03-01-19

UNIT-I

SIGNAL : Signal is a physical quantity which varies with time, space and independent variables.

→ Signal is nothing but which carries some information

EX : ECG [ElectroCardioGram] — Heart

EEG [ElectroEncephaloGram] — Brain

→ If a signal depends on one independent variable, it is called a one dimensional signal. (Speech signal)

→ If a signal depends on two independent variables, it is called two dimensional signal (Image).

CLASSIFICATIONS OF SIGNALS :

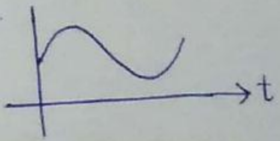
1> Continuous time signal

2> Discrete time signal

3> Digital signal

1> CONTINUOUS TIME SIGNAL :

→ The signals that are defined for every instant of time are known as continuous time signal.



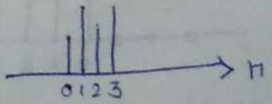
→ In CTS, both time and amplitude are continuous in nature

→ It is denoted by $x(t)$.

2> DISCRETE TIME SIGNAL :

→ The signals that are defined for particular instant of time are known as Discrete time signal.

→ For DTS, Time is discrete and amplitude is continuous



→ It is denoted by $x(n)$.

3> DIGITAL SIGNAL :

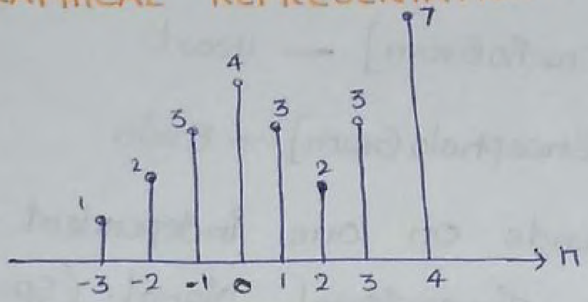
→ For digital signals, both time and amplitude are discrete.

REPRESENTATION OF DISCRETE TIME SIGNALS

1> SEQUENCE REPRESENTATION :

$$x(n) = \left\{ 1, 2, 3, \underset{\uparrow}{4}, 3, 2, 3, 7 \right\}$$

2> GRAPHICAL REPRESENTATION :



3> TABULAR METHOD :

n	-3	-2	-1	0	1	2	3	4
x(n)	1	2	3	4	3	2	3	7

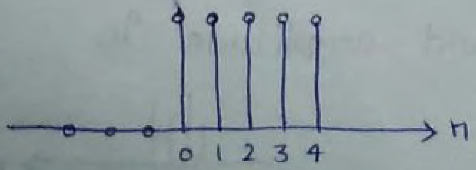
4> FUNCTIONAL METHOD :

$$x(n) = \begin{cases} 1 & \text{for } n = -3 \\ 2 & \text{for } n = \pm 2 \\ 3 & \text{for } n = \pm 1, 3 \\ 4 & \text{for } n = 0 \\ 7 & \text{for } n = 4 \end{cases}$$

STANDARD DISCRETE TIME SIGNALS :

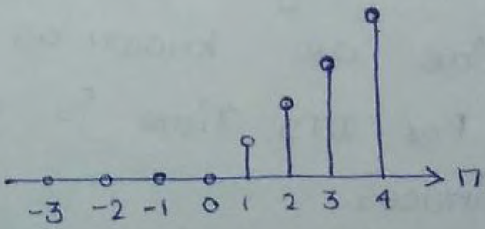
1> UNIT STEP :

$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$



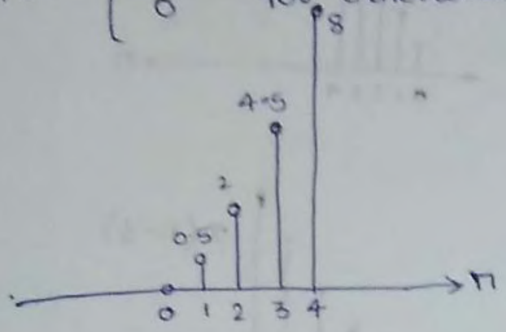
2> UNIT RAMP :

$$r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for otherwise} \end{cases}$$



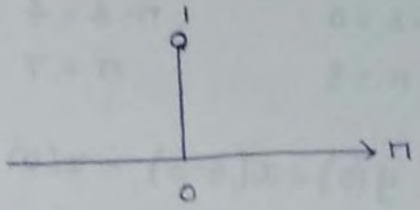
3> UNIT PARABOLA :

$$p(n) = \begin{cases} n^2/2 & \text{for } n \geq 0 \\ 0 & \text{for otherwise} \end{cases}$$



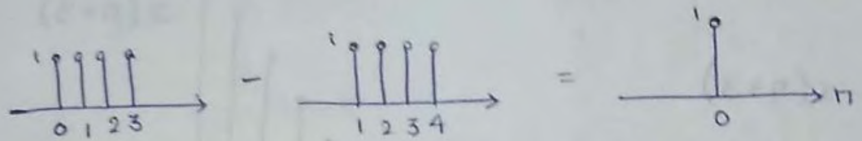
4> UNIT IMPULSE OR UNIT SAMPLE SEQUENCE

$$\delta(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for otherwise} \end{cases}$$



*** SOME PROPERTIES OF UNIT IMPULSE :

1> $u(n) - u(n-1) = \delta(n)$



2> $\delta(n-k) = \begin{cases} 1 & \text{for } n = k \\ 0 & \text{for } n \neq k \end{cases}$

3> $x(n) = \sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k)$

BASIC OPERATIONS OF DISCRETE TIME SIGNALS :

1> Time Shifting
2> Time Reversal / Folding
3> Time Scaling
4> Amplitude Scaling
5> Signal addition
6> Signal multiplication

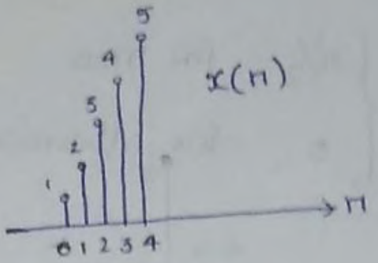
1> TIME SHIFTING :

The Time Shifting of a Signal may result in time delay or time advance.

$$y(n) = x(n-k)$$

Ex : $x(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 5\}$

$k = 3$ $\quad y(n) = x(n-3)$



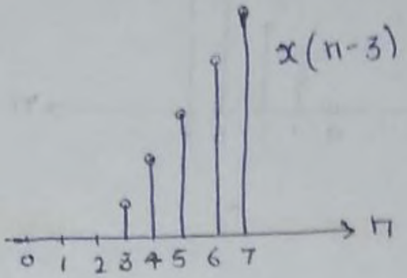
Starting point $\quad$ End point

$n-3 = 0 \qquad n-3 = 4$

$n = 3 \qquad n = 7$

$y(3) = x(3-3) = x(0)$

$y(3) = 1$

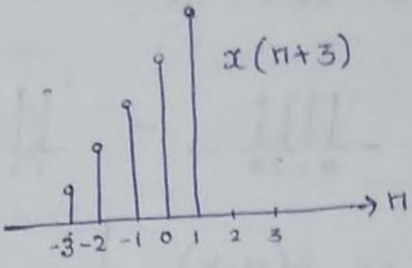


$y(4) = x(4-3) = x(1)$

$y(4) = 2$

$k = -3$

$y(n) = x(n+3)$



$n+3 = 0 \qquad n+3 = 4$

$n = -3 \qquad n = 1$

$y(-3) = x(-3+3) = x(0) = 1$

$y(-2) = x(-2+3) = x(1) = 2$

→ If K = +ve, It is delayed and shifted to the right side by K units.

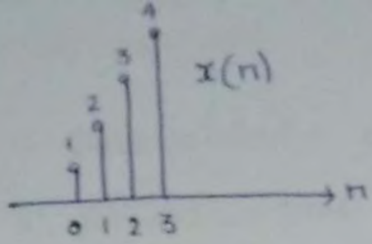
→ If K = −ve, It is advanced and shifted to the left side by K units.

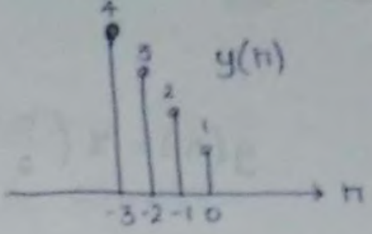
2> TIME REVERSAL/TIME FOLDING :

The Time reversal Signal is the reflection of the original signal and It is obtained by replacing the independent variable 'n' by '−n'.

$x(n) = \{\underset{\substack{\uparrow \\ 0}}{1}, 2, 3, 4\}$

$y(n) = x(-n) = \{4, 3, 2, \underset{\substack{\uparrow \\ 0}}{1}\}$





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4> AMPLITUDE SCALING :

$y(n) = a\,x(n)$ $\quad x(n) = \{1, 2, 3, 4\}$

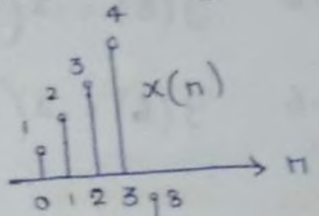
$\downarrow$

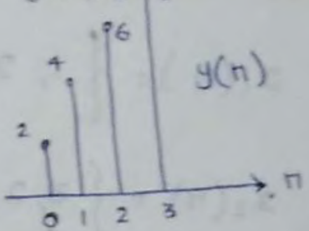
constant

$a = 2$

$y(n) = 2 \cdot \{1, 2, 3, 4\}$

$= \{2, 4, 6, 8\}$





3> TIME SCALING :

$y(n) = x(an)$

$a > 1 \longrightarrow$ Time compression

$a < 1 \longrightarrow$ Time expansion

a>1

$a = 2$ $\quad n = 0$

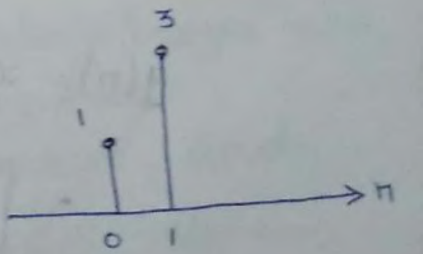
$y(0) = x(2 \cdot 0) = x(0) = 1$

$n = 1$

$y(1) = x(2 \cdot 1) = x(2) = 3$

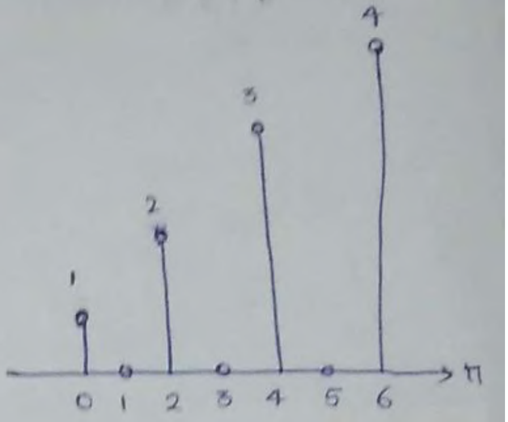
$n = 2$

$y(2) = x(2 \cdot 2) = x(4) = 0$



$a < 1$

$a = \frac{1}{2}$ $\quad\quad y(n) = x\left(\frac{n}{2}\right)$



$n=0 \quad y(0) = x\left(\frac{0}{2}\right) = x(0) = 1$

$n=1 \quad y(1) = x\left(\frac{1}{2}\right) = 0$

$n=2 \quad y(2) = x\left(\frac{2}{2}\right) = x(1) = 2$

$n=3 \quad y(3) = x\left(\frac{3}{2}\right) = 0$

$n=4 \quad y(4) = x\left(\frac{4}{2}\right) = x(2) = 3$

$n=5 \quad y(5) = x\left(\frac{5}{2}\right) = 0$

$n=6 \quad y(6) = x\left(\frac{6}{2}\right) = x(3) = 4$

5> SIGNAL ADDITION:

$x_1(n) = \left\{1, 2, \underset{\uparrow}{3}, 4, 5\right\}$

$x_2(n) = \left\{\underset{\uparrow}{3}, 2, 1, 0, 4\right\}$

$y(n) = x_1(n) + x_2(n)$

$= \left\{1, 2, \underset{\uparrow}{6}, 6, 6, 0, 4\right\}$

6> SIGNAL MULTIPLICATION :

$y(n) = x_1(n) \times x_2(n)$

$= \left\{0, 0, \underset{\uparrow}{9}, 8, 5, 0, 0\right\}$

CLASSIFICATION OF DISCRETE TIME SIGNALS

1> Deterministic and Random signal
2> periodic and Aperiodic signal
3> Energy and power signal
4> causal and Noncausal signal
5> Even and odd signal

1> DETERMINISTIC AND RANDOM SIGNAL

DETERMINISTIC SIGNAL:

A Deterministic signal can be completely represented by mathematical equation.

Ex: Sinusoidal signal, cos signal etc.,

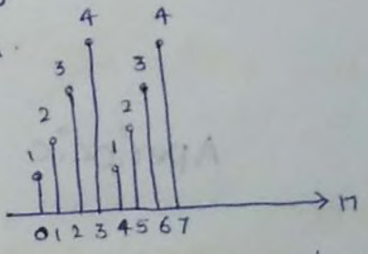
NON-DETERMINISTIC SIGNAL OR RANDOM SIGNAL:

A Random signal cannot be represented by any mathematical equation

Ex: Noise

2> PERIODIC AND APERIODIC SIGNAL

PERIODIC SIGNAL: A signal which has a particular pattern and repeats itself at regular intervals of time is called a periodic signal.



$$x(n) = x(n+N)$$

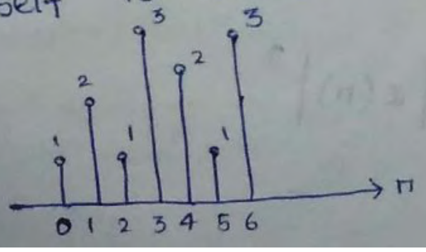
where N = Fundamental period

$$N = \frac{2\pi}{\omega}(m)$$

m – Smallest integer value

ω – frequency

APERIODIC SIGNAL: A signal which doesnot repeats its pattern itself is called a Apeiodic signal.



Ex :

1> $x(n) = e^{j6\pi n}$

periodic $\omega = 6\pi$

$$N = \frac{2\pi}{\omega}(m) = \frac{2\pi}{6\pi}(1)$$

$$N = \frac{1}{3}$$

2> $x(n) = \cos\left(\frac{2\pi}{3}\right)n$

periodic

$$\omega = \frac{2\pi}{3}$$

$$N = \frac{2\pi}{\omega}(m) = \frac{2\pi}{2\pi/3}(1)$$

$$N = 3$$

3> $x(n) = \cos\left(\frac{\pi}{3}\right)n + \cos\left(\frac{3\pi}{4}\right)n$

periodic

$$\omega = \frac{\pi}{3}$$

$$N_1 = \frac{2\pi}{\pi/3}(1)$$

$$N_1 = 6$$

periodic

$$\omega = \frac{3\pi}{4}$$

$$N_2 = \frac{2\pi}{3\pi/4}(3)$$

$$= \frac{8}{3}(3)$$

$$N_2 = 8$$

N = LCM of N_1 and N_2

$$N = 24$$

4> $x(n) = \cos 4n$

Aperiodic

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3> ENERGY AND POWER SIGNALS :

ENERGY SIGNAL :

A signal is said to be an energy signal if and only if its total energy is finite and power is zero

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

EX : $x(n) = \left(\frac{1}{2}\right)^{n} u(n)$

$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

$$\text{Energy, } E = \sum_{n=-\infty}^{\infty} \left| \left(\frac{1}{2}\right)^{n} u(n) \right|^{2} = \sum_{n=0}^{\infty} \left| \left(\frac{1}{2}\right)^{n} \right|^{2} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^{n} = 1 + \frac{1}{4} + \left(\frac{1}{4}\right)^{2} + \left(\frac{1}{4}\right)^{3} + \text{--------}$$

$$= \frac{1}{1-\frac{1}{4}}$$

$$E = \frac{4}{3}$$

$$\text{Power, } P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^{2} = \frac{1}{2\cdot\frac{1}{0}+1} \left(\sum_{n=-N}^{N} |x(n)|^{2} \right)$$

$$P = 0$$

∴ Energy signal

POWER SIGNAL : A signal is said to be power signal if and only if its total power is finite and energy is infinite

$$P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^{2}$$

$$\sum_{n=-N}^{N} 1 = 2N+1$$

EX : $x(n) = e^{j\left(\frac{\pi}{3}n + \frac{\pi}{2}\right)}$

$$\text{Energy, } E = \sum_{n=-\infty}^{\infty} |x(n)|^{2} = \sum_{n=-\infty}^{\infty} \left| e^{j\left(\frac{\pi}{3}n + \frac{\pi}{2}\right)} \right|^{2}$$

$$\left|e^{j(\omega+\theta)}\right| = \left|\cos(\omega+\theta) + j\sin(\omega+\theta)\right| \qquad = \sum_{n=-\infty}^{\infty} 1$$

$$= \sqrt{\cos^{2}(\omega+\theta) + \sin^{2}(\omega+\theta)} \qquad E = \infty$$

$$= 1$$

$$\text{Power, } P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} 1$$

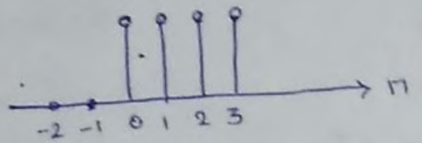
$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \cdot 2N+1$$

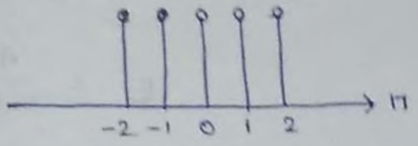
$$= 1$$

∴ power signal

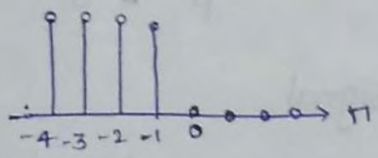
4> CAUSAL AND NON-CAUSAL SIGNALS :

A Discrete time signal $x(n)$ is said to be causal if $x(n) = 0$ for $n < 0$. otherwise Non-causal





A signal that is zero for all $n \geq 0$ is called a anti-causal signal



$x(n) = 0$ for $n \geq 0$

5> EVEN AND ODD SIGNALS :

EVEN SIGNAL: A Discrete time signal $x(n)$ is said to be even or symmetric signal, if it satisfies the condition

$$x(-n) = x(n) \quad \text{for all } n \text{ values}$$

Ex : Cos signal

ODD SIGNAL: A Discrete time signal $x(n)$ is said to be odd or asymmetric signal, if it satisfies the condition.

$$x(-n) = -x(n) \quad \text{for all } n \text{ values}$$

$$x(n) = x_e(n) + x_o(n) \longrightarrow ①$$

Replace n by $-n$

$$x(-n) = x_e(-n) + x_o(-n)$$

$$x(-n) = x_e(n) - x_o(n) \longrightarrow ②$$

① + ②

$$x(n) + x(-n) = 2\, x_e(n)$$

$$x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$

① - ②

$$x(n) - x(-n) = 2\, x_o(n)$$

$$x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$

Ex : $x(n) = \left\{ \overset{-2}{-2}, \overset{-1}{5}, \underset{\uparrow}{\overset{0}{1}}, \overset{1}{3} \right\}$

$$x(-n) = \left\{ \overset{-1}{3}, \underset{\uparrow}{\overset{0}{1}}, \overset{1}{5}, -2 \right\}$$

$$x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$

$$= \frac{1}{2}\left[-2, 8, 2, 8, -2\right]$$

$$\boxed{<\frac{\cancel{1}}{2}\cancel{/}14 \quad \cancel{<}/7} \qquad = \left[-1, 4, 1, 4, -1\right]$$

$$x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$

$$= \frac{1}{2}\left[-2, 2, 0, -2, 2\right]$$

$$= \left[-1, 1, 0, -1, 1\right]$$

SYSTEM

It is a physical device which performs operation on input signal and produces desired output signal.

* Systems are of two types

1> Continuous time system

2> Discrete time system

CLASSIFICATIONS OF DISCRETE TIME SYSTEMS :

1> Static and Dynamic system

2> Causal and Non-causal system

3> Time Variant and Time-Invariant system

4> Linear and Non-linear system

5> Stable and Unstable system

6> FIR (Finite Impulse Response) and IIR (Infinite Impulse Response) system

1> STATIC AND DYNAMIC SYSTEM :

STATIC SYSTEM: A system is said to be Static or memory less system, if the output response depends only on present Input but not on past or future inputs or past outputs.

EX:- $y(n) = x(n)$

$y(n) = 2x^2(n)$

$y(n) = 3x(n)$

DYNAMIC SYSTEM: A system is said to be dynamic or memory system, if the output response depends on past or future inputs or past output

EX :- 1> $y(n) - y(n-1) = x(n) + x(n-1)$

2> $y(n) = y(n-3) + x(n-1) + x(n) + x(n+2)$

3> $y(n) = x(2n)$

2> CAUSAL AND NON-CAUSAL SYSTEMS :

CAUSAL SYSTEM : A System is said to be causal, if the output of the System at instant 'n' depends on present and past values of the input but not on future inputs.

EX:- $y(n) = x(n) + x(n-1)$

$y(n) = 2x(n) + 3x(n-2)$

NON-CAUSAL SYSTEM: A system is said to be non-causal if the output of the system at instant 'n' depends on present , past and future values of the inputs.

EX :- $y(n) = x(n) + x(n-1) + x(n+1)$

$y(n) = x(n) + 2x(n-1) + 3x(n+2)$

3> TIME INVARIANT AND TIME VARIANT SYSTEMS :

TIME INVARIANT SYSTEM : A system is said to be time invariant if the input and output characteristics doesnot change with time.

$$y(n,k) = y(n-k)$$

TIME VARIANT SYSTEM : A system is said to be time variant if the input and output characteristics change with time

$$y(n,k) \neq y(n-k)$$

EX : 1> $y(n) = n\,x(n)$

$$y(n,k) = n\,x(n-k)$$

$n = n-k$

$$y(n-k) = (n-k)\,x(n-k)$$

$$y(n,k) \neq y(n-k)$$

Time variant

2> $y(n) = x(n)$

$$y(n,k) = x(n-k)$$

$$y(n-k) = x(n-k)$$

$$y(n,k) = y(n-k)$$

Time Invariant

3> $y(n) = x^2(n-2)$

$$y(n,k) = x^2(n-k-2)$$

$$y(n-k) = x^2(n-k-2)$$

$$y(n,k) = y(n-k)$$

Time Invariant

4> $y(n) = x\left(\frac{n}{2}\right)$

$$y(n,k) = x\left(\frac{n}{2} - k\right)$$

$$y(n-k) = x\left(\frac{n-k}{2}\right)$$

$$y(k) \neq y(n-k)$$

Time variant

4> LINEAR AND NON - LINEAR SYSTEMS :

LINEAR SYSTEM : A system that satisfies superposition theorem is said to be linear system

$$T\left[a x_1(n) + b x_2(n)\right] = a y_1(n) + b y_2(n)$$

EX : $y(n) = n\,x(n)$

$$y_1(n) = n\,x_1(n)$$

$$y_2(n) = n\,x_2(n)$$

$$a y_1(n) + b y_2(n) = a n x_1(n) + b n x_2(n)$$

$$y_3(n) = T\left[a x_1(n) + b x_2(n)\right]$$

$$= n\left[a x_1(n) + b x_2(n)\right]$$

$$= a n x_1(n) + b n x_2(n)$$

L.H.S = R.HS

Linear System

2) $y(n) = x^2(n)$

$$y_1(n) = x_1^2(n) \qquad y_2(n) = x_2^2(n)$$

$$a\,y_1(n) + b\,y_2(n) = a\,x_1^2(n) + b\,x_2^2(n)$$

$$y_3(n) = T\left[a\,x_1(n) + b\,x_2(n)\right]$$

$$= \left[a x_1(n) + b x_2(n)\right]^2$$

$$= a^2 x_1^2(n) + b^2 x_2^2(n) + 2ab\,x_1(n)\,x_2(n)$$

L·H·S $\neq$ R·H·S

Non - Linear System

5) STABLE AND UNSTABLE SYSTEM :

A System is stable if every bounded input result in bounded output otherwise Unstable system.

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

Ex : $h(n) = \left(\frac{1}{2}\right)^n u(n)$

$$\sum_{n=-\infty}^{\infty} \left|\left(\frac{1}{2}\right)^n u(n)\right| < \infty$$

$$\sum_{n=0}^{\infty} \left|\left(\frac{1}{2}\right)^n\right| < \infty$$

$$1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \text{-----} < \infty$$

$$\frac{1}{1-\frac{1}{2}} < \infty$$

$$\frac{2}{2-1} < \infty$$

$$2 < \infty$$

Stable System

2> $h(n) = u(n)$

$$\sum_{n=-\infty}^{\infty} |u(n)| < \infty$$

$$\sum_{n=0}^{\infty} 1 < \infty$$

$$\infty$$

Unstable system

3> $y(n) = x(n) + \frac{1}{2} x(n-1) + \frac{1}{4} x(n-2)$

$$y(n) = h(n)$$

$$x(n) = \delta(n)$$

$$h(n) = \delta(n) + \frac{1}{2}\delta(n-1) + \frac{1}{4}\delta(n-2)$$

$$\sum_{n=-\infty}^{\infty} \left[\delta(n) + \frac{1}{2}\delta(n-1) + \frac{1}{4}\delta(n-2) \right] < \infty$$

$$1 + \frac{1}{2} + \frac{1}{4} < \infty$$

$$\frac{4+2+1}{8} < \infty$$

$$\frac{7}{8} < \infty$$

stable system

6> FIR [Finite Impulse Response] and IIR [Infinite Impulse Response] SYSTEMS :

FIR SYSTEM: If the Impulse response sequence of the system is of finite duration. then the System is called FIR system

Ex : $h(n) = \{1, 2, 3, 4\}$

$h(n) = \{1, 2, 3, 7, 9, 10\}$

IIR SYSTEM: If the impulse response sequence of the system is of infinite duration, then the system is called IIR system

Ex: $h(n) = a^n u(n)$

CONVOLUTION AND CORRELATION:

The combination of two signals and produces by third signal

$$y(n) = x(n) * h(n)$$

$$y(n) = \sum_{h=-\infty}^{\infty} x(k)\, h(n-k)$$

Convolution is classified into two types

Linear convolution

Circular convolution

LINEAR CONVOLUTION:

Linear convolution is classified into three types

1) Graphical method
2) Tabular method
3) Third / shortcut method

Determine the convolution of two sequences by using graphical method.

$$x(n) = \{\overset{0}{\underset{\uparrow}{3}}, \overset{1}{2}, \overset{2}{1}, \overset{3}{2}\} \rightarrow l=4 \qquad h(n) = \{\overset{-1}{1}, \overset{0}{\underset{\uparrow}{2}}, \overset{1}{1}, \overset{2}{2}\} \rightarrow m=4$$

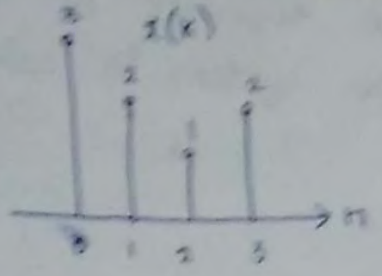
Linear convolution length $y(n) = l + m - 1$

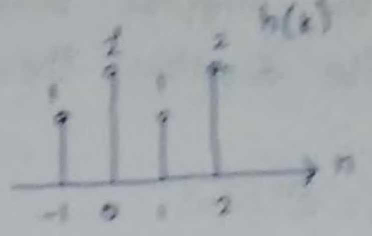
$= 4 + 4 - 1$

$= 7$

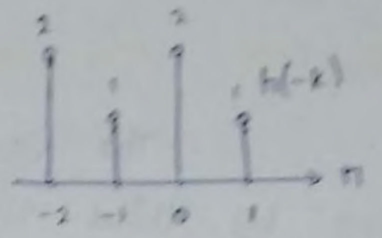
starting point $\longrightarrow n_1 + h_1 = 0 - 1 = -1$

ending point $\longrightarrow n_2 + h_2 = 3 + 2 = 5$





$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\ h(n-k)$$

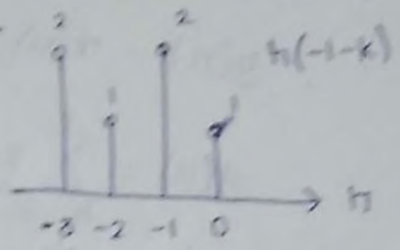


n = -1

$$y(-1) = \sum_{k=-\infty}^{\infty} x(k)\ h(-1-k)$$

$$= 3 \times 1$$

$$y(-1) = 3$$



n = 0

$$y(0) = \sum_{k=-\infty}^{\infty} x(k)\ h(-k)$$

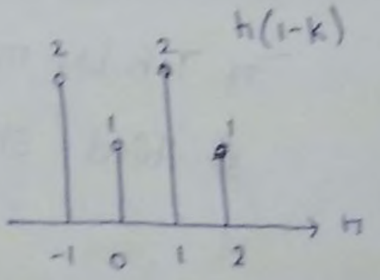
$$= 3 \times 2 + 2 \times 1$$

$$= 6 + 2$$

$$y(0) = 8$$

n = 1

$$y(1) = \sum_{k=-\infty}^{\infty} x(k)\ h(1-k)$$

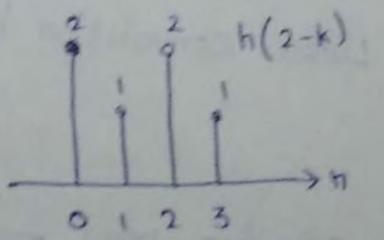


$$= 3 \times 1 + 2 \times 2 + 1 \times 1$$

$$= 3 + 4 + 1$$

$$y(1) = 8$$

n = 2 $$y(2) = \sum_{k=-\infty}^{\infty} x(k)\ h(2-k)$$



$$= 3 \times 2 + 2 \times 1 + 1 \times 2 + 2 \times 1$$

$$= 6 + 2 + 2 + 2$$

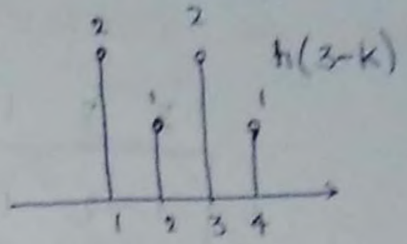
$$y(2) = 12$$

$n=3 \quad y(3) = \sum_{n=-\infty}^{\infty} x(k)\, h(3-k)$

$= 2\times2 + 1\times1 + 2\times2$

$= 4+1+4$

$y(3) = 9$

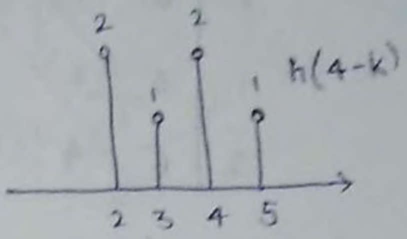


$n=4 \quad y(4) = \sum_{n=-\infty}^{\infty} x(k)\, h(4-k)$

$= 2\times1 + 1\times2$

$= 2+2$

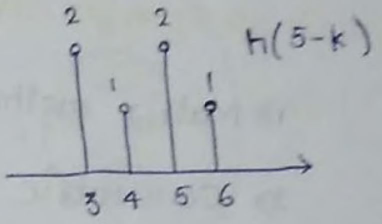
$y(4) = 4$



$n=5 \quad y(5) = \sum_{n=-\infty}^{\infty} x(k)\, h(5-k)$

$= 2\times2$

$y(5) = 4$



TABULAR METHOD :

k	-3	-2	-1	0	1	2	3	4	5	6
x(k)				3	2	1	2			
h(k)			1	2	1	2				
h(-k)		2	1	2	1					
h(-1-k)	2	1	2	1						
h(1-k)			2	1	2	1				
h(2-k)				2	1	2	1			
h(3-k)					2	1	2	1		
h(4-k)						2	1	2	1	
h(5-k)							2	1	2	1

THIRD OR SHORTCUT METHOD

$$\begin{array}{c|cccc} & 3 & 2 & 1 & 2 \\ \hline 1 & 3 & 2 & 1 & 2 \\ 2 & 6 & 4 & 2 & 4 \\ 1 & 3 & 2 & 1 & 2 \\ 2 & 6 & 4 & 2 & 4 \end{array}$$

$$y(n) = \{3, \underset{\uparrow}{8}, 8, 12, 9, 4, 4\}$$

10-01-19

CIRCULAR CONVOLUTION:

1> Matrix method

2> Concentric circle method

3> Using DFT and IDFT

Perform the circular convolution of the following sequences $x(n) = \{\underset{\uparrow}{1}, 1, 2, 1\}$, $h(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

$\llcorner$ length = l $\qquad$ $\llcorner$ length = m

1) MATRIX METHOD:

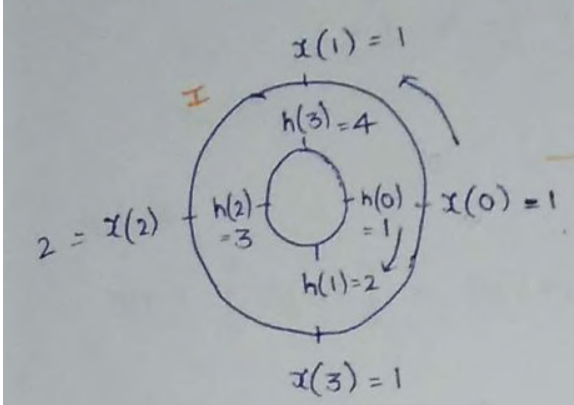
$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

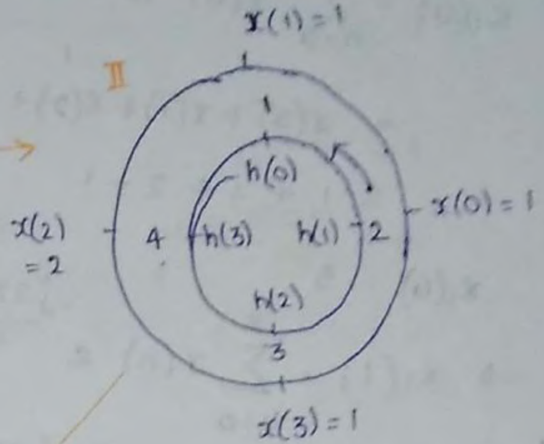
$$y(n)_L = \text{Max}(l, m)$$
$$= \text{Max}(4, 4)$$
$$= 4$$

$$= \begin{bmatrix} 1+4+6+2 \\ 2+1+8+3 \\ 3+2+2+4 \\ 4+3+4+1 \end{bmatrix} = \begin{bmatrix} 13 \\ 14 \\ 11 \\ 12 \end{bmatrix}$$

$$y(n) = [\underset{\uparrow}{13}, 14, 11, 12]$$

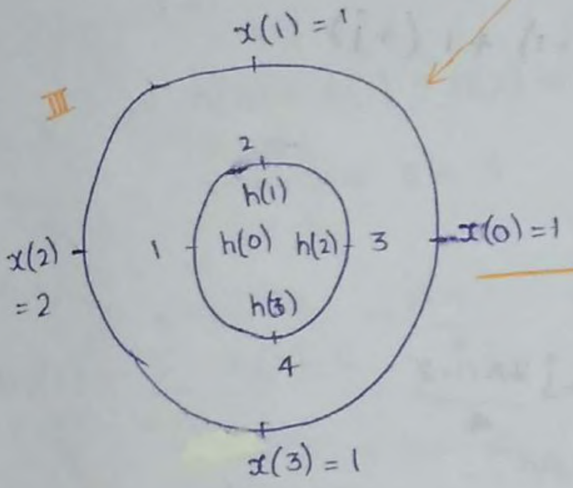
2> CONCENTRIC CIRCLES METHOD :

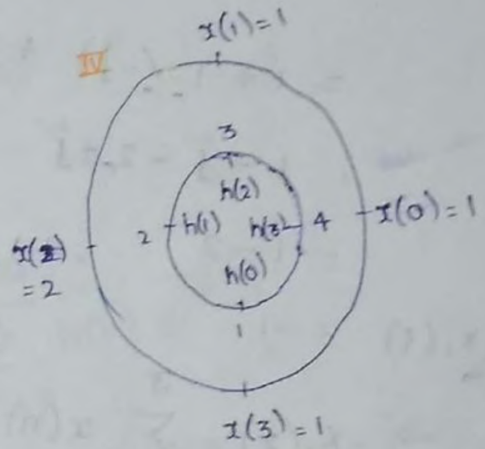




$y(0) = 1 + 4 + 6 + 2 = 13$

$y(1) = 2 + 1 + 8 + 3 = 14$





$y(3) = 3 + 2 + 2 + 4 = 11$

$y(4) = 4 + 3 + 4 + 1 = 12$

$$y(n) = [\underset{\uparrow}{13}, 14, 11, 12]$$

3> USING DFT AND IDFT :

DFT :

$$x(k) = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi nk}{N}}$$

IDFT :

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)\, e^{\frac{j2\pi nk}{N}}$$

N = 4

$$\text{DFT}[x(n)] = X_1(K) = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi nk}{N}}$$

$K = 0, 1, 2, 3$

K=0

$$X_1(0) = \sum_{n=0}^{3} x(n)\, \underbrace{e^{-j\frac{2\pi n\cdot 0}{4}}}_{1}$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1 + 1 + 2 + 1$$

$$X_1(0) = 5$$

K=1 ⇒ $X_1(1) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi n\cdot 1}{4}}$

$$= x(0) + x(1)\, e^{-j\frac{2\pi}{\cancel{4}_2}} + x(2)\, e^{-j\pi} + x(3)\, e^{-j\frac{3\pi}{2}}$$

$$= 1 + 1\cdot(-j) + 2(-1) + 1(+j)$$

$$= 1 - \cancel{j} - 2 + \cancel{j}$$

$$= 1 - 2$$

$$X_1(1) = -1$$

K=2 ⇒ $X_1(2) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi n\cdot \cancel{2}}{\cancel{4}}}$

$$= \sum_{n=0}^{3} x(n)\, e^{-j\pi n}$$

$$= x(0) + x(1)\, e^{-j\pi} + x(2)\, e^{-j\cdot 2\pi} + x(3)\, e^{-j3\pi}$$

$$= 1 + 1\cdot(-1) + 2(1) + 1(-1)$$

$$= 1 - 1 + 2 - 1$$

$$X_1(2) = 1$$

K=3 ⟶ $X_1(3) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{\cancel{2}\pi n\cdot 3}{\cancel{4}2}}$

$$= \sum_{n=0}^{3} x(n)\, e^{-j\frac{3\pi n}{2}}$$

$$= x(0) + x(1)\, e^{-j\frac{3\pi}{2}} + x(2)\, e^{-j3\pi} + x(3)\, e^{-j\frac{9\pi}{2}}$$

$= 1 + 1(j) + 2(-1) + 1(-j)$

$= 1 + j - \cancel{2} - \cancel{j}$

$x_1(3) = -1$

$N = 4$

$$DFT\left[h(n)\right] = H_2(k) = \sum_{n=0}^{N-1} h(n)\, e^{-j\frac{2\pi nk}{N}}$$

$K = 0, 1, 2, 3$

$$H_2(0) = \sum_{n=0}^{3} h(n)\, e^{-j\frac{2\pi n \cdot 0}{4}} = \sum_{n=0}^{3} h(n)$$

$= h(0) + h(1) + h(2) + h(3)$

$= 1 + 2 + 3 + 4$

$= 10$

$$H_2(1) = \sum_{n=0}^{3} h(n)\, e^{-j\frac{2\pi n \cdot 1}{\cancel{4}2}} = \sum_{n=0}^{3} h(n)\, e^{-j\frac{\pi n}{2}}$$

$= h(0) + h(1)\, e^{-j\pi/2} + h(2)\, e^{-j\pi} + h(3)\, e^{-j3\pi/2}$

$= 1 + 2(-j) + 3(-1) + 4(+j)$

$= 1 - 2j - 3 + 4j$

$= -2 + 2j$

$$H_2(2) = \sum_{n=0}^{3} h(n)\, e^{-j\frac{2\pi n \cdot \cancel{2}}{\cancel{4}}} = \sum_{n=0}^{3} h(n)\, e^{-j\pi n}$$

$= h(0) + h(1)\, e^{-j\pi} + h(2)\, e^{-j2\pi} + h(3)\, e^{-j3\pi}$

$= 1 + 2(-1) + 3(1) + 4(-1)$

$= 1 - 2 + 3 - 4$

$= \cancel{4} - 2 - \cancel{4}$

$= -2$

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$$H_2(3) = \sum_{K=0} h(n)\, e^{-j\frac{2\pi n\cdot 3}{4_2}} = \sum_{K=0}^{3} h(n)\, e^{-j\frac{3\pi n}{2}}$$

$$= h(0) + h(1)\, e^{-j\frac{3\pi}{2}} + h(2)\, e^{-j3\pi} + h(3)\, e^{-j\frac{9\pi}{2}}$$

$$= 1 + 2(j) + 3(-1) + 4(-j)$$

$$= 1 + 2j - 3 - 4j$$

$$= -2 - 4j$$

$$Y_3(K) = X_1(K)\cdot H_1(K)$$

$$= \left\{ \underset{\uparrow}{5}, -1, 1, -1 \right\} \cdot \left\{ \underset{\uparrow}{10}, -2+2j, -2, -2-2j \right\}$$

$$Y_3(k) = \left\{ 50, 2-2j, -2, 2+2j \right\}$$

$$\text{IDFT}\left(Y_3(k) \right)$$

$$y_3(n) = \frac{1}{N} \sum_{K=0}^{N-1} Y_3(k)\, e^{\frac{j2\pi nk}{N}}$$

$n = 0$

$$y_3(0) = \frac{1}{4} \sum_{K=0}^{3} Y_3(k)\, e^{\frac{j2\pi K\cdot 0}{N}}$$

$$= \frac{1}{4}\left[y_3(0) + Y_3(1) + Y_3(2) + Y_3(3) \right]$$

$$= \frac{1}{4}\left[50 + \cancel{2} - \cancel{2j} - \cancel{2} + 2 + \cancel{2j} \right]$$

$$= \frac{1}{4}(52)$$

$$y_3(0) = 13$$

n = 1

$$y_3(1) = \frac{1}{4}\sum_{K=0}^{3} y_3(k)\, e^{\frac{j2\pi k}{N=4}}$$

$$= \frac{1}{4}\left[y_3(0) + y_3(1)\, e^{j\frac{2\pi\cdot 1}{\cancel{4}\,2}} + y_3(2)\, e^{j\frac{2\pi\cdot 2}{4}} + y_3(3)\, e^{j\frac{2\pi\cdot 3}{\cancel{4}\,2}} \right]$$

$$= \frac{1}{4}\left[50 + (2-2j)(j) + (-2)(-1) + (2+2j)(-j) \right]$$

$$= \frac{1}{4}\left[50 + \cancel{2j} - 2j^2 + 2 - \cancel{2j} - 2j^2 \right]$$

$$= \frac{1}{4}\left[50 + 2 + 2 + 2 \right]$$

$$= \frac{1}{4}\left[56 \right]$$

$$y_3(1) = 14$$

n = 2

$$y_3(2) = \frac{1}{4}\sum_{K=0}^{3} y_3(K)\, e^{\frac{j\cdot\cancel{2}\pi n\cdot K}{\cancel{4}\,2}}$$

$$= \frac{1}{4}\sum_{K=0}^{3} y_3(k)\, e^{\frac{j\cdot\pi\cancel{2}\cdot K}{\cancel{2}}}$$

$$= \frac{1}{4}\sum_{K=0}^{3} y_3(k)\, e^{j\pi k}$$

$$= \frac{1}{4}\left[y_3(0) + y_3(1)\, e^{j\pi} + y_3(2)\, e^{j2\pi} + y_3(3)\, e^{j3\pi} \right]$$

$$= \frac{1}{4}\left[50 + (2-2j)(-1) + (-2)(1) + (2+2j)(-1) \right]$$

$$= \frac{1}{4}\left[50 - 2 + \cancel{2j} - 2 - 2 - \cancel{2j} \right]$$

$$= \frac{1}{4}\left[50 - 6 \right]$$

$$= \frac{1}{4}(44)$$

$$y_3(3) = \frac{1}{4}\sum_{k=0}^{3} y_3(k)\, e^{j\frac{2\pi nk}{\cancel{4}2}}$$

$$= \frac{1}{4}\sum_{k=0}^{3} y_3(k)\, e^{j\frac{3\pi}{2}k}$$

$$= \frac{1}{4}\left[y_3(0) + y_3(1)e^{j\frac{3\pi}{2}} + y_3(2)e^{j3\pi} + y_3(3)e^{j9}\right.$$

$$= \frac{1}{4}\left[50 + (2-2j)(-j) + (-2)(-1) + (2+2j)(j)\right]$$

$$= \frac{1}{4}\left[50 - \cancel{2j} + 2j^2 + 2 + \cancel{2j} + 2j^2\right]$$

$$= \frac{1}{4}\left[50 - \cancel{2} + 2 - 2\right]$$

$$= \frac{1}{4}(48)$$

$$y_3(3) = 12$$

$$y(n) = \{\underset{\uparrow}{13}, 14, 11, 12\}$$

17-01-19

CORRELATION :

* The relationship between two signals is called correlation.

* There are two types of correlation

1> Auto - correlation

2> Cross - correlation

AUTO - CORRELATION :

$$\gamma_{xx} = x(n) * x(-n) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-\ell)$$

EX : Find Auto correlation of given sequence

$$x(n) = \{1, 2, 4, 6\}$$

$$x(n) = \{\underset{\uparrow}{1}, 2, 4, 6\} \qquad x(-n) = \{6, 4, 2, \underset{\uparrow}{1}\}$$

$$\gamma_{xx} = x(n) * x(-n)$$

$$= \{\underset{\uparrow}{1}, 2, 4, 6\} * \{6, 4, 2, \underset{\uparrow}{1}\}$$

	1	2	4	6
6	6	12	24	36
4	4	8	16	24
2	2	4	8	12
1	1	2	4	6

$$y(n) = \{6, 16, 34, \underset{\uparrow}{57}, 34, 16, 6\}$$

CROSS CORRELATION :

$$\gamma_{xh} = x(n) * h(-n)$$

or

$$\gamma_{xh} = x(-n) * h(n)$$

Ex : Find cross-correlation of given sequence

$x(n) = \{1, 2, 7, 1\}$, $h(n) = \{1, 3, 2, 2\}$

$x(n) = \{\underset{\uparrow}{1}, 2, 7, 1\}$ $h(-n) = \{2, 2, 3, \underset{\uparrow}{1}\}$

	1	2	7	1
2	2	4	14	2
2	2	4	14	2
3	3	6	21	3
1	1	2	7	1

$$y(n) = \{2, 6, 21, \underset{\uparrow}{23}, 25, 10, 1\}$$

19-01-19

LINEAR CONSTANT COEFFICIENT DIFFERENCE EQUATION:

General form of difference an n^{th} order of LTI-DT system is

$$\sum_{K=0}^{N} a_K y(n-k) = \sum_{k=0}^{M} b_K x(n-k)$$

$$y(n) = -\sum_{K=0}^{N} a_K y(n-k) + \sum_{K=0}^{M} b_K x(n-k)$$

$$\text{Total Response} = \underset{\text{Zero input Response}}{\text{Natural Response}} + \underset{\text{Zero state Response}}{\text{Forced Response}}$$

The zero i/p response of the system depends only on the initial states of the system i.e., the input is zero

The zero state response of the system is the response of the system due to input alone, when the initial state of the system is zero

NATURAL RESPONSE:

For a DT-System, Natural Response is solution of homogenous equation.

$$\sum_{K=0}^{N} a_K y(n-k) = \underbrace{\sum_{K=0}^{M} b_K x(n-k)}_{\text{Input is zero}}$$

$$\sum_{k=0}^{N} a_K y(n-k) = 0$$

$$a_0\, y(n) + a_1\, y(n-1) + a_2\, y(n-2) + \cdots + a_N\, y(n-N) = 0$$

$$\{ a_0 = 1 \}$$

$$y(n) + a_1\, y(n-1) + a_2\, y(n-2) + \cdots + a_N\, y(n-N) = 0$$

$$\boxed{y(n) = \lambda^n}$$

$$\lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \cdots + a_N \lambda^{n-N} = 0$$

$$\lambda^{n-N} \left[\lambda^N + a_1 \lambda^{N-1} + a_2 \lambda^{N-2} + \cdots + a_N \right] = 0$$

$$\lambda^N + a_1 \lambda^{N-1} + a_2 \lambda^{N-2} + \cdots + a_N = 0$$

CASE (I) : Roots are in distinct form

Ex : $\lambda_1 = 2$, $\lambda_2 = 3$

$$y_h(n) = c_1 (2)^n + c_2 (3)^n$$

CASE (II) : Roots are repeated

Ex : $\lambda_1 = 1$, $\lambda_2 = 1$

$$y_h(n) = (c_1 + c_2 n)\,(1)^n$$

CASE (III) : Roots are complex

Ex : $\lambda_1 = a + jb$ $\lambda_2 = a - jb$

$$y_h(n) = \gamma^n \left[A_1 \cos n\theta + A_2 \sin n\theta \right]$$

$A_1, A_2 \rightarrow$ constants

$$\gamma = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

PROBLEM :

Find the Natural Response of the system defined by difference equation i.e., given by

$y(n) + 2\, y(n-1) + y(n-2) = x(n) + x(n-1)$; with initial conditions $y(-1) = y(-2) = 1$ ①

By using Homogenous equation, Input terms should be zero

The Homogenous equation is obtained by quating the input terms to zero. That is

$$y(n) + 2y(n-1) + y(n-2) = 0 \longrightarrow ②$$

The homogenous solution is of the form

$$y(n) = \lambda^n \longrightarrow (2a)$$

Substituting eqn (2a) in eqn ②

$$\lambda^n + 2\lambda^{n-1} + \lambda^{n-2} = 0$$

$$\lambda^{n-2}\left[\lambda^2 + 2\lambda + 1\right] = 0$$

$$\lambda^2 + 2\lambda + 1 = 0$$

$$\lambda^2 + \lambda + \lambda + 1 = 0$$

$$\lambda(\lambda+1) + 1(\lambda+1) = 0$$

$$(\lambda+1)(\lambda+1) = 0$$

$$\lambda_1 = -1, \quad \lambda_2 = -1$$

Roots are repeated then the general form of homogenous solution is

$$y_h(n) = (C_1 + C_2 n)(-1)^n \longrightarrow ③$$

Substituting $n=0$ in eqn ③

$$y_h(0) = (C_1 + C_2(0))(-1)^0$$

$$y_h(0) = C_1 \longrightarrow ④$$

Substituting $n=1$ in eqn ③

$$y_h(1) = (C_1 + C_2)(-1)$$

$$y_h(1) = -C_1 - C_2 \longrightarrow ⑤$$

Substituting n=0 in eqn ②

$$y(0) + 2y(-1) + y(-2) = 0$$

$$y(0) + 2\cdot 1 + 1 = 0$$

$$y(0) = -3 \longrightarrow ⑥$$

Substituting n=1 in eqn ②

$$y(1) + 2y(0) + y(-1) = 0$$

$$y(1) + 2(-3) + 1 = 0$$

$$y(1) - 6 + 1 = 0$$

$$y(1) = 5 \longrightarrow ⑦$$

from eqn ④

$$y(0) = c_1$$

$$\boxed{c_1 = -3}$$

from eqn ⑤

$$y(1) = -c_1 - c_2$$

$$5 = -(-3) - c_2$$

$$5 = 3 - c_2$$

$$\boxed{c_2 = -2}$$

Substituting c_1, c_2 values in eqn ③

The natural response

$$y_n(n) = (-3-2n)(-1)^n \quad n \geq 0$$

$$= (-3-2n)(-1)^n u(n)$$

Find the Natural Response of the System defined by difference equation i.e., given by

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$ with initial conditions $y(-1) = y(-2) = 1$

$$y(n) - 4\,y(n-1) + 4\,y(n-2) = x(n) - x(n-1) \longrightarrow ①$$

$$y(-1) = y(-2) = 1$$

By using homogeneous equation, input terms should be zero

$$y(n) - 4\,y(n-1) + 4y(n-2) = 0 \longrightarrow ②$$

$$y(n) = \lambda^n$$

$$\lambda^n - 4\lambda^{n-1} + 4\lambda^{n-2} = 0$$

$$\lambda^{n-2}\left[\lambda^2 - 4\lambda + 4\right] = 0$$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda^2 - 2\lambda - 2\lambda + 4 = 0$$

$$\lambda(\lambda-2) - 2(\lambda-2) = 0$$

$$(\lambda-2)(\lambda-2) = 0$$

$$\lambda_1 = 2, \ \lambda_2 = 2$$

Roots are repeated, then

$$y_h(n) = (c_1 + c_2 n)(2)^n \longrightarrow ③$$

Substituting $n=0$ in eqn ③

$$y(0) = (c_1 + 0)\,2^0$$

$$y(0) = c_1 \longrightarrow ④$$

Substituting $n=1$ in eqn ③

$$y(1) = (c_1 + c_2)(2)$$

$$y(1) = 2c_1 + 2c_2 \longrightarrow ⑤$$

Substituting $n=0$ in eqn ②

$$y(0) - 4\,y(-1) + 4\,y(-2) = 0$$

$$y(0) - 4\cdot 1 + 4\cdot 1 = 0$$

$y(0) - 4 + 4 = 0$

$y(0) = 0 \longrightarrow ⑥$

substituting $n=1$ in eqn ②

$y(1) - 4\,y(0) + 4\,y(-1) = 0$

$y(1) - 4 \cdot 0 + 4 \cdot 1 = 0$

$y(1) = -4 \longrightarrow ⑦$

from eqn ④

$y(0) = c_1$

$0 = c_1$

$c_1 = 0$

from eqn ⑤

$-4 = 2 \cdot 0 + 2c_2$

$2c_2 = -4$

$c_2 = -2$

Substituting c_1, c_2 in eqn ③

$y_h(n) = (0 - 2 \cdot n)\,(2)^n$

$y_h(n) = -2n(2)^n$

21-01-19

FORCED RESPONSE :

The forced Response is obtained by summing the particular solution and homogeneous solution.

$$y_f(n) = y_p(n) + y_h(n)$$

$x(n) \rightarrow$ I/P SIGNAL $\qquad y_p(n) \rightarrow$ PARTICULAR SOLUTION

1) Step signal (A) $\longrightarrow$ K

2) $A M^n \longrightarrow K M^n$

3) $\delta(n) \longrightarrow 0$

4> $An^M \xrightarrow{\hspace{3cm}} K_0 n^M + K_1 n^{M-1} + \cdots K$

5> $\left.\begin{matrix} A\cos\omega n \\ A\sin\omega n \end{matrix}\right\} \xrightarrow{\hspace{2cm}} C_1 \cos\omega n + C_2 \sin\omega n$

where A, M, K, C_1, C_2 are constants

Find forced Response of the system describe by difference eqn. $y(n) + 2y(n-1) + y(n-2) = x(n) + x(n-$ and input is $x(n) = \left(\frac{1}{2}\right)^n u(n)$

$\downarrow$
①

The forced Response

$$y_f(n) = y_p(n) + y_h(n)$$

the particular solution is of the form
for input $x(n) = \left(\frac{1}{2}\right)^n u(n)$,

$$y_p(n) = \left(\frac{1}{2}\right)^n K \longrightarrow ②$$

Substituting eqn ② in eqn ①

$$\left(\frac{1}{2}\right)^n K + 2\left(\frac{1}{2}\right)^{n-1} K + \left(\frac{1}{2}\right)^{n-2} K = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1)$$

$n = 2$

$$\frac{1}{4}\cdot K + 2\cdot\frac{1}{2}\cdot K + 1\cdot K = \frac{1}{4}\cdot u(2) + \frac{1}{2}\cdot u(1)$$

$$K\left[\frac{1}{4} + 1 + 1\right] = \frac{1}{4} + \frac{1}{2} \qquad \left\{ \because u(n) = 1 \text{ for } n \geq 0 \right.$$

$$K\left(\frac{9}{\cancel{4}}\right) = \frac{3}{\cancel{4}}$$

$$K = \frac{3}{9}$$

$$\boxed{K = \frac{1}{3}}$$

$$\boxed{y_p(n) = \left(\frac{1}{2}\right)^n \cdot \frac{1}{3}} \longrightarrow ③$$

$y_h(n) = ?$

$$y(n) + 2\,y(n-1) + y(n-2) = 0$$

$$y(n) = \lambda^n$$

$$\lambda^n + 2\,\lambda^{n-1} + \lambda^{n-2} = 0$$

$$\lambda^{n-2}\left[\lambda^2 + 2\lambda + 1\right] = 0$$

$$\lambda^2 + 2\lambda + 1 = 0$$

$$\lambda^2 + \lambda + \lambda + 1 = 0$$

$$\lambda(\lambda+1) + 1(\lambda+1) = 0$$

$$(\lambda+1)(\lambda+1) = 0$$

$$\lambda = -1, -1$$

Roots are repeated, then

$$y_h(n) = (c_1 + c_2 n)(-1)^n \longrightarrow ④$$

$$y_f(n) = y_p(n) + y_h(n)$$

The forced Response

$$y_f(n) = \left(\frac{1}{2}\right)^n \cdot \frac{1}{3} + (c_1 + c_2 n)(-1)^n \longrightarrow ⑤$$

$n = 0$

$$y(0) = 1 \cdot \frac{1}{3} + (c_1 + 0) \cdot 1$$

$$y(0) = \frac{1}{3} + c_1 \longrightarrow ⑥$$

$n = 1$

$$y(1) = \frac{1}{2} \cdot \frac{1}{3} + (c_1 + c_2)(-1)$$

$$y(1) = \frac{1}{6} - c_1 - c_2 \longrightarrow ⑦$$

In forced response, states is zero.

Substituting $n=0$ in eqn ①

$$y(0) + 2\, y(-1) + y(-2) = \left(\frac{1}{2}\right)^{0} u(0) + \left(\frac{1}{2}\right)^{0-1} u(0-1)$$

$$\boxed{y(-1) = y(-2) = 0} \longrightarrow \text{zero state Response}$$

$\left\{ \because u(n) = 0 \text{ for } n < 0 \right.$

$$y(0) = 1 + 0$$

$$y(0) = 1 \longrightarrow ⑧$$

Substituting $n=1$ in eqn ①

$$y(1) + 2\, y(0) + y(-1) = \left(\frac{1}{2}\right)^{1} u(1) + \left(\frac{1}{2}\right)^{1-1} u(1-1)$$

$$y(1) + 2 \cdot 1 + 0 = \frac{1}{2} + 1$$

$\left\{ \begin{array}{l} \because u(n) = 1 \text{ for } n \geq 0 \\ \qquad\qquad 0 \text{ for } n < 0 \end{array} \right.$

$$y(1) + 2 = \frac{3}{2}$$

$$y(1) = \frac{-1}{2} \longrightarrow ⑨$$

from eqn ⑥

$$y(0) = \frac{1}{3} + c_1$$

$$1 = \frac{1}{3} + c_1 \Longrightarrow c_1 = 1 - \frac{1}{3}$$

$$c_1 = \frac{2}{3}$$

from eqn ⑦

$$y(1) = \frac{1}{6} - c_1 - c_2$$

$$\frac{-1}{2} = \frac{1}{6} - \frac{2}{3} - c_2$$

$$c_2 = \frac{1}{6} - \frac{2}{3} + \frac{1}{2} = \frac{1 - 4 + 3}{6}$$

$$c_2 = 0$$

$\therefore$ Forced Response

$$y_f(n) = \left(\frac{1}{2}\right)^n \cdot \frac{1}{3} + (c_1 + c_2 n)(-1)^n$$

$$\boxed{y_f(n) = \left(\frac{1}{2}\right)^n \cdot \frac{1}{3} + \frac{2}{3}(-1)^n} \quad n \geq 0 \quad \text{(or)} \quad \boxed{\text{mul } u(n)}$$

Find Forced Response of the System described by difference equation $y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$ and input is $x(n) = (-1)^n u(n)$

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1) \longrightarrow ①$$

Forced Response $y_f(n) = y_p(n) + y_h(n)$

$$y_p(n) = (-1)^n \cdot K \longrightarrow ②$$

Substituting eqn ② in eqn ①

$$(-1)^n K - 4(-1)^{n-1} K + 4(-1)^{n-2} \cdot K = (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

$n = 2$

$$1 \cdot K - 4(-1)K + 4 \cdot 1 \cdot K = 1 \cdot u(2) - (-1)u(1)$$

$$K + 4K + 4K = 1 + 1 \qquad \left\{ \because u(n) = 1 \text{ for } n \geq 0 \right\}$$

$$9K = 2$$

$$K = \frac{2}{9}$$

$$\boxed{y_p(n) = (-1)^n \cdot \frac{2}{9}} \longrightarrow ③$$

$y_h(n) = ?$

$$y(n) - 4y(n-1) + 4y(n-2) = 0$$

$$y(n) = \lambda^n$$

$$\lambda^n - 4\lambda^{n-1} + 4\lambda^{n-2} = 0$$

$$\lambda^{n-2}\left[\lambda^2 - 4\lambda + 4\right] = 0$$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda^2 - 2\lambda - 2\lambda + 4 = 0$$

$$\lambda(\lambda-2) - 2(\lambda-2) = 0$$

$$(\lambda-2)(\lambda-2) = 0$$

$$\lambda = 2, 2$$

Roots are repeated, then

$$y_h(n) = (c_1 + c_2 n) \cdot 2^n \longrightarrow ④$$

forced Response $y_f(n) = (-1)^n \cdot \frac{2}{9} + (c_1 + c_2 n) \cdot 2^n$ $\longrightarrow ⑤$

$n = 0$

$$y(0) = 1 \cdot \frac{2}{9} + (c_1 + 0) \cdot 1$$

$$y(0) = \frac{2}{9} + c_1 \longrightarrow ⑥$$

$n = 1$

$$y(1) = -1 \cdot \frac{2}{9} + (c_1 + c_2)\, 2$$

$$y(1) = -\frac{2}{9} + 2c_1 + 2c_2 \longrightarrow ⑦$$

Substituting $n=0$ in eqn ①

$$y(0) - 4\,y(-1) + 4\,y(-2) = (-1)^0\, u(0) - (-1)^{0-1}\, u(0-1)$$

$$y(-1) = y(-2) = 0 \quad \left\{ \because u(n) = 0 \text{ for } n < 0 \right\}$$

$$y(0) = 1 - 0$$

$$y(0) = 1 \longrightarrow ⑧$$

Substituting $n=1$ in eqn ①

$$y(1) - 4\,y(0) + 4\,y(-1) = (-1)^1 u(1) - (-1)^0 u(0)$$

$$y(1) - 4\cdot 1 + 0 = -1 - 1 \qquad \left\{ \because u(n) = \begin{matrix} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{matrix} \right\}$$

$$y(1) = -2 + 4$$

$$y(1) = 2 \longrightarrow ⑨$$

from eqn ⑤

$$y(0) = \frac{2}{9} + c_1$$

$$1 = \frac{2}{9} + c_1$$

$$c_1 = 1 - \frac{2}{9}$$

$$c_1 = \frac{7}{9}$$

from eqn ⑦

$$y(1) = -\frac{2}{9} + 2\cdot\frac{7}{9} + 2C_2$$

$$2 = -\frac{2}{9} + \frac{14}{9} + 2C_2$$

$$2 = \frac{\cancel{12}^{4}}{\cancel{9}_{3}} + 2C_2$$

$$2 = \frac{4}{3} + 2C_2$$

$$2C_2 = 2 - \frac{4}{3}$$

$$2C_2 = \frac{2}{3}$$

$$C_2 = \frac{1}{3}$$

∴ Forced Response $y_f(n) = (-1)^n \frac{2}{9} + (c_1 + c_2 n)\cdot 2^n$

$$\boxed{y_f(n) = (-1)^n\cdot\frac{2}{9} + \left(\frac{7}{9} + \frac{1}{3}n\right)\cdot 2^n}$$

24-01-19

TOTAL RESPONSE :

Total Response = Natural Response + Forced Response

It is obtained by adding Natural Response and forced Response

Find Response of the system described by difference equation $y(n) + 2y(n-1) + y(n-2) = x(n) + x(n-1)$ → ① and Input response $x(n) = (\frac{1}{2})^n u(n)$ with initial conditions $y(-1) = y(-2) = 1$

$$T.R = N.R + F R$$

$$= (-3-2n)(-1)^n + \left(\frac{1}{2}\right)^n \cdot \frac{1}{3} + \frac{2}{3}(-1)^n$$

$$= \left(-3+\frac{2}{3}\right)(-1)^n - 2n(-1)^n + \left(\frac{1}{2}\right)^n \cdot \frac{1}{3}$$

$$= \left(-\frac{7}{3}\right)(-1)^n - (2n)(-1)^n + \left(\frac{1}{2}\right)^n \frac{1}{3}$$

$$T.R = \left(-\frac{7}{3} - 2n\right)(-1)^n + \left(\frac{1}{2}\right)^n \cdot \frac{1}{3}$$

METHOD -II

NOTE : To calculate total Response, use actual initial conditions for forced response.

$$y(0) = \frac{1}{3} + c_1 \longrightarrow ②$$

$$y(1) = \frac{1}{6} - c_1 - c_2 \longrightarrow ③$$

Sub $n=0$ in eqn ①

$$y(0) + 2y(-1) + y(-2) = \left(\frac{1}{2}\right)^0 u(0) + \left(\frac{1}{2}\right)^{0-1} u(0-1)$$

$$y(0) + 2\cdot 1 + 1 = 1\cdot 1 + 0$$

$$y(0) = 1-3$$

$$y(0) = -2$$

Sub $n=1$ in eqn ①

$$y(1) + 2y(0) + y(-1) = \left(\frac{1}{2}\right)^{1} u(1) + \left(\frac{1}{2}\right)^{1-1} u(1-1)$$

$$y(1) + 2(-2) + 1 = \frac{1}{2} + 1$$

$$y(1) - 3 = \frac{3}{2}$$

$$y(1) = \frac{3}{2} + 3$$

$$y(1) = \frac{9}{2}$$

from eqn ②

$$-2 = \frac{1}{3} + C_1$$

$$C_1 = -2 - \frac{1}{3}$$

$$\boxed{C_1 = \frac{-7}{3}}$$

from eqn ③

$$\frac{9}{2} = \frac{1}{6} + \frac{7}{3} - C_2$$

$$\frac{9}{2} = \frac{1+14}{6} - C_2$$

$$C_2 = \frac{15}{6} - \frac{9}{2}$$

$$C_2 = \frac{15-27}{6}$$

$$C_2 = \frac{-12}{6}$$

$$\boxed{C_2 = -2}$$

$C_1 = \frac{-7}{3}$ $C_2 = -2$

$$y_t(n) = \left(\frac{1}{2}\right)^n \cdot \frac{1}{3} + (C_1 + C_2 n)(-1)^n$$

$$y_t(n) = \left(\frac{1}{2}\right)^n \cdot \frac{1}{3} + \left(\frac{-7}{3} - 2n\right)(-1)^n$$

Find total response of $y(n) + 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$ $\downarrow$ ① with initial conditions with input $x(n) = (-1)^n u(n)$ $y(-1) = y(-2) = 1$

$y(n) = y_h(n) + y_p(n)$

$$y_f(n) = y_p(n) + y_h(n)$$

$$y_p(n) = (-1)^n \cdot K\, u(n) \longrightarrow ②$$

Sub eqn ② in eqn ①

$$(-1)^n K - 4(-1)^{n-1} K + 4(-1)^{n-2} K = (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

$n=2$

$$(-1)^2 K - 4(-1)^1 K + 4\cdot 1\cdot K = (-1)^2 u(2) - (-1)\, u(1)$$

$$K + 4K + 4K = 1 + 1$$

$$9K = 2$$

$$\boxed{K = \frac{2}{9}}$$

$$y_p(n) = (-1)^n \cdot \frac{2}{9}$$

$$y_h(n) = ?$$

$$y(n) - 4y(n-1) + 4y(n-2) = 0$$

$$y(n) = \lambda^n$$

$$\lambda^n - 4\lambda^{n-1} + 4\lambda^{n-2} = 0$$

$$\lambda^{n-2}\left[\lambda^2 - 4\lambda + 4\right] = 0$$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda^2 - 2\cdot 2\lambda + 2^2 = 0$$

$$(\lambda - 2)^2 = 0$$

$$\lambda = 2, 2$$

$n=0$

$$y(0) = \frac{2}{9} + (c_1 + 0)2^0$$

$$y(0) = \frac{2}{9} + c_1 \longrightarrow ③$$

$n=1$

$$y(1) = -\frac{2}{9} + (c_1 + c_2)2$$

$$y(1) = -\frac{2}{9} + 2c_1 + 2c_2 \hookrightarrow ④$$

Roots are repeated, then

$$y_h(n) = (c_1 + c_2 n)\, 2^n$$

forced Response $y_f(n) = (-1)^n \cdot \frac{2}{9} + (c_1 + c_2 n)\, 2^n$

Substituting $n=0$ in eqn ①

$$y(0) - 4y(-1) + 4y(-2) = (-1)^0 u(0) - (-1)^{0-1} u(0-1)$$

$$y(0) - 4\cdot 1 + 4\cdot 1 = 1 - 0$$

$$y(0) = 1$$

Substituting $n=1$ in eqn ①

$$y(1) - 4y(0) + 4y(-1) = (-1)^1 u(1) - (-1)^{1-1} u(1-1)$$

$y(1) - \cancel{4}\cdot 1 + \cancel{4}\cdot 1 = -1 - 1$

$y(1) = -2$

from eqn ③

$1 = \frac{2}{9} + c_1$

$c_1 = 1 - \frac{2}{9}$

$\boxed{c_1 = \frac{7}{9}}$

from eqn ④

$-\cancel{2} = -\frac{\cancel{2}}{9} + 2\cdot\frac{7}{9} + \cancel{2}c_2$

$c_2 = -1 + \frac{1}{9} - \frac{7}{9}$

$c_2 = \frac{-9+1-7}{9}$

$c_2 = \frac{-16+1}{9}$

$c_2 = \frac{-15}{9}$

$\boxed{c_2 = \frac{-5}{3}}$

$y_h(n) = (c_1 + c_2 n)\, 2^n$

$= \left(\frac{7}{9} - \frac{5}{3}n\right) 2^n$

$y_h(n) = \left(\frac{7}{9} - \frac{5}{3}n\right) 2^n$

$y_f(n) = y_p(n) + y_h(n)$

$y_f(n) = \frac{2}{9}(-1)^n + \left(\frac{7}{9} - \frac{5}{3}n\right) 2^n$

Determine the impulse response of the system
$y(n) = 0.6\ y(n-1) - 0.08\ y(n-2) + x(n)$ (1.93)

$$y(n) = 0.6\, y(n-1) - 0.08\ y(n-2) + x(n) \longrightarrow ①$$

from Linear time Invariant discrete time system

$$\sum_{K=0}^{M} a_K\, y(n-K) = \sum_{K=0}^{N} b_K\, x(n-K)$$

$M \neq N$

$$y(n) = y_p(n) + y_h(n)$$

$$y_p(n) = \delta(n)$$

$$y_p(n) = 0$$

$$y(n) = 0 + y_h(n)$$

$$y(n) = y_h(n)$$

By using homogenous solution, Input terms must be zero

$$y(n) = 0.6\, y(n-1) - 0.08\ y(n-2) + 0$$

$$y(n) = \lambda^n$$

$$\lambda^n - 0.6\,\lambda^{n-1} + 0.08\,\lambda^{n-2} = 0$$

$$\lambda^{n-2}\left[\lambda^2 - 0.6\,\lambda + 0.08\right] = 0$$

$$\boxed{\begin{aligned} &\lambda^2 - \frac{6}{10}\lambda + \frac{8}{100} = 0 \\ &100\lambda^2 - 60\lambda + 8 = 0 \end{aligned}}$$

$$\lambda^2 - 0.6\lambda + 0.08 = 0$$

$$\lambda_1 = 0.4 \quad \lambda_2 = 0.2$$

$y_h(n) = c_1(0.4)^n + c_2(0.2)^n$

forced response

$y(n) = 0 + c_1(0.4)^n + c_2(0.2)^n \longrightarrow$ ②

Substituting $n=0$ in above equation

$y(0) = c_1 + c_2 \longrightarrow$ ③

Substituting $n=1$ in eqn ②

$y(1) = 0.4\,c_1 + 0.2\,c_2 \longrightarrow$ ④

Substituting $n=0$ in eqn ①

$y(0) = 0.6\,y(-1) - 0.08\,y(-2) + x(0)$

$\{\ x(0) = \delta(0) = 1$

$y(0) = 0 - 0 + 1$

$y(0) = 1 \longrightarrow$ ⑤

Substituting $n=1$ in eqn ①

$y(1) = 0.6\,y(0) - 0.08\,y(-1) + x(1)$

$y(1) = 0.6 \cdot 1 - 0 + 0$

$y(1) = 0.6 \longrightarrow$ ⑥

from eqn ③

$1 = c_1 + c_2$

from eqn ④

$0.6 = 0.4\,c_1 + 0.2\,c_2$

$0.4\,\cancel{c_1} + 0.4\,c_2 = 0.4$

$0.4\,c_1 + 0.2\,c_2 = 0.6$

$(-) \quad (-) \quad (-)$

$0.2\,c_2 = -0.2$

$\boxed{c_2 = -1}$

$$c_1 + c_2 = 1$$

$$c_1 - 1 = 1$$

$$\boxed{c_1 = 2}$$

$$y_h(n) = c_1 (0.4)^n + c_2 (0.2)^n$$

$$y_h(n) = 2 (0.4)^n - 1 (0.2)^n$$

Determine impulse response described by the difference equation $y(n) + y(n-1) - 2y(n-2) = x(n-1) + 2x(n-2)$ (1.95)

$$y(n) + y(n-1) - 2y(n-2) = x(n-1) + 2x(n-2) \longrightarrow ①$$

from linear time invariant discrete time system

$$\sum_{K=0}^{M} a_K \, y(n-K) = \sum_{K=0}^{M} b_K \; (n-k)$$

$$y(n) = y_p(n) + y_h(n)$$

$$y(n) = 0 + y_h(n)$$

$$y(n) = y_h(n)$$

$\therefore$ In Impulse response, particular solution is zero

By using homogeneous solution, the input terms must be zero

$$y(n) + y(n-1) - 2y(n-2) = 0$$

$$y(n) = \lambda^n$$

$$\lambda^n + \lambda^{n-1} - 2\lambda^{n-2} = 0$$

$$\lambda^{n-2}\left[\lambda^2 + \lambda - 2\right] = 0$$

$$\lambda^2 + \lambda - 2 = 0$$

$$\lambda^2 + 2\lambda - \lambda - 2 = 0$$

$$\lambda(\lambda+2) - 1(\lambda+2) = 0$$

$(\lambda+2)\ (\lambda-1)=0$

$\lambda = -2, 1$

Roots are distinct

$y_h(n) = c_1(1)^n + c_2(-2)^n \longrightarrow ②$

Here $M=N$, then impulse response is added to the homogeneous solution

$y_h(n) = c_1(1)^n + c_2(-2)^n + A\,\delta(n) \longrightarrow ③$

Sub $n=0$ in eqn ③

$y(0) = c_1 + c_2 + A \longrightarrow ④$

substituting $n=1$ in eqn

$y(1) = c_1 - 2c_2 + 0$

$y(1) = c_1 - 2c_2 \longrightarrow ⑤$

Substituting $n=2$ in eqn ③

$y(2) = c_1 + c_2 \cdot 4 + 0$

$y(2) = c_1 + 4c_2 \longrightarrow ⑥$

Substituting $n=0$ in eqn ①

$y(0) + y(-1) - 2\,y(-2) = x(-1) + 2\,x(-2)$

$y(0) + 0 - 0 = 0$

$y(0) = 0$

$\left\{ \begin{array}{l} x(-1) = \delta(-1) = 0 \\ x(-2) = \delta(-2) = 0 \end{array} \right.$

Substituting $n=1$ in eqn ①

$y(1) + y(0) - 2\,y(-1) = x(0) + 2\,x(-1)$

$y(1) + 0 - 0 = 1 + 0$

$y(1) = 1$

Substituting $n=2$ in eqn ⑦

$$y(2)+y(1)-2y(0) = x(1)+2x(0) \qquad \left\{ x(0) = \delta(0) = 1 \right.$$

$$y(2)+1-0 = 0+2$$

$$y(2) = 1$$

from eqn ④

$$c_1+c_2+A = 0$$

from eqn ⑤

$$c_1-2c_2 = 1$$

from eqn ⑥

$$c_1+4c_2 = 1$$

$$\cancel{c_1}-2C_2 = 1$$

$$\cancel{c_1}+4C_2 = 1$$

$$(-) \quad (-) \quad (-)$$

$$-6C_2 = 0$$

$$\boxed{C_2 = 0}$$

$$\rightarrow C_1 - 0 = 1$$

$$\boxed{C_1 = 1}$$

$$\rightarrow C_1+C_2+\hat{A} = 0$$

$$1+0+A = 0$$

$$\boxed{A = -1}$$

$c_1 = 1$

$c_2 = 0$

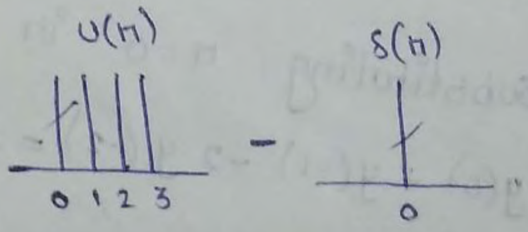
$A = -1$

$$\therefore \quad y(n) = 1\cdot(1)^n + 0\,(-2)^n + (-1)\,\delta(n)$$

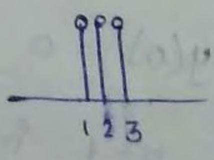
$$y(n) = (1)^n - \delta(n)$$

$$y(n) = (1)^n - \delta(n)$$

$$= u(n) - \delta(n)$$



$$\boxed{y(n) = u(n-1)}$$



28-01-19

FREQUENCY RESPONSE ANALYSIS OF DISCRETE TIME SYSTEM :

The output y(n) of any LTI-DT's to an input x(n) can be obtained by convolution sum

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$h(n) \longrightarrow$ Impulse response of system

Let us consider a complex exponential input signal

$$x(n) = e^{j\omega n}$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot e^{j\omega(n-k)}$$

$$= e^{j\omega n} \sum_{k=-\infty}^{\infty} h(k) e^{-j\omega k}$$

$$y(n) = \underset{\substack{\uparrow \\ \text{Input signal}}}{e^{j\omega n}} \cdot \underset{\substack{\uparrow \\ \text{frequency response}}}{H(e^{j\omega})}$$

$$\left[X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\omega n} \right]$$

$$H(e^{j\omega}) = \underset{\substack{\uparrow \\ \text{Magnitude} \\ \text{response}}}{|H(e^{j\omega})|} \; \underset{\substack{\uparrow \\ \text{phase} \\ \text{response}}}{e^{j\theta(\omega)}}$$

Determine and plot the magnitude and phase response of $y(n) = \frac{1}{2}[x(n) + x(n-2)]$

$$y(n) = \frac{1}{2}[x(n) + x(n-2)]$$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \frac{1}{2}\left[x(n)+x(n-2)\right] e^{-j\omega n}$$

$$= \frac{1}{2}\sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} + \frac{1}{2}\sum_{n=-\infty}^{\infty} x(n-2)\, e^{-j\omega n}$$

$e^{-j\omega n+j2\omega-j2\omega}$

$e^{-j(n-2)\omega} \cdot e^{-j2\omega}$

$$= \frac{1}{2} X(e^{j\omega}) + \frac{1}{2} e^{-j2\omega} X(e^{j\omega})$$

$$Y(e^{j\omega}) = X(e^{j\omega})\left[\frac{1}{2} + \frac{1}{2} e^{-j2\omega}\right]$$

$$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{2}\left[1+e^{-j2\omega}\right]$$

$$H(e^{j\omega}) = \frac{1}{2}\left[1+\cos 2\omega - j\sin 2\omega\right]$$

Magnitude response

$$\left|H(e^{j\omega})\right| = \frac{1}{2}\sqrt{(1+\cos 2\omega)^2 + \sin^2 2\omega}$$

$$= \frac{1}{2}\sqrt{1+\cos^2 2\omega + 2\cos 2\omega + \sin^2 2\omega}$$

$$= \frac{1}{2}\sqrt{2+2\cos 2\omega}$$

$$= \frac{1}{2}\sqrt{2}\cdot\sqrt{1+\cos 2\omega}$$

$$= \frac{1}{\sqrt{2}}\sqrt{2\cos^2\omega}$$

$$= \frac{1}{\sqrt{2}}\cdot\sqrt{2}\cos\omega$$

$$\left|H(e^{j\omega})\right| = \cos\omega$$

phase response $\underline{/H(e^{j\omega})} = \tan^{-1}\left(\frac{b}{a}\right)$

$$= \tan^{-1}\left[\frac{-\sin 2\omega}{1+\cos 2\omega}\right] = \tan^{-1}\left[\frac{-\cancel{2}\sin\omega \cancel{\cos\omega}}{\cancel{2}\cos^{\cancel{2}}\omega}\right]$$

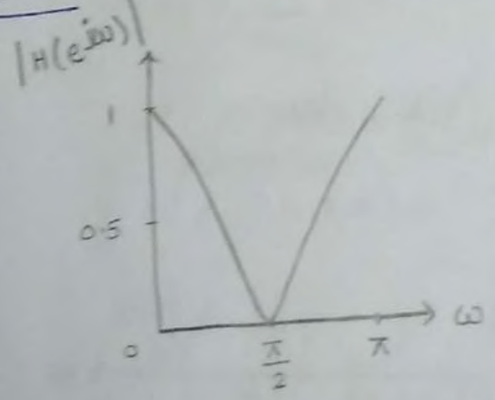
$$= \tan^{-1}\left[\frac{-\sin\omega}{\cos\omega}\right] = \tan^{-1}\left[-\tan\omega\right]$$

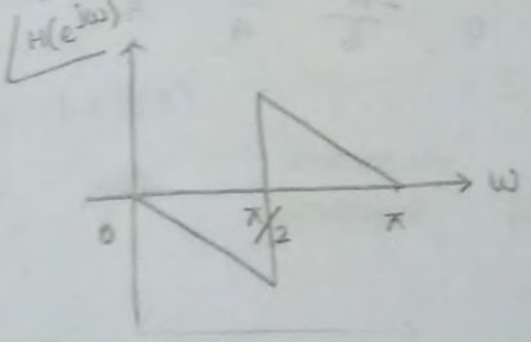
$\underline{/H(e^{j\omega})} = -\omega$

$= -\omega$ for $H(e^{j\omega}) > 0$

$-\omega + \pi$ for $H(e^{j\omega}) < 0$

ω	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π
$H(e^{j\omega})$	1	0.86	0.707	0.5	0	-0.5	-0.707	-1
$\lvert H(e^{j\omega})\rvert$	1	0.86	0.707	0.5	0	0.5	0.707	1
$\underline{/H(e^{j\omega})}$	0	$-\pi/6$	$-\pi/4$	$-\pi/3$	$\pi/2$	$\pi/3$	$\pi/4$	0





2> A discrete time system has a unit sample response $h(n) = \frac{1}{2}\delta(n) + \delta(n-1) + \frac{1}{2}\delta(n-2)$. Find the system frequenc response plot Magnitude and phase response.

$$h(n) = \frac{1}{2}\delta(n) + \delta(n-1) + \frac{1}{2}\delta(n-2)$$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{1}{2}\,\delta(n) + \delta(n-1) + \frac{1}{2}\,\delta(n-2) \right] e^{-j\omega n}$$

$$= \frac{1}{2}\sum_{n=-\infty}^{\infty} \delta(n)\overset{e^{-j\omega n}}{} + \sum_{n=-\infty}^{\infty} \delta(n-1)\, e^{-j\omega n} + \frac{1}{2}\sum_{n=-\infty}^{\infty} \delta(n-2)\, e^{-j\omega n}$$

$$= \frac{1}{2} + e^{-j\omega} + \frac{1}{2}\, e^{-j\omega 2}$$

$$= e^{-j\omega}\left[\frac{1}{2}\, e^{j\omega} + 1 + \frac{1}{2}\, e^{-j\omega} \right]$$

$$\frac{e^{j\omega} + e^{-j\omega}}{2} = \cos\omega$$

$$H(e^{j\omega}) = e^{-j\omega}\left[1 + \cos\omega\right]$$

$$\left|H(e^{j\omega})\right| = 1 + \cos\omega$$

$$r\, e^{j\theta} \longrightarrow \text{phase}$$

r ↓ Magnitude

$$\angle H(e^{j\omega}) = -\omega$$

$\cos\omega - j\sin\omega\,(1+\cos\omega)$

$\cos\omega + \cos^2\omega - j\sin\omega - j\sin\omega\cos\omega$

$\cos\omega + \cos^2\omega - j(\sin\omega + \sin\omega\cos\omega)$

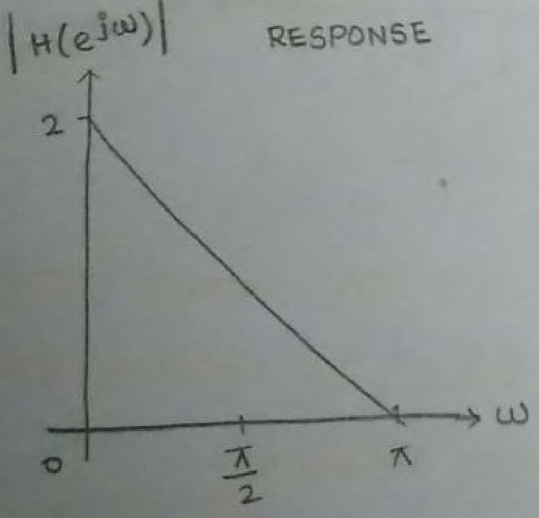
$\cos\omega(1+\cos\omega) - j(\sin\omega(1+\cos\omega))$

$\tan^{-1}(-\tan\omega)$

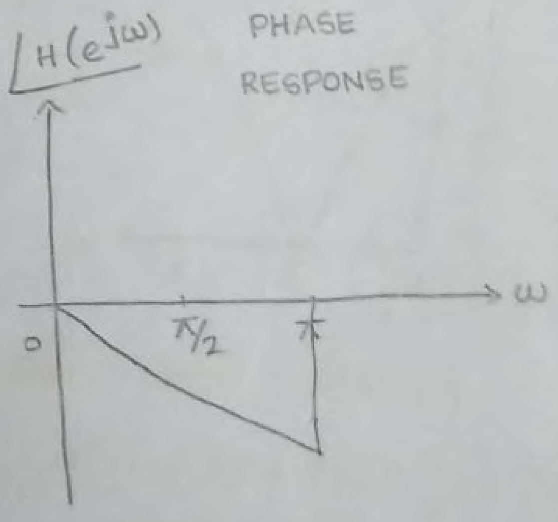
$-\omega$

ω	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	π
$\left\|H(e^{j\omega})\right\|$	2	1.866	1.707	1.5	1	0.5	0.293	0
$\angle H(e^{j\omega})$	0	$-\frac{\pi}{6}$	$-\frac{\pi}{4}$	$-\frac{\pi}{3}$	$-\frac{\pi}{2}$	$-\frac{2\pi}{3}$	$-\frac{3\pi}{4}$	$-\pi$

MAGNITUDE RESPONSE



PHASE RESPONSE



29-01-19

DFT [DISCRETE FOURIER TRANSFORM]:

for N-point

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi nk}{N}}$$

$K = 0, 1, ------, N-1$

$x(n) = (-1)^n \quad 0 \leq N \leq 3$

$0, 1, 2, 3$

$\{1, -1, 1, -1\}$

$x(n) = (-1)^N \quad N = 4$

$0 ---- N-1$

$0, 1, 2, 3$

$\{1, -1, 1, -1\}$

IDFT

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(k)\, e^{\frac{j2\pi nk}{N}}$$

$n = 0, 1, 2, ------, N-1$

1) Find DFT of Sequence $x(n) = \{1, 1, 1, 1\}$

$N = 4$

$K = 0, 1, ----4-1$

$K = 0, 1, 2, 3$

$x(n) = \left(\frac{1}{4}\right)^n$

$N = 16$

$K = 0 ---- N-1$

$= 0, 1, ---- 15$

$K=0$

$$X(0) = \sum_{n=0}^{3} x(n)\, e^{-0}$$

$= x(0) + x(1) + x(2) + x(3)$

$= 1 + 1 + 1 + 1$

$X(0) = 4$

$K=0$

$$x(0) = \sum_{n=0}^{15} x(n)\, e^{-j\frac{2\pi n \cdot 0}{15}}$$

$x(n) = \frac{1}{3} \quad 0 \leq n \leq 2$

$\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0\right)$

$1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3$

$= \frac{1}{1-\frac{1}{4}}$

$\frac{4}{3}$

$k=1$

$$X(1) = \sum_{n=0}^{3} x(1)\, e^{-j\cdot\frac{2\pi n \cdot 1}{\cancel{4}\,2}}$$

$$= \sum_{n=0}^{3} x(1)\, e^{-jn\pi/2}$$

$$= x(0)\, e^{0} + x(0)\, e^{-j\pi/2} + x(1)\, e^{-j\pi} + x(2)\, e^{-j3\pi/2} + x(3)\, e^{-j2\pi}$$

$= 1 + (-j) + (-1 + 0) + (0 + j)$

$= 0$

$k=2$

$$X(2) = \sum_{n=0}^{3} x(n)\, e^{\frac{-j2\pi n 2}{4}}$$

$$= \sum_{n=0}^{3} x(n) e^{-jn\pi}$$

$$= x(0)\, e^{0} + x(1) e^{-j\pi} + x(2) e^{-j2\pi} + x(3) e^{-j3\pi}$$

$$= 1\cdot 1 + 1\cdot(-1+0) + 1(1+0) + 1(-1)$$

$$= 1-1+1-1$$

$$X(2) = 0$$

$K=3$

$$X(3) = \sum_{n=0}^{3} x(n)\, e^{\frac{-j2\pi n\cdot 3}{\cancel{4}\,2}}$$

$$= \sum_{n=0}^{3} x(n) e^{-jn3\pi/2}$$

$$= x(0)e^{0} + x(1)\, e^{-j3\pi/2} + x(2)\, e^{-j3\pi} + x(3)\, e^{-j9\pi/2}$$

$$= 1\cdot 1 + 1\cdot(0-j(-1)) + 1(-1+0) + 1(0-j(1))$$

$$= 1+j-1-j$$

$$X(3) = 0$$

$$X(k) = \{4,0,0,0\}$$

2> Find DFT of the Sequence $x(n) = \{1,1,0\,0\}$

$N=4$

$K = 0, 1, \text{----}\ 4-1$

$K = 0, 1, 2, 3$

$k=0$

$$X(0) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi n\cdot 0}{4}}$$

$$= \sum_{n=0}^{3} x(n)\, e^{0}$$

$$= x(1) + x(2) + x(3) + x(4)$$

$$= 1 + 1 + 0 + 0$$

$$X(0) = 2$$

$k=1$

$$X(1) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi n\cdot 1}{4}}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-jn\pi/2}$$

$$= x(0)\,e^{0} + x(1)\, e^{-j\pi/2} + x(2)\, e^{-j\pi} + x(3)\, e^{-j3\pi/2}$$

$$= 1\cdot 1 + 1\cdot\left(0 - j(1)\right) + 0 + 0$$

$$X(1) = 1 - j$$

$k=2$

$$X(2) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{\cancel{2}\pi n\cdot \cancel{2}}{\cancel{4}}}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-jn\pi}$$

$$= x(0)\,e^{0} + x(1)\,e^{-j\pi} + x(2)\,e^{-j2\pi} + x(3)\,e^{-j3\pi}$$

$$= 1\cdot 1 + 1\cdot\left(-1 - j(0)\right) + 0 + 0$$

$$= 1 - 1$$

$$X(2) = 0$$

$K = 3$

$$X(3) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{\cancel{2}\pi n\cdot 3}{\cancel{4}\,2}}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-jn\,3\pi/2}$$

$$= x(0)\,e^{0} + x(1)\,e^{-j\,3\pi/2} + x(2)\,e^{-j\,3\pi} + x(3)\,e^{-j\,9\pi/2}$$

$$= 1\cdot 1 + 1\cdot\big(0 - j(-1)\big) + 0 + 0$$

$$X(3) = 1 + j$$

$$X(k) = \{2,\ 1-j,\ 0,\ 1+j\}$$

3> Find IDFT of Sequence $X(k) = \{2, 1-j, 0, 1+j\}$

$N = 4$

$k = 0, 1, \text{---}\ 4-1$

$n = 0, 1, 2, 3$

$$\left\{ x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{\frac{j2\pi nk}{N}} \right.$$

$n = 0$

$$x(0) = \frac{1}{N}\sum_{k=0}^{3} X(k)\, e^{\frac{j2\pi\cdot 0\cdot k}{4}}$$

$$= \frac{1}{4}\sum_{k=0}^{3} X(k)\, e^{0}$$

$$= \frac{1}{4}\Big[X(0) + X(1) + X(2) + X(3)\Big]$$

$$= \frac{1}{4}\Big[2 + 1 - \cancel{j} + 0 + 1 + \cancel{j}\Big]$$

$$= \frac{1}{4}\cdot 4$$

$$x(0) = 1$$

$n = 1$

$$x(1) = \frac{1}{4}\sum_{K=0}^{3} x(k)\, e^{j\frac{2\pi k}{4}}$$

$$= \frac{1}{4}\sum_{K=0}^{3} x(k)\, e^{jK\cdot\pi/2}$$

$$= \frac{1}{4}\left[x(0)e^{0} + x(1)e^{j\pi/2} + x(2)e^{j\pi} + x(3)e^{j3\pi/2} \right]$$

$$= \frac{1}{4}\left[2\cdot 1 + (1-j)(0+j) + 0 + (1+j)(0-j) \right]$$

$$= \frac{1}{4}\left[2 + \cancel{j} - j^2 - \cancel{j} - j^2 \right]$$

$$= \frac{1}{4}\left[2 + 1 + 1 \right]$$

$$= \frac{1}{4}\cdot 4$$

$$x(1) = 1$$

$n = 2$

$$x(2) = \frac{1}{4}\sum_{K=0}^{3} x(k)\, e^{j\frac{2\pi\cdot 2\cdot k}{4}}$$

$$= \frac{1}{4}\sum_{K=0}^{3} x(k)\, e^{jk\pi}$$

$$= \frac{1}{4}\left[x(0)e^{0} + x(1)e^{j\pi} + x(2)e^{j2\pi} + x(3)e^{j3\pi} \right]$$

$$= \frac{1}{4}\left[2\cdot 1 + (1-j)(-1+0) + 0 + (1+j)(-1+0) \right]$$

$$= \frac{1}{4}\left[2 - 1 + \cancel{j} - 1 - \cancel{j} \right]$$

$$= \frac{1}{4}\left[2 - 2 \right]$$

$$x(2) = 0$$

$n = 3$

$$x(3) = \frac{1}{4} \sum_{k=0}^{3} X(k)\, e^{j\frac{2\pi 3k}{\cancel{4}\,2}}$$

$$= \frac{1}{4} \sum_{k=0}^{3} X(k)\, e^{j\frac{3\pi}{2}k}$$

$$= \frac{1}{4}\left[X(0)\, e^{0} + X(1)\, e^{j3\pi/2} + X(2)\, e^{j3\pi} + X(3)\, e^{j9\pi}\right.$$

$$= \frac{1}{4}\left[2 \cdot 1 + (1-j)(0-j) + 0 + (1+j)(0+j)\right]$$

$$= \frac{1}{4}\left[\;2 - \cancel{j} + j^{2} + \cancel{j} + j^{2}\right]$$

$$= \frac{1}{4}\left[\;2 - 1 - 1\right]$$

$$= \frac{1}{4}\left[2 - 2\right]$$

$$x(3) = 0$$

$$x(n) = \{1, 1, 0, 0\}$$

4> Find DFT of sequence $x(n) = \{1, 1, -1, -1\}$

$N = 4$

$K = 0, 1, --- 4-1$

$K = 0, 1, 2, 3$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi nk}{N}}$$

$K = 0$

$$X(0) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi n \cdot 0}{4}}$$

$$= \sum_{n=0}^{3} x(n)\, e^{0}$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1 + 1 - 1 - 1$$

$$X(0) = 0$$

$k = 1$

$$X(1) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi n\cdot 1}{4}}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-jn\pi/2}$$

$$= x(0)\, e^{0} + x(1)\, e^{-j\pi/2} + x(2)\, e^{-j\pi} + x(3)\, e^{-j3\pi/2}$$

$$= 1\cdot 1 + 1\cdot(0 - j(1)) + (-1)(-1 - j(0)) + (-1)(0 - j(-1))$$

$$= 1 - j + 1 - j$$

$$X(1) = 2 - 2j$$

$k = 2$

$$X(2) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{\cancel{2}\pi n\cdot \cancel{2}}{\cancel{4}}}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-jn\pi}$$

$$= x(0)\, e^{0} + x(1)\, e^{-j\pi} + x(2)\, e^{-j2\pi} + x(3)\, e^{-j3\pi}$$

$$= 1\cdot 1 + 1\cdot(-1 + 0) + (-1)(1 - 0) + (-1)(-1 - 0)$$

$$= 1 - 1 - 1 + 1$$

$$X(2) = 0$$

$k = 3$

$$X(3) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{\cancel{2}\pi n\cdot 3}{\cancel{4}\,2}}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-jn3\pi/2}$$

$$= x(0)\,e^{0} + x(1)\,e^{-j3\pi/2} + x(2)\,e^{-j3\pi} + x(3)\,e^{-j9\pi/2}$$

$$= 1\cdot 1 + 1\cdot\left(0-j(-1)\right) + (-1)\left(-1-0\right) + (-1)\left(0-j(1)\right)$$

$$= 1 + j + 1 + j$$

$$X(3) = 2+2j$$

$$X(k) = \left\{0,\ 2-2j,\ 0,\ 2+2j\right\}$$

5) Find DFT of the sequence $x(n) = \delta(n) + \delta(n-2)$.

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi nk}{N}}$$

$$= \sum_{n=0}^{N-1} \left[\delta(n) + \delta(n-2)\right] e^{-j\frac{2\pi nk}{N}}$$

$$= \sum_{n=0}^{N-1} \delta(n)\, e^{-j\frac{2\pi nk}{N}} + \sum_{n=0}^{N-1} \delta(n-2)\, e^{-j\frac{2\pi nk}{N}}$$

$$= \delta(0)\, e^{-j\frac{2\pi\cdot 0\cdot k}{N}} + \delta(2-2)\, e^{-j\frac{2\pi\cdot 2\cdot k}{N}}$$

$$= 1\cdot e^{0} + \delta(0)\, e^{-j\frac{4\pi k}{N}}$$

$$X(k) = 1 + e^{-j\frac{4\pi k}{N}}$$

Find DFT of the sequence $x(n) = \begin{cases} 1 & \text{for } 0 \le n \le 2 \\ 0 & \text{otherwise} \end{cases}$

for $N = 4$, $N = 8$

$N = 4$

$$x(n) = \{1, 1, 1, 0\}$$

$K = 0, 1, 2, 3$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi nk}{N}}$$

K=0

$$X(0) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi n\cdot 0}{4}}$$

$$= \sum_{n=0}^{3} x(n) \cdot e^{0}$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1+1+1+0$$

$$X(0) = 3$$

K=1

$$X(1) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi n\cdot 1}{4}}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-jn\pi/2}$$

$$= x(0)\, e^{0} + x(1)\, e^{-j\pi/2} + x(2)\, e^{-j\pi} + x(3)\, e^{-j3\pi/2}$$

$$= 1\cdot 1 + 1\cdot(0-j(1)) + 1(-1-0) + 0$$

$$= \cancel{1} - j - \cancel{1}$$

$$X(1) = -j$$

K = 2 ⟹ $X(2) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi n\cdot \cancel{2}}{\cancel{4}}}$

$$= \sum_{n=0}^{3} x(n)\, e^{-jn\pi}$$

$$= x(0)e^{0} + x(1)\, e^{-j\pi} + x(2)\, e^{-j2\pi} + x(3)e^{-j3\pi}$$

$$= 1\cdot 1 + 1\cdot(-1-0) + 1(1-0) + 0$$

$$= 1 - 1 + 1$$

$$X(2) = 1$$

$$k=3 \implies X(3) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{\cancel{2}\pi n\cdot 3}{\cancel{4}\,2}}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-jn\,3\pi/2}$$

$$= x(0)\,e^{0} + x(1)\,e^{-j\,3\pi/2} + x(2)\,e^{-j\,3\pi} + x(3)\,e$$

$$= 1\cdot 1 + 1\cdot(0 - j(-1)) + 1\,(-1-0) + 0$$

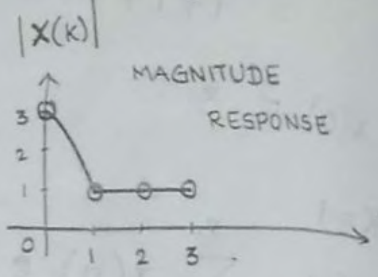
$$= 1 + j - 1$$

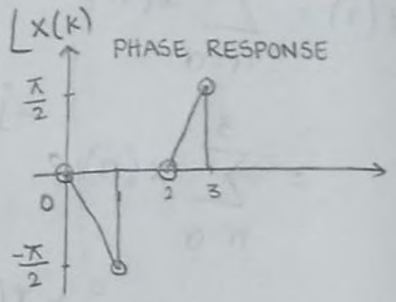
$$X(3) = j$$

$$X(k) = \{3, -j, 1, j\}$$

$$|X(k)| = \{3, 1, 1, 1\}$$

$$\angle X(k) = \{0, -\pi/2, 0, \pi/2\}$$





$$N = 8$$

$$x(n) = \{1, 1, 1, 0, 0, 0, 0, 0\}$$

$$k=0 \implies X(0) = \sum_{n=0}^{7} x(n)\, e^{-j\frac{2\pi n\cdot 0}{N}}$$

$$= \sum_{n=0}^{7} x(n)\, e^{0}$$

$$= x(0) + x(1) + x(2) + x(3) + x(4) + x(5) + x(6) + x($$

$$= 1 + 1 + 1 + 0 + 0 + 0 + 0 + 0$$

$$X(0) = 3$$

$$k=1 \implies X(1) = \sum_{n=0}^{7} x(n)\, e^{-j\frac{2\pi n\cdot 1}{8}}$$

$$= \sum_{n=0}^{7} x(n)\, e^{-jn\pi/4}$$

$$= x(0)e^{0} + x(1)e^{-j\pi/4} + x(2)e^{-j\pi/2} + 0+0+0+0+0$$

$$= 1\cdot 1 + 1\cdot\left(0.707 - j(0.707)\right) + 1\left(0 - j(1)\right)$$

$$= 1 + 0.707 - 0.707j - j$$

$$X(1) = 1.707 - 1.707j$$

$$K=2 \Rightarrow X(2) = \sum_{n=0}^{7} x(n)\, e^{-j\frac{2\pi n\cdot 2}{8_2}}$$

$$= \sum_{n=0}^{7} x(n)\, e^{-jn\pi/2}$$

$$= x(0)e^{0} + x(1)e^{-j\pi/2} + x(2)e^{-j\pi} + 0+0+0+0+0$$

$$= 1\cdot 1 + 1\cdot\left(0 - j(1)\right) + 1\left(-1 - j(0)\right) + 0+0+0+0+0$$

$$= 1 - j - 1$$

$$X(2) = -j$$

$$K=3 \Rightarrow X(3) = \sum_{n=0}^{7} x(n)\, e^{-j\frac{2\pi n\cdot 3}{8_4}}$$

$$= \sum_{n=0}^{7} x(n)\, e^{-jn3\pi/4}$$

$$= x(0)e^{0} + x(1)e^{-j3\pi/4} + x(2)e^{-j3\pi/2} + 0+0+0+0+0$$

$$= 1\cdot 1 + 1\cdot\left(-0.707 - j(0.707)\right) + 1\left(0 - j(-1)\right) + 0$$

$$= 1 - 0.707 - 0.707j + j$$

$$X(3) = 0.293 + 0.293j$$

$$K=4 \Rightarrow X(4) = \sum_{n=0}^{7} x(n)\, e^{-j\frac{2\pi n\cdot \cancel{4}}{\cancel{8}_2}}$$

$$= \sum_{n=0}^{7} x(n)\, e^{-jn\pi}$$

$$= x(0)e^{0} + x(1)e^{-j\pi} + x(2)e^{-j2\pi} + 0 + 0 + 0 + 0 + 0$$

$$= 1\cdot 1 + 1\cdot\left(-1 - j(0)\right) + 1\left(1 - j(0)\right) + 0$$

$$= 1 - 1 + 1$$

$$X(4) = 1$$

$$K = 5 \Longrightarrow X(5) = \sum_{n=0}^{7} x(n)\, e^{-j\frac{2\pi n\cdot 5}{\cancel{8}4}}$$

$$= \sum_{n=0}^{7} x(n)\, e^{-jn\frac{5\pi}{4}}$$

$$= x(0)e^{0} + x(1)e^{-j\frac{5\pi}{4}} + x(2)\, e^{-j\frac{5\pi}{2}} + 0 + 0 + 0 + 0 + 0$$

$$= 1\cdot 1 + 1\left(-0.707 - j(-0.707)\right) + 1\left(0 - j(1)\right)$$

$$= 1 - 0.707 + 0.707j - j$$

$$X(5) = 0.293 - 0.293j$$

$$K = 6 \Longrightarrow X(6) = \sum_{n=0}^{7} x(n)\, e^{-j\frac{2\pi n\cdot \overset{3}{\cancel{6}}}{\cancel{8}\cancel{4}_{2}}}$$

$$= \sum_{n=0}^{7} x(n)\, e^{-jn\,3\pi/2}$$

$$= x(0)e^{0} + x(1)\, e^{-j\,3\pi/2} + x(2)\, e^{-j3\pi} + 0 + 0 + 0 + 0 + 0$$

$$= 1\cdot 1 + 1\cdot\left(0 - j(-1)\right) + 1\left(-1 - j(0)\right)$$

$$= 1 + j - 1$$

$$X(6) = j$$

$$K = 7 \longrightarrow X(7) = \sum_{n=0}^{7} x(n)\, e^{-j\frac{2\pi n\cdot 7}{\cancel{8}4}}$$

$$= \sum_{n=0}^{7} x(n)\, e^{-jn\,7\pi/4}$$

$$= x(0)e^{0} + x(1)\, e^{-j\,7\pi/4} + x(2)\, e^{-j\,7\pi/2} + 0 + 0 + 0 + 0 + 0$$

$$= 1\cdot 1 + 1\cdot\left(0\cdot707 - j\left(-0\cdot707\right)\right) + 1\left(0 - j(-1)\right) + 0 + 0 + 0 + 0 + 0$$

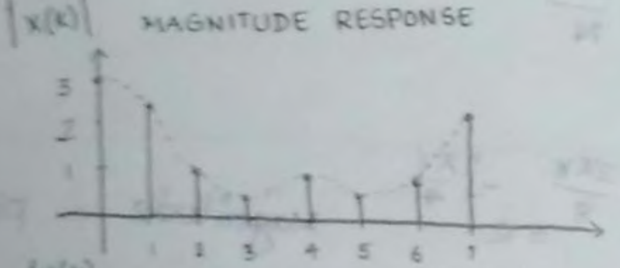
$$= 1 + 0\cdot707 + 0\cdot707j + j$$

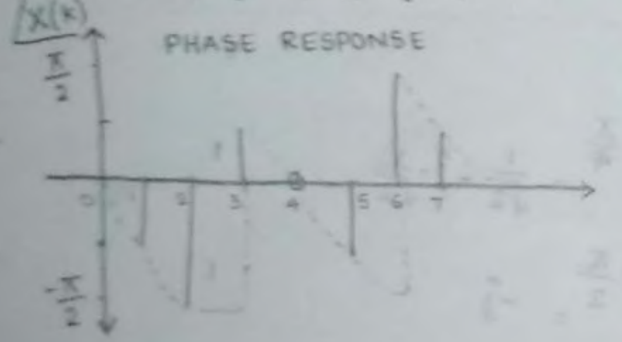
$$x(7) = 1\cdot707 + 1\cdot707j$$

$$x(k) = \left\{ 3,\ 1\cdot707 - 1\cdot707j,\ -j,\ 0\cdot293 + 0\cdot293j,\ 1,\ 0\cdot293 - 0\cdot293j,\ j,\ 1\cdot707 + 1\cdot707j \right\}.$$

$$|x(k)| = \left\{ 3,\ 2\cdot414,\ 1,\ 0\cdot414,\ 1,\ 0\cdot414,\ 1,\ 2\cdot414 \right\}$$

$$\angle x(k) = \left\{ 0,\ \frac{-\pi}{4},\ \frac{-\pi}{2},\ \frac{\pi}{4},\ 0,\ \frac{-\pi}{4},\ \frac{\pi}{2},\ \frac{\pi}{4} \right\}$$





In N=4, it is difficult to extrapolate the frequency spectrum for low values of N the spacing between successive samples is high which results in poor resolution

In N=8, we can observe that it is possible to extrapolate the frequency spectrum i.e., Zero pading gives a high density frequency spectrum and provides a better displayed version for plotting.

TWIDDLE FACTOR :

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi nk}{N}} \longrightarrow ①$$

$$W_N^{nk} = e^{-j\frac{2\pi nk}{N}} \longrightarrow ②$$

$$W_N = e^{-j\frac{2\pi}{N}}$$

$$W_N^{n} = e^{-j\frac{2\pi n}{N}}$$

Let $nk = r$

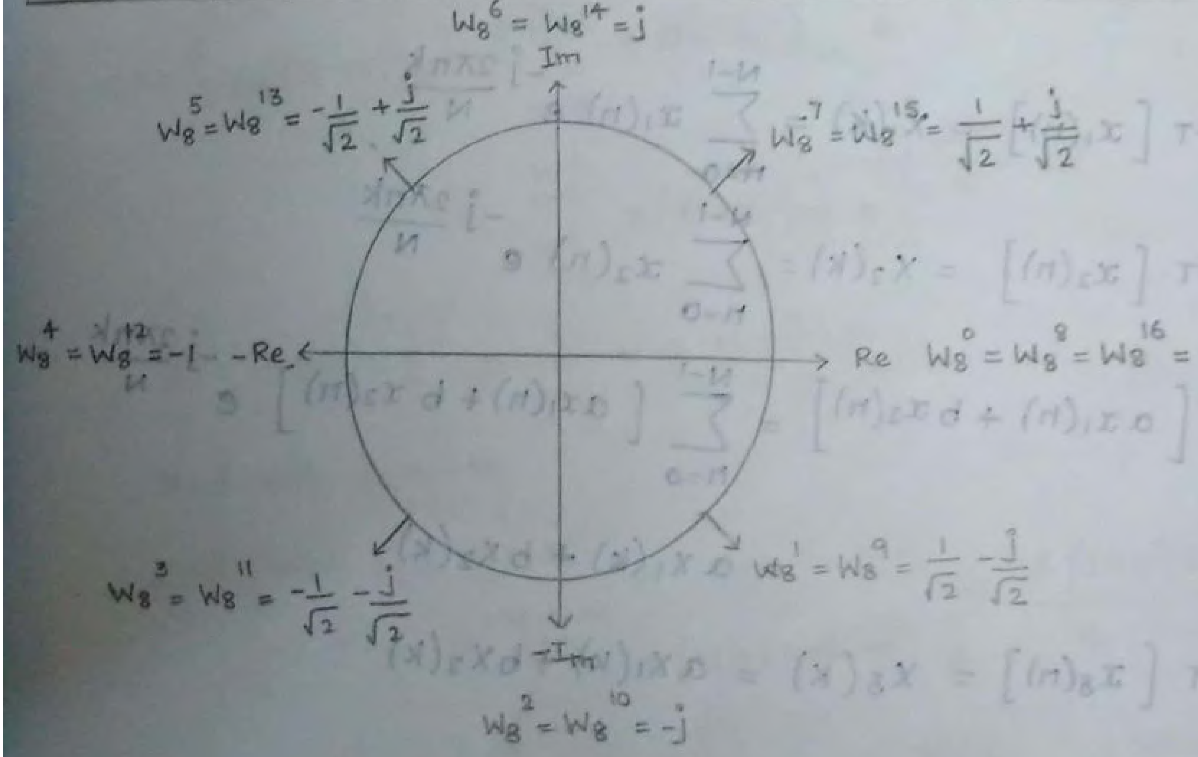
from eq ②

$$W_N^{r} = e^{-j\frac{2\pi r}{N}}$$

For $N=8$

$nk=r$	$W_8^r = e^{-j\frac{2\pi r}{8}} = e^{-j\frac{\pi r}{4}}$	Magnitude	phase
0	$W_8^0 = 1$	1	0
1	$W_8^1 = e^{-j\frac{\pi}{4}} = \frac{1}{\sqrt2} - j\cdot\frac{1}{\sqrt2}$	1	$-\pi/4$
2	$W_8^2 = e^{-j\frac{\pi}{2}} = -j$	1	$-\pi/2$
3	$W_8^3 = e^{-j\frac{3\pi}{4}} = \frac{-1}{\sqrt2} - j\frac{1}{\sqrt2}$	1	$-3\pi/4$
4	$W_8^4 = e^{-j\pi} = -1$	1	$-\pi$
5	$W_8^5 = e^{-j\frac{5\pi}{4}} = \frac{-1}{\sqrt2} + j\frac{1}{\sqrt2}$	1	$-5\pi/4$
6	$W_8^6 = e^{-j\frac{3\pi}{2}} = j$	1	$-3\pi/2$
7	$W_8^7 = e^{-j\frac{7\pi}{4}} = \frac{1}{\sqrt2} + j\cdot\frac{1}{\sqrt2}$	1	$-7\pi/4$
8	$W_8^8 = e^{-j2\pi} = 1$	1	-2π

nk = x	$W_8^x = e^{-j\frac{2\pi x}{8}} = e^{-j\frac{\pi x}{4}}$	magnitude	phase
9	$W_8^9 = e^{-j\frac{9\pi}{4}} = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$	1	$-\frac{9\pi}{4}$
10	$W_8^{10} = e^{-j\frac{5\pi}{2}} = -j$	1	$-\frac{5\pi}{2}$
11	$W_8^{11} = e^{-j\frac{11\pi}{4}} = -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$	1	$-\frac{11\pi}{4}$
12	$W_8^{12} = e^{-j3\pi} = -1$	1	-3π
13	$W_8^{13} = e^{-j\frac{13\pi}{4}} = -\frac{1}{\sqrt{2}} + j\cdot\frac{1}{\sqrt{2}}$	1	$-\frac{13\pi}{4}$
14	$W_8^{14} = e^{-j\frac{7\pi}{2}} = j$	1	$-\frac{7\pi}{2}$
15	$W_8^{15} = e^{-j\frac{15\pi}{4}} = \frac{1}{\sqrt{2}} + j\cdot\frac{1}{\sqrt{2}}$	1	$-\frac{15\pi}{4}$
16	$W_8^{16} = e^{-j4\pi} = 1$	1	-4π



PROPERTIES OF TWIDDLE FACTOR :

1> $W_8^0 = W_8^8 = W_8^{16} = 1$

$$W_8^1 = W_8^9 = W_8^{17} = \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$$

$$W_N^r = W_N^{r+N} = W_N^{r+2N} \longrightarrow \text{periodic property}$$

2> $$\left.\begin{aligned} W_8^1 &= -W_8^5 \\ W_8^2 &= -W_8^6 \end{aligned}\right\} \; W_N^r = -W_N^{r+N/2} \longrightarrow \text{Symmetric property}$$

DFT PROPERTIES :

1) LINEARITY PROPERTY :

If two finite duration Sequence $x_1(n)$ and $x_2(n)$ are linearly combined by

$$x_3(n) = a\,x_1(n) + b\,x_2(n)$$

Then

$$\text{DFT}\left[x_3(n)\right] = X_3(k) = a X_1(k) + b X_2(k)$$

PROOF :

$$\text{DFT}\left[x_1(n)\right] = X_1(k) = \sum_{n=0}^{N-1} x_1(n)\, e^{-j\frac{2\pi nk}{N}}$$

$$\text{DFT}\left[x_2(n)\right] = X_2(k) = \sum_{n=0}^{N-1} x_2(n)\, e^{-j\frac{2\pi nk}{N}}$$

$$\text{DFT}\left[a x_1(n) + b x_2(n)\right] = \sum_{n=0}^{N-1}\left[a x_1(n) + b\,x_2(n)\right] e^{-j\frac{2\pi nk}{N}}$$

$$= a X_1(k) + b X_2(k)$$

$$\text{DFT}\left[x_3(n)\right] = X_3(k) = a X_1(k) + b X_2(k)$$

2> CIRCULAR TIME SHIFT PROPERTY :

In general the Shifted Version $x(n)$

$$x(n) = \left\{x(0), x(1), x(2), \ldots\ldots\ldots, x(N-2), x(N-1)\right\}$$

$$x((n-1))_N = \{x(N-1), x(0), x(1), x(2), \text{--------}, x(N-2)\}$$

$$x((n-2))_N = \{x(N-2), x(N-1), x(0), x(1), x(2), \text{-----}, x(N-3)\}$$

$$x((n-k))_N = \{x(N-k), x(N-k+1), \text{----------}, x(N-k-1)\}$$

$$x((n-N))_N = \{x(0), x(1), x(3), \text{-----------} x(-1)\}$$

$$\boxed{x(n) = x((n-N))_N} = x(\cancel{N}+n-\cancel{N})$$

or

$$x((n-m))_N = x(N+n-m)$$

STMT :

$$\text{DFT}[x(n)] = X(k)$$

$$\text{DFT}[x((n-m))_N] = e^{-j\frac{2\pi mk}{N}} \cdot X(k)$$

PROOF :

$$\text{DFT}[x((n-m))_N] = \sum_{n=0}^{N-1} x((n-m))_N \, e^{-j\frac{2\pi nk}{N}}$$

$$= \sum_{n=0}^{N-1} x(N+n-m) \, e^{-j\frac{2\pi nk}{N}}$$

let $l = n-m \implies n = l+m$

$n=0 \implies l_1 = -m$

$n=N-1 \implies l_2 = N-1-m$

$$= \sum_{l=-m}^{N-1-m} x(N+l+\cancel{m}-\cancel{m}) \, e^{-j\frac{2\pi(l+m)k}{N}}$$

$$= \underbrace{\sum_{l=0}^{N-1} x((l))_N \, e^{-j\frac{2\pi lk}{N}}} \cdot e^{-j\frac{2\pi mk}{N}}$$

$$= X(k) \, e^{-j\frac{2\pi mk}{N}}$$

3) TIME REVERSAL :

STMT :

$$DFT[x(n)] = X(k)$$

$$DFT[x((-n))_N] = DFT[x(N-n)] = X(N-k) = X((-k))_N$$

PROOF :

$$DFT[x((-n))_N] = \sum_{n=0}^{N-1} x((-n))_N . e^{-j\frac{2\pi nk}{N}}$$

$$= \sum_{n=0}^{N-1} x(N-n)\, e^{-j\frac{2\pi nk}{N}}$$

let $N-n = l \implies n = N-l$

$n=0 \implies l_1 = N$

$n=N-1 \implies l_2 = N-N+1 = 1$

$$= \sum_{l=N}^{1} x(l)\, e^{-j\frac{2\pi(N-l)k}{N}}$$

$$= \sum_{l=0}^{N-1} x(l)\cdot e^{-j\frac{2\pi \cancel{N} k}{\cancel{N}}} \cdot e^{j\frac{2\pi lk}{N}}$$

$$= \sum_{l=0}^{N-1} x(l)\cdot \underbrace{e^{-j2\pi k}}_{1}\, e^{j\frac{2\pi lk}{N}}$$

$$= \sum_{l=0}^{N-1} x(l)\cdot e^{-j\frac{2\pi l(N-k)}{N}}$$

$$\underbrace{e^{-j\frac{2\pi l \cancel{N}}{\cancel{N}}}}_{1} \cdot e^{j\frac{2\pi lk}{N}}$$

$$= X(N-k)$$

$$= X((-k))_N$$

$$DFT[x((-n))_N] = X((-k))_N$$

4> CIRCULAR FREQUENCY SHIFT :

STMT :

$$DFT\left[x(n)\right] = X(k)$$

$$DFT\left[x(n)\, e^{j\frac{2\pi l n}{N}}\right] = X((k-l))_N$$

PROOF :

$$DFT\left[x(n)\, e^{j\frac{2\pi l n}{N}}\right] = \sum_{n=0}^{N-1} x(n)\, e^{j\frac{2\pi l n}{N}}\, e^{-j\frac{2\pi n k}{N}}$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi (k-l)n}{N}}$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi (k-l)n}{N}} \cdot \underbrace{e^{-j\frac{2\pi n N}{N}}}_{1}$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi n(N+k-l)}{N}}$$

$$= X(N+k-l)$$

$$= X((k-l))_N$$

$$\therefore DFT\left[x(n)\, e^{j\frac{2\pi l n}{N}}\right] = X((k-l))_N$$

5) COMPLEX CONJUGATE :

STMT :

$$DFT\left[x(n)\right] = X(k)$$

$$DFT\left[x^*(n)\right] = X^*(N-k) = X^*((-k))_N$$

PROOF :

$$DFT\left[x^*(n)\right] = \sum_{n=0}^{N-1} x^*(n)\, e^{-j\frac{2\pi n k}{N}}$$

$$= \sum_{n=0}^{N-1} \left[x(n)\, e^{+j\frac{2\pi nk}{N}} \right]^*$$

$$= \sum_{n=0}^{N-1} \left[x(n)\, e^{j\frac{2\pi nk}{N}}\, e^{-j\frac{2\pi nN}{N}} \right]^*$$

$$= \sum_{n=0}^{N-1} \left[x(n)\, e^{-j\frac{2\pi n(N-k)}{N}} \right]^*$$

$$= X^*(N-k)$$

$$= X^*((-k))_N$$

$$\text{DFT}\left[x^*(n) \right] = X^*((-k))_N$$

6> CONVOLUTION PROPERTY :

STMT :

$$\text{DFT}\left[x(n) \right] = X(k)$$

$$\text{DFT}\left[x_1(n) * x_2(n) \right] = X_1(k)\, X_2(k)$$

PROOF :

$$\text{DFT}\left[x_1(n) * x_2(n) \right] = \sum_{n=0}^{N-1} x_1(n) * x_2(n)\; e^{-j\frac{2\pi nk}{N}}$$

$$= \sum_{n=0}^{N-1} \left[\sum_{m=0}^{N-1} x_1(m)\, x_2(n-m) \right] e^{-j\frac{2\pi nk}{N}}$$

$$= \sum_{m=0}^{N-1} x_1(m) \underbrace{\sum_{n=0}^{N-1} x_2(n-m)_N\; e^{-j\frac{2\pi nk}{N}}}$$

$$= \sum_{m=0}^{N-1} x_1(m)\; e^{-j\frac{2\pi mk}{N}} \cdot X_2(k)$$

{ from circular time shift property

$$= \underbrace{\sum_{m=0}^{N-1} x_1(m)\, e^{-j\frac{2\pi mk}{N}}} \cdot X_2(k)$$

$$= X_1(k)\, X_2(k)$$

$$\therefore \text{DFT}\left[x_1(n) * x_2(n)\right] = X_1(k)\, X_2(k)$$

7) MULTIPLICATION PROPERTY :

STMT :

$$\text{DFT}\left[x(n)\right] = X(k)$$

$$\text{DFT}\left[x_1(n)\, x_2(n)\right] = \frac{1}{N}\left[X_1(k) * X_2(k)\right]$$

PROOF :

$$\text{DFT}\left[x_1(n)\, x_2(n)\right] = \sum_{n=0}^{N-1} x_1(n)\, x_2(n)\, e^{-j\frac{2\pi nk}{N}}$$

$$= \sum_{n=0}^{N-1} \cdot \frac{1}{N} \sum_{l=0}^{N-1} X_1(l)\, e^{j\frac{2\pi nl}{N}} \cdot x_2(n)\, e^{-j\frac{2\pi nk}{N}}$$

$$= \frac{1}{N} \sum_{l=0}^{N-1} X_1(l) \underbrace{\sum_{n=0}^{N-1} x_2(n)\, e^{-j\frac{2\pi n(k-l)}{N}}}$$

$$= \underbrace{\frac{1}{N} \sum_{l=0}^{N-1} X_1(l)\, X_2(k-l)}$$

$$= \frac{1}{N}\left[X_1(k) * X_2(k)\right]$$

$$\text{DFT}\left[x_1(n)\, x_2(n)\right] = \frac{1}{N}\left[X_1(k) * X_2(k)\right]$$

04-02-19

PARSEVAL'S THEOREM PROPERTY:

STMT: If DFT$[x(n)] = X(k)$

DFT$[y(n)] = Y(k)$

then $$\sum_{n=0}^{N-1} x(n) . y^*(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\ Y^*(k)$$

PROOF:

$$\sum_{n=0}^{N-1} x(n)\, y^*(n) = \sum_{n=0}^{N-1} \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi nk}{N}}\ y^*(n) \Big\{ \text{By ID}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X(k) \sum_{n=0}^{N-1} y^*(n)\ e^{j\frac{2\pi nk}{N}}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X(k) \underbrace{\sum_{n=0}^{N-1} \left[y(n)\ e^{-j\frac{2\pi nk}{N}} \right]^*}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X(k) . Y^*(k)$$

$$\sum_{n=0}^{N-1} x(n)\, y^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\ Y^*(k)$$

FILTERING LONG DURATION SEQUENCE:

Suppose $x(n)$ is long duration sequence and it is to be processed with a system having impulse response of finite duration by convolving two sequences. Because of length of input sequence it ca not be practical to store it all before performing Linear convolution.

Therefore, the input sequence must be divided into bl

The successive blocks are processed separately and the results are combined to desired output sequence which is identical to the sequence obtained by Linear convolution

Two methods commonly used are

- * overlap Save method
- * Overlap Add method

Find the output y(n) of a filter ~~to the~~ whose impulse response $h(n) = \{1, 1, 1\}$ and input $x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$ using overlap save method and overlap add method.

OVERLAP SAVE METHOD:

Given $h(n) = \{1, 1, 1\}$ $\longrightarrow$ $M = 3$

$x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$

Assume $l = 3$

$\therefore$ length $= l + M - 1 = 3 + 3 - 1 = 5$

$$x_1(n) = \{\underbrace{0, 0}_{\text{(M-1) 0's are added}}, \underbrace{3, -1, 0}_{\text{first M data points}}\}$$

$$x_2(n) = \{\underbrace{-1, 0}_{\text{last (M-1) data points of previous block}}, 1, 3, 2\}$$

$$x_3(n) = \{\underbrace{3, 2}, 0, 1, 2\}$$

$$x_4(n) = \{1, 2, 1, \underbrace{0\, 0}\}$$

$h(n) = \{1, 1, 1\}$

$h(n) = \{1, 1, 1, 0, 0\}$

$y_1(n) = x_1(n) \textcircled{N} h(n)$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 3 \\ 2 \\ 2 \end{bmatrix}$$

$y_1(n) = \{-1, 0, 3, 2, 2\}$

$y_2(n) = x_2(n) \textcircled{N} h(n)$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -1+0+0+3+2 \\ -1+0+0+0+2 \\ -1+0+1+0+0 \\ 0+0+1+3+0 \\ 0+0+1+3+2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 4 \\ 6 \end{bmatrix}$$

$y_2(n) = \{4, 1, 0, 4, 6\}$

$y_3(n) = x_3(n) \textcircled{N} h(n)$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3+0+0+1+2 \\ 3+2+0+0+2 \\ 3+2+0+0+0 \\ 0+2+0+1+0 \\ 0+0+0+1+2 \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 5 \\ 3 \\ 3 \end{bmatrix}$$

$y_3(n) = \{6, 7, 5, 3, 3\}$

$y_4(n) = x_3(n) \textcircled{N} h(n)$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+0+0+0+0 \\ 1+2+0+0+0 \\ 1+2+1+0+0 \\ 0+2+1+0+0 \\ 0+0+1+0+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 4 \\ 3 \\ 1 \end{bmatrix}$$

$y_4(n) = \{1, 3, 4, 3, 1\}$

$\boxed{-1 \quad 0} \; 3 \quad 2 \quad 2$

$\qquad \boxed{4, 1,} \; 0, 4, 6$

$\qquad\qquad \boxed{6, 7,} \; 5, 3 \; 3$

$\qquad\qquad\qquad \boxed{1, 3,} \; 4, 3, 1$

$y(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1\}$

SHORTCUT METHOD:

$$\begin{array}{c|cccccccccc} & 3 & -1 & 0 & 1 & 3 & 2 & 0 & 1 & 2 & 1 \\ \hline 1 & 3 & -1 & 0 & 1 & 3 & 2 & 0 & 1 & 2 & 1 \\ 1 & 3 & -1 & 0 & 1 & 3 & 2 & 0 & 1 & 2 & 1 \\ 1 & 3 & -1 & 0 & 1 & 3 & 2 & 0 & 1 & 2 & 1 \end{array}$$

$y(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1\}$

OVERLAP ADD METHOD:

$M = 3$

Assume $l = 3$

length $\longrightarrow$ $l + M - 1 = 3 + 3 - 1 = 5$

$x_1(n) = \{ \underbrace{3, -1, 0}_{l \text{ data points}}, \underbrace{0, 0}_{(M-1) \text{ zeros}} \}$

$x_2(n) = \{ \underbrace{1, 3, 2}_{l \text{ data points}}, 0, 0 \}$

$x_3(n) = \{0, 1, 2, 0, 0\}$

$x_4(n) = \{1, 0, 0, 0, 0\}$

$h(n) = \{1, 1, 1\}$,

$h(n) = \{1, 1, 1, 0, 0\}$

$y_1(n) = x_1(n) \textcircled{N} h(n)$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3+0+0+0+0 \\ 3-1+0+0+0 \\ 3-1+0+0+0 \\ 0-1+0+0+0 \\ 0+0+0+0+0 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 2 \\ -1 \\ 0 \end{bmatrix}$$

$y_1(n) = \{3, 2, 2, -1, 0\}$

$y_2(n) = x_2(n) \textcircled{N} h(n)$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+0+0+0+0 \\ 1+3+0+0+0 \\ 1+3+2+0+0 \\ 0+3+2+0+0 \\ 0+0+2+0+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 6 \\ 5 \\ 2 \end{bmatrix}$$

$y_2(n) = \{1, 4, 6, 5, 2\}$

$y_3(n) = x_3(n) \textcircled{N} h(n)$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+0+0+0+0 \\ 0+1+0+0+0 \\ 0+1+2+0+0 \\ 0+1+2+0+0 \\ 0+0+2+0+0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 3 \\ 2 \end{bmatrix}$$

$y_3(n) = \{0, 1, 3, 3, 2\}$

$y_4(n) = x_4(n) \textcircled{N} h(n)$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+0+0+0+0 \\ 1+0+0+0+0 \\ 1+0+0+0+0 \\ 0+0+0+0+0 \\ 0+0+0+0+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$y_4(n) = \{1, 1, 1, 0, 0\}$

$$\boxed{3 \quad 2 \quad 2 \quad -1 \quad 0}$$

$$+\boxed{1 \quad 4 \quad 6 \quad 5 \quad 2}$$

$$+\boxed{0 \quad 1 \quad 3 \quad 3 \quad 2}$$

$$+\boxed{1 \quad 1 \quad 1 \quad 0 \quad 0}$$

$$y(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1\}$$

Find the output $y(n)$ of a filter whose impulse response $h(n) = \{1, 2\}$ and input $x(n) = \{1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1\}$ using overlap save method and overlap add method.

OVERLAP SAVE METHOD:

Given $h(n) = \{1, 2\} \longrightarrow M = 2$

$$x(n) = \{1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1\}$$

Assume $l = 2$

length $= l + M - 1 = 2 + 2 - 1 = 3$

$$x_1(n) = \{\underbrace{0}_{\substack{(M-1)\text{ 0's} \\ \text{are added}}}, \underbrace{1, 2}_{\substack{\text{first M} \\ \text{data points}}}\}$$

$$x_2(n) = \{2, -1, 2\}$$

$$x_3(n) = \{2, 3, -2\}$$

$$x_4(n) = \{-2, -3, -1\}$$

$$x_5(n) = \{-1, 1, 1\}$$

$$x_6(n) = \{1, 2, -1\}$$

$$x_7(n) = \{-1, 0, 0\}$$

$$h(n) = \{1, 2\}$$

$$h(n) = \{1, 2, 0\}$$

$$y_1(n) = x_1(n) \; ⓝ \; h(n)$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0+0+4 \\ 0+1+0 \\ 0+2+2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 4 \end{bmatrix}$$

$y_1(n) = \{4, 1, 4\}$

$y_2(n) = x_2(n) \textcircled{N} h(n)$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2+0+4 \\ 4-1+0 \\ 0-2+2 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 0 \end{bmatrix}$$

$y_2(n) = \{6, 3, 0\}$

$y_3(n) = x_3(n) \textcircled{N} h(n)$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 2+0-4 \\ 4+3+0 \\ 0+6-2 \end{bmatrix} = \begin{bmatrix} -2 \\ 7 \\ 4 \end{bmatrix}$$

$y_3(n) = \{-2, 7, 4\}$

$y_4(n) = x_4(n) \textcircled{N} h(n)$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ -3 \\ -1 \end{bmatrix} = \begin{bmatrix} -2+0-2 \\ -4-3+0 \\ 0-6-1 \end{bmatrix} = \begin{bmatrix} -4 \\ -7 \\ -7 \end{bmatrix}$$

$y_4(n) = \{-4, -7, -7\}$

$y_5(n) = x_5(n) \textcircled{N} h(n)$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1+0+2 \\ -2+1+0 \\ 0+2+1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$$

$y_5(n) = \{1, -1, 3\}$

$y_6(n) = x_6(n) \textcircled{N} h(n)$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1+0-2 \\ 2+2+0 \\ 0+4-1 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$$

$y_6(n) = \{-1, 4, 3\}$

$\boxed{\textcircled{4}, 1, 4}$

$\boxed{\textcircled{6}, 3, 0}$

$\boxed{\textcircled{-2}, 7, 4}$

$\boxed{\textcircled{-4}, -7, -7}$

$\boxed{\textcircled{1}, -1, 3}$

$\boxed{\textcircled{-1}, 4, 3}$

$\boxed{\textcircled{-1}, 2, 0}$

$y_7(n) = x_7(n) \textcircled{N} h(n)$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1+0+0 \\ -2+0+0 \\ 0+0+0 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix}$$

$y_7(n) = \{-1, 2, 0\}$

$y(n) = \{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2\}$

$x(n) - l - 12$

$h(n) - m - 2$

$l + m - 1$

$14 - 1$

13

SHORTCUT METHOD

$$\begin{array}{c|cccccccccccc} & 1 & 2 & -1 & 2 & 3 & -2 & -3 & -1 & 1 & 1 & 2 & -1 \\ \hline 1 & 1 & 2 & -1 & 2 & 3 & -2 & -3 & -1 & 1 & 1 & 2 & -1 \\ 2 & 2 & 4 & -2 & 4 & 6 & -4 & -6 & -2 & 2 & 2 & 4 & -2 \end{array}$$

$y(n) = \{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2\}$

OVERLAP ADD METHOD

$M = 2$

Assume $l = 2$

length $\longrightarrow l + M - 1 = 2 + 2 - 1 = 3$

$x_1(n) = \{1, 2, 0\}$

$x_2(n) = \{-1, 2, 0\}$

$x_3(n) = \{3, -2, 0\}$

$x_4(n) = \{-3, -1, 0\}$

$x_5(n) = \{1, 1, 0\}$

$x_6(n) = \{2, -1, 0\}$

$x_7(n) = \{0, 0, 0\}$

$h(n) = \{-1, 2\}$

$h(n) = \{1, 2, 0\}$

$y_1(n) = x_1(n) \textcircled{N} h(n)$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+0+0 \\ 2+2+0 \\ 0+4+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix}$$

$y_1(n) = \{1, 4, 4\}$

$y_2(n) = x_2(n) \textcircled{N} h(n)$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1+0+0 \\ -2+2+0 \\ 0+4+0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 4 \end{bmatrix}$$

$y_2(n) = \{-1, 0, 4\}$

$y_3(n) = x_3(n) \textcircled{N} h(n)$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 3+0+0 \\ 6-2+0 \\ 0-4+0 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ -4 \end{bmatrix}$$

$y_3(n) = \{3, 4, -4\}$

$y_4(n) = x_4(n) \textcircled{N} h(n)$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -3+0+0 \\ -6-1+0 \\ 0-2+0 \end{bmatrix} = \begin{bmatrix} -3 \\ -7 \\ -2 \end{bmatrix}$$

$y_4(n) = \{-3, -7, -2\}$

$y_5(n) = x_5(n) \textcircled{N} h(n)$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+0+0 \\ 2+1+0 \\ 0+2+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

$y_5(n) = \{1, 3, 2\}$

$y_6(n) = x_6(n) \textcircled{N} h(n)$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2+0+0 \\ 4-1+0 \\ 0-2+0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$$

$y_6(n) = \{2, 3, -2\}$

$$y_7(n) = x_7(n) \textcircled{N} h(n)$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+0+0 \\ 0+0+0 \\ 0+0+0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{1 \quad 4 \quad 4}$$

$$+$$

$$\boxed{-1 \quad 0 \quad 4}$$

$$+$$

$$\boxed{3 \quad 4 \quad -4}$$

$$+$$

$$\boxed{-3 \quad -7 \quad -2}$$

$$+$$

$$\boxed{1 \quad 3 \quad 2}$$

$$+$$

$$\boxed{2 \quad 3 \quad -2}$$

$$+$$

$$\boxed{0 \quad 0 \quad 0}$$

$$y(n) = \{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2\}$$

Find DFT of $x(n) = \{1, 1, 1, 1\}$

$N = 4$

$$X(k) = W_N\, x(n)$$

$$= W_4\, x(n)$$

$$W_4 = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ 1 & W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ 2 & W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ 3 & W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{array}$$

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$X(k) = W_4\, x(n)$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+1+1 \\ \cancel{1} - \cancel{j} - \cancel{1} + \cancel{j} \\ \cancel{1} - \cancel{1} + \cancel{1} - \cancel{1} \\ \cancel{1} + \cancel{j} - \cancel{1} - \cancel{j} \end{bmatrix}$$

$$X(k) = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$X(k) = \{4, 0, 0, 0\}$$

$$r = nk$$

$$W_N^r = e^{-j\frac{2\pi r}{N}}$$

$$W_4^0 = e^0 = 1$$

$$W_4^1 = e^{-j\frac{2\pi}{4}} = e^{-j\frac{\pi}{2}} = -j$$

$$W_N^r = -W_N^{r \pm \frac{N}{2}}$$

$$W_N^r = W_N^{r \pm N}$$

$$W_4^{1 \pm \frac{4}{2}} = W_4^3 = -(-j)$$

$$W_4^3 = j$$

$$W_4^2 = e^{-j\frac{4\pi}{4}} = e^{-j\pi} = -1$$

$$W_4^2 = -1$$

$$W_4^{2 \pm \frac{4}{2}} = W_4^{2 \pm 2} = W_4^4$$

$$= -(-1)$$

$$W_4^4 = 1$$

$$W_4^6 = e^{-j\frac{2\pi \cdot \cancel{6}^{3}}{\cancel{4}_{2}}} = e^{-j3\pi}$$

$$W_4^6 = -1$$

$$W_4^{6 \pm \frac{4}{2}} = W_4^8 = -(-1)$$

$$W_4^8 = 1$$

$$W_4^9 = e^{-j\frac{2\pi \cdot 9}{\cancel{4}_{2}}} = e^{-j\frac{9\pi}{2}}$$

$$W_4^9 = -j$$

Scanned by CamScanner

06-02-19

RELATIONSHIP OF DFT TO OTHER TRANSFORM :

1> RELATIONSHIP TO THE Z-TRANSFORM :

$$X(z) = \sum_{n=0}^{N-1} x(n)\, z^{-n}$$

IDFT $$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{\frac{j2\pi nk}{N}}$$

$$X(z) = \sum_{n=0}^{N-1} \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{\frac{j2\pi nk}{N}}\, z^{-n}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k) \sum_{n=0}^{N-1} \left[e^{j\frac{2\pi k}{N}} . z^{-1}\right]^{n}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k)\left[\frac{1-\left(e^{j\frac{2\pi k}{N}} . z^{-1}\right)^{\substack{N\cancel{-1}\\ \cancel{+1}}}}{1-e^{j\frac{2\pi k}{N}} . z^{-1}}\right]$$

$a > 1$

$$\sum_{n=0}^{N} a^{n} = \frac{a^{N+1}-1}{a-1}$$

$a < 1$

$$\sum_{n=0}^{N} a^{n} = \frac{1-a^{N+1}}{1-a}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k)\left[\frac{1-e^{j\frac{2\pi k}{\cancel{N}} . \cancel{N}} . z^{-N}}{1-e^{j\frac{2\pi k}{N}} . z^{-1}}\right]$$

$$X(z) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\left[\frac{1-z^{-N}}{1-e^{j\frac{2\pi k}{N}} . z^{-1}}\right]$$

2> RELATIONSHIP TO THE DTFT :

$$X(e^{j\omega}) = \sum_{n=0}^{N-1} x(n)\, e^{-j\omega n}$$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi nk}{N}}$$

$$\boxed{X(k) = X(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{N}} \qquad k = 0, 1, \text{---}\ N-1}$$

05-02-19

UNIT - II

FAST FOURIER TRANSFORM

↓ Pattern Recognition

→ It requires less no. of Computation

✓ FFT Reduction factor more than 100 over

N = 1024 $\boxed{N = 2^M}$

DFT

mul

N^2

$1048576 \sim 10^6$

complex additions

$N(N-1)$

FFT

mul

$\frac{N}{2}\log_2 N = \frac{1024}{2}\log_2 1024$

$\boxed{5120}$ $= \frac{1024}{2}\log_2 2^{10}$

$= \overset{5}{\cancel{10}} \cdot \frac{1024}{\cancel{2}}$

$= 5120 \simeq 5000$

add

$N \log_2 N$

Reduction factor $= \frac{10^6}{5 \times 10^3} = \frac{1000}{5}$

$= 200$

Fast Fourier Transform

↓ DIT - FFT [Decimation in Time - FFT]

↓ DIF - FFT [Decimation in frequency - FFT]

$x(n) = \{\overset{0}{1}, \overset{1}{1}, \overset{2}{1}, \overset{3}{1}\}$

$X(K) = \{4, 0, 0, 0\}$

$x(n) = \{\overset{0}{1}, \overset{1}{3}, \overset{2}{7}, \overset{3}{8}, \overset{4}{3}, \overset{5}{5}, \overset{6}{3}, \overset{7}{2}\}$

DIT - FFT

1,7,3,3 — even places

3, 8 ,5, 2 — odd places

DIF- FFT

1, 3, 7, 8

3, 5, 3, 2

07-02-19

DECIMATION IN TIME - FAST FOURIER TRANSFORM [DIT-FF

This is also known as radix-2 DIT-FFT.

$$N = 2^M$$

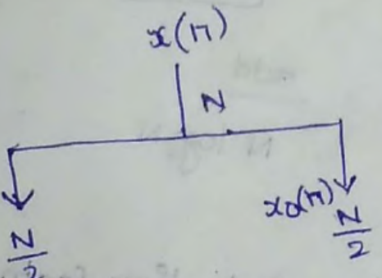
where M → integer

✓ let $x(n)$ be 'N' point sequence, N is assumed

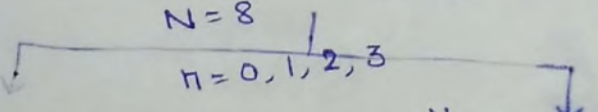
be power of 2.

✓ Decimate or break this sequence into two subse

of each length 'N/2'.



$$x_e(n) = x(2n) \quad \text{for } n = 0, 1, ----, \frac{N}{2}-1$$



$$x_o(n) = x(2n+1) \quad \text{for } n = 0, 1, ----, \frac{N}{2}-1$$

[N-point DFT of $x(n)$]

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{nk} \quad ; \; e^{-j\frac{2\pi nk}{N}} = W_N^{nk}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_N^{2nk} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_N^{(2n+1)k}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_N^{2nk} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_N^{2nk} \cdot W_N^{k}$$

$$W_N^{2nk} = e^{-j\frac{2\pi \cdot 2nk}{N}} = e^{-j\frac{2\pi nk}{N/2}}$$

$$= W_{\frac{N}{2}}^{nk}$$

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x_e(n) W_{\frac{N}{2}}^{nk} + W_N^k \sum_{n=0}^{\frac{N}{2}-1} x_o(n) W_{\frac{N}{2}}^{nk}$$

$$\boxed{X(k) = X_e(k) + W_N^k X_o(k)} \longrightarrow ① \qquad 0 \le k \le \frac{N}{2}-1$$

from Symmetric property (Twiddle factor)

$$\boxed{W_N^{k \pm \frac{N}{2}} = -W_N^k}$$

$$W_N^k = -W_N^{k - N/2}$$

Substituting $k = k - \frac{N}{2}$ in eqn ①

$$X(k) = X_e\left(k-\frac{N}{2}\right) - W_N^{k-\frac{N}{2}} X_o\left(k-\frac{N}{2}\right) \longrightarrow ② \qquad \frac{N}{2} \le k \le N-1$$

For $N=8$, $X_e(k)$ and $X_o(k)$ are 4-point dft of $x_e(n)$ and $x_o(n)$ respectively.

$N = 8$

$x(n) = \{x(0), x(1), x(2), \ldots\ldots, x(7)\}$

$x_e(n) = \{x(0), x(2), x(4), x(6)\}$

$x_o(n) = \{x(1), x(3), x(5), x(7)\}$

$X(0) = X_e(0) + W_8^0 X_o(0)$

$X(1) = X_e(1) + W_8^1 X_o(1)$

$X(2) = X_e(2) + W_8^2 X_o(2)$

$X(3) = X_e(3) + W_8^3 X_o(3)$

$X(4) = X_e(0) - W_8^0 X_o(0)$

$X(5) = X_e(1) - W_8^1 X_o(1)$

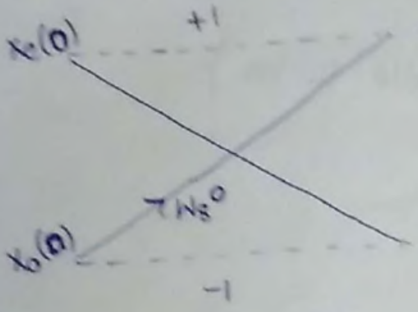
$X(6) = X_e(2) - W_8^2 X_o(2)$

$X(7) = X_e(3) - W_8^3 X_o(3)$

X(0) is obtained by multiplying $X_o(0)$ with W_8^0 and adding the product to $X_e(0)$.

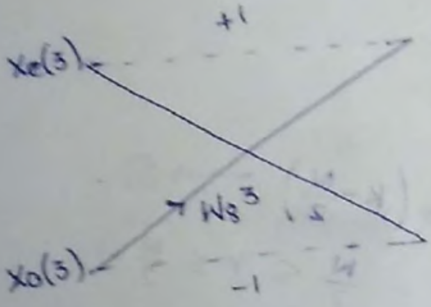
Ily X(4) is obtained by multiplying $X_o(0)$ with W_8^0 and subtracting the product to $X_e(0)$.

BUTTERFLY DIAGRAMS



$X(0) = X_e(0) + W_8^0 X_o(0)$

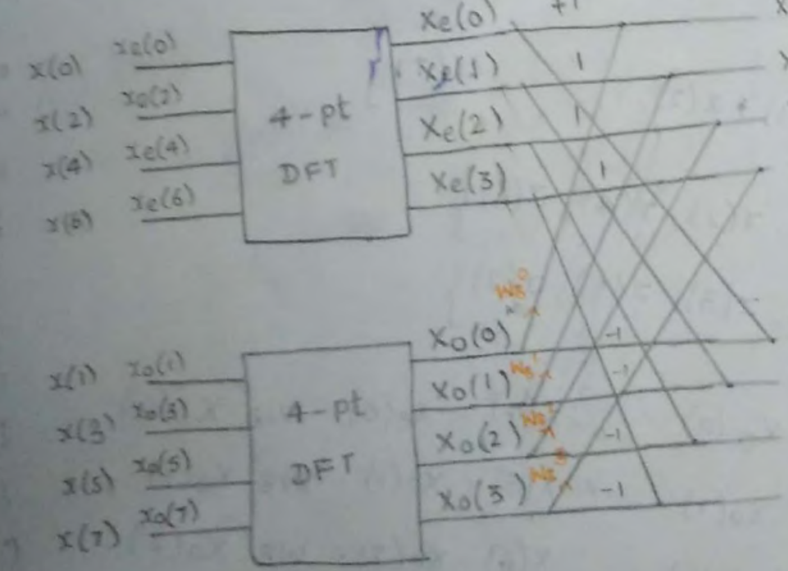
$X(4) = X_e(0) - W_8^0 X_o(0)$



$X(3) = X_e(3) + W_8^3 X_o(3)$

$X(7) = X_e(3) - W_8^3 X_o(3)$

8-point DFT flow graph can be constructed from two 4-point DFTS.



$X(0) = X_e(0) + W_8^0 X_o(0)$

$X(1) = X_e(1) + W_8^1 X_o(1)$

$X(2) = X_e(2) + W_8^2 X_o(2)$

$X(3) = X_e(3) + W_8^3 X_o(3)$

$X(4) = X_e(0) - W_8^0 X_o(0)$

$X(5) = X_e(1) - W_8^1 X_o(1)$

$X(6) = X_e(2) - W_8^2 X_o($

$X(7) = X_e(3) - W_8^3 X_o($

12-02-19

STEPS TO FOLLOW IN RADIX-2 DIT-FFT:

1> No. of input samples $N = 2^m$

2> Input sequence is bit reversal or shuffled and output sequence is natural order.

BIT REVERSAL ORDER

N = 8

$0 \longrightarrow 000 \longrightarrow 000 \longrightarrow 0$

$1 \longrightarrow 001 \longrightarrow 100 \longrightarrow 4$

$2 \longrightarrow 010 \longrightarrow 010 \longrightarrow 2$

$3 \longrightarrow 011 \longrightarrow 110 \longrightarrow 6$

$4 \longrightarrow 100 \longrightarrow 001 \longrightarrow 1$

$5 \longrightarrow 101 \longrightarrow 101 \longrightarrow 5$

$6 \longrightarrow 110 \longrightarrow 011 \longrightarrow 3$

$7 \longrightarrow 111 \longrightarrow 111 \longrightarrow 7$

N = 4 0, 1, 2, 3

BIT REVERSAL

$0 \longrightarrow 00 \longrightarrow 00 \longrightarrow 0$

$1 \longrightarrow 01 \longrightarrow 10 \longrightarrow 2$

$2 \longrightarrow 10 \longrightarrow 01 \longrightarrow 1$

$3 \longrightarrow 11 \longrightarrow 11 \longrightarrow 3$

3> No. of stages in flow graph $\boxed{M = \log_2 N}$

4> Each stage consists of $\frac{N}{2}$ butterflies

5> No. of complex multiplications $\frac{N}{2} \log_2 N$

6> No. of complex additions $N \log_2 N$

7> Twiddle factor exponents are a function of a stage index 'm' is given as

$$\boxed{K = \frac{Nt}{2^m}}, \quad t = 0, 1, 2, \cdots 2^{m-1} - 1$$

8> No. of sets of butterflies in each stage is 2^{M-m}

9> Exponent Repeat factor (ERF) $= 2^{M-m}$

1) Find DFT of sequence $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$ Using DIT-FFT.

$x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$

i) $N = 8 = 2^3 \longrightarrow$ No. of input samples

ii) i/p ⟶ Bit reversal order

o/p ⟶ Natural order

iii) No. of stages in flowgraph is

$M = \log_2 N = \log_2 8 = 3$

iv) Each stage consists of $\frac{N}{2} = \frac{8}{2} = 4$ butterflies

v) No. of complex multiplications are $\frac{N}{2} \log_2 N$

$= \frac{8}{2} \log_2 8 = 4 \cdot 3 = 12$

vi) No. of complex additions are $N \log_2 N$

$= 8 \log_2 8 = 8 \cdot 3 = 24$

vii) Twiddle factor

$K = \frac{Nt}{2^m}$ where $t = 0, 1, 2, ------ 2^{m-1} - 1$.

stage 1 : $m = 1$

$t = 0, 1, 2, --- 2^{1-1} - 1$

$t = 0$

$K = \frac{8(0)}{2^1} = 0 \longrightarrow W_8^0$

stage 2 : $m = 2$

$t = 0, 1, 2, ---- 2^{2-1} - 1$

$t = 0, 1$

$t = 0 \implies K = \frac{8(0)}{2^2} = 0 \longrightarrow W_8^0$

$t = 1 \implies K = \frac{8(1)}{2^2} = 2 \longrightarrow W_8^2$

stage 3 : $m = 3$

$t = 0, 1, 2, \text{----} \; 2^{3-1} - 1$

$t = 0, 1, 2, 3$

$t = 0 \implies K = \frac{8(0)}{2^3} = 0 \longrightarrow W_8^0$

$t = 1 \implies K = \frac{8(1)}{2^3} = 1 \longrightarrow W_8^1$

$t = 2 \implies K = \frac{8(2)}{2^3} = 2 \longrightarrow W_8^2$

$t = 3 \implies K = \frac{8(3)}{2^3} = 3 \longrightarrow W_8^3$

viii> No. of sets of butterflies in each stage is 2^{M-m}

stage 1 : $2^{3-1} = 2^2 = 4$

stage 2 : $2^{3-2} = 2^1 = 2$

stage 3 : $2^{3-3} = 2^0 = 1$

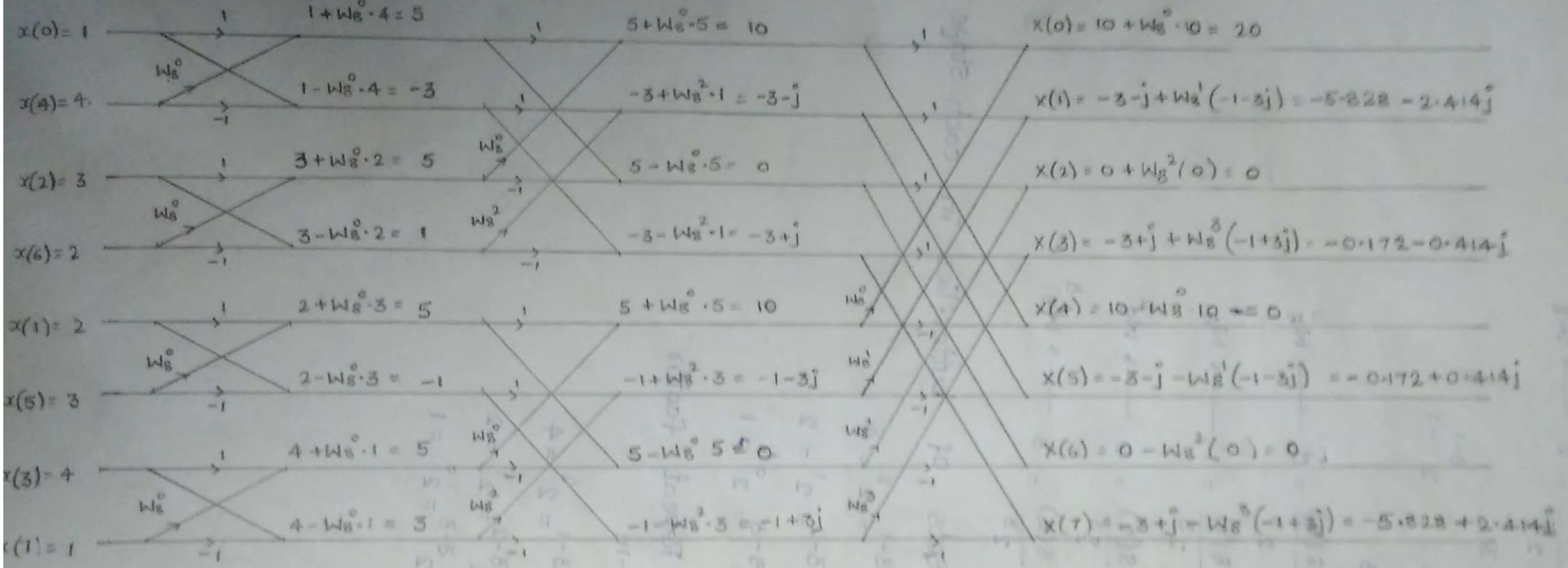
ix> Exponent Repeat factor

$ERF = 2^{M-m}$

stage 1 : $2^{3-1} = 2^2 = 4$

stage 2 : $2^{3-2} = 2^1 = 2$

stage 3 : $2^{3-3} = 2^0 = 1$



$$X(K) = \{20, -5.828 - 2.414j, 0, -0.172 - 0.414j, 0, -0.172 + 0.414j, 0, -5.828 + 2.414j\}$$

13-02-19

Find DFT of the Sequence $x(n) = \{0.5, 0.5, 0.5, 0.5, 0, 0, 0, 0\}$ using DIT-FFT.

$$x(n) = \{0.5, 0.5, 0.5, 0.5, 0, 0, 0, 0\}$$

i) No. of Input Samples $N = 8 = 2^3$

ii) i/p ⟶ Bit reversal order

o/p ⟶ Natural order

iii) No. of stages in flow graph is

$$M = \log_2 N = \log_2 8 = 3$$

iv) Each stage consists of $\frac{N}{2} = \frac{8}{2} = 4$ butterflies

v) No. of complex multiplications are $\frac{N}{2} \log_2 N$

$$= \frac{8}{2} \log_2 8 = 4 \cdot 3 = 12$$

vi) No. of complex additions are $N \log_2 N$

$$= 8 \log_2 8 = 8 \cdot 3 = 24$$

vii) Twiddle factor

$$K = \frac{Nt}{2^m} \quad \text{where } t = 0, 1, 2, \text{------} 2^{m-1} - 1$$

Stage 1 : $m = 1$

$$K = \frac{8(0)}{2^1} = 0 \longrightarrow W_8^0 \qquad t = 0, 1, 2, \text{----} 2^{1-1} - 1$$

$t = 0$

Stage 2 : $m = 2$

$$t = 0, 1, \text{----} 2^{2-1} - 1$$

$t = 0, 1$

$$t=0 \Longrightarrow K = \frac{8(0)}{2^2} = 0 \longrightarrow W_8^0$$

$$t=1 \Longrightarrow K = \frac{8(1)}{2^2} = 2 \longrightarrow W_8^2$$

stage 3 : $m = 3$

$t = 0, 1, \text{----} 2^{3-1} - 1$

$t = 0, 1, 2, 3$

$t = 0 \Longrightarrow K = \frac{8(0)}{2^3} = 0 \longrightarrow W_8^0$

$t = 1 \Longrightarrow K = \frac{8(1)}{2^3} = 1 \longrightarrow W_8^1$

$t = 2 \Longrightarrow K = \frac{8(2)}{2^3} = 2 \longrightarrow W_8^2$

$t = 3 \Longrightarrow K = \frac{8(3)}{2^3} = 3 \longrightarrow W_8^3$

viii> No. of sets of butterflies in each stage is 2^{M-m}

stage 1 : $2^{3-1} = 2^2 = 4$

stage 2 : $2^{3-2} = 2^1 = 2$

stage 3 : $2^{3-3} = 2^0 = 1$

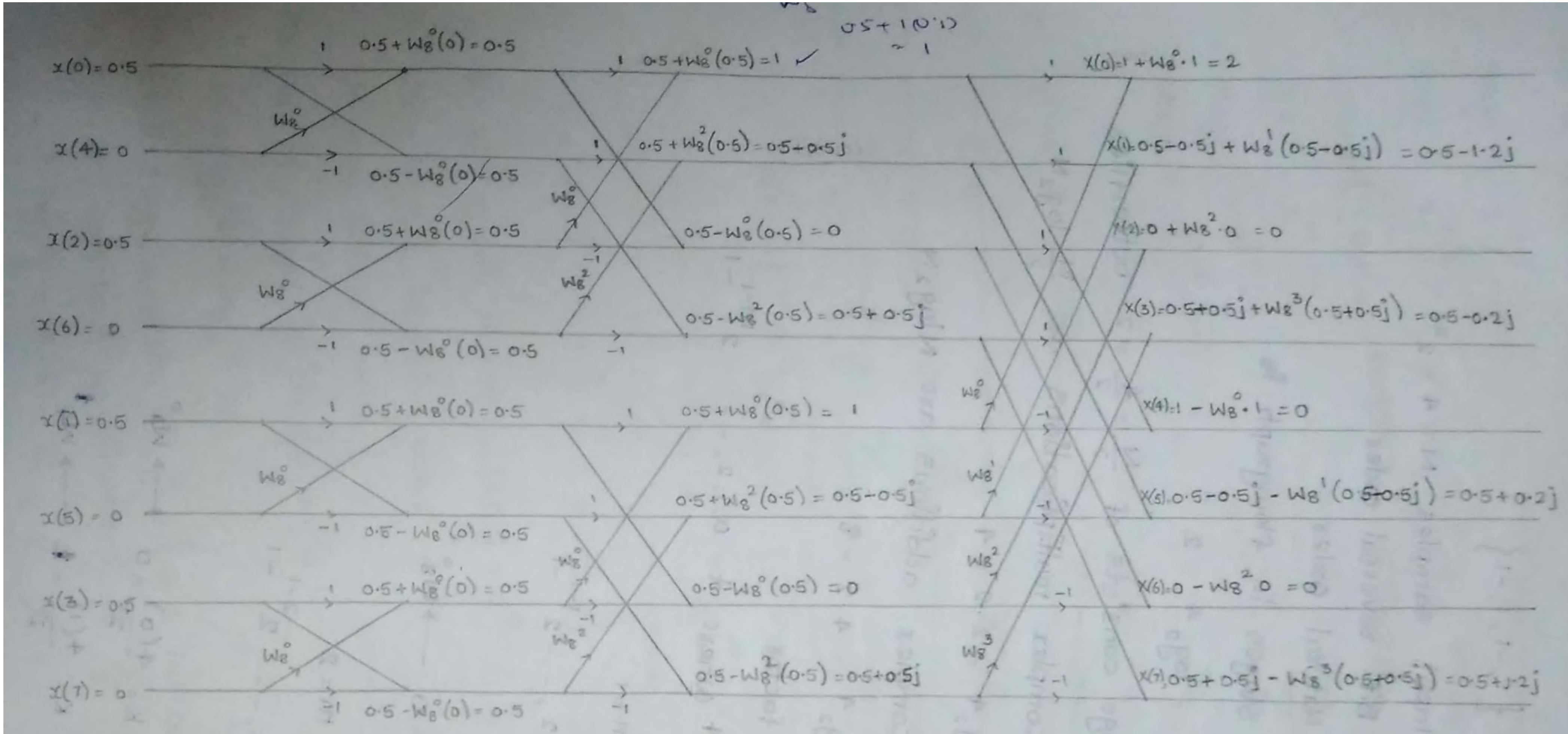
ix> Exponent Repeat factor

$ERF = 2^{M-m}$

stage 1 : $2^{3-1} = 2^2 = 4$

stage 2 : $2^{3-2} = 2^1 = 2$

stage 3 : $2^{3-3} = 2^0 = 1$



$$X(k) = \{2,\ 0.5-1.2j,\ 0,\ 0.5-0.2j,\ 0,\ 0.5+0.2j,\ 0,\ 0.5+1.2j\}$$

Find DFT of the sequence $x(n) = \{1,-1,1,-1\}$ using DIT-FFT

$x(n) = \{1,-1,1,-1\}$

i) No. of input samples $N = 4 = 2^2$

ii) I/P ⟶ Bit reversal order

O/P ⟶ Natural order

iii) No of stages in flowgraph is

$M = \log_2 N = \log_2 4 = 2$

iv) Each stage consists of $\frac{N}{2} = \frac{4}{2} = 2$ butterflies

v) No. of complex multiplications are $\frac{N}{2} \log_2 N$

$= \frac{4}{2} \log_2 4 = 2 \cdot 2 = 4$

vi) No. of complex additions are $N \log_2 N$

$= 4 \log_2 4 = 4 \cdot 2 = 8$

vii) Twiddle factor

$K = \frac{Nt}{2^m}$ where $t = 0,1,2, \ldots 2^{m-1}-1$

Stage 1 : $m = 1$

$t = 0,1,2, \ldots 2^{1-1}-1$

$t = 0$

$K = \frac{4(0)}{2^1} = 0 \longrightarrow W_4^0$

Stage 2 : $m = 2$

$t = 0,1,2, \ldots 2^{2-1}-1$

$t = 0,1$

$t = 0 \Rightarrow K = \frac{4(0)}{2^2} = 0 \longrightarrow W_4^0$

$t = 1 \longrightarrow K = \frac{4(1)}{2^2} = 1 \longrightarrow W_4^1$

viii> No. of sets of butterflies in each stage is 2^{M-m}

Stage 1 : $2^{2-1} = 2$

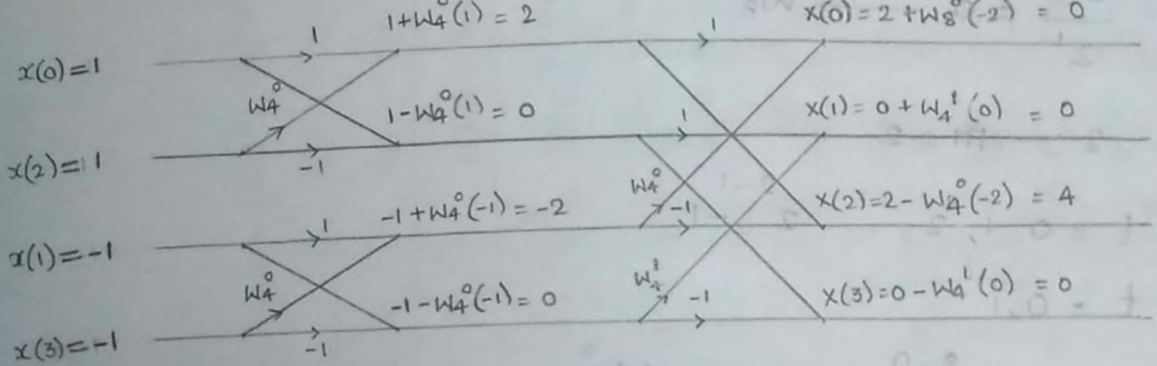
Stage 2 : $2^{2-2} = 1$

ix > Exponent Repeat factor

$ERF = 2^{M-m}$

Stage 1 : $2^{2-1} = 2$

Stage 2 : $2^{2-2} = 1$



$X(K) = \{0, 0, 4, 0\}$

14-02-19

Find DFT of the sequence $x(n) = \{5, 0, 1-j, 0, 1, 0, 1+j, 0\}$ using DIT-FFT.

$x(n) = \{5, 0, 1-j, 0, 1, 0, 1+j, 0\}$

i> No. of Input samples $N = 8 = 2^4$

ii> I/P ⟶ Bit Reversal order

O/P ⟶ Natural order

iii> No. of stages in flowgraph is

$M = \log_2 N = \log_2 8 = 3$

iv> Each stage consists of $\frac{N}{2} = \frac{8}{2} = 4$ butterflies

v> No. of complex multiplications of $\frac{N}{2} \log_2 N$

$= \frac{8}{2} \log_2 8 = 4 \cdot 3 = 12$

vi> No. of complex additions are $N \log_2 N$

$= 8 \log_2 8 = 8 \cdot 3 = 24$

vii> Twiddle factor

$K = \frac{Nt}{2^m}$ where $t = 0, 1, 2, \ldots 2^{m-1} - 1$

Stage 1 : $m = 1$

$t = 0, 1, 2, \ldots 2^{1-1} - 1$

$t = 0$

$K = \frac{8 \cdot 0}{2^1} = 0 \longrightarrow W_8^0$

Stage 2 : $m = 2$

$t = 0, 1, 2, \ldots 2^{2-1} - 1$

$t = 0, 1$

$t = 0 \Longrightarrow K = \frac{8 \cdot 0}{2^2} = 0 \longrightarrow W_8^0$

$t = 1 \Longrightarrow K = \frac{8 \cdot 1}{2^2} = 2 \longrightarrow W_8^2$

Stage 3 : $m = 3$

$t = 0, 1, 2, \ldots 2^{3-1} - 1$

$t = 0, 1, 2, 3$

$t = 0 \Longrightarrow K = \frac{8 \cdot 0}{2^3} = 0 \longrightarrow W_8^0$

$t = 1 \Longrightarrow K = \frac{8 \cdot 1}{2^3} = 1 \longrightarrow W_8^1$

$t = 2 \Longrightarrow K = \frac{8 \cdot 2}{2^3} = 2 \longrightarrow W_8^2$

$t = 3 \Longrightarrow K = \frac{8 \cdot 3}{2^3} = 3 \longrightarrow W_8^3$

viii> No. of sets of butterflies in each stage is 2^{M-m}

stage 1 : $2^{3-1} = 2^2 = 4$

stage 2 : $2^{3-2} = 2^1 = 2$

stage 3 : $2^{3-1} = 2^0 = 1$

Scanned by CamScanner

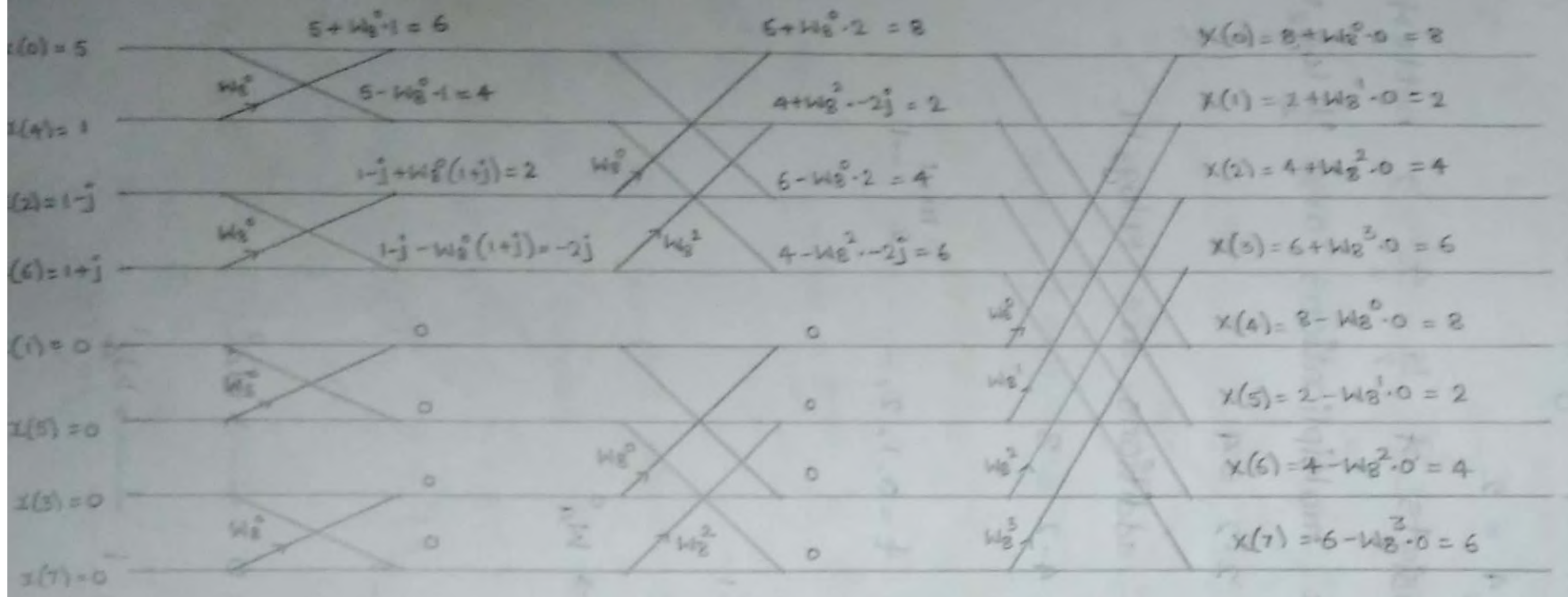
ix> Exponent Repeat factor

$$ERF = 2^{M-m}$$

Stage 1 : $2^{3-1} = 2^2 = 4$

Stage 2 : $2^{3-2} = 2^1 = 2$

Stage 3 : $2^{3-3} = 2^0 = 1$



$$X(k) = \{8, 2, 4, 6, 8, 2, 4, 6\}$$

$$W_N^r = e^{-j\frac{2\pi r}{N}}$$

$$W_8^0 = e^{-j\frac{2\pi 0}{8}} = e^0 = 1$$

$$W_8^1 = e^{-j\frac{2\pi \cdot 1}{8}} = e^{-j\pi/4} = 0.707 - j\,0.707$$

$$W_8^2 = e^{-j\frac{2\pi \cdot 2}{8}} = e^{-j\pi/2} = 0 - j \cdot 1 = -j$$

$$W_8^3 = e^{-j\frac{2\pi \cdot 3}{8}} = e^{-j\frac{3\pi}{4}} = -0.707 - 0.707j$$

19-02-19

DECIMATION IN FREQUENCY - FAST FOURIER TRANSFORM (DIF - FFT) :

→ In this algorithm $x(n)$ is divided into two parts of length $\frac{N}{2}$.

→ The first $\frac{N}{2}$ Samples ~~are~~ of $x(n)$ is $x_1(n)$

→ The second $\frac{N}{2}$ Samples of $x(n)$ is $x_2(n)$

Then $x_1(n) = x(n)$, $n = 0, 1, 2, ----, \frac{N}{2}-1$

$x_2(n) = x(n)$, $n = \frac{N}{2}, ------ N-1$

N-point DFT of $x(n)$

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi nk}{N}}$$

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{nk} \longrightarrow ①$$

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x(n)\, W_N^{nk} + \sum_{n=\frac{N}{2}}^{N-1} x(n)\, W_N^{nk}$$

2^{nd} term modified as

$$n = m + \frac{N}{2}$$

$$n = \frac{N}{2} \longrightarrow m = 0$$

$$n = N-1 \longrightarrow m = \frac{N}{2} - 1$$

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x(n)\, W_N^{nk} + \sum_{m=0}^{\frac{N}{2}-1} x\left(m+\frac{N}{2}\right) W_N^{\left(m+\frac{N}{2}\right)k}$$

Replacing 'm' with 'n'

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x(n)\, W_N^{nk} + \sum_{n=0}^{\frac{N}{2}-1} x\left(n+\frac{N}{2}\right) W_N^{\left(n+\frac{N}{2}\right)k}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(n)\, W_N^{nk} + \sum_{n=0}^{\frac{N}{2}-1} x\left(n+\frac{N}{2}\right) W_N^{nk}\; W_N^{\frac{N}{2}k}$$

$$\left\{ \; W_N^{\frac{N}{2}k} = e^{\frac{-j\, 2\pi \frac{N}{2} k}{N}} = e^{-j\pi k} \right.$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(n)\, W_N^{nk} + e^{-j\pi k} \sum_{n=0}^{\frac{N}{2}-1} x\left(n+\frac{N}{2}\right) W_N^{nk}$$

$$\boxed{X(k) = \sum_{n=0}^{\frac{N}{2}-1} x_1(n)\, W_N^{nk} + e^{-j\pi k} \sum_{n=0}^{\frac{N}{2}-1} x_2(n)\, W_N^{nk}} \longrightarrow ②$$

CASE (1) k is even, eqn ② becomes

$$X(2k) = \sum_{n=0}^{\frac{N}{2}-1} x_1(n)\, W_N^{2nk} + \underbrace{e^{-j2\pi k}}_{=1} \sum_{n=0}^{\frac{N}{2}-1} x_2(n)\, W_N^{2nk}$$

$$\left\{ W_N^{2nk} = W_{\frac{N}{2}}^{nk} \right.$$

$$X(2k) = \sum_{n=0}^{\frac{N}{2}-1} x_1(n)\, W_{\frac{N}{2}}^{nk} + \sum_{n=0}^{\frac{N}{2}-1} x_2(n)\, W_{\frac{N}{2}}^{nk}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \left[x_1(n) + x_2(n) \right] W_{\frac{N}{2}}^{nk}$$

let $f(n) = x_1(n) + x_2(n)$

$$X(2k) = \sum_{n=0}^{\frac{N}{2}-1} f(n)\, W_{\frac{N}{2}}^{nk} \longrightarrow ③$$

CASE (ii) k is odd, eqn ② becomes

$$X(2k+1) = \sum_{n=0}^{\frac{N}{2}-1} x_1(n)\, W_N^{n(2k+1)} + e^{-j\pi(2k+1)} \sum_{n=0}^{\frac{N}{2}-1} x_2(n)\, W_N^{n(2k+1)}$$

$$\left\{ e^{-j\pi(2k+1)} = e^{-j2\pi k} \cdot e^{-j\pi} = 1 \cdot -1 = -1 \right.$$

$$X(2k+1) = \sum_{n=0}^{\frac{N}{2}-1} x_1(n)\, W_N^{2nk} \cdot W_N^{n} - \sum_{n=0}^{\frac{N}{2}-1} x_2(n)\, W_N^{2nk} \cdot W_N^{n}$$

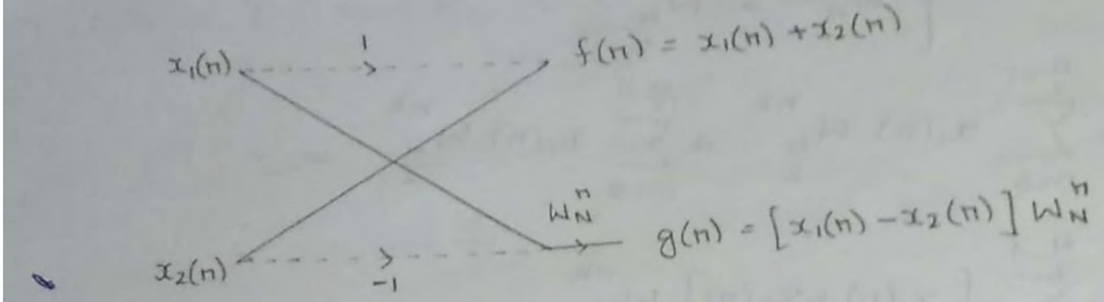
$$\left\{ W_N^{2nk} = W_{\frac{N}{2}}^{nk} \right.$$

$$X(2k+1) = \sum_{n=0}^{\frac{N}{2}-1} x_1(n)\, W_{\frac{N}{2}}^{nk} \cdot W_N^{n} - \sum_{n=0}^{\frac{N}{2}-1} x_2(n)\, W_{\frac{N}{2}}^{nk} \cdot W_N^{n}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \left[x_1(n) - x_2(n) \right] W_{\frac{N}{2}}^{nk} \cdot W_N^{n}$$

let $g(n) = \left[x_1(n) - x_2(n) \right] W_N^{n}$

$$X(2k+1) = \sum_{n=0}^{\frac{N}{2}-1} g(n) \cdot W_{\frac{N}{2}}^{nk} \longrightarrow ④$$



STEPS FOR RADIX-2 DIF-FFT ALGORITHM :

1> No. of Input Samples $N = 2^M$

2> Input sequence is Natural order

output sequence is Bit reversal order

3> No. of stages in flowgraph $M = \log_2 N$

4> Each stage consists of $\frac{N}{2}$ butterflies

5> No. of complex multiplications are $\frac{N}{2} \log_2 N$

6> No. of Complex additions are $N \log_2 N$

7> Twidde factor exponents are functions of stage index 'm' and is given by

$$K = \frac{Nt}{2^{M-m+1}}, \quad t = 0, 1, \text{-------} 2^{M-m} - 1$$

8> No. of sets of butterflies in each stage is 2^{m-1}

9> Exponent Repeat factor

$$ERF = 2^{m-1}$$

Find DFT of sequence $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$ by using DIF-FFT.

1> No. of Input Samples $N = 8 = 2^3$

2> Input ⟶ Natural order
output ⟶ Bit reversal order

3> No. of stages in flowgraph is

$$M = \log_2 N = \log_2 8 = 3$$

4> Each stage consists of $\frac{N}{2} = \frac{8}{2} = 4$ butterflies

5> No. of complex multiplications are $\frac{N}{2} \log_2 N$

$$= \frac{8}{2} \log_2 8 = 4 \cdot 3 = 12$$

6> No. of complex ~~mult~~ additions are $N \log_2 N$

$$= 8 \log_2 8 = 8 \cdot 3 = 24$$

7> Twiddle factor :

$$K = \frac{Nt}{2^{M-m+1}}, \quad t = 0, 1, 2, ----- 2^{M-m} - 1$$

Stage 1 : $m = 1$

$$t = 0, 1, 2, --- 2^{3-1} - 1$$

$$t = 0, 1, 2, 3$$

$$t=0 \Rightarrow K = \frac{8 \cdot 0}{2^{3-1+1}} = 0 \longrightarrow W_8^0$$

$$t=1 \Longrightarrow K = \frac{8 \cdot 1}{2^{3-1+1}} = \frac{8}{8} = 1 \longrightarrow W_8^1$$

$$t=2 \Longrightarrow K = \frac{8 \cdot 2}{2^{3-1+1}} = \frac{8 \cdot 2}{8} = 2 \longrightarrow W_8^2$$

$$t=3 \Longrightarrow K = \frac{8 \cdot 3}{2^{3-1+1}} = \frac{8 \cdot 3}{8} = 3 \longrightarrow W_8^3$$

stage 2 : $m = 2$

$t = 0, 1, \ldots 2^{3-2} - 1$

$t = 0, 1$

$t = 0 \longrightarrow k = \frac{8 \cdot 0}{2^{3-2+1}} = 0 \longrightarrow W_8^0$

$t = 1 \Longrightarrow k = \frac{8 \cdot 1}{2^{3-2+1}} = \frac{8}{4} = 2 \longrightarrow W_8^2$

stage 3 : $m = 3$

$t = 0, 1, \ldots, 2^{3-3} - 1$

$t = 0$

$k = \frac{8 \cdot 0}{2^{3-3+1}} = 0 \longrightarrow W_8^0$

8> No. of sets of butterflies in each stage is 2^{m-1}

stage 1 : $2^{1-1} = 1$

stage 2 : $2^{2-1} = 2$

stage 3 : $2^{3-1} = 4$

$\frac{Nt}{2^{M-m+1}}$

$k = 0, 1, \ldots 2^{M-m} - 1$

$\frac{Nt}{2^{m}}$

$k = 0, 1, \ldots 2^{m-1} - 1$

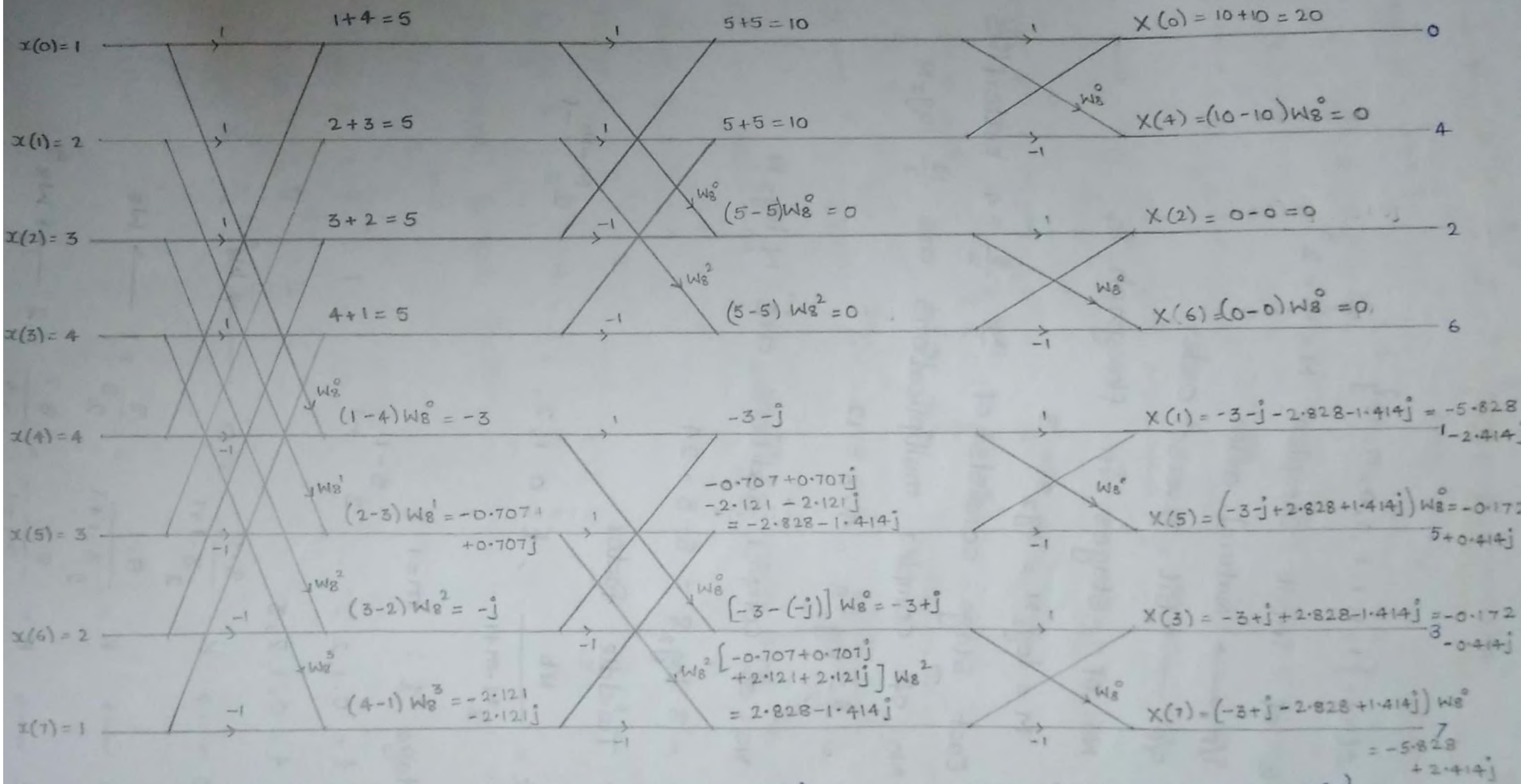
9> Exponent Repeat factor

$ERF = 2^{m-1}$

Stage 1 : $2^{1-1} = 1$

Stage 2 : $2^{2-1} = 2$

stage 3 : $2^{3-1} = 4$



$X(K)=\{20, -5.828-2.414j, 0, -0.172-0.414j, 0, -0.172+0.414j, 0, -5.828+2.414j\}$

Find DFT of the Sequence $x(n) = \{1, -1, 1, -1\}$ using DIF-FFT.

i> No. of input Samples $N = 4 = 2^2$

ii> Input $\longrightarrow$ Natural order

output $\longrightarrow$ Bit Reversal order

iii> No. of stages in flow graph is

$$M = \log_2 N = \log_2 4 = 2$$

iv> Each stage consists of $\frac{N}{2} = \frac{4}{2} = 2$ butterflies

v> No. of complex multiplications are $\frac{N}{2} \log_2 N$

$$= \frac{4}{2} \log_2 4 = 2 \cdot 2 = 4$$

vi> No. of complex additions are $N \log_2 N$

$$= 4 \log_2 4 = 4 \cdot 2 = 8$$

vii> Twiddle factor

$$K = \frac{Nt}{2^{M-m+1}}, \quad t = 0, 1, 2, \text{-----} 2^{M-m} - 1$$

Stage 1 : $m = 1$

$$t = 0, 1, 2, \text{---} 2^{2-1} - 1$$

$$t = 0, 1$$

$$t=0 \Longrightarrow K = \frac{4 \cdot 0}{2^{2-1+1}} = 0 \longrightarrow W_4^0$$

$$t=1 \Longrightarrow K = \frac{4 \cdot 1}{2^{2-1+1}} = \frac{4}{4} = 1 \longrightarrow W_4^1$$

Stage 2 : $m = 2$

$$t = 0, 1, 2, \text{---} 2^{2-2} - 1$$

$$t = 0$$

$$t=0 \Longrightarrow K = \frac{4 \cdot 0}{2^{2-2+1}} = 0 \longrightarrow W_4^0$$

viii> No. of Sets of butterflies in each stage is 2^{m-1}

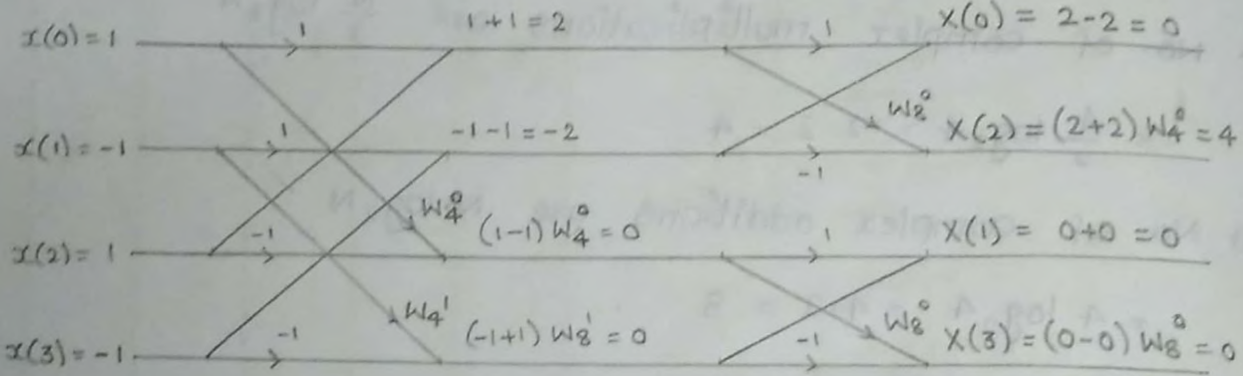
Stage 1 : $2^{1-1} = 1$

Stage 2 : $2^{2-1} = 2$

ix> Exponent Repeat Factor ERF = 2^{m-1}

Stage 1 : $2^{1-1} = 1$

Stage 2 : $2^{2-1} = 2$



$X(K) = \{0, 0, 4, 0\}$

06-03-19

Find IDFT of Sequence $X(K) = \{7, -0.707 - j\,0.707, -j, 0.707 - j\,0.707, 1, +0.707 + j\,0.707, j, -0.707 + j\,0.707\}$ by using DIF-FFT?

$X(K) = \{7, -0.707 - j\,0.707, -j, 0.707 - j\,0.707, 1, 0.707 + j\,0.707,$
$j, -0.707 + j\,0.707\}$

$X^*(K) = \{7, -0.707 + j\,0.707, j, 0.707 + j\,0.707, 1, 0.707 - j\,0.707,$
$-j, -0.707 - j\,0.707\}$

1> No. of Input Samples $N = 8 = 2^3$

2> Input ⟶ Natural order

Output ⟶ Bit Reversal order

3> No. of stages in flowgraph is

$M = \log_2 N = \log_2 8 = 3$

4> Each stage consists of $\frac{N}{2} = \frac{8}{2} = 4$ butterflies

5> No. of complex multiplications are $\frac{N}{2} \log_2 N$

$$= \frac{8}{2} \log_2 8 = 4 \cdot 3 = 12$$

6> No. of complex additions are $N \log_2 N$

$$= 8 \log_2 8 = 8 \cdot 3 = 24$$

7> Twiddle factor

$$K = \frac{Nt}{2^{M-m+1}} , \quad t = 0, 1, 2, \ldots \ldots \ldots 2^{M-m} - 1$$

Stage 1 : m = 1

$$t = 0, 1, 2, \ldots \ldots 2^{3-1} - 1$$

$$t = 0, 1, 2, 3$$

$$t=0 \Longrightarrow K = \frac{8 \cdot 0}{2^{3-1+1}} = 0 \longrightarrow W_8^0$$

$$t=1 \Longrightarrow K = \frac{8 \cdot 1}{2^{3-1+1}} = \frac{8}{2^3} = 1 \longrightarrow W_8^1$$

$$t=2 \Longrightarrow K = \frac{8 \cdot 2}{2^{3-1+1}} = \frac{8 \cdot 2}{2^3} = 2 \longrightarrow W_8^2$$

$$t=3 \Longrightarrow K = \frac{8 \cdot 3}{2^{3-1+1}} = \frac{8 \cdot 3}{2^3} \cdot 3 \longrightarrow W_8^3$$

Stage 2 : m = 2

$$t = 0, 1, 2, \ldots \ldots 2^{3-2} - 1$$

$$t = 0, 1$$

$$t=0 \Longrightarrow K = \frac{8 \cdot 0}{2^{3-2+1}} = 0 \longrightarrow W_8^0$$

$$t=1 \longrightarrow K = \frac{8 \cdot 1}{2^{3-2+1}} = \frac{8}{4} = 2 \longrightarrow W_8^2$$

Stage 3 : m = 3

$$t = 0, 1, 2, \ldots \ldots 2^{3-3} - 1$$

$t = 0$

$K = \frac{8 \cdot 0}{2^{3-3+1}} = 0 \longrightarrow W_8^0$

viii> No. of sets of butterflies in each stage is 2^{m-1}

stage 1 : $2^{1-1} = 1$

Stage 2 : $2^{2-1} = 2$

stage 3 : $2^{3-1} = 4$

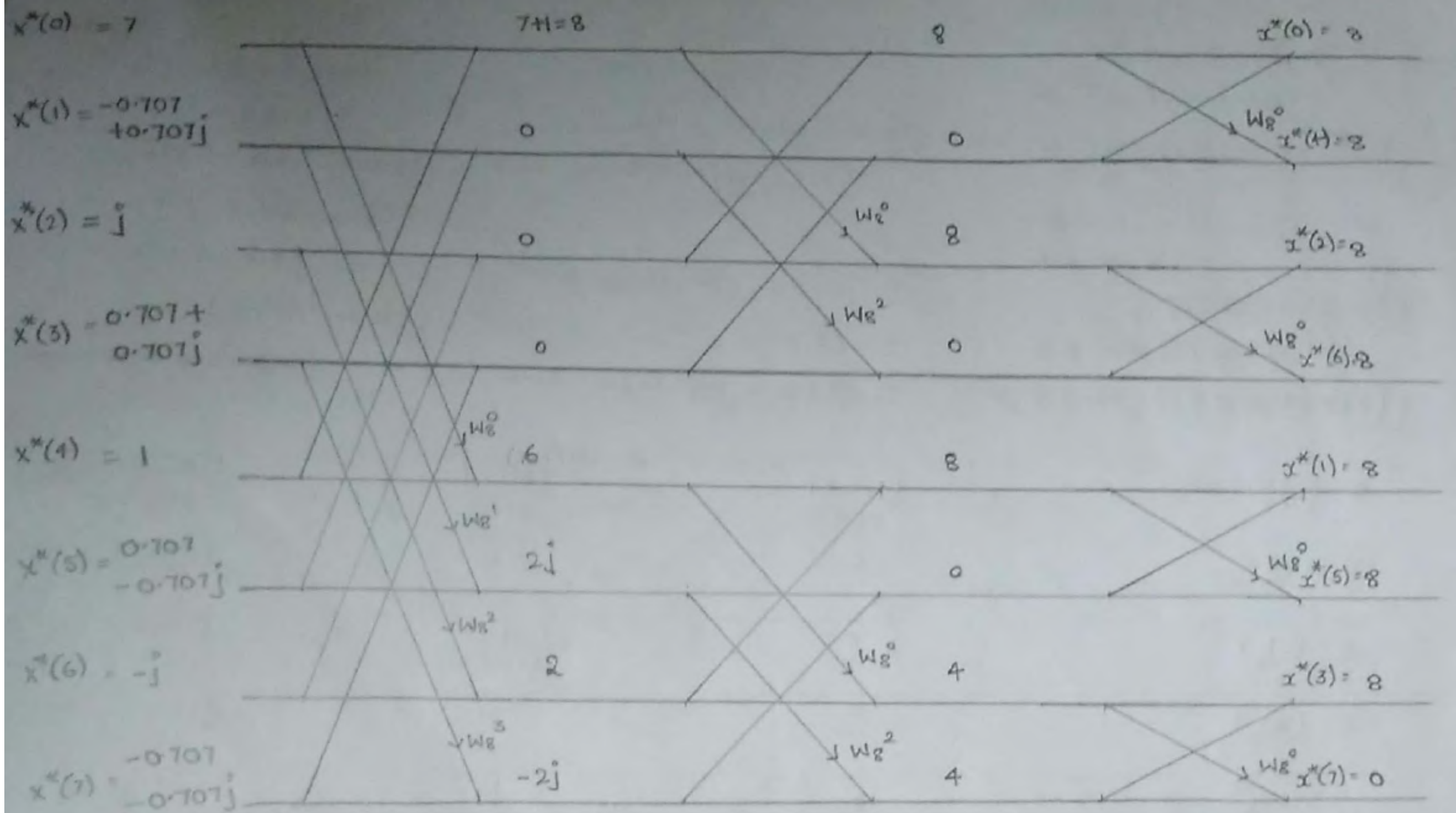
ix> Exponent Repeat factor

ERF = 2^{m-1}

stage 1 : $2^{1-1} = 1$

stage 2 : $2^{2-1} = 2$

stage 3 : $2^{3-1} = 4$



$(7-1)\ W_8^0 = 6$

$(-1.414 + 1.414j)\ W_8^1 = (-1.414 + 1.414j)$

$(0.707 - j\ 0.707)$

$= 2j$ $\quad j$

$(j + j)\ W_8^2 = 2j \cdot -j = 2$

$(1.414 + 1.414j)\ W_8^3 = (1.414 + 1.414j)$

$(-0.707 - 0.707j)$

$= -2j$

$4j \cdot -j$

$=$

$$x(n) = \frac{1}{N}\left[x^*(k)\right]$$

$$= \frac{1}{8}\left[8, 8, 8, 8, 8, 8, 8, 0\right]$$

$$= \left\{1, 1, 1, 1, 1, 1, 1, 0\right\}$$

07-03-19

SPLIT - RADIX :

The Computation process of FFT algorithm can be further reduced by combining two radix algorithms i.e., Radix-2 and Radix-4.

Radix-2 DIT-FFT, the even number of samples of N point DFT are given by

$$\boxed{x(2k) = \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) + x(n+N/2) \right] W_{\frac{N}{2}}^{nk}}$$

Radix-4 DIT-FFT algorithm, odd number of samples of N-point DFT are given as

$$X(4K+1) = \sum_{n=0}^{\frac{N}{4}-1} \left[\left(x(n) - x\left(n+\frac{N}{2}\right)\right) - j\left(x\left(n+\frac{N}{4}\right) - x\left(n+\frac{3N}{4}\right)\right)\right] \cdot W_{\frac{N}{4}}^{nk} \cdot W_N^{n}$$

$$X(4k+3) = \sum_{n=0}^{\frac{N}{4}-1} \left[\left(x(n) - x\left(n+\frac{N}{2}\right)\right) + j\left(x\left(n+\frac{N}{4}\right) - x\left(n+\frac{3N}{4}\right)\right)\right] \cdot W_{\frac{N}{4}}^{nk} \cdot W_N^{3n}$$

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk}$$

$$= \sum_{n=0}^{\frac{N}{4}-1} x(n) W_N^{nk} + \sum_{n=\frac{N}{4}}^{\frac{N}{2}-1} x(n) W_N^{nk} + \sum_{n=\frac{N}{2}}^{\frac{3N}{4}-1} x(n) W_N^{nk} +$$

$$\sum_{n=\frac{3N}{4}}^{N-1} x(n) W_N^{nk}$$

$$= \sum_{n=0}^{\frac{N}{4}-1} x(n)\, W_N^{nk} + \sum_{n=0}^{\frac{N}{4}-1} x\left(n+\frac{N}{4}\right) W_N^{\left(n+\frac{N}{4}\right)k} + \sum_{n=0}^{\frac{N}{4}-1} x\left(n+\frac{N}{2}\right) W_N^{\left(n+\frac{N}{2}\right)k} +$$

$$\sum_{n=0}^{\frac{N}{4}-1} x\left(n+\frac{3N}{4}\right) W_N^{\left(n+\frac{3N}{4}\right)k}$$

$$= \sum_{n=0}^{\frac{N}{4}-1} \left[x(n)\, W_N^{nk} + x\left(n+\frac{N}{4}\right) W_N^{nk} \cdot W_N^{\frac{NK}{4}} + x\left(n+\frac{N}{2}\right) W_N^{nk} \cdot W_N^{\frac{N}{2}k} \right.$$

$$\left. + x\left(n+\frac{3N}{4}\right) W_N^{nK} \cdot W_N^{\frac{3N}{4}K} \right]$$

$$\left\{ \begin{aligned} W_N^{\frac{NK}{4}} &= e^{-j\frac{\frac{2\pi \cancel{N}K}{\cancel{4}\,2}}{\cancel{N}}} \\ &= e^{-j\frac{\pi}{2}K} \\ &= \left(\cos\frac{\pi}{2} - j\sin\frac{\pi}{2}\right)^K \\ &= (-j)^K \end{aligned} \right.$$

$$X(K) = \sum_{n=0}^{\frac{N}{4}-1} \left[x(n) + x\left(n+\frac{N}{4}\right)(-j)^K + x\left(n+\frac{N}{2}\right)(-1)^K + x\left(n+\frac{3N}{4}\right)(j)^K \right] \cdot W_N^{nK}$$

then

$$X(4K+1) = \sum_{n=0}^{\frac{N}{4}-1} \left[x(n) + x\left(n+\frac{N}{4}\right)(-j)^{4K+1} + x\left(n+\frac{N}{2}\right)(-1)^{4K+1} + \right.$$

$$\left. x\left(n+\frac{3N}{4}\right)(j)^{4K+1} \right] W_N^{n(4K+1)}$$

$$\left\{ \begin{aligned} (-j)^{4K+1} &= (-j)^{4K} \cdot -j \\ &= 1 \cdot -j \quad \{ K=0 \\ &= -j \end{aligned} \right.$$

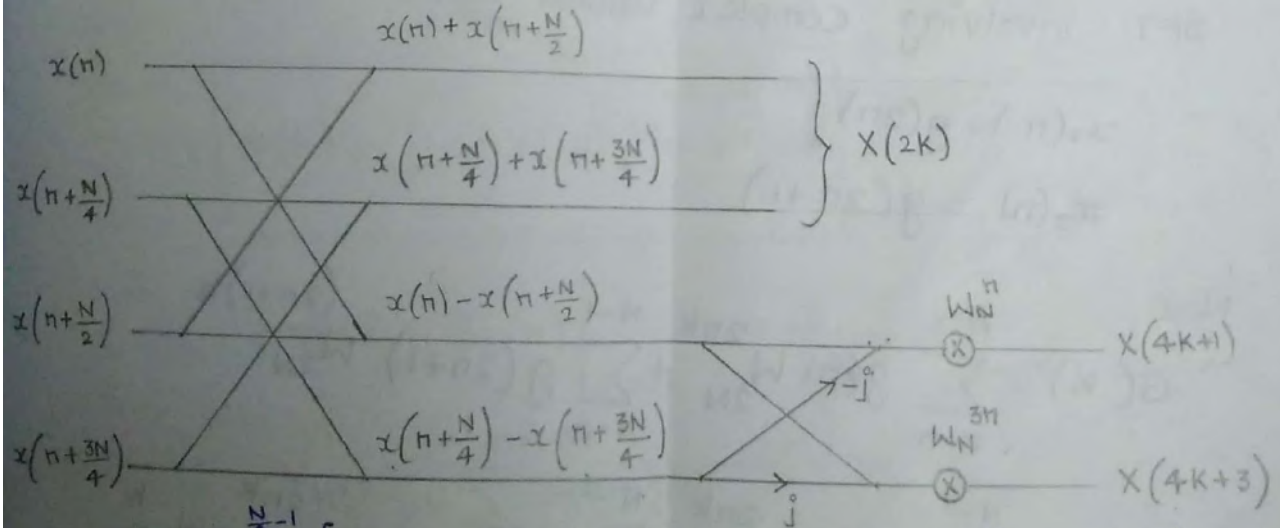
$$X(4K+1) = \sum_{n=0}^{4} \left[x(n) - j\,x\left(n+\frac{N}{4}\right) - x\left(n+\frac{N}{2}\right) + j\,x\left(n+\frac{3N}{4}\right) \right] W_N^{n} \cdot W_{\frac{N}{4}}^{nk}$$

then

$$X(4K+3) = \sum_{n=0}^{\frac{N}{4}-1} \left[x(n) + x\left(n+\frac{N}{4}\right)(-j)^{4K+3} + x\left(n+\frac{N}{2}\right)(-1)^{4K+3} + x\left(n+\frac{3N}{4}\right)(j)^{4K+3} \right] W_N^{n(4K+3)}$$

$$\left\{ \begin{aligned} (-j)^{4K+3} &= (-j)^{4K} \cdot (-j)^{3} \\ &= 1 \cdot +j \quad \{ K=0 \\ &= +j \end{aligned} \right.$$

$$X(4K+3) = \sum_{n=0}^{\frac{N}{4}-1} \left[x(n) + j\,x\left(n+\frac{N}{4}\right) - x\left(n+\frac{N}{2}\right) - j\,x\left(n+\frac{3N}{4}\right) \right] W_N^{3n} \cdot W_{\frac{N}{4}}^{nk}$$



$$X(4K+1) = \sum_{n=0}^{\frac{N}{4}-1} \left[x(n) - x\left(n+\frac{N}{2}\right) - j\,x\left(n+\frac{N}{4}\right) + j\,x\left(n+\frac{3N}{4}\right) \right] W_N^{n} \cdot W_{\frac{N}{4}}^{nk}$$

$$X(4K+3) = \sum_{n=0}^{\frac{N}{4}-1} \left[x(n) - x\left(n+\frac{N}{2}\right) + j\,x\left(n+\frac{N}{4}\right) - j\,x\left(n+\frac{3N}{4}\right) \right] W_N^{3n} \cdot W_{\frac{N}{4}}^{nk}$$

12-03-19

EFFICIENT COMPUTATION OF DFT OF TWO REAL SEQUENCES:

Let us consider a complex valued sequence

$$x(n) = x_1(n) + j x_2(n) \quad [0 \leq n \leq N-1]$$

$$x_1(n) = \frac{x(n) + x^*(n)}{2}$$

$$x_2(n) = \frac{x(n) - x^*(n)}{2j}$$

$$\text{DFT}[x_1(n)] = X_1(K) = \frac{X(K) + X^*(N-K)}{2}$$

$$\text{DFT}[x_2(n)] = X_2(K) = \frac{X(K) - X^*(N-K)}{2j}$$

By perform a single DFT in complex valued sequence $x(n)$ to obtain DFT of two real sequences i.e., $X_1(K)$ and $X_2(K)$.

EFFICIENT COMPUTATION OF DFT OF A 2N- POINT REAL SEQUENCES:

→ $g(n)$ is a 2N- point

→ obtain 2N- point DFT of $g(n)$ from one N-point DFT involving complex valued data

$$x_1(n) = g(2n)$$

$$x_2(n) = g(2n+1)$$

Now,

$$G(K) = \sum_{n=0}^{N-1} g(2n) W_{2N}^{2nk} + \sum_{n=0}^{N-1} g(2n+1) W_{2N}^{(2n+1)k}$$

$$= \sum_{n=0}^{N-1} g(2n) W_{2N}^{2nk} + \sum_{n=0}^{N-1} g(2n+1) W_{2N}^{2nk} \cdot W_{2N}^{K}$$

$$W_{2N}^{2nk} = W_{\frac{2N}{2}}^{nk} = W_N^{nk}$$

$$= \sum_{n=0}^{N-1} g(2n)\, W_N^{nk} + W_{2N}^{k} \sum_{n=0}^{N-1} g(2n+1)\, W_N^{nk}$$

$$\boxed{G(K) = X_1(K) + W_{2N}^{K} X_2(K)}$$

$$G(K+N) = X_1(K+N) + W_{2N}^{K+N} X_2(K+N)$$

By periodicity property

$$= X_1(K) + W_{2N}^{K} \cdot W_{2N}^{N} X_2(K)$$

$$\left\{ W_{2N}^{N} = e^{-j\frac{2\pi N}{2N}} = e^{-j\pi} = -1 \right.$$

$$\boxed{G(K+N) = X_1(K) - W_{2N}^{K} X_2(K)}$$

GOERTZEL METHOD :

$$X(K) = \sum_{m=0}^{N-1} x(m)\, W_N^{mk}$$

$$= \underbrace{W_N^{-Nk}}_{1} \sum_{m=0}^{N-1} x(m)\, W_N^{mk}$$

$$\boxed{X(K) = \sum_{m=0}^{N-1} x(m)\, W_N^{-K(N-m)}}$$

$$X(k) = y_K(n)\Big|_{n=N}$$

$$y_K(n) = \sum_{m=0}^{N-1} x(m)\, W_N^{-k(n-m)}$$

$\Big\{$ By convolution.

$$y(n) = \sum_{k=0}^{N-1} x(k)\, h(n-k)$$

$$h(n) = W_N^{-kn} u(n)$$

$$= \frac{1}{1 - W_N^{-kn} e^{-j\omega}}$$

$$\left\{ a^n u(n) = \frac{1}{1 - a^n e^{-j\omega}} \right.$$

system Transfer function

$$H(z) = \frac{1}{1 - W_N^{-kn} \cdot z^{-1}}$$

$$\frac{Y(z)}{X(z)} = \frac{1}{1 - W_N^{-kn} \cdot z^{-1}}$$

$$Y(z) - W_N^{-kn} z^{-1} Y(z) = X(z)$$

Time domain

$$\boxed{y(n) - W_N^{-kn} y(n-1) = x(n)}$$

CHIRP-Z TRANSFORM :

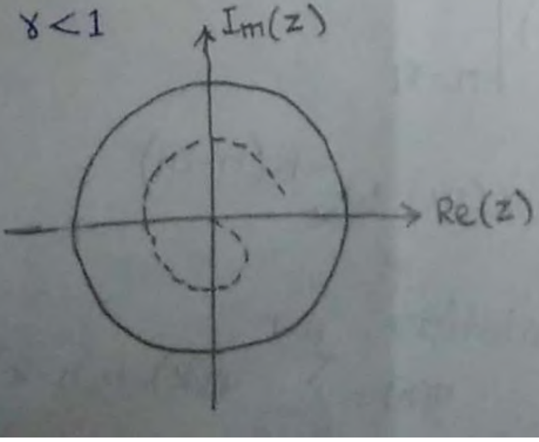
$$X(z_k) = \sum_{n=0}^{N-1} x(n) z^{-n}$$

If the contour is a circle of radius 'r'.

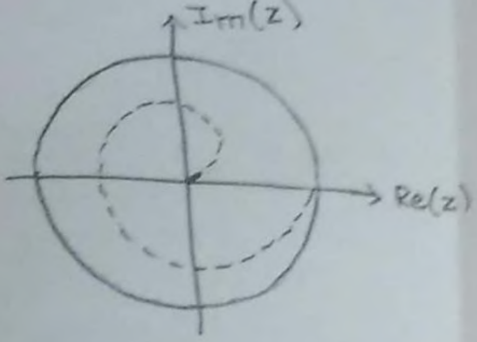
$$Z_K = r \cdot e^{j\frac{2\pi K}{N}}$$

$$\boxed{X(z_K) = \sum_{n=0}^{N-1} x(n) r^{-n} \cdot e^{-j\frac{2\pi nk}{N}}}$$

i> if $r<1$



$r<1$, the points fall on a contour that spirals towards the origin ii > if r > 1



r > 1, the points fall on a contour that spirals away from the origin

APPLICATIONS OF FFT ALGORITHM:

- Correlation
- Spectral Analysis
- Linear Filtering

★ FIR is non-Recursive system

★ IIR is Recursive System

13-03-19

UNIT - III

IMPLEMENTATION OF DISCRETE TIME SYSTEMS
OR
STRUCTURES FOR THE REALIZATION OF DISCRETE-TIME SYSTEMS

FIR is Non-Recursive

↓

present output depends on present input and past inputs

$$y(n) = x(n) + x(n-1)$$

IIR is Recursive

↓

Present output depends on present input, past input and past outputs

$$y(n) = x(n) + x(n-1) + y(n-1)$$

FIR	IIR
Direct form	Direct form - I
Cascade form	Direct form - II
Lattice Realization	Signal flow graph
	Cascade form
	Parallel form
	Lattice - Ladder

INTRODUCTION:

Realizations of discrete time systems are classified into two types

1) FIR Realization

2) IIR Realization

FIR is a Non-Recursive Realization in which present output depends on present input and past input

IIR is a Recursive Realization in which present output depends on present input, Past inputs and Past outputs

IIR can be realized in many forms

1> Direct form - I realization

2> Direct form - II realization

3> Transpose direct form realization

4> Cascade form realization

5> parallel form realization

6> Lattice ladder realization

FIR can be realized in many forms

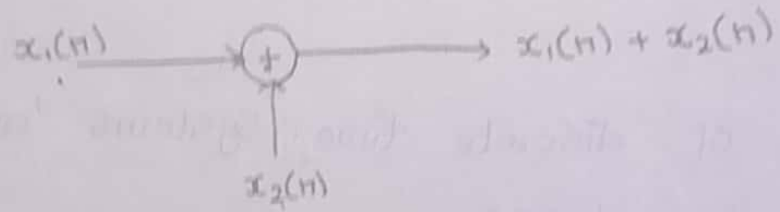
1> Direct form realization

2> Cascade form realization

3> Lattice realization

BASIC ELEMENTS :

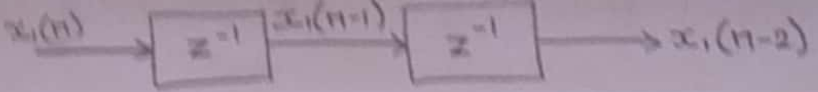
1>



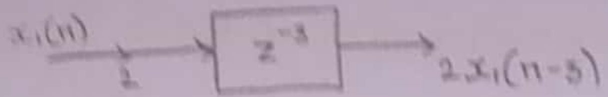
2>

3>

i)



ii)



IIR :

i) DIRECT FORM - I REALIZATION :

Let us consider an LTI recursive system described by difference equation

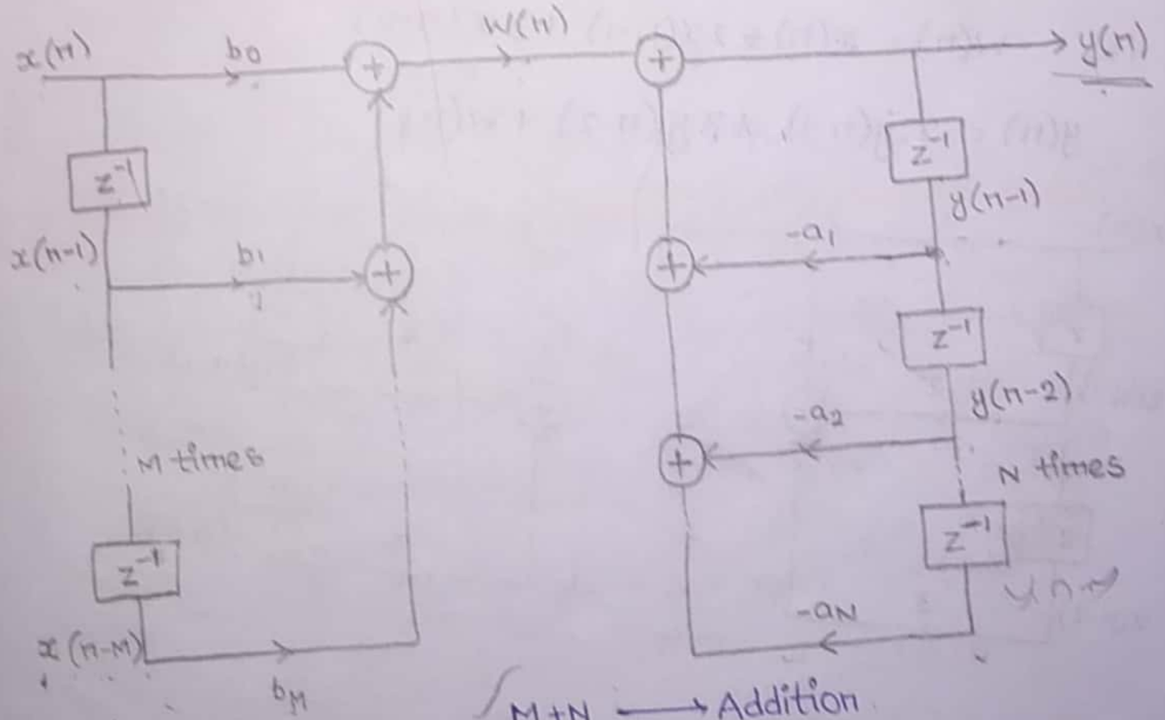
$$y(n) = -\sum_{K=1}^{N} a_K\, y(n-k) + \sum_{K=0}^{M} b_K\, x(n-k)$$

$$= -a_1 y(n-1) - a_2\, y(n-2) - \cdots - a_N\, y(n-N) +$$

$$b_0\, x(n) + b_1\, x(n-1) + b_2\, x(n-2) + \cdots + b_M x(n-M)$$

Let $w(n) = b_0 x(n) + b_1\, x(n-1) + \cdots + \underline{b_M}\, x(n-M)$

$y(n) = -a_1 y(n-1) - a_2\, y(n-2) - \cdots - a_N y(n-N) + \underline{w(n)}$



M+N ⟶ Addition

M+N+1 ⟶ Multiplication

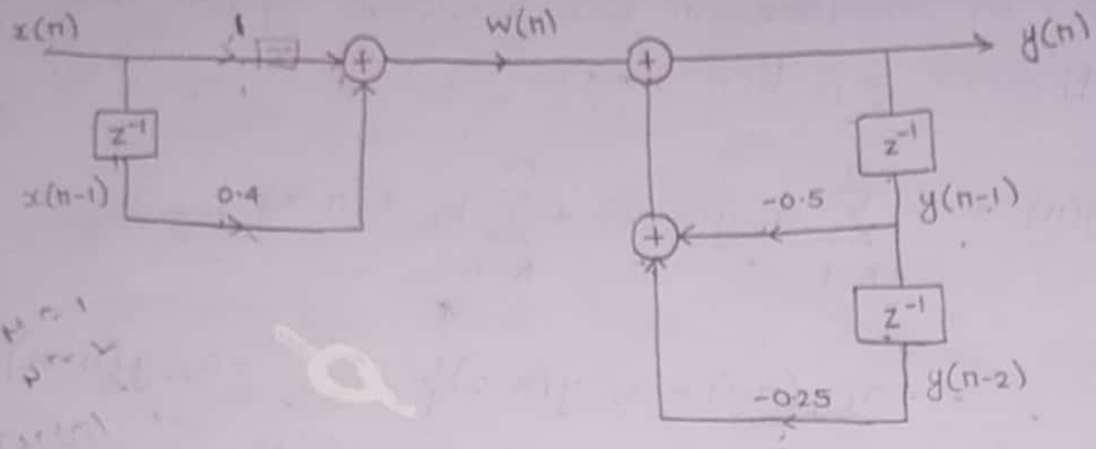
M+N+1 ⟶ Memory locations or delay elements

1> Determine Direct form - I realization for the system described by difference equation

$y(n) = -0.5\, y(n-1) - 0.25\, y(n-2) + x(n) + 0.4\, x(n-1)$

let $w(n) = x(n) + 0.4\, x(n-1)$

$y(n) = -0.5\, y(n-1) - 0.25\, y(n-2) + w(n)$

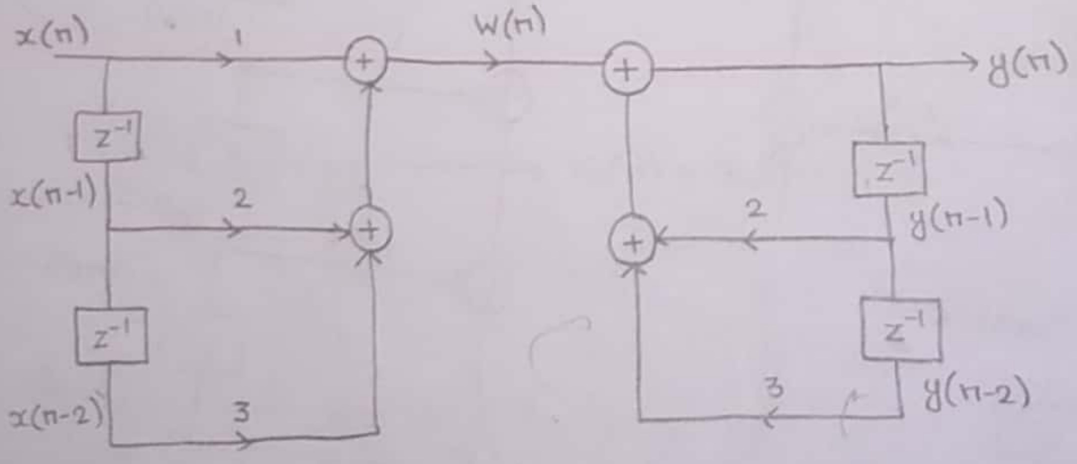


2> Determine Direct form - I realization for the system described by difference equation

$y(n) = 2\, y(n-1) + 3\, y(n-2) + x(n) + 2\, x(n-1) + 3\, x(n-2)$.

let $w(n) = x(n) + 2\, x(n-1) + 3\, x(n-2)$

$y(n) = 2\, y(n-1) + 3\, y(n-2) + w(n)$

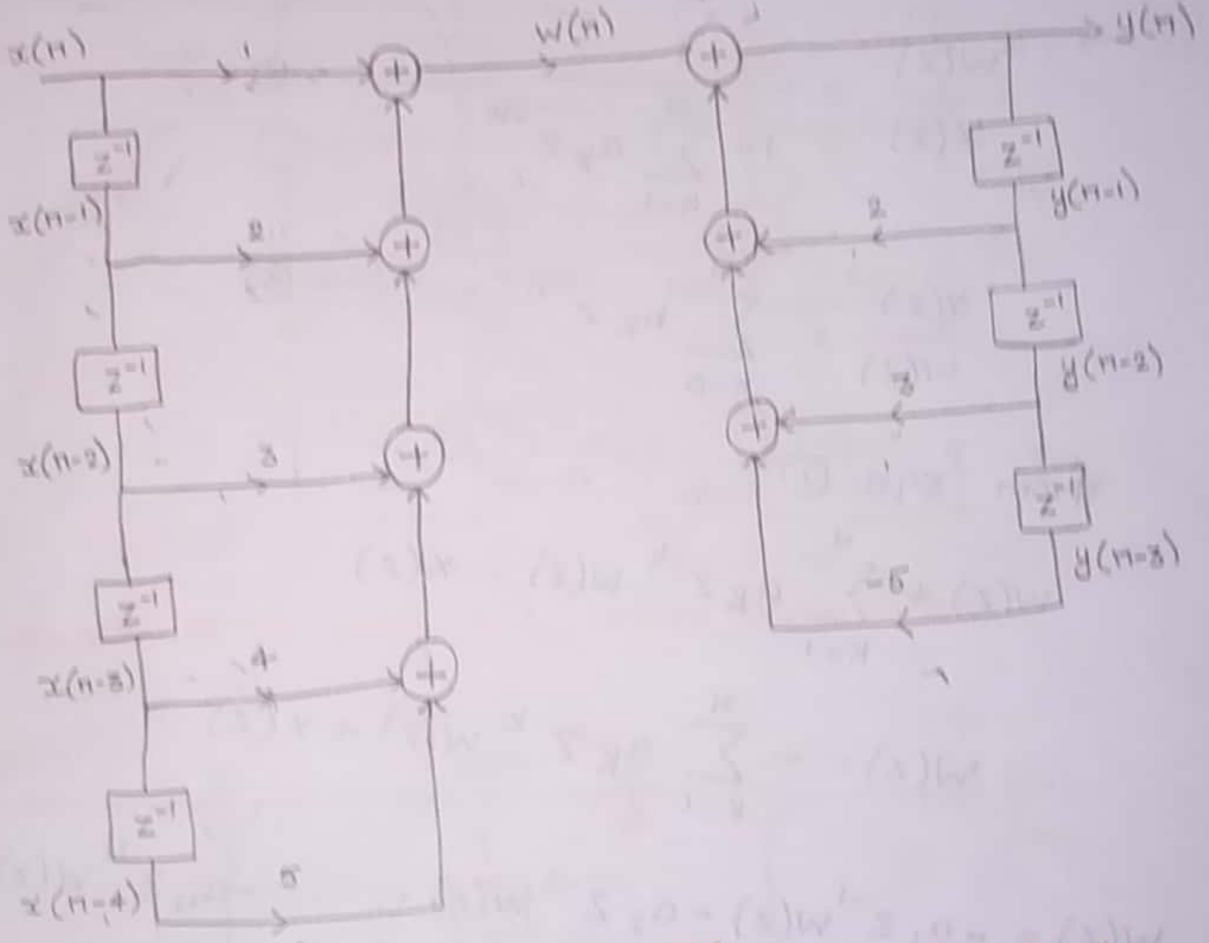


3) Determine Direct form-I realization for the system described by difference equation

$$y(n) = 2\,y(n-1) + 3\,y(n-2) - 5\,y(n-3) + 2\,x(n-1) + 3\,x(n-2) + x(n) + 4\,x(n-3) + 5\,x(n-4)$$

Let $W(n) = x(n) + 2\,x(n-1) + 3\,x(n-2) + 4\,x(n-3) + 5\,x(n-4)$

$$y(n) = 2\,y(n-1) + 3\,y(n-2) - 5\,y(n-3) + W(n)$$



DIRECT FORM - II REALIZATION : (CANONICAL FORM)

$$y(n) = -\sum_{K=1}^{N} a_K\, y(n-K) + \sum_{K=0}^{M} b_K\, x(n-K) \longrightarrow ①$$

Apply z-transform on both sides in eqn①

$$Y(z) = -\sum_{K=1}^{N} a_K\, z^{-k}\, Y(z) + \sum_{K=0}^{M} b_K\, z^{-k}\, X(z)$$

$$Y(z)\left[1 + \sum_{K=1}^{N} a_K\, z^{-k}\right] = \sum_{K=0}^{M} b_K\, z^{-k}\, X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{\sum_{K=0}^{M} b_K z^{-K}}{1+\sum_{K=1}^{N} a_K z^{-K}}$$

$$\frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)} = \frac{\sum_{K=0}^{M} b_K z^{-k}}{1+\sum_{K=1}^{N} a_K z^{-k}}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1+\sum_{K=1}^{N} a_K z^{-k}} \longrightarrow ② \checkmark$$

$$\frac{Y(z)}{W(z)} = \sum_{K=0}^{M} b_K z^{-k} \longrightarrow ③ \checkmark$$

from eqn ②

$$W(z) + \sum_{K=1}^{N} a_K z^{-k} W(z) = X(z)$$

$$W(z) = -\sum_{K=1}^{N} a_K z^{-k} W(z) + X(z)$$

$$W(z) = -a_1 z^{-1} W(z) - a_2 z^{-2} W(z) \cdots\cdots - a_N z^{-N} W(z) + X(z)$$

Apply Inverse z-tranform

$$W(n) = -a_1 W(n-1) - a_2 W(n-2) - \cdots\cdots - a_N W(n-N) + x(n) \longrightarrow ④$$

from eqn ③

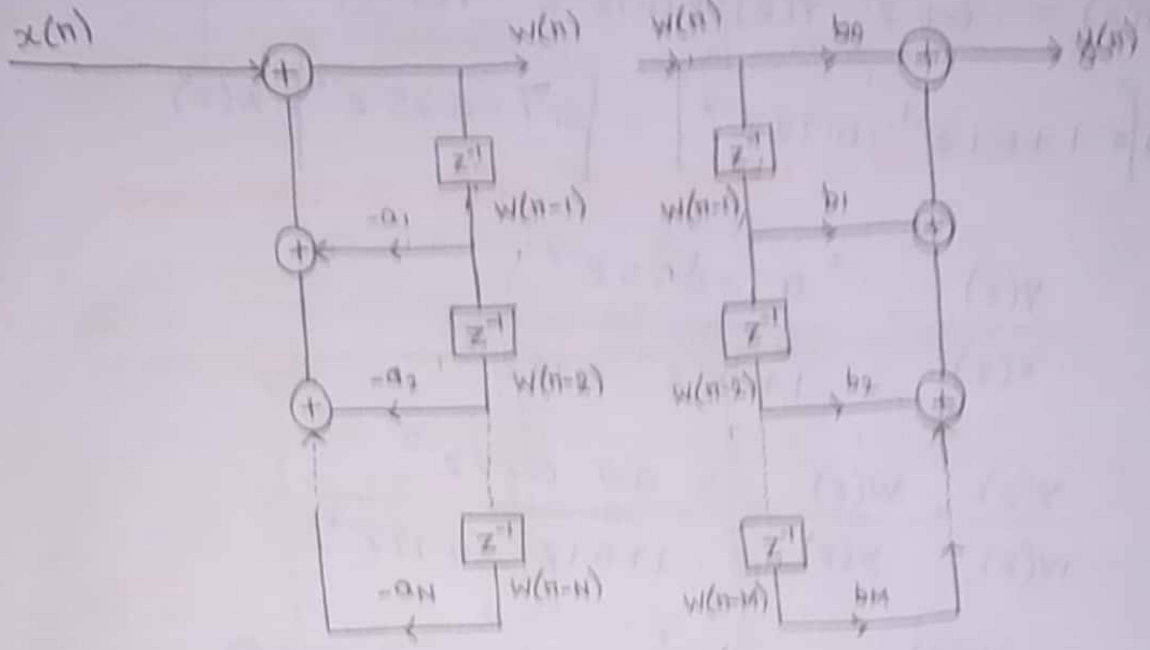
$$Y(z) = \sum_{K=0}^{M} b_K z^{-k} W(z)$$

$$Y(z) = b_0 W(z) + b_1 z^{-1} W(z) + \cdots\cdots\cdots + b_M z^{-M} W(z)$$

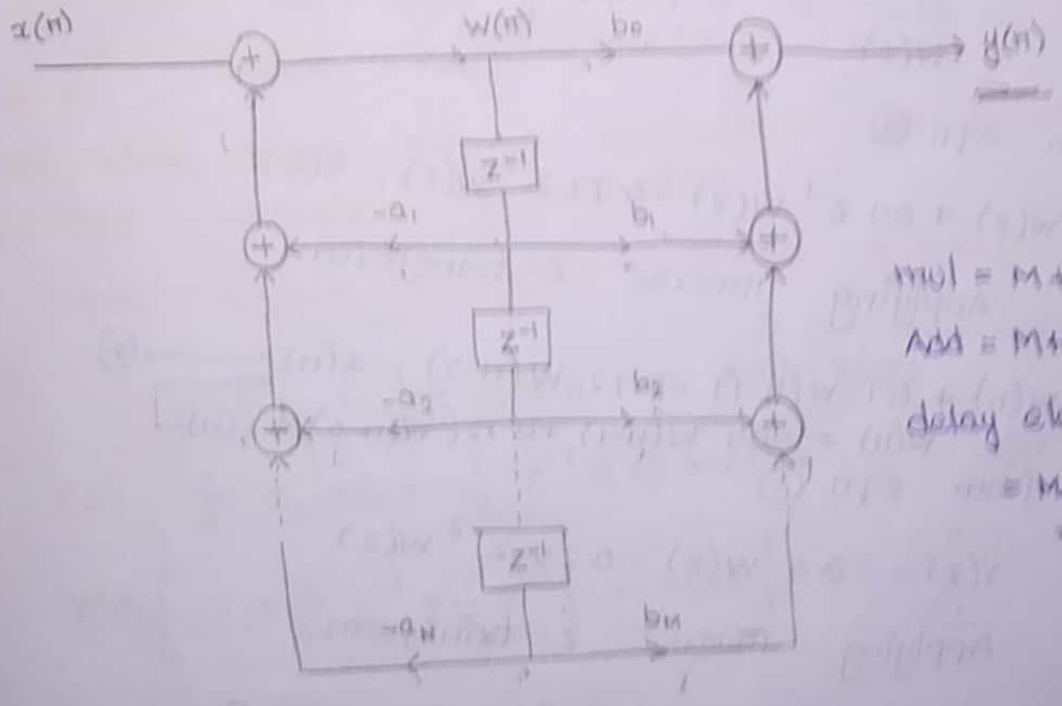
Applying Inverse Z-transform

$$y(n) = b_0\, w(n) + b_1\, w(n-1) + b_2\, w(n-2) + \cdots + b_M\, w(n-M) \longrightarrow ⑤$$

REALIZATION STRUCTURE FOR EQ ④ & ⑤



COMBINATION OF BOTH REALIZATION STRUCTURES :



mul = M+N+1

Add = M+N

delay element = Max(M,N)

1) Determine Direct form-II realization for the System described by difference equation

$$y(n) = -0.1\, y(n-1) + 0.72\, y(n-2) + 0.7\, x(n) - 0.25\, x(n-2) \longrightarrow ①$$

Apply Z-transform on both sides

$$Y(z) = -0.1\, z^{-1}\, Y(z) + 0.72\, z^{-2}\, Y(z) + 0.7\, X(z) - 0.25\, z^{-2}\, X(z$$

$$Y(z)\left[1 + 0.1\, z^{-1} - 0.72\, z^{-2}\right] = \left[0.7 - 0.25\, z^{-2}\right] X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{0.7 - 0.25\, z^{-2}}{1 + 0.1 z^{-1} - 0.72 z^{-2}}$$

$$\frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)} = \frac{0.7 - 0.25\, z^{-2}}{1 + 0.1 z^{-1} - 0.72 z^{-2}}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 + 0.1\, z^{-1} - 0.72\, z^{-2}} \longrightarrow ②$$

$$\frac{Y(z)}{W(z)} = 0.7 - 0.25\, z^{-2} \longrightarrow ③$$

from eqn ②

$$W(z) + 0.1\, z^{-1}\, W(z) - 0.72\, z^{-2}\, W(z) = X(z)$$

Applying inverse Z-transform

$$w(n) + 0.1\, w(n-1) - 0.72\, w(n-2) = x(n) \longrightarrow ④$$

$$w(n) = -0.1\, w(n-1) + 0.72\, w(n-2) + x(n)$$

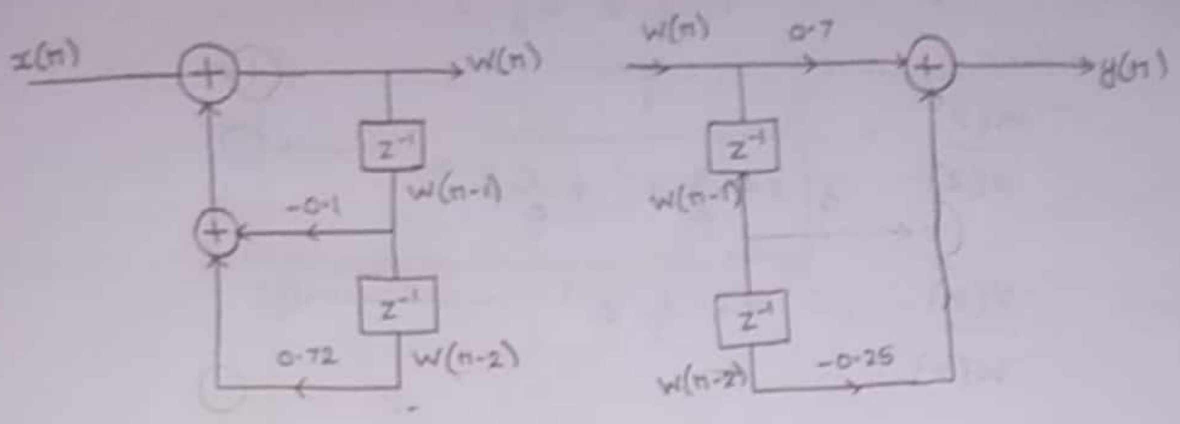
from eqn ③

$$Y(z) = 0.7\, W(z) - 0.25\, z^{-2}\, W(z)$$

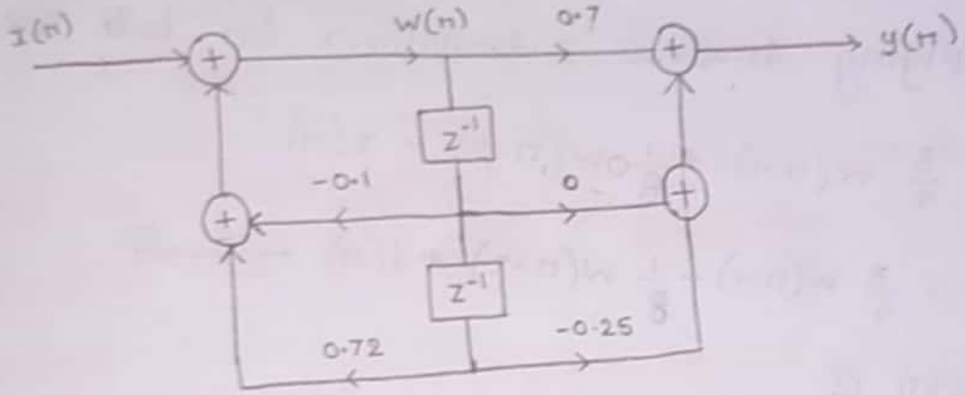
Applying inverse Z-transform

$$y(n) = 0.7\, w(n) - 0.25\, w(n-2) \longrightarrow ⑤$$

REALIZATION STRUCTURE FOR EQ(4) & EQ(5)



COMBINATION OF BOTH REALIZATION STRUCTURES



mul = 2 + 2+1= 5

add = 2+2 = 4

delay element = Max (2,2) = 2

14-03-19

2) Determine Direct-form-II realization for the system described by difference equation

$$y(n) = \frac{3}{4} y(n-1) - \frac{1}{8} y(n-2) + x(n) + \frac{1}{2} x(n-1) \longrightarrow ①$$

Apply z-transform on both sides

$$Y(z) = \frac{3}{4} z^{-1} Y(z) - \frac{1}{8} z^{-2} Y(z) + X(z) + \frac{1}{2} z^{-1} X(z)$$

$$Y(z)\left[1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}\right] = X(z)\left[1 + \frac{1}{2} z^{-1}\right]$$

$$\frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{2} z^{-1}}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

$$\frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)} = \frac{1+\frac{1}{2}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} \longrightarrow ②$$

$$\frac{Y(z)}{W(z)} = 1+\frac{1}{2}z^{-1} \longrightarrow ③$$

from eqn ②

$$W(z) - \frac{3}{4}z^{-1}W(z) + \frac{1}{8}z^{-2}W(z) = X(z)$$

Applying inverse z-transform on both sides

$$w(n) - \frac{3}{4}w(n-1) + \frac{1}{8}w(n-2) = x(n)$$

$$w(n) = \frac{3}{4}w(n-1) - \frac{1}{8}w(n-2) + x(n) \longrightarrow ④$$

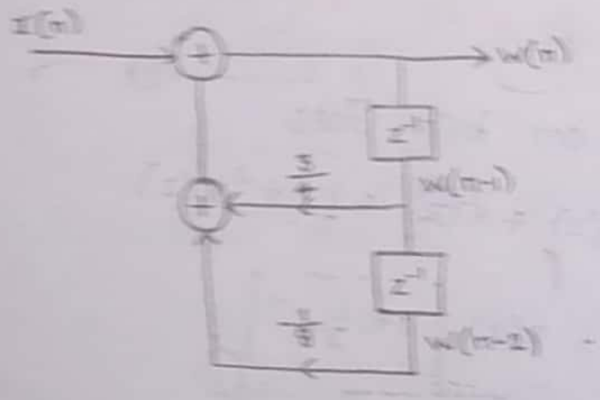
from eqn ③

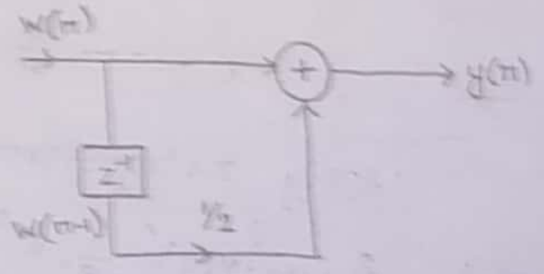
$$Y(z) = W(z) + \frac{1}{2}z^{-1}W(z)$$

Applying inverse z-transform on both sides

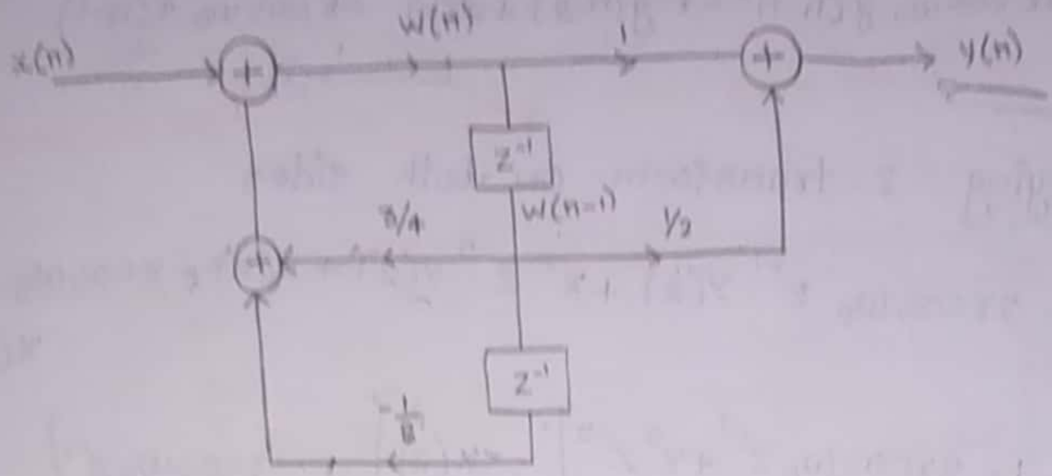
$$y(n) = w(n) + \frac{1}{2}w(n-1) \longrightarrow ⑤$$

REALIZATION STRUCTURES FOR EQ(4) & EQ(5)





COMBINATION OF BOTH REALIZATION STRUCTURES



mul = 2+1+1 = 4

add = 2+1 = 3

delay element = Max(2,1) = 2

16-03-19

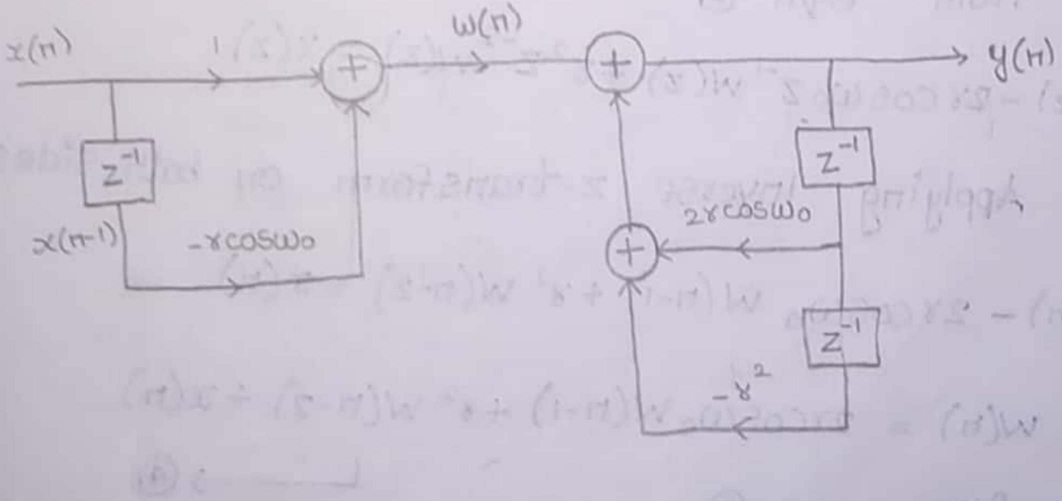
3) Determine Direct form-I and Direct form-II realization for the system described by difference equation.

$$y(n) = 2\gamma \cos\omega_0 y(n-1) - \gamma^2 y(n-2) + x(n) - \gamma \cos\omega_0 x(n-1)$$

DIRECT FORM -I

let $W(n) = x(n) - \gamma\cos\omega_0 x(n-1)$

$$y(n) = 2\gamma\cos\omega_0 y(n-1) - \gamma^2 y(n-2) + W(n)$$



DIRECT FORM - II :

$$y(n) = 2r\cos\omega_0\, y(n-1) - r^2 y(n-2) + x(n) - r\cos\omega_0\, x(n-1) \longrightarrow ①$$

Applying z-transform on both sides

$$Y(z) = 2r\cos\omega_0\, z^{-1} Y(z) - r^2 z^{-2} Y(z) + X(z) - r\cos\omega_0\, z^{-1} X(z)$$

$$Y(z)\left[1 - 2r\cos\omega_0\, z^{-1} + r^2 z^{-2}\right] = X(z)\left[1 - r\cos\omega_0\, z^{-1}\right]$$

$$\frac{Y(z)}{X(z)} = \frac{1 - r\cos\omega_0\, z^{-1}}{1 - 2r\cos\omega_0\, z^{-1} + r^2 z^{-2}}$$

$$\frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)} = \frac{1 - r\cos\omega_0\, z^{-1}}{1 - 2r\cos\omega_0\, z^{-1} + r^2 z^{-2}}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 - 2r\cos\omega_0\, z^{-1} + r^2 z^{-2}} \longrightarrow ②$$

$$\frac{Y(z)}{W(z)} = 1 - r\cos\omega_0\, z^{-1} \longrightarrow ③$$

from eqn ②

$$W(z) - 2r\cos\omega_0\, z^{-1} W(z) + r^2 z^{-2} W(z) = X(z)$$

Applying inverse z-transform on both sides

$$W(n) - 2r\cos\omega_0\, W(n-1) + r^2 W(n-2) = x(n)$$

$$w(n) = 2r\cos\omega_0\, W(n-1) - r^2 W(n-2) + x(n) \longrightarrow ④$$

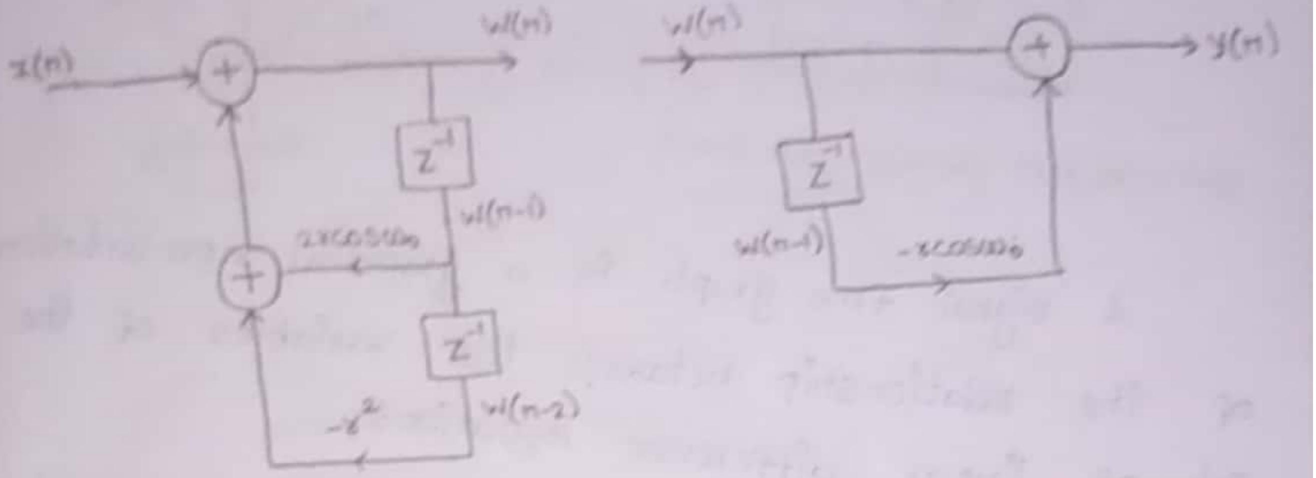
from eqn ③

$$Y(z) = W(z) - r\cos\omega_0\, z^{-1} W(z)$$

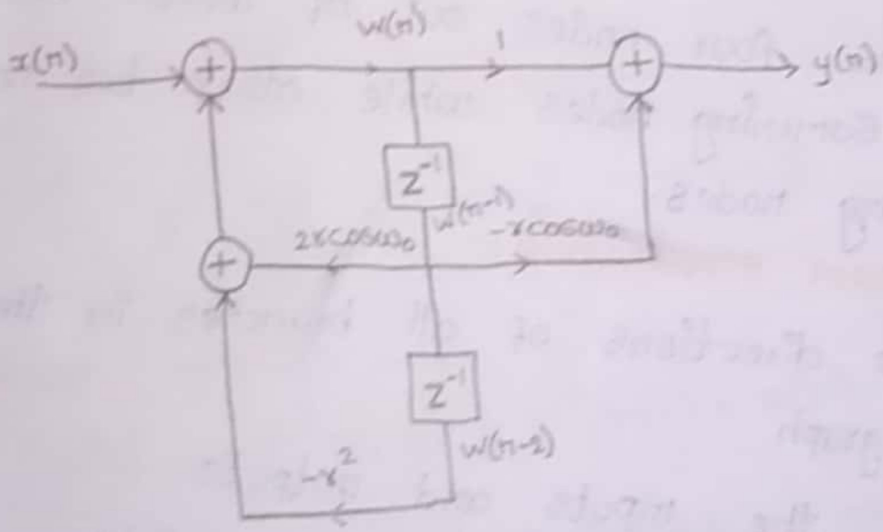
Applying inverse z-transform on both sides

$$y(n) = w(n) - r\cos\omega_0\, w(n-1) \longrightarrow ⑤$$

REALIZATION STRUCTURES FOR EQN ④ & ⑤



COMBINATION OF BOTH REALIZATION STRUCTURES :

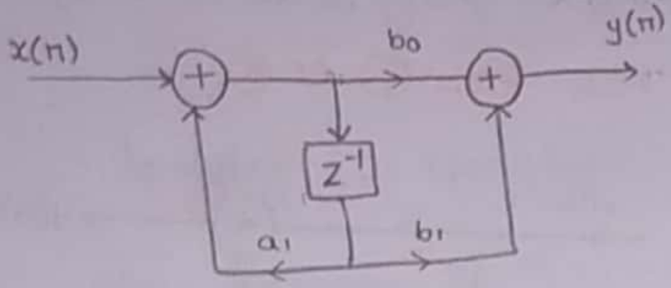


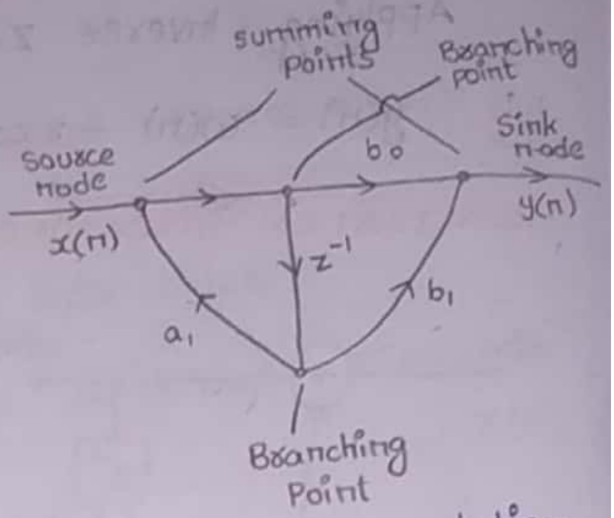
mul = 2+1+1 = 4

add = 2+1 = 3

Delay element = Max(2,1) = 2

SIGNAL FLOW GRAPH :





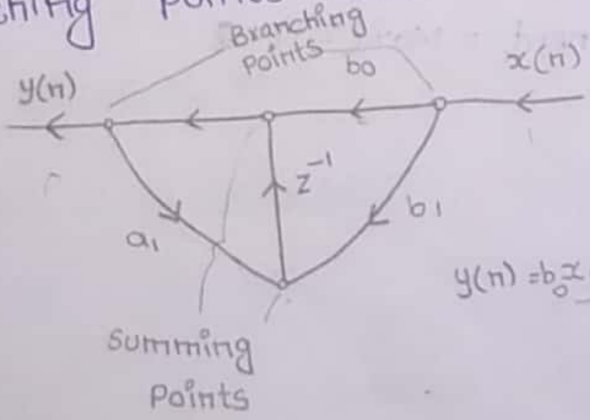
$y(n) = b_0 x(n) + b_1 x(n-1) + a_1 y(n-1)$

A signal flow graph is a graphical representation of the relationship between the variables of the set of linear difference equations

The system block diagram can be converted to the signal flow graph to find that the flow-graph contains four nodes out of which two nodes are summing nodes while other two nodes are branching nodes.

TRANSPOSED FORM REALIZATION :

1> Reverse the directions of all branches in the signal flowgraph
2> Interchange the inputs and outputs
3> Summing points become branching points
4> Branching points become summing points.



$y(n) = \underline{b_0 x(n)} + b_1 x(n-1) + a_1 y(n-1)$

18-08-19

1) Determine the direct form-II and Transposed direct form-II for the given systen

$$y(n) = \frac{1}{2} y(n-1) - \frac{1}{4} y(n-2) + x(n) + x(n-1) \longrightarrow ①$$

Applying z-transform on both sides

$$Y(z) = \frac{1}{2} z^{-1} Y(z) - \frac{1}{4} z^{-2} Y(z) + X(z) + z^{-1} X(z)$$

$$Y(z)\left[1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}\right] = X(z)\left[1 + z^{-1}\right]$$

$$\frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}$$

$$\frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)} = \frac{1+z^{-1}}{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}} \longrightarrow ②$$

$$\frac{Y(z)}{W(z)} = 1+z^{-1} \longrightarrow ③$$

from eqn ②

$$W(z) - \frac{1}{2} z^{-1} W(z) + \frac{1}{4} z^{-2} W(z) = X(z)$$

Applying Inverse z-transform on both sides

$$w(n) - \frac{1}{2} w(n-1) + \frac{1}{4} w(n-2) = x(n)$$

$$w(n) = \frac{1}{2} w(n-1) - \frac{1}{4} w(n-2) + x(n) \longrightarrow ④$$

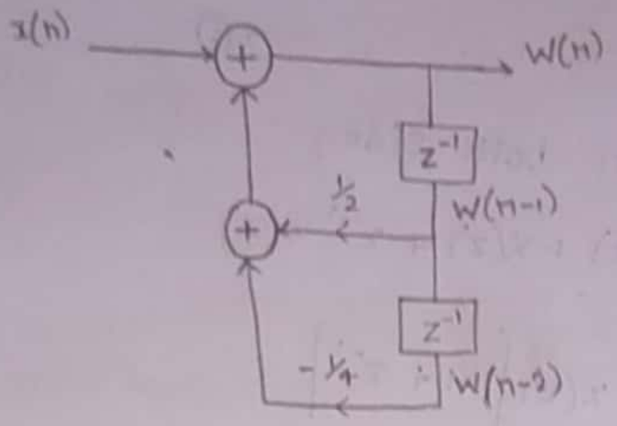
from eqn ③

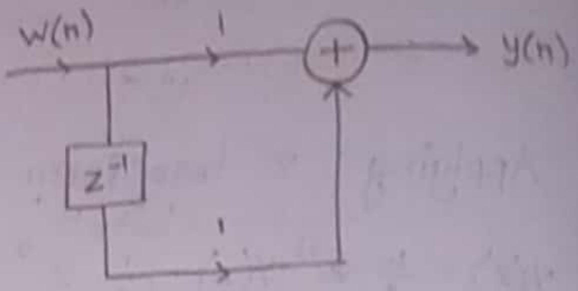
$$Y(z) = W(z) + z^{-1} W(z)$$

Applying inverse z-transform on both sides

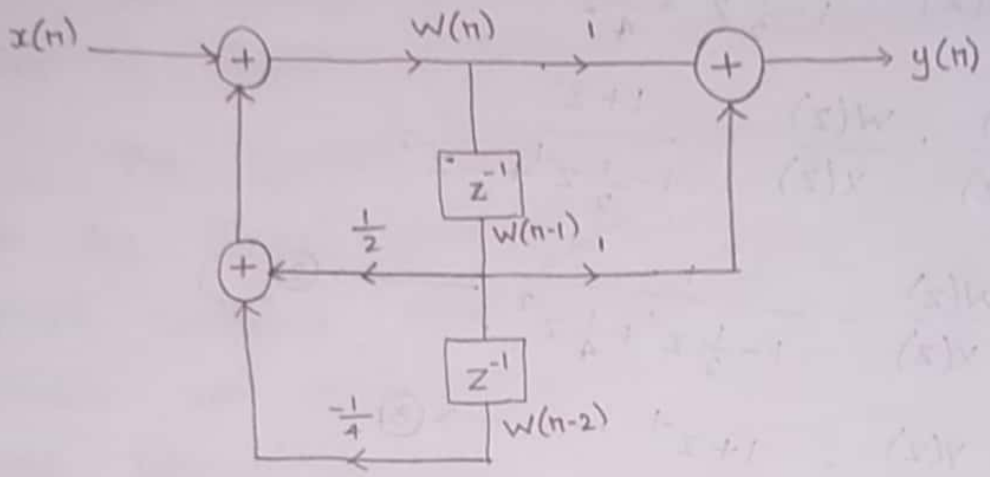
$$y(n) = \omega(n) + \omega(n-1) \longrightarrow ⑤$$

REALIZATION STRUCTURES FOR EQ④ AND ⑤

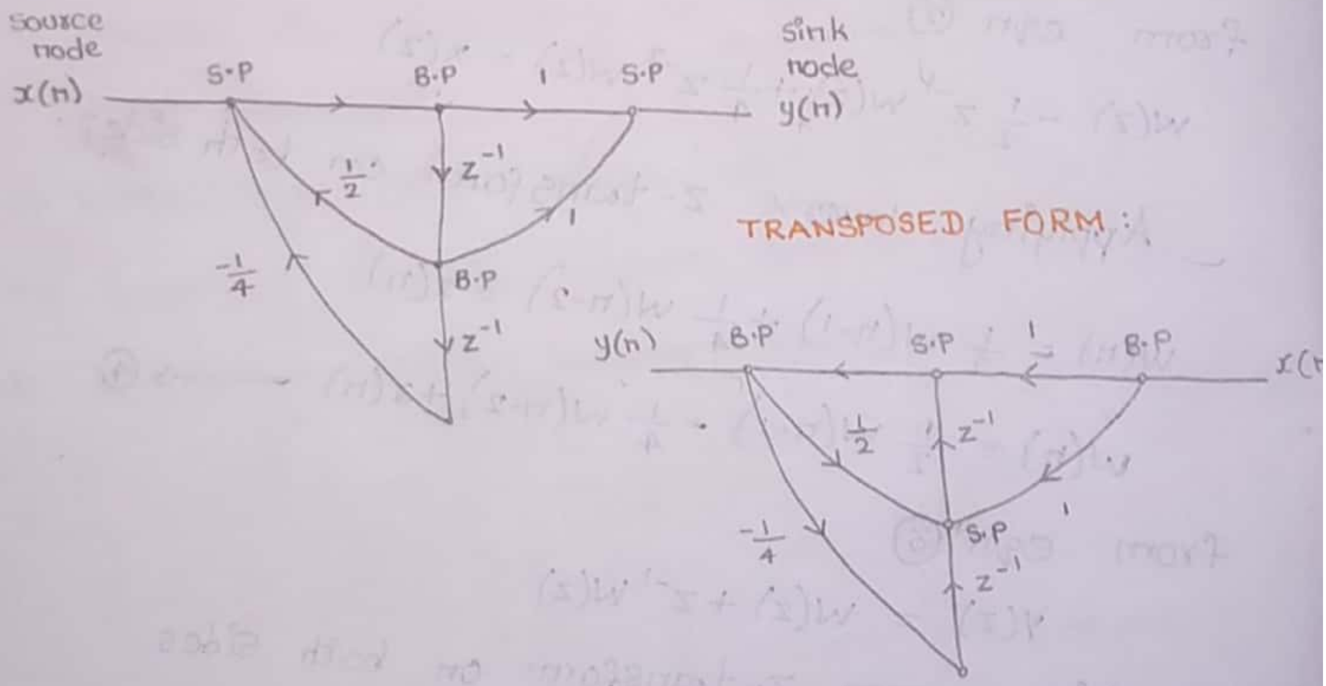




COMBINATION OF BOTH REALIZATION STRUCTURES

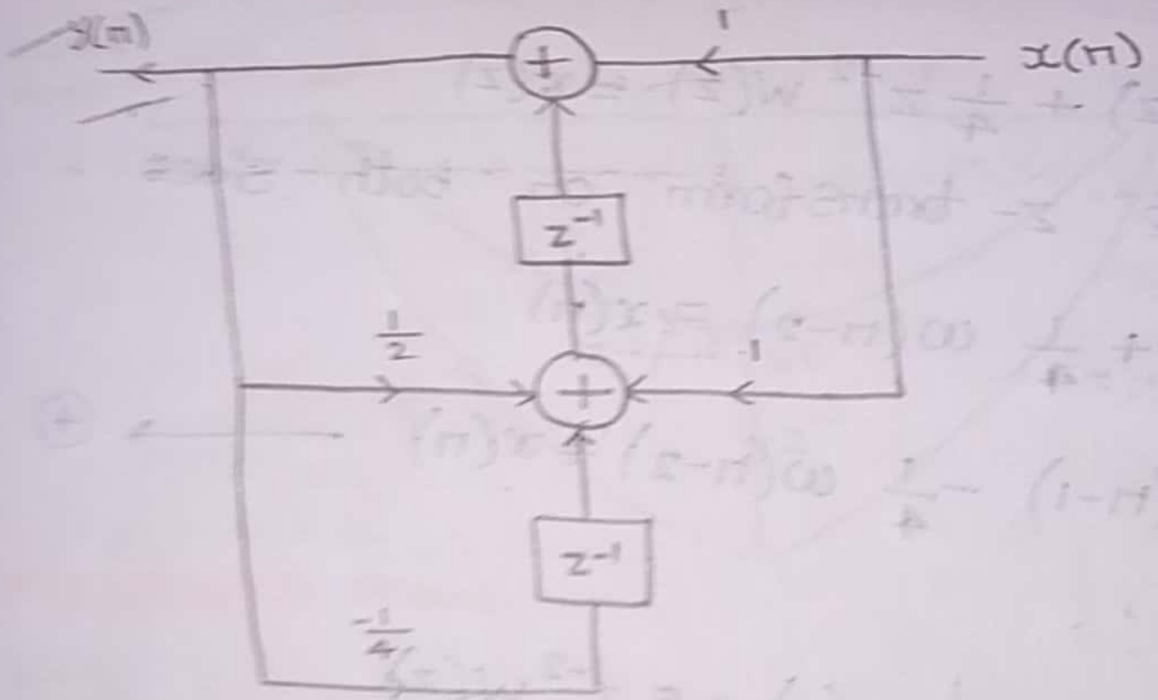


SIGNAL FLOW GRAPH



TRANSPOSED FORM :

REALIZATION STRUCTURE :



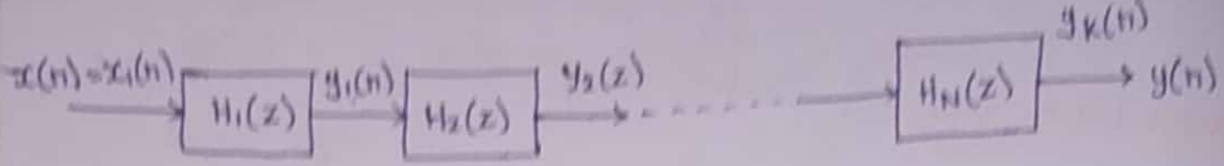
$$y(n) = \frac{1}{2}\, y(n-1) - \frac{1}{4}\, y(n-2) + x(n) + x(n-1)$$

19-03-19

CASCADE FORM REALIZATION:

Let us consider Linear Time Invariant LTI system with system function

$$H(z) = H_1(z) \cdot H_2(z) \cdot \ldots\ldots H_N(z)$$



1) Realize the system with difference equation

$$y(n) = \frac{3}{4} y(n-1) - \frac{1}{8} y(n-2) + x(n) + \frac{1}{3} x(n-1)$$

$\longrightarrow$ ①

in cascade form Realization

Apply z-transform on both sides

$$Y(z) = \frac{3}{4} z^{-1} Y(z) - \frac{1}{8} z^{-2} Y(z) + X(z) + \frac{1}{3} z^{-1} X(z)$$

$$Y(z)\left[1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}\right] = X(z)\left[1 + \frac{1}{3} z^{-1}\right]$$

$$\frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{3} z^{-1}}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

$$H(z) = \frac{1 + \frac{1}{3} z^{-1}}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

$$= \frac{1 + \frac{1}{3} z^{-1}}{1 - \frac{2}{4} z^{-1} - \frac{1}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

$$= \frac{1 + \frac{1}{3} z^{-1}}{1 - \frac{1}{2} z^{-1} - \frac{1}{4} z^{-1}\left(1 - \frac{1}{2} z^{-1}\right)}$$

$$H(z) = \frac{1 + \frac{1}{3} z^{-1}}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{2} z^{-1}\right)}$$

$$H(z) = H_1(z) \cdot H_2(z)$$

$$H(z) = \frac{1 + \frac{1}{3} Z^{-1}}{1 - \frac{1}{2} Z^{-1}} \cdot \frac{1}{1 - \frac{1}{4} Z^{-1}}$$

$$H_1(z) = \frac{Y_1(z)}{X_1(z)} = \frac{1 + \frac{1}{3} z^{-1}}{1 - \frac{1}{2} z^{-1}}$$

$$\frac{Y_1(z)}{W_1(z)} \cdot \frac{W_1(z)}{X_1(z)} = \frac{1 + \frac{1}{3} z^{-1}}{1 - \frac{1}{2} z^{-1}}$$

$$\frac{W_1(z)}{X_1(z)} = \frac{1}{1 - \frac{1}{2} z^{-1}} \longrightarrow ②$$

$$\frac{Y_1(z)}{W_1(z)} = 1 + \frac{1}{3} z^{-1} \longrightarrow ③$$

from eqn ②

$$W_1(z) - \frac{1}{2} z^{-1} W_1(z) = X_1(z)$$

Applying Inverse z-transform on b.s

$$W_1(n) - \frac{1}{2} W_1(n-1) = X_1(n)$$

$$W_1(n) = \frac{1}{2} W_1(n-1) + x_1(n) \longrightarrow ④$$

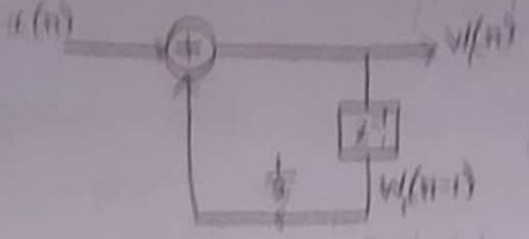
from eqn ③

$$Y_1(z) = W_1(z) + \frac{1}{3} z^{-1} W_1(z)$$

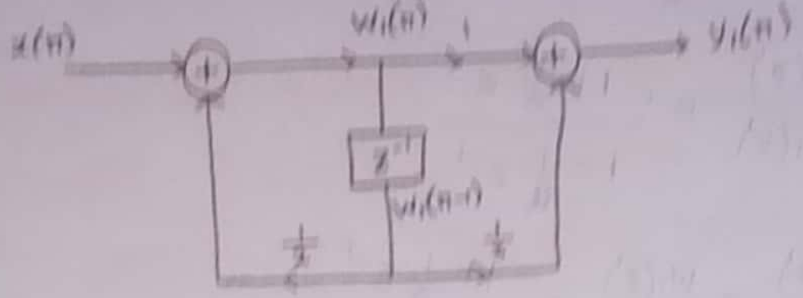
Applying Inverse Z-transform on b.s

$$y_1(n) = W_1(n) + \frac{1}{3} W_1(n-1) \longrightarrow ⑤$$

REALIZATION STRUCTURES OF EQN ④ & ⑤



COMBINATION OF BOTH REALIZATION STRUCTURES :



$$H_2(z) = \frac{1}{1-\frac{1}{4}z^{-1}}$$

$$\frac{Y(z)}{Y_1(z)} = \frac{1}{1-\frac{1}{4}z^{-1}}$$

$$\frac{Y(z)}{W_2(z)} \cdot \frac{W_2(z)}{Y_1(z)} = \frac{1}{1-\frac{1}{4}z^{-1}}$$

$$\frac{W_2(z)}{Y_1(z)} = \frac{1}{1-\frac{1}{4}z^{-1}} \longrightarrow ⑥$$

$$\frac{Y(z)}{W_2(z)} = 1 \longrightarrow ⑦$$

from eqn ⑥

$$W_2(z) - \frac{1}{4}z^{-1}W_2(z) = Y_1(z)$$

Applying inverse Z-transform on both sides

$$w_2(n) - \frac{1}{4}w_2(n-1) = y_1(n)$$

$$w_2(n) = \frac{1}{4}w_2(n-1) + y_1(n) \longrightarrow ⑧$$

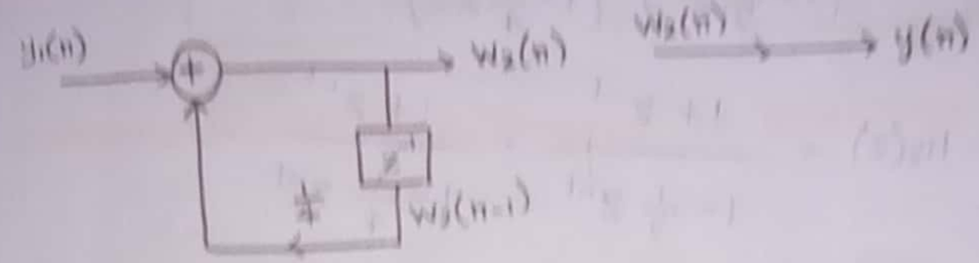
from eqn ⑦

$$Y(z) = W_2(z)$$

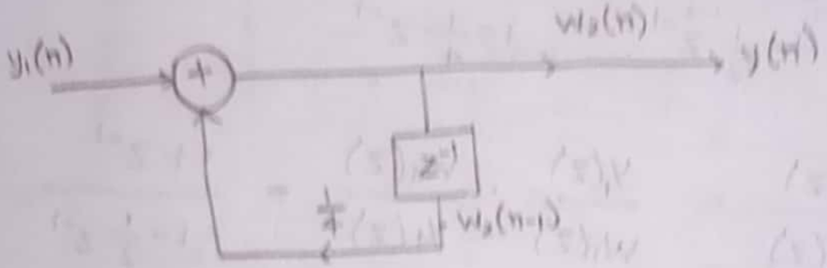
Applying inverse z-transform on both sides

$$y(n) = w_2(n) \longrightarrow ⑨$$

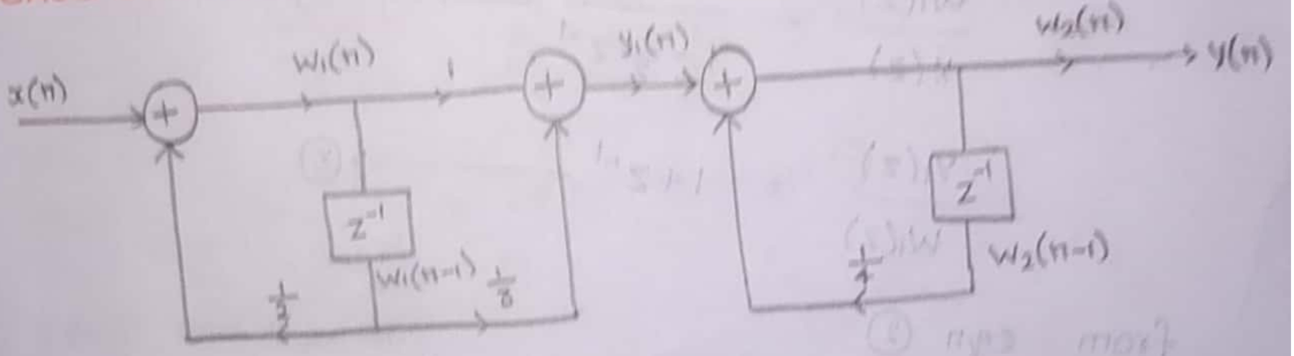
REALIZATION STRUCTURES OF EQN ⑧ AND ⑨



COMBINATION OF BOTH REALIZATION STRUCTURES :



CASCADED FORM REALIZATION :



2) Obtain cascade form of the system performing equation

$$H(z) = \frac{1+2z^{-1}+z^{-2}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

$$H(z) = \frac{1+z^{-1}+z^{-1}+z^{-2}}{1-\frac{2}{4}z^{-1}-\frac{1}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

$$= \frac{1+z^{-1}+z^{-1}(1+z^{-1})}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

$$H(z) = \frac{(1+z^{-1})\,(1+z^{-1})}{\left(1-\frac{1}{2}z^{-1}\right)\,\left(1-\frac{1}{4}z^{-1}\right)}$$

$$H_1(z)\cdot H_2(z) = \frac{1+z^{-1}}{1-\frac{1}{2}z^{-1}} \cdot \frac{1+z^{-1}}{1-\frac{1}{4}z^{-1}}$$

$$\frac{Y_1(z)}{X(z)} \cdot \frac{Y(z)}{Y_1(z)} = \frac{1+z^{-1}}{1-\frac{1}{2}z^{-1}} \cdot \frac{1+z^{-1}}{1-\frac{1}{4}z^{-1}}$$

$$H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{Y_1(z)}{W_1(z)} \cdot \frac{W_1(z)}{X(z)} = \frac{1+z^{-1}}{1-\frac{1}{2}z^{-1}}$$

$$\frac{W_1(z)}{X(z)} = \frac{1}{1-\frac{1}{2}z^{-1}} \longrightarrow ②$$

$$\frac{Y_1(z)}{W_1(z)} = 1+z^{-1} \longrightarrow ③$$

from eqn ②

$$W_1(z) - \frac{1}{2}z^{-1}W_1(z) = X(z)$$

Applying inverse z-transform on both sides

$$w_1(n) - \frac{1}{2}w_1(n-1) = x(n)$$

$$w_1(n) = \frac{1}{2}w_1(n-1) + x(n) \longrightarrow ④$$

from eqn ③

$$Y_1(z) = W_1(z) + z^{-1}W_1(z)$$

Applying inverse z-transform on both sides

$$y_1(n) = w_1(n) + w_1(n-1) \longrightarrow ⑤$$

REALIZATION STRUCTURES OF EQN ④ & ⑤

COMBINATION OF BOTH REALIZATION STRUCTURES :

$$H_2(z) = \frac{Y(z)}{Y_1(z)} = \frac{Y(z)}{W_2(z)} \cdot \frac{W_2(z)}{Y_1(z)} = \frac{1+z^{-1}}{1-\frac{1}{4}z^{-1}}$$

$$\frac{W_2(z)}{Y_1(z)} = \frac{1}{1-\frac{1}{4}z^{-1}} \longrightarrow ⑥$$

$$\frac{Y(z)}{W_2(z)} = 1+z^{-1} \longrightarrow ⑦$$

from eqn ⑥

$$W_2(z) - \frac{1}{4}z^{-1}W_2(z) = Y_1(z)$$

Applying inverse z-transform on both sides

$$w_2(n) - \frac{1}{4}w_2(n-1) = y_1(n)$$

$$w_2(n) = \frac{1}{4}w_2(n-1) + y_1(n) \longrightarrow ⑧$$

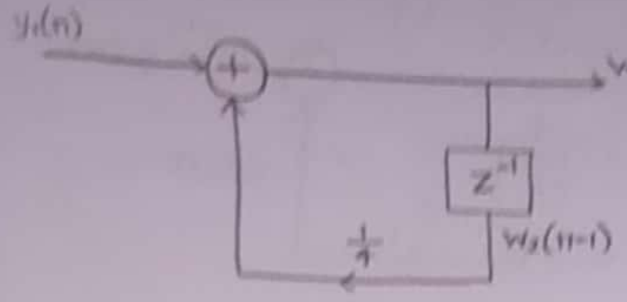
from eqn ⑦

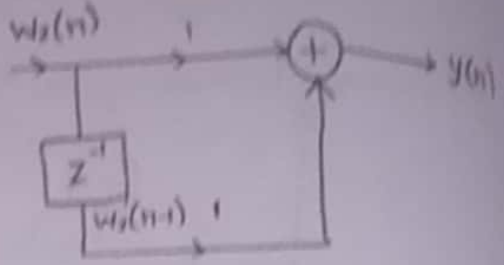
$$Y(z) = W_2(z) + z^{-1}W_2(z)$$

Applying inverse z-transform on both sides

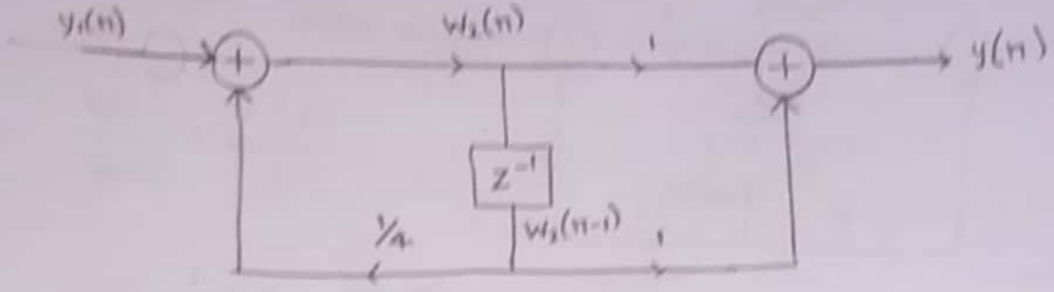
$$y(n) = W_2(n) + W_2(n-1) \longrightarrow ⑨$$

REALIZATION STRUCTURES OF EQN ⑧ AND ⑨

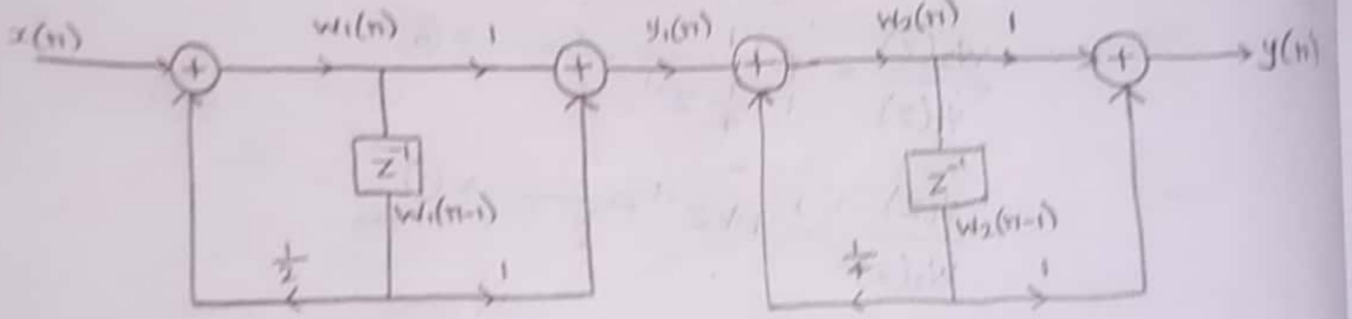




COMBINATION OF BOTH REALIZATION STRUCTURES:



CASCADED FORM REALIZATION STRUCTURE:



PARALLEL FORM REALIZATION:

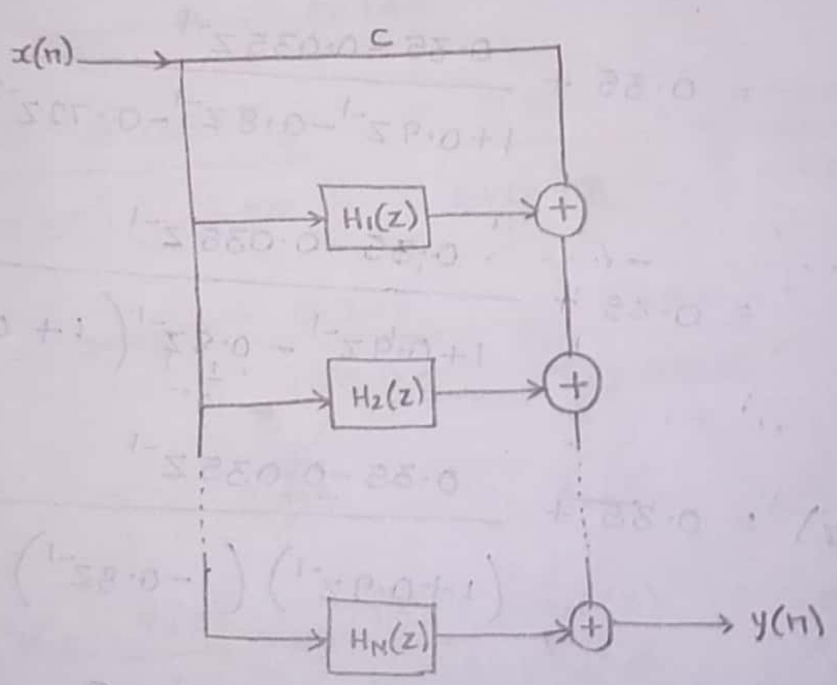
A parallel form Realization structure of an IIR system can be obtained by performing a partial expansion.

$$H(z) = C + \sum_{K=1}^{N} \frac{C_K}{1-P_K z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)} = C + \frac{C_1}{1-P_1 z^{-1}} + \frac{C_2}{1-P_2 z^{-1}} + \cdots + \frac{C_N}{1-P_N z^{-1}}$$

$$Y(z) = C X(z) + \frac{C_1}{1-P_1 z^{-1}} X(z) + \frac{C_2}{1-P_2 z^{-1}} X(z) + \cdots + \frac{C_N}{1-P_N z^{-1}} X(z)$$

$$Y(z) = C X(z) + H_1(z) X(z) + H_2(z) X(z) + \cdots + H_N(z) X(z)$$



parallel form Realization

1) Realize a system with difference equation

$$y(n) = -0.1\ y(n-1) + 0.72\ y(n-2) + 0.7\ x(n) - 0.25\ x(n-2) \longrightarrow ①$$

Applying z-transform on both sides

$$Y(z) = -0.1\ z^{-1} Y(z) + 0.72\ z^{-2} Y(z) + 0.7\ X(z) - 0.25\ z^{-2} X(z)$$

$$Y(z)\left[1+0.1z^{-1}-0.72z^{-2}\right]=X(z)\left[0.7-0.25z^{-2}\right]$$

$$\frac{Y(z)}{X(z)}=\frac{0.7-0.25z^{-2}}{1+0.1z^{-1}-0.72z^{-2}}$$

20-03-19

$$\begin{array}{r|l} & 0.35 \\ \hline -0.72z^{-2}+0.1z^{-1}+1 & \cancel{-0.25z^{-2}}+0.7 \\ & \cancel{-0.25z^{-2}}+0.035z^{-1}+0.35 \\ & (+)\qquad (-)\qquad (-) \\ \hline & -0.035z^{-1}+0.35 \end{array}$$

$$H(z)=0.35+\frac{0.35-0.035z^{-1}}{1+0.1z^{-1}-0.72z^{-2}}$$

$$=0.35+\frac{0.35-0.035z^{-1}}{1+0.9z^{-1}-0.8z^{-1}-0.72z^{-2}}$$

$$=0.35+\frac{0.35-0.035z^{-1}}{1+0.9z^{-1}-0.8z^{-1}(1+0.9z^{-1})}$$

$$H(z)=0.35+\frac{0.35-0.035z^{-1}}{(1+0.9z^{-1})(1-0.8z^{-1})}$$

$$\frac{0.35-0.035z^{-1}}{(1+0.9z^{-1})(1-0.8z^{-1})}=\frac{A}{1+0.9z^{-1}}+\frac{B}{1-0.8z^{-1}}$$

$$0.35-0.035z^{-1}=A(1-0.8z^{-1})+B(1+0.9z^{-1})$$

$$z=-0.9\Longrightarrow z^{-1}=\frac{-1}{0.9}$$

$$0.35 - 0.035 \cdot \frac{-1}{0.9} = A\left(1 - 0.8\left(\frac{-1}{0.9}\right)\right) + B\left(1 + 0.9\left(\frac{-1}{0.9}\right)\right)$$

$$0.35 + 0.0388 = A\left(1 + 0.889\right) + B\left(1 - 1\right)$$

$$0.3888 = A(1.889)$$

$$A = 0.2058$$

$$A = 0.206$$

$$z = 0.8 \implies z^{-1} = \frac{1}{0.8}$$

$$0.35 - 0.035\left(\frac{1}{0.8}\right) = A\left(1 - 0.8\left(\frac{1}{0.8}\right)\right) + B\left(1 + 0.9\left(\frac{1}{0.8}\right)\right)$$

$$0.35 - 0.04375 = A(1 - 1) + B(1 + 1.125)$$

$$0.30625 = B(2.125)$$

$$B = 0.144$$

$$\therefore \frac{0.35 - 0.035z^{-1}}{(1 + 0.9z^{-1})(1 - 0.8z^{-1})} = \frac{0.206}{1 + 0.9z^{-1}} + \frac{0.144}{1 - 0.8z^{-1}}$$

$$H(z) = 0.35 + \underbrace{\frac{0.206}{1 + 0.9z^{-1}}}_{H_1(z)} + \underbrace{\frac{0.144}{1 - 0.8z^{-1}}}_{H_2(z)}$$

$$H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{Y_1(z)}{W_1(z)} \cdot \frac{W_1(z)}{X(z)} = \frac{0.206}{1 + 0.9z^{-1}}$$

$$\frac{W_1(z)}{X(z)} = \frac{1}{1 + 0.9z^{-1}} \longrightarrow ②$$

$$\frac{Y_1(z)}{W_1(z)} = 0.206 \longrightarrow ③$$

from eqn ② $W_1(z) + 0.9\, z^{-1} W_1(z) = X(z)$

Applying Inverse z-transform

$$\omega_1(n) + 0.9\,\omega_1(n-1) = x(n)$$

$$\omega_1(n) = -0.9\,\omega_1(n-1) + x(n) \longrightarrow ④$$

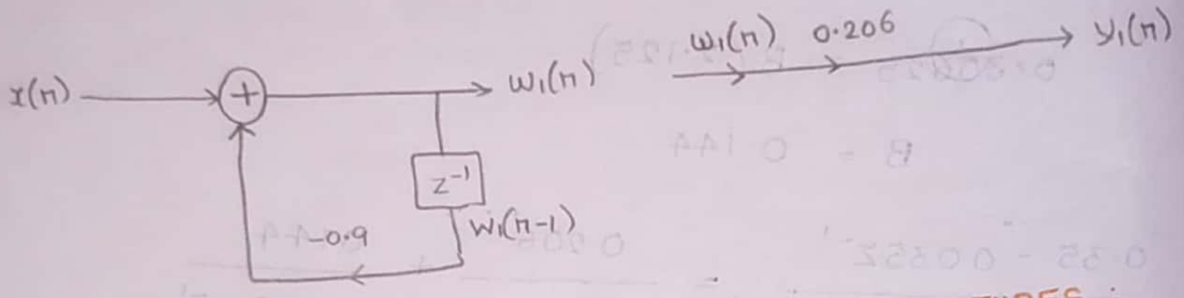
from eqn ③

$$y_1(z) = 0.206\,W_1(z)$$

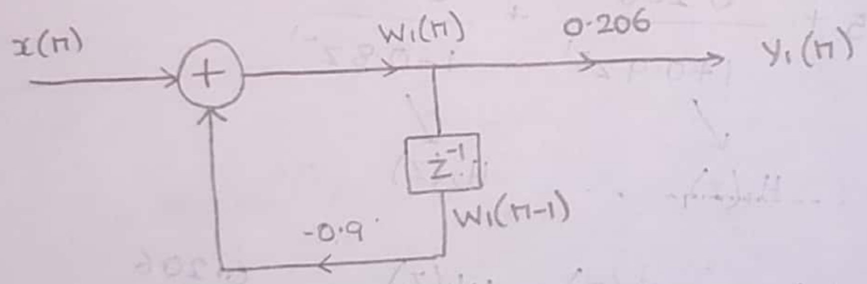
Applying Inverse z-transform

$$y_1(n) = 0.206\,\omega_1(n) \longrightarrow ⑤$$

REALIZATION STRUCTURES OF EQN ④ & ⑤



COMBINATION OF BOTH REALIZATION STRUCTURES :



$$H_2(z) = \frac{Y_2(Z)}{X(Z)} = \frac{Y_2(z)}{W_2(Z)} \cdot \frac{W_2(z)}{X(Z)} = \frac{0.144}{1-0.8z^{-1}}$$

$$\frac{W_2(Z)}{X(Z)} = \frac{1}{1-0.8z^{-1}} \longrightarrow ⑥$$

$$\frac{Y_2(Z)}{W_2(Z)} = 0.144 \longrightarrow ⑦$$

from eqn ⑥

$W_2(z) - 0.8 z^{-1} W_2(z) = x(z)$

Applying inverse z-transform

$w_2(n) - 0.8 \, w_2(n-1) = x(n)$

$w_2(n) = 0.8 \, w_2(n-1) + x(n) \longrightarrow$ ⑧

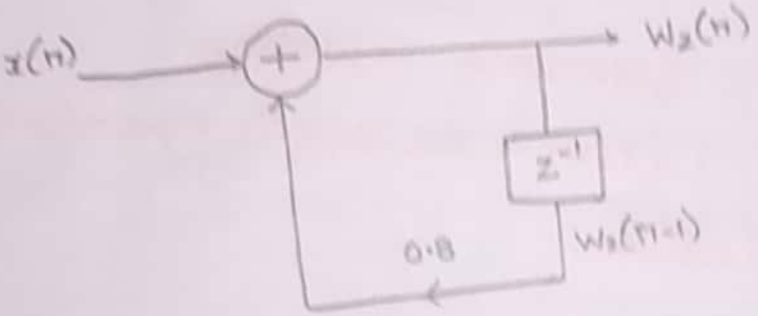
from eqn ⑦

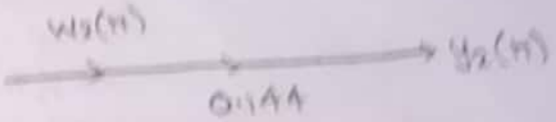
$Y_2(z) = 0.144 \, W_2(z)$

Applying inverse z-transform

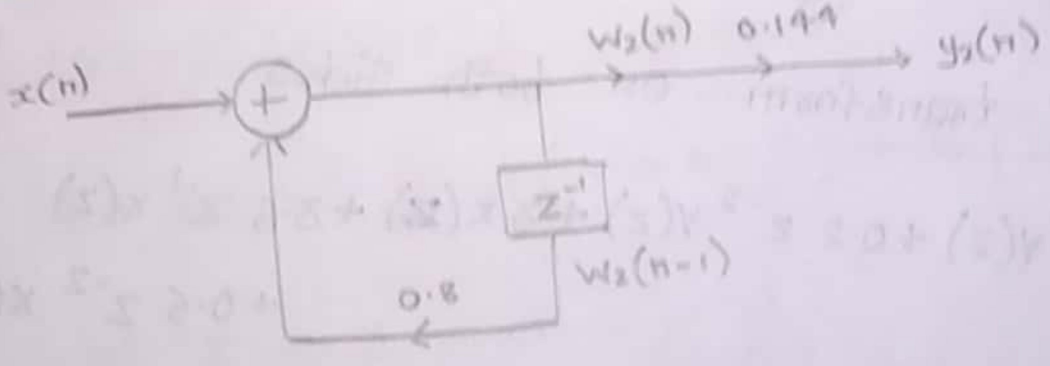
$y_2(n) = 0.144 \, w_2(n) \longrightarrow$ ⑨

REALIZATION STRUCTURES OF EQN ⑧ & ⑨

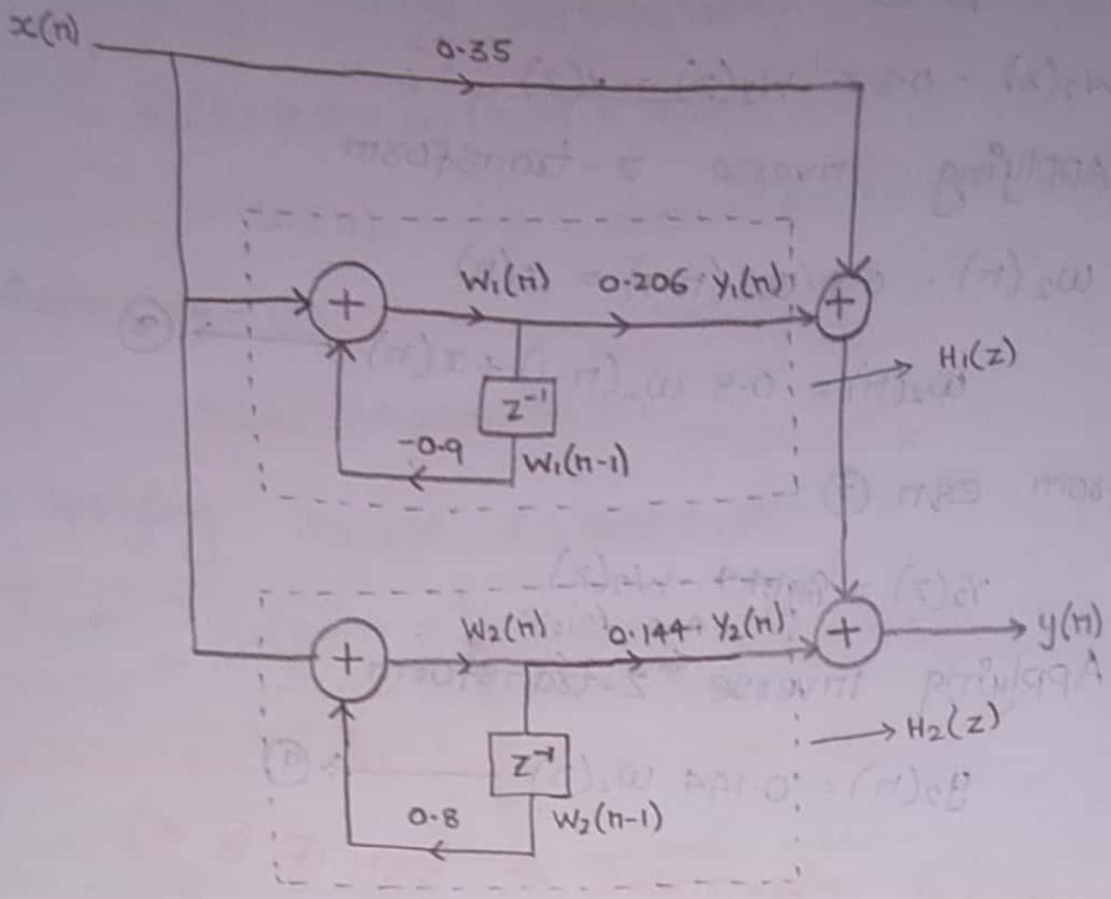




COMBINATION OF BOTH REALIZATION STRUCTURES:



PARALLEL FORM REALIZATION:



2> Realize a system with difference equation $y(n) = -0.1\,y(n-1) + 0.2y(n-2) + 3x(n) + 3.6\,x(n-1) + 0.6\,x(n-2)$ using parallel form Realization.

$$y(n) = -0.1\,y(n-1) + 0.2\,y(n-2) + 3x(n) + 3.6\,x(n-1) + 0.6\,x(n-2)$$

Applying z-transform on both sides

$$Y(z) = -0.1\,z^{-1}Y(z) + 0.2\,z^{-2}Y(z) + 3\,X(\boxtimes) + 3.6\,z^{-1}X(z) + 0.6\,z^{-2}X(z)$$

$$Y(z)\left[1 + 0.1\,z^{-1} - 0.2\,z^{-2}\right] = X(z)\left[3 + 3.6\,z^{-1} + 0.6\,z^{-2}\right]$$

$$\frac{Y(z)}{X(z)} = \frac{3 + 3.6\,z^{-1} + 0.6\,z^{-2}}{1 + 0.1\,z^{-1} - 0.2\,z^{-2}}$$

$$\begin{array}{r|l} & \quad -3 \\ \hline -0.2z^{-2}+0.1z^{-1}+1 & 0.6z^{-2}+3.6z^{-1}+3 \\ & +0.6z^{-2}-0.3z^{-1}-3 \\ \hline & \qquad\quad 3.9z^{-1}+6 \end{array}$$

$$H(z) = -3 + \frac{3.9z^{-1}+6}{-0.2z^{-2}+0.1z^{-1}+1}$$

$$= -3 + \frac{3.9z^{-1}+6}{-0.2z^{-2}-0.4z^{-1}+0.5z^{-1}+1}$$

$$= -3 + \frac{3.9z^{-1}+6}{-0.4z^{-1}\left(0.5z^{-1}+1\right)+1\left(0.5z^{-1}+1\right)}$$

$$H(z) = -3 + \frac{6+3.9z^{-1}}{\left(1+0.5z^{-1}\right)\left(1-0.4z^{-1}\right)}$$

$$\frac{6+3.9z^{-1}}{\left(1+0.5z^{-1}\right)\left(1-0.4z^{-1}\right)} = \frac{A}{1+0.5z^{-1}} + \frac{B}{1-0.4z^{-1}}$$

$$6+3.9z^{-1} = A\left(1-0.4z^{-1}\right) + B\left(1+0.5z^{-1}\right)$$

let $z=-0.5 \longrightarrow z^{-1} = \frac{-1}{0.5}$

$$6+3.9\left(\frac{-1}{0.5}\right) = A\left(1-0.4\cdot\frac{-1}{0.5}\right) + B\left(1+0.5\cdot\frac{-1}{0.5}\right)$$

$$6-7.8 = A\left(1+0.8\right) + B\left(1-1\right)$$

$$-1.8 = A\left(1.8\right)$$

$$A = -1$$

let $z = 0.4 \longrightarrow z^{-1} = \frac{1}{0.4}$

$$6 + 3.9\left(\frac{1}{0.4}\right) = A\left(1 - 0.4\left(\frac{1}{0.4}\right)\right) + B\left(1 + 0.5\left(\frac{1}{0.4}\right)\right)$$

$$6 + 9.75 = A(1-1) + B(1 + 1.25)$$

$$15.75 = B(2.25)$$

$$B = 7$$

$$\frac{6 + 3.9z^{-1}}{(1+0.5z^{-1})(1-0.4z^{-1})} = \frac{-1}{1+0.5z^{-1}} + \frac{7}{1-0.4z^{-1}}$$

$$H(z) = -3 + \underbrace{\frac{-1}{1+0.5\,z^{-1}}}_{H_1(z)} + \underbrace{\frac{7}{1-0.4\,z^{-1}}}_{H_2(z)}$$

$$H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{Y_1(z)}{W_1(z)} \cdot \frac{W_1(z)}{X(z)} = \frac{-1}{1+0.5\,z^{-1}}$$

$$\frac{W_1(z)}{X(z)} = \frac{1}{1+0.5z^{-1}} \longrightarrow ②$$

$$\frac{Y_1(z)}{W_1(z)} = -1 \longrightarrow ③$$

from eqn ②

$$W_1(z) + 0.5\,z^{-1}W_1(z) = X(z)$$

Applying inverse z-transform on both sides

$$W_1(n) + 0.5\,W_1(n-1) = x(n)$$

$$W_1(n) = -0.5\,W_1(n-1) + x(n) \longrightarrow ④$$

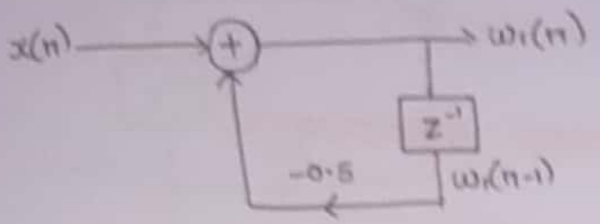
from eqn ③

$$Y_1(z) = -W_1(z)$$

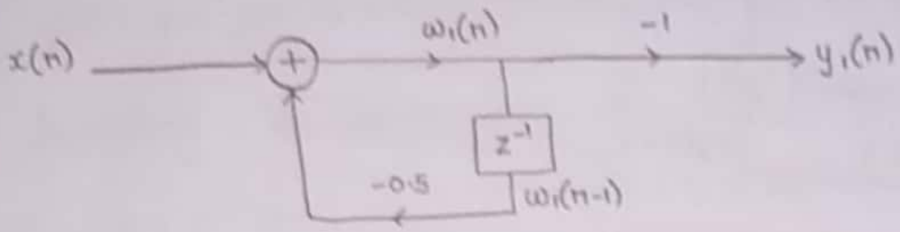
Applying inverse z-transform on both sides

$y_1(n) = -\omega_1(n) \longrightarrow$ ⑤

REALIZATION STRUCTURES FOR EQN ④ & ⑤



COMBINATION OF BOTH REALIZATION STRUCTURES :



$$H_2(z) = \frac{Y_2(z)}{X(z)} = \frac{Y_2(z)}{W_2(z)} \cdot \frac{W_2(z)}{X(z)} = \frac{7}{1-0.4z^{-1}}$$

$\frac{W_2(z)}{X(z)} = \frac{1}{1-0.4z^{-1}} \longrightarrow$ ⑥

$\frac{Y_2(z)}{W_2(z)} = 7 \longrightarrow$ ⑦

from eqn ⑥

$$W_2(z) - 0.4z^{-1}W_2(z) = X(z)$$

Applying ~~w~~ inverse z-transform on both sides

$$\omega_2(n) - 0.4\,\omega_2(n-1) = x(n)$$

$\omega_2(n) = 0.4\,\omega_2(n-1) + x(n) \longrightarrow$ ⑧

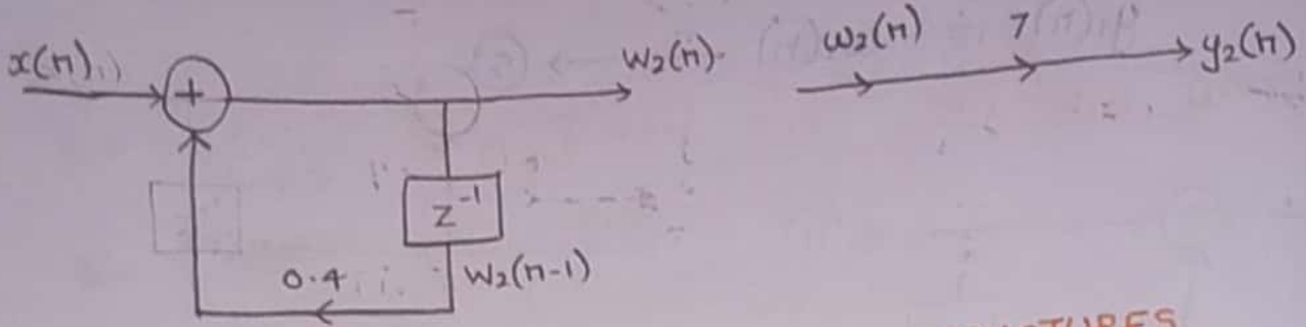
from eqn ⑦

$$Y_2(z) = 7W_2(z)$$

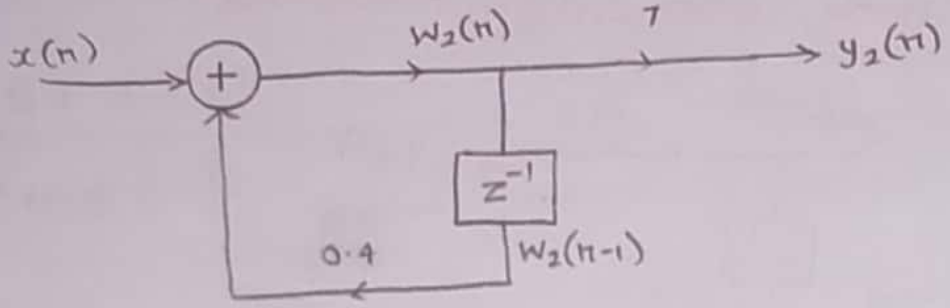
Applying inverse z-transform on both sides

$Y_2(n) = 7\,\omega_2(n) \longrightarrow$ ⑨

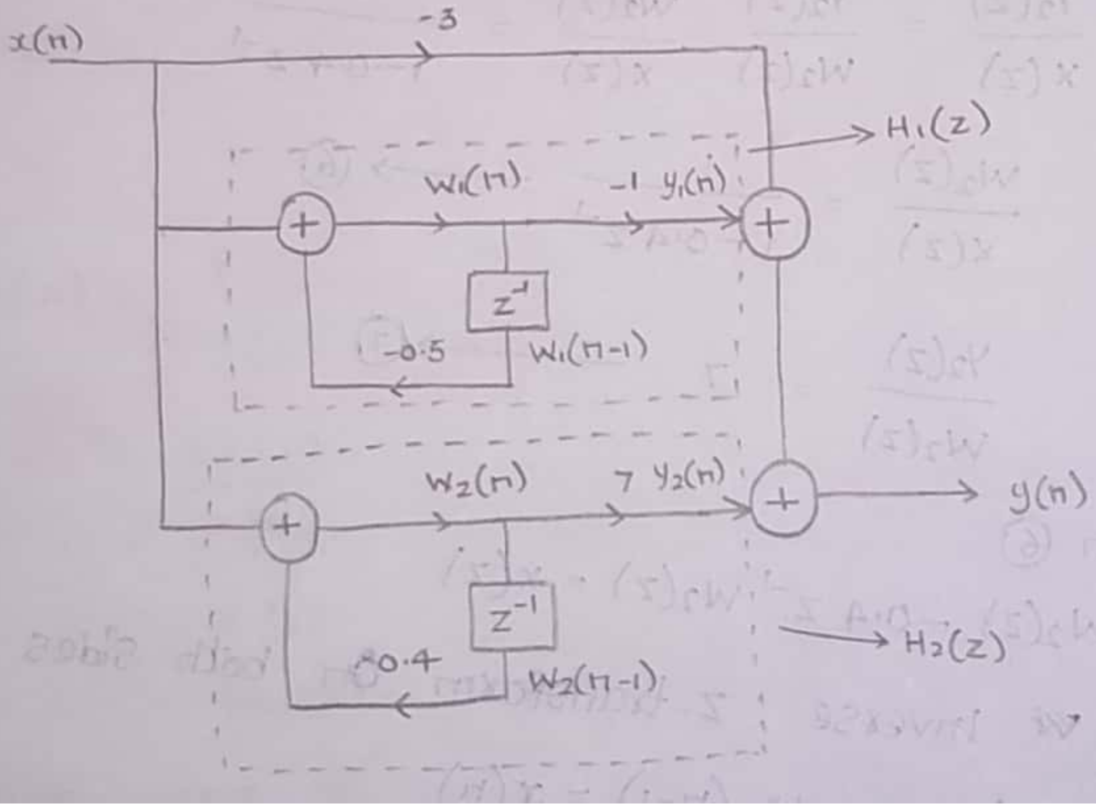
REALIZATION STRUCTURES FOR EQN ⑧ & ⑨



COMBINATION OF BOTH REALIZATION STRUCTURES



PARALLEL FORM REALIZATION:



23-03-

REALIZATION OF FIR:

1) DIRECT FORM REALIZATION / TRANSVERSAL STRUCTURE:

$$\boxed{H(z) = \sum_{n=0}^{N-1} h(n) \, z^{-n}}$$

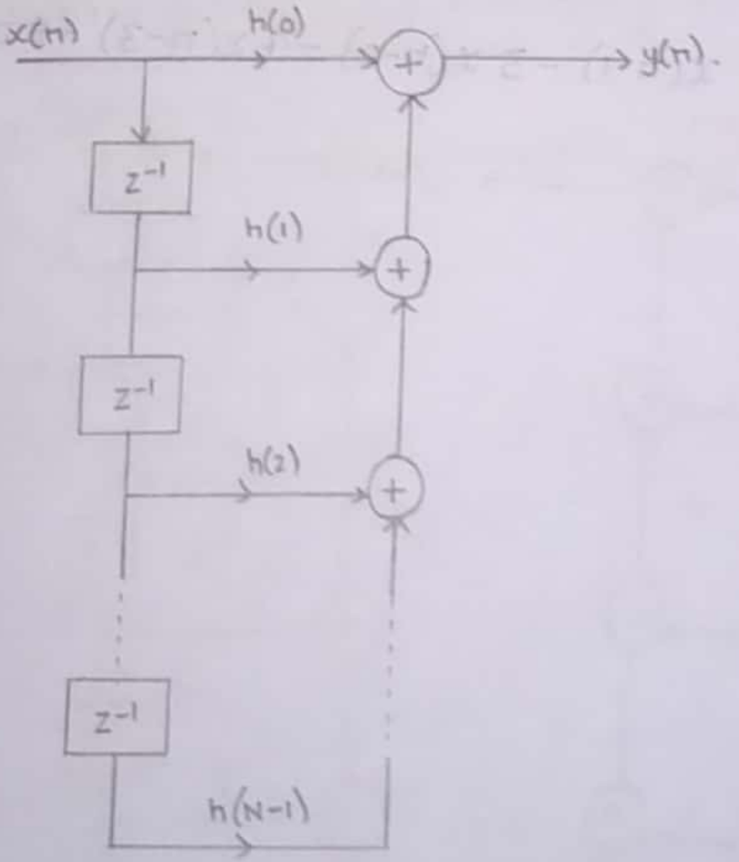
$$\frac{Y(z)}{X(z)} = h(0) + h(1) z^{-1} + h(2) z^{-2} + \text{------} + h(N-1) z^{-(N-1)}$$

$$Y(z) = h(0) \, X(z) + h(1) z^{-1} X(z) + h(2) z^{-2} X(z) + \text{-----}$$
$$\text{----} + h(N-1) \, z^{-(N-1)} X(z)$$

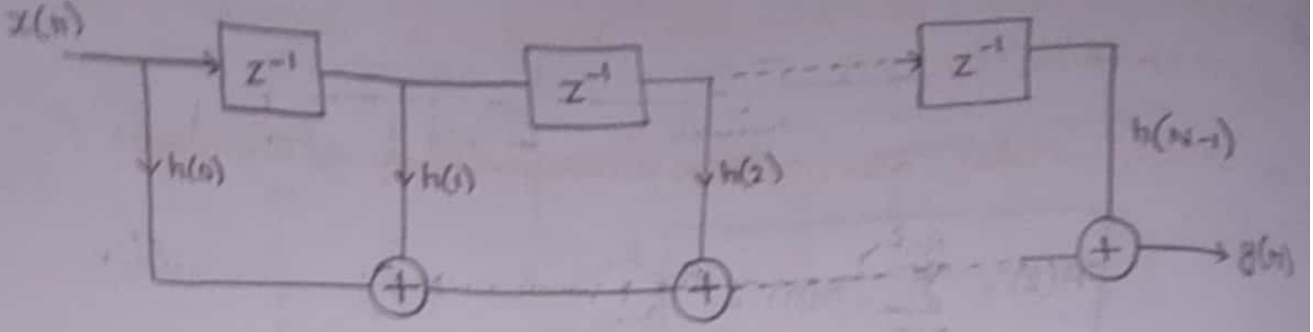
Applying Inverse z-transform

$$y(n) = h(0) \, x(n) + h(1) \, x(n-1) + h(2) \, x(n-2) + \text{------} + h(N-1) \, x(n-N+1)$$

REALIZATION STRUCTURE:



(OR)



1) Determine the direct form Realization of the Syste function $H(z) = 1 + 2z^{-1} - 3z^{-2} - 4z^{-3} + 8z^{-4}$

$$H(z) = 1 + 2z^{-1} - 3z^{-2} - 4z^{-3} + 8z^{-4}$$

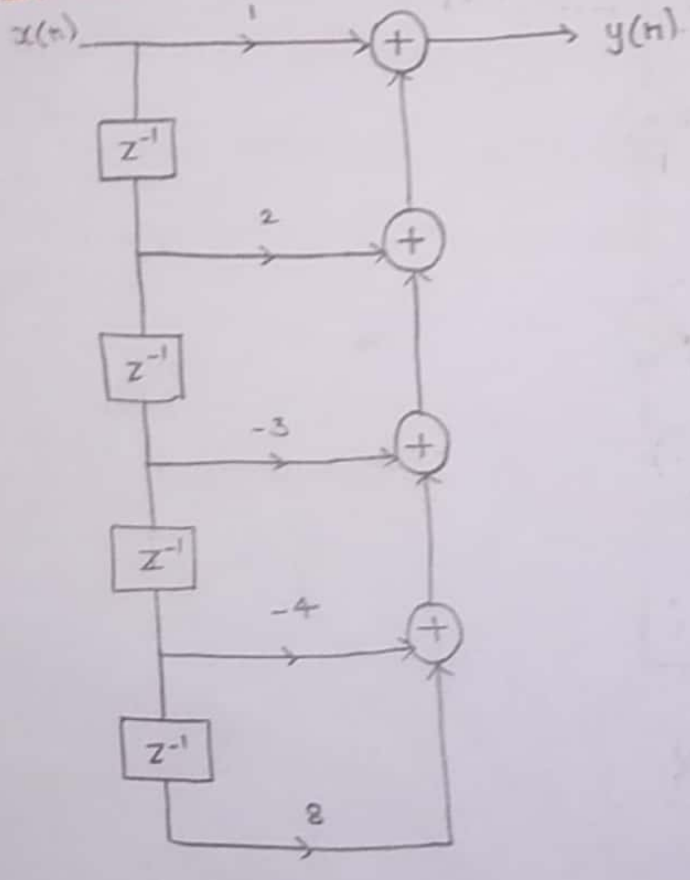
$$\frac{Y(z)}{X(z)} = 1 + 2z^{-1} - 3z^{-2} - 4z^{-3} + 8z^{-4}$$

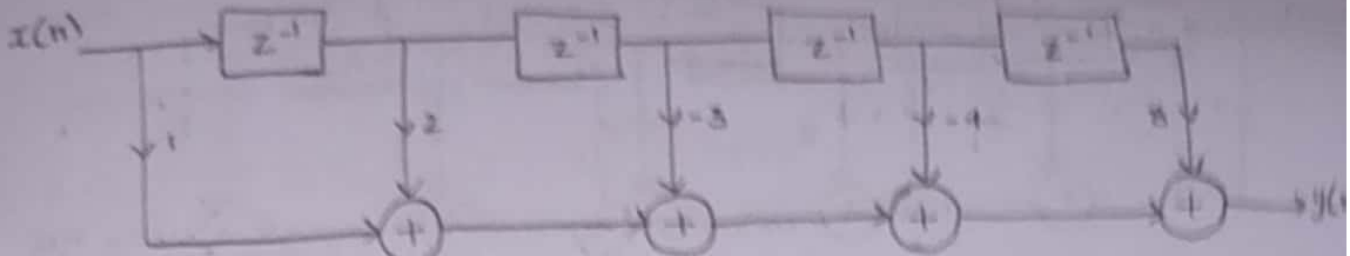
$$Y(z) = X(z) + 2z^{-1}X(z) - 3z^{-2}X(z) - 4z^{-3}X(z) + 8z^{-4}X(z$$

Applying Inverse Z-transform on both sides

$$y(n) = x(n) + 2x(n-1) - 3x(n-2) - 4x(n-3) + 8x(n-4)$$

REALIZATION STRUCTURE :





2> Determine Direct form Realization of the system function $H(z) = 1+\frac{1}{5}z^{-1}+\frac{1}{8}z^{-2}+\frac{1}{3}z^{-4}+\frac{1}{5}z^{-5}$

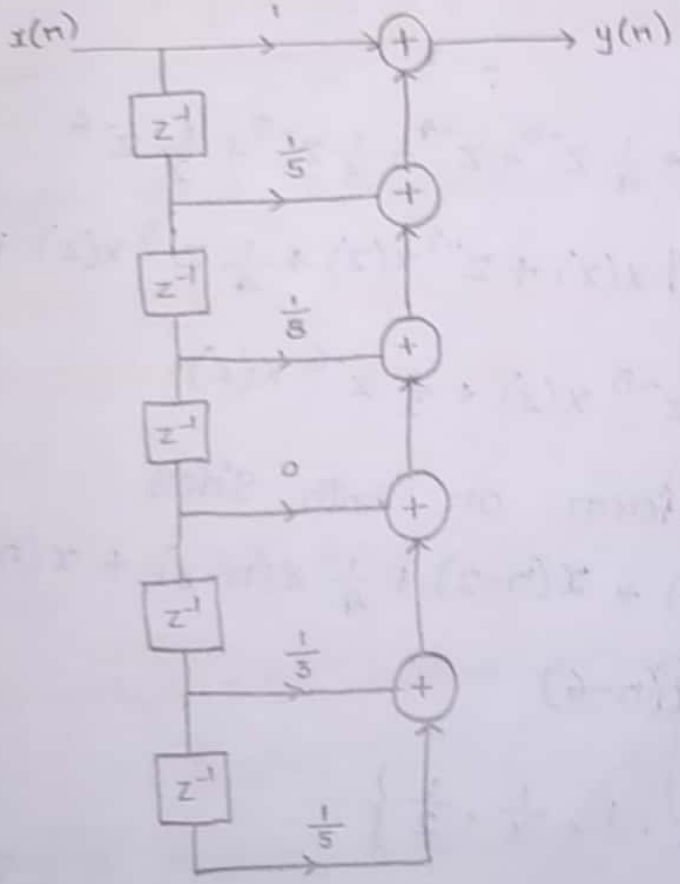
$$H(z) = 1+\frac{1}{5}z^{-1}+\frac{1}{8}z^{-2}+\frac{1}{3}z^{-4}+\frac{1}{5}z^{-5}$$

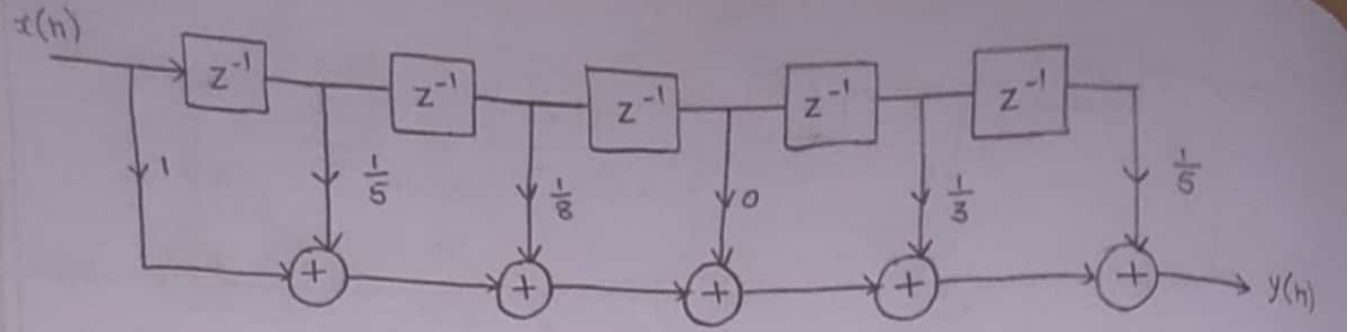
$$\frac{Y(z)}{X(z)} = 1+\frac{1}{5}z^{-1}+\frac{1}{8}z^{-2}+\frac{1}{3}z^{-4}+\frac{1}{5}z^{-5}$$

$$Y(z) = X(z)+\frac{1}{5}z^{-1}X(z)+\frac{1}{8}z^{-2}X(z)+\frac{1}{3}z^{-4}X(z)+\frac{1}{5}z^{-5}X(z)$$

Applying inverse z-transform on both sides

$$y(n) = x(n)+\frac{1}{5}x(n-1)+\frac{1}{8}x(n-2)+\frac{1}{3}x(n-4)+\frac{1}{5}x(n-5)$$





25-03-19

LINEAR PHASE REALIZATION OR MINIMUM NUMBER OF MULTIPLIER'S :

For Linear phase FIR filter

$$h(n) = h(N-1-n)$$

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

1) obtain Direct form Realization with min. no. of multipliers for the system transfer function

$$H(z) = \frac{1}{2} + \frac{1}{3} z^{-1} + z^{-2} + \frac{1}{4} z^{-3} + z^{-4} + \frac{1}{3} z^{-5} + \frac{1}{2} z^{-6}$$

$$\frac{Y(z)}{X(z)} = \frac{1}{2} + \frac{1}{3} z^{-1} + z^{-2} + \frac{1}{4} z^{-3} + z^{-4} + \frac{1}{3} z^{-5} + \frac{1}{2} z^{-6}$$

$$Y(z) = \frac{1}{2} X(z) + \frac{1}{3} z^{-1} X(z) + z^{-2} X(z) + \frac{1}{4} z^{-3} X(z) +$$

$$z^{-4} X(z) + \frac{1}{3} z^{-5} X(z) + \frac{1}{2} z^{-6} X(z)$$

Apply Inverse z-transform on both sides

$$y(n) = \frac{1}{2} x(n) + \frac{1}{3} x(n-1) + x(n-2) + \frac{1}{4} x(n-3) + x(n-4) +$$

$$\frac{1}{3} x(n-5) + \frac{1}{2} x(n-6)$$

$$h(n) = \left\{ \frac{1}{2}, \frac{1}{3}, 1, \frac{1}{4}, 1, \frac{1}{3}, \frac{1}{2} \right\}$$

$$h(n) = h(N-1-n)$$

$N = 7$

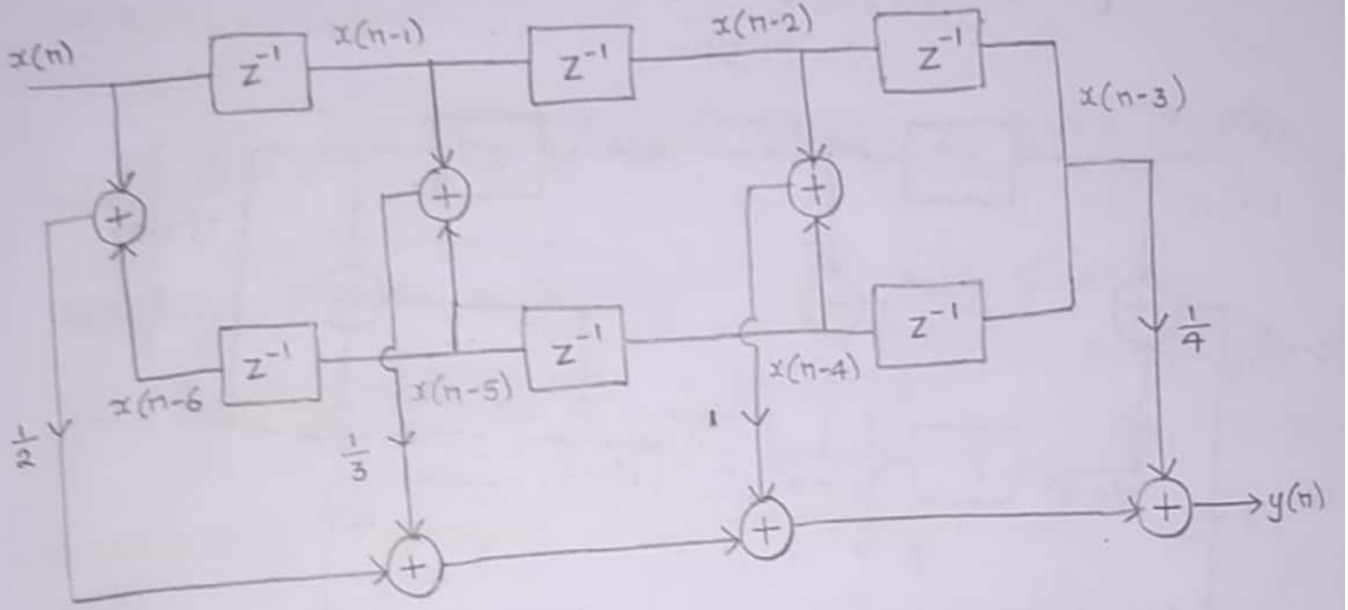
$n=0 \longrightarrow h(0) = h(7-1-0) = h(6)$

$n=1 \longrightarrow h(1) = h(7-1-1) = h(5)$

$n=2 \longrightarrow h(2) = h(7-1-2) = h(4)$

$n=3 \longrightarrow h(3) = h(7-1-3) = h(3)$

$$y(n) = \frac{1}{2}\left[x(n) + x(n-6)\right] + \frac{1}{3}\left[x(n-1) + x(n-5)\right] + 1\left[x(n-2) + x(n-4)\right] + \frac{1}{4}x(n-3)$$



* Linear phase Realization requires $\frac{N+1}{2}$ multipliers.

2> Obtain Direct form Realization with minimum number of multipliers for the system transfer function

$$H(Z) = 1 + 3Z^{-1} + 4Z^{-2} + 4Z^{-3} + 3Z^{-4} + Z^{-5}$$

$$\frac{Y(Z)}{X(Z)} = 1 + 3Z^{-1} + 4Z^{-2} + 4Z^{-3} + 3Z^{-4} + Z^{-5}$$

$$Y(Z) = X(Z) + 3Z^{-1}X(Z) + 4Z^{-2}X(Z) + 4Z^{-3}X(Z) + 3Z^{-4}X(Z) + Z^{-6}X(Z)$$

Applying Inverse Z-transform on both sides

$$y(n) = x(n) + 3x(n-1) + 4x(n-2) + 4x(n-3) + 3x(n-4) + x(n-5)$$

$h(n) = \{1, 3, 4, 4, 3, 1\}$

$h(n) = h(N-1-n)$

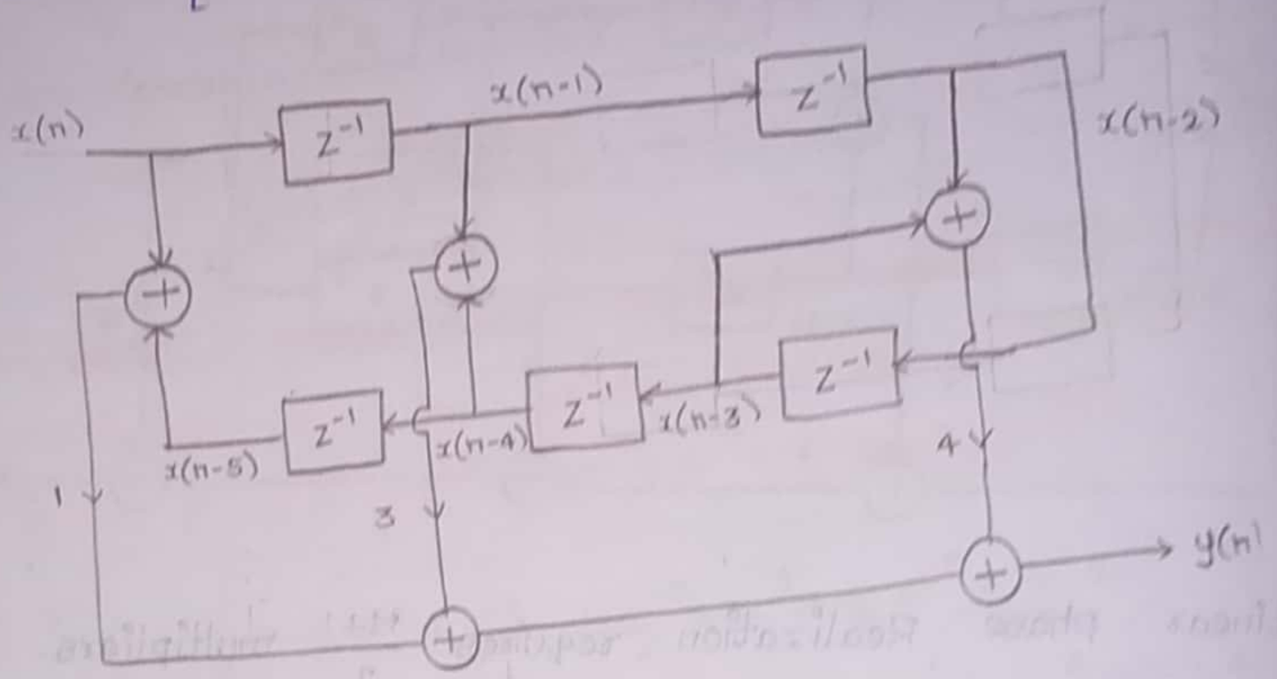
$N = 6$

$n=0 \longrightarrow h(0) = h(6-1-0) = h(5)$

$n=1 \longrightarrow h(1) = h(6-1-1) = h(4)$

$n=2 \longrightarrow h(2) = h(6-1-2) = h(3)$

$y(n) = 1\left[x(n) + x(n-5)\right] + 3\left[x(n-1) + x(n-4)\right] + 4\left[x(n-2) + x(n-3)\right]$



NOTE :

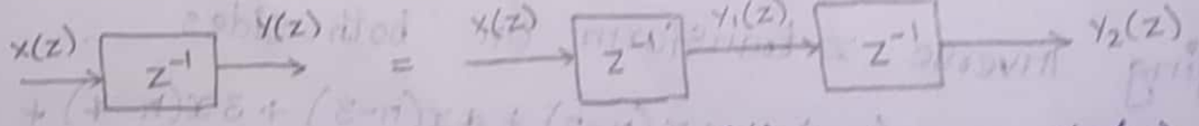
In Linear phase Realization, If

$N = \text{Odd} \longrightarrow$ it requires $\frac{N+1}{2}$ Multipliers

$N = \text{even} \longrightarrow$ it requires $\frac{N}{2}$ Multipliers

CASCADE FORM REALIZATION :

$H(z) = H_1(z) \cdot H_2(z)$



$H(z) = \frac{Y(z)}{X(z)}$ $\qquad H_1(z) = \frac{Y_1(z)}{X(z)} \cdot H_2(z) = \frac{Y_2(z)}{Y_1(z)}$

1> Find the Cascade form Realization for the system Transfer function $H(z) = (1+2z^{-1}-z^{-2})(1+z^{-1}-z^{-2})$

$$H(z) = (1+2z^{-1}-z^{-2})(1+z^{-1}-z^{-2})$$

$$H(z) = H_1(z) \cdot H_2(z)$$

$$\frac{Y(z)}{X(z)} = \frac{Y_1(z)}{X(z)} \cdot \frac{Y_2(z)}{Y_1(z)}$$

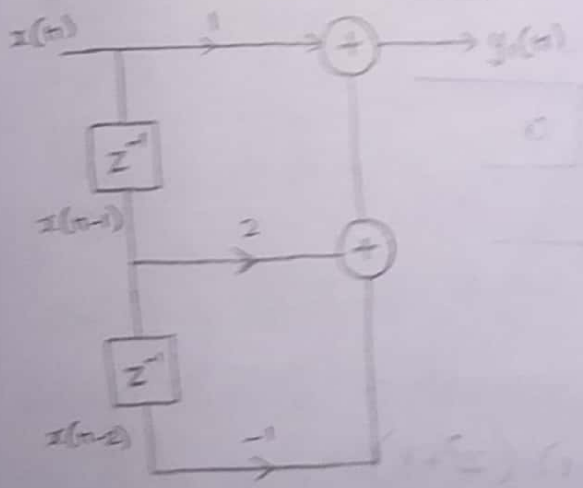
$$\frac{Y_1(z)}{X(z)} = 1+2z^{-1}-z^{-2} \qquad \frac{Y_2(z)}{Y_1(z)} = 1+z^{-1}-z^{-2}$$

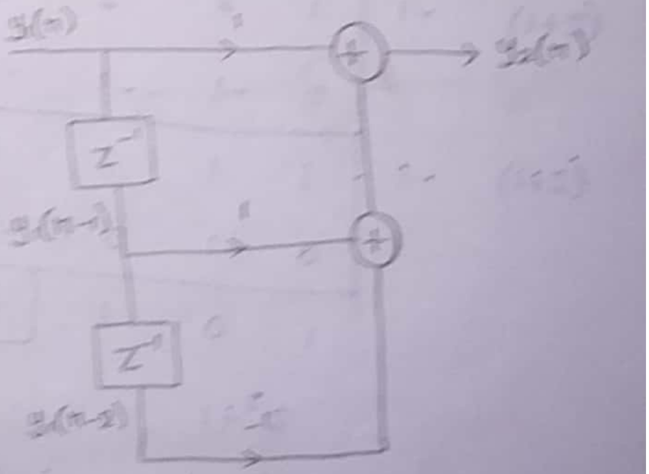
$$Y_1(z) = X(z) + 2z^{-1}X(z) - z^{-2}X(z) \qquad Y_2(z) = Y_1(z) + z^{-1}Y_1(z) - z^{-2}Y_1(z)$$

Applying inverse z-transform on both sides

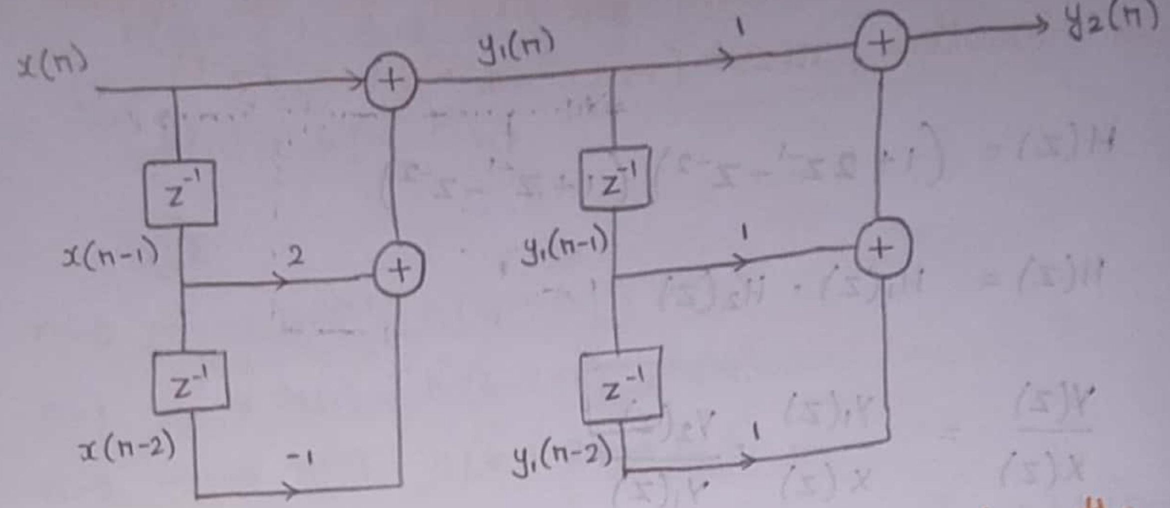
$$y_1(n) = x(n) + 2x(n-1) - x(n-2) \longrightarrow ② \qquad y_2(n) = y_1(n) + y_1(n-1) - y_1(n-2) \longrightarrow ③$$

REALIZATION STRUCTURES FOR EQN ② & ③





COMBINATION OF BOTH REALIZATION STRUCTURES :



2> Obtain the Cascade form Realization for the system transfer function $H(z) = 1+3z^{-1}+4z^{-2}+4z^{-3}+3z^{-4}+z$

$$H(z) = 1+3z^{-1}+4z^{-2}+4z^{-3}+3z^{-4}+z^{-5} \longrightarrow ①$$

$$= z^{-5}\left[z^{5}+3z^{4}+4z^{3}+4z^{2}+3z^{1}+1\right]$$

By using Remainder value theorem

		1	3	4	4	3	1
(z+1)	-1	0	-1	-2	-2	-2	-1
		1	2	2	2	1	0
(z+1)	-1	0	-1	-1	-1	-1	
		1	1	1	1	0	
(z+1)	-1	0	-1	0	-1		
		1	0	1	0		

z^2+1

$$H(z) = z^{-5}(z+1)(z+1)(z+1)(z^2+1)$$

$$H(z) = z^{-1}(z+1)\, z^{-1}(z+1)\, z^{-1}(z+1)\, z^{-2}(z^2+1)$$

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$$H(z) = H_1(z)\, H_2(z)\, H_3(z)\, H_4(z)$$

$$\frac{Y(z)}{X(z)} = \frac{Y_1(z)}{X(z)} \cdot \frac{Y_2(z)}{Y_1(z)} \cdot \frac{Y_3(z)}{Y_2(z)} \cdot \frac{Y_4(z)}{Y_3(z)}$$

$$\frac{Y_1(z)}{X(z)} = z^{-1}(z+1)$$

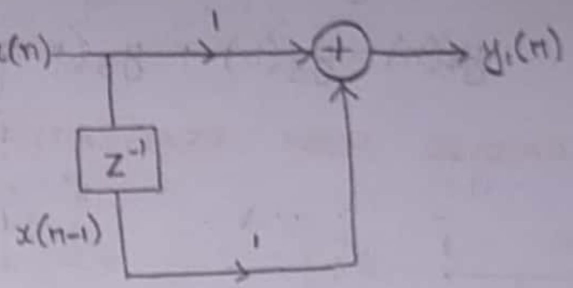
$$\frac{Y_1(z)}{X(z)} = 1+z^{-1}$$

$$Y_1(z) = X(z) + z^{-1}X(z)$$

Apply Inverse z-transform

$$y_1(n) = x(n) + x(n-1) \longrightarrow ②$$

REALIZATION STRUCTURE FOR EQN ②



$$\frac{Y_2(z)}{Y_1(z)} = z^{-1}(z+1)$$

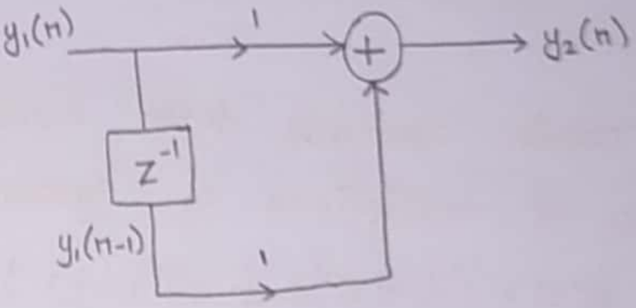
$$\frac{Y_2(z)}{Y_1(z)} = 1+z^{-1}$$

$$Y_2(z) = Y_1(z) + z^{-1}Y_1(z)$$

Apply Inverse z-transform

$$y_2(n) = y_1(n) + y_1(n-1) \longrightarrow ③$$

REALIZATION STRUCTURE FOR EQN ③



$$\frac{Y_3(z)}{Y_2(z)} = z^{-1}(z+1)$$

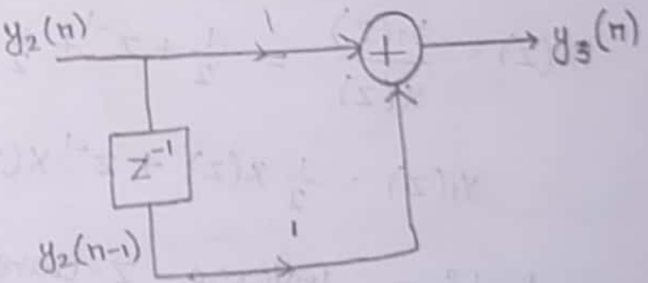
$$\frac{Y_3(z)}{Y_2(z)} = 1+z^{-1}$$

$$Y_3(z) = Y_2(z) + z^{-1}Y_2(z)$$

Apply Inverse z-transform

$$y_3(n) = y_2(n) + y_2(n-1) \longrightarrow ④$$

REALIZATION STRUCTURE FOR EQN ④

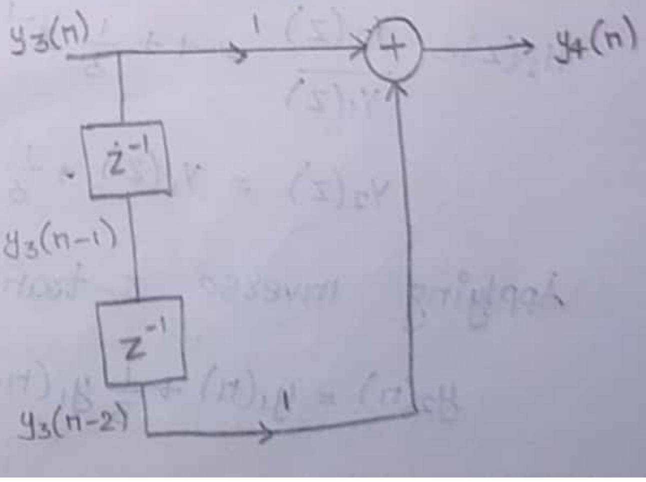


$$\frac{Y_4(z)}{Y_3(z)} = z^{-2}(z^2+1)$$

$$\frac{Y_4(z)}{Y_3(z)} = 1+z^{-2}$$

$$Y_4(z) = Y_3(z) + z^{-2}Y_3(z)$$

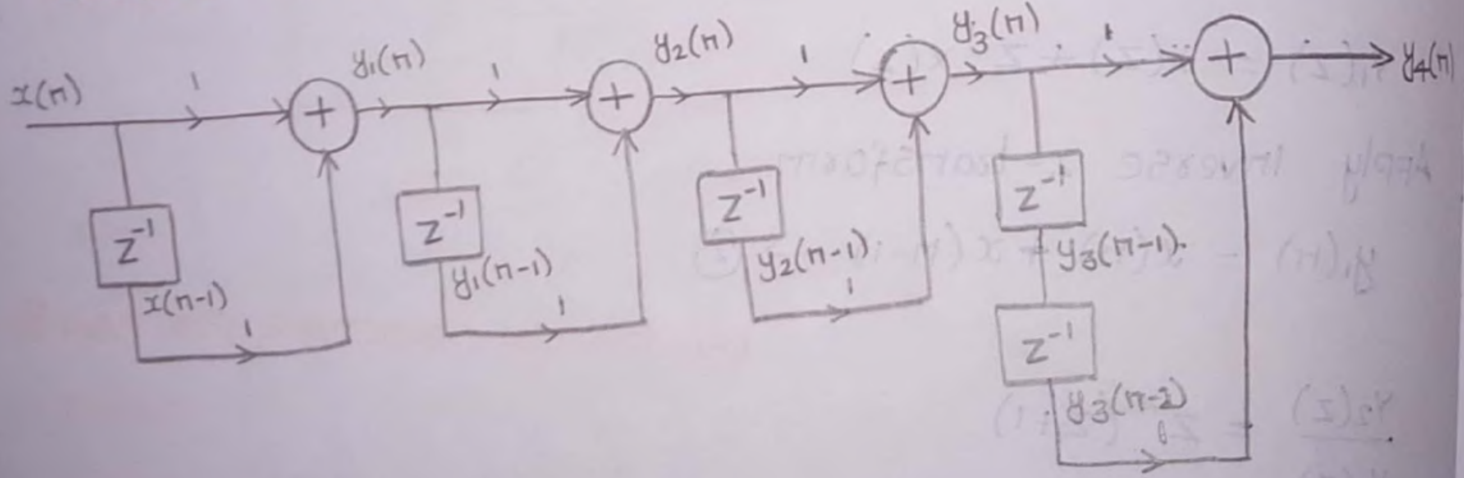
REALIZATION STRUCTURE FOR EQN ⑤



Apply Inverse Z- transform

$$y_4(n) = y_3(n) + y_3(n-2) \longrightarrow ⑤$$

CASCADE FORM REALIZATION STRUCTURE:



FIR LATTICE REALIZATION STRUCTURE:

$$H(z) = A_M(z) = 1 + \sum_{K=1}^{m} a_m(K) z^{-k} \longrightarrow ①$$

$$\frac{Y(z)}{X(z)} = 1 + \sum_{K=1}^{m} a_m(K) z^{-k}$$

$$Y(z) = X(z) + \sum_{K=1}^{m} a_m(K) z^{-K} X(z)$$

Applying Inverse z- transform

$$y(n) = x(n) + \sum_{K=1}^{m} a_m(k)\, x(n-k)$$

$m=1$

$$y(n) = x(n) + a_1(1)\, x(n-1) \longrightarrow ②$$

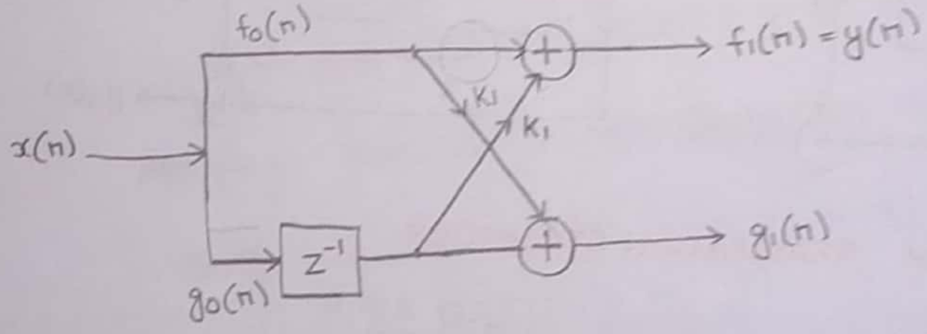
$$\boxed{a_1(0) = 1}$$

$m=2$

$$y(n) = x(n) + a_2(1)\, x(n-1) + a_2(2)\, x(n-2) \longrightarrow ③$$

$$\boxed{a_2(0) = 1}$$

SINGLE STAGE LATTICE:



$$x(n) = f_0(n) = g_0(n)$$

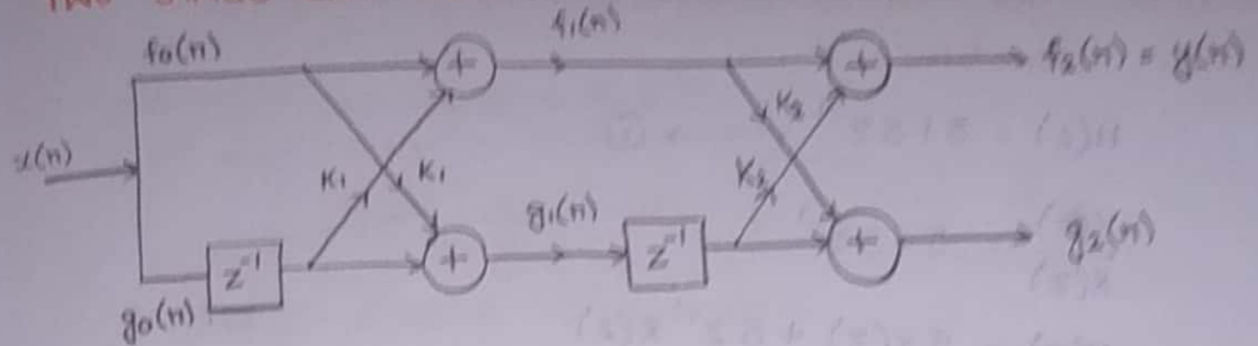
$$f_1(n) = f_0(n) + K_1\, g_0(n-1)$$

$$y(n) = x(n) + K_1\, x(n-1) \longrightarrow ④$$

$$g_1(n) = K_1\, f_0(n) + g_0(n-1)$$

$$g_1(n) = K_1\, x(n) + x(n-1) \longrightarrow ⑤$$

TWO STAGE LATTICE :



$$f_2(n) = f_1(n) + K_2\, g_1(n-1)$$

$$f_2(n) = f_0(n) + K_1\, g_0(n-1) + K_2\, g_1(n-1)$$

$$y(n) = x(n) + K_1\, x(n-1) + K_2\left[K_1\, x(n-1) + x(n-2)\right]$$

$$y(n) = x(n) + K_1\, x(n-1) + K_1 K_2\, x(n-1) + K_2\, x(n-2)$$

$y(n) = x(n) + K_1(1+K_2)\, x(n-1) + K_2\, x(n-2) \longrightarrow$ ⑥

$$g_2(n) = K_2\, f_1(n) + g_1(n-1)$$

$$g_2(n) = K_2\left[f_0(n) + K_1\, g_0(n-1)\right] + K_1\, f_0(n-1) + g_0(n-2)$$

$$g_2(n) = K_2 f_0(n) + K_1K_2\, g_0(n-1) + K_1\, f_0(n-1) + g_0(n-2)$$

$$g_2(n) = K_2\, x(n) + K_1K_2\, x(n-1) + K_1\, x(n-1) + x(n-2)$$

$g_2(n) = K_2\, x(n) + K_1(1+K_2)\, x(n-1) + x(n-2) \longrightarrow$ ⑦

PROCEDURE TO REALIZE LATTICE STRUCTURE OF FIR SYSTEM :

1>. If coefficient of present input $x(n)$ is not unity then convert it to unity by taking common in the coefficients of present input.

2> Find order of difference equation and compare of coefficients of given difference equation with coefficients of same order lattice structure

3> Assign calculated values of K1, K2, ----- and construct the lattice structure.

1) Realize the System $H(z) = 5+3z^{-1}$ using FIR lattice structure?

$$H(z) = 5+3z^{-1} \longrightarrow ①$$

$$\frac{Y(z)}{X(z)} = 5+3z^{-1}$$

$$Y(z) = 5X(z) + 3z^{-1}X(z)$$

Apply inverse Z- transform

$$y(n) = 5x(n) + 3x(n-1)$$

$$y(n) = 5\left[x(n) + \frac{3}{5}x(n-1)\right] \longrightarrow ②$$

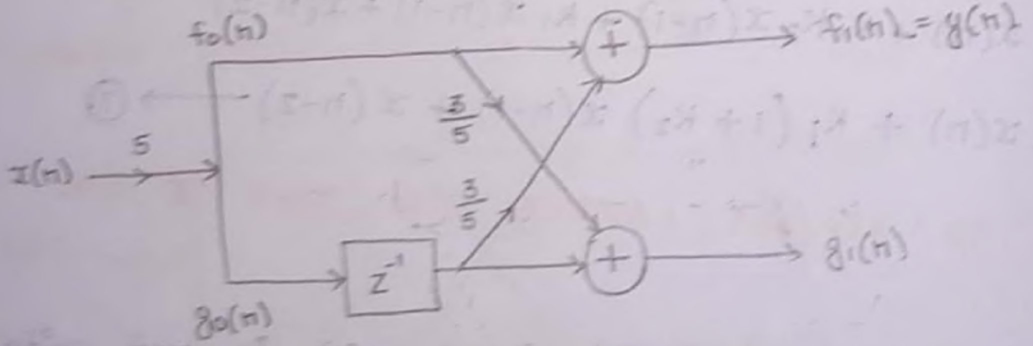
From Single stage lattice

$$y(n) = x(n) + K_1 x(n-1) \longrightarrow ③$$

compare eqn ② and eqn ③

$$\boxed{K_1 = \frac{3}{5}}$$

SINGLE STAGE LATTICE :



$$y(n) = 5\left[x(n) + \frac{3}{5}x(n-1)\right]$$

$$g_1(n) = 5\left[\frac{3}{5}x(n) + x(n-1)\right]$$

2) Determine lattice coefficient corresponding to FIR system with system function

$$H(z) = 1 + \frac{7}{9}z^{-1} + \frac{3}{5}z^{-2}$$

$$H(z) = 1 + \frac{7}{9} z^{-1} + \frac{3}{5} z^{-2} \longrightarrow ①$$

$$\frac{Y(z)}{X(z)} = 1 + \frac{7}{9} z^{-1} + \frac{3}{5} z^{-2}$$

$$Y(z) = X(z) + \frac{7}{9} z^{-1} X(z) + \frac{3}{5} z^{-2} X(z)$$

Apply inverse Z-transform

$$y(n) = x(n) + \frac{7}{9} x(n-1) + \frac{3}{5} x(n-2) \longrightarrow ②$$

From two stage Lattice

$$y(n) = x(n) + K_1 (1+K_2) x(n-1) + K_2 x(n-2) \longrightarrow ③$$

compare eqn ② and eqn ③

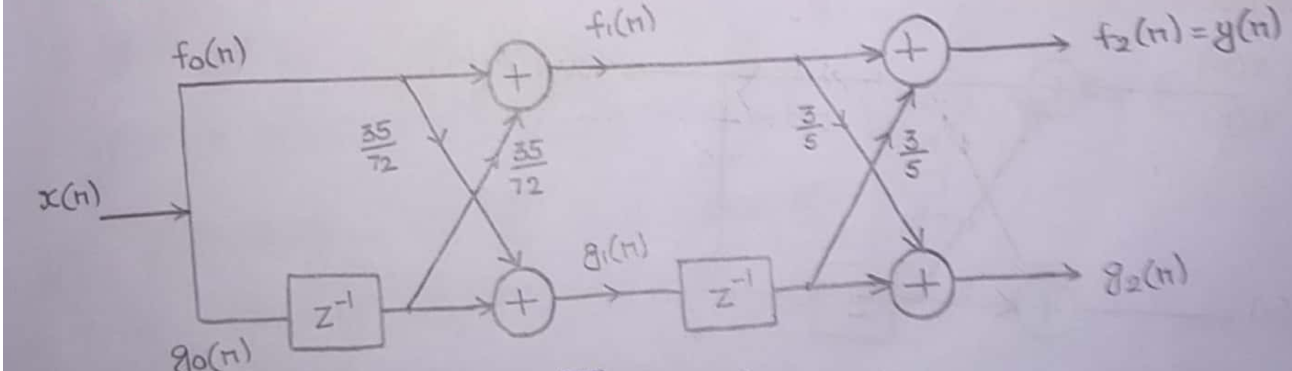
$$K_1(1+K_2) = \frac{7}{9} \qquad K_2 = \frac{3}{5}$$

$$K_1 \left(1 + \frac{3}{5}\right) = \frac{7}{9}$$

$$K_1 \cdot \frac{8}{5} = \frac{7}{9}$$

$$\boxed{K_1 = \frac{35}{72}}$$

TWO STAGE LATTICE :



$$f_1(n) = x(n) + \frac{35}{72} x(n-1)$$

$$g_1(n) = \frac{35}{72} x(n) + x(n-1)$$

$$y(n) = \left[x(n) + \frac{35}{72} x(n-1)\right] + \frac{3}{5}\left[\frac{35}{72} x(n-1) + x(n-2)\right]$$

$$g_2(n) = \frac{3}{5}\left[x(n) + \frac{35}{72} x(n-1)\right] + \left[\frac{35}{72} x(n) + x(n-1)\right]$$

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LATTICE STRUCTURE OF IIR :

Let us consider an all pole - system with system eqn.

$$H(z) = \frac{1}{A_N(z)} = \frac{1}{1+\sum_{K=1}^{N} a_N(k) z^{-k}} \longrightarrow ①$$

$$\frac{Y(z)}{X(z)} = \frac{1}{1+\sum_{K=1}^{N} a_N(K) z^{-k}}$$

$$Y(z) + \sum_{K=1}^{N} a_N(K) z^{-k} Y(z) = X(z)$$

$$Y(z) = X(z) - \sum_{K=1}^{N} a_N(K) z^{-k} Y(z)$$

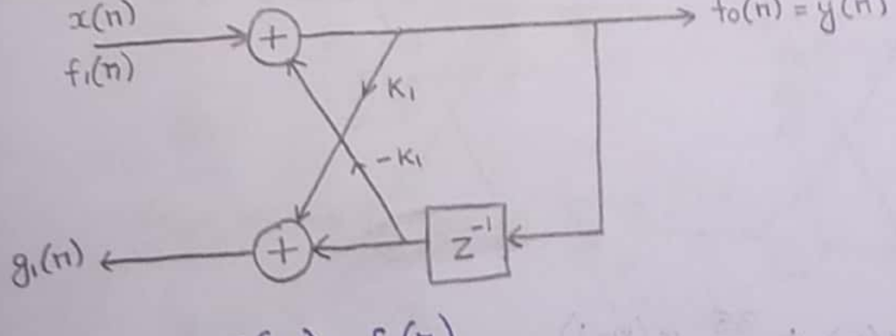
Apply Inverse z - transform

$$y(n) = x(n) - \sum_{K=1}^{N} a_N(K) y(n-k)$$

For N = 1

$$y(n) = x(n) - a_1(1) y(n-1) \longrightarrow ②$$

SINGLE STAGE LATTICE STRUCTURE :



$$x(n) = f_1(n)$$

$$y(n) = f_0(n) = g(n)$$

$$y(n) = f_1(n) - K_1 g_0(n-1)$$

$$y(n) = x(n) - K_1 y(n-1) \longrightarrow ③$$

comparing eqns ② and ③

$$K_1 = a_1(1)$$

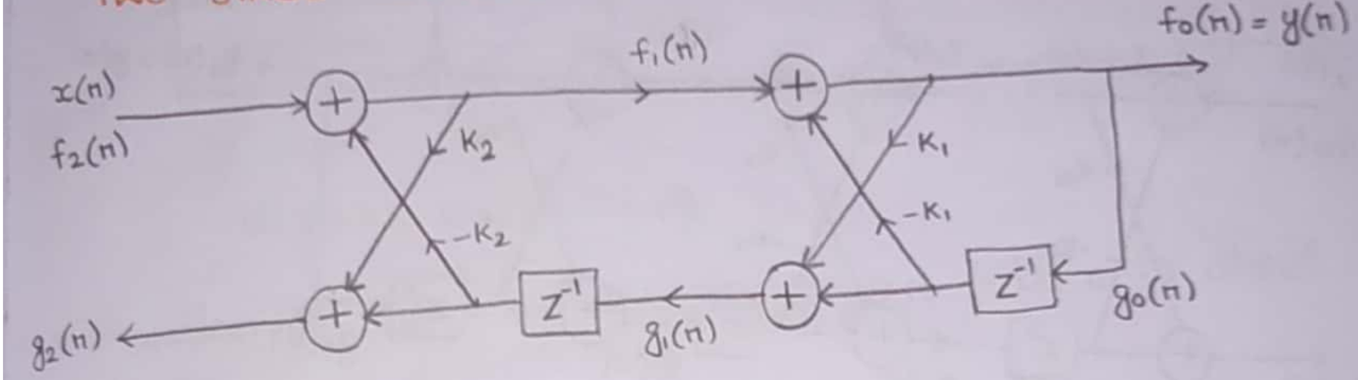
$$g_1(n) = g_0(n-1) + K_1 f_0(n)$$

$$g_1(n) = y(n-1) + K_1 y(n) \longrightarrow ④$$

For $N=2$

$$y(n) = x(n) - a_2(1)\, y(n-1) - a_2(2)\, y(n-2) \longrightarrow ⑤$$

TWO STAGE LATTICE STRUCTURE :



$$y(n) = f_0(n) = g_0(n)$$

$$y(n) = f_1(n) + g_0(n-1)(-K_1)$$

$$f_1(n) = f_2(n) - K_2\, g_1(n-1)$$

$$g_1(n) = K_1 f_0(n) + g_0(n-1)$$

$$y(n) = f_2(n) - K_2\, g_1(n-1) - K_1\, g_0(n-1)$$

$$y(n) = f_2(n) - K_2\left[K_1 f_0(n-1) + g_0(n-2)\right] - K_1\, g_0(n-1)$$

$$y(n) = x(n) - K_1 K_2\, y(n-1) - K_2\, y(n-2) - K_1\, y(n-1)$$

$$y(n) = x(n) - K_1(1+K_2)\, y(n-1) - K_2\, y(n-2) \longrightarrow ⑥$$

Compare eqn ⑤ and eqn ⑥

$$K_1(1+K_2) = a_2(1) \qquad K_2 = a_2(2)$$

$$g_2(n) = K_2\, f_1(n) + g_1(n-1)$$

$$f_0(n) = f_1(n) - K_1\, g_0(n-1)$$

$$f_1(n) = f_0(n) + K_1\, g_0(n-1)$$

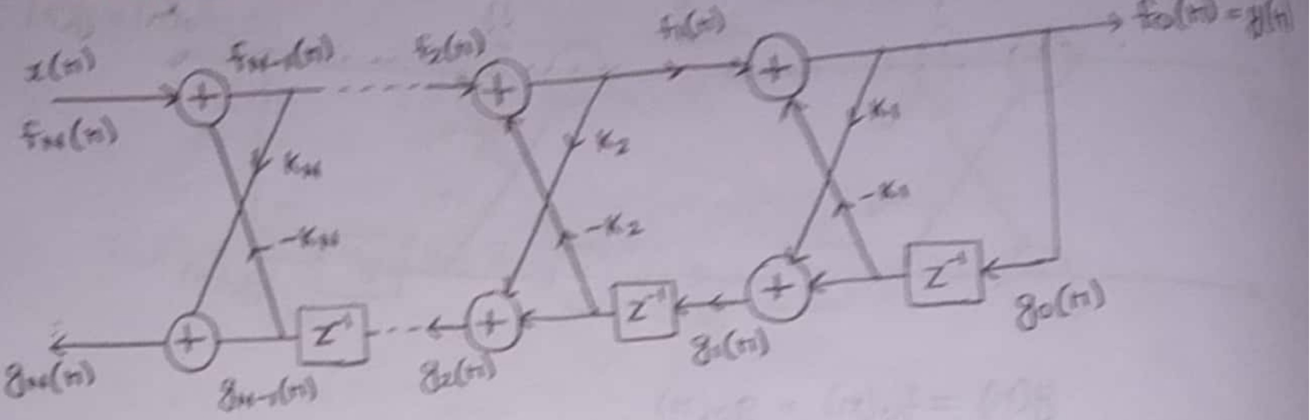
$$g_1(n) = K_1 f_0(n) + g_0(n-1)$$

$$g_2(n) = K_2\left[f_0(n) + K_1 g_0(n-1)\right] + K_1 f_0(n-1) + g_0(n-2)$$

$$g_2(n) = K_2 y(n) + K_1 K_2 y(n-1) + K_1 y(n-1) + y(n-2)$$

$$g_2(n) = K_2 y(n) + K_1(1+K_2) y(n-1) + y(n-2)$$

N-STAGE LATTICE STRUCTURE:



LATTICE - LADDER IIR STRUCTURE:

$$H(z) = \frac{B_M(z)}{A_N(z)} = \frac{\sum_{K=0}^{M} b_M(K) z^{-K}}{1 + \sum_{K=1}^{N} a_N(K) z^{-K}} \longrightarrow ①$$

Ladder part $\boxed{y(n) = \sum_{m=0}^{M} c_m . g_m(n)}$

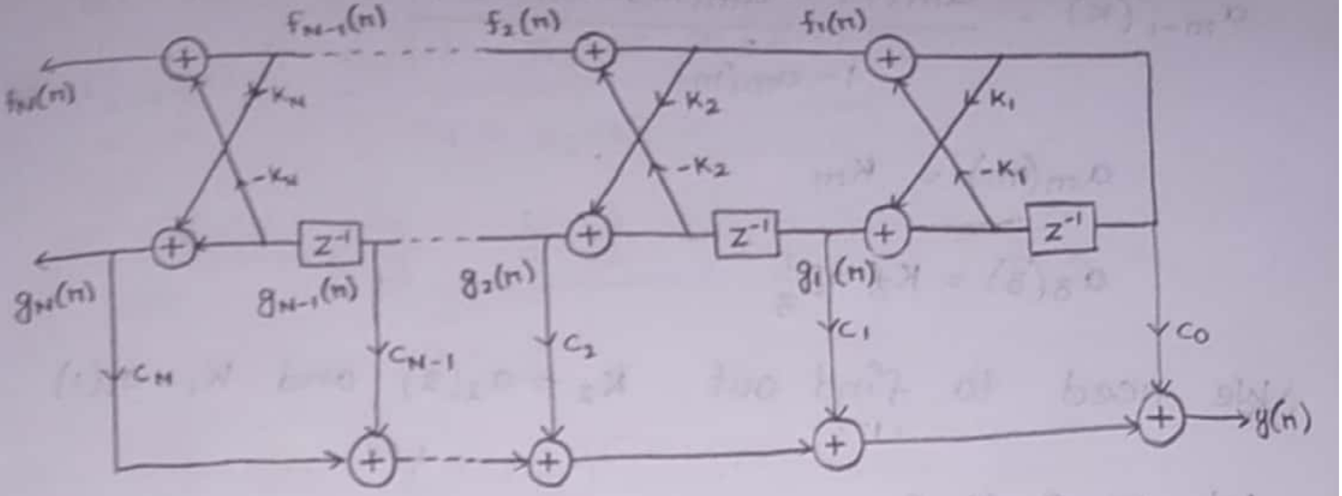
$c_m \longrightarrow$ Ladder Co-efficients

$g_m(n) \longrightarrow$ Weighted linear Combinations.

To find Ladder Co-efficients :

$$c_m = b_m - \sum_{i=m+1}^{M} c_i a_i(i-m)$$

where $m = M, M-1, ---- 0$



$$a_m(m) = K_m \longrightarrow ②$$

Equation ② can be used to convert lattice to direct form.

$$a_{m-1}(K) = \frac{a_m(K) - a_m(m)\, a_m(m-K)}{1 - a_m^2(m)} \longrightarrow ③$$

Equation ③ can be used to convert direct form to Lattice structure.

1> Obtain Lattice ladder structure for system transfer function

$$H(Z) = \frac{1+2Z^{-1}+2Z^{-2}+Z^{-3}}{1+\frac{13}{24}Z^{-1}+\frac{5}{8}Z^{-2}+\frac{1}{3}Z^{-3}}$$

$$H(z) = \frac{B_M(z)}{A_N(z)} = \frac{1+2Z^{-1}+2Z^{-2}+Z^{-3}}{1+\frac{13}{24}Z^{-1}+\frac{5}{8}Z^{-2}+\frac{1}{3}Z^{-3}}$$

Here $B_M(z) = 1+2Z^{-1}+2Z^{-2}+Z^{-3}$

$$A_N(Z) = 1+\frac{13}{24}Z^{-1}+\frac{5}{8}Z^{-2}+\frac{1}{3}Z^{-3}$$

$a_3(0) = 1$, $a_3(1) = \frac{13}{24}$, $a_3(2) = \frac{5}{8}$, $a_3(3) = \frac{1}{3}$

$$a_{m-1}(k) = \frac{a_m(k) - a_m(m)\, a_m(m-k)}{1 - a_m^2(m)}$$

$$a_m(m) = K_m$$

$$a_3(3) = K_3 = \frac{1}{3}$$

We need to find out $K_2 = a_2(2)$ and $K_1 = a_1(1)$

let $m = 3,\ k = 2$

$$a_{3-1}(2) = \frac{a_3(2) - a_3(3)\, a_3(3-2)}{1 - a_3^2(3)}$$

$$a_2(2) = \frac{\frac{5}{8} - \frac{1}{3} \cdot \frac{13}{24}}{1 - \left(\frac{1}{3}\right)^2} = \frac{\frac{45-13}{3 \cdot 24}}{1 - \frac{1}{9}} = \frac{\frac{\overset{4}{\cancel{32}}}{\cancel{3} \cdot \cancel{24}\ 8}}{\frac{\cancel{8}}{\cancel{9}\ 3}}$$

$$= \frac{4}{8}$$

$$a_2(2) = \frac{1}{2}$$

let $m = 3,\ k = 1$

$$a_{3-1}(1) = \frac{a_3(1) - a_3(3)\, a_3(3-1)}{1 - a_3^2(3)} = \frac{\frac{13}{24} - \frac{1}{3} \cdot \frac{5}{8}}{1 - \left(\frac{1}{3}\right)^2}$$

$$a_2(1) = \frac{\frac{13-5}{24}}{\frac{9-1}{9}} = \frac{\frac{\cancel{8}}{24}}{\frac{\cancel{8}}{9}}$$

$$a_2(1) = \frac{3}{8}$$

let $m=2,\ k=1$

$$a_{2-1}(1) = \frac{a_2(1) - a_2(2)\ a_2(2-1)}{1 - a_2^2(2)}$$

$$= \frac{\frac{3}{8} - \frac{1}{2}\cdot\frac{3}{8}}{1-\frac{1}{4}}$$

$$= \frac{{}_2\frac{\cancel{3}}{\cancel{8}}\left(1-\frac{1}{2}\right)}{\frac{\cancel{3}}{4}}$$

$$= \frac{1}{2}\cdot\frac{1}{2}$$

$$a_1(1) = \frac{1}{4}$$

$$K_1 = \frac{1}{4}\ ,\quad K_2 = \frac{1}{2}\ ,\quad K_3 = \frac{1}{3}$$

from $B_M(z) = 1 + 2z^{-1} + 2z^{-2} + z^{-3}$

$b_0 = 1 \quad b_1 = 2 \quad b_2 = 2 \quad b_3 = 1$

Ladder co-efficients

$$c_m = b_m - \sum_{i=m+1}^{M} c_i\ a_i(i-m)$$

where $m = M, M-1, \ldots 0$

$m = M = 3$

$$c_3 = b_3 - \underbrace{\sum_{i=4}^{3} c_i\ a_i(i-3)}_{0}$$

$$c_3 = b_3$$

$$c_3 = 1$$

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$$c_2 = b_2 - \sum_{i=3}^{3} c_i a_i (i-2)$$

$$= b_2 - c_3 a_3 (3-2)$$

$$= b_2 - c_3 a_3 (1)$$

$$= 2 - 1 \cdot \frac{13}{24}$$

$$= 2 - 0.541$$

$$c_2 = 1.45$$

$$c_1 = b_1 - \sum_{i=2}^{3} c_i a_i (i-1)$$

$$= b_1 - c_2 a_2 (1) - c_3 a_3 (2)$$

$$= 2 - 1.45 \left(\frac{3}{8}\right) - 1 \left(\frac{5}{8}\right)$$

$$= 2 - 0.54375 - 0.625$$

$$c_1 = 0.83$$

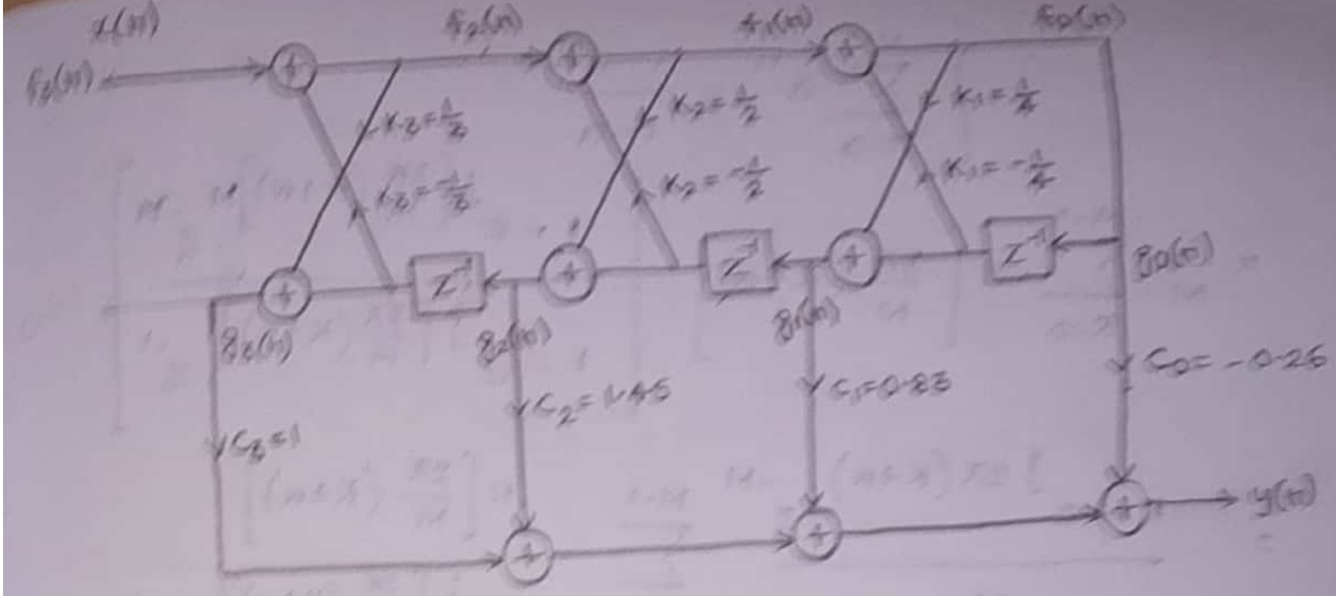
$$c_0 = b_0 - \sum_{i=1}^{3} c_i a_i (i)$$

$$= b_0 - c_1 a_1 (1) - c_2 a_2 (2) - c_3 a_3 (3)$$

$$= 1 - 0.83 \left(\frac{1}{4}\right) - 1.45 \left(\frac{1}{2}\right) - 1 \left(\frac{1}{3}\right).$$

$$= 1 - 0.2075 - 0.725 - 0.33$$

$$c_0 = -0.26$$



30-03-19

FREQUENCY SAMPLING STRUCTURE:

To desire frequency sampling structure. So that the specify the desired frequency response

$$w_k = \frac{2\pi}{M}(k+\alpha) \qquad \text{where } k = 0, 1, \text{-----} M-1$$

$$\alpha = 0 \text{ or } \frac{1}{2}$$

$$H(\omega) = \sum_{n=0}^{M-1} h(n)\, e^{-j\omega n}$$

$$H\left[\frac{2\pi}{M}(k+\alpha)\right] = \sum_{n=0}^{M-1} h(n)\, e^{-j\left[\frac{2\pi}{M}(k+\alpha)\right]n}$$

$$h(n) = \frac{1}{M}\sum_{k=0}^{M-1} H\left[\frac{2\pi}{M}(k+\alpha)\right] e^{j\left[\frac{2\pi}{M}(k+\alpha)\right]n}$$

$$\boxed{H(z) = \sum_{n=0}^{M-1} h(n)\, z^{-n}}$$

$$H(z) = \sum_{n=0}^{M-1} \frac{1}{M}\sum_{k=0}^{M-1} H\left[\frac{2\pi}{M}(k+\alpha)\right] e^{j\left[\frac{2\pi}{M}(k+\alpha)\right]n} \cdot z^{-n}$$

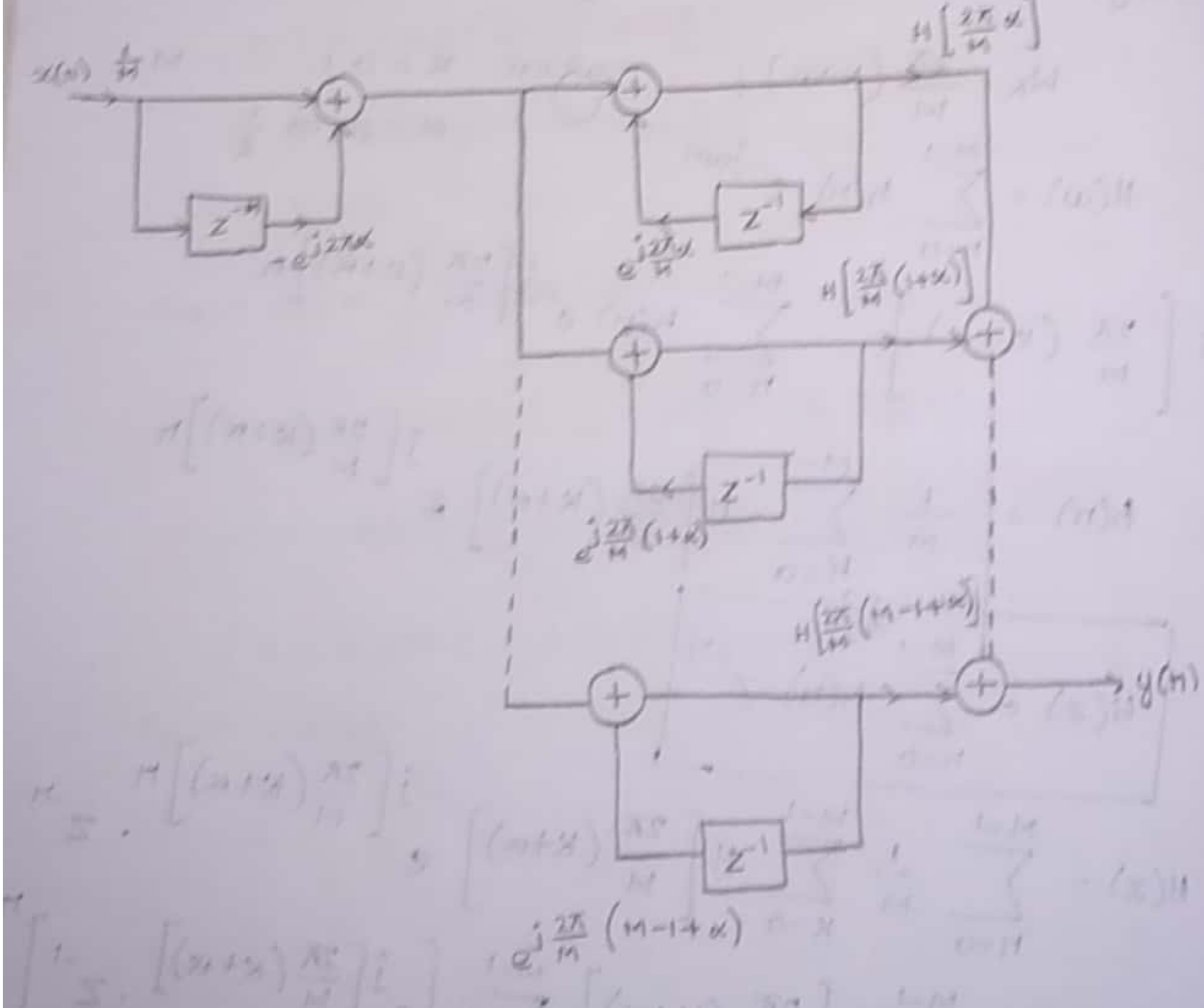
$$= \frac{1}{M}\sum_{k=0}^{M-1} H\left[\frac{2\pi}{M}(k+\alpha)\right] \sum_{n=0}^{M-1}\left[e^{j\left[\frac{2\pi}{M}(k+\alpha)\right]} \cdot z^{-1}\right]^{n}$$

$$\left\{ \sum_{n=0}^{M-1} a^n = \frac{1-a^M}{1-a} \right.$$

$$= \frac{1}{M} \sum_{k=0}^{M-1} H\left[\frac{2\pi}{M}(k+\alpha)\right] \left[\frac{1-e^{j\left[\frac{2\pi}{M}(k+\alpha)\right]M} . z^{-M}}{1-e^{j\left[\frac{2\pi}{M}(k+\alpha)\right]} . z^{-1}} \right]$$

$$= \frac{1-e^{j2\pi(k+\alpha)} . z^{-M}}{M} \sum_{k=0}^{M-1} \frac{H\left[\frac{2\pi}{M}(k+\alpha)\right]}{1-e^{j\left[\frac{2\pi}{M}(k+\alpha)\right]} . z^{-1}}$$

$$= \frac{1}{M}\left[1-e^{j2\pi\alpha} . z^{-M}\right] \sum_{k=0}^{M-1} \frac{H\left[\frac{2\pi}{M}(k+\alpha)\right]}{1-e^{j\left[\frac{2\pi}{M}(k+\alpha)\right]} . z^{-1}}$$



-04-19

UNIT - IV

DESIGN OF DIGITAL FILTERS

ANALOG LOWPASS BUTTERWORTH FILTER:

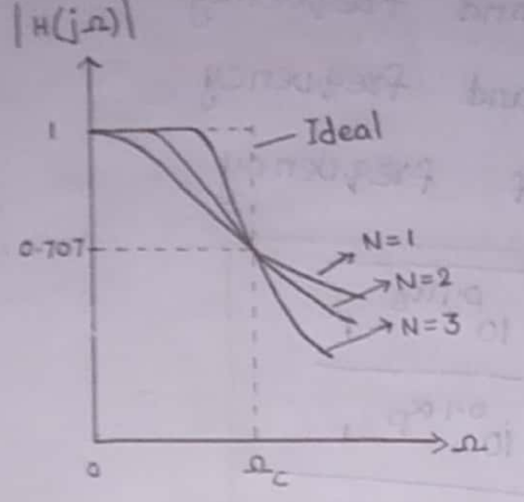


fig: Frequency response of Analog low pass Butterworth filter

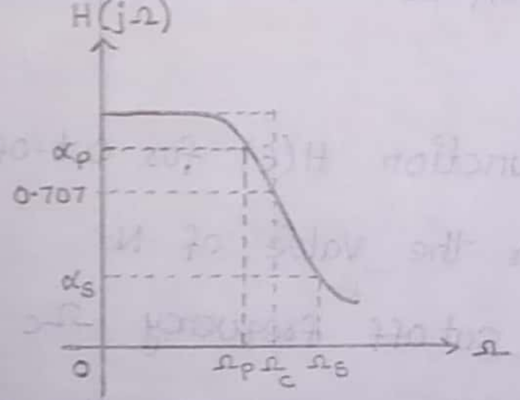
poles of H(S)

$$\boxed{P_K = \pm \Omega_c \, e^{\frac{j(2k+N+1)\pi}{2N}}}$$

K = 0, 1, ------ N-1

where N is filter order

ORDER OF THE FILTER:



Pass band attenuation, $\alpha_p = \frac{1}{\sqrt{1+\varepsilon^2}}$

stop band attenuation, $\alpha_s = \frac{1}{\sqrt{1+\lambda^2}}$

Here $\varepsilon \rightarrow$ parameter Specifying allowable Pass band

$\lambda \rightarrow$ parameter specifying allowable Stop band

$\Omega_p \rightarrow$ pass band frequency

$\Omega_s \rightarrow$ stop band frequency

$\Omega_c \rightarrow$ cut-off frequency

$$\boxed{N \geq \frac{\log\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\left(\dfrac{\Omega_s}{\Omega_p}\right)}}$$

$$\boxed{\Omega_c = \frac{\Omega_p}{\left(10^{0.1\alpha_p}-1\right)^{\frac{1}{2N}}} = \frac{\Omega_s}{\left(10^{0.1\alpha_s}-1\right)^{\frac{1}{2N}}}}$$

STEPS TO DESIGN AN ANALOG LOW PASS BUTTERWORTH FILTER.

1. From the given Specifications, find the order of filter, N.
2. Find the Transfer function H(s) for cut-off frequen Ω_c ($\Omega_c = 1$ rad/sec) for the value of N.
3. Calculate the value of cutoff frequency Ω_c.
4. Find the transfer function Ha(s) for the above value of cutoff frequency (Ω_c) by substituting $s \rightarrow \frac{s}{\Omega_c}$ in H(s)

1. Design an analog low pass butterworth filter that has 2 dB pass band attenuation at a frequency of 20 rad/sec and atleast 10 dB stop band attenuation at a frequency of 30 rad/sec

1. order of the filter $N \geq \dfrac{\log\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\left(\dfrac{\Omega_s}{\Omega_p}\right)}$

$\alpha_p = 2\text{dB}$

$\alpha_s = 10\text{ dB}$

$\Omega_p = 20\text{ rad/sec}$

$\Omega_s = 30\text{ rad/sec}$

$$N \geq \frac{\log\sqrt{\dfrac{10^{0.1(10\text{dB})}-1}{10^{0.1(2\text{ dB})}-1}}}{\log\left(\dfrac{30}{20}\right)}$$

$N \geq 3.37$

$N = 4$

2. Poles of $H(s)$

$$P_K = \pm\,\Omega_c\, e^{\frac{j(2K+N+1)\pi}{2N}}$$

$K = 0, 1, 2, \text{----} N-1$

$K = 0, 1, 2, \text{---} 4-1$

$K = 0, 1, 2, 3$

$\Omega_c = 1$ rad/sec

$K=0$

$$P_0 = \pm\, 1 \cdot e^{\frac{j(0+4+1)\pi}{2\cdot 4}} = \pm\, e^{j\cdot\frac{5\pi}{8}}$$

$$P_0 = \pm(-0.38 + j\,0.92)$$

$$K=1 \Longrightarrow P_1 = \pm 1 \cdot e^{\frac{j(2\cdot 1+4+1)\pi}{2\cdot 4}} = \pm\, e^{j\frac{7\pi}{8}}$$

$$P_1 = \pm(-0.92 + j\,0.38)$$

$$K=2 \Longrightarrow P_2 = \pm 1 \, e^{\frac{j(2.2+4+1)\pi}{2.4}} = \pm e^{j.\frac{9\pi}{8}}$$

$$P_2 = \pm\left(-0.92 - j\,0.38\right)$$

$$K=3 \Longrightarrow P_3 = \pm 1 \cdot e^{\frac{j(2.3+4+1)\pi}{2.4}} = \pm e^{j\frac{11\pi}{8}}$$

$$P_3 = \pm\left(-0.38 - 0.92j\right)$$

For filter stability, we take left half of S-plane of poles.

$$H(S) = \frac{1}{(S-P_0)\,(S-P_1)\,(S-P_2)\,(S-P_3)}$$

$$= \frac{1}{(S+0.38-j\,0.92)\,(S+0.92-j\,0.38)\,(S+0.92+j0.3\;\;(S+0.38+j\,0.92)}$$

$s^2+0.76S+0.1444$
$+0.8464$
$s^2+0.76S+1$

$s^2+1.84S+0.8464+0.1444$
$s^2+1.84S+1$

$$H(S) = \frac{1}{(s^2+0.76S+1)\,(s^2+1.84S+1)}$$

3. cut-off frequency

$$\Omega_c = \frac{\Omega p}{\left(10^{0.1\alpha p}-1\right)^{\frac{1}{2N}}} = \frac{20}{\left(10^{0.1\times 2}-1\right)^{\frac{1}{2.4}}}$$

$$= \frac{20}{0.9351}$$

$$\Omega_c = 21.38 \text{ rad/sec}$$

4. Transfer function $H_a(s)$

$$s \longrightarrow \frac{s}{\Omega_c}$$

$$H_a(s) = H(s)\Big]_{s \to \frac{s}{\Omega_c}}$$

$$H_a(s) = \frac{1}{\left[\left(\frac{s}{\Omega_c}\right)^2 + 0.76\frac{s}{\Omega_c} + 1\right]\left[\left(\frac{s}{\Omega_c}\right)^2 + 1.24\frac{s}{\Omega_c} + 1\right]}$$

$$H_a(s) = \frac{1}{\left[\frac{s^2}{457} + 0.76\frac{s}{21.38} + 1\right]\left[\frac{s^2}{457} + 1.24\frac{s}{21.38} + 1\right]}$$

$$H_a(s) = \frac{457}{(s^2 + 16.24s + 457)(s^2 + 29.33s + 457)}$$

2) Determine the specifications of $\alpha_p = 1dB$, $\alpha_s = 30dB$, $\Omega_p = 200$ rad/sec, $\Omega_s = 600$ rad/sec. Determine the order of the filter and design analog low pass butterworth filter.

1. Order of the filter $N \geq \dfrac{\log\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\left(\dfrac{\Omega_s}{\Omega_p}\right)}$

$$N \geq \frac{\log\sqrt{\dfrac{10^{0.1(30)}-1}{10^{0.1(1)}-1}}}{\log\left(\dfrac{600}{200}\right)}$$

$$N \geq 3.758 \longrightarrow N = 4$$

2. Poles of H(S)

$$P_K = \pm \Omega_c \, e^{\frac{j(2K+N+1)\pi}{2N}}$$

$K = 0, 1, 2, \text{----} N-1$

$K = 0, 1, 2, \text{----} 4-1$

$K = 0, 1, 2, 3$

$\Omega_c = 1$ rad/sec

$$K=0 \Longrightarrow P_0 = \pm 1 \cdot e^{\frac{j(0+4+1)\pi}{2 \cdot 4}} = \pm e^{j\frac{5\pi}{8}}$$

$$P_0 = \pm(-0{\cdot}38 + j\,0{\cdot}92)$$

$$K=1 \Longrightarrow P_1 = \pm 1 \cdot e^{\frac{j(2+4+1)\pi}{2 \cdot 4}} = \pm e^{j\frac{7\pi}{8}}$$

$$P_1 = \pm(-0{\cdot}92 + j\,0{\cdot}38)$$

$$K=2 \Longrightarrow P_2 = \pm 1 \cdot e^{\frac{j(2 \cdot 2+4+1)\pi}{2 \cdot 4}} = \pm e^{j\frac{9\pi}{8}}$$

$$P_2 = \pm(-0{\cdot}92 - j\,0{\cdot}38)$$

$$K=3 \Longrightarrow P_3 = \pm 1 \cdot e^{\frac{j(2 \cdot 3+4+1)\pi}{2 \cdot 4}} = \pm e^{j\frac{11\pi}{8}}$$

$$P_3 = \pm(-0{\cdot}38 - 0{\cdot}92j)$$

For filter stability, we take left half of S-plane of poles

$$H(s) = \frac{1}{(s-P_0)(s-P_1)(s-P_2)(s-P_3)}$$

$$= \frac{1}{(s+0{\cdot}38-j\,0{\cdot}92)(s+0{\cdot}92-j\,0{\cdot}38)(s+0{\cdot}92+j\,0{\cdot}3 \quad (s+0{\cdot}38+0{\cdot}92j)}$$

$$H(s) = \frac{1}{(s^2+0.76s+1)(s^2+1.84s+1)}$$

3> Cut-off frequency

$$\Omega_c = \frac{\Omega p}{\left(10^{0.1\alpha p}-1\right)^{\frac{1}{2N}}} = \frac{200}{\left(10^{0.1\times1}-1\right)^{\frac{1}{2.4}}}$$

$$\Omega_c = 236.8 \text{ rad/sec}$$

4> Transfer function $H_a(s)$

$$s \longrightarrow \frac{s}{\Omega_c}$$

$$H_a(s) = H(s)\Big]_{s \longrightarrow \frac{s}{\Omega_c}}$$

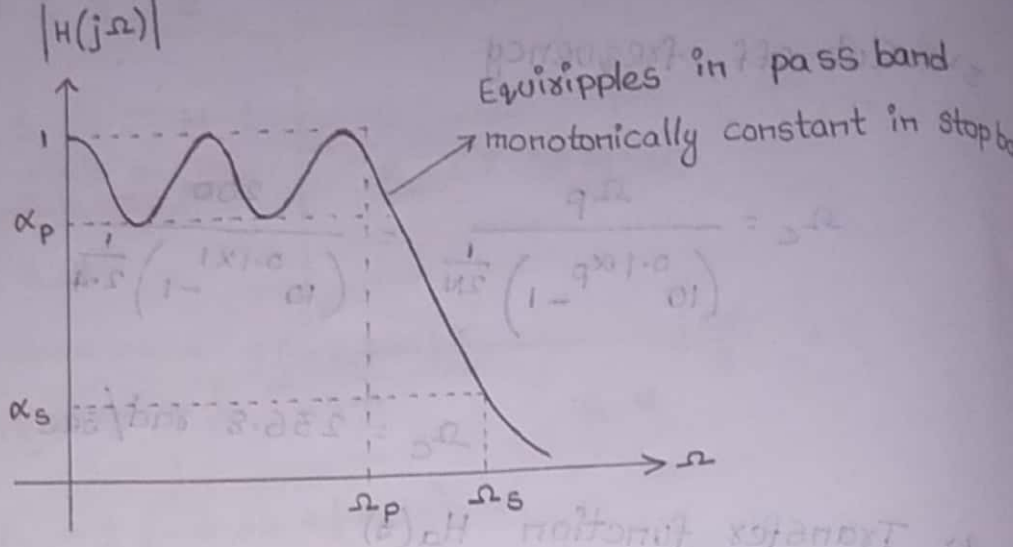
$$H_a(s) = \frac{1}{\left[\left(\frac{s}{\Omega_c}\right)^2 + 0.76\frac{s}{\Omega_c} + 1\right]\left[\left(\frac{s}{\Omega_c}\right)^2 + 1.84\frac{s}{\Omega_c} + 1\right]}$$

$$H_a(s) = \frac{1}{\left[\frac{s^2}{56075} + 0.76\frac{s}{236.8} + 1\right]\left[\frac{s^2}{56075} + 1.84\frac{s}{236.8} + 1\right]}$$

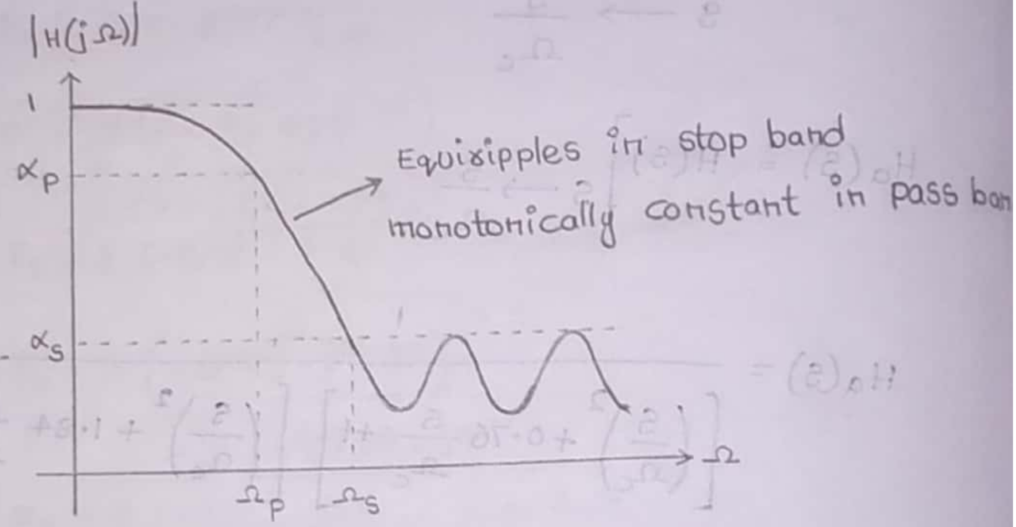
$$H_a(s) = \frac{56075}{(s^2+179.96s+56075)(s^2+435.71s+56075)}$$

22-04-19

ANALOG LOWPASS CHEBYSEV FILTER:



TYPE-1



TYPE-2

The specification of chebysev filter is given as

$$\alpha_p \leq |H(j\Omega)| \leq 1 \quad \text{for} \quad 0 \leq \Omega \leq \Omega_p$$

$$|H(j\Omega)| \leq \alpha_s \quad \text{for} \quad \Omega_s \leq \Omega$$

STEPS TO DESIGN AN ANALOG LOW PASS CHEBYSEV FILTER:

1. From the given specifications, find the order of filter, N.

$$N \geq \frac{\cos h^{-1}\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cos h^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$\alpha_p = \frac{1}{\sqrt{1+\varepsilon^2}} \qquad \alpha_s = \frac{1}{\sqrt{1+\lambda^2}}$$

2.

Poles $\boxed{P_K = \sigma_K + j\Omega_K}$

where $K = 0, 1, ---- N-1$

Here $\sigma_K = a \cos \phi_K$, $\Omega_K = b \sin \phi_K$

$$a = \Omega_p \left(\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right)$$

$$b = \Omega_p \left(\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right)$$

$$\boxed{\mu = \frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}} \qquad \boxed{\phi_K = \frac{(2K+N+1)\pi}{2N}}$$

3.

$$H_a(S) = \frac{K}{(S-P_0)(S-P_1)(S-P_2)----}$$

$$= \frac{K}{S^N + b_{N-1}S^{N-1} + ----- + b_1 S + b_0}$$

Here

$$K = \begin{cases} b_0 & \text{for } N = \text{odd} \\ \dfrac{b_0}{\sqrt{1+\varepsilon^2}} & \text{for } N = \text{even} \end{cases}$$

For Normalized chebysev filter, pass band frequency $\Omega_p = 20$ rad/sec, pass band attenuation $\alpha_p = 2.5$dB, stop band frequency $\Omega_s = 50$ rad/sec. stop band attenuation $\alpha_s = 30$dB. Design chebysev low pass filter.

$\Omega_p = 20$ rad/sec

$\alpha_p = 2.5$ dB

$\Omega_s = 50$ rad/sec

$\alpha_s = 30$ dB

1> Order of the filter

$$N \geq \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\left(\dfrac{\Omega_s}{\Omega_p}\right)}$$

$$\cosh^{-1}x = \ln\left(x + \sqrt{x^2+1}\right)$$

$$N \geq \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1(30)}-1}{10^{0.1(2.5)}-1}}}{\cosh^{-1}\left(\dfrac{50}{20}\right)}$$

$$\geq \frac{\cosh^{-1} 35.827}{\cosh^{-1} 2.5}$$

$$N \geq \frac{\ln\left(35.827 + \sqrt{35.827^2+1}\right)}{\ln\left(2.5 + \sqrt{2.5^2+1}\right)}$$

$$N \geq \frac{4.272}{1.647}$$

$$N \geq 2.593$$

$$N \cong 3$$

2) $P_K = \sigma_K + j\Omega_K$

where $K = 0, 1, --- N-1$

$K = 0, 1, --- 3-1$

$K = 0, 1, 2$

$K=0$

$P_0 = \sigma_0 + j\Omega_0$

$= a\cos\phi_0 + j\, b\sin\phi_0$

$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1}$

$= \sqrt{10^{0.1(2.5)} - 1}$

$\varepsilon = 0.88$

$$\mu = \frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}$$

$$= \frac{1+\sqrt{1+(0.88)^2}}{0.88}$$

$\mu = 2.65$

$\lambda = \sqrt{10^{0.1\alpha_s} - 1}$

$= \sqrt{10^{0.1(30)} - 1}$

$\lambda = 31.6$

$$a = \Omega_p\left(\frac{\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}}{2}\right) = 20\left(\frac{(2.65)^{\frac{1}{3}} - (2.65)^{-\frac{1}{3}}}{2}\right)$$

$a = 6.6$

$$b = \Omega_p\left(\frac{\mu^{\frac{1}{N}} + \mu^{-\frac{1}{N}}}{2}\right) = 20\left(\frac{(2\cdot65)^{\frac{1}{3}} + (2\cdot65)^{-\frac{1}{3}}}{2}\right)$$

$$b = 21\cdot06$$

$$\phi_K = \frac{(2K+N+1)\pi}{2N} \qquad K = 0, 1, 2$$

$$\phi_0 = \frac{(0+3+1)\pi}{2\cdot3} = \frac{4\pi}{2\cdot3} = \frac{2\pi}{3}$$

$$\phi_1 = \frac{(2+3+1)\pi}{2\cdot3} = \frac{6\pi}{6} = \pi$$

$$\phi_2 = \frac{(4+3+1)\pi}{2\cdot3} = \frac{8\pi}{2\cdot3} = \frac{4\pi}{3}$$

$K=0 \Longrightarrow$ $P_0 = \sigma_0 + j\Omega_0$

$$P_0 = a\cos\phi_0 + j\, b\sin\phi_0$$

$$= 6\cdot6\cos\left(\frac{2\pi}{3}\right) + j\ 21\cdot06\sin\left(\frac{2\pi}{3}\right)$$

$$= 6\cdot6(-0\cdot5) + j\ 21\cdot06\,(0\cdot866)$$

$$= -3\cdot3 + j\ 18\cdot23$$

$K=1 \longrightarrow$ $P_1 = \sigma_1 + j\Omega_1$

$$= a\cos\phi_1 + j\ b\sin\phi_1$$

$$= 6\cdot6\cos(\pi) + j\ 21\cdot06\ \sin(\pi)$$

$$= 6\cdot6(-1) + j\ 21\cdot06\,(0).$$

$$P_1 = -6\cdot6$$

$K=2 \Longrightarrow$ $P_2 = \sigma_2 + j\Omega_2$

$$= a\cos\phi_2 + j\ b\sin\phi_2$$

$$= 6.6 \cos\left(\frac{4\pi}{3}\right) + j\, 21.06 \sin\left(\frac{4\pi}{3}\right)$$

$$= 6.6(-0.5) + j\, 21.06(-0.866)$$

$$P_2 = -3.3 - j\, 18.23$$

iii) Transfer function

$$H_a(s) = \frac{K}{(s-P_0)(s-P_1)(s-P_2)}$$

$$= \frac{K}{(\underbrace{s+3.3}-j\,18.23)(s+6.6)(\underbrace{s+3.3}+j\,18.23)}$$

$$= \frac{K}{(s+6.6)\left((s+3.3)^2-(j\,18.23)^2\right)}$$

$$= \frac{K}{(s+6.6)(s^2+6.6s+10.89+332.3329)}$$

$$= \frac{K}{(s+6.6)(s^2+6.6s+343.2229)}$$

$$= \frac{K}{s^3+6.6s^2+343.2229s+6.6s^2+43.56s + 2265.27114}$$

$$H_a(s) = \frac{K}{s^3+13.2s^2+386.7829s+2265.27114}$$

As $N = \text{odd}$ $(N=3)$ then $K = b_0$

$$K = 2265.27114$$

$$H_a(s) = \frac{2265.27114}{s^3+13.2s^2+386.7829s+2265.27114}.$$

For given specifications $\frac{1}{\sqrt{2}} \leq |H(j\Omega)| \leq 1 \; ; \; 0 \leq \Omega \leq 2$ and $|H(j\Omega)| \leq 0.1 \; ; \; \overset{rads/sec}{4} \leq \Omega$. Determine filter order and design chebysev filter.

$\alpha_p = \frac{1}{\sqrt{2}}$ $\qquad$ $\alpha_s = 0.1$

$\Omega_p = 2$ rad/sec $\qquad$ $\Omega_s = 4$ rad/sec

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1}$$

$$\varepsilon^2 + 1 = 10^{0.1\alpha_p}$$

$$0.1\alpha_p = \log(\varepsilon^2+1)$$

$$\alpha_p = 10 \log(\varepsilon^2+1)$$

$$\alpha_p = -10 \log\left(\frac{1}{\varepsilon^2+1}\right)$$

$$\alpha_p = -20 \log\left(\frac{1}{\sqrt{\varepsilon^2+1}}\right)$$

$\alpha_p = -20 \log\left(\frac{1}{\sqrt{2}}\right)$ $\alpha_s = -20 \log(0.1)$

$\alpha_p = 3.01 \text{ dB}$ $\alpha_s = 20$

1) Order of the filter

$$N \geq \frac{\cosh^{-1}\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$\cosh^{-1}x = \ln\left(x + \sqrt{x^2+1}\right)$

$$N \geq \frac{\cosh^{-1}\sqrt{\frac{10^{0.1(20)}-1}{10^{0.1(3.01)}-1}}}{\cosh^{-1}\left(\frac{4}{2}\right)}$$

$$N \geq \frac{\cosh^{-1}(9.95)}{\cosh^{-1} 2}$$

$$N \geq \frac{\ln\left(9.95 + \sqrt{9.95^2+1}\right)}{\ln\left(2 + \sqrt{2^2+1}\right)}$$

$$N \geq \frac{2.9932}{1.4436}$$

$N \geq 2.0734$

$N \cong 3$

2) POLES

$$\alpha p = \frac{1}{\sqrt{1+\varepsilon^2}} = \frac{1}{\sqrt{2}}$$

$1+\varepsilon^2 = 2$

$\varepsilon^2 = 1$

$\varepsilon = 1$

$$\mu = \frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon} = \frac{1+\sqrt{1+1}}{1} = 1+\sqrt{2}$$

$$= 1+1.414$$

$$\mu = 2.414$$

$$a = \Omega_p\left(\frac{\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}}{2}\right) = 2\left(\frac{(2.414)^{\frac{1}{3}} - (2.414)^{-\frac{1}{3}}}{2}\right)$$

$$a = 0.596$$

$$b = \Omega_p\left(\frac{\mu^{\frac{1}{N}} + \mu^{-\frac{1}{N}}}{2}\right) = 2\left(\frac{(2.414)^{\frac{1}{3}} + (2.414)^{-\frac{1}{3}}}{2}\right)$$

$$b = 2.0869$$

$$\phi_K = \frac{(2K+N+1)\pi}{2N}$$

$K = 0, 1, \dots N-1$

$K = 0, 1, \dots 3-1$

$K = 0, 1, 2$

$$K=0 \implies \phi_0 = \frac{(0+3+1)\pi}{2 \cdot 3} = \frac{4\pi}{2 \cdot 3} = \frac{2\pi}{3}$$

$$K=1 \implies \phi_1 = \frac{(2+3+1)\pi}{2 \cdot 3} = \frac{6\pi}{6} = \pi$$

$$K=2 \implies \phi_2 = \frac{(4+3+1)\pi}{2 \cdot 3} = \frac{8\pi}{2 \cdot 3} = \frac{4\pi}{3}$$

$$K=0 \implies P_0 = \sigma_0 + j\Omega_0$$

$$P_0 = a\cos\phi_0 + j\, b\sin\phi_0$$

$$P_0 = 0.596\cos\left(\frac{2\pi}{3}\right) + j\ 2.0869\sin\left(\frac{2\pi}{3}\right)$$

$P_0 = 0.596\ (-0.5) + j\ 2.0869\ (0.866)$

$P_0 = -0.298 + j\ 1.8072$

$K=1 \Longrightarrow P_1 = \sigma_1 + j\Omega_1$

$P_1 = a\cos\phi_1 + j\ b\sin\phi_1$

$P_1 = 0.596\ \cos\pi + j\ 2.0869\ \sin\pi$

$P_1 = 0.596\ (-1) + j\ 2.0869\ (0)$

$P_1 = -0.596$

$K=2 \Longrightarrow P_2 = \sigma_2 + j\Omega_2$

$P_2 = a\cos\phi_2 + j\ b\sin\phi_2$

$P_2 = 0.596\ \cos\left(\frac{4\pi}{3}\right) + j\ 2.0869\ \sin\left(\frac{4\pi}{3}\right)$

$P_2 = 0.596\ (-0.5) + j\ (2.0869)\ (-0.866)$

$P_2 = -0.298 - j\ 1.8072$

3> Transfer function

$$H_a(s) = \frac{K}{(s-P_0)\ (s-P_1)\ (s-P_2)}$$

$$= \frac{K}{(\underbrace{s+0.298}_{}-j\ 1.8072)\ (s+0.596)\ (\underbrace{s+0.298}_{}+j\ 1.8072)}$$

$$H_a(s) = \frac{K}{(s+0.596)\ (s^2+0.596s+0.0888+3.2659)}$$

$$H_a(s) = \frac{K}{(s+0.596)\ (s^2+0.596s+3.3547}$$

$$H_a(s) = \frac{K}{s^3 + 0.596s^2 + 3.3547s + 0.596s^2 + 0.3552s + 1\cdot}$$

$$H_a(s) = \frac{K}{s^3 + 1.192s^2 + 3.7099s + 1.9994}$$

For the given specifications of $\alpha_p = 3\text{dB}$, $\alpha_s = 16\text{dB}$, p = 1KHZ, $f_s = 2\text{KHZ}$. Determine the filter order and esign analog lowpass chebyshev filter.

$$\alpha_p = 3\text{ dB}$$

$$\Omega_p = 2\pi \times f_p = 2\pi \cdot 1000 = 2000\pi \text{ rad/sec}$$

$$\alpha_s = 16\text{ dB}$$

$$\Omega_s = 2\pi \times f_s = 2\pi \cdot 2000 = 4000\pi \text{ rad/sec}$$

› Order of the filter

$$N \geq \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\left(\dfrac{\Omega_s}{\Omega_p}\right)} \qquad \cosh^{-1} x = \ln(x + \sqrt{x^2+}$$

$$N \geq \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1(16)}-1}{10^{0.1(3)}-1}}}{\cosh^{-1}\left(\dfrac{4000\pi}{2000\pi}\right)}$$

$$N \geq \frac{\cosh^{-1} 6.2446}{\cosh^{-1} 2}$$

As N is odd (N=3)
then $K = b_0$
$\rightarrow K = 1.9994$

$$H_a(s) = \frac{1.9994}{s^3 + 1.192s^2 + 3.7099s + 1.9994}$$

$$N \geq \frac{\ln\left(6.2446 + \sqrt{6.2446^2 + 1}\right)}{\ln\left(2 + \sqrt{2^2 + 1}\right)}$$

$$N \geq \frac{2.5312}{1.4436}$$

$$N \geq 1.7533$$

$$N \cong 2$$

2) POLES:

~~$\alpha_p = \frac{1}{\sqrt{1+\varepsilon^2}} = 3$~~

~~$\sqrt{1+\varepsilon^2} = \frac{1}{3}$~~

~~$1+\varepsilon^2 = \frac{1}{9} \Longrightarrow \varepsilon^2 = \frac{1}{9} - 1$~~

~~$\varepsilon^2 =$~~

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1}$$

$$= \sqrt{10^{0.1(3)} - 1}$$

$$\varepsilon = 0.9976$$

$$\mu = \frac{1 + \sqrt{1+\varepsilon^2}}{\varepsilon} = \frac{1 + \sqrt{1 + 0.9976^2}}{0.9976} = 2.418$$

$$a = \Omega_p \left(\frac{\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}}{2} \right) = 2000\pi \left(\frac{2.418^{\frac{1}{2}} - 2.418^{-\frac{1}{2}}}{2} \right)$$

$$= 1000\pi \left(0.9119\right)$$

$$a = 911.9\pi$$

$$b = \Omega_p \left(\frac{\mu^{\frac{1}{N}} + \mu^{-\frac{1}{N}}}{2} \right) = 2000\pi \left(\frac{2.418^{\frac{1}{2}} + 2.418^{-\frac{1}{2}}}{2} \right)$$

$$= 1000\pi \left(2.198\right)$$

$$b = 2198\pi$$

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$$\phi_K = \frac{(2K+N+1)\pi}{2N} \qquad K = 0, 1, --- N-1$$

$= 0, 1, --- 2-1$

$K = 0, 1$

$$K=0 \Rightarrow \phi_0 = \frac{(0+2+1)\pi}{2 \cdot 2} = \frac{3\pi}{4}$$

$$K=1 \Rightarrow \phi_1 = \frac{(2+2+1)\pi}{2 \cdot 2} = \frac{5\pi}{4}$$

$K=0 \longrightarrow P_0 = \sigma_0 + j\Omega_0$

$= a \cos\phi_0 + j\, b \sin\phi_0$

$= 911.9\pi \cos\left(\frac{3\pi}{4}\right) + j\; 2198\pi \sin\left(\frac{3\pi}{4}\right)$

$= 911.9\pi\,(-0.7071) + j\; 2198\pi\,(0.7071)$

$P_0 = -644.804\pi + j\; 1554.205\pi$

$K=1 \longrightarrow P_1 = \sigma_\phi + j\Omega_1$

$= a \cos\phi_1 + j\, b \sin\phi_1$

$= 911.9\pi \cos\left(\frac{5\pi}{4}\right) + j\; 2198\pi \sin\left(\frac{5\pi}{4}\right)$

$= 911.9\pi\,(-0.7071) + j\; 2198\pi\,(-0.7071)$

$P_1 = -644.804\pi - j\; 1554.205\pi$

3> Transfer function

$$H_a(S) = \frac{K}{(S-P_0)\,(S-P_1)}$$

$$= \frac{K}{(S + 644.804\pi - j\,1554.205\pi)\,(S + 644.804\pi + j\,1554.205\pi)}$$

$$= \frac{K}{s^2 + 415772.1984\pi^2 + 1289.608\pi s + 2415553.182\pi^2}$$

$$H_a(S) = \frac{K}{s^2 + 1289.608\pi s + 2831325.38\pi^2}$$

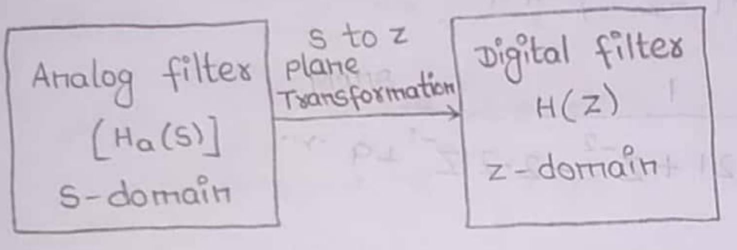
AS $N =$ even $(N=2)$ then $K = \frac{b_0}{\sqrt{1+\varepsilon^2}} = \frac{2831325.38\pi^2}{\sqrt{1+0.9976^2}}$

$K = 2004453.275\pi^2$

$$H_a(S) = \frac{2004453.275\pi^2}{s^2 + 1289.608\pi S + 2831325.38\pi^2}$$

24-04-19

DESIGN OF IIR FILTER FROM ANALOG FILTER :



1. Approximation derivate method or Backward difference method.
2. Impulse Invariance
3. Bilinear Transformation

BACKWARD DIFFERENCE METHOD :-

$$S = \frac{1 - Z^{-1}}{T} \quad \left\{ T = 1 \text{ sec} \right.$$

Use the backward difference method, convert analog filter to digital filter. The system function $H(S) = \frac{1}{S+2}$

$$H(S) = \frac{1}{S+2} = \frac{1}{\frac{1-Z^{-1}}{T} + 2} = \frac{1}{\frac{1-Z^{-1}}{1} + 2}$$

$$H(S) = \frac{1}{3 - Z^{-1}}$$

Use the backward difference method, convert analog filter to digital filter. The system function $H(s) = \frac{1}{(s+0.1)^2 +}$

$$H(S) = \frac{1}{(S+0.1)^2+9}$$

$$H(Z) = \frac{1}{\left(\frac{1-Z^{-1}}{T}+0.1\right)^2+9}$$

$$= \frac{1}{(1-Z^{-1}+0.1)^2+9}$$

$$= \frac{1}{(1.1-Z^{-1})^2+9}$$

$$= \frac{1}{1.21+Z^{-2}-2.2Z^{-1}+9}$$

$$H(Z) = \frac{1}{Z^{-2}-2.2Z^{-1}+10.21}$$

IMPULSE INVARINCE METHOD:

let $H_a(S)$ is the system function of analog fi

partial fraction. This can be expressed in partial fraction

$$H_a(S) = \sum_{K=1}^{N} \frac{C_K}{S-P_K}$$

where $P_K \longrightarrow$ poles of an analog filter

$C_K \longrightarrow$ Coefficients in the partial fraction

Inverse Laplace Transform

$$h_a(t) = \sum_{K=1}^{N} C_K e^{P_K t}$$

$h_a(t)$ periodically at $t = nT$

$$h(n) = h_a(nT)$$

$$h(n) = \sum_{K=1}^{N} C_K e^{P_K nT}$$

$$H(z) = \sum_{n=0}^{\infty} h(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} \sum_{K=1}^{N} C_K e^{P_K nT} z^{-n}$$

$$= \sum_{K=1}^{N} C_K \sum_{n=0}^{\infty} \left(e^{P_K T} z^{-1}\right)^{n}$$

$$H(z) = \sum_{K=1}^{N} \frac{C_K}{1 - e^{P_K T} \cdot z^{-1}}$$

STEPS TO DESIGN A DIGITAL FILTER

STEP 1 : For the given specifications, find $H_a(s)$

STEP 2 : Select the sampling rate of the digital filter T sec/samples.

STEP 3 : Express the analog filter transfer function

$$H_a(s) = \sum_{K=1}^{N} \frac{C_K}{s - P_K}$$

STEP 4 : Compute the z-transform of digital filter

$$H(z) = \sum_{K=1}^{N} \frac{C_K}{1 - e^{P_K T} \cdot z^{-1}}$$

For the analog Transfer function $H(s) = \frac{2}{(s+1)(s+2)}$ Determine $H(z)$ by using Impulse Invariance method assuming $T = 1$ sec

$$H(s) = \frac{2}{(s+1)(s+2)} \qquad T = 1\ \text{sec}$$

$$\frac{2}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$2 = A(s+2) + B(s+1)$$

$$s = -1 \implies 2 = A(-1+2) + B(-1+1)$$

$$2 = A(1) + B(0)$$

$$2 = A + 0$$

$$A = 2$$

$$s = -2 \implies 2 = A(-2+2) + B(-2+1)$$

$$2 = 0 + B(-1)$$

$$2 = -B$$

$$B = -2$$

$$H(s) = \frac{2}{s+1} - \frac{2}{s+2}$$

$$= \frac{2}{s-(-1)} - \frac{2}{s-(-2)}$$

$$\left\{ \begin{array}{l} c_1 = A = 2 \\ c_2 = B = -2 \end{array} \right.$$

There are two poles

$P_1 = -1 \qquad P_2 = -2$

$$H(z) = \sum_{K=1}^{N} \frac{c_K}{1 - e^{P_K T} . z^{-1}}$$

$$= \sum_{K=1}^{2} \frac{c_K}{1 - e^{P_K T} . z^{-1}}$$

$$= \frac{c_1}{1-e^{P_1 T}\cdot z^{-1}} + \frac{c_2}{1-e^{P_2 T}\cdot z^{-1}}$$

$$= \frac{2}{1-e^{-T}\cdot z^{-1}} + \frac{-2}{1-e^{-2T}\cdot z^{-1}}$$

$$= \frac{2}{1-e^{-1}\cdot z^{-1}} + \frac{-2}{1-e^{-2} z^{-1}}$$

$$= \frac{2}{1-0.3678 Z^{-1}} + \frac{-2}{1-0.1353 Z^{-1}}$$

$$= 2\left[\frac{1}{1-0.3678 Z^{-1}} - \frac{1}{1-0.1353 Z^{-1}}\right]$$

$$= 2\left[\frac{\cancel{1}-0.1353 Z^{-1}-\cancel{1}+0.3678 Z^{-1}}{1-0.1353 Z^{-1}-0.3678 Z^{-1}+0.0497 Z^{-2}}\right]$$

$$= 2\left[\frac{0.2325 Z^{-1}}{1-0.5031 Z^{-1}+0.0497 Z^{-2}}\right]$$

$$H(z) = \frac{0.465 Z^{-1}}{1-0.5031 Z^{-1}+0.0497 Z^{-2}}$$

For the analog Transfer function $H(s) = \frac{10}{S^2+7S+10}$

Determine H(z) by using Impulse Invariance method assuming T=0.2 sec.

$$H(s) = \frac{10}{s^2+7s+10} \qquad T = 0.2 \text{ sec}$$

$$\frac{10}{s^2+7s+10} = \frac{10}{s^2+5s+2s+10} = \frac{10}{s(s+5)+2(s+5)}$$

$$\frac{10}{s^2+7s+10} = \frac{10}{(s+2)(s+5)}$$

$$\frac{10}{s^2+7s+10} = \frac{C_1}{s+2} + \frac{C_2}{s+5}$$

$$10 = C_1(s+5) + C_2(s+2)$$

$$s = -2 \longrightarrow \quad 10 = C_1(-2+5) + C_2(-2+2)$$

$$10 = C_1 \cdot 3 + 0$$

$$C_1 = \frac{10}{3}$$

$$s = -5 \longrightarrow \quad 10 = C_1(-5+5) + C_2(-5+2)$$

$$10 = 0 + C_2 \cdot -3$$

$$C_2 = -\frac{10}{3}$$

$$H(s) = \frac{10/3}{s+2} + \frac{-10/3}{s+5}$$

$$H(s) = \frac{10/3}{s-(-2)} - \frac{10/3}{s-(-5)}$$

∴ There are two poles

$P_1 = -2 \qquad P_2 = -5$

$$H(z) = \sum_{k=1}^{N} \frac{C_k}{1-e^{P_kT}\cdot z^{-1}} = \sum_{k=1}^{2} \frac{C_k}{1-e^{P_kT}\cdot z^{-1}}$$

$$= \frac{C_1}{1-e^{P_1T}\cdot z^{-1}} + \frac{C_2}{1-e^{P_2T}\cdot z^{-1}}$$

$$= \frac{10/3}{1-e^{-2(0.2)}\cdot z^{-1}} + \frac{-10/3}{1-e^{-5(0.2)}\cdot z^{-1}}$$

$$= \frac{10/3}{1-e^{-0.4}z^{-1}} - \frac{10/3}{1-e^{-1}\cdot z^{-1}}$$

$$= \frac{10}{3}\left[\frac{1}{1-0.6703z^{-1}} - \frac{1}{1-0.3678z^{-1}}\right]$$

$$= \frac{10}{3}\left[\frac{1-0.3678z^{-1}-1+0.6703z^{-1}}{1-0.3678z^{-1}-0.6703z^{-1}+0.2465z^{-2}}\right]$$

$$= \frac{10}{3}\left[\frac{0.3025z^{-1}}{1-1.0381z^{-1}+0.2465z^{-2}}\right]$$

$$H(z) = \frac{3.025z^{-1}}{3-3.1143z^{-1}+0.7395z^{-2}}$$

DESIGN OF IIR FILTER USING BILINEAR TRANSFORMATIO

Let us consider a analog filter with system func

$$H(s) = \frac{b}{s+a} \longrightarrow ①$$

$$\frac{Y(s)}{X(s)} = \frac{b}{s+a}$$

$$s\,Y(s) + a\,Y(s) = b\,x(s)$$

This can be characterized by differential equation

$$\frac{d}{dt} y(t) + a\,y(t) = b\,x(t) \longrightarrow ②$$

y(t) can be treated by trapezoidal formula

$$y(t) = \int_{t_0}^{t} y'(\tau)\,d\tau + y(t_0)$$

where $y'(\tau)$ is the derivative of $y(t)$

The approximation of the integral by the trapezoid formula at $t = nT$ and $t_0 = nT - T$

$$\left\{ \int_a^b f(x)\,dx = \frac{b-a}{2}\left[f(b) + f(a)\right] \right.$$

$$y(nT) = \frac{T}{2}\left[y'(nT) + y'(nT-T)\right] + y(nT-T) \longrightarrow ③$$

from ②

$$y'(nT) = -a\,y(nT) + b\,x(nT) \longrightarrow ④$$

$$y'(nT-T) = -a\,y(nT-T) + b\,x(nT-T) \longrightarrow ⑤$$

sub eqn ④ and ⑤ in eqn ③

$$y(nT) = \frac{T}{2}\left[-a\,y(nT) + b\,x(nT) - a\,y(nT-T) + b\,x(nT-T)\right] + y(nT-T)$$

$$= \frac{-aT}{2}\,y(nT) + \frac{bT}{2}\,x(nT) - \frac{aT}{2}\,y(nT-T) + \frac{bT}{2}\,x(nT-T) + y(nT-T)$$

$$y(nT) + \frac{aT}{2}\,y(nT) + \frac{aT}{2}\,y(nT-T) - y(nT-T) = \frac{bT}{2}\left[x(nT) + x(nT-T)\right]$$

with $y(n) = y(nT)$ and $x(n) = x(nT)$

$$y(n) + \frac{aT}{2}\,y(n) + \frac{aT}{2}\,y(n-1) - y(n-1) = \frac{bT}{2}\left[x(n) + x(n-1)\right] \longrightarrow ⑥$$

Apply Z-Transform for eqn ⑥

$$Y(z) + \frac{aT}{2}\,Y(z) + \frac{aT}{2}z^{-1}Y(z) - z^{-1}Y(z) = \frac{bT}{2}\left[X(z) + z^{-1}X(z)\right]$$

$$Y(z)\left[1 + \frac{aT}{2} + \frac{aT}{2}z^{-1} - z^{-1}\right] = \frac{bT}{2}\left[1 + z^{-1}\right]X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{\frac{bT}{2}(1+z^{-1})}{1 + \frac{aT}{2} + \frac{aT}{2}z^{-1} - z^{-1}}$$

$$H(z) = \frac{\frac{bT}{2}(1+z^{-1})}{1 - z^{-1} + \frac{aT}{2}(1+z^{-1})}$$

$$H(z) = \frac{b}{\frac{1-z^{-1}}{\frac{T}{2}(1+z^{-1})} + \frac{\frac{aT}{2}\cancel{(1+z^{-1})}}{\frac{T}{2}\cancel{(1+z^{-1})}}}$$

$$H(z) = \frac{b}{\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+a} \longrightarrow ⑦$$

$$\boxed{S = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

The relationship between S and Z is known as Bilinear transformation.

Let $Z = re^{j\omega}$ and $S = \sigma + j\Omega$

$$S = \frac{2}{T}\left[\frac{1-Z^{-1}}{1+Z^{-1}}\right]$$

$$= \frac{2}{T}\left[\frac{Z-1}{Z+1}\right]$$

$$= \frac{2}{T}\left[\frac{re^{j\omega}-1}{re^{j\omega}+1}\right]$$

$$= \frac{2}{T}\left[\frac{r\cos\omega - 1 + jr\sin\omega}{r\cos\omega + 1 + jr\sin\omega}\right]\left[\frac{r\cos\omega + 1 - jr\sin}{r\cos\omega + 1 - jr\sin}\right.$$

$$= \frac{2}{T}\left[\frac{r^2\cos^2\omega + \cancel{r\cos\omega} - jr^2\cancel{\sin\omega\cos\omega} - \cancel{r\cos\omega} - 1 + jr\sin\omega + jr^2\cancel{\sin\omega\cos\omega} + jr\sin\omega + r^2\sin^2\omega}{r^2\cos^2\omega + 1 + 2r\cos\omega + r^2\sin^2\omega}\right.$$

$$= \frac{2}{T}\left[\frac{r^2(\cos^2\omega + \sin^2\omega) - 1 + 2jr\sin\omega}{r^2(\cos^2\omega + \sin^2\omega) + 1 + 2r\cos\omega}\right]$$

$$= \frac{2}{T}\left[\frac{r^2-1+2jr\sin\omega}{r^2+1+2r\cos\omega}\right]$$

$$S = \frac{2}{T}\left[\frac{r^2-1}{1+r^2+2r\cos\omega} + j\,\frac{2r\sin\omega}{1+r^2+2r\cos\omega}\right]$$

comparing with $S = \sigma + j\Omega$

$$\sigma = \frac{2}{T}\left[\frac{r^2-1}{1+r^2+2r\cos\omega}\right]$$

$$\Omega = \frac{2}{T}\left[\frac{2r\sin\omega}{1+r^2+2r\cos\omega}\right]$$

Filter stability

$r = 1$ $\quad\sigma = 0$

$$\Omega = \frac{2}{T}\left[\frac{2\cdot 1\sin\omega}{1+1+2\cos\omega}\right]$$

$$= \frac{2}{T}\left[\frac{2\sin\omega}{2+2\cos\omega}\right]$$

$$= \frac{2}{T}\left[\frac{\cancel{2}\sin\omega}{\cancel{2}(1+\cos\omega)}\right]$$

$$= \frac{2}{T}\left[\frac{\cancel{2}\sin\frac{\omega}{2}\cancel{\cos\frac{\omega}{2}}}{\cancel{2}\cos^{\cancel{2}}\frac{\omega}{2}}\right]$$

$$= \frac{2}{T}\left[\frac{\sin\frac{\omega}{2}}{\cos\frac{\omega}{2}}\right]$$

$$\boxed{\Omega = \frac{2}{T}\tan\frac{\omega}{2}} \longrightarrow ⑧$$

$$\tan\frac{\omega}{2} = \frac{\Omega T}{2}$$

$$\frac{\omega}{2} = \tan^{-1}\frac{\Omega T}{2}$$

$$\boxed{\omega = 2\tan^{-1}\frac{\Omega T}{2}} \longrightarrow ⑨$$

for small value of θ

$$\tan\theta = \theta$$

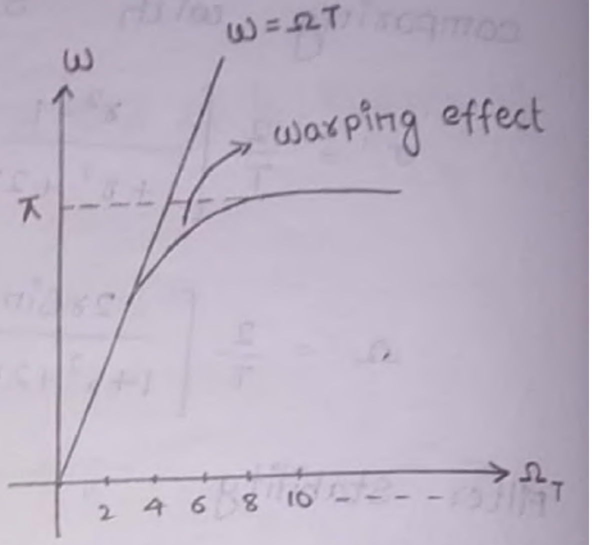
$\{\tan 0 = 0\}$

Eqn ⑧

$$\Omega = \frac{2}{T}\cdot\frac{\omega}{2}$$

$$\Omega = \frac{\omega}{T}$$

$$\boxed{\omega = \Omega T}$$



RELATIONSHIP BETWEEN Ω AND ω

STEPS

1. For the given Specifications, find analog filter $H_a(s$
2. Select the sampling rate of the digital filter T sec/Sa
3. Substitute $S = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$

Applying Bilinear transformation $H(s) = \frac{2}{(s+1)(s+2)}$

with T= 1 sec.

$$H(s) = \frac{2}{(s+1)(s+2)}$$

$$H(z) = \frac{2}{\left[\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+1\right]\left[\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+2\right]}$$

$$= \frac{2}{\left[2\left(\frac{z-1}{z+1}\right)+1\right]\left[2\left(\frac{z-1}{z+1}\right)+2\right]}$$

$$= \frac{2}{\frac{2z-2+z+1}{z+1} \cdot \frac{2z-2+2z+2}{z+1}}$$

$$= \frac{\cancel{2}(z+1)^2}{(3z-1)(\overset{}{\underset{2}{\cancel{4}}}z)}$$

$$= \frac{(z+1)^2}{2z(3z-1)}$$

$$= \frac{z^2+2z+1}{6z^2-2z}$$

$$= \frac{\cancel{z^2}\left(1+\frac{2}{z}+\frac{1}{z^2}\right)}{\cancel{z^2}\left(6-\frac{2}{z}\right)}$$

$$H(z) = \frac{1+2z^{-1}+z^{-2}}{6-2z^{-1}}$$

Applying Bilinear Transformation $H(s) = \frac{s^2+4.525}{s^2+0.692s+0.504}$ with T=1 sec.

$$H(S) = \frac{S^2+4.525}{S^2+0.692S+0.504}$$

$$H(z) = \frac{\left[\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + 4.525}{\left[\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + 0.692\left[\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right] + 0.504}$$

$$= \frac{\frac{4}{1} \cdot \frac{1-2z^{-1}+z^{-2}}{1+2z^{-1}+z^{-2}} + 4.525}{\frac{4}{1} \cdot \frac{1-2z^{-1}+z^{-2}}{1+2z^{-1}+z^{-2}} + 0.692\left(\frac{2}{1} \cdot \frac{1-z^{-1}}{1+z^{-1}}\right) + 0.504}$$

$$= \frac{\frac{4-8z^{-1}+4z^{-2}+4.525+9.05z^{-1}+4.525z^{-2}}{1+\cancel{2z^{-1}}+z^{-2}}}{\frac{4-8z^{-1}+4z^{-2}+0.692(2-2z^{-1})(1+z^{-1}) + 0.504+1.008z^{-1}+0.504z^{-2}}{1+\cancel{2z^{-1}}+z^{-2}}}$$

$$= \frac{8.525 + 1.05z^{-1} + 8.525z^{-2}}{4.504 - 6.992z^{-1} + 4.504z^{-2} + 0.692\left(2+\cancel{2z^{-1}}-\cancel{2z^{-1}} - 2z^{-2}\right)}$$

$$= \frac{8.525 + 1.05z^{-1} + 8.525z^{-2}}{4.504 - 6.992z^{-1} + 4.504z^{-2} + 1.384 - 1.384z^{-2}}$$

$$H(z) = \frac{8.525 + 1.05z^{-1} + 8.525z^{-2}}{5.888 - 6.992z^{-1} + 3.12z^{-2}}$$

1/4/19

UNIT - V

MULTIRATE SIGNAL PROCESSING

The systems that use single sampling rate from A/D converter to D/A converter are known as "single rate system". The Discrete time system that process the data at more than one sampling rate known as "Multi Rate System". There are many cases where multirate signal processing is used. They are

In Audio signal processing, for example, a CD (Compact Disc) is sampled at 44.1KHZ but DAT (Digital Audio Tape) is sampled at 48KHZ. Conversion between CD and DAT (Since studio recordings are mostly made on DAT) use Multirate signal processing techniques

To convert CD to DAT, we are using Interpolation (increasing the sampling rate).

To convert DAT to CD, we are using Decimation (Decreasing Sampling rate)

In video, PAL (Phase Alternative Live) and NTSC (National Television System Community) run at different sampling rates. Therefore to watch an American programme in Europe, one needs a sampling rate converter. In Transmultiplexers

Two basic operations in Multirange signal processing

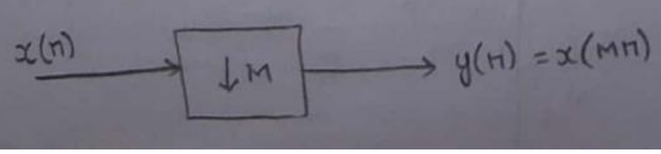
1> Decimation

2> Interpolation.

1> DECIMATION :

Decimation is nothing but decreasing Sampling rate

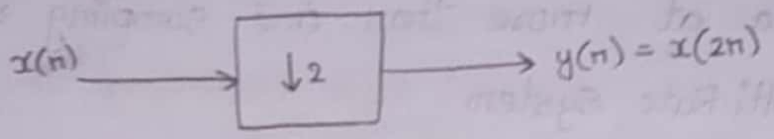
DOWN SAMPLING :



The Sampling rate of discrete time Signal $x(n)$ can be reduced by a factor M by taking every Mth value of Signal.

Here M = Down Sampling factor

Ex : 1> $x(n) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, M = 2



$n=0 \Longrightarrow y(0) = x(0) = 1$

$n=1 \Longrightarrow y(1) = x(2) = 3$

$n=2 \Longrightarrow y(2) = x(4) = 5$

$n=3 \longrightarrow y(3) = x(6) = 7$

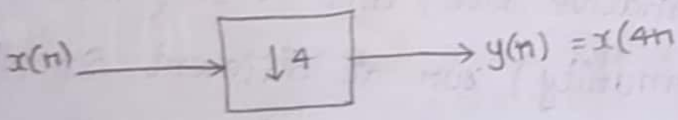
$n=4 \Longrightarrow y(4) = x(8) = 9$

That is, we left one sample in between samples of $x(n)$ to generate $y(n)$

$y(n) = x(2n) = \{1, 3, 5, 7, 9\}$

compare to $x(n)$ and $y(n)$, $(M-1)$ samples are discarded

2> $x(n) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, M = 4



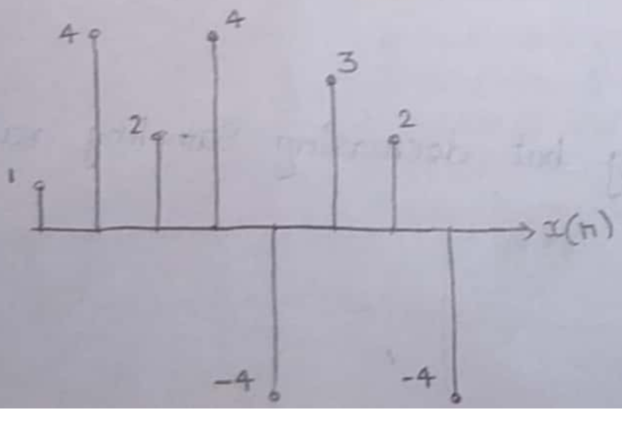
$n=0 \Longrightarrow y(0) = x(0) = 1$

$n=1 \Longrightarrow y(1) = x(4) = 5$

$n=2 \Longrightarrow y(2) = x(8) = 9$

$y(n) = x(4n) = \{1, 5, 9\}$

3>

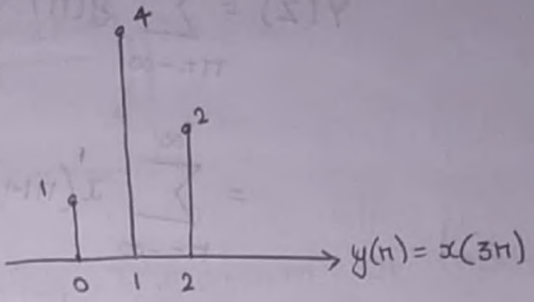


M = 3

$x(n) = \{1, 4, 2, 4, -4, 3, 2, -4\}$

As $(M-1)$ samples are to be discarded i.e., 2 samples.

then $y(n) = \{1, 4, 2\}$



$n=0 \Rightarrow y(0) = x(0) = 1$

$n=1 \Rightarrow y(1) = x(3) = 4$

$n=2 \Rightarrow y(2) = x(6) = 2$

02-04-19

SPECTRUM OF DOWN SAMPLED SIGNAL:

Z-Transform of input signal $x(n)$

$$X(Z) = \sum_{n=-\infty}^{\infty} x(n)\, Z^{-n} \longrightarrow ①$$

Down sampled signal $y(n)$ is obtained by multiplying sequence $x(n)$ with periodic train of impulses $p(n)$ with a period 'M' and then leaving out M-1 samples

$$P(n) = \begin{cases} 1 & n = 0, \pm M, \pm 2M \\ 0 & \text{otherwise} \end{cases}$$

$$x'(n) = x(n) \cdot P(n)$$

$$\text{that is } x'(n) = \begin{cases} x(n) & n = 0, \pm M, \pm 2M \\ 0 & \text{otherwise} \end{cases}$$

Leave M-1 samples we get

$$y(n) = x'(nM)$$

$$= x(nM) \cdot \underbrace{P(nM)}_{1}$$

$$\boxed{y(n) = x(nM)} \longrightarrow ②$$

Z-Transform of output signal y(n)

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n)\, z^{-n} \longrightarrow ③$$

$$= \sum_{n=-\infty}^{\infty} x'(nM) \cdot z^{-n}$$

let $nM = p \longrightarrow n = p/M$

$$= \sum_{p=-\infty}^{\infty} x'(p) \cdot z^{-p/M}$$

$$Y(z) = \sum_{n=-\infty}^{\infty} x'(n)\, z^{-n/M}$$

$$\boxed{Y(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot p(n) \cdot z^{-n/M}} \longrightarrow ④$$

Discrete fourier series representation of the signal P(n

$$P(n) = \frac{1}{M} \sum_{K=0}^{M-1} e^{j\frac{2\pi kn}{M}} \longrightarrow ⑤$$

Substitute eqn ⑤ in eqn ④

$$Y(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot \frac{1}{M} \sum_{K=0}^{M-1} e^{j\frac{2\pi kn}{M}} \cdot Z^{-n/M}$$

$$= \frac{1}{M} \sum_{K=0}^{M-1} \underbrace{\left[\sum_{K=-\infty}^{\infty} x(n) \cdot \left[e^{\frac{-j2\pi k}{M}} \cdot Z^{1/M} \right]^{-n} \right]}$$

$$= \frac{1}{M} \sum_{K=0}^{M-1} x\left(e^{\frac{-j2\pi k}{M}} \cdot Z^{\frac{1}{M}} \right)$$

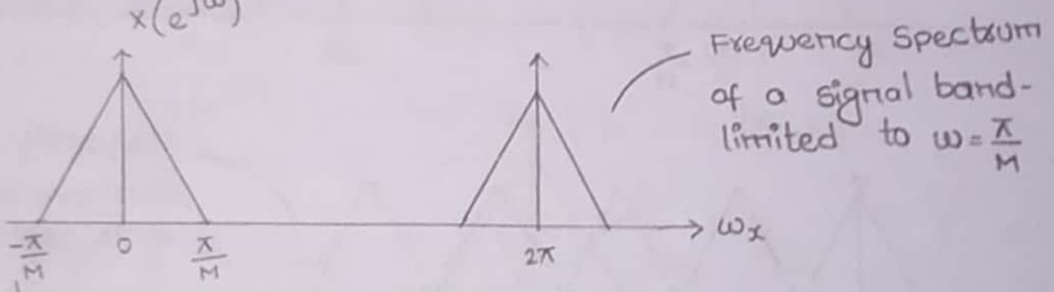
$$Y(e^{j\omega}) = \frac{1}{M}\sum_{K=0}^{M-1} X\left(e^{\frac{-j2\pi k}{M}} e^{\frac{j\omega}{M}}\right) \qquad \left\{ z = e^{j\omega} \right.$$

$$\boxed{Y(e^{j\omega}) = \frac{1}{M}\sum_{K=0}^{M-1} X\left(e^{\frac{j(\omega-2\pi k)}{M}}\right)}$$

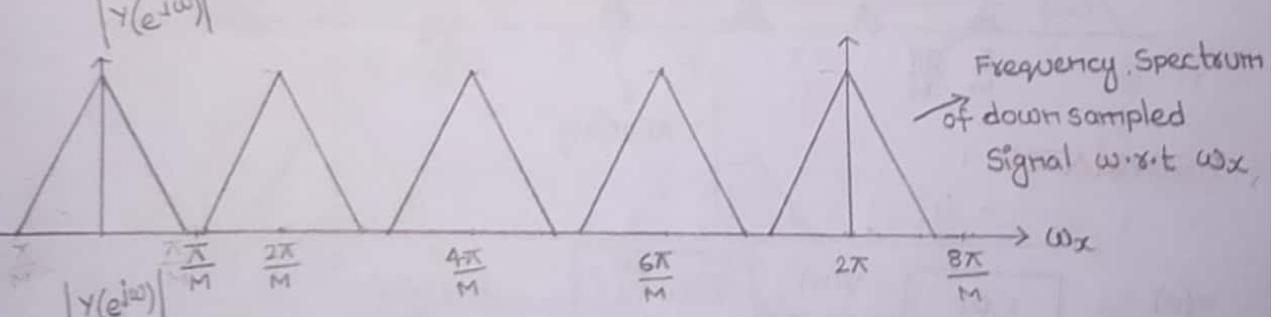
From above relation, Fourier transform of input signal $x(n)$ is $X(e^{j\omega})$ and Fourier transform of output signal $y(n)$ is $Y(e^{j\omega})$ where it is a sum of M uniformly shifted & streched versions of $X(e^{j\omega})$ and Scaled by a factor of $\frac{1}{M}$.

The spectrum of original signal $X(e^{j\omega})$ is band limited to $\omega = \pm\frac{\pi}{M}$

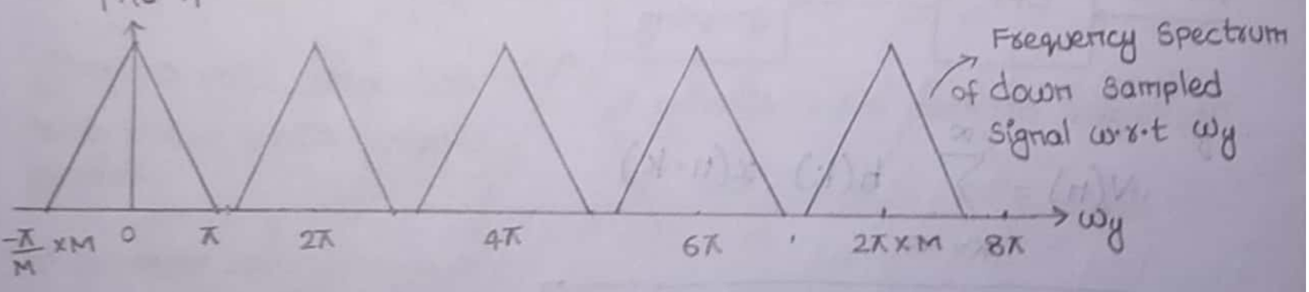
The Spectrum being periodic with period of 2π.



Frequency Spectrum of a signal band-limited to $\omega = \frac{\pi}{M}$



Frequency Spectrum of down sampled Signal w.r.t ω_x

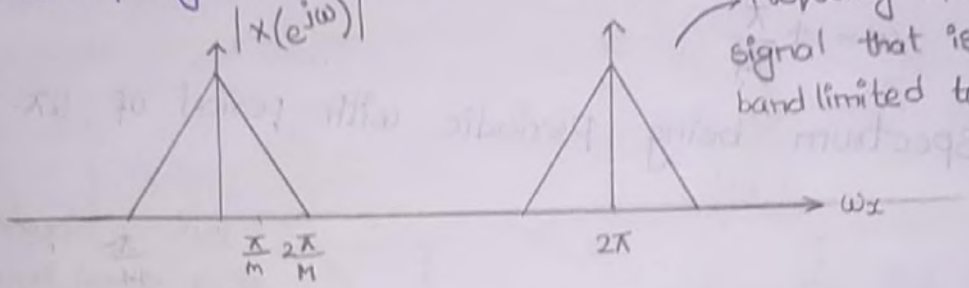


Frequency Spectrum of down sampled Signal w.r.t ω_y

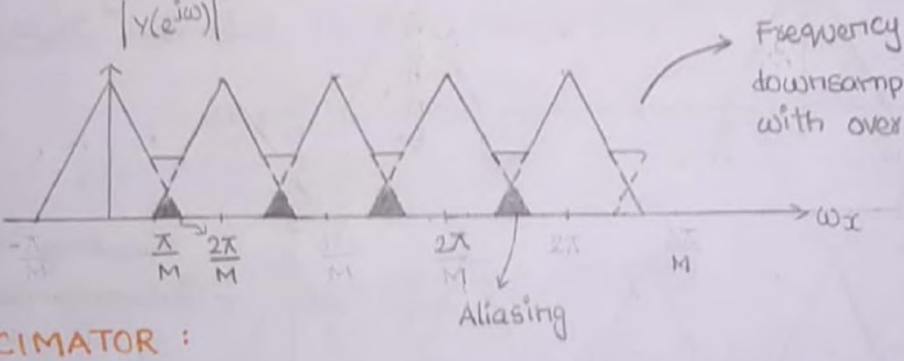
ALIASING FILTER :

The spectrum obtained after downsampling ~~is~~ will overlap if the original ~~signal~~ spectrum is not bandlimited to $\omega = \pm\frac{\pi}{M}$. This overlap causes ~~overla~~ aliasing. Therefore aliasing due to downsampling a signal by a factor of M is absent if and only if the signal $x(n)$ is bandlimited to $\pm\frac{\pi}{M}$. If the signal $x(n)$ is not bandlimited to $\omega = \pm\frac{\pi}{M}$ then a low pass filter with cutoff frequency $\frac{\pi}{M}$ is used prior to before downsampling. This filter is known as Anti-Aliasing filter.

The complete process i.e., of filtering and then downsampling is referred to called as "Decimation".

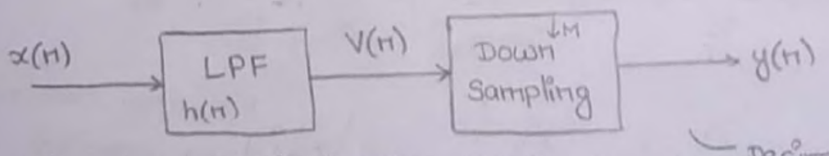


Frequency spectrum of signal that is not band limited to $\omega_c = \frac{\pi}{M}$



Frequency spectrum of downsampled signal with overlapping

DECIMATOR :



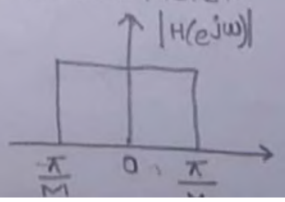
Decimator with an anti-aliasing filter and a down-sampler

The signal obtained after filtering is

$$V(n) = \sum_{K=-\infty}^{\infty} h(K)\; x(n-K)$$

$$\boxed{y(n) \overset{=V(nM)}{=} \sum_{K=-\infty}^{\infty} h(K)\; x(nM-K)}$$

IDEAL MAGNITUDE RESPONSE OF ANTI-ALIASING FILTER



1) Consider a factor of 2 downsampler and draw Spectrum

$M = 2$

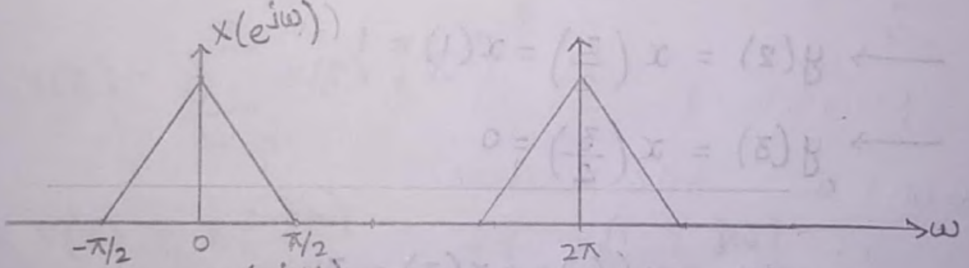
$$Y(e^{j\omega}) = \frac{1}{M}\sum_{K=0}^{M-1} X\left(e^{\frac{j(\omega-2\pi K)}{M}}\right)$$

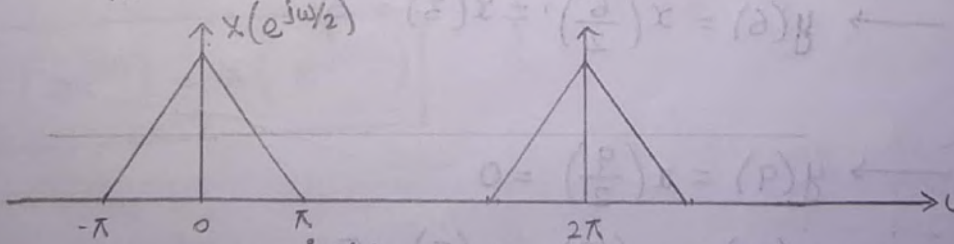
$$= \frac{1}{2}\sum_{K=0}^{1} X\left(e^{\frac{j(\omega-2\pi K)}{2}}\right)$$

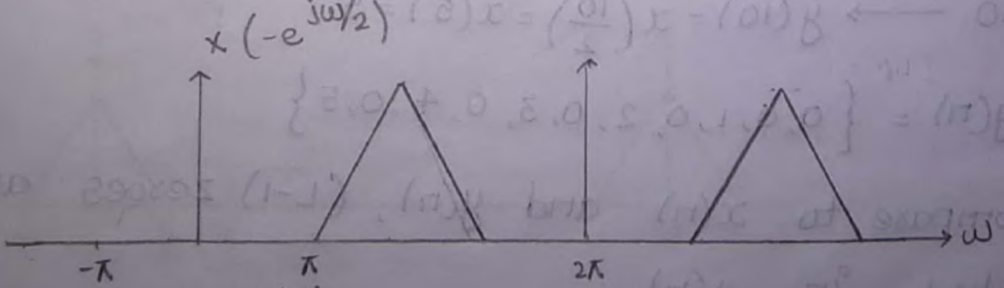
$$= \frac{1}{2}\left[X\left(e^{\frac{j\omega}{2}}\right) + X\left(e^{\frac{j(\omega-2\pi)}{2}}\right)\right]$$

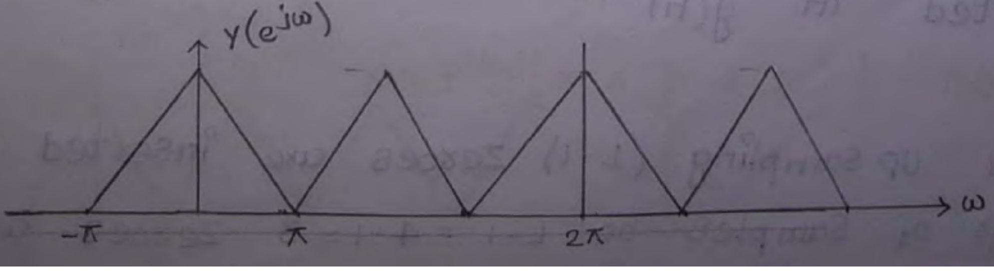
$$= \frac{1}{2}\left[X\left(e^{\frac{j\omega}{2}}\right) + X\left(e^{\frac{j\omega}{2}}\cdot \underbrace{e^{-j\pi}}_{-1}\right)\right]$$

$$Y(e^{j\omega}) = \frac{1}{2}\left[X\left(e^{j\omega/2}\right) + X\left(-e^{j\omega/2}\right)\right]$$





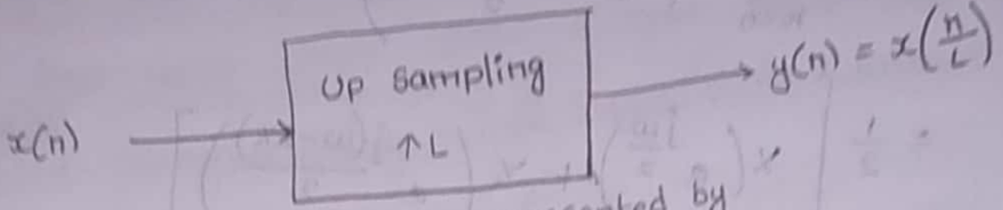




3-04-19

UP SAMPLING :

The Up Sampling rate of discrete time signal can b increased by a factor L by placing (L-1) equally spaced zeroes between each pair, of Samples.



Mathematically upsampling is represented by

$$y(n) = \begin{cases} x\left(\frac{n}{L}\right) & n = 0, \pm L, \pm 2L \cdots \\ 0 & \text{otherwise} \end{cases}$$

Ex : $x(n) = \{0, 1, 2, 3, 4, 5\}$, $L = 2$

$n=0 \longrightarrow y(0) = x\left(\frac{0}{2}\right) = x(0) = 0$

$n=1 \longrightarrow y(1) = x\left(\frac{1}{2}\right) = 0$

$n=2 \longrightarrow y(2) = x\left(\frac{2}{2}\right) = x(1) = 1$

$n=3 \longrightarrow y(3) = x\left(\frac{3}{2}\right) = 0$

⋮

$n=6 \longrightarrow y(6) = x\left(\frac{6}{2}\right) = x(3) = 3$

⋮

$n=9 \longrightarrow y(9) = x\left(\frac{9}{2}\right) = 0$

$n=10 \longrightarrow y(10) = x\left(\frac{10}{2}\right) = x(5) = 5$

$\therefore\ y(n) = \{0, 0, 1, 0, 2, 0, 3, 0, 4, 0, 5\}$

compare to $x(n)$ and $y(n)$, $(L-1)$ zeroes are inserted in $y(n)$

2> $x(n) = \{1, 2, 4, 6, 3, 2\}$, $L = 4$

In up sampling. (L-1) zeroes are inserted between a pair of Samples. So $L-1 = 4-1 = 3$ Zeroes are to be inserted

$y(n) = \{1, 0, 0, 0, 2, 0, 0, 0, 4, 0, 0, 0, 6, 0, 0, 0, 3, 0, 0, 0, 2\}$

SPECTRUM OF UP SAMPLING:

Z- Transform of output Signal y(n)

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n}$$

$$y(n) = x\left(\frac{n}{L}\right)$$

$$Y(z) = \sum_{n=-\infty}^{\infty} x\left(\frac{n}{L}\right) z^{-n}$$

let $\frac{n}{L} = p$

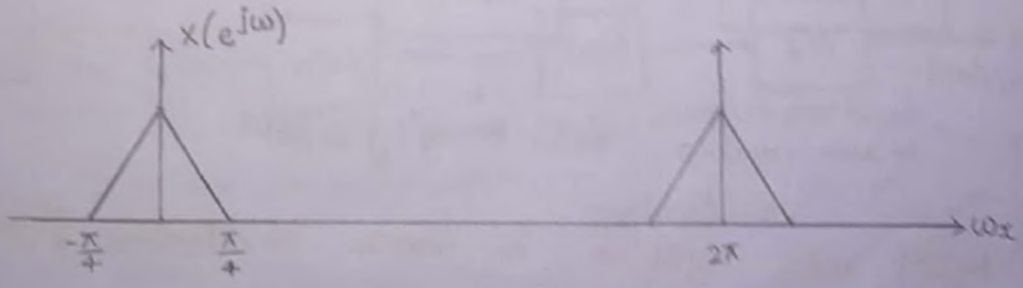
$n = pL$

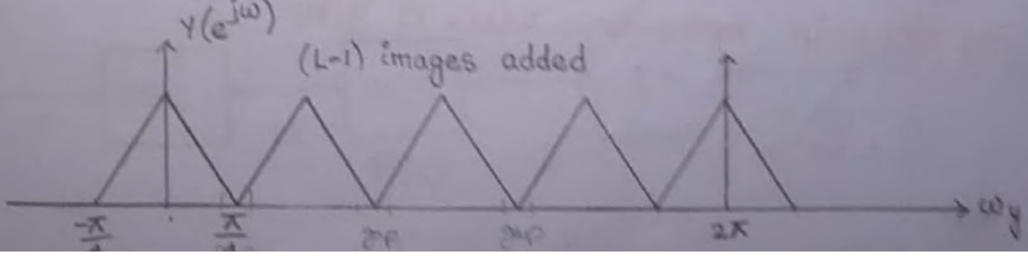
$$Y(z) = \sum_{p=-\infty}^{\infty} x(p) z^{-pL}$$

$$Y(z) = \sum_{p=-\infty}^{\infty} x(p) (z^{L})^{-p}$$

$$Y(z) = X(z^{L})$$

$$\boxed{Y(e^{j\omega}) = X(e^{j\omega L})} \qquad \left\{\because z = e^{j\omega}\right\}$$



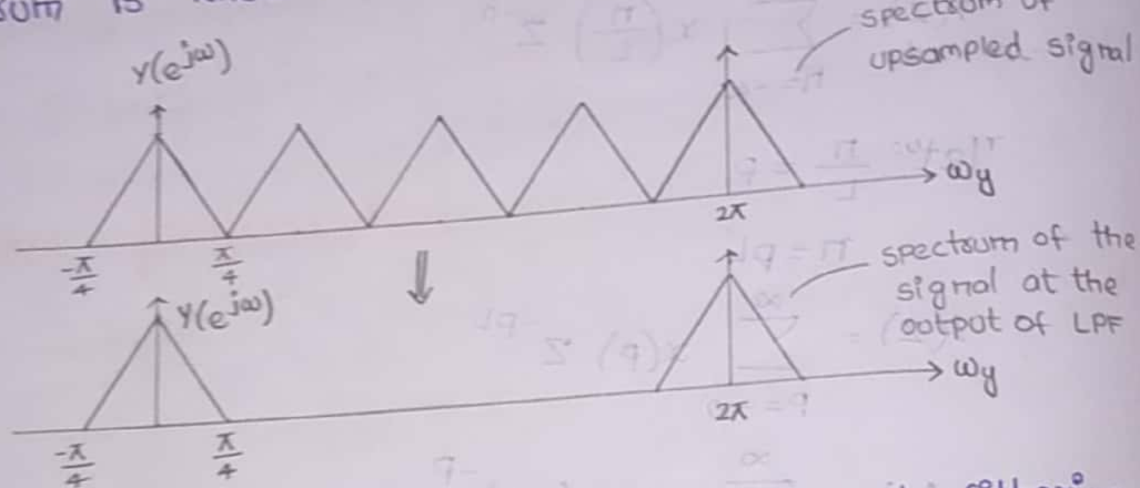


ANTI - IMAGING FILTERS :

Frequency Spectrum of up sampling signal y(n) with factor 'L' contains (L-1) additional images of Input Spectrum. These (L-1) images are due to the addition of L-1 zero samples b/w successive samples of x(n). since we are not interested in image spe

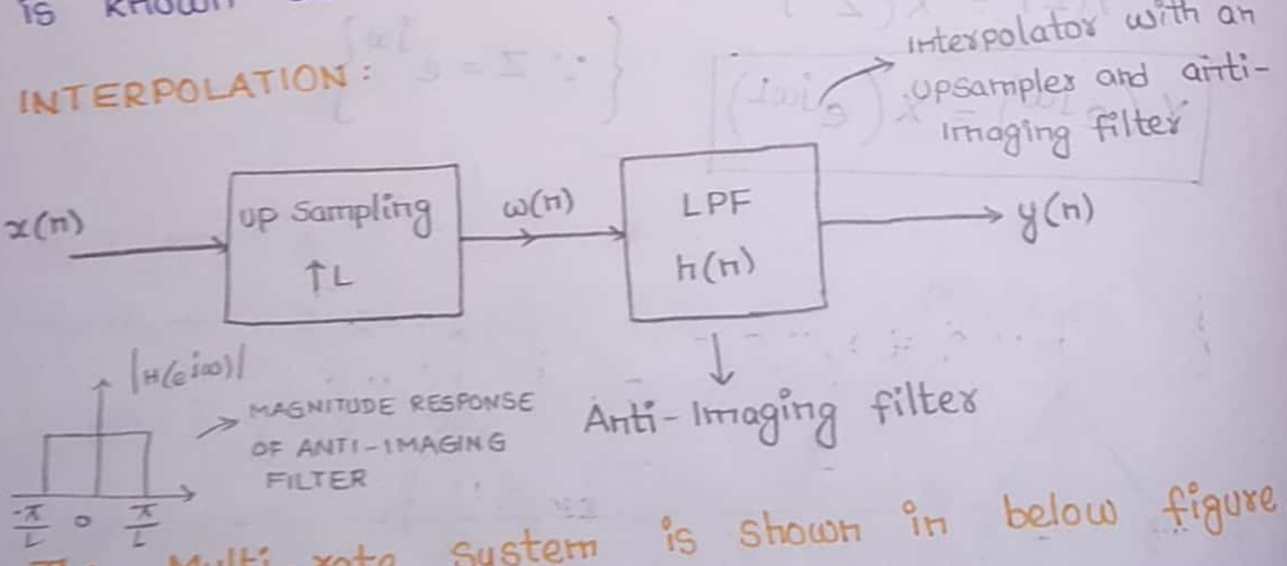
A low pass filter with a cut-off frequency of $\pm \frac{\pi}{L}$ can be used after up-Sampling.

The filter which is used to remove image Spectrum is known as Anti-Imaging filter.

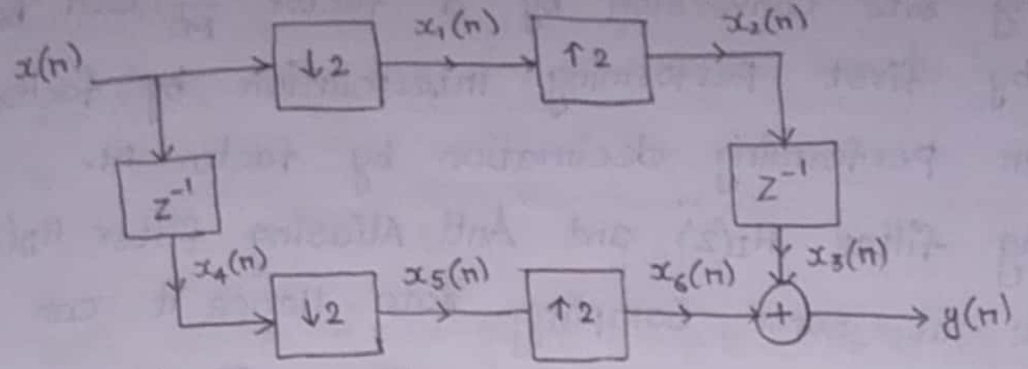


The Complete process of up Sampling and filtering is known as "interpolation".

INTERPOLATION :



1). The Multi-rate system is shown in below figure. Find Relationship between x(n) & y(n).



$x(n) = x_0 \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_7 \quad x_8 \quad x_9 \quad x_{10} \; ---$

$x_1^*(n) = x_0 \quad x_2 \quad x_4 \quad x_6 \quad x_8 \quad x_{10} \quad x_{12} \quad x_{14} \quad x_{16} \quad x_{18} \quad x_{20} \; ---$

$x_2(n) = x_0 \quad 0 \quad x_2 \quad 0 \quad x_4 \quad 0 \quad x_6 \quad 0 \quad x_8 \quad 0 \quad x_{10} \; ---$

Interpolation

$x_3(n) = 0 \quad x_0 \quad 0 \quad x_2 \quad 0 \quad x_4 \quad 0 \quad x_6 \quad 0 \quad x_8 \quad 0 \; ---$

Interpolation

$x_4(n) = x_{-1} \quad x_0 \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_7 \quad x_8 \quad x_9 \; ---$

⊕ Add

$x_5(n) = x_{-1} \quad x_1 \quad x_3 \quad x_5 \quad x_7 \quad x_9 \quad x_{11} \quad x_{13} \quad x_{15} \quad x_{17} \quad x_{19} \; ---$

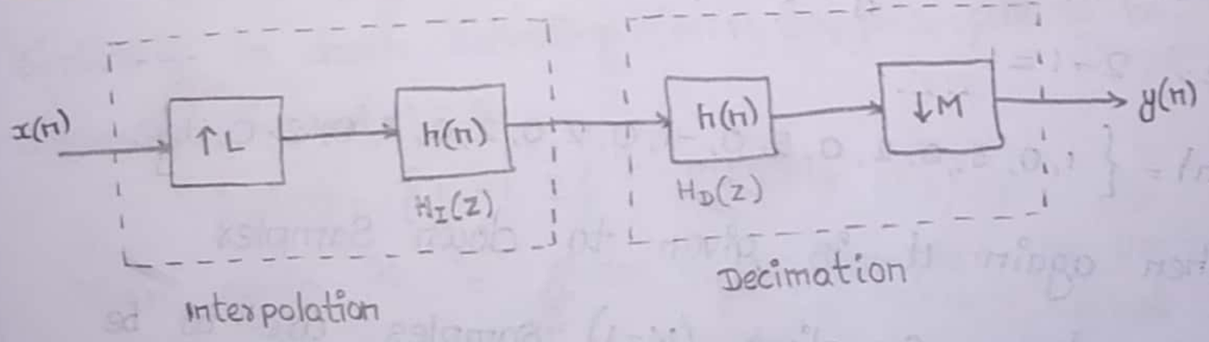
$x_6(n) = x_{-1} \quad 0 \quad x_1 \quad 0 \quad x_3 \quad 0 \quad x_5 \quad 0 \quad x_7 \quad 0 \quad x_9 \; ---$

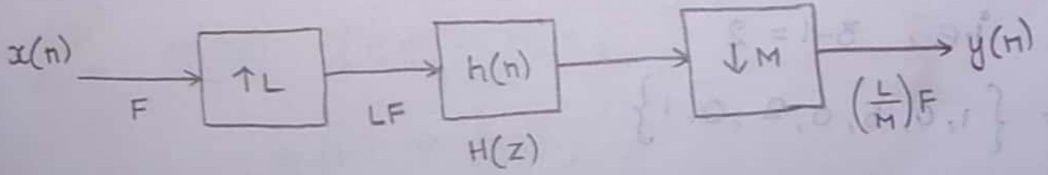
$y(n) = x_1 \quad x_0 \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_7 \quad x_8 \quad x_9 \; ---$

The Relationship between $x(n)$ & $y(n)$ is the input signal is delay by Z^{-1}

8-04-19

CASCADING SAMPLING RATE CONVERSION :

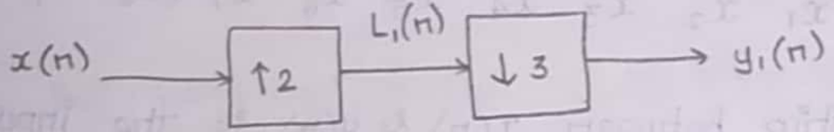


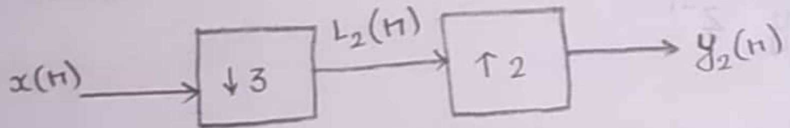


Sampling rate Conversion by a factor $\frac{L}{M}$ can be achieved by first performing Interpolation by factor L and then performing decimation by factor M. Anti Imaging filter $H_I(z)$ and Anti Aliasing filter $H_D(z)$ are operate at same Sampling rate. Hence it can be replaced by Simple low pass filter with transfer function $H(z)$. Low pass filter has cutoff frequency of $\left(\frac{\pi}{L}, \frac{\pi}{M}\right)$.

The cascade factor M down samples and factor of L up Sampler is interchangeable with no change in input and output relation if and only if L and M are co-primes

Ex : $x(n) = \{1, 3, 2, 5, -1, 2, 2, 3, 2, 1\}$





For up Sampling, (L-1) zeroes are to be inserted

i.e., $2-1=1$

$L_1(n) = \{1, 0, 3, 0, 2, 0, 5, 0, -1, 0, 2, 0, 2, 0, 3, 0, 2, 0, 1\}$

Then again it is given to down Sampler

In down sampling, (M-1) samples are to be discarded i.e., $3-1 = 2$

$y_1(n) = \{1, 0, 5, 0, 2, 0, 1\}$

Similarly,

In down sampling, (M-1) samples are to be discarded

i.e., 3-1 = 2

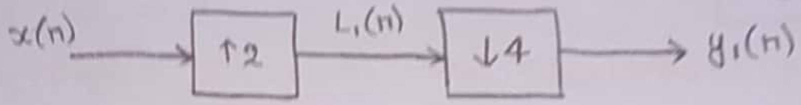
$L_2(n) = \{1, 5, 2, 1\}$

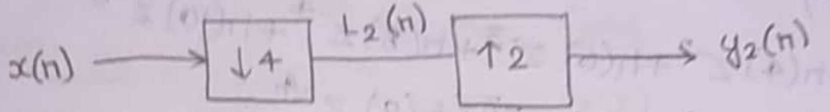
Then again it is given to up sampler. In up sampling, (L-1) zeroes are to be inserted i.e., 2-1=1

$y_2(n) = \{1,0,5,0,2,0,1\}$

$y_1(n) = y_2(n)$

2> $x(n) = \{1, 3, 2, 5, -1, 2, 2, 3, 2, 1\}$





For up sampling, (L-1) zeroes are to be inserted i.e., 2-1=1

$L_1(n) = \{1, 0, 3, 0, 2, 0, 5, 0, -1, 0, 2, 0, 2, 0, 3, 0, 2, 0, 1\}$

Then again it is given to down sampler. So, down sampler discards (M-1) samples i.e., 4-1 = 3

$y_1(n) = \{1, 2, -1, 2, 2\}$

similarly, In down sampling (M-1) samples are to be discarded i.e., 4-1 = 3

$L_2(n) = \{1, -1, 2\}$

Then again it is given to up sampler. So up sampler inserts (L-1) zeroes i.e., 2-1 = 1

$y_2(n) = \{1, 0, -1, 0, 2\}$

$y_1(n) \neq y_2(n)$

IMPLEMENTATION OF SAMPLING RATE CONVERSION :

We consider the efficient implementation of sampling rate conversion system using polyphase filter structure.

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n} \quad \text{let } N = 11 \longrightarrow H(z) = \sum_{n=0}^{10} h(n) z^{-n}$$

$$H(z) = h(0) \underbrace{z^{-0}}_{1} + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(4) z^{-4} + h(5) z^{-5} +$$

$$h(6) z^{-6} + h(7) z^{-7} + h(8) z^{-8} + h(9) z^{-9} + h(10) z^{-10}$$

picking alternate terms

$$H_1(z) = h(0) + h(2) z^{-2} + h(4) z^{-4} + h(6) z^{-6} + h(8) z^{-8} + h(10) z^{-10}$$

$$H_2(z) = h(1) z^{-1} + h(3) z^{-3} + h(5) z^{-5} + h(7) z^{-7} + h(9) z^{-9}$$

$$H_2(z) = z^{-1} [h(1) + h(3) z^{-2} + h(5) z^{-4} + h(7) z^{-6} + h(9) z^{-8}]$$

let $H_0(z) = E_0(z^2)$

$H_1(z) = z^{-1} E_1(z^2)$

$$E_0(z) = h(0) + h(2) z^{-1} + h(4) z^{-2} + h(6) z^{-3} + h(8) z^{-4} + h(10) z^{-5}$$

$$E_1(z) = h(1) + h(3) z^{-1} + h(5) z^{-2} + h(7) z^{-3} + h(9) z^{-4}$$

$$H(z) = H_0(z) + H_1(z)$$

$$H(z) = E_0(z^2) + z^{-1} E_1(z^2) \longrightarrow ①$$

For 3 terms

$$H(z) = E_0(z^3) + z^{-1} E_1(z^3) + z^{-2} E_2(z^3) \longrightarrow ②$$

For M terms

$$H(z) = E_0(z^M) + z^{-1} E_1(z^M) + z^{-2} E_2(z^M) + \cdots + z^{-(M-1)} E_{M-1}(z^M) \longrightarrow ③$$

From eqn ① $\frac{Y(z)}{X(z)} = E_0(z^2) + z^{-1} E_1(z^2)$

$$Y(z) = E_0(z^2) X(z) + z^{-1} E_1(z^2) X(z)$$

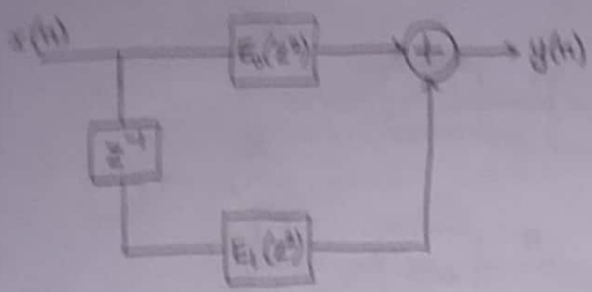
Apply Inverse Z-Transform

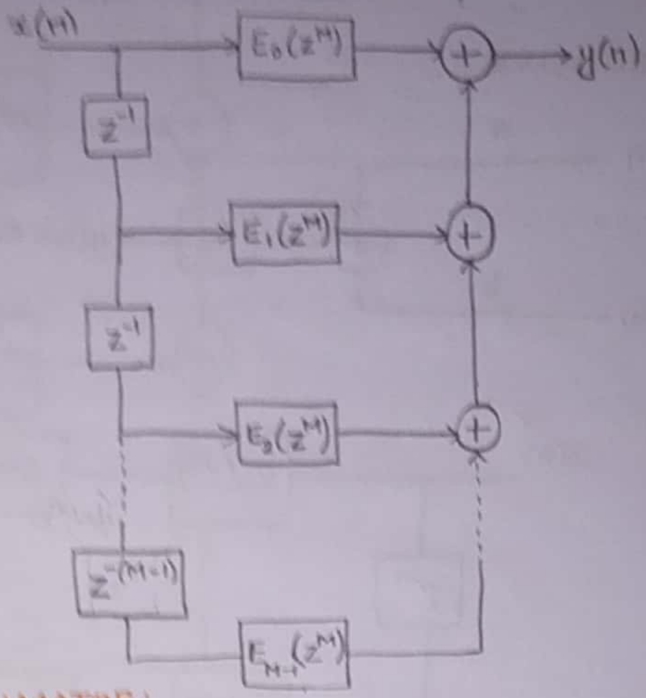
$$y(n) = E_0(z^2) x(n) + E_1(z^2) x(n-1)$$

Scanned by CamScanner

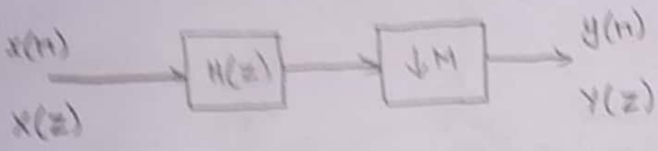
09.04.19

DIRECT FORM REALIZATION FOR EQN① AND EQN②

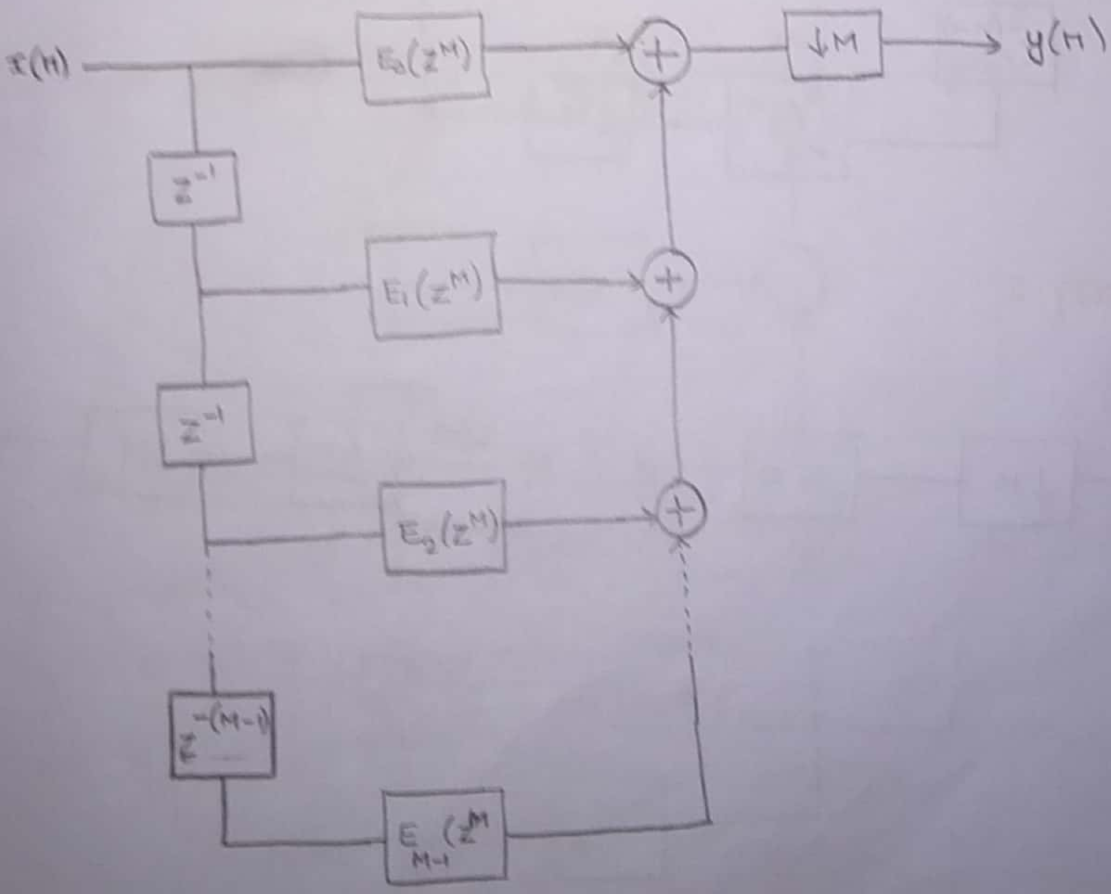




POLYPHASE STRUCTURE OF DECIMATOR:



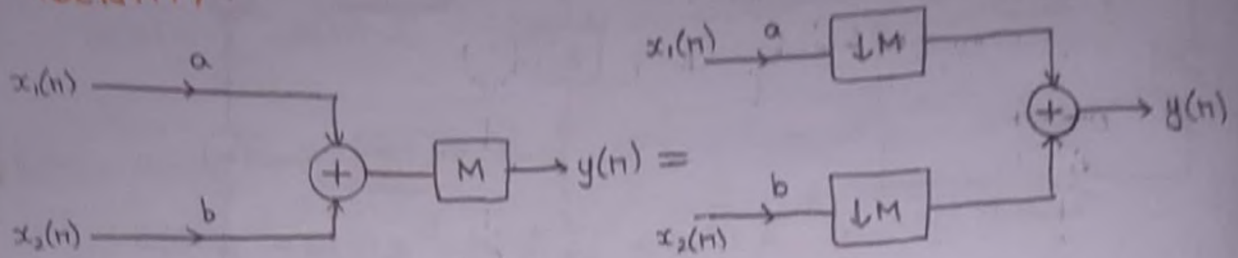
$$H(z) = E_0(z^M) + z^{-1}E_1(z^M) + \cdots\cdots + z^{-(M-1)} E_M(z^M)$$

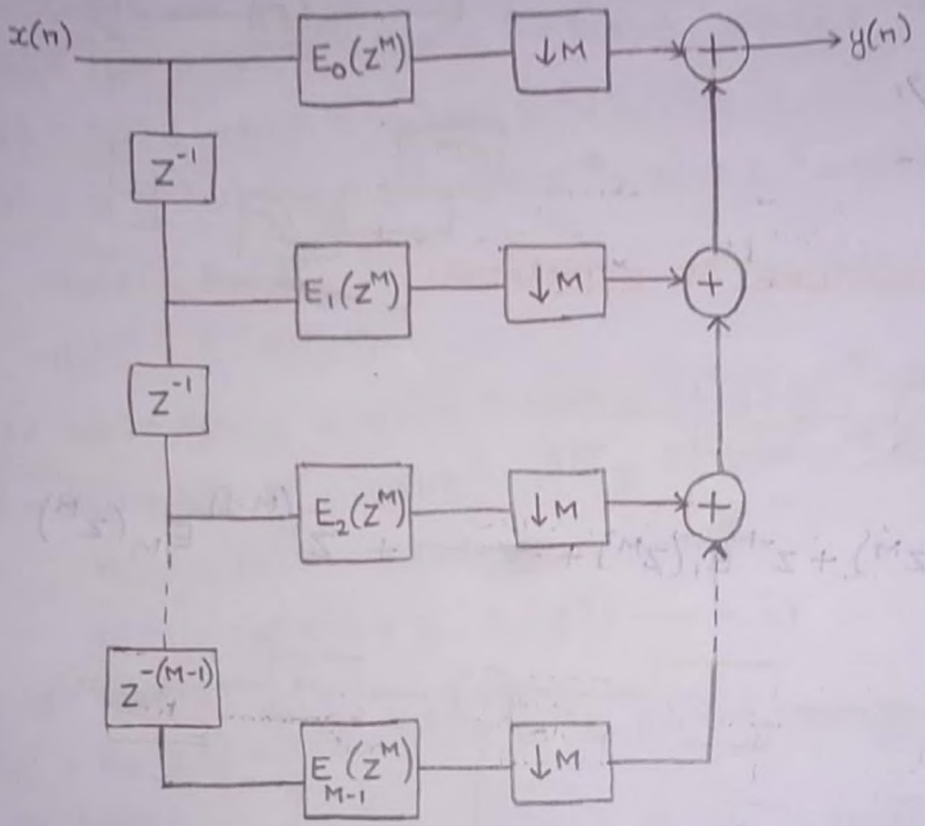


Each one of the term can be treated separately

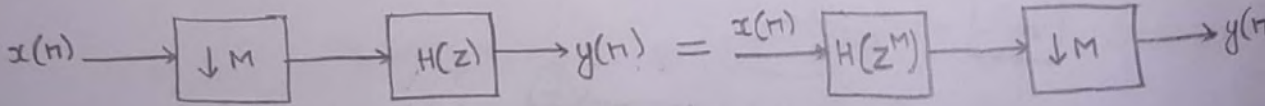
We put down Sampler before adder.

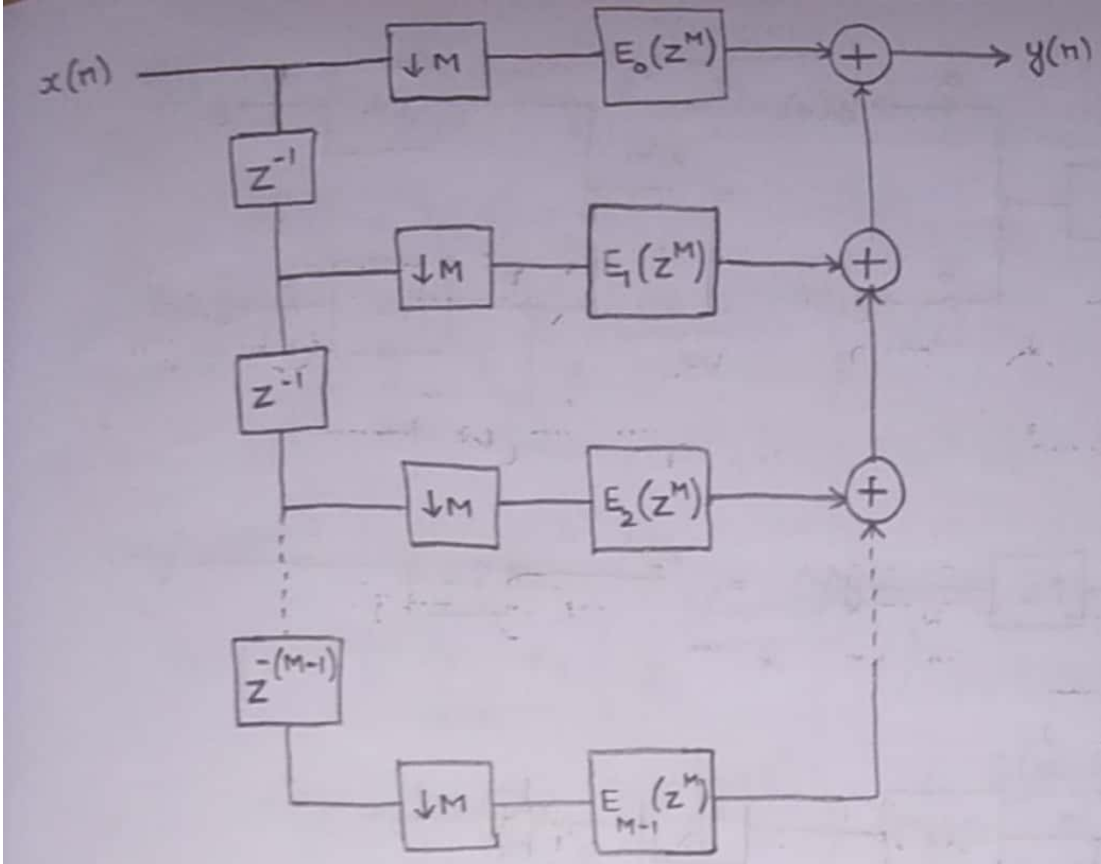
IDENTITY :



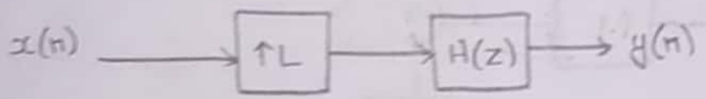


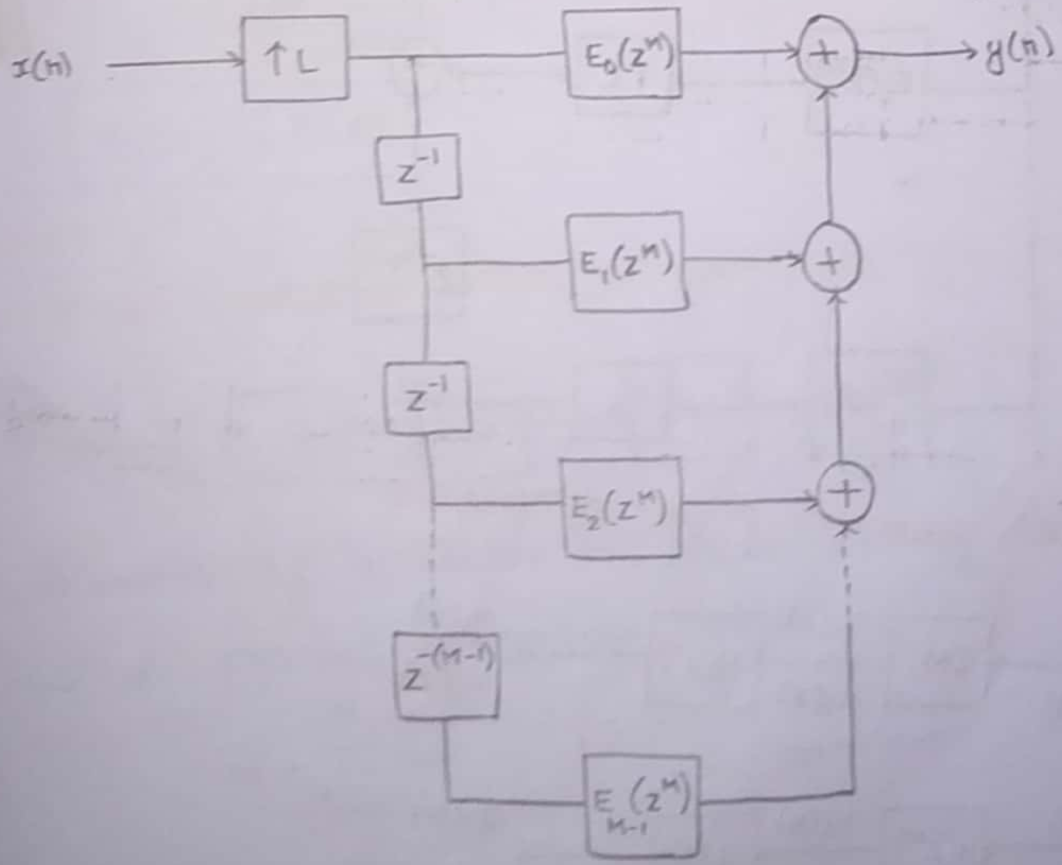
IDENTITY 2 :



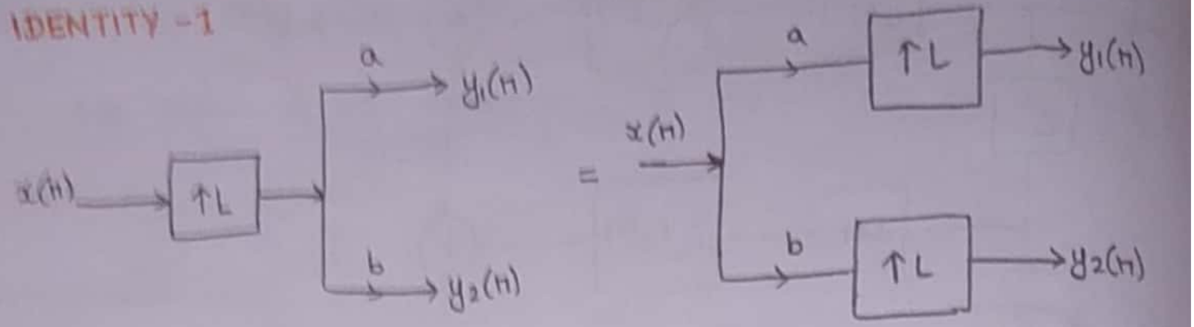


POLYPHASE STRUCTURE OF INTERPOLATION :

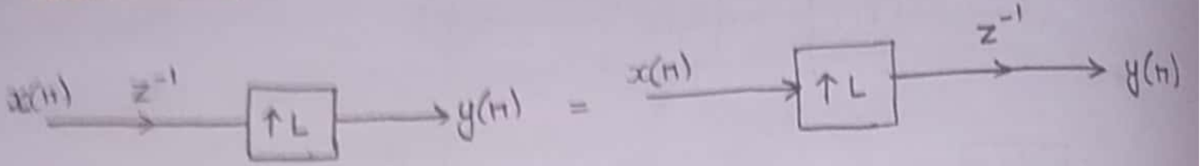


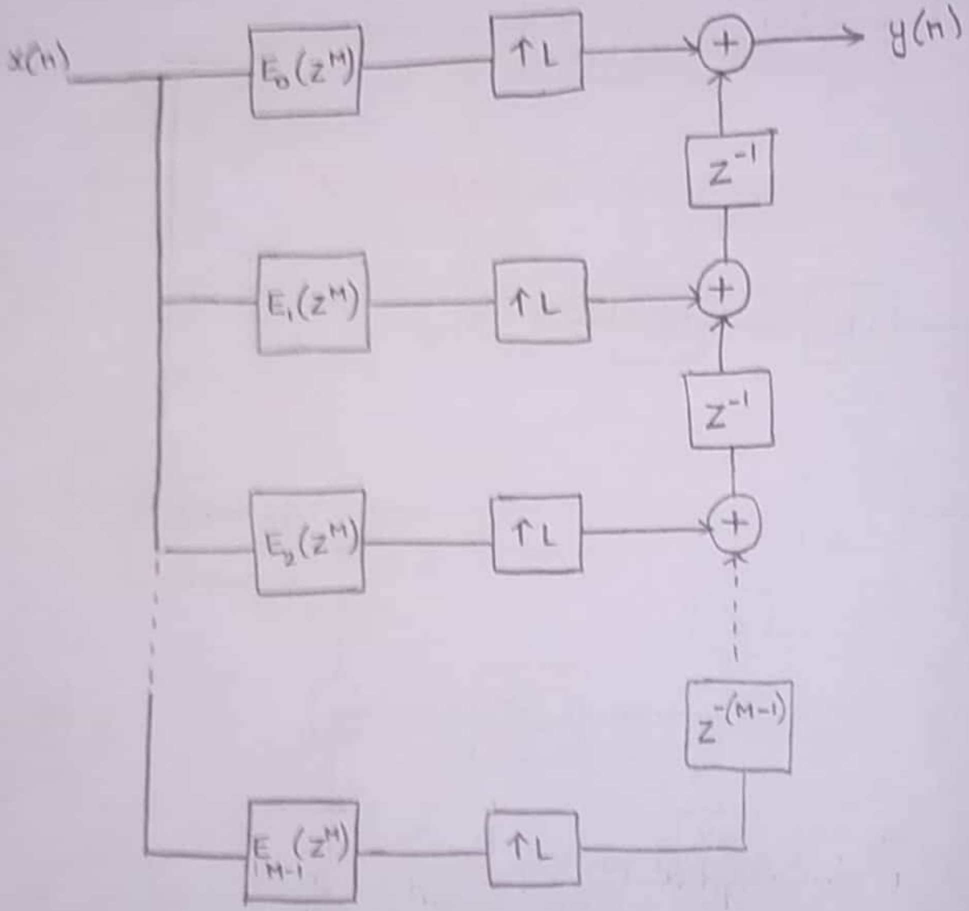


IDENTITY -1

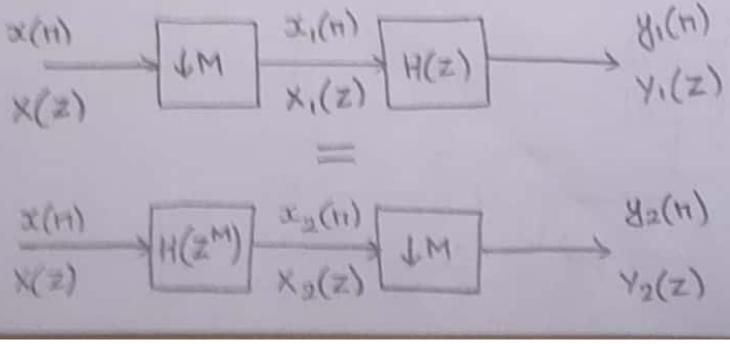


IDENTITY-2





PROOF FOR IDENTITY-2 IN DECIMATOR:



$$X_1(Z) = \frac{1}{M}\sum_{K=0}^{M-1} X\left(e^{\frac{j(\omega-2\pi k)}{M}}\right)$$

$$Y_1(Z) = X_1(Z)\cdot H(Z)$$

$$Y_1(Z) = \frac{1}{M}\sum_{K=0}^{M-1} X\left(e^{\frac{j(\omega-2\pi k)}{M}}\right) H(Z)$$

$$X_2(Z) = X(Z)\cdot H(Z^M)$$

$$Y_2(Z) = \frac{1}{M}\sum_{K=0}^{M-1} X_2\left(e^{\frac{j(\omega-2\pi k)}{M}}\right)$$

$$= \frac{1}{M}\sum_{K=0}^{M-1} X\left(e^{\frac{j(\omega-2\pi k)}{M}}\right) H\left(e^{\frac{j(\omega-2\pi k)}{\cancel{M}}}\right)^{\cancel{M}}$$

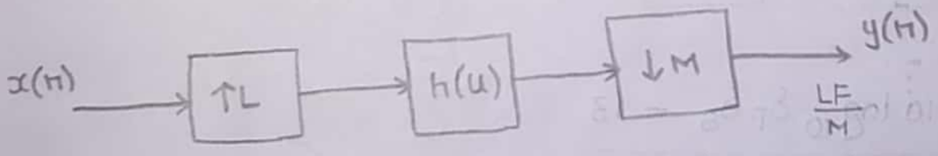
$$= \frac{1}{M}\sum_{K=0}^{M-1} X\left(Z^{\frac{1}{M}}\cdot W_M^k\right)\cdot H\left(e^{j\omega}\cdot e^{-j2\pi k}\right)$$

$$Y_2(Z) = \frac{1}{M}\sum_{K=0}^{M-1} X\left(Z^{1/M}\cdot W_M^k\right) H(Z)$$

$$\boxed{Y_2(Z) = Y_1(Z)}$$

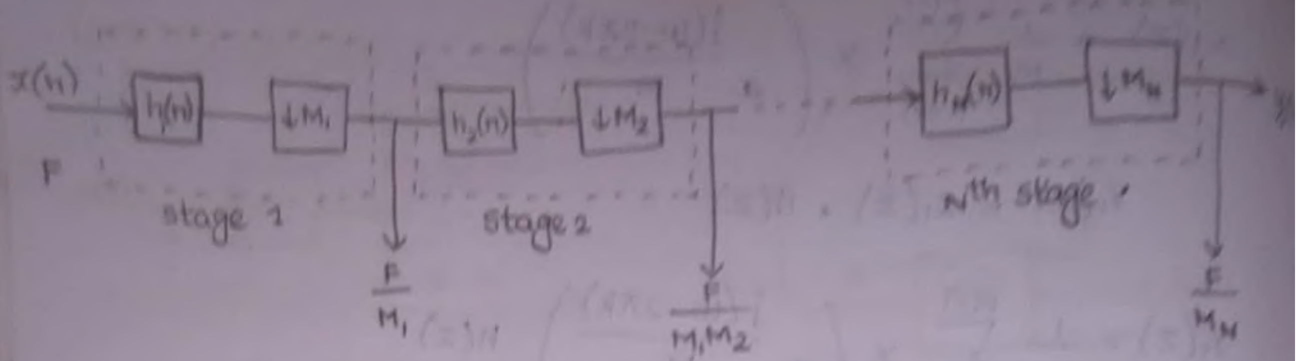
15-04-19

MULTISTAGE IMPLEMENTATION OF SAMPLING RATE CONVERSION :



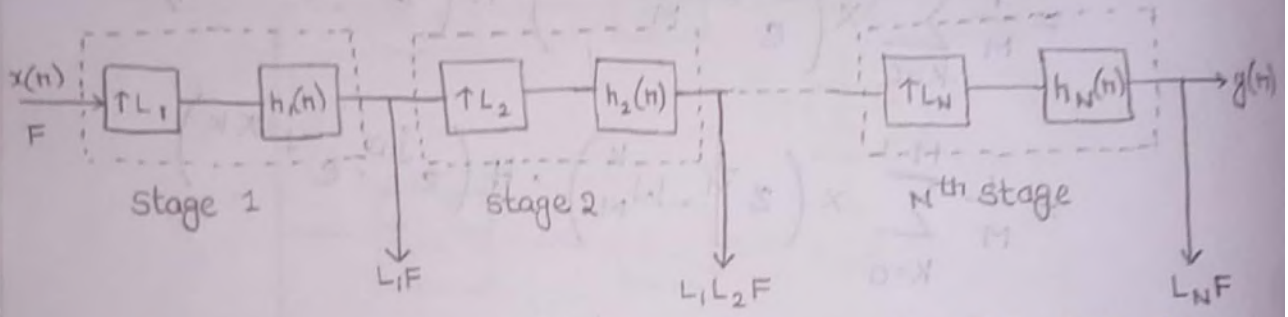
Decimation by a factor 'M', where 'M' may be factored into a product of positive integer as

$$M = \prod_{i=1}^{N} M_i$$

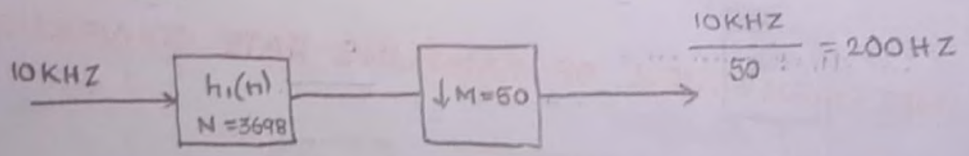


Interpolation by a factor 'L', where L may be factored into a product of positive integers as

$$\boxed{L = \prod_{i=1}^{N} L_i}$$



1> Design one stage decimator to meet the following specifications: Decimation factor M=50, transition band 50 ≤ f ≤ 55, Input sampling frequency 10 KHZ, pass band ripple $\delta_p = 10^{-1}$, stop band ripple $\delta_s = 10^{-3}$.



$$N = \frac{-10\log_{10}\delta_p\delta_s - 13}{14.6\ \Delta f}$$

$$\Delta f = \frac{\overset{\text{Final}}{55} - \overset{\text{Initial}}{50}}{\underset{\text{input sampling frequency}}{10\,KHZ}} = \frac{5}{10\,KHZ} = 5\times10^{-4}$$

$$N = \frac{-10\log_{10}(10^{-1})(10^{-3}) - 13}{14.6\,(5\times10^{-4})}$$

$$= \frac{-10\log_{10} 10^{-4} - 13}{14.6\,(5\times10^{-4})}$$

$$= \frac{-10\cdot -4 \cdot \log_{10}^{10} - 13}{14.6\,(5\times10^{-4})}$$

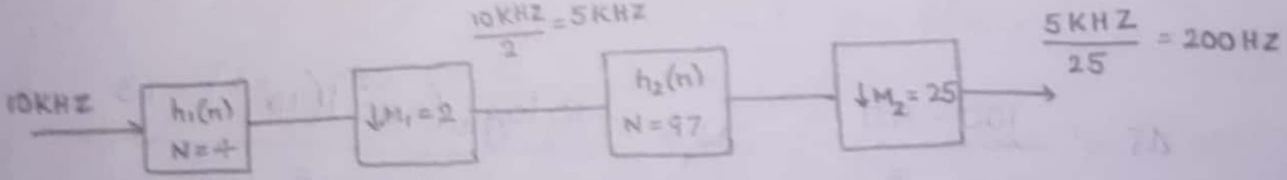
$$= \frac{40-13}{14.6\,(5\times10^{-4})}$$

$$= \frac{27}{14.6\,(5\times10^{-4})}$$

$N = 3698.63$

$N = 3699$

2) Design two stage decimator for same Specifications of above problem



Stage 1 :

Transition band

$50 \le f \le f_1 - 55$

$50 \le f \le 4.945$ KHZ

$$\Delta f = \frac{4.945\times10^{3} - 50}{10\times10^{3}}$$

$\Delta f = 0.4895$

Stage 2 :

$50 \le f \le f_2 - 55$

$50 \le f \le 145$

$$\Delta f = \frac{145-50}{5\times10^{3}}$$

$\Delta f = 0.019$

$$N_1 = \frac{-10\log_{10}(10^{-1})(10^{-3}) - 13}{14.6 \times 0.4895}$$

$$= \frac{-10\log_{10}10^{-4} - 13}{14.6 \times 0.4895}$$

$$= \frac{40-13}{14.6 \times 0.4895}$$

$$= \frac{27}{14.6 \times 0.4895}$$

$N_1 = 3.7779$

$N_1 \cong 4$

$$N_2 = \frac{-10\log_{10}(10^{-1})(10^{-3}) - 1}{14.6 \times 0.019}$$

$$= \frac{-10\log_{10}10^{-4} - 13}{14.6 \times 0.019}$$

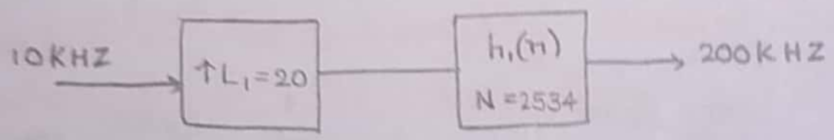
$$= \frac{40-13}{14.6 \times 0.019}$$

$$= \frac{27}{14.6 \times 0.019}$$

$N_2 = 97.3323$

$N_2 \cong 98$

Design one stage and two stage interpolation to mee the following specification : interpolation factor 20 and transition band $90 \leq f \leq 100$, Input sampling frequency 10KHZ, pass band ripple 10^{-2}, stop band ripple 10^{-3}.



$$\Delta f = \frac{100-90}{10 \times 10^3}$$

$$= \frac{10}{10 \times 10^3}$$

$\Delta f = 10^{-3}$

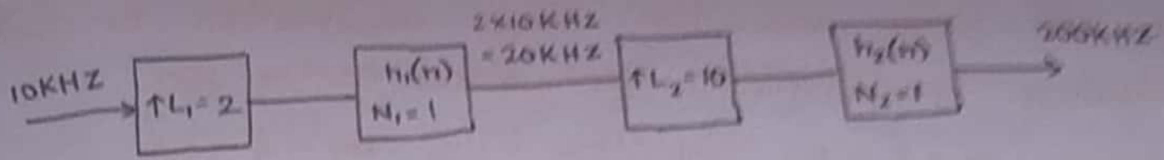
$$N_1 = \frac{-10\log_{10}(10^{-2})(10^{-3}) - 13}{14.6 \times 10^{-3}}$$

$$= \frac{-10\log_{10}(10^{-5}) - 13}{14.6 \times 10^{-3}}$$

$$= \frac{50-13}{14.6 \times 10^{-3}} = \frac{37}{14.6 \times 10^{-3}}$$

$N_1 = 2534.24$

$N_1 = 2534$



stage 1

Transition band

$90 \le f \le f_1 - 100$

$90 \le f \le 20\times10^3 - 100$

$90 \le f \le 19900$

$90 \le f \le 19.9\ KHZ$

$$\Delta f = \frac{19.9\times10^3 - 90}{10\times10^3}$$

$= 1.981$

$$N_1 = \frac{-10\log_{10}(10^{-2})(10^{-3}) - 13}{14.6\times1.981}$$

$$= \frac{-10\log_{10}10^{-5} - 13}{14.6\times1.981}$$

$$= \frac{50 - 13}{14.6\times1.981}$$

$$= \frac{37}{14.6\times1.981}$$

$N_1 = 1.2792$

$N_1 \cong 1$

stage 2

$90 \le f \le f_2 - 100$

$90 \le f \le 200\times10^3 - 100$

$90 \le f \le 199900$

$90 \le f \le 199.9\ KHZ$

$$\Delta f = \frac{199.9\times10^3 - 90}{20\times10^3}$$

$= 9.9905$

$$N_2 = \frac{-10\log_{10}(10^{-2})(10^{-3}) - 13}{14.6\times9.9905}$$

$$= \frac{-10\log_{10}10^{-5} - 13}{14.6\times9.9905}$$

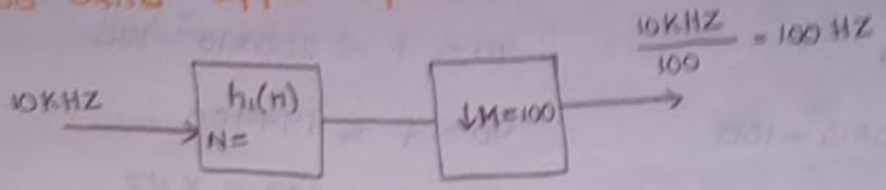
$$= \frac{50 - 13}{14.6\times9.9905}$$

$$= \frac{37}{14.6\times9.9905}$$

$N_2 = 0.2536$

$N_2 \cong 1$

Design one stage and two stage decimation to me
the following Specifications : Decimation factor 100
pass band $0 \le f \le 50$, Transition band $50 \le f \le 55$,
Input Sampling frequency 10KHZ, stop band ripple $\delta_s =$
pass band ripple $\delta_p = 10^{-3}$

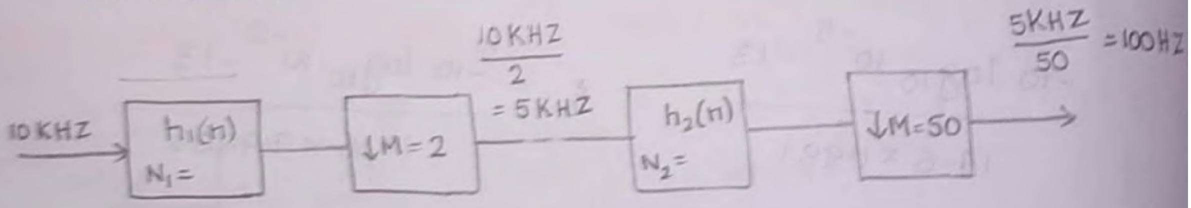


$$\Delta f = \frac{55-50}{10KHZ} = \frac{5}{10KHZ} = 5 \times 10^{-4}$$

$$N = \frac{-10\log_{10}(10^{-1})(10^{-3}) - 13}{14.6 \times 5 \times 10^{-4}} = \frac{-10\log_{10}(10^{-4}) - 13}{14.6 \times 5 \times 10^{-4}}$$

$$= \frac{40-13}{14.6 \times 5 \times 10^{-4}} = \frac{27}{14.6 \times 5 \times 10^{-4}} = 3698.6$$

$N = 3699$



Stage 1 :

Transition band

$50 < f < f_1 - 55$

$50 < f < 5 \times 10^3 - 55$

$50 < f < 4.945\ KHZ$

$$\Delta f = \frac{4945 - 50}{10 \times 10^3}$$

$\Delta f = 0.4895$

Stage 2

Transition band

$50 < f < f_2 - 55$

$50 < f < 100 - 55$

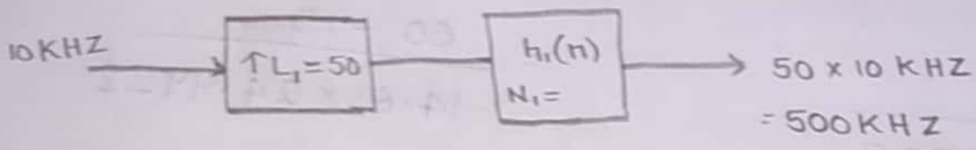
$50 < f < 45$

$$\Delta f = \frac{45 - 50}{5 \times 10^3}$$

$\Delta f = -1 \times 10^{-3}$

16-04-19

Design one stage and second stage interpolation to meet the following specifications : interpolation factor 50, pass band $0 \leq f \leq 75$. stop band $0 \leq f \leq 80$, Input sampling frequency 10KHZ, Stop band ripple $\delta_s = 10^{-4}$, pass band ripple $\delta_p = 10^{-2}$.



$$\Delta f = \frac{80-75}{10 \times 10^3}$$

$$\Delta f = 5 \times 10^{-4}$$

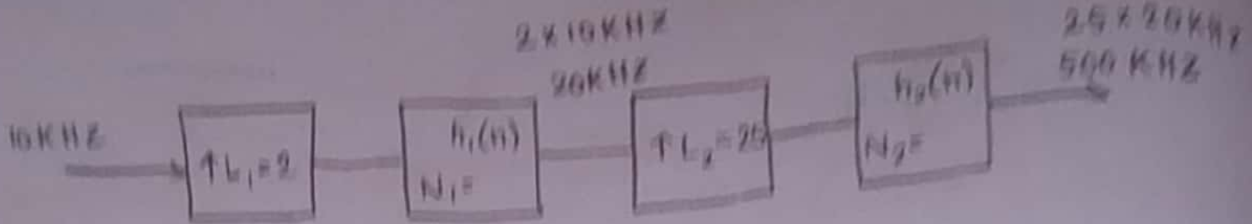
$$N_1 = \frac{-10 \log_{10} (10^{-2})(10^{-4}) - 13}{14 \cdot 6 \times 5 \times 10^{-4}}$$

$$= \frac{-10 \log_{10} 10^{-6} - 13}{14 \cdot 6 \times 5 \times 10^{-4}}$$

$$= \frac{60 - 13}{14 \cdot 6 \times 5 \times 10^{-4}} = \frac{47}{14 \cdot 6 \times 5 \times 10^{-4}}$$

$$= 6438 \cdot 35$$

$$N_1 = 6439$$



Stage 1 :

Transition band

$75 \leq f \leq f_1 - 80$

$75 \leq f \leq 20 \times 10^3 - 80$

$75 \leq f \leq 19920$

$$\Delta f = \frac{19920 - 75}{10 \times 10^3}$$

$\Delta f = 1.9845$

$$N_1 = \frac{-10 \log_{10} (10^{-2})(10^{-4}) - 13}{14.6 \times 1.9845}$$

$$= \frac{-10 \log_{10} (10^{-6}) - 13}{14.6 \times 1.9845}$$

$$= \frac{60 - 13}{14.6 \times 1.9845}$$

$N_1 = 1.6221$

$N_1 \cong 2$

Stage 2 :

Transition band

$75 \leq f \leq f_2 - 80$

$75 \leq f \leq 500 \times 10^3 - 80$

$75 \leq f \leq 499920$

$$\Delta f = \frac{499920 - 75}{20 \times 10^3}$$

$\Delta f = 24.9922$

$$N_2 = \frac{-10 \log_{10} (10^{-2})(10^{-4}) - 13}{14.6 \times 24.9922}$$

$$= \frac{-10 \log_{10} (10^{-6}) - 13}{14.6 \times 24.9922}$$

$$= \frac{60 - 13}{14.6 \times 24.9922}$$

$N_2 = 0.1288$

$N_2 \cong 1$

SAMPLING RATE CONVERSION OF BAND PASS SIGNAL:

Decimate by a factor 'M' an integer positioned band pass signal with Spectrum range is

$$(k-1)\frac{\pi}{M} < |\omega| < \frac{k\pi}{M}$$

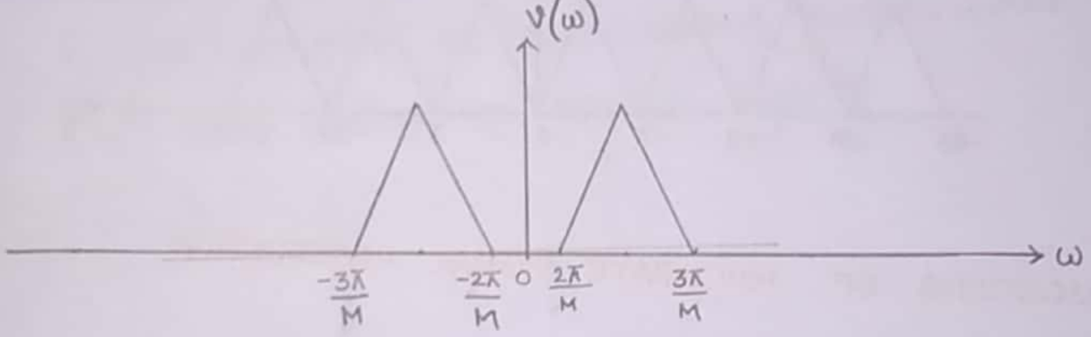
where k is positive integer

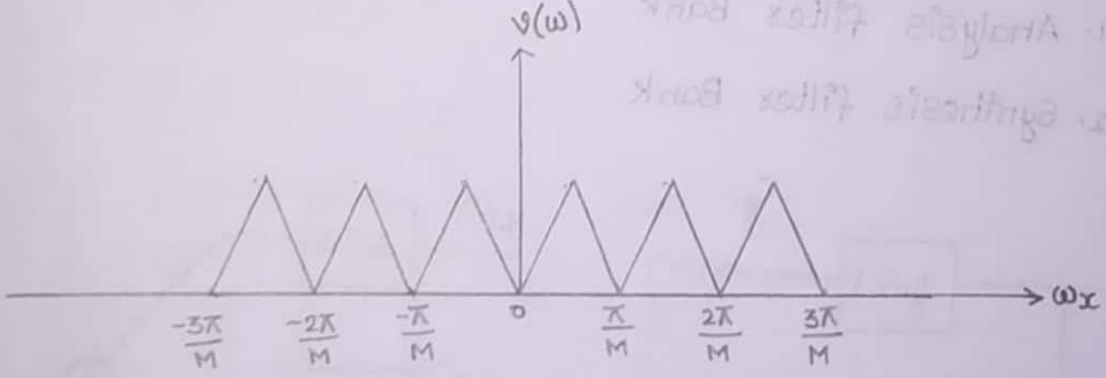
A Bandpass filter

$$H_{BP}(\omega) = \begin{cases} 1 & \text{for } (k-1)\frac{\pi}{M} < |\omega| < \frac{k\pi}{M} \\ 0 & \text{otherwise} \end{cases}$$

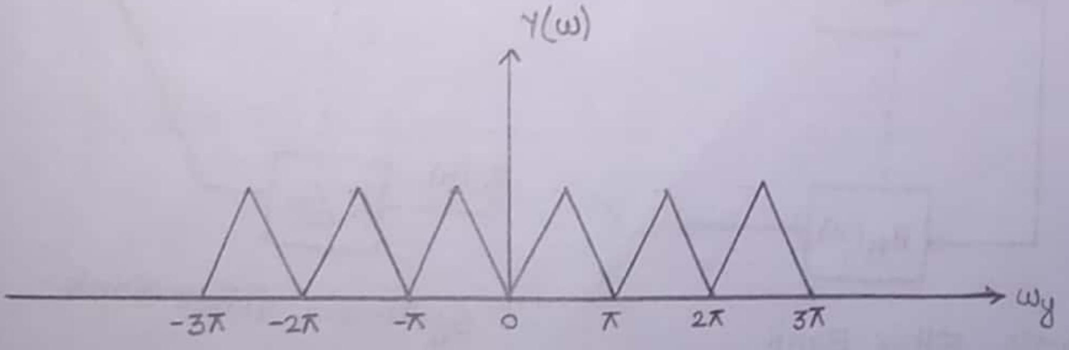
For $k = \text{odd}$

let $K = 3$ $\qquad \frac{2\pi}{M} < |\omega| < \frac{3\pi}{M}$



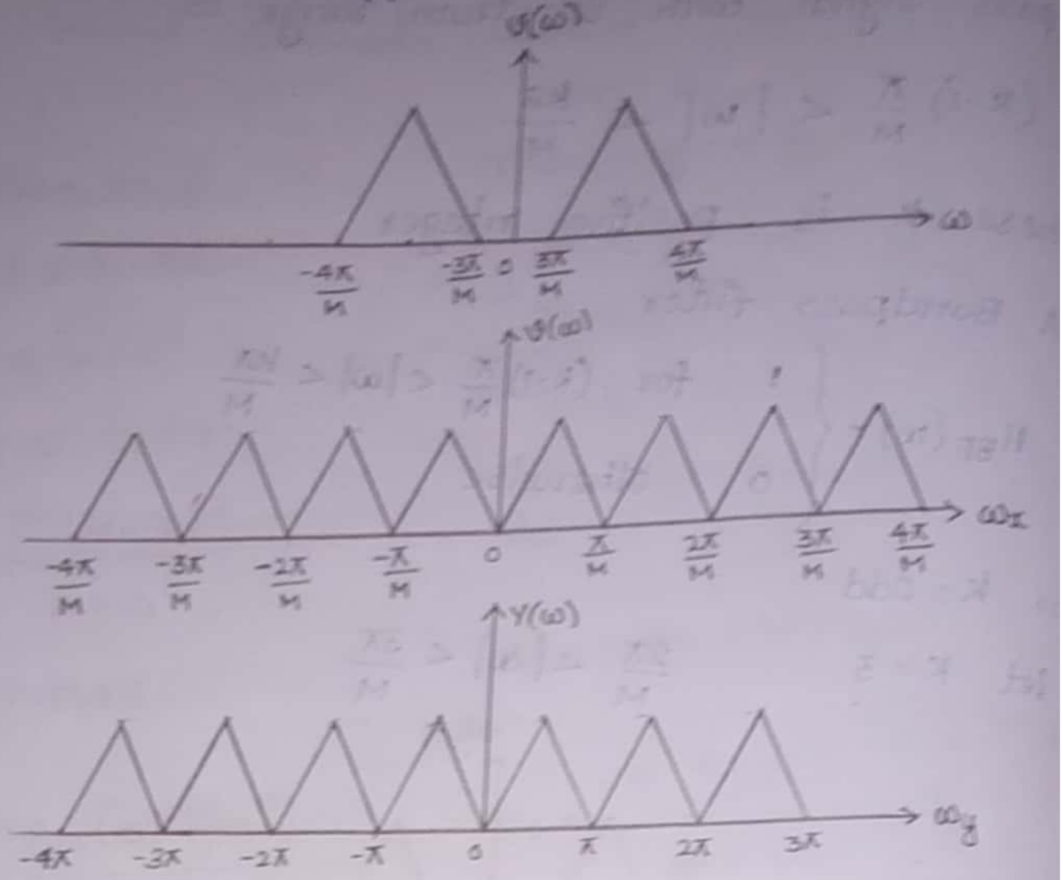


$y(\omega)$ is normalized Scaling $\omega_y = M\omega_x$



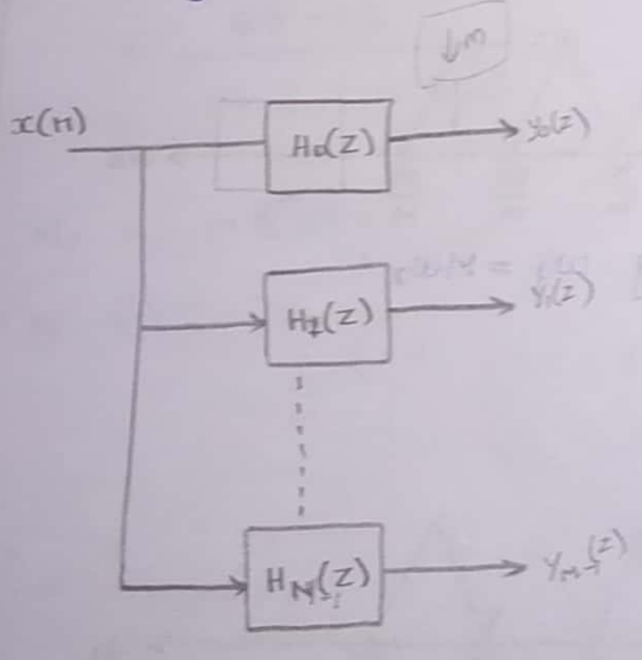
for K = even

let K = 4 $\frac{3\pi}{M} \le |\omega| \le \frac{4\pi}{M}$

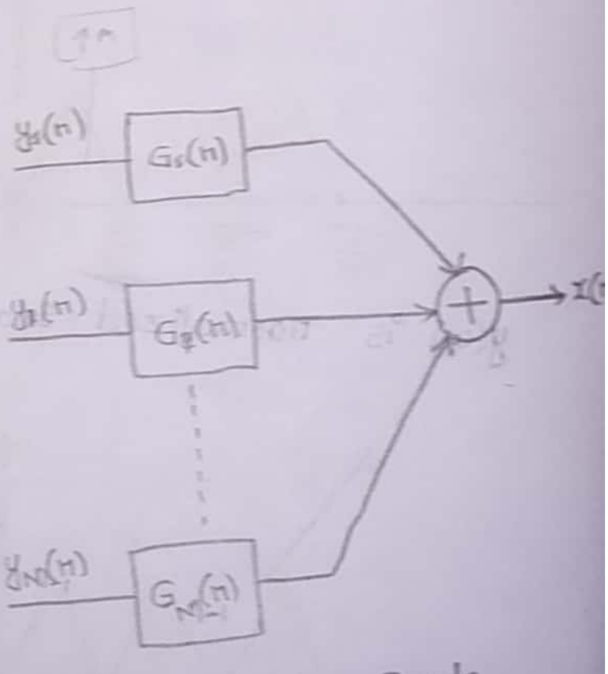


APPLICATIONS OF MULTIRATE SIGNAL PROCESSING:

1. Analysis filter Bank
2. Synthesis filter Bank.



Analysis Filter Bank



Synthesis filter Bank

The Analysis filter bank is used for spectrum analysis in which a signal is divided into set of sub band signals. All the Subband signals contains Same frequency and Same Value.

The Synthesis filter bank is a set of filters are used to combine or synthesize a number of sub-band signals into a single signal.

APPLICATIONS

- Design of Phase shifters
- Interfacing of Digital systems with Different Sampling Rates
- Implementation of Narrowband lowpass filter
- Sub-band coding of speech signals