which are separated by dielectric material

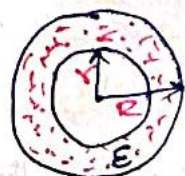
R = Radius of outer cylinder

r = Radius of inner cylinder

l = length of the cylinders

∴ Capacitance

$$C = \frac{2\pi \epsilon l}{\ln\left(\frac{R}{r}\right)} \text{ F}$$



* LVDT (Linear Variable Differential Transformer) :-

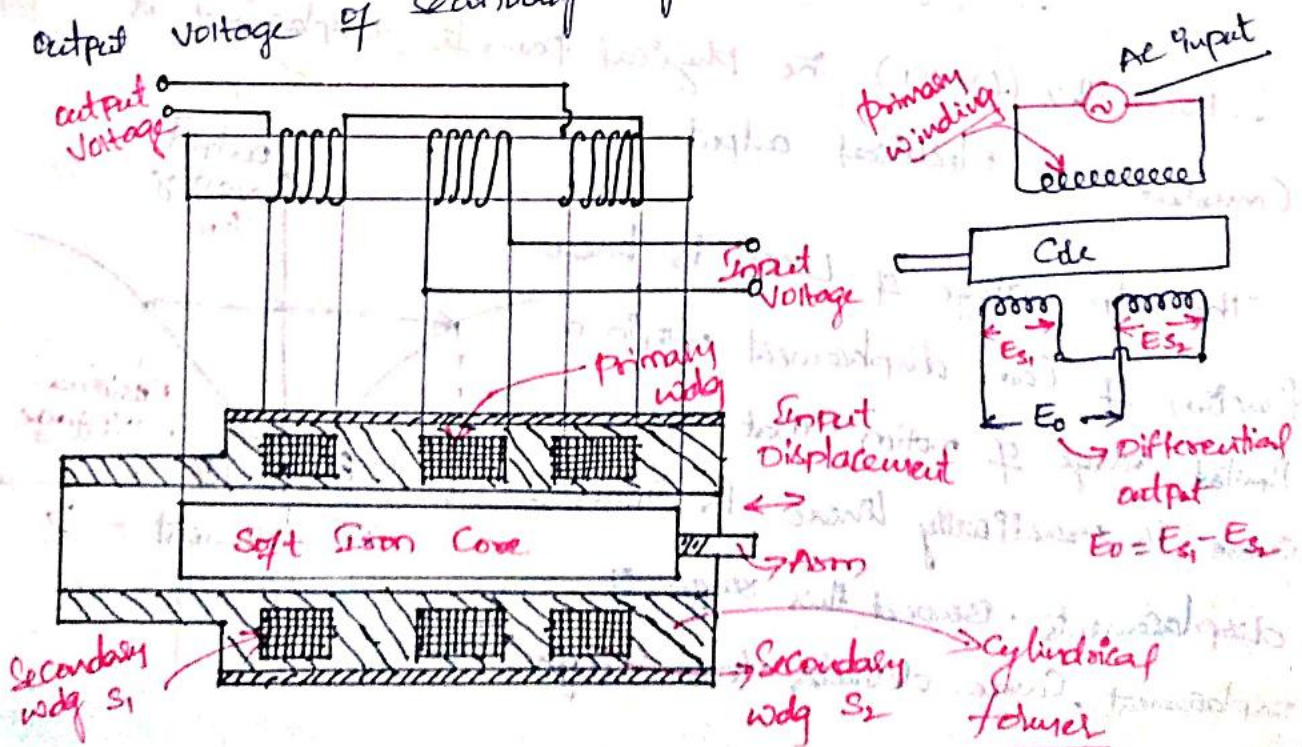
The most widely used inductive transducer to translate the linear motion into electrical signals is "linear variable differential transformer". The basic construction of LVDT is shown below. It consists of a single primary winding and two secondary windings wound on a cylindrical former.

The secondary windings have equal number of turns and are identically placed on either side of primary winding.

A primary winding is connected to the alternating current source. A movable soft iron core is placed inside the former. In practice the core is made up of nickel iron and gives low harmonics and high sensitivity. The frequency of applied ac source may be between 50Hz to 20kHz.

Since, the primary winding is excited by an alternating source, it produces an alternating magnetic field which induces an alternating voltage in two secondary windings (S_1, S_2).

The output voltage of secondary wdg (S_1) is E_{S1} and similarly, output voltage of secondary wdg (S_2) is E_{S2} .



In order to Convert the outputs from S_1 and S_2 into a single voltage signal, the two secondary terminals S_1 and S_2 are connected in Series Opposition as shown in above figure.

Therefore, the output voltage of a transducer is the difference of two voltages E_{S_1} and E_{S_2} .

$$\therefore E_0 = E_{S_1} - E_{S_2}$$

When the core is at its normal position then the flux linking in both the windings are same. Thus, the output voltage (E_0) is zero at Null position of soft iron core.

If the core moved to left of the Null position then the flux linkages at S_1 is more and less at S_2 . Accordingly output voltage of S_1 (E_{S_1}) is more than E_{S_2} .

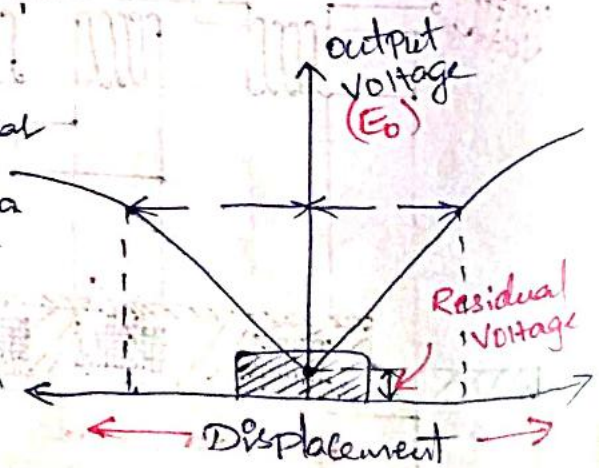
$$\therefore E_0 = E_{S_1} - E_{S_2}$$

Similarly, if the core is moved to right of Null position then flux linking at S_2 is more and less at S_1 . Now, voltage at secondary wdg S_2 (E_{S_2}) is more than E_{S_1} .

$$\therefore E_0 = E_{S_2} - E_{S_1}$$

\therefore From this (LVDT) the physical parameter Displacement is converted to Electrical output.

The o/p voltage of LVDT is linear function of core displacement within a limited range of motion. That means, curve is practically linear for small displacements. Beyond this range of displacement, curve deviates from straight line.



Advantages :-

1. LVDT has good linearity and high sensitivity
2. It consumes less power.
3. LVDT is rugged in construction and can withstand shocks and vibrations.

Disadvantages :-

1. Has less dynamic response
2. LVDT is sensitive to stray magnetic fields.

Application :-

Generally used to measure displacement, pressure, force.

* Strain Gauge :-

Strain Gauges are the devices whose resistance changes under the application of force (or) strain. It is often easy to measure the parameters like length, displacement, weight etc, which can be expressed physically. But the parameters like strain, stress that cannot be really sensed directly by any instrument. For such cases special devices called Strain Gauges.

If a metal conductor is stretched (or) compressed, its resistance changes on account of the fact that both the length and diameter of the conductor changes. Also, there is a change in the value of resistivity of conductor when subjected to strain, a property called "Piezo-Resistive" effect.

⇒ Positive strain :-

When a strain gauge is subjected to tension, it is said to be positively strained. When strain gauge subjected to positive strain, its length increases and Area of cross-section decreases.

→ Negative Strain :

When a strain gauge is subjected to compression, it is said to be negatively strained. Under this condition, the conductor length decreases and area of cross section increases.

Now, whenever a positive strain is applied to strain gauge, its length increases and area of cross section decreases. As we know that Resistance is directly proportional to length and is inversely proportional to Area of cross section. So, there will be a change in Resistance. Due to that change an output is obtained in the form of Electrical quantity by a transducer known as strain gauge.

This can be explained by a Wheatstone bridge by placing strain gauge in the place of unknown resistance R.

from the given Wheatstone's bridge,

P, Q are Ratio arms. S = Standard Resistance and a strain gauge is considered and designed with a Resistance R such that the bridge is in balanced condition.



Now whenever the force weight, velocity etc, the physical input is applied to strain gauge, it experiences a stress and therefore the resistance of given strain gauge will be changes. Now according to the change in Resistance of the strain gauge the voltmeter reads the deflection which gives Electrical output quantity namely Voltage.

* Gauge factor :-

The Gauge factor of a strain gauge is defined as the ratio of change in Resistance to the change in length.

$$G_f = \frac{\Delta R/R}{\Delta L/L}$$

As we know that, strain gauges are works based on the principle of Resistive transducer.

$$\therefore R = \frac{PL}{A} \quad \text{--- (1)}$$

Now, differentiate (1) w.r.t. stress 'S'.

$$\therefore \frac{dR}{dS} = \frac{P}{A} \frac{dL}{dS} - \frac{PL}{A^2} \frac{dA}{dS} + \frac{L}{A} \frac{dP}{dS} \quad \text{--- (2)}$$

Dividing on both sides the eq (2) with R.

$$\therefore \frac{1}{R} \frac{dR}{dS} = \frac{1}{L} \frac{dL}{dS} - \frac{1}{A} \frac{dA}{dS} + \frac{1}{P} \frac{dP}{dS} \quad \text{--- (3)}$$

from the above equation

$\frac{dL}{L}$ = change in length, $\frac{dA}{A}$ = change in Area,

$\frac{dP}{P}$ = change in Resistivity.

\therefore The change in Resistance $\frac{dR}{R}$ is due to any of 3 parameters

given above

As we know that, $A = \frac{\pi}{4} D^2$ --- (4)

$$\Rightarrow \frac{dA}{dS} = 2 \frac{\pi}{4} D \frac{dD}{dS} \quad \text{--- (5)}$$

$$\Rightarrow \frac{(5)}{(4)} \Rightarrow \frac{1}{A} \cdot \frac{dA}{dS} = \frac{2 \frac{\pi}{4} D \frac{dD}{dS}}{\frac{\pi}{4} D^2} = \frac{2}{D} \frac{dD}{dS}$$

$$\frac{1}{A} \cdot \frac{dA}{dS} = \frac{2}{D} \frac{dD}{dS} \quad \text{--- (6)}$$

Now, Substitute (6) in (3)

$$\Rightarrow (3) \Rightarrow \frac{1}{R} \frac{dR}{dS} = \frac{1}{L} \frac{dL}{dS} - \frac{2}{D} \frac{dD}{dS} + \frac{1}{P} \frac{dP}{dS} \quad \text{--- (7)}$$

Now, from the Poisson's Ratio $\nu = - \frac{\text{lateral strain}}{\text{longitudinal strain}}$

$$\Rightarrow \nu = - \frac{dD/D}{dL/L} \quad \left[\because dD = \text{change in diameter} \right]$$

$$\Rightarrow \frac{dD}{D} = -\nu \frac{dL}{L} \quad \text{--- (8)}$$

Substituting (8) in (7) we get,

$$\Rightarrow \frac{1}{R} \frac{dR}{ds} = \frac{1}{L} \frac{dL}{ds} + 2\nu \frac{dL}{ds} + \frac{1}{P} \frac{dP}{ds}$$

Now, for small variations of ds , the above equation will be

$$\Rightarrow \frac{\Delta R}{R} = \frac{\Delta L}{L} + 2\nu \frac{\Delta L}{L} + \frac{\Delta P}{P} \quad \text{--- (9)}$$

As we know that, $G_f = \frac{\Delta R/R}{\Delta L/L}$

$$\Rightarrow \frac{\Delta R}{R} = G_f \cdot \frac{\Delta L}{L} \quad \left[\because \epsilon = \frac{\Delta L}{L} = \text{strain} \right]$$

$$= G_f \cdot \epsilon$$

\therefore Gauge factor $G_f = \frac{\Delta R/R}{\epsilon}$

$$= \frac{\frac{\Delta L}{L} + 2\nu \frac{\Delta L}{L} + \frac{\Delta P}{P}}{\frac{\Delta L}{L}}$$

$$\therefore G_f = \frac{1 + 2\nu + \left(\frac{\Delta P}{P}\right)}{\epsilon}$$

Note :-

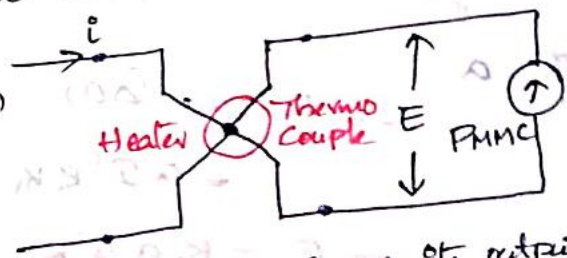
1. The strain gauges work on the principle of change in Resistance due to applied physical quantity.
2. The principle of change in Resistance can also be used for the calibration of force, pressure, displacement, weight etc.
3. As we have positive strain and Negative strain. In both of the cases the resistance of a strain gauge (or) Metallic Conductor changes. Due to this change in Resistance the output (Electrical quantity) will be obtained.

* Thermo Couples :-

The most common Electrical method of temperature measurement uses the Thermo - Electric sensor, also known as Thermocouple. The basic principle of temperature measurement using a Thermo Electric sensor was discovered by Seebeck in 1821.

According to this principle when two dissimilar metals having different temperatures are placed together, a voltage is generated at the junction which is nearly proportional to the temperature. This junction is known as Thermocouple. This principle is used to convert heat energy to Electrical Energy at the junction of two conductors.

The Heat at the junction is produced by the Electrical Current flowing in the Heater Element while the Thermocouple produces an emf at its output terminals which can be measured with the help of PMMC.



The EMF produced is proportional to the temperature. The Thermocouple type of instruments can be used for both AC and DC applications. These instruments are very accurate above a frequency of 50 MHz.

Now, the thermal EMF developed in a circuit composed of two dissimilar metals with junctions kept at absolute temperatures of T_1 and T_2 may be written as

$$E = a(T_1 - T_2) + b(T_1 - T_2)^2$$

where, a, b are constant values of two dissimilar metals.

$$\Rightarrow E = a(\Delta\theta) + b(\Delta\theta)^2 \quad \text{--- (1)}$$

where $\Delta\theta = T_1 - T_2$ = difference in temperatures of hot and cold junctions.

As we know that, $\Delta\theta = T_1 - T_2$

where T_1 = Temperature of hot metal which is directly proportional to $I^2 R$.

T_2 = Temperature of cold junction which is usually zero.

$$\therefore \Delta\theta = T_1 \propto I^2 R \quad \text{--- (2)} \Rightarrow \Delta\theta = K_1 I^2 R$$

From the equation (1) $\Rightarrow E = a(\Delta\theta) + b(\Delta\theta)^2$

Usually, the value of b is very small as compared to the value of a .

$$\therefore E = a(\Delta\theta) = a I^2 R K_1$$

$$\Rightarrow E = K_1 a I^2 R \quad \text{--- (3)} \Rightarrow \text{Amount of EMF developed at the junction.}$$

Now, the deflection of PMMC (θ) is directly proportional to the voltage generated.

$$\therefore \theta \propto E$$

$$\Rightarrow \theta = K_2 E$$

$$= K_2 K_1 a I^2 R$$

$$\Rightarrow \theta = K_3 I^2$$

$$\left[\text{where } K_3 = K_1 K_2 a R \right]$$

$$\Rightarrow \theta \propto I^2$$

From the above relation, we can say that deflection of PMMC is directly proportional to square of current.

Advantages :-

1. Thermocouples are cheaper than Resistance Thermometers.
2. Thermocouples are Very convenient for measuring the temperature.

Disadvantages :-

1. They have low accuracy.
2. As the Thermocouple is placed remote from measuring devices, the Circuitry is very complex.

* Thermistors :-

Thermistor is a contraction term of Thermal Resistors. Thermistors are generally composed of semi-conductor materials. Although positive temperature coefficient resistances are available, most thermistors have a negative temperature coefficient resistance. Positive coefficient temperature resistances increase with increase in temperature, where negative coefficient temperature resistances decrease with increase in temperature.

The negative temperature coefficient of resistance can be as large as several percent per degree Celsius. This allows the thermistor circuits to detect very small changes in temperature.

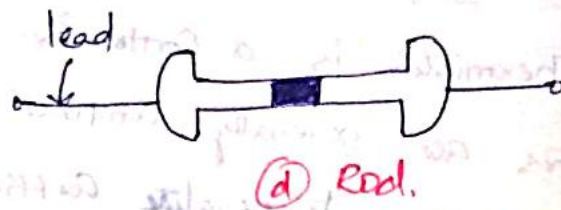
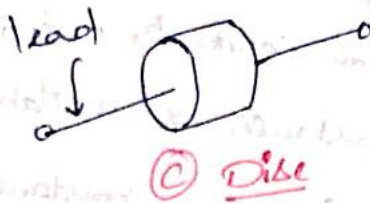
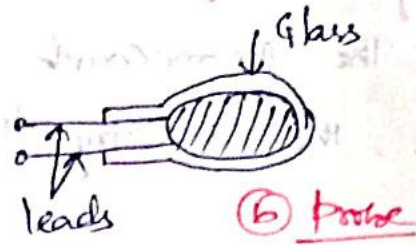
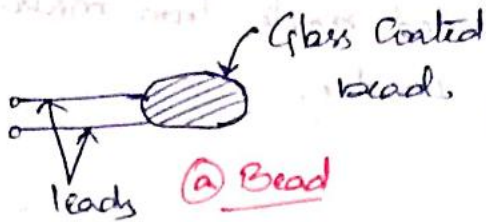
Thermistors are widely used in applications which involve measurements in the range of -60°C to 15°C . The resistance of thermistors ranges from 0.5Ω to $0.75\text{M}\Omega$.

The resistance of thermistor in the form of bead is smallest in size. Beads may be sealed in the tips of solid glass rods to form 'probes' which may be easier to insert than the beads.

Construction :-

Thermistors are composed of mixture of metallic oxides such as manganese, nickel, cobalt, copper, iron and uranium. They are available in variety of sizes and shapes.

Thermistors may be in the form of beads, rods, discs.



Thermistors are mainly used in the applications of

1. Measurement of temperature
2. Measurement of Thermal Conductivity
3. Measurement of Vacuum
4. Measurement of level, flow & pressure of liquids.

* Piezo - Electric Transducer :-

A Piezo-Electric material is one in which an electric potential appears across certain surfaces of crystal, if the dimensions of the crystal are changed by a application of mechanical force. This potential is produced by the displacement of charges. If a varying potential is applied to proper axis of the crystal, it will change the dimensions of the crystal.

Common Piezo Electric materials include Rochelle Salt, Lithium Sulphate, Quartz, dipotassium tartrate. The materials that exhibit a significant and useful piezo-electric effect are divided into two categories, (i) Natural group (ii) Synthetic group.

Quartz, Rochelle Salt are belong to Natural group while the materials like Lithium Sulphate, dipotassium tartrate belong to Synthetic group.

A Piezo Electric Element used for converting mechanical motion to Electrical signal. mechanical deformation generates a charge and this charge appears as voltage across the Electrodes

∴ The voltage $E = \frac{Q}{C}$

A Piezo Electric Element is a direction sensitive. A tensile force produces a voltage of one polarity and while a compressive force produces a voltage of opposite polarity.

The Polarity of induced charges depends upon the direction of applied force

∴ charge $Q = d \times F$ Coloumbs ①

where d = charge sensitivity

F = applied force

This force causes a change in thickness of crystal.

⇒ $F = \frac{AE}{t} \Delta t$ Newton ②

where A = Area of crystal

E = Young's modulus

t = thickness of crystal

from the above equation

$$(2) \Rightarrow E = \frac{Ft}{A \Delta t} \text{ N/m}^2$$

Now, Substitute (2) in (1)

$$(1) \Rightarrow Q = dAE (\Delta t/t) \quad (3)$$

As we know that, the charge at electrodes gives rise to output voltage E_o .

$$\therefore E_o = \frac{Q}{C_p} \quad (4)$$

where C_p = capacitance between the electrodes

$$= \frac{\epsilon_o \epsilon_r \cdot A}{t} \quad (5)$$

Substitute (1) and (5)

in (4)

$$\therefore E_o = \frac{dF}{\epsilon_r \epsilon_o A/t}$$

$$= \frac{dt}{\epsilon_o \epsilon_r} \cdot \frac{F}{A}$$

But, $\frac{F}{A} = \text{Pressure}$

$$\therefore E_o = \frac{dt}{\epsilon_o \epsilon_r} \cdot P$$

$$\Rightarrow E_o = g \cdot t \cdot P$$

output voltage b/w the crystals

where g = voltage sensitivity of a crystal.

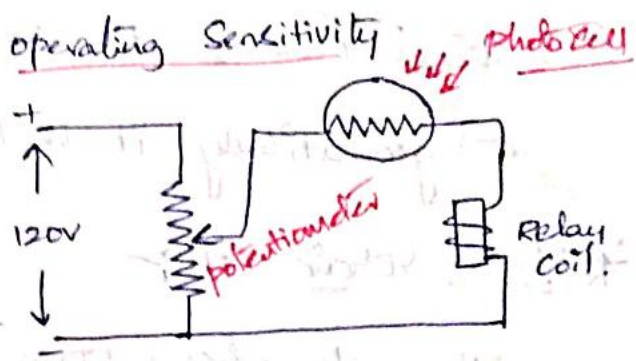
* Photo Conductive Cell :-

Another Piezo-Electric Effect that has proved very useful in photo conductive effect, which is known as photo-conductive cell.

In this type of device, the Electrical Resistance of the material varies with amount of light energy striking it.

A typical control circuit utilizing a photoconductive cell is shown in the figure. A potentiometer is used to make adjustments to compensate for manufacturing tolerances in the photo cell sensitivity and relay operating sensitivity.

When the photocell has the appropriate light incident upon it, its resistance is low and current through the relay is consequently high to operate the relay.



When the light is interrupted (or) shut off partially (or) completely, the resistance of photocell increases thereby reducing the current through the relay. The current may drop to a level where it will not be able to operate the relay and thereby de-energizing the relay.

* Photo Diode :-

A reverse biased semi-conductor diode that passes a small leakage current if the junction is not exposed to light. Under illumination, however, the current rises almost in direct proportion to the light intensity. Thus, a photo diode is used in applications similar to those in a photoconductive cell.

When the device operates with a reverse voltage applied, it functions as photoconductive device. When operating without reverse voltage, it functions as photo voltaic cell. It is also possible to arrange for a photo diode to change from photo-conductive mode to photo voltaic mode.

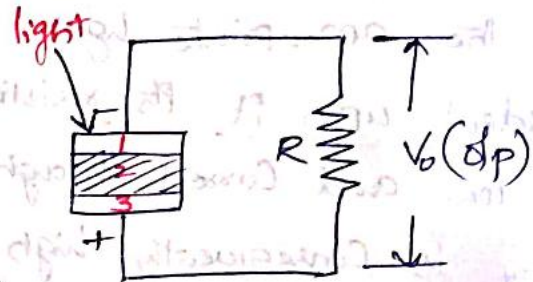
Photo diode has many advantages over photo conductive cell. one of the advantage is Response time is much faster, so that it may be used in applications in which light fluctuations occurs due to high frequencies.

Diagrammatically, it is represented as,



* Photo-voltaic Cell :-

These transducers works on the principle of photo-voltaic effect i.e. when the light strikes



a junction of certain dissimilar metals, a voltage is generated.

As shown in above figure, it has a sandwich construction, consisting of metal base plate (3), a semi-conductor material (2), a transparent thin metal layer (1). A voltage is generated due to incidence of light.

The semi-conductor material used for the construction of photo-voltaic cell is of either Silicon (or) Selenium.