UNIT - I ANTENNA AND WAVE PROPAGATION

ANTENNA BASICS AND DIPOLE ANTENNA

Introduction:

⇒ Antenna was invented by "Galileo Marconi" in the period of radio inventions

⇒ Antena is a electrochemical device which converts the electrical signals into electromagnetic waves (or) radio waves

⇒ The Marconi was classified the antennas into two types They are

1) Marconi Antenna and
2) Hertz antenna

1) Marconi Antenna - Below 1MHz
2) Hertz antenna - Above 1MHz or 2MHz

⇒ which antenna radiates the uniformly in all direction that antenna is called isotropic antenna

⇒ which antenna radiates only one direction that antenna is called directional antenna (or) Omny directional anten

⇒ which antenna radiates in only specified directional direction is called directional antenna

⇒ which antenna radiates in any two directions that type of antenna is called bidirectional antenna

* Antenna Basics

⇒ A radio antenna can be defined as the structure associated with the region of transition between the wave and freespace (or) viceversa

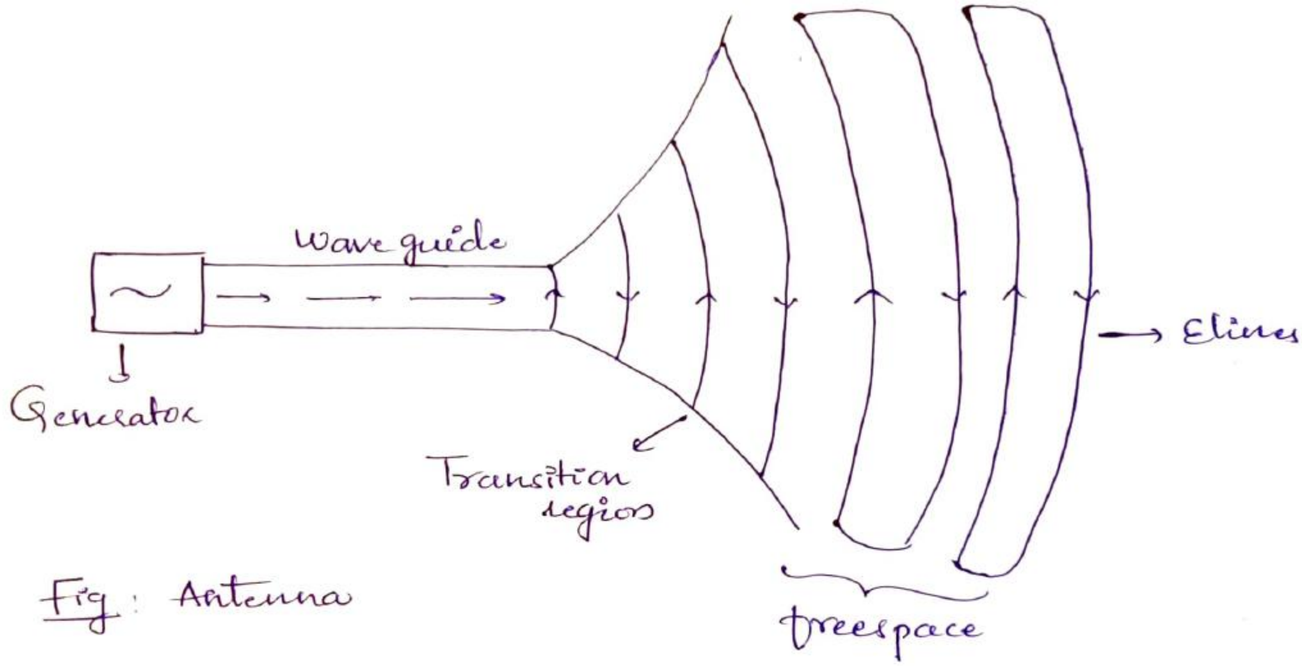


Fig: Antenna

→ An antenna is a transition device (or) transducer b/w a guided wave and free space wave (or) vice-versa

⇒ Then the radiation is produced by the accelerated and decelerated of charge

⇒ The basic Equation of radiation may be expressed as

$$IL = QV$$

where

I = time changing current (Amp/sec)

L = length of current Element (in mts)

Q = charge (in coloumbs)

V = time change of velocity.

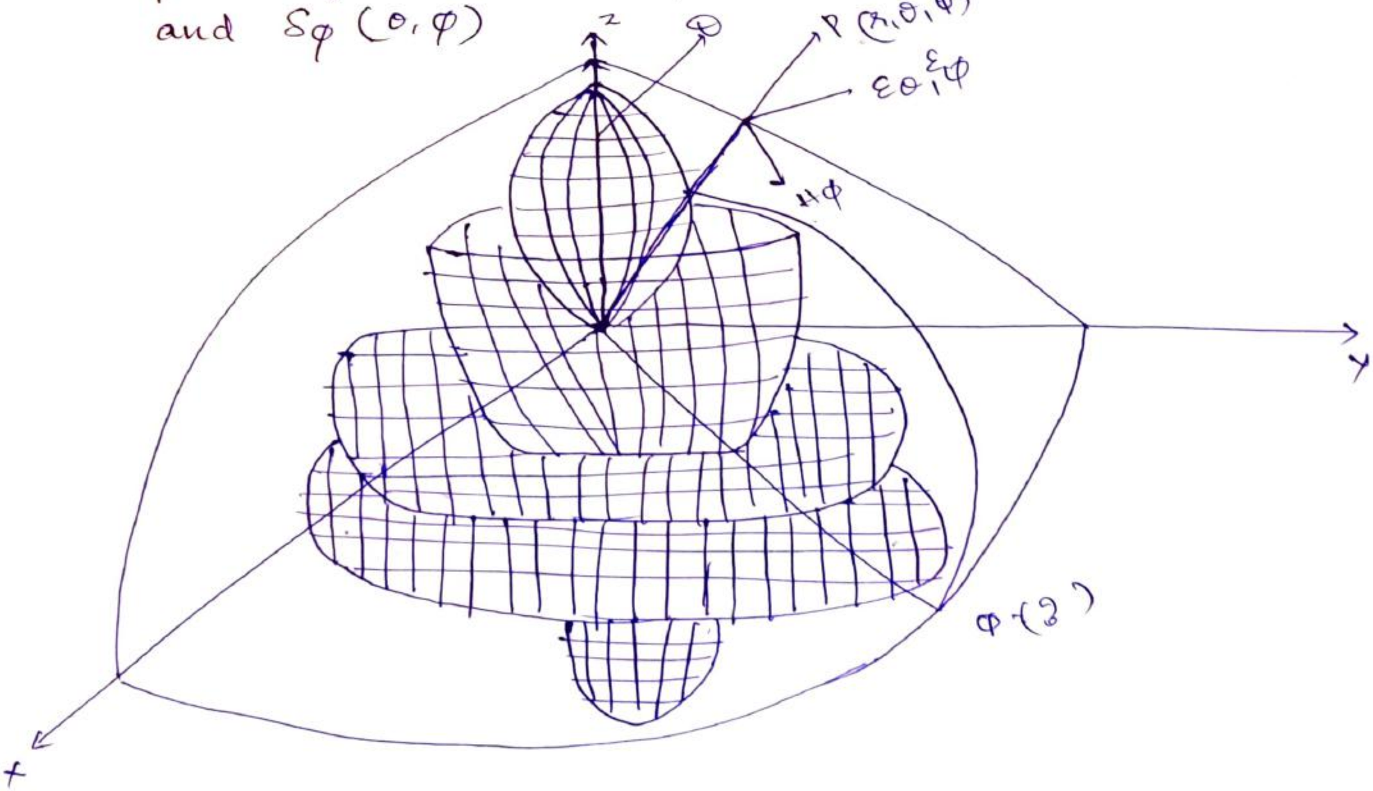
Patterns:-

⇒ A three dimensional field pattern with pattern radius proportional to the field intensity in the direction of θ & ϕ

⇒ The pattern has its main lobe in the z-direction with minor lobes in other directions

⇒ To completely specify the radiation pattern with respect to field intensity and polarization requires the three patterns.

1. The θ component of the Electric field as a function of angles θ and ϕ are $E_\theta(\theta,\phi)$
2. The ϕ components of the Electric field as a function of angles θ and ϕ are $E_\phi(\theta,\phi)$
3. The phases of fields as a function of angles θ, ϕ are $\delta_\theta(\theta,\phi)$ and $\delta_\phi(\theta,\phi)$



→ Any field pattern can be represented in spherical coordinate system only

→ The plane cuts through the main lobe of two cuts at right angle that plane is called principal plane

→ The angular beam width at the half power level (or) half power beam width (HPBW) - 3db and beam width b/w first nulls (FNBW) are important parameters of the pattern

→ The normalized are relative field pattern for the electric field is obtained from dividing a field component by its maximum value, so therefore the normalised field pattern

$$E_\theta(\theta,\phi)_n = \frac{E_\theta(\theta,\phi)}{E_\theta(\theta,\phi)_{max}}$$

→ The half power level occurs at the angles θ & ϕ for $E_\theta(\theta,\phi)$ &

$$E_\theta(\theta,\phi) = \frac{1}{\sqrt{2}} \Rightarrow 0.707$$

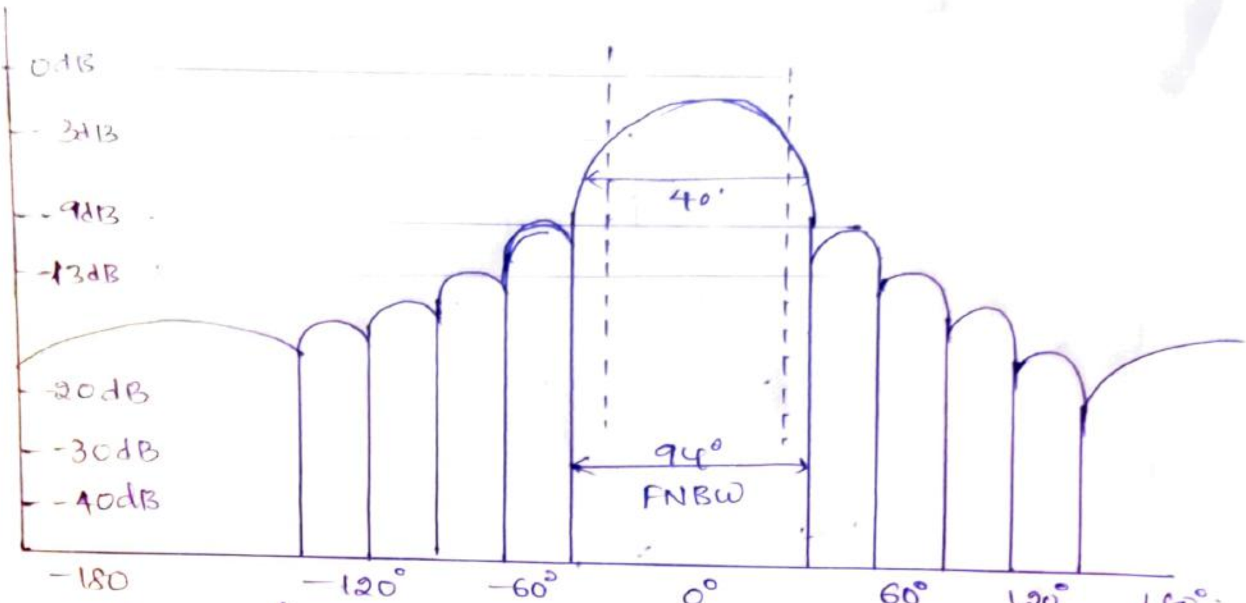
→ The pattern can be expressed in terms of power per unit area (or) pointing area $S(\theta,\phi)$ is given by

∴ Normalised power pattern $= P_n(\theta,\phi)_n$

$$= \frac{S(\theta,\phi)}{S(\theta,\phi)_m}$$

where

$$S(\theta,\phi) = \frac{\left[E_\theta^2(\theta,\phi) + E_\phi^2(\theta,\phi)\right]}{Z_0} \text{ watts/meter}$$



Antenna Parameters

Physical quantities
- size
- weight

Circuit quantities
* Antenna impedance (Z_A)
* Radiation resistance (R_r)
* Antenna temperature (T_A)

Current distribution

Antenna transition region

- Fields $\begin{cases} E_\theta(\theta,\phi) \\ E_\phi(\theta,\phi) \\ \delta(\theta,\phi) \end{cases}$
- polarisation (LP, CP, EP)
- power patterns ($P_n(\theta,\phi)$)
- Beam area (Ω_A)
- Directivity (D)
- Gain (G)
  Effective Aperture (A_e)
- Antenna cross (σ)

Beam area (or) Solid angle (Ω_A) :

=) In polar two dimensional coordinates an incremental area dA on the surface of sphere is product of

Here Zo - Intrinsic impedance of the free space

$Z_0 = 376.7\ \Omega$

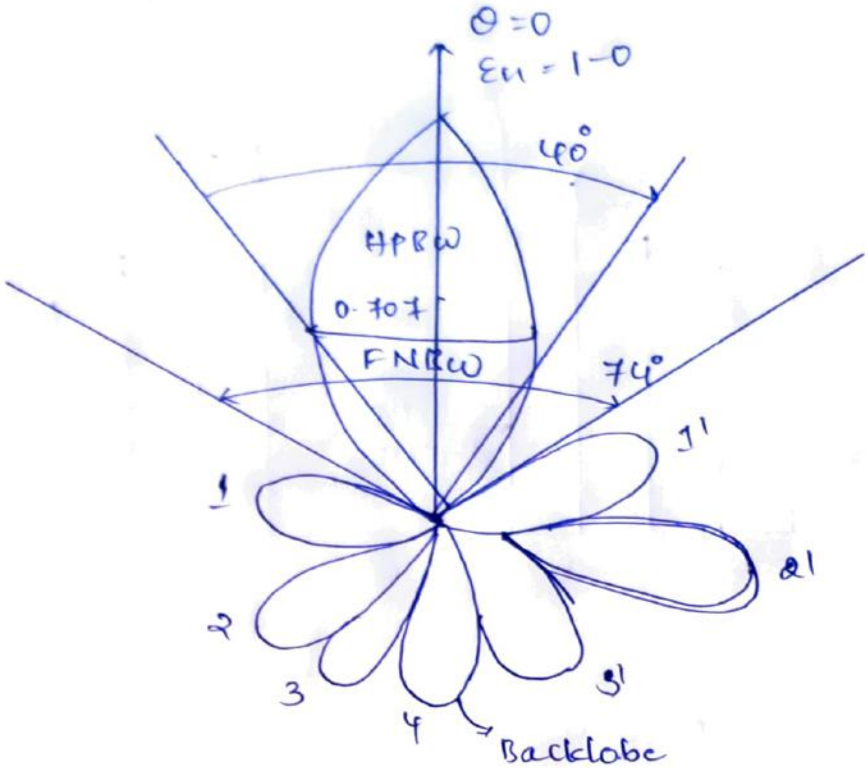


Fig: Normalised electric field pattern

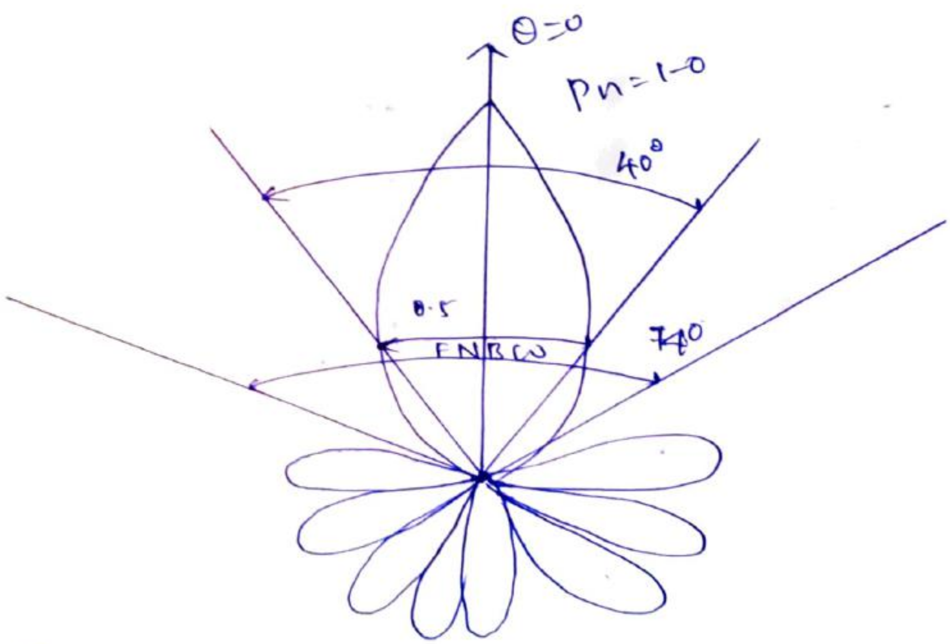
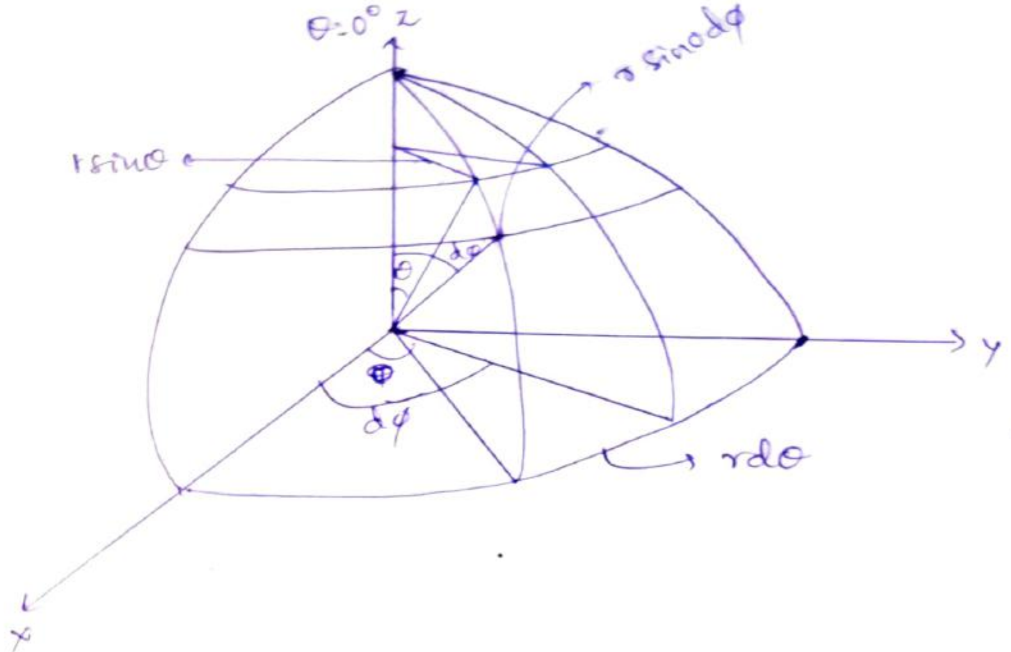


Fig: Normalised power field pattern.

The length $rd\theta$ in θ direction and $r\sin\theta\, d\phi$ in ϕ direction

$$dA = (r\,d\theta)\cdot(r\sin\theta\, d\phi)$$
$$= r^2 \sin\theta\, d\theta\, d\phi$$
$$= r^2\, d\Omega$$

where $d\Omega$ is solid angle expressed in steradians



→ The beam area (or) beam solid angle of an antenna is given by the integral of the normalised power pattern over a sphere

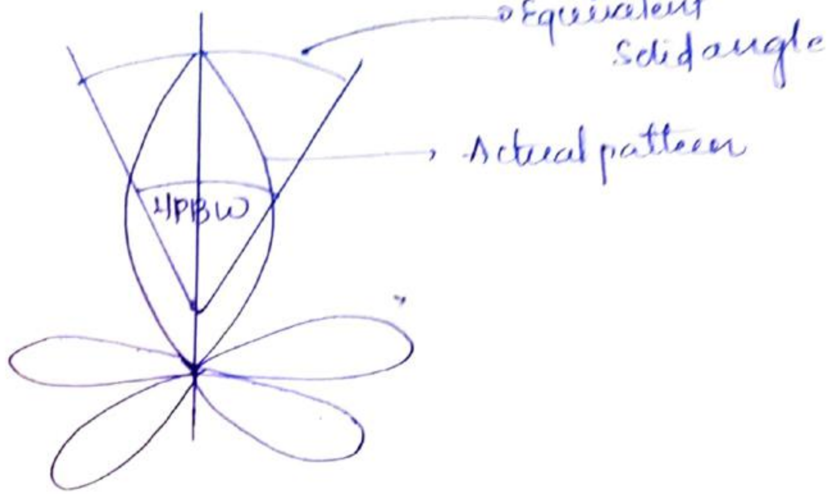
$$\Omega_A = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_n(\theta,\phi)\, \underline{\sin\theta\, d\theta\, d\phi}$$

$$\Omega_A = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_n(\theta,\phi)\, d\Omega \quad sr.$$

Second definition

The beam area of an antenna in terms of angles substanded by the half power points of main lobe in the two principal planes minor lobes being neglected

Beam area $\Omega_A = \phi_{HP} \theta_{HP}$ Sr



<u>Radiation Intensity (U):</u>

⇒ The power radiated from an antenna per unit solid angle is called radiation intensity (U)

$$U(\theta, \phi) = \frac{\text{Radiated power}}{\text{unit solid angle}} \text{ watts/sr}$$

→ The radiation intensity interms of normalised power pattern

$$P_n(\theta, \phi) = \frac{U(\theta, \phi)}{U(\theta, \phi)_{max}}$$

<u>Beam Efficiency:</u>

⇒ The total beam area Ω_A consist of mainlobe area Ω_M plus Ω_m (minor lobe)

$$\therefore \Omega_A = \Omega_M + \Omega_m \text{ Sr}$$

⇒ The rate of main lobe area to the total beam area is called Beam Efficiency

$$\text{Beam Efficiency} = \frac{\Omega_M}{\Omega_A}, \varepsilon_M$$

The ratio of minor lobe area to the total beam area is called Stray factor

$$\text{Stray factor} = \frac{\Omega_m}{\Omega_A} = \varepsilon_m$$

* Directivity:

⇒ Directivity is defined as the rate of maximum radiation Intensity to the average radiation intensity. It is denoted by D

$$\therefore D = \frac{U_{max}}{U_{avg}} = \frac{\text{Maximum radiation Intensity}}{\text{Average radiation Intensity}}$$

⇒ Directivity in terms of radiated power

$$D = 4\pi \times \frac{\text{Maximum radiation intensity}}{\text{Total radiated power.}}$$

⇒ The directivity of antenna is Equal to the rate of maximum power density to average power density

$$D = \frac{P(\theta,\phi)}{P(\theta,\phi)_{avg}}$$

→ The directivity of an antenna is dimensionless and it is greater than (or) equal to 1 (≥ 1)

→ The average power density is given by

$$P(\theta,\phi)_{avg} = \frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P(\theta,\phi)\sin\theta\, d\theta\, d\phi$$

$$= \frac{1}{4\pi}\iint_{4\pi} P(\theta,\phi)\, d\Omega$$

$\therefore$ Directivity $D = \dfrac{P(\theta,\phi)\ max}{\frac{1}{4\pi}\iint_{4\pi} P(\theta,\phi)\, d\Omega}$

$$= \frac{4\pi}{\iint_{4\pi} \frac{P(\theta,\phi)}{P(\theta,\phi)_{max}}\, d\Omega}$$

$$D = \frac{4\pi}{\Omega_A} \quad (\because \text{from the beam area})$$

+ Gain :-

-> The gain (or) power gain of an antenna in a certain direction (θ,ϕ) is defined as

$$G(\theta,\phi) = \frac{\text{Radiation intensity}}{\text{Total input power}/4\pi}$$

$$= \frac{4\pi U(\theta,\phi)}{P_{in}}$$

(or)

The gain can be measured by comparing the maximum power density of an antenna under the test with reference antenna of known gain

$$\text{Gain } G = \frac{P_{max}(\text{Antenna under test})}{P_{max}(\text{Reference antenna})} \times (\text{Gain of reference antenna})$$

The ratio of the power radiated in a particular direction θ,ϕ to the actual power input to the antenna is called power gain of antenna. It is denoted by $G_p(\theta,\phi)$

* Antenna Effeceency factor

→ The antenna Effeciency factor is defined as the ratio of gain to the directivity

$$\therefore \frac{G}{D} = K$$

where K efficiency factor $(0 \le K \le 1)$ it is dimensionless

→ **Relation of G and D**

→ The relation of gain and directivity is

$$G(\theta,\phi) = \varepsilon D(\theta,\phi)$$

from the definition of gain $G(\theta,\phi)$ is

$$G(\theta,\phi) = \frac{\text{Radiation Intensity}}{\text{Total i/p power}/4\pi}$$

$$= \frac{4\pi\, U(\theta,\phi)}{P_{in}}$$

Here P_{in} is i/p Power of antenna. It is related to radiated power P_{rad}

$$\therefore P_{in} = \varepsilon P_{rad}$$

∴ From the above two definitions

$$\therefore G_0 = \frac{4\pi\, U(\theta,\phi)}{P_{in}} = K D_0$$

Here

G_0 = maximum gain of antenna

D_0 = maximum directivity

* Directivity resolution (or) redirectivity gain resolution

→ The directivity resolution is defined as the half beam width of first two nulls

$$\text{Directivity resolution} = \frac{FNBW}{2}$$

→ It is approximately Equal to HPBW i.e,

$$HPBW \simeq \frac{FNBW}{2}$$

→ The directivity resolution is defined as the product of the $\left[\frac{FNBW}{2}\right]$ of two principle planes.

$$\therefore \text{Directivity resolution} = \left(\frac{FNBW}{2}\right)_\theta \cdot \left(\frac{FNBW}{2}\right)_\phi$$

→ If N no of antennas (or). point sources in a certain region, the total average, radiated power of an antenna is

$$P_{avg} = \frac{4\pi}{N}$$

Interms of beam area . $N = \frac{4\pi}{\Omega_A}$

⇒ From the definition of directivity D is

$$D = \frac{4\pi}{\Omega_A}$$

from the above two Equations $\boxed{N = D}$

Effective Height,

→ Effective height of antenna is defined as the ratio of incident i/p voltage to incident Electric field

$$\therefore h = \frac{V}{E}$$

here h = Effective height of antenna

→ Measurement of Effective height is meters

Front to Back ratio (FBR)

⇒ The ratio of radiated power in the desired direction to the opposite direction

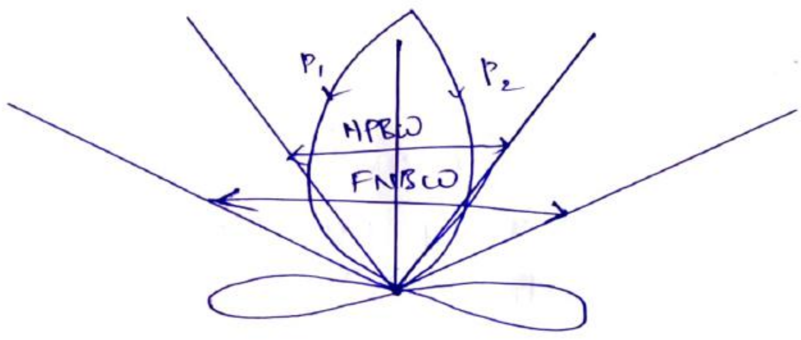
$$\therefore FBR = \frac{\text{Radiated power in desired direction}}{\text{Radiated power in opposite direction}}$$

* Atenna Beamwidth

⇒ Antenna beam width is defined as the angular width between the two point on major lobe of radiation pattern where the radiated power decreases to the half of its maximum value

→ The beam width also called halfpower beam width - Because it is measured b/w two points on the mainlobe where the power is half of its maximum value.

– It is also called as 3dB beam width



→ The antenna radiates pattern is described in terms of angular width b/w first two nulls (or) first side lobes. is called beam width b/w first nulls (or) first null beam width

* Effective aperture :-

→ The effective aperture is ability of an antenna extract energy from EM waves.

→ It is also called effective area (A_e)

→ Effective aperture is defined as the ratio of power recieved in the load to the average power density produced at point

$$\therefore A_e \frac{P_{recieved}}{P_{avg}} m^2 \qquad A_e = \frac{I_{rms}^2 \cdot R_L}{P_{avg}}$$

→ Types of aperture

→ Aperture are classified into three types. They are

1) Scattering aperture
2) loss aperture
3) collective aperture

1) Scattering Aperture :-

→ Ratio of power recieved by radiation resistance (Rrad) to the average power density produced at point. It is called scattering aperture (As)

$$\therefore A_s = \frac{I_{rms}^2 \cdot R_{rad}}{P_{avg}}$$

2) Loss aperture

→ It is defined as the ratio of power dissipated by the loss of resistance of an antenna to the average power density at point

$$\therefore A_L = \frac{I_{rms}^2 \cdot R_{loss}}{P_{avg}}$$

3) collective Aperture

The collective aperture is summation of effective aperture, scattering aperture and loss aperture

$$\therefore A_c = A_e + A_s + A_l$$

$$\therefore A_c = \left[\frac{I_{rms}^2 R_L}{P_{avg}} + \frac{I_{rms}^2 R_{rad}}{P_{avg}} + \frac{I_{rms}^2 R_{loss}}{P_{avg}}\right]$$

physical aperture (or) antenna cross section (or) physical area :-

⇒ It is defined as the actual physical cross section of an antenna (normal) to the direction of propagation of EM waves towards an antenna.

⇒ The ratio of maximum effective aperture to the physical aperture of an antenna is known as absorption ratio.

$$\therefore \gamma = \frac{(A_e)_{max}}{A_p}$$

Here $0 \leq \gamma \leq \infty$

* Radiation Resistance

An antenna radiates power into free space in the form of electromagnetic waves so the power dissipated is given by $P_d = I^2 R$

Assume total power dissipation in the form of EM waves is $R = \frac{P_d}{I^2}$

* Radio Communication link

→ Prof. H.T friis developed the friis transmission formula in 1946 at bell laboratories

→ This formula is useful to obtaining the power recieved by reciever over radio communication link

→ This formula is based on the concept of effective apeture

→ consider a lossless and matched antennas a radio-communication link b/w transmitter and reciever antennas there are separated by a 'r'

→ Let P_t the power radiated from transmitting antenna and A_{et} is the effective apeture of the transmitting antenna

→ Similarly P_r is the radiated power from the recierer and A_{er} is the effective apeture of the recierer.

→ Assume transmitter antenna is isotropic the power recieved per unit area at the recieving antenna is given by

$$S_r = \frac{P_t}{4\pi r^2}$$

Here S_r is power per unit area.

If antenna has G_t (transmission gain), the power per unit area available at eeciering antenna will be increased in the propogation

$$S_r = \frac{P_t \cdot G_t}{4\pi r^2}$$

⟹ If antenna has G_t (transmission gain), the power per unit area available at receiving antenna will be increased in the propogation

$$S_r = \frac{P_t G_t}{4\pi r^2}$$

→ Now power collected by the receiving antenna is

$$P_r = S_r A_{er}$$

$$\therefore P_r = \frac{P_t G_t A_{er}}{4\pi r^2}$$

→ The gain of transmitting antenna is given by

$$G_t = \frac{4\pi A_{et}}{\lambda^2}$$

∴ Power collected at reciever antenna is

$$P_r = \frac{P_t \frac{4\pi A_{et}}{\lambda^2} A_{er}}{4\pi r^2}$$

$$= \frac{P_t \cdot A_{et} \cdot A_{er}}{\lambda^2 r^2}$$

$$\frac{P_r}{P_t} = \frac{A_{et} \cdot A_{er}}{\lambda^2 r^2}$$

Polarization :

⇒ Polarization is nothing but physical orientation of Electromagnetic wave in free space

⇒ The polarization varies with the distance from the center of the antenna

⇒ The polarization of EM wave describe the time varying direction and relative magnitude of the electric field vector

⇒ The polarization of Electric field can be obtained by observing the field along the direction of propagation.

⇒ The polarization can be classified as

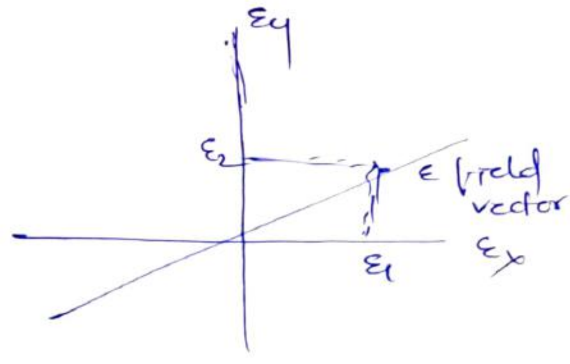
1) Linear polarization
2) circular polarization
3) Elliptical polarization.

1) Linear polarisation

⇒ when the electric field vector at any point in the freespace is the function of time

⇒ If it is directed always along the line the polarization is called linear polarization. and the field said to be linearly polarised field

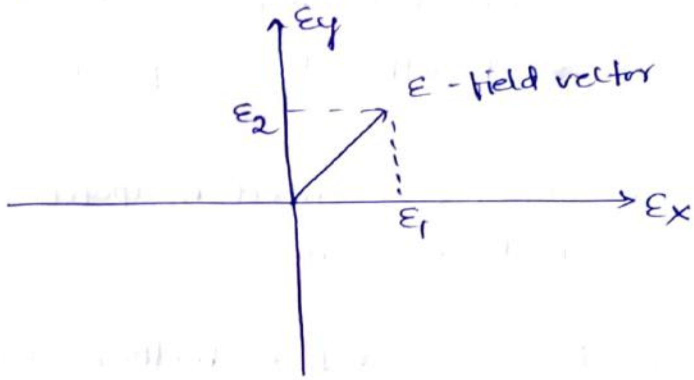
⇒ when the electric field vector E lies in the vertical plane then the wave is said to be vertically polarized wave



⇒ when the electric field vector $\bar{E}$ lies in the horizontal plane then the wave said to be horizontally polarised wave

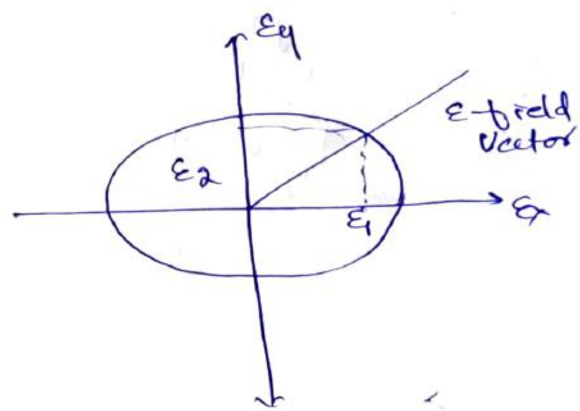
2) <u>Circular Polarization</u> :

→ when the instantaneous electric field vector $\bar{E}$ locus is in a circle then the polarization is called circular polarization



3) <u>Elliptical Polarization</u>

⇒ when the instantaneous Electric field vector $\bar{E}$ locus in the ellipse then the polarisation is called Elliptical polarization



* <u>Fields from oscillating dipole</u>

⇒ Let us consider a dipole antenna with two Equal and opposite charges oscillating up and down with simple harmonic motion.

⇒ Consider that the lobe" be the maximum separation between two equal and opposite charges.

→ The charges undergoes a oscillation and deoscillation along the dipole then the dipole is radius.

i) The Equal and opposite charges are maximum separation 'lo' at $t=0$ hence oscillation of charges is maximum and they are in reverse direction, here current is zero

ii) Let T be the period of oscillation then the period $t = T/8$ the charges move towards each other the shape of field line is different

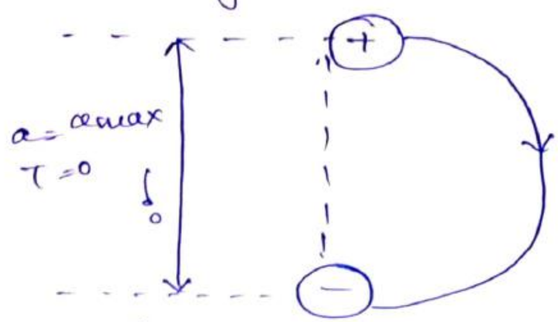
iii) After period $t = T/4$ the charges reach at midpoint of dipole at they instant the oscillation of charges is zero with current is maximum

Then the field line crosses to each other and start detaching

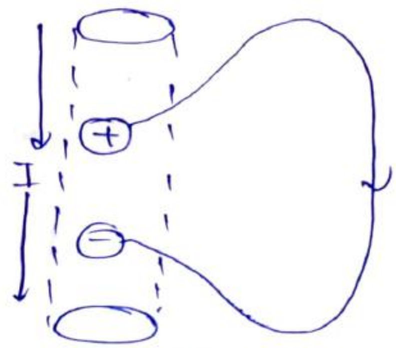
iv) After period $t = \frac{3T}{8}$ a field line completely detached and released from the dipole, after detachment a new field line starts

v) After $t = T/2$ the charges are reach to the exactly opposite ends of the dipole and charges are get maximum oscillation

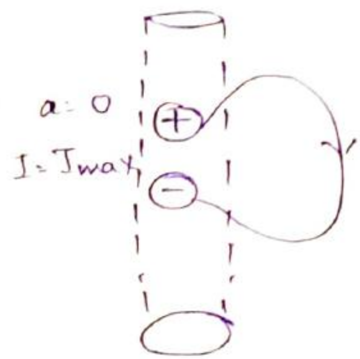
→ In this way the field lines are detached into freespace by oscillating dipole



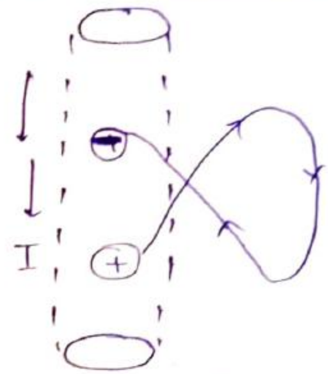
At $t=0$



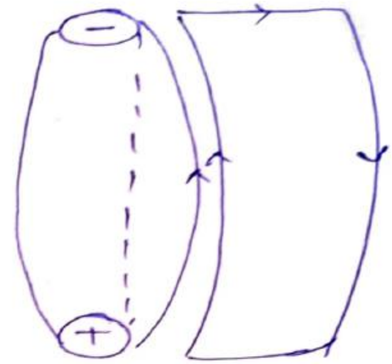
At $t = T/8$



At $t = T/4$



At $t = 3T/8$



Antenna field zones

⇒ The field around the antenna may be divided into two principle regions. They are

1) Near field
2) Far field

⇒ The near field is also called as fresnel zone and far field also known as fraunhofer zone.

⇒ These two zones are divided by a one boundary line. This boundary line radius from the antenna is $R = \frac{2L^2}{\lambda}$

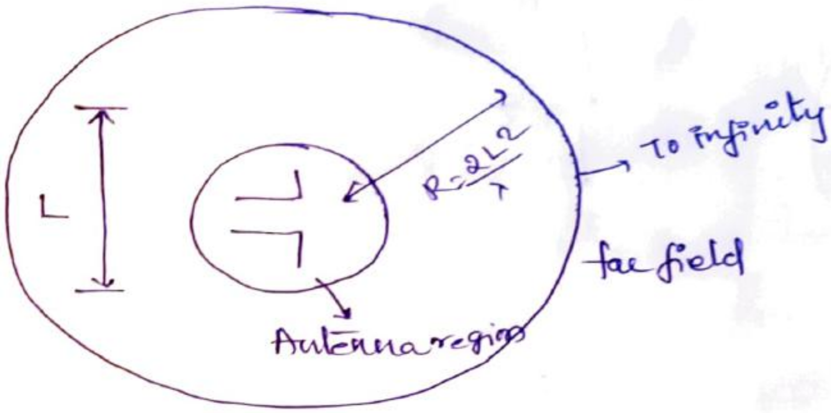
here L = maximum dimension of antenna
λ = wavelength

⇒ In far field measurable components are transfer to the radial direction from the antenna and total power flow is directed radially outward

⇒ In far field the shape of field pattern is independent of distance

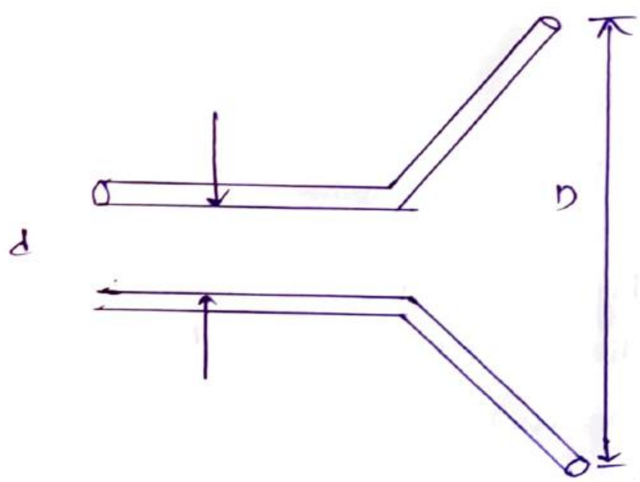
⇒ In near field the longitudinal component of the electric field may be significant and powerflow is not entirely radial

⇒ In the near field shape of the field pattern depends on the distance

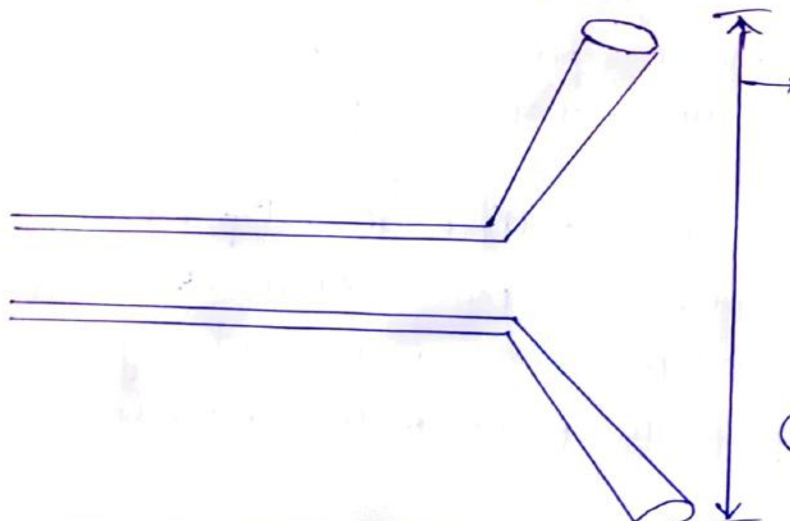


Shape Impedance consideration

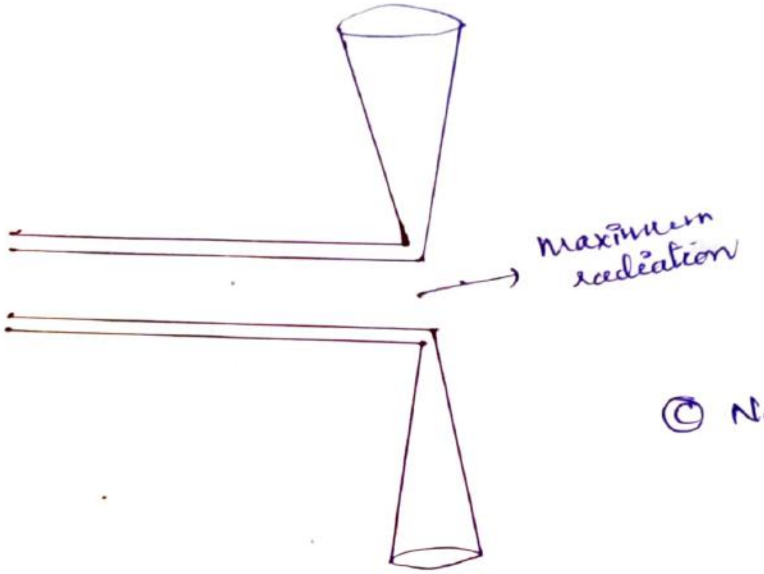
⟹ It is possible in many cases to decide qualitative behaviour of an antenna from its shape



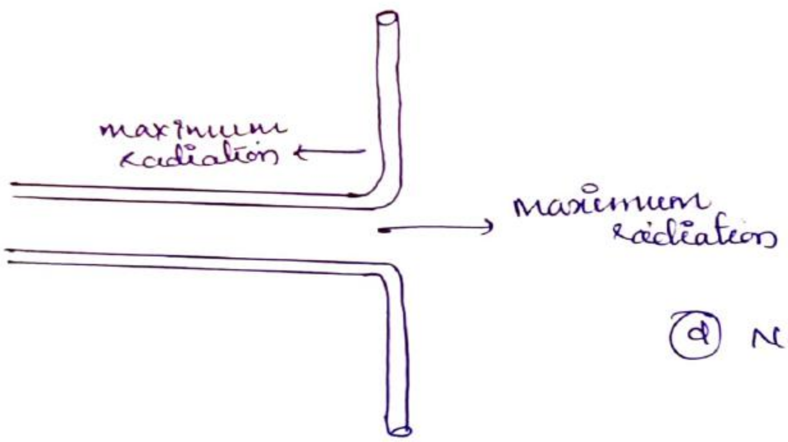
ⓐ wideband width D/d



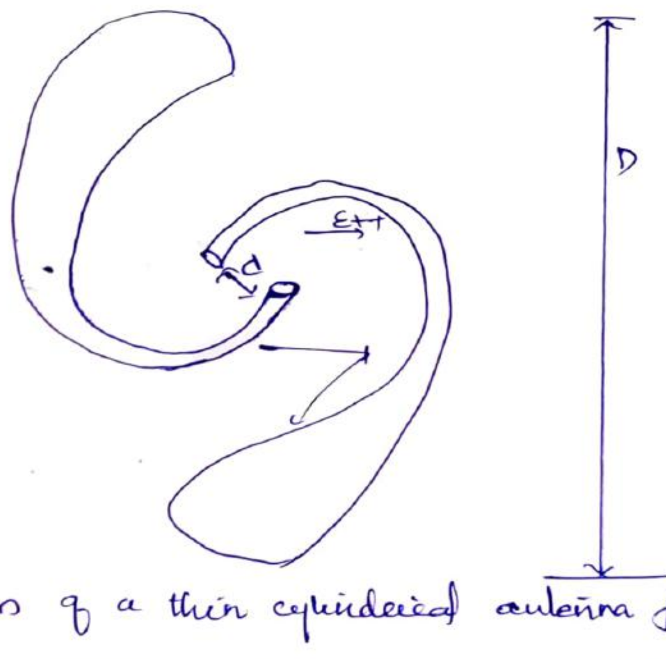
ⓑ Narrow band width



(c) Narrower bandwidth



(d) Narrow band width = 1.5/1



(e) wide bandwidth D/d.

Fig: Evaluation of a thin cylinderical antenna from open Ended out the line

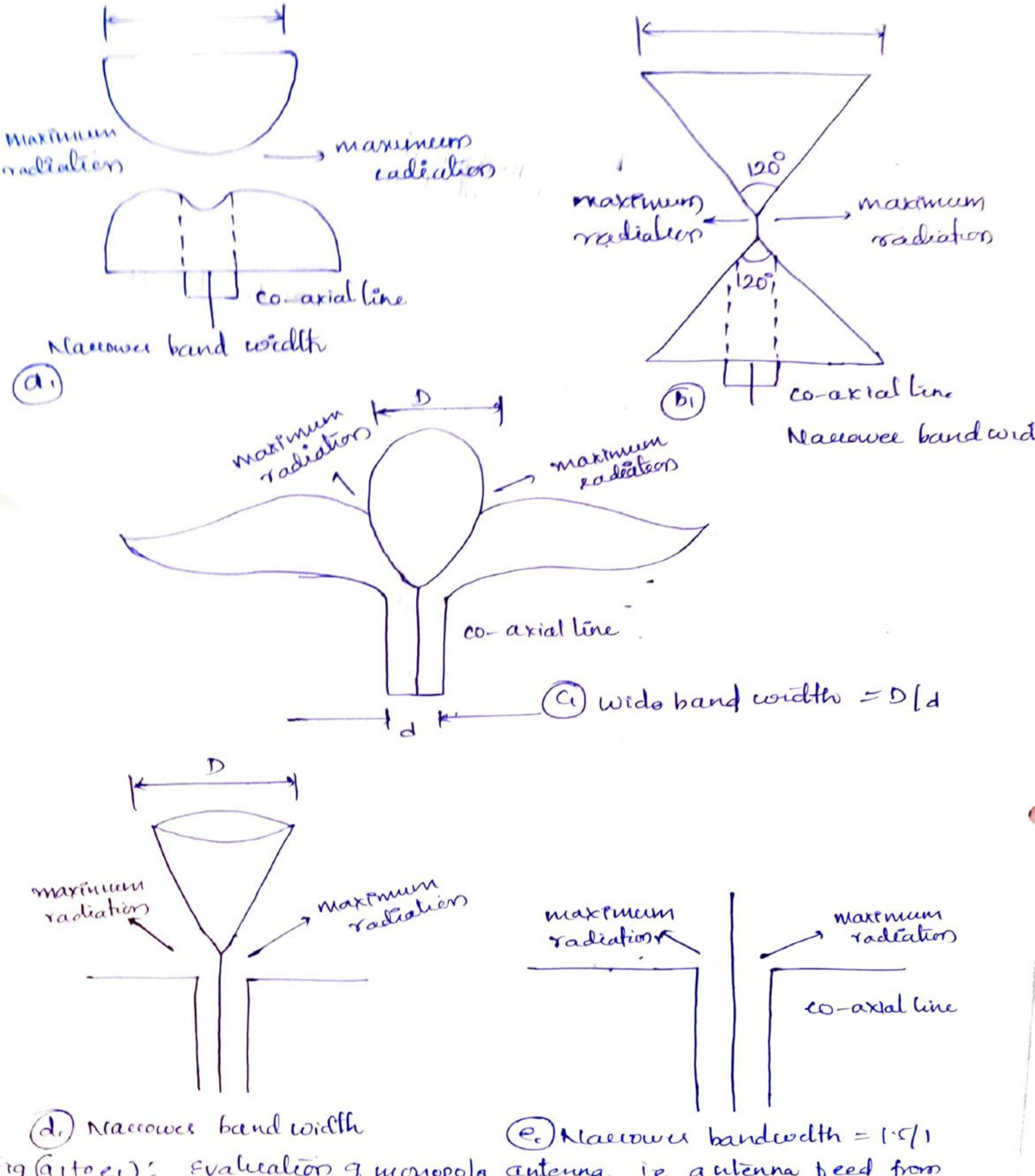


Fig (a1 to e1): Evaluation of monopole antenna ie antenna feed from co-axial transmission line

Antenna Temperature (TA)

→ Antenna temperature is defined as the total temperature around the antenna equipments and heat generated at the end of the both input and o/p terminals of an antenna

⇒ It is denoted by T_A

⇒ The measurements of antenna Temperature is °C Farenheit kelvin

* Antenna Impedance

⇒ The antenna impedance is defined as the ratio of voltage induced at antenna input terminal to the antenna i/p current

* Antenna Transition region

⇒ The area which converts electrical signals to EM waves then that area is called antenna transition region

Part-B Dipole Antenna

* Basic Maxwell Equations :

⇒ For time varying fields the maxwell equations in the form of differential is given by

$$\nabla \times \bar{E} = -\mu \frac{\partial H}{\partial t} \longrightarrow (1)$$

$$\nabla \times \bar{H} = \bar{J} + \varepsilon \cdot \frac{\partial \bar{E}}{\partial t} \longrightarrow (2)$$

$$\nabla \cdot \bar{E} = \frac{\rho_v}{\varepsilon} \longrightarrow (3)$$

$$\nabla \cdot \bar{H} = 0 \longrightarrow (4)$$

from Eq(4) it is clear that the divergence of $\bar{H}$ is '0' but

From the vector identity the divergence of curl of a vector field is 0

This it must be expressed as a curl of some vector so we define the vector potential $\bar{A}$

$$\mu \bar{H} = \nabla \times \bar{A} \longrightarrow (5)$$

$$\boxed{\bar{H} = \frac{\nabla \times \bar{A}}{\mu}}$$

Put the value of $\bar{H}$ in Eq (1) then

$$\nabla \times \bar{E} = -\mu \frac{\partial}{\partial t}\left(\frac{\nabla \times \bar{A}}{\mu}\right)$$

$$\nabla \times \bar{E} = \left[\nabla \times \frac{\partial \bar{A}}{\partial t}\right]$$

$$\nabla \times \bar{E} + \nabla \times \frac{\partial \bar{A}}{\partial t} = 0$$

$$\nabla \times \left[\bar{E} + \frac{\partial \bar{A}}{\partial t}\right] = 0 \longrightarrow (6)$$

According to the vector identity the curl of gradient of a scalar is always zero. So from Eq (6) $\left[\bar{E} + \frac{\partial \bar{A}}{\partial t}\right]$ is defined as a gradient of scalar

Let us introduce the scalar potential 'V'

$$\therefore \bar{E} + \frac{\partial \bar{A}}{\partial t} = \nabla V \longrightarrow (7)$$

Then the Electric field strength

$$\boxed{\bar{E} = -\nabla V - \frac{\partial \bar{A}}{\partial t}} \longrightarrow (8)$$

from Eq (5) & Eq (8) It is clear that the Electric and magnetic fields $\bar{E}$ & $\bar{H}$ can be expressed interms of a scalar field V & Vector field $\bar{A}$

→ substituting the values of $\bar{E}$ and $\bar{H}$ forms Eq ⑦ & Eq ⑧ respectively in Eq ②

$$\nabla \times \left(\frac{\nabla \times \bar{A}}{\mu}\right) = \bar{J} + \varepsilon \frac{\partial}{\partial t}\left(-\nabla v - \frac{\partial \bar{A}}{\partial t}\right)$$

$$\nabla \times (\nabla \times \bar{A}) = \mu \bar{J} + \mu\varepsilon \left(-\nabla \frac{\partial v}{\partial t} - \frac{\partial^2 \bar{A}}{\partial t^2}\right) \longrightarrow ⑨$$

$$\boxed{\nabla \cdot (\nabla \cdot \bar{A}) - \nabla^2 \bar{A} = \mu \bar{J} - \mu \varepsilon \nabla \frac{\partial v}{\partial t} - \mu\varepsilon \frac{\partial^2 A}{\partial t^2}} \longrightarrow 10$$

Substituting the value of $\bar{E}$ from Eq(8) & in Eq(3)

$$\Rightarrow \nabla \left(-\nabla v - \frac{\partial \bar{A}}{\partial t}\right) = \frac{\rho_v}{\varepsilon}$$

$\nabla \times \nabla \times \bar{A} = \nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}$

$$\Rightarrow \nabla^2 v - \nabla \frac{\partial \bar{A}}{\partial t} = \frac{\rho_v}{\varepsilon}$$

$$\Rightarrow \boxed{\nabla^2 v + \nabla \frac{\partial \bar{A}}{\partial t} = -\frac{\rho_v}{\varepsilon}} \longrightarrow ⑪$$

* Helmholtz Theorem

Statement:

⟹ It states that any vector field can be defined uniquely if the curl and divergence of the field both are known at any point.

Proof:

The curl of $\bar{A}$ is already specified in Eq ⑦. we may choose divergence of $\bar{A}$ from Eq ⑩

$$\nabla \bar{A} = -\mu\varepsilon \cdot \frac{\partial v}{\partial t} \longrightarrow ⑫$$

The relation b/w $\bar{A}$ and v is known Lorentz gauge condition.

from eq (10)

$$\nabla^2 \bar{A} - \nabla(\nabla \bar{A}) = -\mu \bar{J} + \mu\varepsilon \nabla \frac{\partial V}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{A}}{\partial t^2}$$

$$\nabla^2 \bar{A} - \nabla\left(-\mu\varepsilon \cdot \frac{\partial V}{\partial t}\right) = -\mu \bar{J} + \mu\varepsilon \nabla \frac{\partial V}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{A}}{\partial t^2}$$

$$\nabla^2 \bar{A} + \cancel{\mu\varepsilon \nabla \frac{\partial V}{\partial t}} = -\mu \bar{J} + \cancel{\mu\varepsilon \nabla \frac{\partial V}{\partial t}} + \mu\varepsilon \frac{\partial^2 \bar{A}}{\partial t^2}$$

$$\nabla^2 \bar{A} = -\mu \bar{J} + \mu\varepsilon \frac{\partial^2 \bar{A}}{\partial t^2}$$

$$\nabla^2 \bar{A} - \mu\varepsilon \frac{\partial^2 \bar{A}}{\partial t^2} = -\mu \bar{J} \longrightarrow (13)$$

Similarly

from eq (11) $\Rightarrow$

$$\nabla^2 V + \nabla \frac{\partial \bar{A}}{\partial t} = \frac{-\rho_v}{\varepsilon}$$

$$\nabla^2 V + \frac{\partial}{\partial t}(\nabla \bar{A}) = \frac{-\rho_v}{\varepsilon}$$

$$\nabla^2 V + \frac{\partial}{\partial t}\left(-\mu\varepsilon \frac{\partial V}{\partial t}\right) = \frac{-\rho_v}{\varepsilon}$$

$$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = \frac{-\rho_v}{\varepsilon} \longrightarrow (14)$$

The Equations (13) & (14) are standard wave equations including with source terms

$$A(r,t) = \frac{\mu}{4\pi} \int_V \frac{\bar{J}(r', t - R/u)}{} \, dr'$$

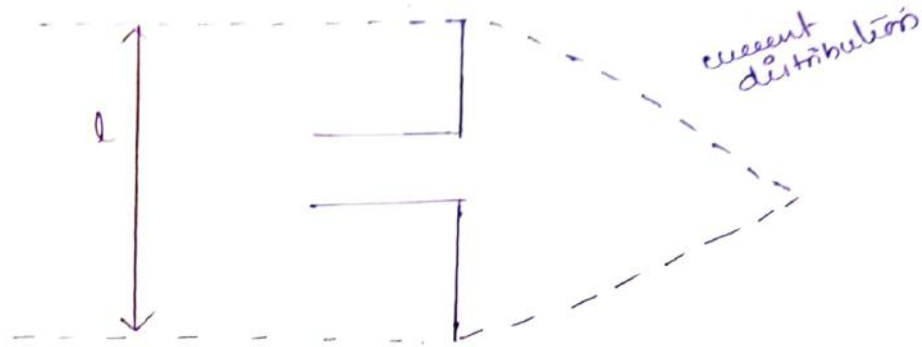
$$V(r,t) = \frac{1}{4\pi\varepsilon} \int_V \frac{\rho_v(r', t - R/u)}{R} \, dv'$$

* Radiation from Short Electric dipole

→ The length of an antenna is very short in wavelength the current amplitude on a such antenna is maximum at the center and it is decreased uniformly to zero at the ends.

→ If we consider the same current I flows through the antenna and current element the dipole antenna radiates only one quarter (1/4) of power i.e, radiated by current element.

→ Because of the field strengths at every point on the short dipole reduces to half of the values for the current element
⇒ Hence the power density to ¼ (one quarter)
∴ The radiation resistance for short dipole is ¼th times that of the current element



⇒ The radiation resistance of short dipole antenna is given by

$$R_{rad} = \frac{\left[80\pi^2 \left(l/\lambda\right)^2\right]}{4}$$

$$\boxed{R_{rad} = 20\pi^2 \left(l/\lambda\right)^2}$$

where l = length of short dipole antenna
λ = wavelength of antenna

Power radiation from short electric dipole is

$$\boxed{P_{rad} = \frac{V_{in}^2}{R_{rad}}}$$

Field components

The field components can refer the field which formed by same physical quantities

The fields can be classified as.

electric field (E- field)
Magnetic field (H- field)
EH-field (or) EM (field)

E-field:

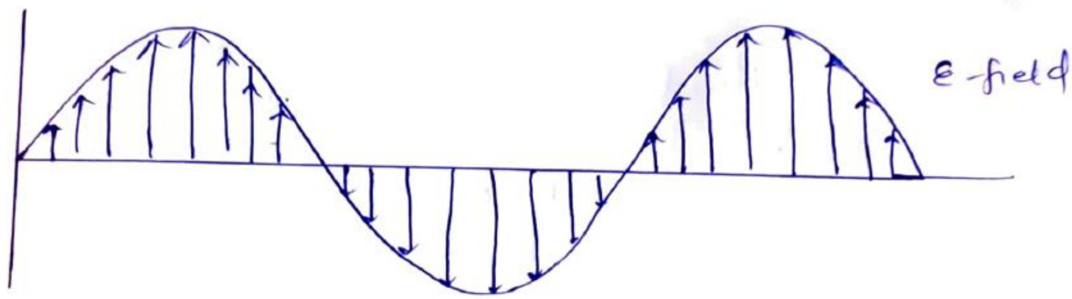
The Electric field can be formed by oscillation of range. The field measured by θ and φ components of a principal plane

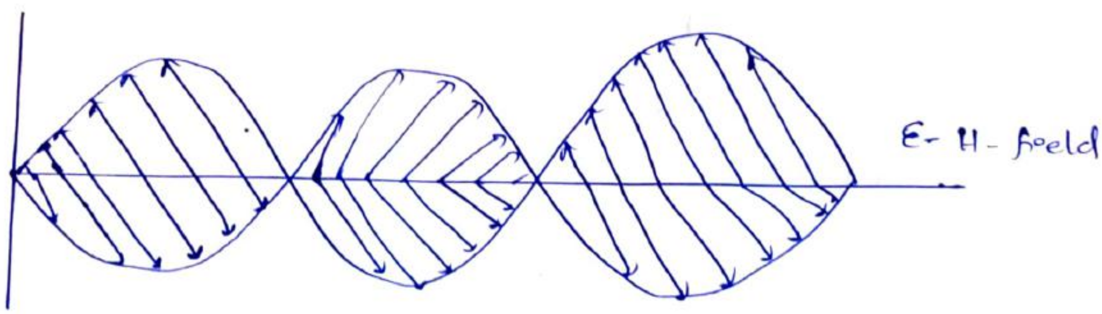
H-field

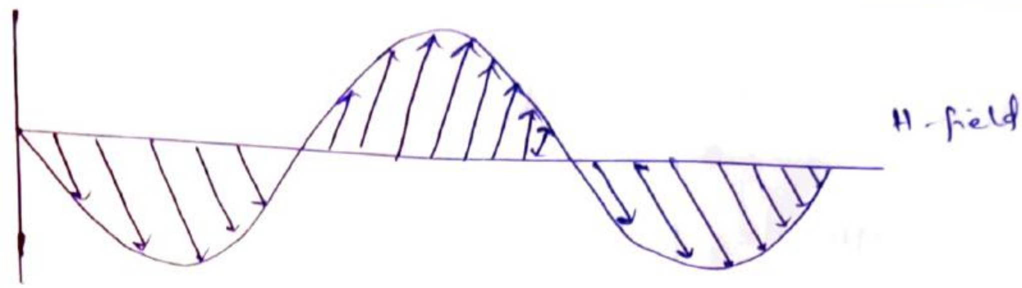
The H-field can be formed by current flow. It is measured by θ & φ components in a principle plane

3) EH field (or) EM field

-> It is formed by the combination of E-field and H-field
=> E field and H field are perpendicular to each other







Power radiated by halfwave dipole and quarter wave monopole

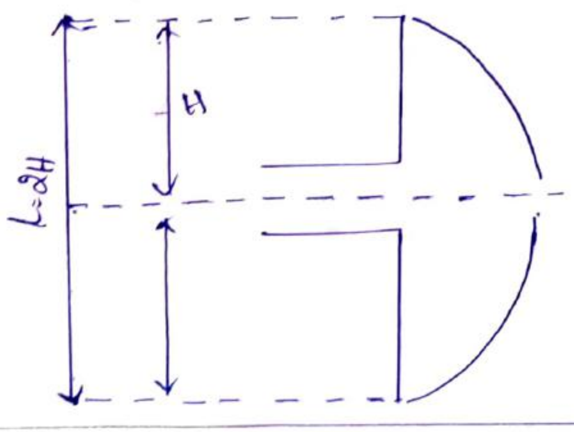
⇒ A dipole antenna is a vertical radiator fed in center, it produces maximum radiation in the plane normal to the axis

⇒ The vertical antenna of height $h = \lambda/2$ produces the radiation characteristics

- In general an antenna requires large current to radiate large amount of power to generate large current at radiofrequencies are practically impossible

⇒ The halfwave dipole consist of two legs each of the length is $\lambda/2$. The physical length of halfwave dipole at frequency of operation is $\lambda/2$ in free space

⇒ The quarter wave monopole consist of single leg erected on the perfect ground i.e, perfect conductor the length of the leg of the quarter wave monopole is $\lambda/4$



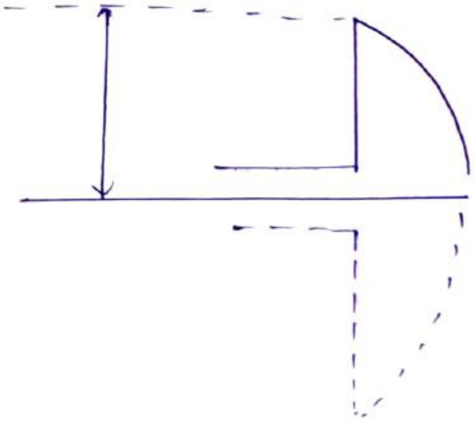


Fig: Sinusoidal current distribution in half wave dipole

Fig: Sinusoid current distribution in quarter wave monopole

⇒ The magnitude of the magnetic field strength for the radiation field of a halfwave dipole (or) quarterwave monopole is given by

$$\boxed{|H_\phi| = \frac{I_m}{2\pi r}\left[\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}\right]}$$

The relation b/w E field & H field is given by

$$\frac{|E_\theta|}{|H_\phi|} = 120\pi \text{ (for freespace)}$$

$$|E_\theta| = (120\pi)|H_\phi|$$

$$= (\cancel{120}^{60}\pi)\frac{I_m}{\cancel{2\pi} r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]$$

$$|E_\theta| = \frac{60 I_m}{r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]$$

The field components E_θ and H_ϕ are in time phase the maximum value of pointing vector can be obtain by multiplying the magnitudes of E_θ and H_ϕ

$$\therefore P_{max} = |E_\theta|\,|H_\phi|$$

$$= \left(\frac{60 I_m}{r}\left[\frac{\cos(\frac{\pi}{2})\cos\theta}{\sin\theta}\right]\right)\cdot\left(\frac{I_m}{2\pi r}\left(\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right)\right)$$

$$= \frac{60 I_m^2}{2\pi r^2}\left[\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}\right]^2$$

The average value of the power is half of its maximum value $\therefore P_{avg} = \frac{P_{max}}{2}$

$$P_{avg} = \frac{\frac{60 I_m^2}{2\pi r^2}\left[\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}\right]^2}{2}$$

$$= \frac{15 I_m^2}{\pi r^2}\left[\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}\right]^2$$

The RMS current is given by

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$I_m = I_{rms}\sqrt{2}$$

Substitute I_m in P_{avg}, then

$$P_{avg} = \frac{15 (I_{rms}\sqrt{2})^2}{\pi r^2}\left[\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}\right]^2$$

$$= \frac{30 I_{rms}^2}{\pi r^2}\left[\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}\right]^2$$

The total power radiated through the surface of radius r is given by

$$P = \oint_s P_{avg} \cdot ds$$

$$= \oint_s \frac{30 I_{rms}^2}{\pi r^2}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2 r^2 \sin\theta \, d\theta \, d\phi$$

$$= \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{30 I_{rms}^2}{\pi} \frac{\cos^2(\pi/2 \cos\theta)}{\sin\theta} \, d\theta$$

$$\int_{\theta=0}^{\pi} (2\pi) \frac{30 I_{rms}^2}{\pi} \frac{\cos^2(\pi/2 \cos\theta)}{\sin\theta} d\theta$$

$$P = 60 I_{rms}^2 \int_0^{\pi} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta$$

By Simpson's rule the integral value is 0.609

$$\therefore \int_0^{\pi} \frac{\cos^2(\pi/2 \cos\theta)}{\sin\theta} d\theta = 0.609$$

$\therefore$ power radiated $= 60 I_{rms}^2 (0.609)$

$= 36.54 I_{rms}^2$

The radiation resistance is given by

$R_{rad} = 36.5\,\Omega$ for quarter wave monopole

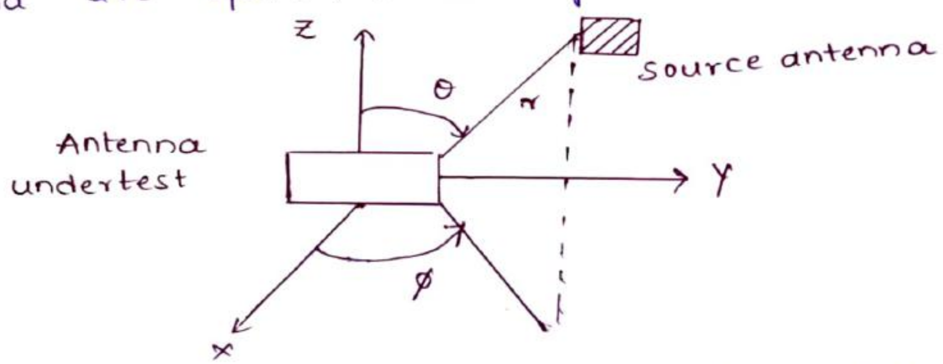
$R_{rad} = 73\,\Omega$ for halfwave dipole

ANTENNA MEASUREMENT

The antenna measurements are carried out using the test antenna in the receiving mode. It is benificial if the test antenna is reciprocal because the receiving mode characteristics of such reciprocal antennas are identical to the characteristics in the Transmitting mode.

Basic concepts of Antenna measurement:

The important measurement parameters of the antenna are Gain, directivity, radiationpattern etc.



→ The antenna undertest (AUT) is consider to be located at origin of the co-ordinate system. The source antenna is placed at different locations with respect to the AUT.

→ Note that the source antenna may be transmitting or receiving at different locations. The number of samples of the pattern are obtained. To acheive different locations, Generally AUT is rotated.

→ To acheive sharp sample of pattern, it is necessary that there exists single direct signal path between the AUT and Source antenna.

Reciprocity:

→ Generally antenna can either act as a transmitter or receiver there exists a reciprocal relationship between the Transmitting and receiving properties of the antenna.

→ The reciprocal relationship is very useful in the antenna measurement. This relationship allows to obtain the the characterisation of an antenna from Transmitting tests or receiving test.

→ It is necessary to study two important consequences helpful in antenna measurement. They are

i) The transmitting and receiving patterns of antenna are same.

ii) The powerflow is the same in transmitting mode and receiving mode.

→ when the AUT is used in a huge transmitter or receiver the direction of the signal can be defined easily. Practically while using reciprocity relation-ship following conditions must go fulfilled.

a) the emf's at the terminals of the transmitting or receiving antennas should be of same frequency.

b) The powerflow should be equal to that is due to matched impedance

c) The media should be linear, isotropic and passive

→ The Reciprocity principle is not only applicable to the antenna patterns but also to the other characteri-stics of the antenna except current destributions. Thus one can conviniently use antenna in some cases as radiator or as receiver in some other cases.

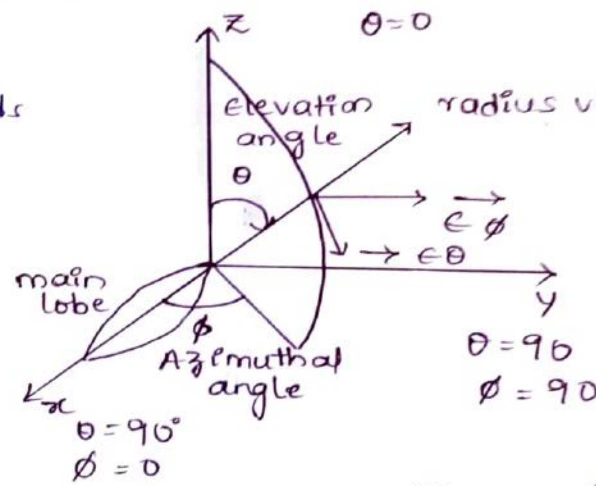
Near field and far field:

→ There are three main regions of the radiated field of the antenna. The region very close to antenna is called NEAR FIELD

→ The region next to near field which is called far field, the near field is also called Fresnel region and far field is also called fraunhofer region.

→ According to Huygen's principle it hardly matters where the field with sufficient information is samples at surface in any region of the radiated field But practically there is a limit on the distance between the surface and the AUT

Coordinates



* According to IEEE standards the spherical co-ordinate system as the figure is to universally for the antenna measurement.
* The angle measured from the z-axis is called 'ELEVATION ANGLE' and it denoted by θ. The angle measured from the projection of the radius vector to the horizontal x-y plane. is called 'AZIMUTH ANGLE' which is denoted by ϕ.
* Depending upon the mechanical structure of the antenna the coordinate system is defined as the the peak radiation takes places along x-axis is general.

* when the source antenna is moved along lines F, constant θ, the cut obtained are called conical cuts or ∅ cuts, when the source antenna is moved along lines of constant ∅ the cuts obtained are called great circle θ cuts or θ-cuts, if takes along the equator with θ = π/2 then such a cut is called θ-cut as well as ∅-cut. The two principle plane cuts are the orthogonal great circle cuts through the axis of the main lobe.

* Source of error in antenna measurement:

For the antenna measurement in far field region is plane wave with uniform phase and amplified is ideal recuirement but practically they are deviations of the plane wave. Due to the finite distance between π and source antenna amplitude taper

The advantages of the far field are as follows:

→ computing and multiplying reflections are least significant far field region.

→ The far field region, only power measurements is done and its purpose is to obtain power pattern.

→ At any point in the far field region, the field pattern measured is valid.

→ In the far field region there is no remarkable effect on the result even in the location of a phase entering the antenna is changed. It helps in antenna measurement error when AUT is rotated.

→ But the major drawback of the far field measurement of the antenna is a the request large in distance and between receiving and antennas, thus it demands large antennas in the distance may increase such a extend that the result of the measurement the far field region suffers by atmospheric attenuation.

→ You general, the reactive here field region is located very close to the AUT also due to the mutual impedance as a result of the reactive coupling between two antennas, the measurement complicated. Hence practically the reactive near field region is also not used for a antenna measurements.

→ so the obvious remaining for the choice for the antenna measurement is the measurement in the radians effect the main, lobe of the antenna significantly. Due to the reflections from the surroundings ripple in the amplitude and phase is observed. these ripples effect the accuracy of the side lobe significantly following are different sources of errors in antenna measurement.

1. Source Due to finite measurement distance between antennas:

→ when the distance between the antennas is very small, the field received by AUT at different points will be with different phases causing Quadric phase errors.

→ The Quadric phase error, affects by reducing the measured gain and increasing the side lobes as compared to the ideal uniform plane condition.

→ Due to the small distance between the antennas the amplitude gets affected the amplitude errors are of two types transverse amplitude error and longitudinal amplitude error.

2. Reflection from surroundings:

→ The reflections from surroundings is another important source of the error because reflections cause amplitude ripple as well as phase ripple.

→ The ripples occur in a region due to the interference between the direct wave and reflected wave.

→ Note that the powers of the waves are not added but always the field of the waves are added. so due to very small reflection also large measurement error may be caused.

3. Errors due to coupling in the reactive near field:

→ The reactive near field causes significant error at low frequencies. If the distance is greater than 10λ, then the coupling is negligible.

4. Errors due to measurement effects:

→ Basically the antenna measurement is a 3-dimensional vector measurement so any misalignment of the source antenna causes amplitude errors due to mis alignment the pattern cut cannot be properly taken.

5. Errors due to manmade interface:

→ On outdoor ranges, when the man-made interference gets coupled with receiver at the frequency same as the measurement frequency or any other frequency. harmonic distortion takes place.

→ on indoor ranges, in anechoic chambers the reflections from the walls floor and ceiling are significant.

6. Error due to atmospheric effects:

→ Due to atmospheric effects such as variation of refractive index of atmosphere multihop propogation takes place which finally results in significant amplitude variation during measurement

→ At higher frequencies the attenuation of the atmosphere is very high which results in amplitude variations.

7 Errors due to cables:

→ If the cables used for the connection donot have proper sheilding, leakage occurs and the cables acts as antennas producing measurement errors. the incorrect use of cable also cause errors.

8. Errors due to impedance mismatch:

→ If the antenna impedance is not properly matched with the measurement impedance errors occur in the gain measurement.

9. Errors due to imperfections of instruments:

→ Due to the imperfections of the instruments in the measurements such as transmitter, receiver, positioner

Measurement of Radiation patterns:

→ The radiation capabilites of an antenna are characterized by the characteristics of an antenna such as the radiation pattern, polarization and gain.

→ All these quantities are measured on the surface of a sphere with constant radius. Any point 'p' on such sphere can be described using spherical co-ordinate system.

→ Basically for representation of a point on the surface only θ and ϕ specifications are sufficient because sphere with constant radius is considered

→ Thus the radiation characteristics of the antenna as a function of θ and ϕ for constant radius and frequency is called radiation pattern of an antenna, for horizon-tal antenna following patterns are required.

i) The ϕ component of electric field as a function of ϕ is measured in x-y plane ($\theta = 90°$). The field component can be then represented as $E_\phi(\theta = 90°, \phi)$ and it is called E-plane pattern.

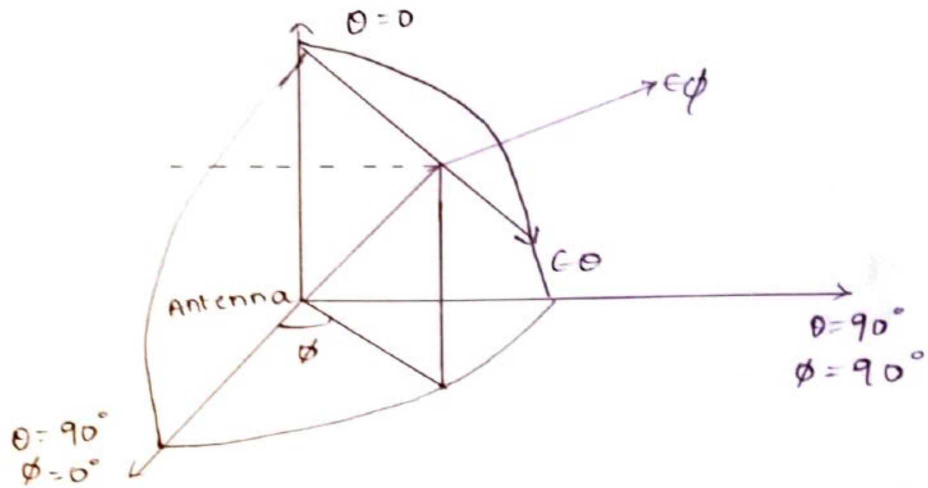
ii) The ϕ component of electric field as a function of θ is measured in x-z plane ($\phi = 0$) it is represented as $E_\phi(\theta ; \phi = 90°)$ and it is called H-plane pattern.

→ The two patterns bisect the major lobe in mutually perpendicular planes providing sufficient information for the measurement.

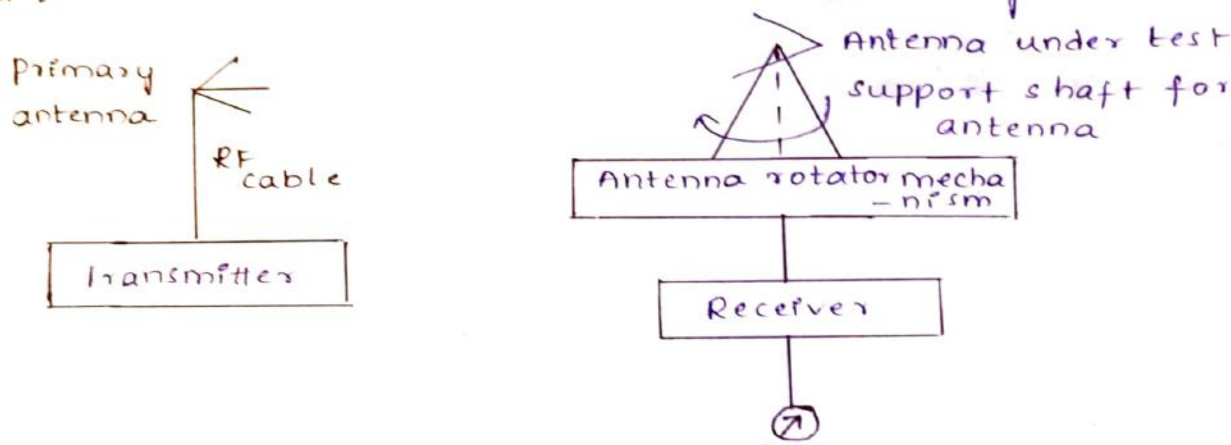
→ for the vertical antenna following patterns are required.

i) The θ component of the electric field is measured in x-y plane ($\theta = 90°$) as a function of ϕ. The component can be represented as $E_\theta(\theta = 90°, \phi)$ and it is called H-plane pattern.

ii) The θ component of the electric field is measured as a function of ϕ in the x-z plane ($\phi = 90°$). Then the field component can be represented by $E_\theta(\theta, \phi = 90)$



Arrangement for measurement of radiation pattern of an antenna :



→ The simple arrangement for the radiation pattern measurement consists primary antenna in transmitting mode, secondary antenna under test

→ The secondary antenna is coupled with rotating shaft and it is rotated using antenna rotator mechanism.

→ To measure the relative amplitude of the received field an indicator is used along with the receiver.

→ Usually the antenna under test is used in the receiving mode it is properly illuminated by the stationary primary antenna.

→ The secondary antenna is rotated about vertical axis for E-plane measurement the antenna support is rotated with the antennas Horizontal, while for H-P pattern measurement the shaft is rotated with both antennas vertical.

Measurement of Gain of an antenna:

→ The performance of any antenna can be described interms of gain of an antenna. depending upon the frequency of operation various methods can be used for the measurement of gain of an antenna.

→ Typically to measure Gain above 1GHz, free space ranges are used and 0.1 - 1GHz range, the ground-reflection ranges are convinently used.

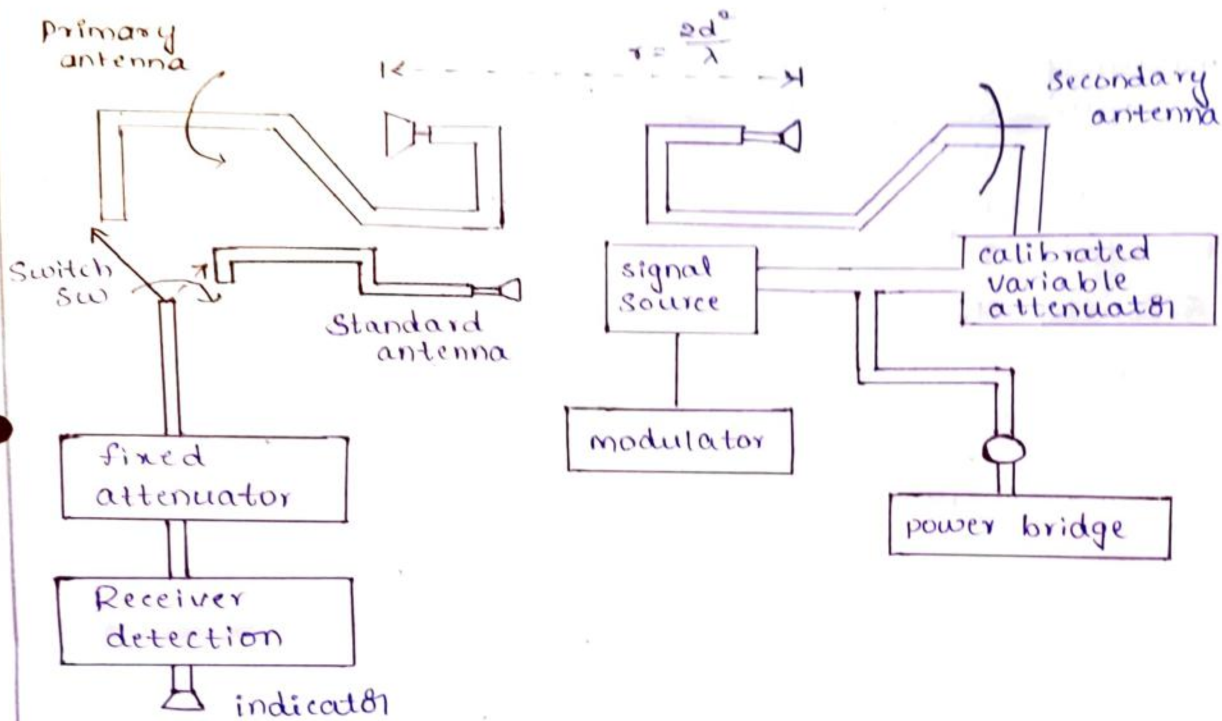
→ Usually below 1MHz frequency antenna gains are not measured, in such cases the measurements are done with the field strength of the radiated ground wave.

→ Basically there are two standard methods, used for the measurement of gain of an antenna.

i) Gain transfer method or direct comparison method.

ii) Absolute Gain method.

Direct comparision method :



→ This method uses two antennas termed as primary and secondary antennas. The secondary antennas is transmitting antenna.

→ The knowledge of gain of the secondary antenna is not necessary. The primary antenna consist two different antennas separated through a switch SW.

→ The primary & secondary antennas are separated with a distance $2d^2/\lambda$.

→ The primary antenna is the standard gain antenna and the subject antenna under test.

→ The gain measurement by the gain-comparison method in two step procedure.

1. Through the switch sw, first standard gain antenna is connected to the receiver the antenna is adjusted in the direction of the secondary antenna to have max signal intensity. The input connected to the secondary antenna is adjust to required level. for this i/p corresponding primary antenna radiating at the receiver is recorded. Corresponding attenuator and power bridge readings are recorded as A_1 and P_1.

2. Secondly the antenna under test is connected to the receiver by changing the position of the switch sw. To get the same reading at the receiver the attenuator is adjusted. Then corresponding attenuator and power bridge readings are recorded as A_2 and P_2.

<u>Case 1:</u> if $P_1 = P_2$ then no connection need

$$\text{power gain} = G_P = \frac{A_2}{A_1}$$

Taking algorithms on both sides

$$\log_{10} G_P = \log_{10}\left(\frac{A_2}{A_1}\right)$$

$$\log_{10} G_P = \log_{10} A_2 - \log_{10} A_1$$

$$\boxed{\underset{(dB)}{G_P} = \underset{(dB)}{A_2} - \underset{(dB)}{A_1}}$$

<u>Case 2:</u> if $P_1 \neq P_2$ then the connection need

let $\frac{P_1}{P_2} = P$

$$\log_{10} \frac{P_1}{P_2} = P(dB)$$

hence power gain is given by

$$G = G_P \times \frac{P_1}{P_2} = \frac{A_2}{A_1} \cdot \frac{P_1}{P_2}$$

$$G = G_p \cdot \frac{P_1}{P_2}$$

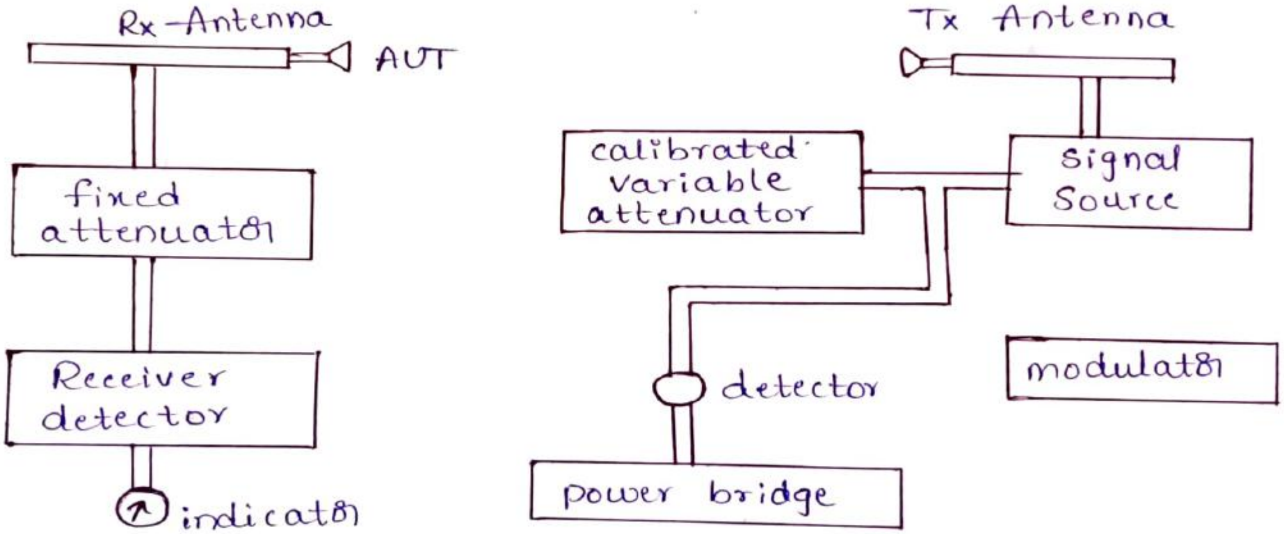
Taking algorithms

$$\log_{10} G = \log_{10}\left(G_p \cdot \frac{P_1}{P_2}\right)$$

$$= \log_{10} G_p + \log_{10}\left(\frac{P_1}{P_2}\right)$$

$$\boxed{G = G_{p\,(dB)} + P_{(dB)}}$$

Measurment of Absolute Gain:-



→ Let the P_t is transmitted power and P_r is receiver power and A_{et} is effective aperature of transmitter and A_{er} is effective aperature of receiver.

$$\therefore \; A_{et} = A_{er} = \frac{G_D \lambda^2}{4\pi r}$$

from frii's transmission equation

$$\frac{P_r}{P_t} = \frac{A_{er} \cdot A_{et}}{\lambda^2 \cdot r^2}$$

$$\frac{P_r}{P_t} = \left(\frac{G_{tD}\lambda}{4\pi r}\right)^2$$

$$\frac{G_{tD}\lambda}{4\pi r} = \sqrt{\frac{P_r}{P_t}}$$

$$\boxed{G_{tD} = \frac{4\pi r}{\lambda}\sqrt{\frac{P_r}{P_t}}}$$

Measurement of directivity:

The directivity can be obtained from the radiation pattern of an antenna. The directivity of an antenna is defined as the radio of max power density to the avg power density.

$$G_{tDmax} = \frac{P_{dmax}}{\frac{P_{rad}}{4\pi r^2}} = D$$

Basically directivity of an antenna is different as less

$$D = \frac{4\pi |E_{max}|^2}{\int_0^{2\pi}\int_0^{\pi} |E(\theta,\phi)|^2 \sin\theta \, d\theta \, d\phi}$$

$$= \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} \frac{|E(\theta,\phi)|^2}{|E_{max}|^2} \sin\theta \, d\theta \, d\phi}$$

$$D = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} P(\theta,\phi) \sin\theta \, d\theta \, d\phi}$$

VHF, UHF and Microwave Antennas - I

UNIT - II

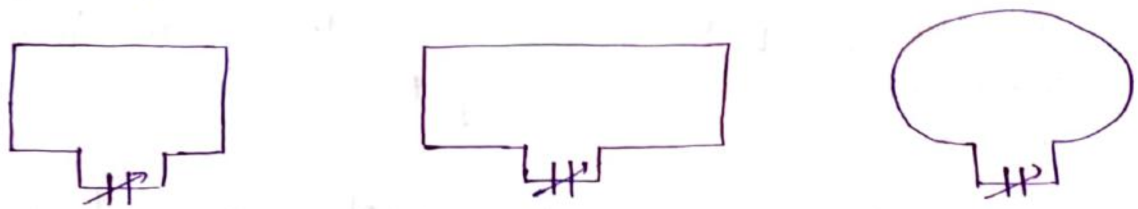
Introduction:

The radio waves propogated from the Transmitter follow the circular route to reach the Receiver to find the direction of radiation of the radio wave from the Transmitter this method is known as 'Direction finding'.

Loop Antenna:

→ A loop antenna is nothing but a radiating coil of any shape with one or more no. of turns carrying a round of current.

→ A loop is formed on a ferrite core (or) aircore



ⓐ Square loop ⓑ Rectangular loop ⓒ circular loop

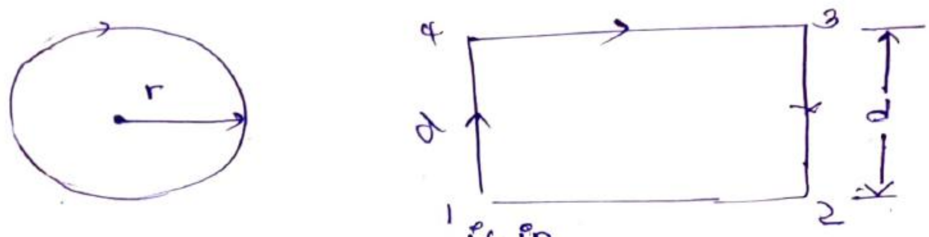
→ Loop may consists of one (or) more loops on a ferrite (or) air core it is commonly called as firrite.

→ Generally the loop antennas are classified into two type based on its dimensions

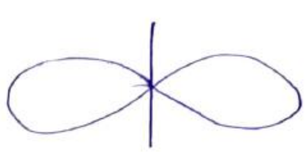
1. The dimensions of the loop are very small as composed with wavelength.
2. The dimensions of the loop are comparable with wave length.

Far field radiation pattern of small loop antenna

* Then consider a circular loop of radius 'r' very less compared with wave length (r<<d). such that the current is in phase through a loop
* Let consider the circular loop is represented by a square loop of dimensions and such that the area of both loops are



* consider that the square loop is in ~~of~~ the coordinate system
* The far field fo the square loop will have only $E\phi$ component.
* To obtain the far field pattern considering all four diople it is sufficient to consider only two dioples such as sides 1,4 and 2,3.
* Then the radiation pattern of the dioples are in horizontal plane and vertical plane
* the radiation pattern in the vertical plane for both dipoles is circular in other words the dioploes 1,4 and 2,3 are behaving like isotropic point source.

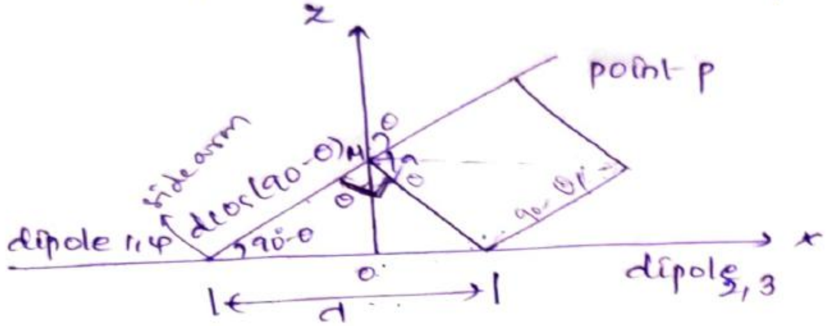


Radiation pattern in Horizantal plane

Radiation pattern

-) Now the dipole due to loop source with reference 8H center 'O' can be represented as

$E\phi$ = far field component due to dipole 1,4   far field component due to dipole 2,3



-) The waves radiated from dipole will be take more time to reach from dipole 1,4 compare to the waves radiated from the dipole 2,3.

-) Because wave radiated from the dipole 1,4 will have to an extra distance from 1 to M compared to that of the waves radiated from the dipole.

This extra distance is called path difference

$$\therefore \text{path difference} = d\cos(90^\circ-\theta)$$

The path difference interms of the wavelength

$$\text{path difference} = \frac{d\cos(90^\circ-\theta)}{\lambda}$$

* Let the phase angle denoted by ψ. then the phase angle and path difference are

$$\text{phase angle } \psi = 2\pi \times \text{path difference}$$

$$\psi = \frac{2\pi}{\lambda} d\cos(90^\circ-\theta)$$

$$\psi = \frac{2\pi}{\lambda} d\sin\theta$$

* In general the field component of any diople is given by

field component = Magnitude $\times e^{j\,(\text{phase angle})}$

* let E_0 be the Magnitude of electric field due to dipoles 1,4

$$E_1 = -E_0 \cdot e^{j\psi/2}$$

due to dipole 2,3

$$E\phi_2 = E_0 \cdot e^{-j\psi/2}$$

$\therefore$ Far field radiation due to the square loop is

$$E\phi = E\phi_1 + E\phi_2$$

$$= -E_0 e^{j\psi/2} + E_0 e^{-j\psi/2}$$

$$= -E_0 \left[e^{j\psi/2} - e^{-j\psi/2} \right]$$

$$= -j2E_0 \left[\frac{e^{j\psi/2} - e^{-j\psi/2}}{j2} \right]$$

$$E\phi = -j2E_0 \sin \psi/2$$

$$\psi = \text{phase angle} = \frac{2\pi}{\lambda} d \sin\theta$$

but we known that the phase angle shift β is given by

$$\beta = \frac{2\pi}{\lambda}$$

$$\psi = \beta$$

* In a free space the feilds are represented by

$$\eta_0 = 120\pi = \frac{E\phi}{H_\theta}$$

$$H_\theta = \frac{E\phi}{\eta_0}$$

$$= \frac{-j2}{120\pi} E_0 \sin\left[\frac{\beta d \sin\theta}{2}\right]$$

$$E_\theta = \frac{-j}{60\pi} E_0 \sin\left[\frac{\beta d \sin\theta}{2}\right] \text{ v/m.}$$

* j operator indicates that the total field component E_0 is in the plane quadrature.

* If length of the side of square is very less than one wave length $(d << \lambda)$

$$\sin\left[\frac{\beta d \sin\theta}{2}\right] \approx \frac{\beta d \sin\theta}{2}$$

∴ the magnitude of the individual field

here $\theta = 90°$ because the field length is measure from x-axis rather than z-axis.

$$E_0 = \frac{j60\pi\,[I]}{r\lambda}$$

$$E_\phi = -j2\left[\frac{j\,60\pi\,[I]\,L}{r\lambda}\right]\left[\frac{\beta d \sin\theta}{2}\right]$$

$$= \frac{120\pi \times \frac{2\pi}{\lambda} \times [I] \cdot L \times d \cdot \sin\theta}{r\lambda \times 2}$$

$$= \frac{120\pi^2 \times [I] \sin\theta [I]}{r\lambda}$$

$$E_\phi = \frac{120\pi^2\,[I] \times \sin\theta}{r\lambda}$$

$$E_\theta = \frac{-j}{60\pi} \cdot E_0 \left[\frac{\sin\theta}{2}\right] \beta d$$

$$= \frac{-j}{60\pi} \times \frac{j60\pi [I].L}{r\lambda} . \beta d \frac{\sin\theta}{2}$$

$$= \frac{[I].l \times d \times 2\pi/\lambda}{r\lambda} . \frac{\sin\theta}{2}$$

$$= \frac{\pi \times [I] \times A \times \sin\theta}{r\lambda^2}$$

$$E\theta = \frac{\pi [I] A \sin\theta}{r\lambda^2}$$

<u>Far field radiation pattern of the short Magnetic dipole:</u>

* let us calculate the far field of a short magnetic dipole
* This method is very much similar to that used for electric dipole.

the magnetic dipole and Magnetic vector potential

The magnetic current is given by

$$F = \frac{\mu}{4\pi} \iiint \frac{[I_m]}{r} dv$$

Let us consider that only z component of $\bar{F}$ is present

$$\bar{F} = F_z \bar{a}_z = \bar{a}_z \left[\frac{\mu}{4\pi} \int_{-l/2}^{l/2} \frac{I_m}{r} dz \right]$$

$$F_z = \frac{\mu}{4\pi} \int_{-l/2}^{l/2} \frac{I_m \, e^{j\omega(t - r/c)}}{r} dz$$

$$= \frac{2\mu I_m}{4\pi r} \int_0^{l/2} e^{j\omega(t-r/c)} dz$$

$$= \frac{\mu I_m}{2\pi r} e^{j\omega(t-r/c)} [z]_0^{l/2}$$

$$F_z = \frac{\mu I m\, e^{j\omega(t - r/c)}}{4\pi r}$$

→ If $r >> L$ and $d >> L$

→ The electric field can be obtained from the retarded potential using relation

$$\bar{E} = \frac{1}{\mu}(\nabla \times \bar{F})$$

→ Thus resolving F_z into spherical co-ordinate and taking curl as the component $E\phi$ is obtained in

$$\therefore E\phi = \frac{I_m L \sin\theta}{4\pi}\left[\frac{j\omega}{cr} + \frac{1}{r^2}\right]$$

for a large distance $r >> L$ and $d >> L$

$$\therefore E\phi = j\,\frac{I m\, L \sin\theta\omega}{4\pi\, cr}$$

$$= j\,\frac{I m L \sin\theta(2\pi f)}{4\pi(f d) r}$$

$\omega = 2\pi$
$d = c/f$

$$E\phi = \frac{j I m L \sin\theta}{r}$$

$$H\theta = \frac{E\phi}{\eta_0}$$

$$= \frac{j I m\, L \sin\theta}{2 d r (120\pi)}$$

$$H\theta = \frac{j I m\, L \sin\theta}{240\pi d r}$$

but $I m . L = -j 240\pi^2 \frac{[I] A}{d}$

$$\therefore E\phi = \frac{j(-j240\pi^2 [I] A)\sin\theta}{2 d r . \lambda}$$

$$= \frac{120\pi^2 [I] A}{d^2 r}\sin\theta$$

$$E_\theta = \frac{j(-j240\pi^2[I]A)}{240\pi d r \cdot d} \sin\theta$$

$$E_\theta = \frac{\pi[I]a}{d} \sin\theta$$

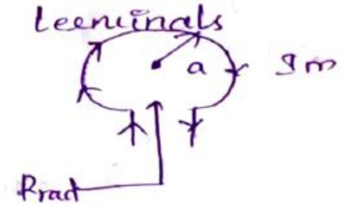
Radiation Resistance of loop antenna :- (small loop)

The poynting vector integrated over a large sphere and this radiated power is equal to square of r.m.s current and multiplied by radiation resistance Rrad

$\bar{D} = I^2_{rms} \cdot R_{rad}$ $\quad \bar{P} = I^2_{rms} R_{rad}$

$= \frac{1}{2} I_m^2 R_{rad}$

The radiation resistance of the loop antenna in the impedance appearing at the loop terminals



$$\bar{P} = \frac{1}{2} Re(\bar{E} \times \bar{H})$$

radial component due to E_θ and H_ϕ

$P_r = \frac{1}{2} Re(E_\theta \bar{H}_\phi)$ $\quad \bar{H}_\phi$ = conjugate of H_ϕ

but $E_\theta = \eta_0 H_\phi$ $\quad \eta_0 = \frac{E_\theta}{H_\phi}$

$$P_r = \frac{1}{2} H^2_\phi \eta_0$$

$$P_r = \frac{1}{2} |H_\phi|^2 \eta_0$$

But according to general case of loop antenna magnetic field

$$H_\theta = \frac{\beta a[I]}{2r} J_1(\beta a \sin\theta)$$

$$= \frac{1}{2} \left| \frac{\beta a[I]}{2r} J_1(\beta a \sin\theta)^2 \right| \cdot 120\pi$$

$$= \frac{1}{2} (\cancel{120}^{60}\pi) \frac{(\beta a I_m)^2}{-4r^2} J_1^2 (\beta a \sin\theta)^2$$

$$P_r = \frac{15\pi(\beta a I_m)^2}{r^2} J_1^2(\beta a \sin\theta)^2$$

To find total power radiated integrating this power over a large sphere

$$P = \int_0^{\pi}\int_0^{2\pi} \bar{\Phi} \cdot d\bar{s}\, \beta \sin\theta$$

$$= \int_0^{\pi}\int_0^{2\pi} 15\pi(\beta a I_m)^2 J_1^2(\beta a \sin\theta)^2$$

$$P = [\phi]_0^{2\pi} \left[15\pi(\beta a I_m)^2 \int_0^{\pi} J_1^2(\beta a \sin\theta)^2 \sin\theta\, d\theta\right.$$

$$= (2\pi)(15\pi)(\beta a I_m)^2 \int_0^{\pi} J_1^2(\beta a \sin\theta)^2 \sin\theta\, d\theta$$

$$P = 30\pi^2(\beta a I_m)^2 \int_0^{\pi} J_1^2(\beta a \sin\theta)^2 \sin\theta\, d\theta$$

For a small loop antenna

$$J_1^2(\beta a \sin\theta)^2 = \left[\frac{\beta a \sin\theta}{2}\right]^2$$

$$\therefore P = 30\pi^2(\beta a I_m)^2 \int_0^{\pi} \left(\frac{\beta a \sin\theta}{2}\right)^2 \sin\theta\, d\theta$$

$$= \frac{30\pi^2}{4}(\beta a I_m)^2 (\beta a)^2 \int_0^{\pi} \sin^2\theta \sin\theta\, d\theta$$

$$= \frac{30\pi^2}{4}(\beta a)^4 (I_m)^2 \cdot 2 \underbrace{\int_0^{\pi/2} \sin^3\theta\, d\theta}_{\rightarrow\, 2/3}$$

$$\boxed{\therefore P = 10\pi^2(\beta a)^4 I_m^2}$$

$$\frac{1}{2} I_m^2 \cdot R_{rad} = 10\pi^2(\beta a)^4 I_m^2$$

$$R_{rad} = 20\pi^2(\beta a)^4$$

$$= 20\pi^2 \beta^4 a^4$$

$$= 20\beta^4(\pi a^2)^2$$

But πa^2 is the area A of the loop

$$R_{rad} = 20\beta^4\pi^2\,\Omega$$

But $\beta = \frac{2\pi}{\lambda}$

$$R_{rad} = 20\left(\frac{2\pi}{\lambda}\right)^4\Omega$$

~~$R_{rad} = 3\pi$~~

Hence approximately

$$R_{rad} = 320\,\pi^4\left(A/\lambda^2\right)^2\Omega$$

All the above expressions are for a ~~loop~~ small loop with a single turn

If the loop consist N turns

$$R_{rad} = 320\,\pi^4\left(A/\lambda^2\right)^2 N^2\,\Omega$$

let c be the circumference of a loop with radius a

then $c = 2\pi a$

$$R_{rad} = 20\left(\frac{2\pi}{\lambda}\right)^2 A^2$$

$$= 20\,\pi^2\left(\frac{2\pi}{\lambda}\right)^2\left(\pi a^2\right)^2$$

$$= 20\,\pi^2\left(\frac{2\pi a}{\lambda}\right)^4$$

$$= 20\pi^2\left(c/\lambda\right)^4\Omega$$

<u>for large loop</u>:-

$$P = 30\pi^2\left(\beta a I_m\right)^2\int_0^{\pi} J_1^2(\beta a\sin\theta)\sin\theta\, d\theta$$

The property of bessel function

$$\int_0^{\pi} J_1^2(x\sin\theta)\sin\theta\, d\theta = \frac{1}{x}\int_0^{2x} J_2(y)\,dy$$

hence the power can be written as

$$P = 30\pi^2\left(\beta a I_m\right)^2\cdot\frac{1}{\beta a}\int_0^{2\beta a} J_2\, y\, dy$$

when $a/\lambda \geq 5$ the loop is consider as large loop when $a/\lambda \geq 5$ the loop is consider as large loop

$$\int_0^{\pi} J_m^2 (\beta a \sin\theta) \sin\theta d\theta = \frac{1}{\beta a} \int_0^{2\beta a} J_2(y) dy \approx \frac{1}{\beta a}$$

$$P = 30\pi^2 (\beta a I_m)^2 \frac{1}{\beta a}$$

$$P = 30\pi^2 I_m^2 \beta a$$

but

$$\beta = \frac{2\pi}{\lambda}$$

$$P = 30\pi^2 \left[\frac{2\pi}{\lambda} a\right] I_m^2$$

$$\boxed{P = 30\pi^2 I_m^2 (c/\lambda)}$$

$$\frac{1}{2} I_m^2 R_{rad} = 30\pi^2 I_m^2 (c/\lambda)$$

$$R_{rad} = 60\pi^2 (c/\lambda)$$

let $c = 2\pi a$

$$\therefore R_{rad} = 60\pi^2 (2\pi a/\lambda)$$

$$= 120\pi^3 (c/\lambda) \qquad \pi = 3.142$$

$$\boxed{R_{rad} = 3720 (c/\lambda)\,\Omega}$$

Directivity of circular loop antenna :-

The directivity of an antenna is defined as the ratio of maximum radiation intensity to the average radiation intensity

$$D = \frac{\text{Maximum radiation intensity}}{\text{Average radiation intensity}} = \frac{(P_r)(r^2)}{\left(\frac{P}{4\pi}\right)}$$

$$\therefore D = \frac{\left\{\frac{15\pi (\beta a I_m)^2}{r^2} J_1^2 (\beta a \sin\theta)\right\}_{max} r^2}{\left\{\frac{30\pi^2 (\beta a I_m)^2 \int_0^{\pi} J_1^2 (\beta a \sin\theta) \sin\theta d\theta}{4\pi}\right\}}$$

$$= \frac{[60\pi^2 (\beta a I_m)^2 J_1^2 (\beta a \sin\theta)]_{max}}{30\pi^2 (\beta a I_m)^2 \int_0^{\pi} J_1^2 (\beta a \sin\theta) \sin\theta d\theta}$$

$$= \frac{2 [J_1^2 (\beta a \sin\theta)]_{max}}{\frac{1}{\beta a} \int_0^{\pi} J_1^2 (\beta a \sin\theta) \sin\theta}$$

$$= \frac{2 \beta a [J_1^2 (\beta a \sin\theta)]_{max}}{\int_0^{2\beta a} J_1^2 (\beta a \sin\theta) \sin\theta d\theta}$$

but $\beta a = C/\lambda$

$$D = \frac{2 C/\lambda [J_1^2 (C/\lambda \sin\theta)]_{max}}{\int_0^{\lambda 2} J_1^2 (C/\lambda \sin\theta) \sin\theta d\theta}$$

$$= \frac{2 C/\lambda [J_1^2 (C/\lambda \sin\theta)]_{max}}{\int_0^{2C/\lambda} J_1 (y) dy}$$

→ This expression is called faster's expression for directivity of circular loop of any circumference and uniform in-phase current

→ Let us consider two cases one for small loop and other for large loop

Case - I small loop $\frac{C}{\lambda} < \cancel{\frac{1}{3}} \frac{1}{3}$

for condition $\frac{C}{\lambda} < \frac{1}{3}$ the directivity of loop is given

$$D = \frac{2 [J_1^2 (\beta a \sin\theta)]_{max}}{\int_0^{\pi} J_1^2 [(\beta a \sin\theta)] \sin\theta d\theta}$$

for

For small loop $J_1(\theta) = \theta/2$

$$= \frac{2\left[\frac{\beta a \sin\theta}{2}\right]^2_{max}}{\int_0^{\pi}\left[\frac{\beta a \sin\theta}{2}\right]\sin\theta\, d\theta}$$

$$= \frac{2\left[\left(\frac{\beta a}{2}\right)^2 \sin^2\theta\right]_{max}}{\left(\frac{\beta a}{2}\right)^2 \int_0^{\pi} \sin^2\theta \sin\theta\, d\theta}$$

$$= \frac{2[\sin^2\theta]_{max}}{\int_0^{\pi} \sin^3\theta\, d\theta}$$

$$= \frac{2(\sin^2\theta)_{max}}{2\int_0^{\pi/2} \sin^3\theta\, d\theta} \qquad \theta = x$$

$$= \frac{2(1)}{2(2/3)}$$

The pattern of short dipole in same as for the small loop and thus the value of directivity is also same for the loop as well as short loop.

<u>case 1:-</u>

for a large loop the directivity is given, by.

$$D = \frac{2(c/\lambda)(J_1^2\, c/\lambda \sin\theta)_{max}}{\int_0^{2c/\lambda} J_2(y)\,dy}$$

but $\int_0^{2c/\lambda} J_2(y)\,dy = 1$

$$\therefore \quad D = 2\frac{c}{d}\left[J_1^2\left(\frac{c}{d}\sin\theta\right)\right]_{max}$$

for condition

$$\frac{c}{d} \geq 1.84 \text{ the maximum}$$

for condition

$$J_1^2\, c/d \,\sin\theta = 0.554$$

$$\therefore D = 2\frac{c}{d}[0.584]^2 = 0.68211(c/d)$$

$$D = 0.682\,(c/d)$$

Applications of loop antenna :

1. A small loop antenna is used as a source for parabolic in many applications.
2. Large loop antenna can be used on direction finder.
3. Loops are mounted at the top of the tower and can be used as Omnidirectional system.
4. For line of sight communication an array of loops with different dimensions are used.

Features of Loop Antenna :-

1. The loop Antenna with circumference of loop less then on at highest frequency are called small loop antennas. This can be used upto 30 MHz.
2. Vertical loop antenna is most widely used for direction of applications. If it is not shielded it receives, bidirectional signal but if the antenna is shielded, it

receives unidirectional signal. due to electrostatic shielding the directional characteristics are improved.

3. The loop antennas are mostly used in LF, MF, HF, VHF and UHF ranges and it shows doublet shaped radiation pattern.

4. The loop antennas when used with ferrite core, effective diameter of a loop can be increased and such loop antenna can be used as broadcast receiver.

Yagi-guda Antenna:-

- Yagi-guda Antenna is a high gain antenna
- These antenna was first invented by Japanese Inventor/ scientist profeccessor S. uda in 1940. then it was described in english by profeccessor H. Yagi.
- These description is in English and it used world wide and the antenna is become popular.
- Hence the antenna name is yagi-uda antenna.
- A basic yagi-guda antenna consists a driven element. one reflector and one or more antenna.
- The driven element is a resonant half wave diode made of a metallic rod.
- In parasitic elements are continously arranged parallel to the driven elements and at the same line of sight.

→ All the elements are placed - parallel to each other and close to each other.

→ The current flow through a driven element the parasitic element receive excitation through the induced emf.

→ The phase and amplitude of the current through the parasitic elements mainly depends on the length of the elements and spaceing, between elements

→ To vary reactance of any element the dimensions of the elements are readjusted.

→ Generally the spacing between the driven and parasitic element is kept nearly 0.1 λ to 0.15 λ.

→ Yagi-uda consist both the radiator (R) and the director (D) elements is same antenna.

→ The elements at the back side of the driven elements is the reflector.

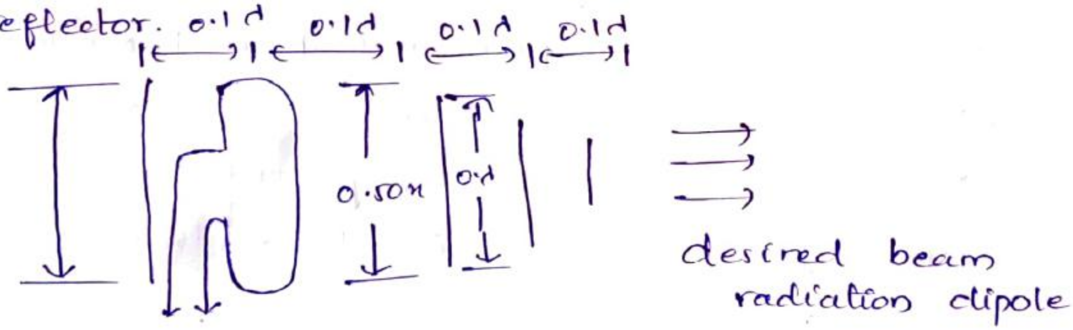


Fig: Yagi-uda antenna

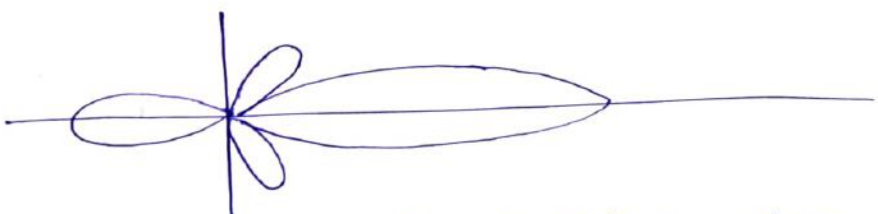


fig: Radiation pattern

→ the element infront of the driven element is the director which is of largest length in all the elements.

→ the length of the different elements can be obtained by using following formulas.

→ It is directional antenna.

→ It has operating freq above 10 GHz

→ It can be used for 40 to 60 km distance

→ It has two types of element

(1) Active element [Driven Element]

(2) Parasitic element [Reflector, Directors]

Directors :- Direct to radiation in required (or desired) direction.

reflector :- reflects radiation in desired direction.

Radiation of yagi-uda antenna.

Driven element. Reflected antenna

⟹ Designing of 3-element in yagi-uda antenna.

→ length of Active element

$$L_a = \frac{478}{f_{MHz}} (f_t)$$

$$L_R = \frac{492}{f_{MHz}} \cancel{(f_t)}$$

$$L_D = \frac{461.2}{f_{MHz}} (f_t)$$

Advantages ① High gain antenna

② High Front to Back ratio
③ light weight
④ High directivity
⑤ Less power wastage
⑥ easy to handling and Maintainance

Unit-II

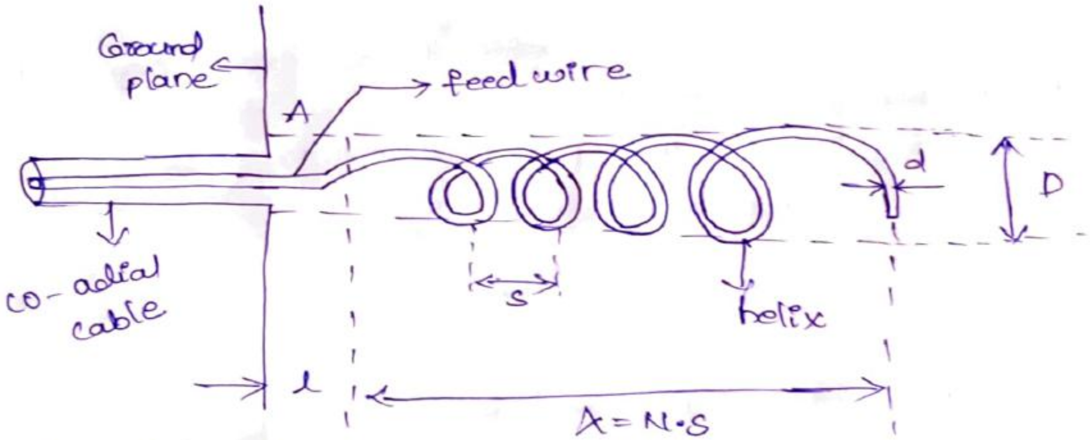
⇒ The Helical antenna was fast Introduced by John D. Utt antenna, which provides circular polarization.

⇒ It consists of thick copper wire wound in the form of a screw thread forming helix.

⇒ The helix of a helical antenna combines three different Geometric shaped like straight line, circle, and cylinder.

⇒ Helical Geometry:-

⇒ The helical antenna is connected between the co-axial cable and ground plane. The ground plane is made of radial and concentric conductors.



⇒ The helix can be described by using following symbols.

N = Number of Turns

D = Diameter of Helix

A = Axial length

C = Circumference of Helix = $\pi \cdot D$

S = Spacing between two turns

λ = length of one Turn

α = pitch angle

d = Diameter of conductor

⇒ The helix is fed by a co-axial cable. one end of the helix is connected to the centre of the cable and on conductor is connected to the ground plane.

⇒ The mode of the radiation of the antenna depends on the diameter of the helix i.e., D, the spacing between turns that is S.

⇒ The circumference of the helix is denoted by C and it is equal to πD. Then the pitch angle is given by

$$\alpha = Tan^{-1}\left(\frac{S}{\pi D}\right)$$

⇒ The axial length $A = N \times S$. Length of one complete turn is denoted by L.

$$L = \sqrt{S^2 + C^2} = \sqrt{S^2 + (\pi D)^2}$$

⇒ The pitch angle is defined as the angle between a line tangential to the helix wire and the plane normal to the axis of helix. The pitch angle then can be expressed as,

$$\tan \alpha = \frac{S}{C} = \frac{S}{\pi D}$$

$$\alpha = \tan^{-1}\left(\frac{S}{\pi D}\right)$$

Normal Mode:-

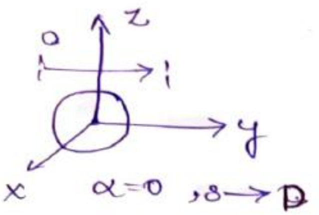
⇒ The radiation is maximum in broadway direction i.e., Name (or) perpendicular to the axis of the helix, hence the mode is called normal mode of radiation.

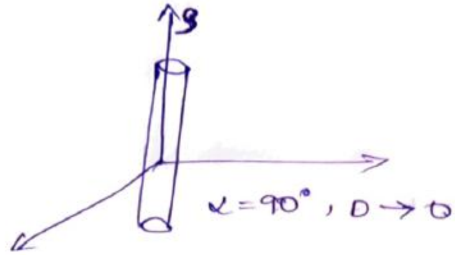
⇒ Normal mode of operation characteristics is obtained when dimensions of helical antenna are very small compared to the operating wavelength.

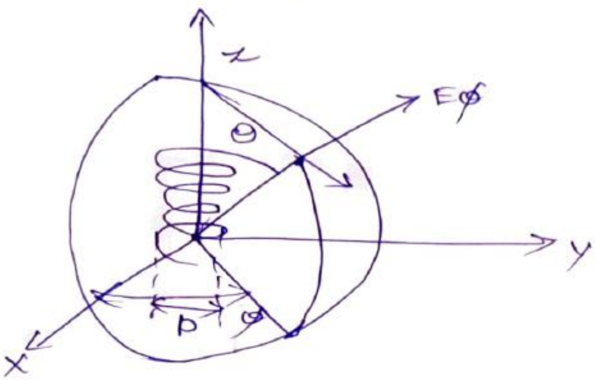
⇒ In this mode antenna band width and efficiency are very lone.

⇒ The above factors can be increased by increasing the antenna size.

⇒ The radiation fields of helical antenna are similar to the loops and short dipoles. Helical antenna is equivalent to the small loops and short dipolex connected in series.







⇒ The far field is of small loop is given by

$$E\phi = \frac{120\pi^2 [I] \sin\theta}{r} \cdot \frac{A}{\lambda^2} \quad \left[\because A = \text{Area of loop } \pi\left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4}\right]$$

||ly The far field of the short dipole is given by

$$E\theta = \frac{60\pi [I] \sin\theta}{r} \cdot \frac{S}{\lambda}$$

⇒ The axial ratio is defined as the ratio of the field due to short dipole to that the field have due to loop.

$$A.R = \frac{E_\theta}{E_\phi}$$

$$= \frac{\frac{60\pi [I]\sin\theta}{r} \cdot \frac{S}{\lambda}}{\frac{120\pi^2 [I]\sin\theta}{r} \cdot \frac{A}{\lambda^2}}$$

$$= \frac{S\lambda}{2\pi A}$$

$$A.R = \frac{S\lambda}{2\pi\left[\frac{\pi D^2}{4}\right]}$$

$$= \frac{4S\lambda}{2\pi^2 D^2}$$

$$\boxed{A.R = \frac{2S\lambda}{\pi^2 D^2}}$$

⇒ Now depending on values of A.R we get three conditions.

condition 1:– AR=0, the elliptical polarization becomes linear horizontal polarization.

condition 2:– AR=∞, the elliptical polarization becomes linear vertical polarization.

condition 3:– AR=1, the elliptical polarization becomes circular polarization.

⇒ Thus the condition for circular polarization is given by

$$A.R = 1 = \frac{E_\theta}{E_\phi} = \frac{2S\lambda}{\pi^2 D^2}$$

i.e., $E_\theta = E_\phi$

$$\Rightarrow 2S\lambda = \pi^2 D^2$$

$$S = \frac{\pi^2 D^2}{2\lambda} = \frac{C^2}{2\lambda}$$

where c = circumference $= \pi D$

⇒ The pitch angle for the circular polarization is given by

$$\alpha = \tan^{-1}\left[\frac{S}{\pi D}\right] = \tan^{-1}\left[\frac{\frac{\pi^2 D^2}{2\lambda}}{\pi D}\right]$$

$$= \tan^{-1}\left[\frac{\pi D}{2\lambda}\right]$$

$$= \tan^{-1}\left[\frac{c}{2\lambda}\right]$$

Axial Mode of Radiation:-

⇒ The helical antenna radiating field maximum in the end fire direction is along the axis of the helix is called axial mode (or) end fire mode helical antenna.

⇒ In the axial mode radiation the polarization of the wave is either circular (or) nearly circular.

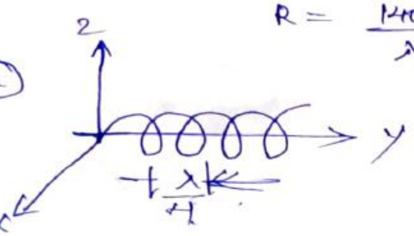
⇒ Helical antenna operated in axial mode when the circumference c and spacing are in the order of one wave-length.

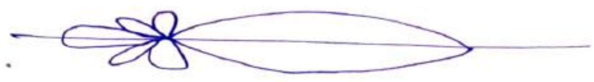
⇒ In axial mode pitch angle lies between 12° to 18° and beam width and antenna gain depends upon helix length NS.

⇒ The i/p impedence is given by

$$R = \frac{140C}{\lambda}$$

(a)





radiation pattern

⇒ The beam width between half power points is given by

$$HPBW = \frac{52}{c}\sqrt{\frac{\lambda^3}{N \cdot S}} \text{ degrees}$$

$$FNBW = \frac{115}{c}\sqrt{\frac{\lambda^3}{NS}} \text{ degrees}$$

⇒ The Maximum directive gain in axial mode is given by

$$G_D = \frac{15NS\lambda^2}{\lambda^3}$$

The axial ratio is

$$A \cdot R = 1 + \frac{1}{2N}$$

⇒ Features of helical Antenna:-

⇒ It is used for circular polarization.

⇒ It is used most widely in VHF, UHF bands.

⇒ The axial mode of helical antenna is most widely used.

⇒ In axial mode it have longer band width and high efficience.

⇒ It's construction is simple and directivity is high.

Applications:-

⇒ These antennas are used to achieve circularly polarized waves over extremely wide band width.

⇒ These are used in the space communication systems such as transmitting telemetry data from moon to the earth.

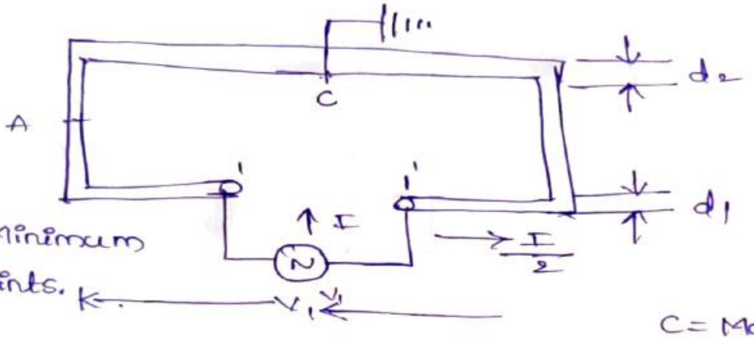
⇒ An array of helical antennas are useful in transmitting (or) receiving VHF signals through the iaso isosphere.

⇒ These are most extensively used in the satilite communications.

Folded dipole antenna:-

⇒ The two half wave dipoles have been folded and joining together, one of the half wave dipoles in contineous while other is split at the center to form a folded dipole.

⇒ The split dipole is fed at the center by a balanced transmission line.



A, B are Minimum current points.

C = Maximum current point.

⇒ The radiation pattern of the conventional half wave dipole and folded dipole is same.

⇒ The folded dipole i/p impedance is high compared with conventional half wave dipole.

⇒ There are two more important factors which differs folded dipole from the conventional half wave dipole, they are directivity and band width.

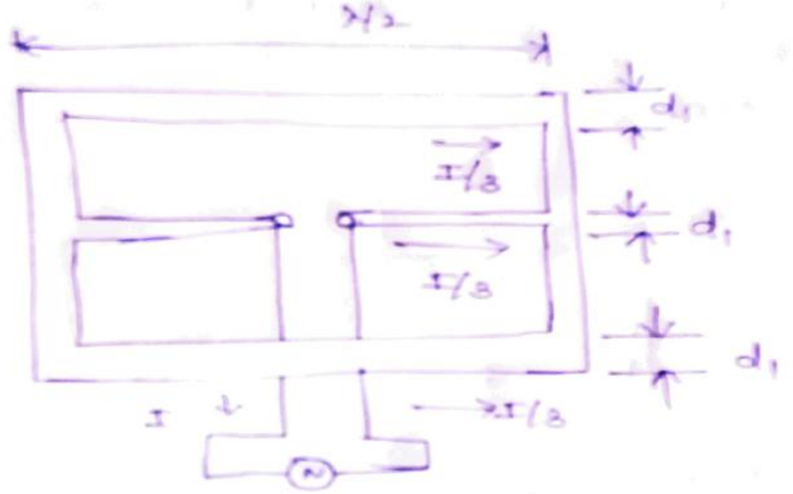
⇒ The power $P = I^2R$ is applied, only half of the current flows in first dipole. Because input impedence is 4 times to the straight dipole.

⇒ If the total current fed at the terminal 1&1' is I, then the each of the dipoles will have current I/2 provided their reduces of are equal ($d_1 = d_2$)

⇒ The i/p impedance of folded dipole is

$$R_{rad} = 2^2 \times 73 = 4 \times 73 = 292\ \Omega$$

⇒ The folded dipole can be fed with a conventional 300 Ω open wire transmission line without any matching device it is called tripole.



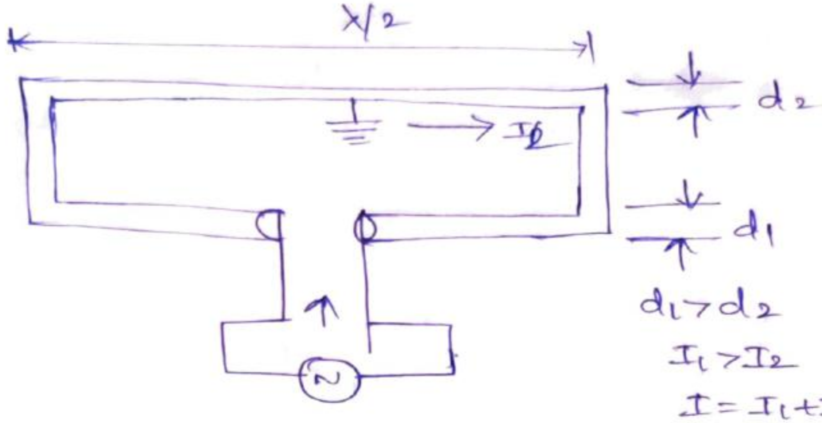
⇒ It is considered only 1/3rd of the radiating current would be supplied at the i/p terminals.

⇒ The i/p impedance (or) the radiation resistance of tripole is given as 9 times greater than impedance of straight dipoles.

$$\therefore R_{rad} = 3^2 (73) = 9 (73) = 657\ \Omega$$

⇒ Folded dipole antenna has built in impedance transforming properties which makes it easy match a transmission line that feeds the antenna.

⇒ Instead of changing the dipoles (i.e., element of the antenna) it is possible to change the input impedance by keeping the radius of the two dipoles unequal.



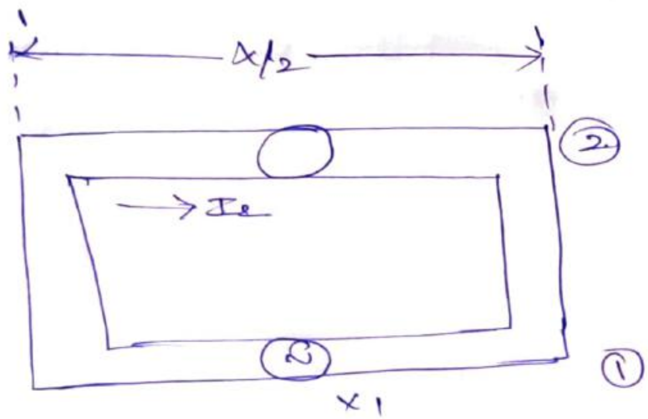
$d_1 > d_2$

$I_1 > I_2$

$I = I_1 + I_2$

I/P impedence of folded dipole antenna:-

⇒ The equivalent circuit of two wire folded dipole of length λ/2, the applied voltage V_1 which is applied across the terminals 1-1' gets equally divided in each dipole as voltage $V_1/2$



$$\therefore \; V_1/2 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow (1)$$

Here,

Z_{11} = self Impedence of dipole ①

Z_{12} = mutual Impedence between dipoles ① and ②

I_1 & I_2 are the current

⇒ Assume both the dipoles of equal radius, then

$I_1 = I_2$

From equation (1)

$$V_1/2 = (Z_{11} + Z_{12}) I_1 \longrightarrow (2)$$

⇒ If the two dipoles are very close to each other such that the spacing between the two is of the order of $\lambda/100$ then we can approximate that self impedence is equal to the multical impedence i.e;

$$Z_{11} = Z_{12}$$

From the equation ②

$$V/2 = (Z_{11} + Z_{11}) I_1$$

$$V_1 = 2(2Z_{11}) I_1$$

$$V_1 = (4Z_{11}) I_1$$

The input impedence of the antenna is given by

$$Z = V/I_1 = 4Z_{11}$$

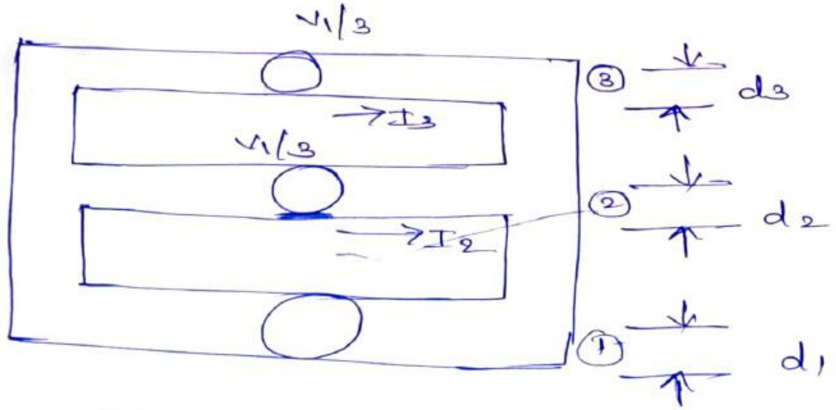
The self impedence of the dipole ① of the length $\lambda/2$ is nothing but it's radiation resistance, which is of value $73\,\Omega$.

$$\therefore Z = 4(73) = 292\,\Omega$$

Let us consider three wire folded dipole (or) tripole is given by

$$V_1/3 = (Z_{11} + Z_{12} + Z_{13}) I_1$$

As $I_1 = I_2 = I_3$



⇒ Assuming that all the elements are placed very closed to each other. Thus

$$Z_{12} = Z_{13} = Z_{11}$$

$$\therefore \quad V_1/3 = (3 \times Z_{11}) I_1$$

$$V_1 = (9 Z_{11}) I_1$$

The input impedance of the tripole is

$$Z = \frac{V_1}{I_1} = 9 Z_{11}$$

The self impedence of the $\lambda/2$ dipole is nothing but its radiation resistance which is also equal to $73\ \Omega$.

$$\therefore Z = 9(73) = 657\ \Omega$$

The folded dipole have ability of transmit forming impedence to the desired value. If the radius of the two dipoles are made unequal, then the input impedence is given by.

$$Z = Z_{11}\left[1 + \frac{r_2}{r_1}\right]^2 = 73\left[1 + \frac{r_2}{r_1}\right]^2$$

Here,

r_1 = Radius of dipole 1

r_2 = Radius of dipole 2

If $r_2 = 2r_1$ then

$$Z = 73\left[1 + \frac{2(r_1)}{}\right]^2 = 9(73) = 157\ \Omega$$

⇒ The impedence transformation depends on the radius industries and distance between the dipoles.

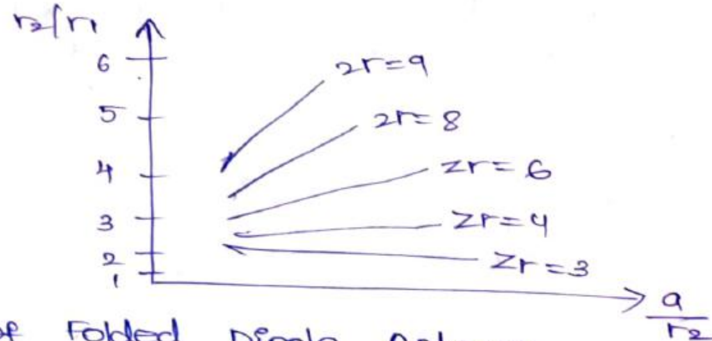
Let be the distance between two dipoles, then according to prof. uda and mushiake, the impedence

$$z = Z_{11}\left[1+\frac{\log\left(\frac{a_0}{r_1}\right)}{\log\left(\frac{a}{r_2}\right)}\right]^2$$

The ratio of antenna input impedence z to the self impedence of dipole is called impedence transformation Ratio (ITR) (or) Impedence step-up ratio.

$$\therefore \text{(ITR)} = \frac{z}{Z_{11}} = \left[1+\frac{\log\left(\frac{a}{r_1}\right)}{\log\left(\frac{a}{r_2}\right)}\right]^2 = z_r$$

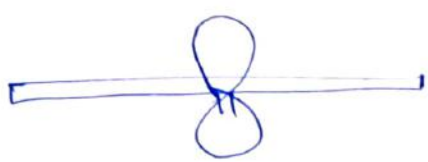
The ITR variation z_r with respect to spacing a and conductor radius r_1 and r_2



⇒ Features of Folded Dipole Antenna

⇒ It is basically a single antenna consulting two (or) three elements.

⇒ The radiation pattern of folded dipole antenna is same as that of straight dipole.



⇒ In a folded dipole antenna total current our is I. But current in each arm is I/2.

⇒ The input impedence of a folded dipole is four times of the straight dipole.

⇒ The folded dipole is used extensively in yagi-uda antenna as an active element.

⇒ The impedence can be transformed by forsts ranging from 1.5 to 25.

⇒ The spacing between the arms of folded dipole is very small.

⇒ Advantages

⇒ It has high input impedence..

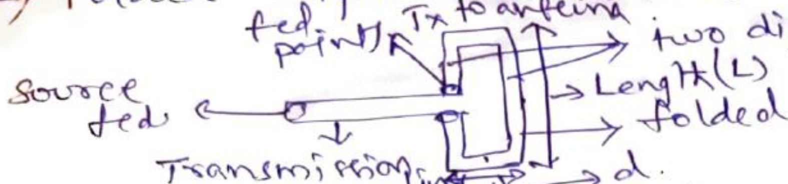
⇒ It has greater band width.

⇒ It acts as built in reactance compensation network.

⇒ It's construction is simple and is cheap

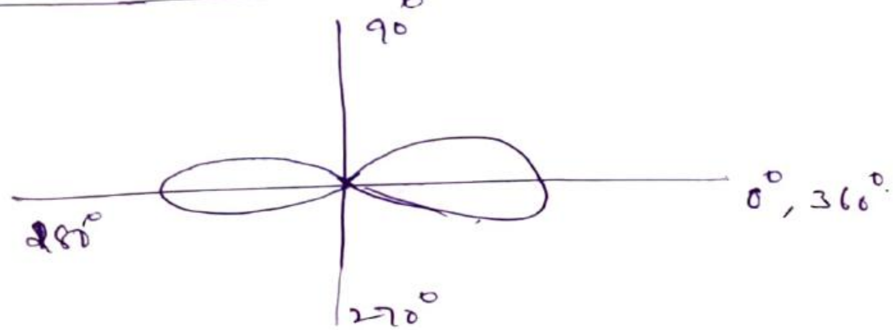
⇒ It has better impedence matching characteristics.

⇒ folded dipole antenna :-



It two Parallel dipoles antennas are connected in a form of wire loop, it is called folded dipole antenna.

Radiation pattern of folded dipole antenna :-



Properties :-

① Impedance increases ; therefore easy to match with the transmission line.

case ① :- $r_1 = r_2$; $Z = n^2 \cdot Z_0$, n = number of antennas, n = 2

ch's impedence is $\boxed{Z = 4Z_0} = 4 \times 73 = 292\,\Omega$

case ② :- $r_1 \neq r_2$; $Z = Z_{11}\left[1 + \dfrac{\log(d/r_1)}{\log(d/r_2)}\right] = Z_{11} \cdot Z_{ratio}$.

Z_{11} = Self impedance.

d = distance B-W two antennas.

r_1, r_2 are radius of 2 antennas.

② It has suitable for FM and TV. Broadcast.

③ It is used in Yagi-uda antenna because it has high gain antenna.

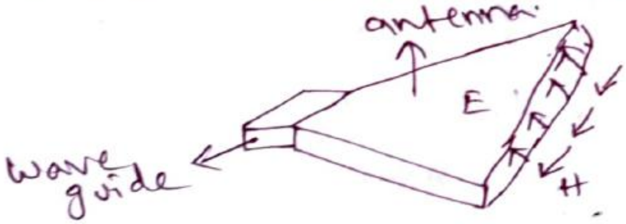
Horn antennas :-

* It is radiating element, which has a shape of horn to direct radio waves in one direction.
* It is a waveguide having one end of signal out.
* Radiation is poor and non-directive pattern results because of the Mismatch between the wave Guide and free space.

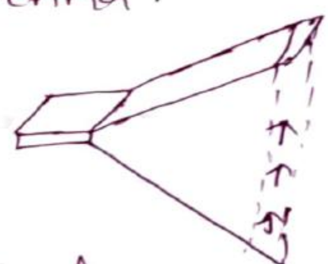
→ * Types of Horn antennas

① Sectoral Horn antenna.

→ It radiating is done only in one direction. is called sectoral Horn antenna.



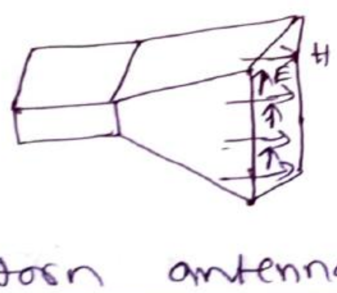
sectoral H - plane.



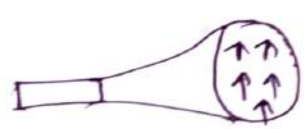
sectoral - E - plane Horn.

→ Pyramidal Horn antennas

→ Radiation is done along both E and H Fields.



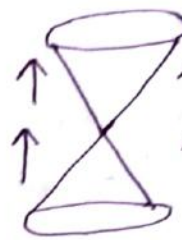
→ Conical Horn antennas



→ Radiation on flaring is done at walls of a circular wave guides.

→ Bi-conical antenna

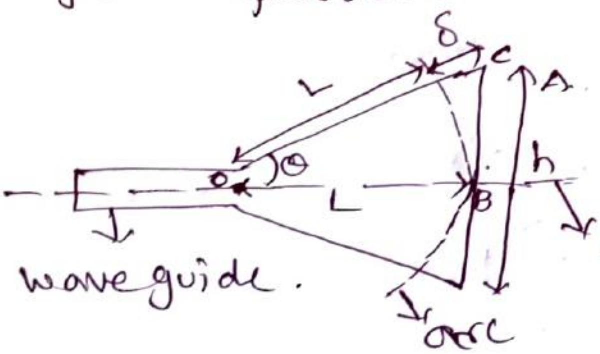
* combination of two conical antennas called Bi-conical antenna.



working principal of Horn antenna

→ Flaring helps to Match the antenna impedance with free space impedance for better radiation.

→ the function of the wave guide horn is to produce a uniform phase front with a larger aperture.



θ → Flare angle
L → Axial length
A → Aperture.
δ →

→ Design parameters.

$\angle COB \Rightarrow \cos\theta = \frac{OB}{OC} = \frac{L}{L+\delta}$

$$\theta = \cos^{-1}\left[\frac{L}{L+\delta}\right] \rightarrow (1)$$

$$\tan\theta = \frac{h/2}{L} \Rightarrow \theta = \tan^{-1}\left[\frac{h}{2L}\right] \rightarrow (2)$$

~~From Eq (1) and (2).~~

From $\angle OBC \Rightarrow (L+\delta)^2 = L^2 + (h/2)^2 \Rightarrow L^2 + \delta^2 + 2L\delta = L^2 + \frac{h^2}{4}$

$\Rightarrow L = h^2/8\delta$ $\quad \delta << L.$

Case (i)
(i) if flare angle 2θ is very large. this will result in non-uniform phase distribution.
→ High Beamwidth.
→ Reduced directivity.

Case (ii) If flare angle 2θ is small; this will results in uniform phase distribution.
→ Beam width is decreased.
→ Directivity is increased.

$$\boxed{D \propto \frac{1}{A}} \quad \text{and} \quad \boxed{A \propto \theta}$$

~~char~~ characteristics

the maximum directivity is achived at largest flare angle.

condition is that 'δ' does not excced the values

0.2 for E-plane Horn.
0.32 for Conical Horn.
0.4 for H-Plane Horn.

→ HPBW for E-direction $\theta_E = \frac{56\lambda}{h}$ degree.

$\theta_H = \frac{67\lambda}{w}$ degree.

→ Directivity.

$$D = \frac{7.5\,(h)\cdot w}{\lambda^2} = \frac{7.5\,(A)}{\lambda^2}.$$

Power gain $(G_p) = \frac{4.5A}{\lambda^2}$.

⇒ Radiated fields:

If the wave is radiating in z direction then E_y and H_x fields are given as,

$$E_y \approx E_1 \cos\left(\frac{\pi}{a}x'\right) e^{-j[ky'^2/2p]}$$

$$H_x \approx \underbrace{\boxed{\frac{E_1}{\eta}}}_{H_1} \cos\left(\frac{\pi}{a}x'\right) e^{-j[ky'^2/2p]}.$$

↳ Max magnetic field.

The complex exponential term is used to represent the quadratic phase variations of the fields over the aperture of the Horn.

⇒ Micro strip antennas:-

These are very Low-profile antennas. A metal Patch mounted at a ground level with a di-electric meterial in Between constitutes a Microstrip (or) patch antenna. these are low size antennas having low radiation.

Frequency range of these antennas are 100MHz and there are used in low profile applications.

→ Features of Micro strip antennas:-

* these antennas are small and light weight.
* these are very comfortable to planar and nonplanar surfaces.
* these antennas are versatile and low cost and Manufactured easily and produces high-quality signals.

⇒ Advantages of microstrip antennas:-

* these antennas operated at Microwave - frequencies.
* these are smaller size type antennas and hence will provide small size end devices.
* These are easily etched on any PCB.

* these antennas are easy to fabricate and comfortable on curved parts of the device. Hence it is easy to integrate them with MICs (or) MMICs (monolithic microwave - Integrated circuits).
* these antennas are having various shapes like rectangular, square, triangular etc...
* they have lower fabrication cost and hence they can be mass manufactured.
* They are capable of supporting multiple frequency bands. (dual, triple).
* they support dual polarization types like linear and circular polarization.
* they are light in weight.
* they are robust when mounted on rigid surfaces of the devices.

⟹ <u>limitations of microstrip antennas:-</u>

* low efficiency
* Narrow Bandwidth of less than 5%.
* Low RF power due to smaller separation between the radiation patch and the ground plane.
*

Rect
Det

Rectangular patch antennas:-

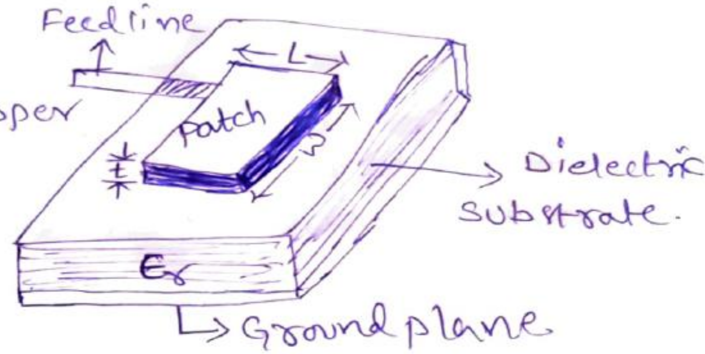
Definition:- A microstrip antenna consists of a radiating metal patch on one side of a dielectric substrate - which has a ground plane on the other side.

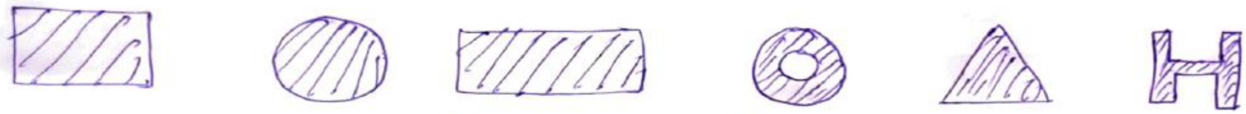
→ It is a kind of internal antenna.

→ The patch is made up of conducting material such as copper (or) gold.

→ It is available in any shape.

→ The radiating patch and the feed lines are photo-etched on the dielectric substrate.



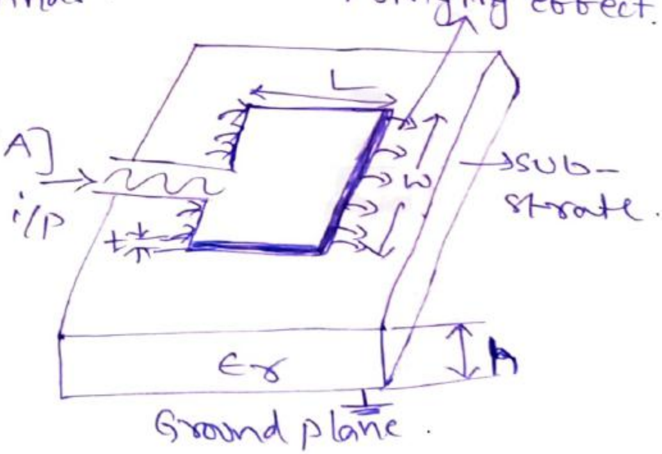


square. Disk Rectangle Ring Triangle H-shape.

Types of microstrip antennas.

→ Radiation Mechanism:-

Consider Rectangular patch [RMSA]



For Rectangular patch.

→ $0.3333\lambda_0 < L < 0.5\lambda_0$

→ $t << \lambda_0$

→ Height of the dielectric substrate. $0.003\lambda_0 \leq h \leq 0.05\lambda_0$

→ $2.2 \leq \epsilon_r \leq 12$.

→ Microstrip antenna [MSA] radiate primarily because of the fringing fields between the patch edge and the ground plane.

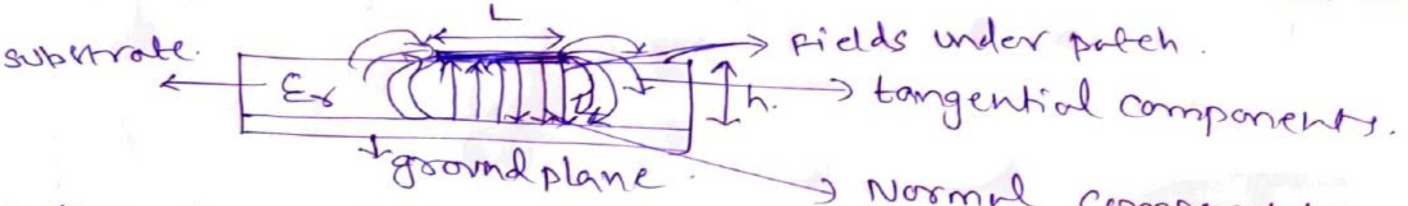
→ Assuming that there is no electric field variation along the width and thickness of the patch.

→ The field varies along the patch length which is about $\lambda/2$.

→ The field at the end can be resolved into the normal and tangential components with respect to ground plane.

→ The normal components are out of phase.

∴ The far field cancels in the normal directions.



→ The tangential components are in phase and the resulting fields combine to give maximum radiated fields.

* Trade-off between the antenna dimensions and Performance:-

→ Thick dielectric substrate having a low dielectric constant → provides Good antenna performance like provides better radiation, larger Bandwidth.

→ But it leads to larger antenna size.

→ In order to design compact patch antenna, substrates with high dielectric constant must be used.

→ Criteria for substrate selection:—

→ surface wave excitation
→ copper loss.
→ Effects of temperature and aging.
→ cost
→ Solderability, weight, elasticity...
Ex: Alumina ($\epsilon_r \to 9.8$), Air ($\epsilon_r = 1$).

→ Feed Techniques of Microstrip antennas:—

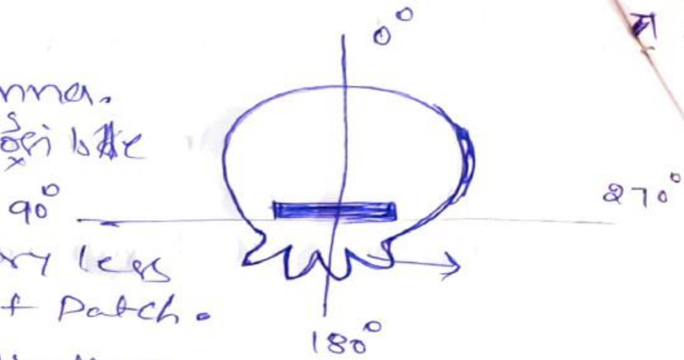
- Contacting Method
  - Microstrip line.
  - Coaxial Probe
- Non-contacting Method.
  - Aperture coupling
  - proximity coupling.

→ Advantages:-
→ light weight, low volume, conformability.
→ Low fabrication cost.
→ Can be easily integrated with microwave-integrated circuits.
→ Easy of installation.
→ Feed lines and Matching Networks can be integrated with antenna structure.

→ Disadvantages of MSA:—
① Narrow Bandwidth
② Lower gain
③ Low power handling capability
④ Excitation of surface waves.

→ Radiation pattern:-

→ It is unidirectional antenna. Maximum Radiation is possible in one direction only.



→ due to improper feeding very less radiation is done in back of patch.

Applications:-

→ used in space craft applications.

→ Aircraft applications.

→ used in Low profile antenna applications.

→ Used in Mobile phones, GPS system.

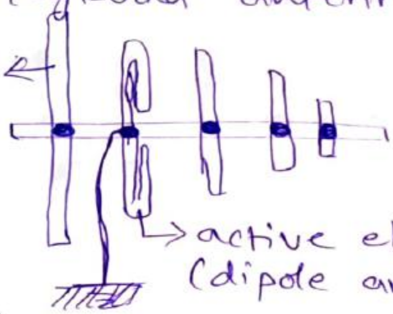
→ used in Biomadical radiator.

Reflector antenna:-

Introduction :-

→ It is highter directional Antenna.
→ It is used to very long distance communication, such as satellite communications.
→ It is operate with microwave frequency range (1-100 GHz).
→ It consists two types of elements.
  1. Active element. (Feed antenna)
  2. Parasitic element. (Reflector).

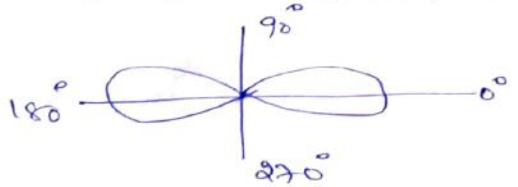
- **Defination** :- Reflector antenna is an antenna used to eliminate the backword radiation and to increase the ~~signification~~ signal radiation in the desired direction.
  For Example we consider the Yagi-Uda antenna.
  Here the reflector ~~is~~ provides uni-direction radiation to words desired direction.



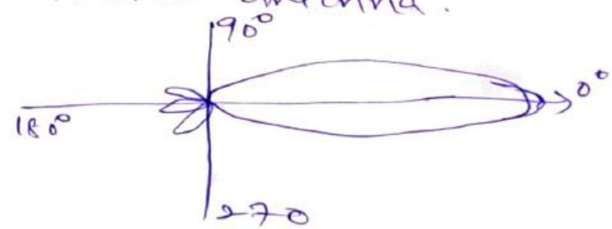
- the Reflector feeds back-word Radiation to desired direction.

Radiation Pattern of dipole and Reflector antennas :-

Di-pole antenna.



Reflector antenna.



→ Reflector is an element that reflects an electro-Magnatic waves.

→ A beam of predetermined characteristics is produced by using a large and suitable at the reflector surface.

Ex:- Dish antenna.

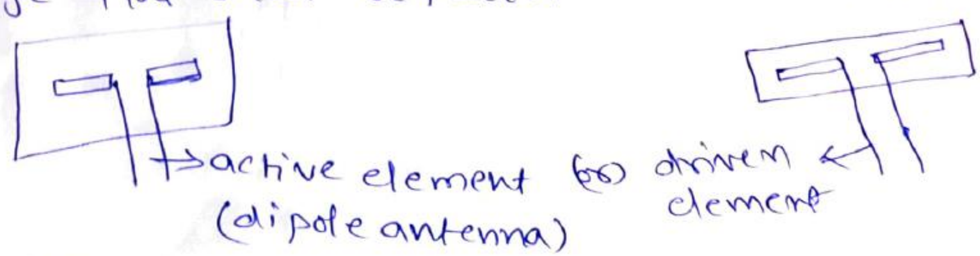
→ **Types of Reflector antenna:-**

It is conductive device and which is available in a dibberent shapes. Based on shapes these are called as follows.

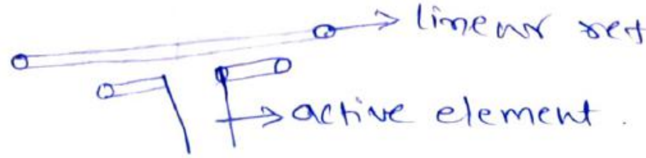
1. Flat sheet Reflector.
   - ↳ ① large flat sheet Reflector
   - ② small flat sheet Reflectors.
2. Thin reflector
3. corner Refleltor.
   - ↳ Active corner Reflector
   - ↳ Passive corner Reflector.
4. Parabolic Reflector.

→ **Flat sheet Reflector:-**

large flat sheet reflector. small flat sheet reflector



→ **thin reflector:-**

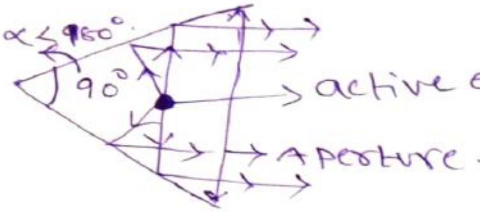


→ This is highly sensitive to frequency change.

→ $L \geq \lambda/2$

'L' length of reflector.

→ corner Reflector

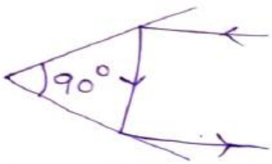


→ with two flat sheets intersecting at an angle ($\alpha < 180°$) a sharper radiation pattern can be obtained. this is called as active corner reflector.

⇒ For better Radiation $\alpha = 90°$.

⇒ Active element radiates it's energy to all directions.

⇒ Passive corner Reflector:-

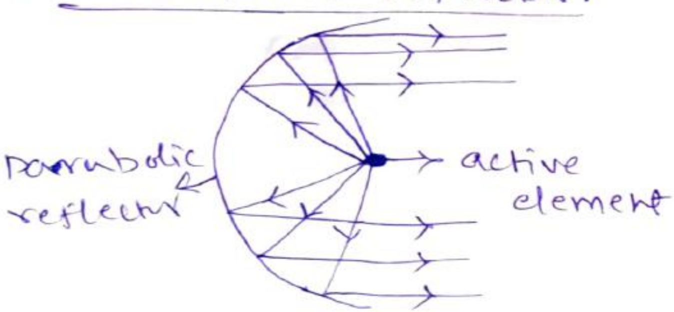


⇒ A corner reflector without an exciting antenna is known as passive reflector.

⇒ In which the incident wave is reflected back towards the source.

→ this doesnot have active source. which ever signal comming towards to reflector that will be reflected back only.

→ Parabolic reflector



⇒ It reflects the wave from the source at the focus into a parallel beam.

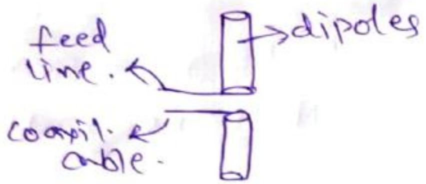
→ provides a highly directional antenna.

→ It is widlly used in all applications

Ex: Dish antenna.

⇒ Feed systems of Reflector antennas

→ the feedline is the that connects the antenna with the transmitter (or) receiver.

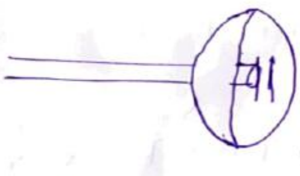


→ Primary radiator or active element which is feed the i/p with coaxil cable.

→ there are 4 feed systems used in reflector.

① Dipole feed
② End fire feed
③ Horn antenna
④ cassegrain feed.

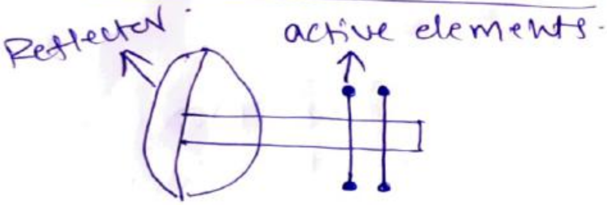
① Dipole feed:—



⇒ this is the simplest feed system.

⇒ Dipole antenna is the primary-radiator which is fed with a coaxial line.

② Endfire feed systems



⇒ the double dipole are together at end of the connecting line.

→ this arrangement produces the end fire Patterns.

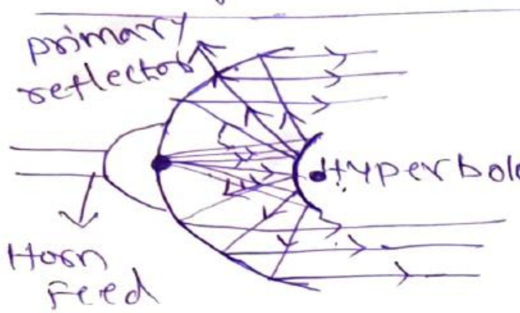
(Max radiation should be in axis of antenna).

③ Horn feed systems

⇒ wave guide Horn antenna is the primary radiator. which is pointing the parabolic antenna.

→ Max. radiation along the parabolic axis.

(4) Cassegrain Feed system:-



→ A primary radiator placed at center of Parabola.

⇒ the second feed is the sub-reflecter placed at the foucus of Hyperboloid.

→ the radiations emitted from primary feeder reach sub-reflector.

→ the sub reflecter reflects and illuminates the main Parabolic reflecter.

→ the main reflector reflects the rays parallel to the axis.

⇒ Advantages:-

1. To minimize the length of the transmission line.
2. Reduction in small loss of radiation.
3. Simple in construction.
4. Low cost.

A lens antenna is an antenna consisting an electromagnetic lens, with a feed. In other words, it is a three dimensional eclectromagnetic device having refractive index n other than unit. It's operation is similar to a glass lens used in optics. The lens antenna can be used in transmitting mode and in receiving mode both. The lens antenna used in transmitting and receiving mode is illustrated in fig (a) and fig (b) respectively.

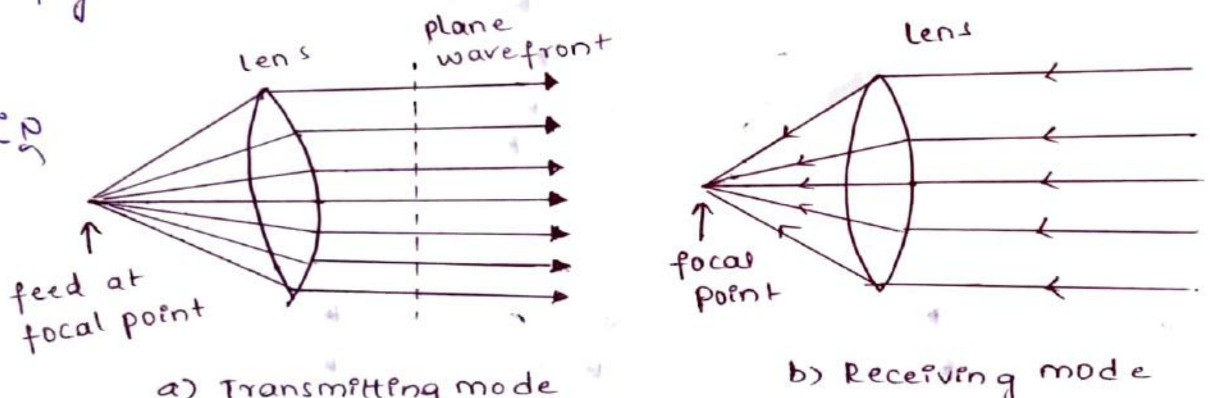


fig: various modes of operation of lens Antenna.

In general the functions of lens antenna are as follows:

i) It controls the illumination of aperture.

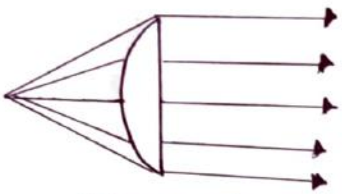
ii) It collimates the electromagnetic rays.

iii) It produces directional characteristics.

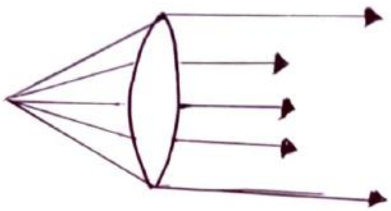
iv) In receiving mode, it converges the incoming wavefront it focus or focal point.

v) It produces plane wavefront from a spherical waveform.

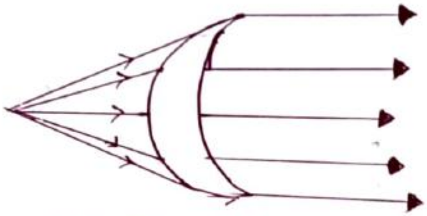
Various lens antenna configurations are as shown in the below fig (a) and (b)



convex plane

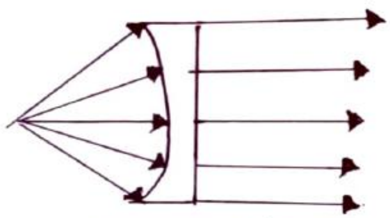


convex - convex

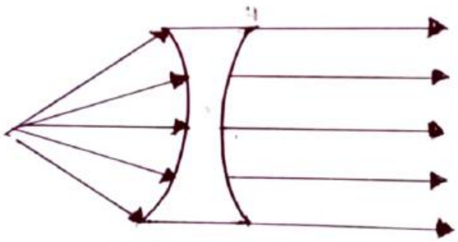


convex - concave

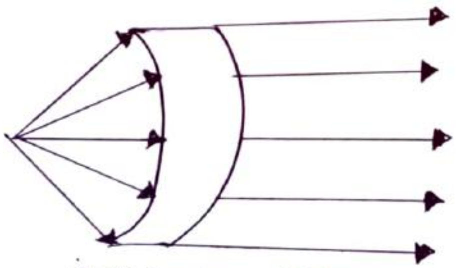
a) Lens antenna with refractive index $n>1$



concave plane



concave - concave



concave - convex

b) Lens antenna with refractive index $n<1$

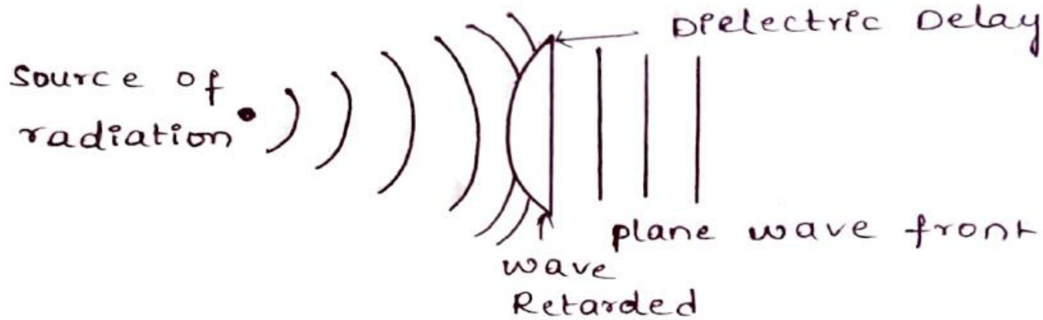
The main application of lenses is to collimate incident divergent energy to prevent it from spreading in undesired directions. These antennas are used to transform the diverged energy into the plane waves by properly choosing lens material and geometrical shape. These are widely used at very high frequency and their dimension and weight become extremely large at. lower frequen-cies. The lens antenna are classified according the material used to construct the lens or accord-ing to the geometrical shape of the lens.

Basically lens antennas can be classified as
i) Delay lens ii) Fast lens.

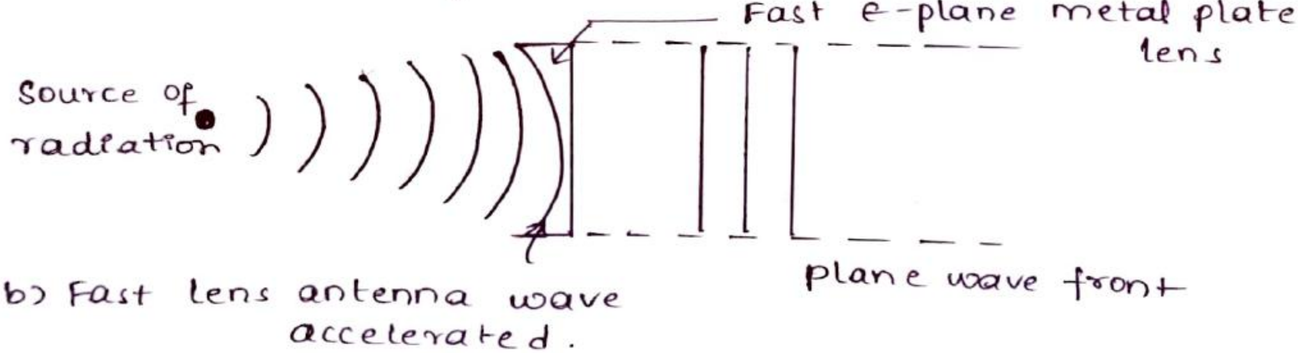
A Delay lens antenna is the antenna in which the electrical path length is increased by the lens medium and the wave is retarded. while a fast lens antenna is the antenna in which electrical path length is decreased by lens medium and the wave is accelerated. The examples of delay lens antennas are dielectric lenses and H-plane metal plate lenses while the example of fast lens antenna.

The E-plane metal plate lenses. The action of Delay and fast antennas are shown in the fig (a) and (b)

The dielectric lens are further classified on the basis of dielectric used. The dielectric lenses are constructed of either metallic or artificial dielectric or non-metallic dielectric such as polystyrene, luctie etc..,



a) Delay lens antenna

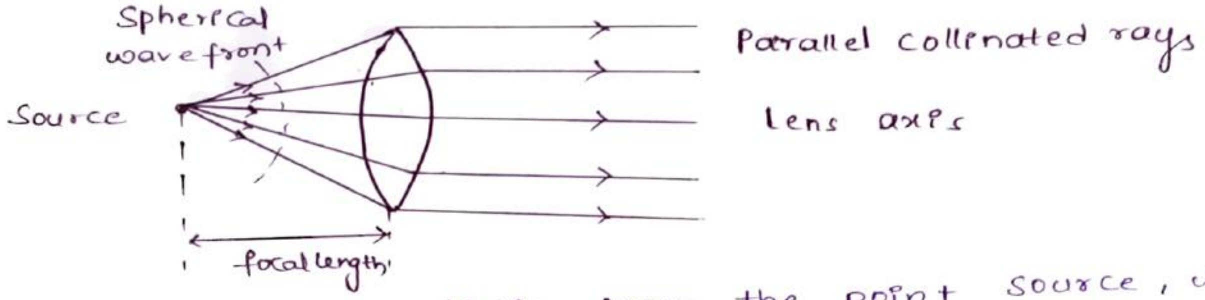


b) Fast lens antenna

fig: Delay and Fast lens antennas.

Principle of lens antenna :

Consider an optical concave lens. If a point source is placed at the focal point of lens which is along the axis of the lens, a fixed distance away from the lens as shown in the below figure.



Due to the radiation from the point source, we get spherical wavefront. when the rays travel to the lens, refraction takes place, due to the refraction index of the lens and thus rays are collimated to obtain plane wavefront of the parallel rays. The refraction is more at the edges than at centre.

To operate a lens at radio frequencies, a dielectric lens is preffered. Such lens with a point source producing spherical wavefront on left side of the lens produces collimated parallel rays to give plane wavefront. This operation illustrate transmitting lens antenna. Now if the parallel rays are incoming from right hand side of the lens, then these rays will converge to a point at the focal point on left hand side of antenna illustrations operation of the receiving lens antenna.

To acheive varying focussing properties at the ratiofrequencies, lens can be constructed of artificial dielectric having refractive index less than unity.

Zoning of lens :

The weight of the lens can be reduced by removing section of lens, which is called zoning of lens. The zoning can be classified as

(a) Curved Surface zoning (b) plane Surface zoning

In general, the zoning of lens is carried out in such a way that particular design frequency, the performance of length of antenna is not affected. The zone step is denoted by z. So in the zoned lens antenna, the thickness z of the lens antenna is such that the electrical length of the thickness z in dielectric is an integral length of λ longer than that in antenna. That means z is dielectric may be $3\lambda_d$ and that in air is $2\lambda_0$. Where λ_d and λ_0 are the wavelength in the dielectric and respectively.

For 1λ difference

$$\frac{z}{\lambda_d} - \frac{z}{\lambda_0} = 1$$

But refractive index $n = \frac{\lambda_0}{\lambda_c}$

$$\therefore \frac{z}{(\lambda_0/n)} - \frac{z}{\lambda_0} = 1$$

$$\therefore \frac{(n-1)z}{\lambda_0} = 1$$

$$\boxed{\therefore z = \frac{\lambda_0}{n-1}}$$

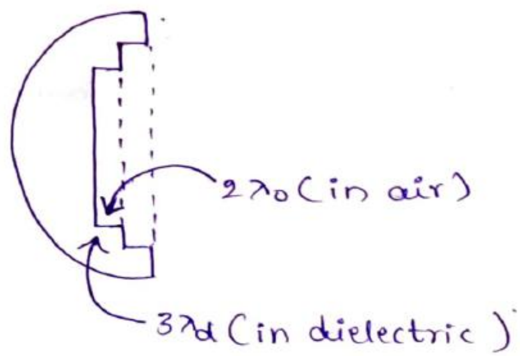


Fig:- Zoning of lens antenna

The difference b/w curved Surface and plane Surface zoning is as follows.

S.No	Curved Surface zoning	Plane Surface zoning
1.	As the zoning is done along the curved surface of lense, it is called curved surface zoning.	As the zoning is done along the plane surface of the lens, it is called plane surface zoning.
2.	It is mechanically stronger than plane surface zoning.	It is mechanically weaker than curved surface zoning.
3.	It has less weight	It has comparatively bulkier.
4.	The power dissipation of curved surface zoning antenna is less.	The power dissipation of curved surface zoning antenna is more.
5.	The curved surface is as shown below in the fig(a).	The curved surface zoning is as shown in fig(b).

a) Curved surface zoining (b) plane surface zoning

Fig:, Types of Surface zoning

Advantages of zoning:

*) The zoning of lens antenna reduces weight of an antenna and makes it comparatively stronger.

*) The zoning of lens antenna makes sure that after emergence the signals are in phase.

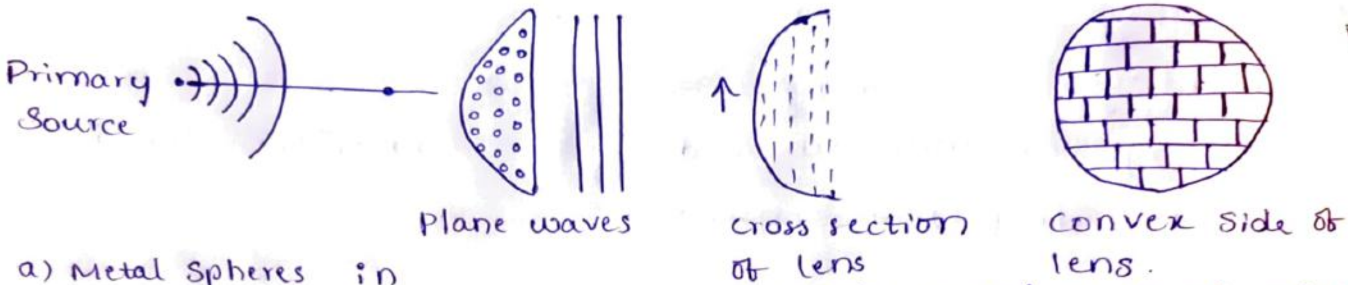
v) The zoned antenna shows less power dissipation compared with that in unzoned antenna.

Disadvantages of zoning:

As compared with unzoned lens antenna, the zoned lens antenna are frequency sensitive.

Metal plane lens antenna:

To make the lens light weight, an artificial or metallic dielectric is used for lens. If we compare the composition of ordinary dielectric which is non-metallic and artificial dielectric which is metallic, we can say that the non-metallic dielectric consists of molecular particles of microscopic size. While artificial, metallic dielectric is madeup of metal particles of macroscopic size. In the metal dielectric, the size of the particle must be dimensionally small as compared with the design wavelength; otherwise the dielectric matter suffers due to the resonance effect of the particles. At the same time, to avoid the diffraction effects, the spacing b/w the particles is less than the wavelength. The particle may be spherical (as shown in the fig (a)), disks, rod or strips, etc. The particles of spherical type are among in the three dimensional array (or) lattice structure a shown in the fig (a).



a) Metal spheres in artificial dielectric

(b) flat metal strips of width in artificial dielectric

Fig:, Artificial dielectric

- Consider that uncharged conducting sphere is ~~physical~~ placed in an electric field $\bar{E}$, which induces positive and negative. charges on metal sphere as shown in the fig. below

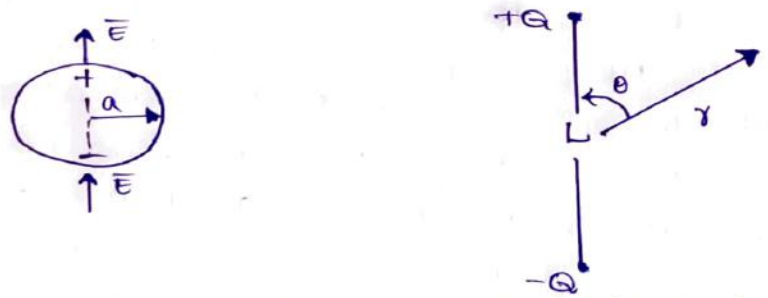


Fig:, Metal sphere in electric field and equivalent dipole.

It is equivalent to the dipole with point charges +Q and −Q separated by distance L. This dipole is electric dipole with dipole moment QL. Assuming $r >> L$, the potential due to dipole is given by,

$$V = \frac{QL\cos\theta}{4\pi\varepsilon_0 r^2} \longrightarrow (1)$$

Let $\bar{P}$ be the polarization of the artificial dielectric. Then it can be expressed interms total charge as,

$$\bar{P} = (NQ)\bar{dL} \longrightarrow ②$$

Where N = Number of sphere per cubic meter

The electric flux density $\bar{D}$ is given by

$$\bar{D} = \varepsilon\bar{E} = \varepsilon_0\bar{E} + \bar{P}$$

$$\varepsilon = \varepsilon_0 + \frac{\bar{P}}{\bar{E}} \longrightarrow ③$$

Where ε is the effective dielectric constant of the artificial dielectric medium which depends on ε_0 i.e., the dielectric constant of free space and polarization $\bar{P}$. In equation ②, we get,

$$\varepsilon = \varepsilon_0 + \frac{(NQ)d\bar{L}}{\bar{E}} \longrightarrow ④$$

For the uniform field, the electrical ~~prop~~ potential is given by,

$$V = -\int_0^r E\cos\theta \, dr = -Er\cos\theta \longrightarrow ⑤$$

Where θ is angle b/w radius and the field vectors. Then outside sphere, the potential is given by,

$$V_0 = -Er\cos\theta + \frac{QL\cos\theta}{4\pi\varepsilon_0 r^2} \longrightarrow ⑥$$

If radius of the sphere be a. Then at the sphere, the potential is given by,

$$0 = -Ea\cos\theta + \frac{QL\cos\theta}{4\pi\varepsilon_0 a^2}$$

$$\therefore \frac{QL\cos\theta}{(4\pi a^2)\varepsilon_0} = (Ea)\cos\theta$$

$$\therefore \frac{QL}{E} = (4\pi a^3)\varepsilon_0 \longrightarrow (7)$$

Putting value for dipole moment per unit applied field, we get,

$$\varepsilon = \varepsilon_0 + N(4\pi a^3)\varepsilon_0$$

$$\boxed{\therefore \varepsilon = \varepsilon_0 + 4\pi\varepsilon_0 N a^3} \longrightarrow (8)$$

But

$\varepsilon = \varepsilon_0 \cdot \varepsilon_r$, thus equation (8) becomes

$$\varepsilon_0\varepsilon_r = \varepsilon_0(1 + 4\pi N a^3)$$

$$\varepsilon_r = 1 + 4\pi N a^3 \longrightarrow (9)$$

Equation (9) indicates the effective relative permitivity of the artificial dielectric.

The effective relative, permeability of the artificial dielectric is given by,

$$\mu_r = 1 - 2\pi N a^3 \longrightarrow (10)$$

Hence the refractive index of the artificial dielectric with conducting spheres is given by,

$$\therefore n = \sqrt{\mu_r\varepsilon_r} = \sqrt{(1 - 2\pi N a^3)(1 + 4\pi N a^3)} \longrightarrow (11)$$

Note that if the relative permeability of the artificial dielectric medium is unity then, the refractive index is given by,

$$\therefore n = \sqrt{\varepsilon_r} = \sqrt{(1 - 4\pi N a^3)} \longrightarrow (12)$$

It is also observed that the relative permeability for the disc or strip type artificial dielectric is approximately, equal to unity. Hence for these artificial dielectrics, the index of refraction is given by eq^n 12 itself.

E-plane Metal plate lens Antenna :-

To study the development of E-plane metal antenna the knowledge of wave guide is required. Consider that TE_{10} mode wave is propagated through two parallel conducting plates of infinite extent. The guide wavelength is denoted by λg. Then the guide wavelength λ_0 through expression,

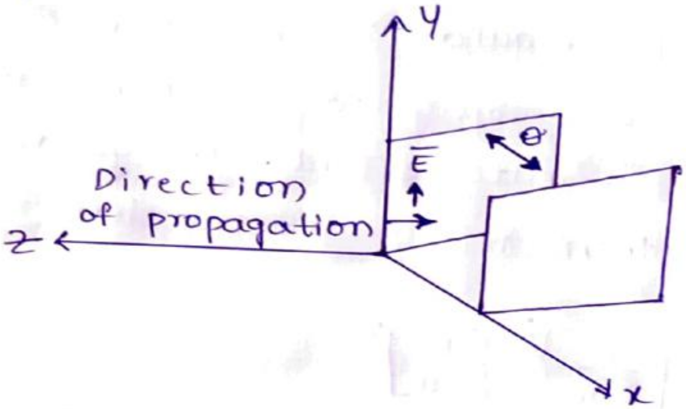
$$\frac{1}{\lambda_g^2} = \left[\frac{1}{\lambda_0}\right]^2 - \left[\frac{1}{2a}\right]^2 \longrightarrow (13)$$

where a = internal dimension of wave guide (or) spacing between two plates.

The phase velocity of the wave in a rectangular guide is always greater than velocity of wave in free space (i.e., $c = 3\times 10^8$ m/s) and is given by,

$$V_p = \frac{c\lambda g}{\lambda_0} \longrightarrow (14)$$

Consider that wave propagates b/w two parallel plane separated by as shown in the fig. T.W The electric field vector is parallel to plates. This indicates that this in the part of a rectangular waveguide with other dimensions infinity large. So the guide wavelength only depends on dimension a i.e., spacing between plates.



Note that one can develope a structure consisting similar parallel plates, many in number and with spacing between two parallel plates same i.e., 'a' such a structure can be regarded as an uniform medium which has effect refractive index n dependent on ratio of velocities. The reffective refractive index is given by,

$$n = \frac{c}{v_p} = \frac{c}{\left[\frac{c\lambda_g}{\lambda_0}\right]} = \frac{\lambda_0}{\lambda_g} \longrightarrow (15)$$

Let us consider equation 13. Multiplying both the Side of the equation by λ_0^2, we get,

$$\lambda_0^2\left[\frac{1}{\lambda_g}\right]^2 = \lambda_0^2\left[\frac{1}{\lambda_0}\right]^2 - \lambda_0^2\left[\frac{1}{2a}\right]^2$$

i.e., $$\left(\frac{\lambda_0}{\lambda_g}\right)^2 = \left(\frac{\lambda_0}{\lambda_0}\right)^2 - \left(\frac{\lambda_0}{2a}\right)^2$$

$$\frac{\lambda_0}{\lambda_g} = \sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2} \longrightarrow 16$$

- But from eq^n 15, the refractive index is the ratio of free space wavelength to the guide wavelength. Hence eq^n 16 can be written as,

$$n = \sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2} \longrightarrow 17$$

The value of n is practically always less than unit,

- The spacing b/w the plates for which the refractive index becomes zero is called critical spacing b/w plate. To obtain the expression for critical spacing, we can write,

$$0 = \sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}$$

Squaring both the sides,

$$0 = 1-\left(\frac{\lambda_0}{2a}\right)^2$$

i.e., $$a = \left(\frac{\lambda_0}{2a}\right)^2$$

$$\therefore (2a)^2 = \lambda_0^2$$

$$\therefore a = \frac{\lambda_0}{2} \longrightarrow (18)$$

Thus in general, a metal plate lens antenna can be developed using principle of waveguides in which many parallel plates with spacing 'a' are used.

The main difference b/w an ordinary lens and metal plate lens is that the ordinary dielectric lens action depends on the retardation of the wave in lens while the metal plate lens action depends on accelerating of the wave in lens. In other words the ordinary dielectric lens slows down the wave front while the metal plate lens speeds up the wavefront. The convergen metal plate lens consists metal plates which are concave in shape and arrangement. Such an arrangement is as shown in the fig.

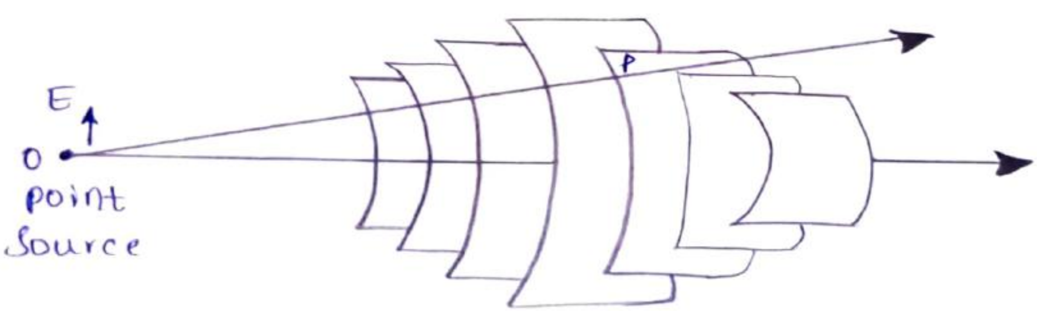


Fig: Convergent E-plane metal plate lens

Now consider a plate which is on the axis of the lens as shown in the fig.

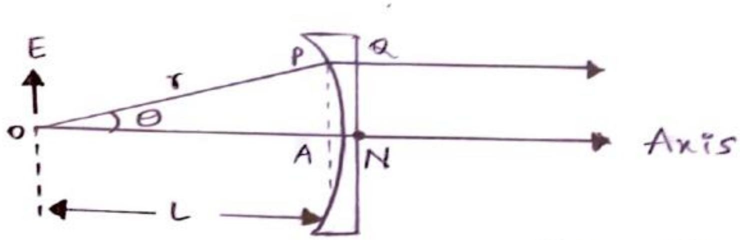


Fig:, A metal plate of lens on the axis of lens

According to Fermat's principle, the shape of the plate is based on the principle of equality of electrical path length.

So, we can write, according to the quality of the electrical path length,

$$OPQ = OAN$$

If 'L' is the focal length then we can write,

$$\frac{L}{\lambda_0} = \frac{r}{\lambda_0} + \frac{L - r\cos\theta}{\lambda_g} \longrightarrow (19)$$

where λ_g = wavelength of wave in lens

λ_0 = wavelength in free space.

Multiplying equation (19) by λ_0, we get,

$$L = r + \left(\frac{\lambda_0}{\lambda_g}\right)(L - r\cos\theta)$$

But the ratio of free space wavelength to the wavelength in the lens is the effective refractive index n (refer eqn (15)). Hence we get,

$$L = r + n(L - r\cos\theta)$$

$$\therefore L = r + nL - nr\cos\theta$$

$$\therefore (L - nL) = r - nr\cos\theta$$

$$\therefore L(1-n) = r(1 - n\cos\theta)$$

$$r = \frac{L(1-n)}{(1-n\cos\theta)} \rightarrow (20)$$

With $\mu < 1$, the equation (8) represents equation of an ellipse. So we can achieve three dimensional concave surface of the concave lens, by rotating the centre plate on the axis. So if we assume primary antenna as a line sources perpendicular to the plane of the page then with all identical plates the surface of the lens takes the form of a elliptical cylinder.

The major drawback of the E-plane metal plate lens antenna compared with the dielectric lens antenna is that the bandwidth of metal plate lens antenna relatively smaller.

H-plane Metal plate lens Antenna :-

A H-plane metal plate lens antenna can be achieved by arranging a stack of metal plates coinciding the orientation of H-plane (or) perpendicular to E-plane as shown in the fig.

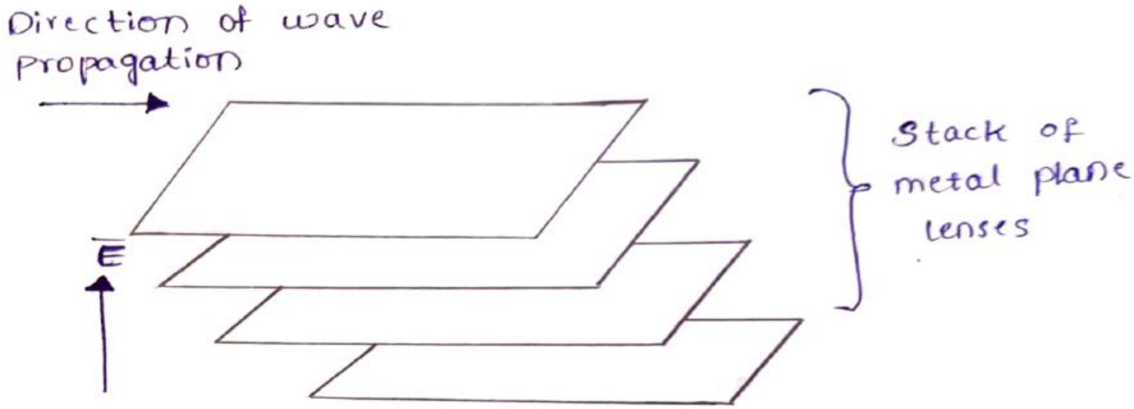
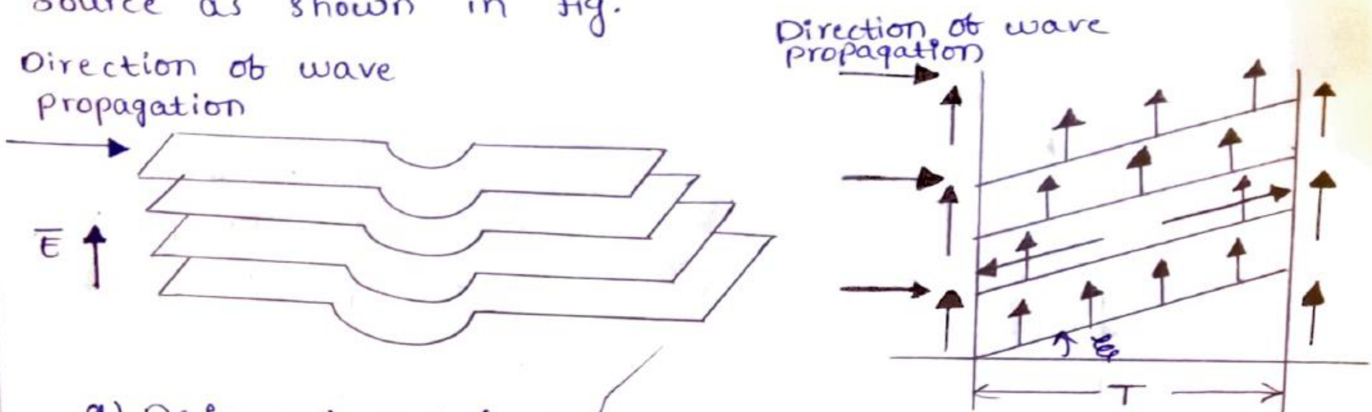


Fig:, H-plane flat metal plates stack

When the metal plates are arranged parallel to H-plane the velocity of the wave remains almost same but the path length decreases. Now to increase path length or to limit wave constraining there are two techniques which are used widely. The first technique is to be form the plates. So that the path length is increased as shown in the fig(a). In the other technique, the plates are arranged in slating mode as shown in the fig (b). In both the techniques, wave constraining is limited and effectively path length is increased.

Let us study how to design a H-plane metal plate be using the metal plates arranged in the slanting plate mode. For this we need to apply the principle of equality of electric path length. Consider that the metal plates are arranged the slanting mode in front of the

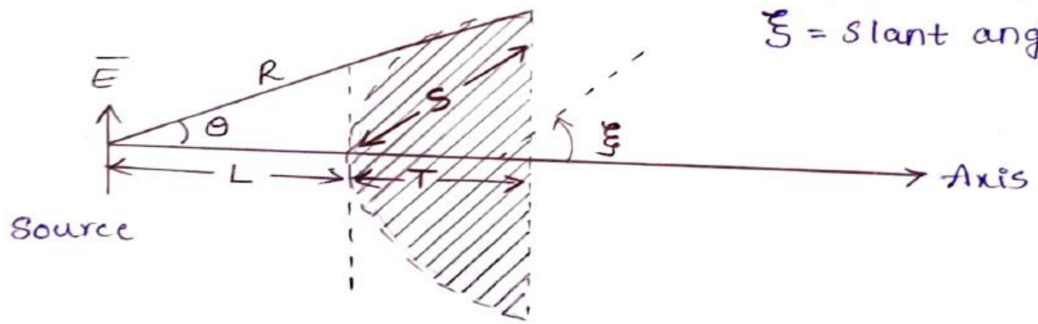
Source as shown in fig.



a) Deformed metal plate

(b) Slanted metal plates

Fig:, Techniques to increase path length in H-plane metal plane lens



S = path length

ξ = Slant angle

Fig:, Design of H-plane metal plate lens.

According to the principle of equality of electrical path length. the condition to be satisfied is given by,

$$R = L + \frac{R\cos\theta - L}{\cos\xi} \longrightarrow (21)$$

Let n be the effective refractive index of slant plate lens medium, then we can write,

$$R + L + \frac{R\cos\theta - L}{(1/n)}$$

$$\therefore R = L + nR\cos\theta - nL$$

$$\therefore R - nR\cos\theta = L - nL$$

$$\therefore R(1 - n\cos\theta) = L - nL$$

$$\therefore R = \frac{(n-1)L}{n\cos\theta - 1} \longrightarrow (22)$$

Above equation is identical to the equation (20) obtained Subsection E-plane metal plate lens antenna. If and only if the index of refraction is greater than or equal to unity. The index of refraction is dependent on the angle ξ called as slant angle. As compared with E-plane metal plate lens, the index of refraction is independent of frequency in case of H-plane metal plate lens. In case of the H-plane metal plate lens, the mos important dimension is path length ξ which gets affected due to variations in other dimensions like T and ξ. With the optimum allowed variation in the electrical path length of $\delta = \frac{\lambda_0}{8}$, the possible tolerance in path length S is given by,

$$\therefore S = \pm 0.06\lambda_0$$

The only disadvantages of the H-plane metal plate lens antenna is that unsymmetrical aperture illumination in E-plane takes place due to this slant plate made construction of the antenna.

Tolerances of lens Antennas:

In general, in dielectric lens antenna, the path length differences, are caused because of,

(i) Deviations in thickness from the ideal contour, and

(ii) Variations in the index of refraction.

Let us assume that maximum allowable variations in both the parameters to be $\frac{\lambda_0}{32}$ rms. Then the thickness tolerance is given by,

$$\frac{\Delta t}{\lambda_d} - \frac{\Delta t}{\lambda_0} = \frac{1}{32}$$

But, n = Index of refraction $= \frac{\lambda_0}{\lambda_d}$

$$\therefore \frac{\Delta t}{(\lambda_0/n)} - \frac{\Delta t}{\lambda_0} = \frac{1}{32}$$

$$\therefore \frac{n\Delta t}{\lambda_0} - \frac{\Delta t}{\lambda_0} = \frac{1}{32}$$

$$\therefore \Delta t = \frac{\lambda_0}{32(n-1)} \cong \frac{0.03\lambda_0}{n-1} \longrightarrow (1)$$

Thus for, $n = 1.5$,

$$\Delta t = 0.06\lambda_0$$

Now for the tolerance of n, we can write,

$$\Delta nt = \frac{\lambda_0}{32}$$

i.e., $\Delta n = \frac{\lambda_0}{32t} = \frac{1}{32(\frac{t}{\lambda})} = \frac{1}{32t_\lambda} = \frac{0.03}{t_\lambda} \longrightarrow (2)$

Where $t_\lambda = \frac{t}{\lambda}$ = Thickness of lens in free space wave length (λ).

Dividing equation (2) by n, we get,

$$\frac{\Delta n}{n} = \frac{0.03}{nt_\lambda} = \frac{3}{nt_\lambda}\% \longrightarrow (3)$$

In case of E-plane metal plate lens, the path length may be affected by the thickness of lens and spacing to b/w lens plate. So again assuming maximum allowable variation of $\frac{\lambda_0}{32}$, the thickness tolerance is given by,

$$\therefore \Delta t = \frac{\lambda_0}{32(1-n)} = \frac{0.03\lambda_0}{1-n} \longrightarrow (4)$$

Then for the tolerance on the spacing b between plates is given by,

$$\therefore \frac{\Delta b}{b} = \frac{3n}{(1-n^2)t_\lambda}\% \longrightarrow (5)$$

As compared to large reflector antenna, for the lens antenna, relatively large amount of warping or twisting tolerated. As the thickness tolerance of a lens is with reference to the thickness of the lens, it not neccessary to the contour of lens should be maintained to this accurence.

-: Antenna Arrays :- Unit - III

Usually a single element (small antenna) provides wide radiation and low directivity, but for long distance communication we need the high directivity.

→ To increase the field strength and directivity in desired direction by using group of antennas excited simultaneously.

→ Such a Group is called Array of antenna or antenna arrays.

→ The Array of Antenna total field strength is determ-ined by vector addition of the field radiated by individual antennas (elements).

→ This individual elements is generally called element of an antenna array.

→ If the elements of an antenna array are equally spaced along a straight line, this array is called uniform linear array.

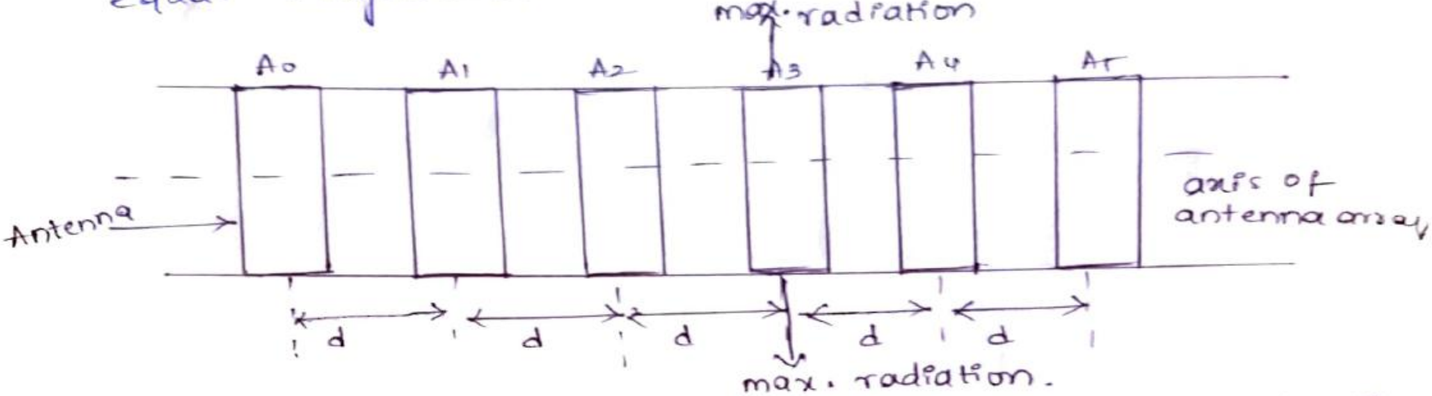
→ The antenna arrays are classified into

1. Broadside array.
2. End fire array.
3. collinear array.
4. Parasitic array.

Broad side Array:

→ A Broad side Array is one of the most commonly used antenna array. it contains no. of identical parallel antennas are arranged along a line and perpendicular to the array axis.

→ In this the individual antennas are equally spaced along a line and each element is fed with current of equal magnitude, all in the same phase.



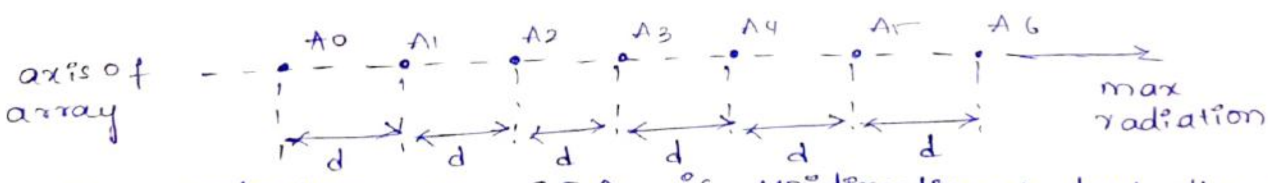
→ The radiation pattern of broadside array is bidirectional which radiates equally well in either direction of max. radiation.

End fire Array :- (EFA)

→ An array contains no. of identical antennas are spaced equally along a line and individual elements are fed with currents of unequal phases is called end fire Array.

→ This array is similar to the broad side Array that individual elements are fed with a phase shift of 180°

→ The direction of radiation is co-incides with the direction of the array axis



→ The radiation of EFA is unidirectional but the end fire array may be bi-directional also.

along of

For example a two element array fed with equal current, 180° out of phase.

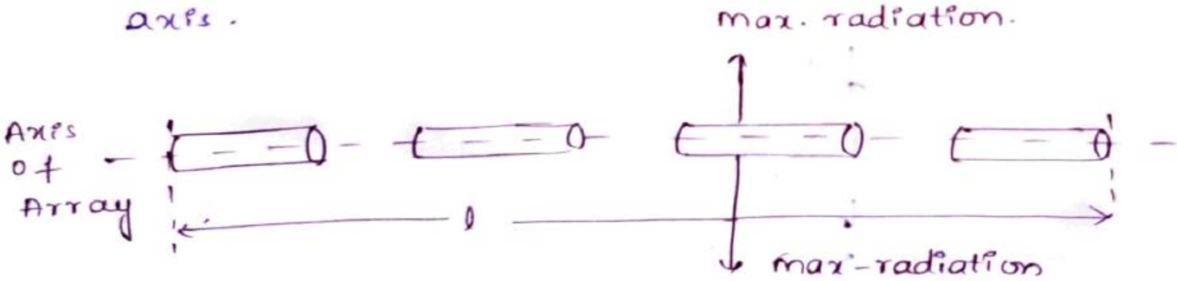
Collinear Array:

→ The Array in which antennas are arranged end to end in a single line is known as collinear Array.

→ the arrangement of collinear array in which one antenna is stacked over the another antenna. similar to that of broadside array.

→ The individual elements of collinear array fed with equal in phase currents, the direction of max. radiation is perpendicular to the line of antenna.

→ The collinear antenna sometimes called as broadcast or omni directional arrays because its radiation pattern has circular symmetry with its main to be every where perpendicular to the principle axis.



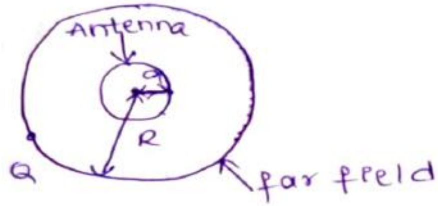
point source :

→ The idea of point source used for describing antenna is based on looking upon the antenna as a volumeless Emitter located at a point from a large distance where only the far field exists.

Antenna Array:

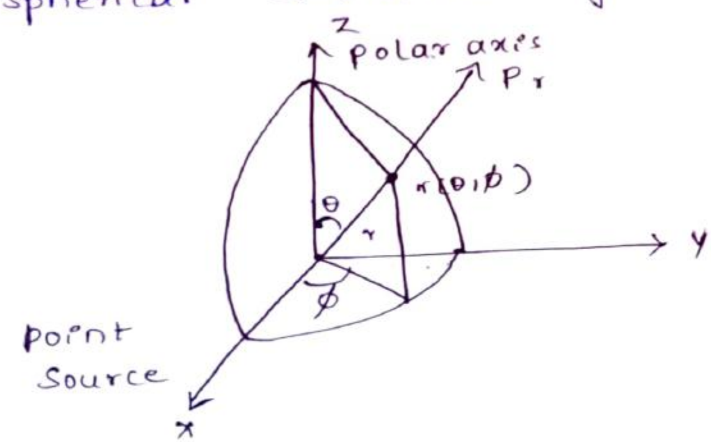
The arrangement in which the combination of antennas grouped and obtain the field strength can be raises in a particular direction. This arrangement is known as Antenna Array.

→ we consider an antenna contained within a volume radius a, and far field of an antenna with radius.



Power Pattern:

→ consider a transmitting antenna, represented by a Point source radiator, located at the origin of a spherical co-ordinate system



→ The radiated power density is the time rate of energy flow per unit area, also known as the poynting vector.

→ The magnitude of poynting vector is P_r. Then the total power $P(w)$ radiated by a point source is the integral over the surface of the sphere of the radial component P_r of the average poynting vector,

$$\therefore P = \oint\!\!\oint \bar{P} \cdot \bar{ds} = \oint\!\!\oint P_r \, ds$$

where ds is an infinitesimal element of area of sphere given by,

$$ds = r^2 \sin\theta \, d\theta \, d\phi$$

→ In case of the isotrophic radiator P_r is independent of θ and ϕ so that,

$$P = P_r \oint\!\!\oint ds$$

$$P_r = \frac{P}{4\pi r^2}$$

$$P = \iint \frac{P}{4\pi r^2} \cdot r^2 \cdot \sin\theta \, d\theta \, d\phi$$

$$P = \iint \frac{P}{4\pi} \sin\theta \, d\theta \, d\phi$$

→ The radiation intensity U, is obtained by multiplying the power P_r by the square of the radius r

$$U = r^2 P_r$$

$$P = \iint U \sin\theta \, d\theta \, d\phi$$

$$= \iint U \, d\Omega$$

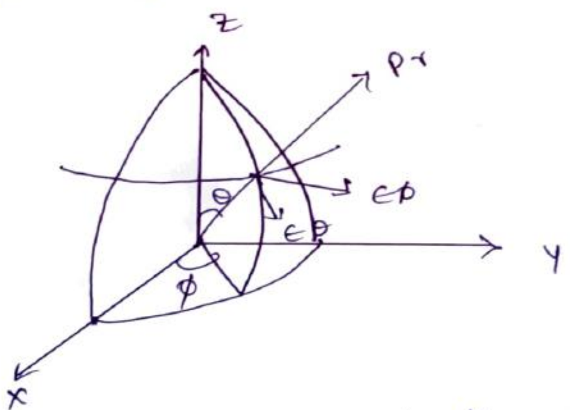
here, $d\Omega$ = element of solid angle.

Field Pattern:

→ To describe the field of a point source to consider the E-field $\bar{E}$ and the magnetic field $\bar{H}$. These are vectors.

→ $\bar{E}$ and $\bar{H}$ are following the relations.

1) Both are entirely transverse to the direction of propogation.
2) They are perpendicular to each other.
3) They are in phase.
4) They are related in magnitude by the intrinsic impedance of the medium (which is η)



→ The poynting vector and the field at the point of the far field are related in same manner they are in a plane wave.

$$\therefore P_r = \frac{1}{2} \frac{E^2}{\eta}$$

here η is the intrinsic impedance of the medium.
$E = \sqrt{E_\phi^2 + E_\theta^2}$. The field may be linearly, elliptical or circularly polarized

→ The direction of maximum field intensity is usually taken as reference direction. then the relative pattern of the E_θ component is,

$$\frac{E_\theta}{E_{\theta m}}$$, similarly E_ϕ component

$$\frac{E_\phi}{E_{\phi m}}$$

where $E_{\theta m}$ and $E_{\phi m}$ are the maximum values of E_θ and E_ϕ.

from the power equation,

$$P_{rm} = \frac{E_m^2}{2Z} \quad (E_m \text{ is max. value of } E)$$

→ The relative field power pattern is given by

$$P_n = \frac{P_r}{P_{rm}} = \frac{U}{U_m} = \left(\frac{E}{E_m}\right)^2$$

⇒ consider an antenna whose far field has only an E_ϕ component in the equitorial plane and zero E_θ component in this field. Let the relative equitorial plane pattern of E_ϕ component is,

$$\frac{E_\phi}{E_{\phi m}} = \cos\phi$$

$$\therefore P_\eta = \frac{P_r}{P_{rm}} = \left(\frac{E_\phi}{E_{\phi m}}\right)^2 = \cos^2\phi$$

⇒ we consider an antenna whose far field has only E_θ components. Let the relative equatorial plane pattern of the E_θ component is,

$$\frac{E_\theta}{E_{\theta m}} = \sin\phi$$

$$P_n = \frac{P_r}{P_{rm}} = \left(\frac{E_\theta}{E_{\theta m}}\right)^2 = \sin^2\phi$$

⇒ The relative pattern of the total field E is

$$P_n = \frac{E}{E_m} = \sqrt{\sin^2\phi + \cos^2\phi} = 1$$

$$P_n = \left(\frac{E}{E_m}\right)^2 = 1$$

Phase pattern:

→ The far field due to a source, in all directions can be completely specified knowing following quantities

a) magnitude of the azimuthal component E_ϕ of the electric field as a function of r, θ and ϕ

b) magnitude of the polar component E_θ of the electric field as a function of r, θ and ϕ

c) phase lag of E_ϕ behind E_θ as a function of θ and ϕ

d) phase lag of either field component behind that of a reference point as a function of r, θ and ϕ

Arrays of point sources:

→ let us consider two isotrophic point sources, with a distance of separation 'd' between them. The polarization of two isotrophic point sources is assumed to be the same

→ They are classified as:

i) Two point sources with currents of equal magnitude and with same phase.

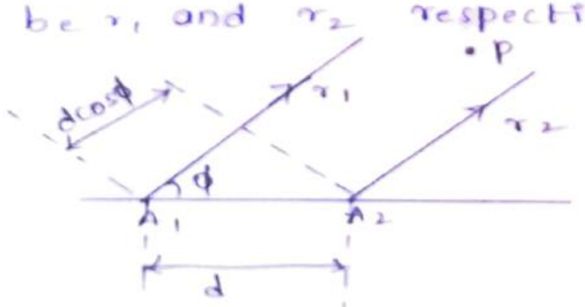
ii) Two point sources with currents of equal magnitude but with opposite phase.

iii) Two point sources with currents of unequal magnitude and with opposite phase.

i) Two point sources with currents equal in magnitude and phase:-

→ consider two point sources A_1 and A_2 seperated by distance 'd' and both point sources are supplied with currents equal in magnitude and phase.

→ Consider P far away from the array. let the distance between point P and point sources A_1 and A_2 be r_1 and r_2 respectively.



→ Let $r_1 = r_2 = r$, the radiation from the point source A_2 will reach earlier at point p than that from point source A_1 because of path difference.

$\therefore$ Path difference $= d\cos\phi$

Expressed in terms of wavelength.

Path difference $= \frac{d\cos\phi}{\lambda}$

$\therefore$ The phase angle ψ is given by,

phase angle $= \psi = 2\pi$ (path difference)

$= \frac{2\pi}{\lambda} d\cos\phi$

but the phaseshift $\beta = \frac{2\pi}{\lambda}$

$\therefore \psi = \beta d\cos\phi$.

→ Let E_1 be the farfield at a distance p due to point source A_1. Similarly E_2 be from A_2.

→ If the phase angle between two fields is $\psi = \beta d\cos\phi$, then the far field component at p due to point source,

$\therefore E_1 = E_0 \cdot e^{-j\psi/2}$

slly, $E_2 = E_0 \cdot e^{j\psi/2}$

The total field at point P,

$E_T = E_1 + E_2 = E_0 \cdot e^{-j\psi/2} + E_0 \cdot e^{j\psi/2}$.

$= E_0 \left[e^{-j\psi/2} + e^{j\psi/2} \right]$

$\therefore E_T = 2E_0 \left[\frac{e^{j\psi/2} + e^{-j\psi/2}}{2} \right]$

from Trigonometry $\frac{e^{j\theta} + e^{-j\theta}}{2} = \cos\theta$

$\therefore E_T = 2E_0 \cos(\psi/2)$

$$\therefore \underbrace{e_T}_{\text{Total field}} = 2E_0 \cos\underbrace{\left(\frac{\beta d \cos\phi}{2}\right)}_{\text{phaseshift}}$$

→ The array factor is the ratio of the magnitude of the resultant field to the magnitude of the maximum field.

$$A.F = \frac{e_T}{e_{max}}$$

but max. field is $e_{max} = 2E_0$

$$\therefore AF = \frac{e_T}{2E_0} = \cos\left(\pi \frac{d}{\ell} \cos\phi\right)$$

Max direction:

→ The total field is maximum when $\cos\left(\frac{\beta d \cos\phi}{2}\right)$ is maximum,

$$\cos\left(\frac{\beta d \cos\phi}{2}\right) = \pm 1.$$

The spacing between two point sources be $\lambda/2$.

$$\cos\left(\frac{\beta(\lambda/2)\cos\phi}{2}\right) = \pm 1$$

i.e., $$\cos\left(\frac{\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cdot \cos\phi}{2}\right) = \pm 1$$

$$\cos\left(\frac{\pi}{2}\cos\phi\right) = \pm 1$$

$$\frac{\pi}{2}\cos\phi_{max} = \cos^{-1}(\pm 1)$$

$$= \pm n\pi$$

if $n = 0$

$$\frac{\pi}{2}\cos\phi_{max} = 0$$

$$\cos\phi_{max} = 0$$

$$\phi_{max} = 90^\circ \text{ (or) } 270^\circ$$

Minima direction:

→ Total field strength is minimum when $\cos\left(\frac{\beta d \cos\phi}{2}\right)$ is minimum,

$$\therefore \cos\left(\frac{\beta d \cos\phi}{2}\right) = 0$$

assume $d = \lambda/2$

$$\cos\left(\frac{\pi}{2}\cos\phi_{min}\right) = 0$$

$$\frac{\pi}{2}\cos\phi_{min} = \cos^{-1}(0)$$

$$= (2n+1)\frac{\pi}{2}$$

if $n = 0$

$$\frac{\pi}{2}\cos\phi_{min} = \pm\frac{\pi}{2}$$

$$\cos\phi_{min} = \pm 1$$

$$\phi_{min} = 0^\circ \text{ (or) } 180^\circ$$

Half power point directions:

→ when the power is half, the voltage or current is $\frac{1}{\sqrt{2}}$ times the max. value hence,

$$\cos\left(\frac{\beta d \cos\phi}{2}\right) = \pm\frac{1}{\sqrt{2}}$$

$$d = \lambda/2, \quad \beta = \frac{2\pi}{\lambda}$$

$$\cos\left(\frac{\pi}{2}\cos\phi\right) = \pm\frac{1}{\sqrt{2}}$$

$$\therefore \frac{\pi}{2}\cos\phi = \cos^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$$

$$= \pm(2n+1)\frac{\pi}{4}$$

if $n=0$

$$\frac{\pi}{2}\cos\phi_{HPPO} = \pm\pi/4$$

$$\cos\phi_{HPPO} = \pm 1/2$$

$$\phi_{HPPO} = \cos^{-1}\left(\pm\frac{1}{2}\right)$$

$$\therefore \phi_{HPPO} = 60^\circ \text{ (or) } 120^\circ$$

- **Two point sources with currents equal in magnitude but opposite in phase :**

consider two point source seperated by distance 'd' and supplied with currents equal in magnitude but opposite in phase. All the conditions are exactly same except the phase of the currents in opposite i.e., 180° with this condition the total field at far point p is given by,

- $$E_T = (-E_1) + E_2 \quad \text{——} \ (1)$$

Assuming equal magnitudes of currents, the field at point p due to the point sources A_1 and A_2 can be written as,

$$E_1 = E_0 e^{-j\psi/2} \quad \text{——} \ (2)$$

and

$$E_2 = E_0 e^{j\psi/2} \quad \text{——} \ (3)$$

substituting values of E_1 & E_2 in eq (1)

$$E_T = -E_0 e^{-j\psi/2} + E_0 e^{j\psi/2}$$

$$E_T = E_0 \left[e^{-j\varphi/2} + e^{j\varphi/2} \right]$$

Rearranging the terms in above eq, we get,

$$E_T = (J2)\, E_0 \left[\frac{e^{j\varphi/2} - e^{-j\varphi/2}}{2} \right] \quad \text{--- (4)}$$

by trigometric identity, $\frac{e^{j\theta} - e^{-j\theta}}{2} = \sin\theta/2$

Hence eq (4) can be written as,

$$E_T = J2E_0 \sin\left(\frac{\varphi}{2}\right) \quad \text{--- (5)}$$

Now as the condition for 2-point sources with current in phase and out of phase is exactly same, phase angle can be written as,

$$\text{phase angle} = \varphi = \beta d \cos\phi . \quad \text{--- (6)}$$

Sub. value of phase angle in eq (5) we get,

$$E_T = J\,(2E_0) \sin\left(\frac{\beta d \cos\phi}{2}\right) \quad \text{--- (7)}$$

Maxima direction:-

from eq (7) the total field is maximum when, $\sin\left[\frac{\beta d \cos\phi}{2}\right]$ is maximum i.e., ± 1

as the maximum value of sin of angle is ± 1.

Hence, condition for maximum is given by,

$$\sin\left[\frac{\beta d \cos\phi}{2}\right] = \pm 1 \quad \text{--- (8)}$$

Let the spacing b/w 2 isotropic point source is equal to $\lambda/2$ ie., $d = \lambda/2$

Substituting $d = \lambda/2$ and $\beta = \frac{2\pi}{\lambda}$ in eq (8) we get,

$$\sin\left[\frac{\pi}{2}\cos\phi\right] = \pm 1.$$

i.e., $\frac{\pi}{2}\cos\phi = \pm(2n+1)\frac{\pi}{2}$, where $n = 0, 1, 2, 3 \ldots$

if $n = 0$ then,

$$\frac{\pi}{2}\cos\phi_{max} = \pm\pi/2$$

i.e., $\cos\phi_{max} = \pm 1$

i.e, $\boxed{\phi_{max} = 0^\circ \text{ (or) } 180^\circ}$ —— (9)

Minima direction :-

Again from eq (7) total field strength is minimum when $\sin\left[\frac{\beta d\cos\phi}{2}\right]$ is minimum i.e., 0.

Hence the condition for minima is given by,

$$\sin\left(\frac{\beta d\cos\phi}{2}\right) = 0 \quad \text{—— (10)}$$

Assuming $d = \lambda/2$ and $\beta = \frac{2\pi}{\lambda}$ in eq (10)

$$\sin\left[\frac{\pi}{2}\cos\phi\right] = 0$$

i.e., $\pi/2\cos\phi = \pm n\pi$ (where, $n = 0, 1, 2, 3 \ldots$)

if $n = 0$, then

$$\frac{\pi}{2}\cos\phi_{min} = 0$$

$\therefore \cos\phi_{min} = 0$

$\boxed{\text{i.e., } \phi_{min} = \pm 90^\circ}$

Half power point direction [HPPD] :-

when power is half of maximum value, the voltage or current equal to $\frac{1}{\sqrt{2}}$ times the respective maximum value. Hence the condition for the half power point can be obtained from eq (7) as,

$$\sin\left(\frac{\beta d \cos\phi}{2}\right) = \pm \frac{1}{\sqrt{2}} \quad — (12)$$

let $d = \lambda/2$, $\beta = \frac{2\pi}{\lambda}$, we can write,

$$\sin\left[\frac{\pi}{2}\cos\phi\right] = \pm\frac{1}{\sqrt{2}}.$$

i.e., $\frac{\pi}{2}\cos\phi = \pm(2n+1)\pi/4$, where $n = 0, 1, 2 \ldots$

if $n=0$, we can write.

$$\frac{\pi}{2}\cos\phi_{HPPD} = \pm\pi/4$$

i.e., $\cos\phi_{HPPD} = \pm 1/2$

$$\phi_{HPPD} = 60^\circ \text{ (or) } 120^\circ$$

Thus from the conditions of maxima and minima and half power points, the field pattern can be drawn as shown

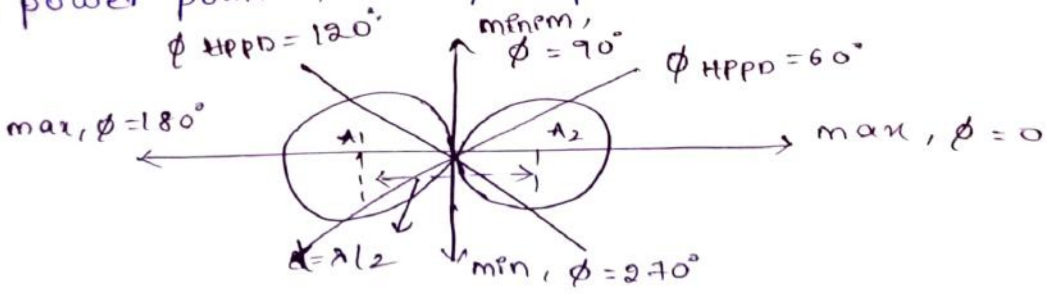
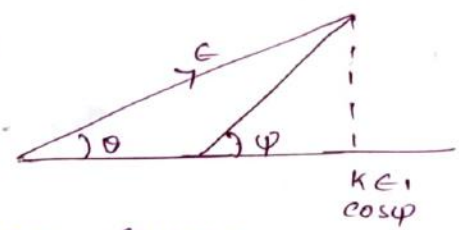


fig: field pattern for 2 point sources with spacing $d = \lambda/2$ and fed with currents equal in magnitude but out of phase by 180°.

Point sources with currents unequal in magnitude and with any phase:

Assume that 2 point sources are seperated by distance d and supplied with currents which are different in magnitudes and with any phase difference say α. Consider that source 1 is assumed to the reference for phase and amplitude of the fields E_1 and E_2 which are due to source 1 and source 2 respectively at the distant point p. let us assume that E_1 is greater than E_2 in magnitude as shown in the vector diagram.



Now, the total phase difference, between the radiations by two point sources at any far point p is given by,

$$\varphi = \frac{2\pi}{\lambda}\cos\phi + \alpha$$

where α is phase angle with which current leads current I, Now if $\alpha = 0$, then the condition is similar to the 2 point sources with currents equal in magnitude and phase. similarly if $\alpha = 180°$, then the condition is similar to the 2-point source with current equal in magnitude but opposite in phase. Assume value of phase difference α as $0 < \alpha < 180°$. Then the resultant field at point 'p' is given by,

$$E_T = E_1 e^{j0} + E_2 e^{j\varphi}$$

$$E_T = E_1 + E_2 e^{j\varphi}$$

$$E_T = E_1 \left(1 + \frac{E_2}{E_1} e^{j\varphi}\right)$$

let $\frac{E_2}{E_1} = k \longrightarrow (2)$

Note that $E_1 > E_2$, the value of k is less than unity. moreover the value of k is given by $0 \leq k$.

$$\boxed{E_T = E_1 \, 1 + k\,(\cos\varphi + J\sin\varphi)} \quad \text{---} \ (3)$$

The magnitude of the resultant field at point at point p is given by,

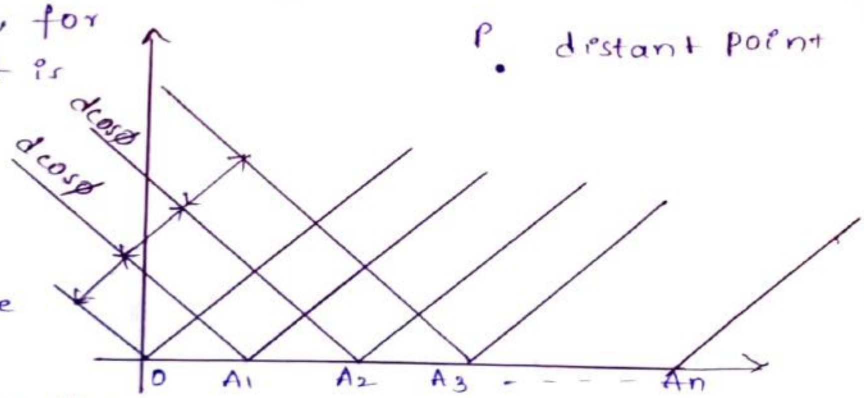
$$|E_T| = E_1 \sqrt{(1+k\cos\varphi)^2 + (k\sin\varphi)^2} \longrightarrow (4)$$

The phase angle b/w two fields at the far point p is given by,

$$\theta = \tan^{-1} \frac{k\sin\varphi}{1+k\cos\varphi} \longleftarrow (5)$$

N- Element uniform linear Arrays :-

At higher frequencies, for point communication, it is necessary to have a pattern with single beam radiation. such highly directive single beam pattern can be obtained by increasing the point sources in the arrow from 2 to n say.



An array of n elements is said to be linear array, if all the individual elements are spaced equally along a line, An array is said to be uniform array if the elements in the array are fed with currents with equal magnitudes and with uniform progressive phase shift along the line.

consider a general 'n' element linear and uniform array with all the individual elements spaced equally,. at distance 'd' from each other and all elements are fed with current equal in magnitude and uniform progressive phase shift along line as shown above figure.

The total resultant field at the distant point 'p' obtained by adding the fields due to n individual sources vectorically. Hence we can write,

$$E_T = E_0 e^{j0} + E_0 e^{j\psi} + E_0 e^{2j\psi} + \cdots + E_0 e^{j(n-1)\psi}$$

$$E_T = E_0 \left[1 + e^{j\psi} + e^{j2\psi} + \cdots + e^{j(n-1)\psi} \right] \quad \text{--- (1)}$$

Note that $\psi = (\beta d \cos\phi + \alpha)$ indicate the total phase difference of the fields from adjacent sources calculated at point P. similarly α is the progressive phase shift b/w 2 adjacent point sources. The value of α may lie b/w 0° and 180°. If $\alpha = 0^\circ$ we get the element uniform linear broadside array. If $\alpha = 180^\circ$ we get n element uniform linear end fire array.

multiplying eq (1) by $e^{j\psi}$, we get

$$E_T e^{j\psi} = E_0 \left[e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \cdots + e^{j(n-1)\psi} \right] \quad —②$$

substracting ② from ①

$$E_T - E_T e^{j\psi} = E_0 \left\{ \left[1 + e^{j\psi} + e^{j2\psi} + \cdots + e^{j(n-1)\psi} \right] - \left[e^{j\psi} + e^{j2\psi} + \cdots e^{jn\psi} \right] \right\}$$

$$E_T \left[1 - e^{j\psi} \right] = E_0 \left[1 - e^{jn\psi} \right]$$

$$E_T = E_0 \left[\frac{1 - e^{jn\psi}}{1 - e^{j\psi}} \right]$$

simplify mathematically,

$$E_T = E_0 \left[\frac{e^{\frac{j\psi n}{2}} \left[e^{\frac{-jn\psi}{2}} - e^{\frac{jn\psi}{2}} \right]}{e^{j\psi/2} \left[e^{-j\psi/2} - e^{j\psi/2} \right]} \right]$$

According to trigonometric identity

$$E_T = E_0 \left[\frac{\left(-j\, 2 \sin \frac{n\psi}{2} \right) e^{\frac{jn\psi}{2}}}{\left(j\, 2 \sin \frac{\psi}{n} \right) e^{j\psi/2}} \right]$$

$$E_T = E_0 \left[\frac{\sin n \frac{\psi}{2}}{\sin \frac{\psi}{2}} \right] e^{j\left(\frac{n-1}{2}\right)\psi}$$

→ The above eqn, indicates the resultant field due to n element array at distant point p.

The magnitude of the resultant field is

$$E_T = E_0 \left[\frac{\sin n \frac{\psi}{2}}{\sin \psi/2} \right]$$

The phase angle θ of the resultant field at point p is

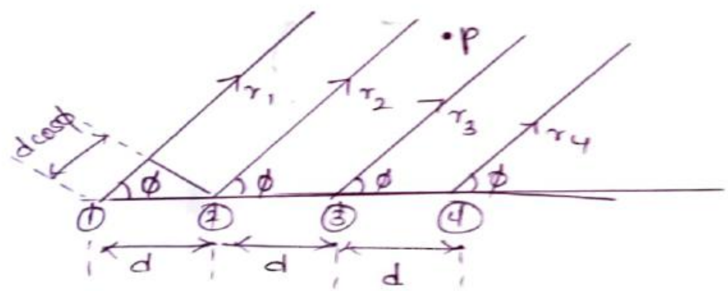
$$\theta = \left(\frac{n-1}{2} \right) \psi = \left(\frac{n-1}{2} \right) \beta d \cos \phi + \alpha$$

Pattern multiplication:-

→ It is a simple method to obtaining the pattern of arrays and it is very useful in the design of array and it makes possible to draw the patterns of complicated arrays rapidly, almost by inspection.

→ In this method. let us consider 4 element array of identical antennas, and spacing between two units be $d = \lambda/2$.

→ Assume that all the elements are supplied with equal magnitude currents which are in phase,



→ At the point p the resultant field has to be obtained is far away, assume the radiation from the antenna in the form of parallel lines.

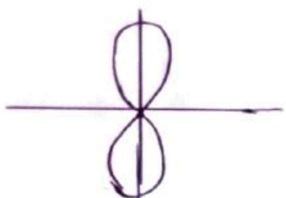
→ The radiation pattern of the antennas ① and ② treated to be operating as a single unit. similarly the radiation pattern of the antennas ③ and ④ treated to be operated as a single unit. spaced between two are $\lambda/2$ and fed with equal current in phase.

→ Now instead of considering two separate elements ① and ② we can replace it by a single antenna located at output mid way between them ($d/2$).

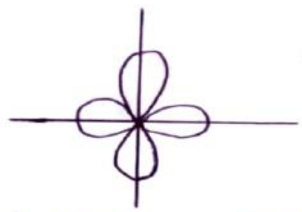
Similarly replacing antennas ③ and ④ by single antennas having same pattern.

→ Now half antennas having bidirectional pattern i.e., figure eight pattern spaced distance λ apart from each other fed with equal currents in phase.

→ Now the resultant radiation of four element array can be obtained as the multiplication of pattern.



Radiation pattern of two antennas spaced $\lambda/2$ destance



Radiation pattern of two antennas space of λ distance

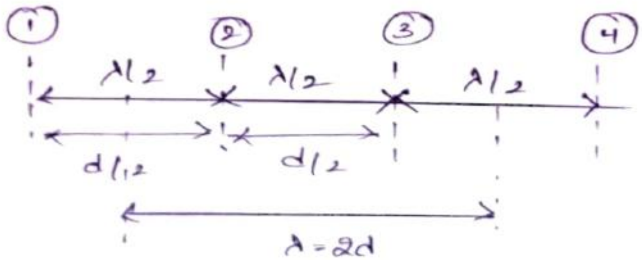
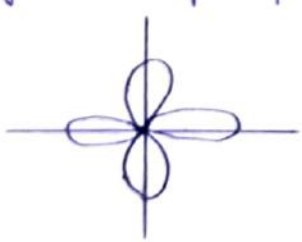
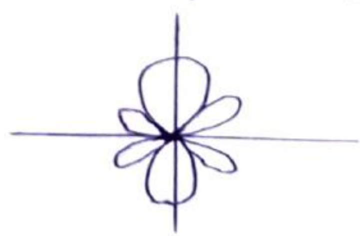


fig: Array of 4 identical elements, replacement of arrays





EFA with increased Directivity :-

For an EFA increased directivity and maximum radiation in $\phi = 0°$ direction, the radiation on intensity for small spacing between elements ($d << \lambda$) is given by,

$$U_0 = \frac{1}{n\beta d}\left(\frac{\pi}{2}\right)^2\left[\frac{\pi}{\sqrt{2}} + \frac{2}{\pi} - 1.8515\right]$$

$$U_0 = \frac{0.878}{n\beta d}$$

multiplying numerator and denominator quantities by 2π

$$U_0 = \frac{0.878 \times 2\pi}{n\beta d \times 2\pi} = \frac{1.756\pi}{2\pi n\beta d}$$

$$= \frac{1.756}{\pi}\left[\frac{\pi}{2n\beta d}\right]$$

$$U_0 = 0.559\left[\frac{\pi}{2n\beta d}\right]$$

Thus the directivity is given by,

$$D = \frac{U_{max}}{U_0}$$

$$= \frac{1}{0.559\left[\frac{\pi}{2n\beta d}\right]}$$

$$= \frac{1}{0.559}\left[\frac{2n\beta d}{\pi}\right] \qquad \beta = \frac{2\pi}{\lambda}$$

$$D = 1.789\left[\frac{2n\left(\frac{2\pi}{\lambda}\right)d}{\pi}\right] \qquad L = (n-1)d \simeq nd$$

$$= 1.789\left[\frac{4nd}{\lambda}\right] = 1.789\left[4\left(\frac{L}{\lambda}\right)\right]$$

Binomial Array:

→ A case of uniform linear array to increase the directivity, the array length has to be increased.

→ when the array length increases, the secondary or side lobes appear in the pattern.

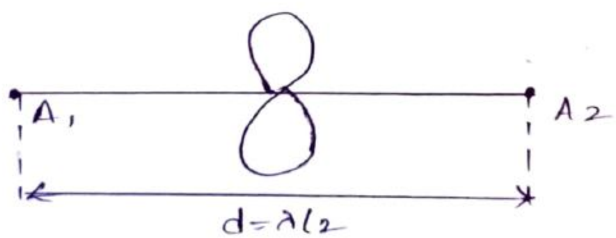
→ In some special applications it is desired to have single main lobe with no. of minor lobes.

→ That means the minor lobes should be eliminated of completely or reduced to minimum levels as compared to main lobe.

→ To acheive such pattern the array is arranged in such a way that the broadside array radiated more strongly at the center than that from edges.

→ let us consider array of the two identical in-phase point sources spaced $\lambda/2$ apart. Then the far field pattern is given by,

$$E = \cos\left(\frac{\pi}{2} \cos\theta\right)$$



The pattern has no minor lobes.

Unit-1

Introduction to microwaves :

Microwaves are electromagnetic waves whose frequencies ranges from 1GHz to 1000 GHz (3mHz to 300 GHz)

⇨ Microwaves (μw's) are so called since they are defined in terms of their wavelength in the sense that micro refers to tinyness - tinyness referring to wavelength and the period of cycle of CM wave.

⇨ Micro wave is a signal that has a wavelength of 1 foot (or) less i.e., $7 \leq 30.5$ cm to 0.03 cm

⇨ The higher frequency edge of microwave borders on the infrared and visible light region

⇨ The Visible light is above infrared region and falls between 430THz to 1PHz

⇨ The various metric prefixes such as mega, micro... is given in the following table.

Table : Metric Prefixes.

prefix	Power of Ten	Symbol	prefix	Power of Ten	symbol
Exa	10^{18}	E	milli	10^{-3}	m
Peta	10^{15}	P	micro	10^{-6}	μ
tera	10^{12}	T	nano	10^{-9}	n
Giga	10^{9}	G	pico	10^{-12}	ρ
mega	10^{6}	M	femto	10^{-15}	f
kilo	10^{3}	K	atto	10^{-18}	a

History:

⇒ As already stated, microwave's are electro magnetic waves Hence, the history of microwaves is embodied in the evolution of electromagnetic waves.

⇒ James clerk maxwell (1831 - 1879) unified all previous known result, experiment and theolitical on electromagnetic waves in four equations and predicted the existence of electromagnetic waves.

⇒ Heinrich "Ruddolf Hertz" experimentally confirmed maxwell's predictions. Guglielmo "marconi" (1874 - 1937) transmitted information on an experimental basis at microwave frequencies.

⇒ George C. south worth (1930) really carried out marconi's experimental basis at the Microwave frequencies.

⇒ During world war - II (1945) based on the previous developments, radar was invented and was exploited for military applications

⇒ The Conventional Vaccum tubes was best set to operate at microwave frequencies but it has some hitches at *(these frequencies but it has son)* these frequencies like inter electrode capacitance (IEC) between the elements within the Vaccum tubes and a longer election transit time

⇒ The IEC effectively shorting at higher frequencies and the longer transit time causing them to be used at lower Operating frequencies.

⇒ The transit time problem was solved by German 2 scientists K. Kurz and H. Barkhausen in the year 1920. They developed a special vaccum tube called generates high frequency of Oscillations.

⇒ In the early 1980's Microwaves devices found applications even in the consumer market with TV receive only (TVRO) broadcast services to home of satellite TV transmission.

⇒ In 1990's microwaves becomes common consumer market products, with the development of microwave ovens, network TV, personal communication system, cell phones, personal communication etc.

⇒ They also found applications in areas other than communication and radar, such as medicine, surveying land, industrial, quality control, GPS, power transmission, space shuttle etc.

Characteristics of microwave:

- Microwave possess certain useful characteristics

1. Microwave wavelength are very small
2. High frequency of microwaves means very large width is available for communication.
3. Microwave radiation penetrates fog and clouds, travels in straight lines and gives reflections hence can be used for distance and direction Measurements (Radar System)

4. Microwaves are necessary for communications through Satellite because they can pass through the ionosphere which reflects lower frequency radio waves.

5. Microwave power is absorbed by water (or) any Other material containing water so that microwaves can be used for heating and drying.

Advantages of Microwave Systems:

There are some unique Advantages of microwave over low frequencies.

1. **Increased bandwidth Availability:**

* Microwaves have large band widths compared to the common bands namely MW, SW and UHF waves. The advantage of large band width is that the frequency range of information channels will be a small percentage of the carrier frequency and more information can be transmitted in a microwave frequency ranges

* Infact, microwave region contains thousand sections of frequency band $0-10^9$ Hz, hence anyone of these thousand sections may be used to transmit all the tv, radio and Other communications

2. **Improved Directive properties:**

* As frequency increases, directivity increases & beam width decreases.

* Hence the beam width of radiation is proportional to λ/D
* At lower frequency bands, the size of antenna becomes very large if it is required get sharp beam of radiation
* However, at microwave frequencies, the antenna size required is very less to get an extremely directed beam. for example we know that

for a Parabolic Antenna $B = \frac{140^\circ}{D(\lambda)}$

Where D = Diameter of Antenna
λ = Wavelength in cm
B = Beamwidth in degrees

At 30 GHz ($\lambda = 1$ cm) for 1° Beam width

$$D = \frac{140^\circ}{B} \times \lambda = \frac{140^\circ}{1^\circ} \times 1\text{ cm} = 140\text{ cm}$$

At 300 MHz ($\lambda = 100$ cm) for 1° Beam width

$$D = \frac{140^\circ}{1} \times 100 = 140\text{ m}$$

from the above example, It is clear that the antenna size required for microwave and frequencies is small.

3. **Fading effect and reliability:**

* fading effect due to variation in the transmission medium is more effective at low frequency. Due to "Line of Sight" (LOS) propagation and high frequencies there is less fading effect and hence microwave communication is more reliable

4. **Power requirements:**

⇒ Transmitted and receiver power requirements are pretty low at microwave frequencies compared to that at short wave band.

5. Microwaves can easily propagate through ionised layers hence Most suited for satellite communication

6. Propagation delay is Negligiable (or) Minimum

7. Signal Cross talk is eliminated

8. Highly reliable system

9. Least Maintainence is required.

Disadvantages of Microwave system:

i. At Microwave frequencies, circuit design is complex

ii. Measurement at Microwave frequencies are difficult.

iii. Line of sight (LOS) propagation limits the use of microwave.

Applications of microwaves:

⇒ Because of certain useful properties that microwave posses it becomes more and more widely used. some applications are discussed here.

1. Drying of wood, paper, printing inks and textiles
2. Destruction of dry root fungus in wood
3. Heating of plastics and rubbers
4. Grinding of minerals
5. Transmission of paper
6. cooking and baking
7. Thickness Measurements of metal sheets in rolling Machines.

1. **Broad Casting:**

⇒ Usually Radio and TV broad casting used the frequencies below the Microwave range. However, increasing congestion of the radio spectrum made reception difficult to some users. There are no frequencies available for increase in Broad casting a radio frequencies any further entry of broad casting will be in the micro-wave region now 12GHz frequency is being used local or satellite TV broad casting with special micro receivers.

2. **Communication:-**

* Increased bandwidth for communication channels requires higher carrier frequencies.
* A Microwave link is a point to point link using the propagation of electromagnetic waves at microwave frequencies through the free space
* Internation telephones and T.V space communication, telemetry communication link railways We use Microwaves
* In satellite communication, microwave frequency are to be used since the ionosphere is opaque to lower frequencies.

3. **Radar:**

* Radar is the traditional use of microwaves the name Radar is derived from the initial letters of radio detection and ranging.

In ... the frequency band is dividend into

* Radars are used to detect Aircraft, track / guide Supersonic Missiles, Observe and track weather pattern, Air traffic control (ATC), Burglar Alarms, garage door Opens, police speed detectors etc.

4. food processing Industry - precoding / cooking, heat frozen / refridgerated precoded meats, Roasting of food grains / beans.

5. Identifying the Objects (or) personel by non-contact method

6. Microwave oven:

* The microwave Oven is an electronically controlled home appliance used for cooking. It uses the principle of dielectric heating at microwave frequencies

* A microwave oven can be represented by the block schematic as shown.

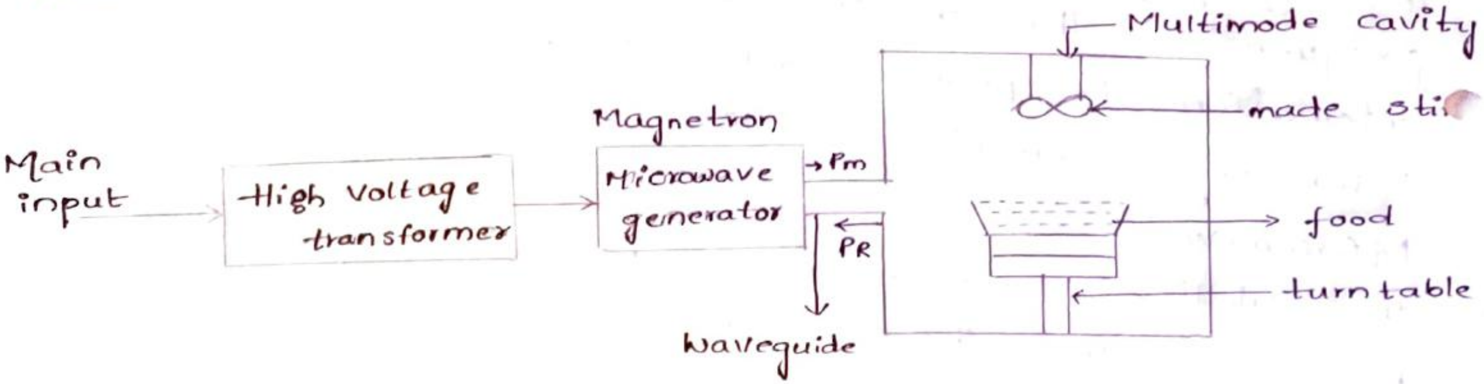


fig: Block schematic of Microwave Oven.

According to CCIR the frequency band is dividend into Several regions. [CCIR = Consultative committee for International Radio) ⑤

Band no.	FREQUENCY RANGE	BAND DESIGNATION
1.	3Hz - 30Hz	Ultra low frequency (ULF)
2.	30Hz - 300Hz	Extra low frequency (ELF)
3.	300Hz - 3kHz	Voice frequency (VF)
4.	3kHz - 30kHz	Very low frequency (VLF)
5.	30kHz - 300kHz	Low frequency (LF)
6.	300kHz - 3MHz	Medium frequency (MF)
7.	3MHz - 30MHz	High frequency (HF)
8.	30MHz - 300MHz	Very High frequency (VHF)
9.	300MHz - 3GHz	Ultra high frequency (UHF)
10.	3GHz - 30GHz	super high frequency (SHF)
11.	30GHz - 300GHz	Extreme high frequency (EHF)
12.	300GHz - 3THz	Infrared light
13.	3THz - 30THz	Infrared light
14.	30THz - 300THz	Infrared light
15.	300THz - 3PHz	visible light
16.	3PHz - 30PHz	Ultra Violet light
17.	30PHz - 300PHz	X- rays
18.	300PHz - 3EHz	Gamma rays
19.	3EHz - 30EHz	cosmic rays.

* According to IEEE (or) Industrial standards

S. no	Band Designation	Range of frequency (GHz)
1	UHF	0.3 - 3
2	L	1.1 - 1.7
3	LS	1.7 - 2.6
4	S	2.6 - 3.9
5	C	3.9 - 8
6	X	8 - 12.5
7	KV	12.5 - 18
8	K	18 - 26
9	Ka	26 - 40
10	Q	33 - 50
11	U	40 - 60
12	M	50 - 75
13	E	60 - 90
14	F	90 - 140
15	G	140 - 220
16	R	220 - 300

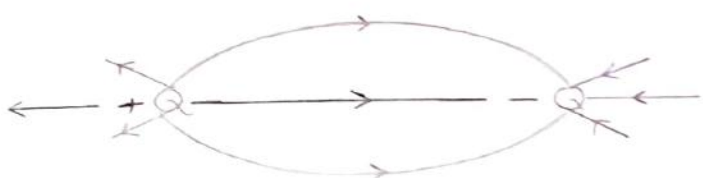
* According to US (or) military microwave frequency bands divided as

S. no	Band designation	Range of frequency (GHz)
1.	A	0.1 - 0.25
2	B	0.25 - 0.5
3	C	0.5 - 1
4	D	1 - 2
5	E	2 - 3
6	F	3 - 4
7	G	4 - 6
8	H	6 - 8
9	I	8 - 10
10	J	10 - 20
11	K	20 - 40
12	L	40 - 60
13	M	60 - 100
14	N	100 - 140

Basic definations :-

Maxwell's Equation :-

Electric flux : (ψ) :-



* Electric flux Originating from a +ve charge and end with a Negative charge in the absence of Negative charge electric flux terminated as infinite
* One coloumb of electric charge would result one coloumb of electric flux.
* The electric flux is passing through the closed surface

- **Electric flux density (D) :-**

The electric flux density ($\bar{D}$) defined as flux per unit area

Gauss Law :

The net electric flux passing through an closed surface is equal to charge enclosed by that surface

$$\psi_{net} = Q_{enclosed} \longrightarrow (1)$$

$$\psi_{net} = \oint \bar{D} \cdot \bar{ds} \longrightarrow (2) \text{ gauss law of integral}$$

$$Q_{enclosed} = \int_v \int_v \cdot dv \longrightarrow (3)$$

from equ (1), (2) & (3)

$$\oint \bar{D} \cdot \bar{ds} = \int_v \cdot \int_v \cdot dv$$

$$\boxed{\nabla \cdot \bar{D} = \int_v} \longrightarrow \text{Gauss Law in pt form.}$$

magnetic flux (ϕ) :-

The amount of magnetic flux passing through a closed cross sectional surface 's' is

$$\phi = \oint_s \bar{B} \cdot \bar{ds}$$

$$\oint_S \bar{B}\cdot d\bar{s} = 0$$

* The zero indicates Non-existance of magnetic charge.

$\nabla\cdot\bar{B} = 0$ [Gauss Law for magnetic field.]

* Unlink the electrics flux, the magnetic flux would not have starting point or ending point. The magnetic flux enters into a closed surface leaves a same closed surface

Ampere's Law:

The line integral of magnetic field intensity around the closed path is equal to the current enclosed by the path

$$\oint_l \bar{H}\cdot d\bar{l} = I_{enc} \longrightarrow \text{Integral form}$$

$$I_{enc} = I = \int_S \bar{J_c}\cdot d\bar{s}$$

$$\oint_l \bar{H}\cdot d\bar{l} = \int_S \bar{J_c}\cdot d\bar{s}$$

$$\nabla\times\bar{H} = \bar{J_c} \longrightarrow \text{point form of Ampere's law.}$$

Faraday's Law of EM Induction :-

When moving conductor cuts by a stationary magnetic flux (or) when a moving magnetic flux cut by a stationary conductor then emf is induced which turns the results a current when a load is connected to it

$$emf = -\frac{d\phi}{dt}$$

$$emf = \oint_L \bar{E} \cdot \bar{dl}$$

$$\phi = \int_S \bar{B} \cdot \bar{ds}$$

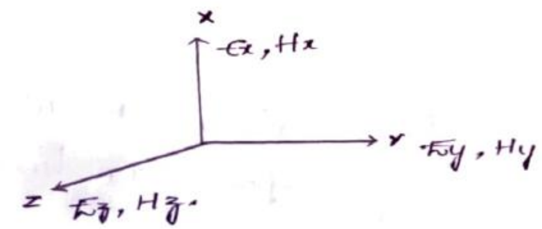
$$\oint_L \bar{E} \cdot \bar{dL} = -\frac{d}{dt}\left(\int_S \bar{B} \cdot \bar{ds}\right)$$

$$= \int_S \left(-\frac{d}{dt}(\bar{B} \cdot \bar{ds})\right)$$

$$\boxed{\nabla \times \bar{E} = -\frac{d}{dt}(\bar{B} \cdot \bar{ds})}$$

- Propagation of electromagnetic waves :-

* The electric field and magnetic field components are mutually perpendicular to the direction of propagation.



- fig: electric and magnetic components

1. Transverse electromagnetic wave :-

* In TEM mode, both electric and magnetic field are purely transverse to the direction of propagation and consequently have no components in z direction.

i.e $E_z = H_z = 0$

2. Transverse electric wave (TE):-

* In TE wave, Only electric field is purely transverse to the direction of propagation and the magnitude field is not purely transverse.

i.e., $E_z = 0$ and $H_z \neq 0$

3. Transverse magnetic (Tm) Waves:-

* In Tm Wave, the magnetic field is transverse to the direction of propagation and the electric field is not purely transverse

i.e., $E_z \neq 0$ and $H_z = 0$

When a wave is travelling along positive z direction then the wave equation for Tm and Te mode can be written as

$$\nabla^2 E_z = -\omega^2 \varepsilon \mu E_z \quad \text{for TM wave} \quad (H_z = 0)$$

$$\nabla^2 H_z = -\omega^2 \mu \varepsilon H_z \quad \text{for TE wave} \quad (E_z = 0)$$

Since all field vectors are varying with time 't', in sinusoidal as manner then,

$$E_z = E_0 \cdot e^{-\gamma z}$$

Where E_{0z} = Maximum electric field in z-direction

γ = propagation constant

$\gamma = \alpha + j\beta$

α = Attenuation constant

β = phase constant $= \frac{2\pi}{\lambda}$

4. Hybrid Wave (HE) :- ⑧

* Here neither electric nor Magnetic fields are purely transverse to the directional propagation i.e.,

$$E_z \neq 0, \quad H_z \neq 0$$

* Now we defined few more relations $\nabla^2 \epsilon$ in rectangular co-ordinates system as

$$\nabla^2 \epsilon = \frac{\delta^2 \epsilon}{\delta x^2} + \frac{\delta^2 \epsilon}{\delta y^2} + \frac{\delta^2 \epsilon}{\delta z^2}$$

Wave Guides:

* Microwaves propagated through various microwave circuits, components and devices that act as section of microwave transmission lines that are broadly called " wave guide".
* The microwave signals travel as electromagnetic waves
* At high frequencies greater than 3GHz, transmission of electromagnetic waves along the transmission lines like twisted pairs, coaxial cables etc becomes difficult mainly due to the losses that are Occur in the conductor. To avoid this losses, we use Wave guides.
* At microwave frequencies, the following transmission lines will be employed.

(i) Multi Conductor line

a) Co-axial lines

b) Strip lines

c) Micro strip lines

d) Slot lines

e) co-planar lines etc

(ii) Single conductor lines (Wave guides)

a) Rectangular Wave guides

b) Circular wave guides

c) Ridge wave guides etc

(iii) Open boundary structures

a) Dielectric rods

b) Open wave guide etc.

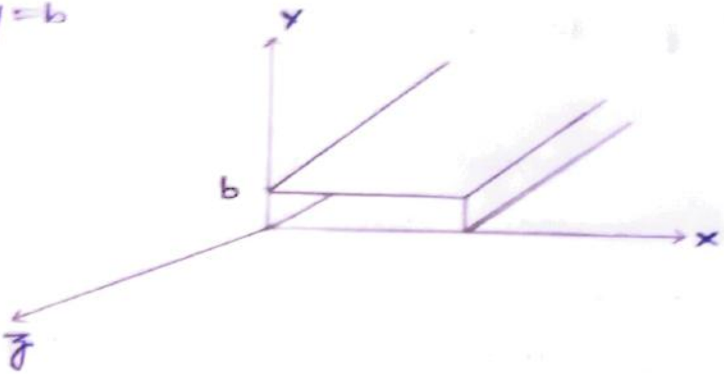
* Multiconductor lines normally support TEM (or) Quasi TEM mode of propagation.
* The single conductor lines support TE (or) TM waves
* The Open boundary structure can supports combination of TE & TM (i.e., HE Mode).

Wave Guide defination:

1. Wave guide is a Hallow metallic tube of uniform cross section for transmitting electro magnetic waves by successive reflections from the inner walls of the tube.
2. Wave guide consists of hallow mettalic tube of rectangular (or) circular shape used to guide an electro magnetic wave. Wave guidal are used principally at frequencies in microwave range
* A wave guide is example of wave propagation through a bounded medium

* The waveguide have the conductor boundaries, the medium b/w the conductor boundaries is non conducting, homogeneous isotropic and charge free medium
* WKT tangential components of E-fields and normal components of H-fields vanishes across the conductor boundaries.
* The TEM wave is cannot exist in the waveguide but TE and TM waves can exist.
* Induced currents in the walls of the conductor give rise to power losses and to minimize these power losses, the wave guide walls resistance is made as low as possible. Hence the inner surface of waveguide is usually coated with either gold (or) silver to improve the conductivity and minimise losses inside the wave guide.
* Any shape of wave guide can support EM wave but irregular shapes are different to Analyze and rarely used, regularly used waveguides are rectangular and circular wave guides
* Rectangular Wave guide:
* consider a rectangular wave guide situated in the rectangular coordinate system with its breadth along x-axis, width along y-axis and length along z-axis

* The waveguide is assumed to propagate along the Z-direction wave guide is filled with air as dielectric.
* The four conducting walls exists at x=0, x=a, y=0 and y=b



* The conducting walls are made of usually brass (or) Aluminium, so the conducting plates have a infinite conductivity. The medium b/w the conducting plates is non-conducting ($\sigma = 0$), Homogenous, Isotropic and charge free ($\rho_v = 0$)
* The wave is propagating through the wave guide by the successive reflections from inner walls of the wave guide
* For evaluating the wave equations, we consider the frequency domain solution
* The maxwell equations for time varying electro-magnetic fields are

1. $\nabla \times E = -\frac{\partial B}{\partial t}$

2. $\nabla \times \bar{H} = I_c + I_D = \sigma E + \frac{dD}{dt}$

3. $\nabla \times \bar{D} = \rho_v$

4. $\nabla \cdot \bar{B} = 0$

* The Medium With in the rectangular wave guide is dielectric (Non conducting, Homogenous, Isotropic and charge free).

* The maxwell equations for above medium is

1. $\nabla \times \bar{E} = -dB/dt \longrightarrow ①$

2. $\nabla \times \bar{H} = dD/dt \quad (\because \sigma = 0) \longrightarrow ②$

3. $\nabla \cdot \bar{D} = 0 \quad (\rho_v = 0) \longrightarrow ③$

4. $\nabla \cdot \bar{B} = 0 \longrightarrow ④$

We know that

$\Rightarrow D = \epsilon E$

$\Rightarrow B = \mu H$

$\Rightarrow \frac{d}{dt}$ can be written As "$j\omega$"

i.e., $\frac{d}{dt} = j\omega$

Let us consider Equ ①

$$\nabla \times \bar{E} = -\frac{dB}{dt}$$

$$= -\frac{d}{dt}(\mu \bar{H})$$

$$= -j\omega\mu\bar{H}$$

$$\nabla \times \bar{E} = -j\omega\mu\bar{H}$$

Take Curl on bothsides.

$$\nabla \times \nabla \times \bar{E} = -j\omega\mu\,(\nabla \times \bar{H})$$

$$\nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E} = -j\omega\mu\,(\nabla \times \bar{H}) \longrightarrow (6)$$

from Equation (3)

$$\nabla \cdot \bar{D} = 0$$

$$\varepsilon\,\nabla \cdot \bar{E} = 0 \longrightarrow (7) \quad (\bar{D} = \varepsilon \bar{E})$$

from Equ (6) and (7)

$$-\nabla^2 \bar{E} = -j\omega\mu\,(\nabla \times \bar{H})$$

$$= -j\omega\mu\left(-\frac{\delta \bar{D}}{\delta t}\right)$$

$$\nabla^2 \bar{E} = j\omega\mu \cdot j\omega \cdot \bar{D}$$

$$= j^2\omega^2 \epsilon E \mu \quad (D = \epsilon E)$$

$$\nabla^2 \bar{E} = -\omega^2 \mu \epsilon \bar{E}$$

* When the wave travelling along z-direction, then wave equation becomes

$$\nabla^2 \bar{E}_z = -\omega^2 \mu \epsilon \bar{E}_z$$

Similarly

$$\nabla^2 \bar{H}_z = -\omega^2 \mu \epsilon \bar{H}_z$$

* In rectangular co-ordinate system the wave equation can be written as

$$\frac{d^2 \bar{E}_z}{dx^2} + \frac{d^2 \bar{E}_z}{dy^2} + \frac{d^2 \bar{E}_z}{dz^2} = -\omega^2 \mu \epsilon \bar{E}_z$$

* We can represent $\frac{d^2}{dz^2}$ at γ^2 Because the wave is propagating along the z-direction.

$$\frac{d^2 E_z}{dx^2} + \frac{d^2 E_z}{dy^2} + \gamma^2 E_z = -\omega^2 \mu \epsilon E_z$$

$$\frac{d^2 E_z}{dx^2} + \frac{d^2 E_z}{dy^2} + (\gamma^2 + \omega^2 \mu \epsilon) E_z = 0$$

$$\frac{d^2 E_z}{dx^2} + \frac{d^2 E_z}{dy^2} + h^2 E_z = 0 \quad (\because \gamma^2 + \omega^2 \mu \epsilon = h^2)$$

Similarly

$$\frac{d^2 H_z}{dx^2} + \frac{d^2 H_z}{dy^2} + h^2 H_z = 0$$

By solving the above partial differential equations, we get solution for E_z & H_z.

* By using maxwell equations, it is possible to find the various components along x and y directions (E_x, H_x, E_y and H_y)

The Maxwell first equation is

$$\nabla \times \bar{H} = j\omega \epsilon \bar{E}$$

Expanding $\nabla \times \bar{H}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ H_x & H_y & H_z \end{vmatrix} = j\omega\epsilon\,[iE_x + jE_y + kE_z]$$

Replacing $\frac{d}{dz} = -\gamma$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & -\gamma \\ H_x & H_y & H_z \end{vmatrix} = j\omega\epsilon \, (\hat{i} E_x + \hat{j} E_y + \hat{k} \cdot E_z)$$

Equating co-efficients of $\hat{i}, \hat{j}$ and $\hat{k}$ after multiplication

$$\hat{i}\left(\frac{d}{dy} H_z + \gamma \cdot H_y\right) - \hat{j}\left(\frac{d}{dx} H_z + \gamma H_x\right) + \hat{k}\left(\frac{d}{dx} H_y - \frac{d}{dy} H_x\right)$$

$$= j\omega\epsilon \, [\hat{i} E_x + \hat{j} E_y + \hat{k} E_z]$$

$$\frac{dH_z}{dy} + \gamma H_y = j\omega\epsilon E_x \longrightarrow (7)$$

$$\frac{d}{dx} H_z + \gamma H_x = -j\omega\epsilon E_y \longrightarrow (8)$$

$$\frac{d}{dx} H_y - \frac{d}{dy} H_x = j\omega\epsilon E_z \longrightarrow (9)$$

Similarly from maxwell's 2nd equation

$$\nabla \times \bar{E} = -j\omega\mu \bar{H}$$

Expanding $\nabla \times \bar{E}$, we get

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu \, [\hat{i} H_x + \hat{j} H_y + \hat{k} H_z]$$

Replace $\frac{d}{dz}$ with "$-\gamma$"

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu \, [\hat{i} H_x + \hat{j} H_y + \hat{k} H_z]$$

Multiplying and Equating the co-efficient of $\hat{i}, \hat{j}$ & $\hat{k}$ we get

$$\frac{dEz}{dy} + \gamma Ey = -j\omega\mu Hx \longrightarrow (10)$$

$$\frac{dEz}{dx} + \gamma . Ex = j\omega\mu Hy \longrightarrow (11)$$

$$\frac{dEy}{dx} - \frac{dEx}{dy} = -j\omega\mu Hz \longrightarrow (12)$$

- from equation (11)

$$Hy = \frac{1}{j\omega\mu}\frac{dEz}{dx} + \frac{\gamma}{j\omega\mu}Ex \longrightarrow (13)$$

By substituting equ (13) in equ (7) we get

$$\frac{d}{dy}(Hz) + \gamma\left(\frac{1}{j\omega\mu} \cdot \frac{dEz}{dx} + \frac{\gamma}{j\omega\mu}Ex\right) = j\omega\epsilon Ex$$

$$\Rightarrow Ex\left[j\omega\epsilon - \frac{\gamma^2}{j\omega\mu}\right] = \frac{\gamma}{j\omega\mu} \cdot \frac{dEz}{dx} + \frac{dHz}{dy}$$

$$\Rightarrow Ex\left[j^2\omega^2\epsilon\mu - \gamma^2\right] = \gamma . \frac{dEz}{dx} + j\omega\mu \frac{dHz}{dy}$$

$$\Rightarrow Ex\left[-h^2\right] = \gamma . \frac{dEz}{dx} + j\omega\mu \cdot \frac{dHz}{dy} \quad (\because \omega^2\epsilon\mu + \gamma^2 = h^2)$$

$$\Rightarrow Ex = \frac{-\gamma}{h^2}\frac{dEz}{dx} - \frac{j\omega\mu}{h^2} \cdot \frac{dHz}{dy} \longrightarrow (14)$$

Similarly

$$Ey = -\frac{\gamma}{h^2} \cdot \frac{dEz}{dy} + \frac{j\omega\mu}{h^2} \cdot \frac{dHz}{dx} \longrightarrow (15)$$

$$H_x = \frac{-\gamma}{h^2} \cdot \frac{dH_z}{dx} + \frac{j\omega\epsilon}{h^2} \cdot \frac{dE_z}{dy} \longrightarrow (16)$$

$$H_y = \frac{-\gamma}{h^2} \frac{dH_z}{dy} - \frac{j\omega\epsilon}{h^2} \frac{dE_z}{dx} \longrightarrow (17)$$

Propagation of TEM Waves:-

* We know that for a TEM wave $E_z=0$ & $H_z=0$

Substitute this value in the equations from 14 to 17

All the field components along x and y direction E_x, E_y, H_x & H_y vanishes and hence a TEM wave cannot exist inside the wave guide.

Propagation of TM wave in rectangular wave guide:

* W.K.T for a TM wave, $H_z=0$; $E_z \neq 0$.
* The wave equations of TM wave is

$$\frac{d^2E_z}{dx^2} + \frac{d^2E_z}{dy^2} + h^2E_z = 0$$

By solving above differential equations by using the variable & separable method we get different field components of E_x, E_y, H_x and H_y

Let us assume

$$E_z = X \cdot Y$$

Where

X is a pure function of x only

Y is a pure function of y only

$$Y\frac{d^2x^2}{dx^2} + X\cdot\frac{d^2y^2}{dy^2} + h^2\cdot XY = 0$$

$$\frac{1}{X}\frac{d^2x}{dx^2} + \frac{1}{Y}\frac{d^2y}{dy^2} + h^2 = 0 \longrightarrow (1)$$

$\frac{1}{X}\cdot\frac{d^2X}{dx^2}$, is a pure function of x Only.

$\frac{1}{Y}\cdot\frac{d^2Y}{dy^2}$, is a pure function of y only.

The Sum of these terms is constant. Hence each term must be equal to constant separately since X and Y are independent Variables.

Let $\frac{1}{X}\cdot\frac{d^2X}{dx^2} = -B^2 \longrightarrow (2)$

$\frac{1}{Y}\cdot\frac{d^2Y}{dy^2} = -A^2 \longrightarrow (3)$

Substituting equation 2 & 3 in (1) then We get

$$-B^2 - A^2 + h^2 = 0$$

$$h^2 = A^2 + B^2$$

(2) & (3) are Ordinary 2nd Order differential equations The solution of these is given as

$$X = c_1 \cos Bx + c_2 \sin Bx \longrightarrow (4)$$

$$Y = c_3 \cos Ay + c_4 \sin Ay \longrightarrow (5)$$

Where c_1, c_2, c_3 and c_4 are constants, which can be evaluated by Applying the "boundary Conditions" The complete Solution is given by

$$E_z = X\cdot Y$$

$$= (c_1 \cos Bx + c_2 \sin Bx)(c_3 \cos Ay + c_4 \sin Ay) \longrightarrow (6)$$

Boundary Conditions:

Note: The tangential components of electric field and Normal components of magnetic fields along the boundaries is zero.

1st Boundary Condition: (Bottom wall)

W.K.T $E_z = 0$, all along the bottom wall

i.e., $E_z = 0$ at $y = 0$ & $x \longrightarrow 0$ to a

Substitute in equation (6)

$$E_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin By)$$

$$0 = (C_1 \cos Bx + C_2 \sin Bx)(C_3 + 0)$$

$$\because \cos 0 = 1, \quad \sin 0 = 0$$

This is true for all $x \longrightarrow 0$ to a

$\rightarrow C_1 \cos Bx + C_2 \sin Bx \neq 0$

$\rightarrow \boxed{C_3 = 0}$

Substitute C_3 Value in equation Number (6)

$$E_z = (C_1 \cos Bx + C_2 \sin Bx)(C_4 \sin Ay) \longrightarrow (7)$$

IInd Boundary Condition: (Left Side)

$E_z = 0$ at $x = 0$ & $y \longrightarrow 0$ to b

Substitute above value in Equ (7)

$$0 = (C_1 \cos(0) + C_2 \sin(0))(C_4 \sin Ay)$$

$$0 = C_1 \cdot C_4 \sin Ay.$$

The above Equation is true for all $y \longrightarrow 0$ to b

$\Rightarrow \sin Ay \neq 0$ and $C_4 \neq 0$

$\Rightarrow \boxed{C_1 = 0}$

Now the equation (7) becomes

$$E_z = C_2 . C_4 \sin Bx \sin Ay \longrightarrow (8)$$

3rd Boundary Condition : (Top plane)

$E_z = 0$ at $y = b$ $x \longrightarrow 0$ to a

from equ (8)

$$0 = C_2 . C_4 \sin Bx \sin Ab$$

Since $\sin Bx \neq 0$, $C_4 \neq 0$ and $C_2 \neq 0$

Otherwise there would be no Solution

$$\sin Ab = 0$$

$$Ab = n\pi$$

Where n is a constant, $n = 0, 1, 2 \ldots\ldots$

$$A = \frac{n\pi}{b}$$

4th Boundary condition: (Right Side plane)

$E_z = 0$ at $x = a$ $y \longrightarrow 0$ to b

Substituting in equation number (8)

$$0 = C_2 . C_4 \sin Ba \cos Ay$$

Here $\sin Ay \neq 0$. $C_2 \neq 0$, $C_4 \neq 0$

$$\sin Ba = 0$$

$$Ba = m\pi$$

Where m is a constant, $m = 0, 1, 2 \ldots$

$$\boxed{B = \frac{m\pi}{a}}$$

Consider equ (8)

$$E_z = C_2 C_4 \sin Bx \cdot \sin Ay$$

$$= C_2 C_4 \sin \left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{n}\right)y \cdot e^{-\gamma z} e^{j\omega t}$$

Where $e^{-\gamma z}$ = propagation along z-direction

$e^{j\omega t}$ = Sinusoidal variation w.r.t "t".

Let $C_2 C_4 = C$

$$E_z = C \cdot \sin \left(\frac{m\pi}{a}\right)x \; \sin\left(\frac{n\pi}{b}\right)y \; e^{j\omega t - \gamma z} \longrightarrow (9)$$

for TM Wave $H_z = 0$

Since E_z and H_z values are known E_x, E_y, H_x and H_y values are given by the

$$E_x = -\frac{\gamma}{h^2} \cdot \frac{dE_z}{dx} - \frac{j\omega\mu}{h^2}\frac{dH_z}{dy}$$

$$E_x = -\frac{\gamma}{h^2}\frac{d}{dx}\left\{C \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \; e^{j\omega t - \gamma z}\right\}$$

$$E_x = \frac{-\gamma}{h^2} C \cdot \frac{m\pi}{a} \cdot \cos\frac{m\pi}{a}x \cdot \sin\frac{n\pi}{b} \cdot y \; e^{j\omega t - \gamma z} \longrightarrow (10)$$

Similarly

$$E_y = \frac{-\gamma}{h^2} \cdot C \cdot \frac{m\pi}{b} \cdot \sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z} \longrightarrow (11)$$

$$H_x = \frac{j\omega t}{h^2} C \cdot \frac{h\pi}{b} \cdot \sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right) \cdot y \; e^{j\omega t - \gamma z} \longrightarrow (12)$$

$$H_y = \frac{j\omega t}{h^2} C \cdot \left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \, e^{j\omega t - \gamma z} \longrightarrow (13)$$

TM modes in Rectangular Wave Guide :-

* The electromagnetic wave inside a wave guide can have an infinate number of patterns which are called mode.
* The mode having the highest cut-off wavelength is called as dominating mode of the wave guide and all other modes are higher modes.
* Depending on the values of m and n, we have various modes in TM waves. In general, we represent the modes as TM_{mn} where m & n are integer's.

Various TM_{mn} Modes :

TM_{00} Mode : $m=0$ and $n=0$

If $m=0$ and $n=0$ are substituted in E_x, E_y, H_x and H_y in equation (10) to (13). We see that all of them vanish and hence TM_{00} cannot exist.

TM_{01} mode : $m=0$ and $n=1$

Again, all the field components vanish and hence TM_{01} mode cannot exist.

TM_{10} Mode : $m=1$ and $n=0$

Even now, all field components vanish and hence TM_{10} mode cannot exist

TM_{11} Mode : $m=1$ and $n=1$

Now we have all the four components E_x, E_y, H_x & H_y

i.e., TM_{11} mode exists and for all higher values of m and n. The components exists, i.e., all higher modes exists.

Cut-off frequency of a wave guide: (or)

Wave guide as a high pass filter.

We know that

$$h^2 = \gamma^2 + \omega^2\mu\epsilon \longrightarrow ①$$

$$h^2 = A^2 + B^2 \longrightarrow ②$$

from equ ① and ②

$$\gamma^2 + \omega^2\mu\epsilon = A^2 + B^2$$

$$= \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon}$$

$$= \alpha + j\beta$$

γ = propagation Constant

β = phase constant

α = attenuation Constant

At low frequency.

$$\omega^2\mu\epsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$\Rightarrow$ "γ" then becomes Real and positive and equal to the attenuation constant "α". i.e., the wave is completely attenuated and there is no phase

Change. Hence the wave Cannot propagated

At high frequency

$$\omega^2 \mu\epsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

'γ' becomes imaginary, there will be phase change 'β' & hence the wave propagates

⇒ The frequency at which 'γ' values just becomes zero is defined as a "Cut-off" frequency

$$-\omega_c^2 \mu\epsilon + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = 0$$

$$\omega_c = \frac{1}{\sqrt{\mu\epsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}$$

$$\boxed{f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}} \qquad \left(\because \frac{1}{\sqrt{\mu\epsilon}} = c\right)$$

⇒ $m = 1, n = 1$

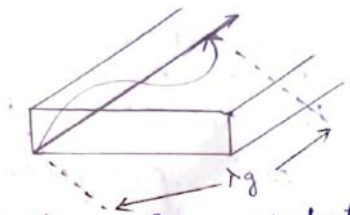
$$\lambda_c = \frac{c}{f_c} = \frac{c}{\frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

$$\lambda_c = \frac{2ab}{\sqrt{m^2 b^2 + n^2 a^2}}$$

* All wave lengths greater than λ_c are attenuated and these wave length is less than λ_c are allowed to propagate inside the wave guide.

Guide Wave length (λ_g):

* It is defined as the distance travelled by wave in Order to undergo a phase shift of 2π radians



* Guide Wavelength is related to the phase constant by the relation.

$$\lambda_g = \frac{2\pi}{\beta}$$

* The Wave length in the wave guide is different from the wave length in the free space.

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda_0^2} - \frac{1}{\lambda_c^2}$$

$$= \frac{\lambda_c^2 - \lambda_0^2}{\lambda_0^2 \lambda_c^2}$$

$$\lambda_g^2 = \frac{\lambda_0^2 \cdot \lambda_c^2}{\lambda_c^2 - \lambda_0^2} = \frac{\lambda_0^2}{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

$$\boxed{\lambda_g = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}}$$

Where λ_0 is free space Wave length

* If. $\lambda_0 << \lambda_c$, The denominator is Approximately equals to '1' and $\lambda_g = \lambda_0$

* If $\lambda_0 >> \lambda_c \Rightarrow \lambda_g$ is imaginary which is nothing but no propagation in the wave guide 17
* If $\lambda_0 = \lambda_c \Rightarrow \lambda_g$ is infinite

Phase Velocity (Vp):

* Wave propagates in the wave guide when guide wave length λ_g is greater than λ_0.
* phase velocity defined as the rate at which the wave changes its phase in terms of the wave guide wavelength

$$V_p = \frac{\lambda_g}{\text{time}} = \lambda_g \cdot f$$

$$= \frac{2\pi f}{2\pi} \cdot \lambda_g$$

$$= 2\pi f / 2\pi / \lambda_g$$

$$\boxed{V_p = \omega/\beta}$$

* V_p is greater than the velocity of the light

⇒ Expression for V_p:

w.k.T $\quad V_p = \frac{\omega}{\beta}$

$$h^2 = \gamma^2 + \omega^2 \mu \epsilon \longrightarrow (1)$$

$$h^2 = A^2 + B^2 \longrightarrow (2)$$

from equ (1) & (2)

$$\gamma^2 + \omega^2 \mu \epsilon = A^2 + B^2$$

$$= \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

for wave propagation $\gamma = j\beta$ $(\because \alpha = 0)$

$$\gamma^2 = (j\beta)^2$$

$$= \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon \longrightarrow (1)$$

at $f = f_c \Rightarrow \omega = \omega_c$ & $\gamma = 0$

$$\omega_c^2 \mu \epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \longrightarrow (2)$$

Substitute equ (2) in equ (1)

$$\gamma^2 = (j\beta)^2 = \omega_c^2 \mu \epsilon - \omega^2 \mu \epsilon$$

$$\beta^2 = \omega^2 \mu \epsilon - \omega_c^2 \mu \epsilon$$

$$\beta = \sqrt{\mu \epsilon} \cdot \sqrt{\omega^2 - \omega_c^2}$$

$$V_p = \frac{\omega}{\beta}$$

$$= \frac{\omega}{\sqrt{\mu \epsilon}} \cdot \frac{1}{\omega \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$$

$$\boxed{V_p = \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}}$$

$$f_c = \frac{c}{\lambda_c}, \quad f = \frac{c}{\lambda_o}$$

$$\boxed{V_p = \frac{c}{\sqrt{1 - \left(\frac{\lambda_o}{\lambda_c}\right)^2}}}$$

Group Velocity (V_g):

* It is defined as the rate at which the wave propagate through the wave guide and is given as

$$V_g = \frac{d\omega}{d\beta}$$

Expression for V_g:

W.K.T $V_g = \frac{d\omega}{d\beta}$

$$\beta = \sqrt{\mu\epsilon}\sqrt{\omega^2 - \omega_c^2}$$

$$\frac{d\beta}{d\omega} = \sqrt{\mu\epsilon}\,\frac{1}{2\sqrt{\omega^2-\omega_c^2}}\cdot 2\omega$$

$$= \frac{\sqrt{\mu\epsilon}}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}$$

$$V_g = \frac{d\omega}{d\beta} = \frac{1}{\sqrt{\mu\epsilon}}\cdot\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\boxed{V_g = c\cdot\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\boxed{V_g = c\cdot\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}} \qquad [\because f = c/\lambda]$$

Relation between V_p & V_g:

$$V_p \cdot V_g = \frac{c}{\sqrt{1-\left(\frac{\lambda_o}{\lambda_c}\right)^2}}\cdot c\cdot\sqrt{1-\left(\frac{\lambda_o}{\lambda_c}\right)^2}$$

$$\boxed{V_p \cdot V_g = c^2}$$

Dominate mode in TM mode :

We know that

$$\lambda_{c\,m,n} = \frac{2ab}{\sqrt{m^2b^2 + n^2a^2}}$$

TM$_{11}$: $\lambda_{c_{11}} = \frac{2ab}{\sqrt{a^2+b^2}}$

TM$_{12}$: $\lambda_{c_{12}} = \frac{2ab}{\sqrt{a^2+4b^2}}$

TM$_{21}$: $\lambda_{c_{21}} = \frac{2ab}{\sqrt{4a^2+b^2}}$

$\lambda_{c_{11}} > \lambda_{c_{12}} > \lambda_{c_{11}} > \lambda_{c_{21}} \cdots\cdots\cdots$

So TM_{11} mode is a "dominating mode" in the TM mode

propagating of TE waves in rectangular wave guide :

We know that In TE mode

$$E_z = 0 \text{ and } H_z \neq 0$$

The wave equation for TE wave is given by

$$\nabla^2 H_z = -\omega^2 \mu \epsilon H_z$$

$$\frac{d^2 H_z}{dx^2} + \frac{d^2 H_z}{dy^2} + \frac{d^2 H_z}{dz^2} = -\omega^2 \mu \epsilon H_z$$

$$\frac{d^2 H_z}{dx^2} + \frac{d^2 H_z}{dy^2} + \gamma^2 \cdot H_z = -\omega^2 \mu \epsilon \cdot H_z$$

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$$\frac{d^2H_z}{dx^2} + \frac{d^2H_z}{dy^2} + (\gamma^2+\omega^2\mu\epsilon)H_z = 0$$

$[\because h^2=\gamma^2+\omega^2\mu\epsilon]$

$$\frac{d^2H_z}{dx^2} + \frac{d^2H_z}{dy^2} + h^2 . H_z = 0 \longrightarrow (1)$$

The above equation is second Order differential equation that can be solved by Variable and Sufferable method.

Assume $H_z = X \cdot Y$

X is purely function of x

Y is purely function of y

from equation (1)

$$Y \cdot \frac{d^2X}{dx^2} + X \cdot \frac{d^2Y}{dy^2} + h^2 . XY = 0$$

Divide by XY

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + h^2 = 0 \longrightarrow (2)$$

Assume

$$\frac{1}{X} \cdot \frac{d^2X}{dx^2} = -B^2 \longrightarrow (3)$$ [pure function of x]

$$\frac{1}{Y} \cdot \frac{d^2Y}{dy^2} = -A^2 \longrightarrow (4)$$ [pure function of y]

Substitute equ (3) and (4) in equ (2)

$$A^2 + B^2 = h^2 \longrightarrow (5)$$

By solving equ (3) and (4)

$$X = C_1 \cos Bx + C_2 \sin Bx$$
$$Y = C_3 \cos Ay + C_4 \sin Ay$$

W.K.T $\quad H_z = X \cdot Y$

$$H_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay) \longrightarrow ⑥$$

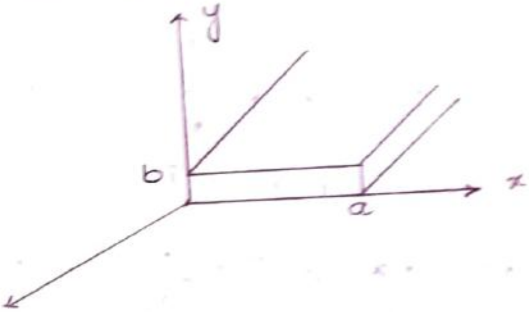
Where C_1, C_2, C_3 and C_4 are constants which can be evaluated by Applying Boundary conditions

Boundary Condition:

1st boundary condition: (Bottom plane)

$E_x = 0$, $E_z = 0$

at $y = 0$, $x \longrightarrow 0$ to a



W.K.T

$$E_x = \frac{-\gamma}{h^2} \cdot \frac{dE_z}{dx} - \frac{j\omega\mu}{h^2} \cdot \frac{dH_z}{dy}$$

Substituting E_x & E_y values in the above equation

$$0 = \frac{-\gamma}{h^2} \cdot \frac{dE_z}{dx} - \frac{j\omega\mu}{h^2} \cdot \frac{dH_z}{dy}$$

$$0 = \frac{j\omega\mu}{h^2} \cdot \frac{d}{dy} \left\{ (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay) \right\}$$

$$0 = \frac{j\omega\mu}{h^2} \left\{ (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cdot A(-\sin Ay) + C_4 \cdot A \cos Ay) \right\}$$

By substituting $y = 0$ in the above equation

$$0 = \frac{j\omega\mu}{h^2} \left\{ (C_1 \cos Bx + C_2 \sin Bx) \cdot A \cdot C_4) \right\}$$

from the above Equation

$$\boxed{C_4 = 0}$$

Substitute C_4 Value in Equation ⑥

$$H_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay) \longrightarrow ⑦$$

2nd Boundary condition : (Left Side)

$E_y = 0$, $E_z = 0$, $x = 0$, $y \longrightarrow 0$ to b

$$E_y = \frac{-\gamma}{h^2} \cdot \frac{dE_z}{dy} + \frac{j\omega\mu}{h^2} \cdot \frac{dH_z}{dx}$$

Since $E_z = 0$, $E_y = 0$

$$0 = \frac{j\omega\mu}{h^2} \frac{d}{dx}(H_z) = 0$$

$$0 = \frac{j\omega\mu}{h^2} \cdot \frac{d}{dx}\{(C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay)\}$$

$$= \frac{j\omega\mu}{h^2}\{C_3 \cos Ay \cdot (C_1 B(-\sin Bx) + C_2 B \cos Bx)\}$$

Substitute $x = 0$ in the above equation

$$0 = \frac{j\omega\mu}{h^2}\{C_3 \cos Ay \cdot BC_2\}$$

from above equation $C_2 = 0$

Substitute C_2 Value in the equation ⑦

$$H_z = C_1 \cos Bx \cdot C_3 \cos Ay \longrightarrow ⑧$$

3rd boundary Condition : (Top plane)

$E_x = 0$, $E_z = 0$ $y = b$, $x \longrightarrow 0$ to a

W.K.T

$$E_x = \frac{-\gamma}{h^2} \cdot \frac{dE_z}{dx} - \frac{j\omega\mu}{h^2} \frac{dH_z}{dy}$$

$$0 = -\frac{j\omega\mu}{h^2} \cdot \frac{dH_z}{dy}$$

$$0 = -\frac{j\omega\mu}{h^2} \cdot \frac{d}{dy}(C_1 \cdot C_3 \cos Bx \cos Ay)$$

$$0 = \frac{j\omega\mu}{h^2} C_1 \cdot C_3 \cos Bx\,(-\sin Ay \cdot A)$$

By substituting $y=b$ in the above equation

$$0 = \frac{j\omega\mu}{h^2} \cdot C_1 C_3 \cos Bx\, A \sin Ab$$

$$\sin Ab = 0$$

$$Ab = n\pi$$

$$\boxed{A = \frac{n\pi}{b}}$$

4th Boundary condition : (Right side plane)

$E_y = 0,\ E_z = 0,\ x = a,\ y \longrightarrow 0$ to b

$$E_y = \frac{-\gamma}{h^2} \cdot \frac{dE_z}{dy} + \frac{j\omega\mu}{h^2} \cdot \frac{dH_z}{dx}$$

$$0 = \frac{j\omega\mu}{h^2} \frac{d}{dx}(C_1 C_3 \cos Bx \cos Ay)$$

$$0 = \frac{j\omega\mu}{h^2} \cdot C_1 C_3 (-\sin Bx) \cdot B \cos Ay$$

Substitute $x=a$ in the above equation

$$0 = \frac{j\omega\mu}{h^2} C_1 C_3\, B \cos Ba \cdot \cos Ay$$

$$\Rightarrow \cos Ba = 0$$

$$Ba = m\pi$$

$$\boxed{B = m\pi/a}$$

Substitute A & B Values in the above equation

$$\boxed{H_z = C\cos\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)\cdot y \cdot e^{j\omega t - \gamma z}} \quad (\because C_1 \cdot C_2 = C)$$

Field Components:

$$E_x = \frac{j\omega\mu}{h^2} \cdot C \cdot \left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{(j\omega t - \gamma z)}$$

$$E_y = -\frac{j\omega\mu}{h^2} \cdot C \cdot \left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{(j\omega t - \gamma z)}$$

$$H_x = -\frac{\gamma}{h^2} \cdot \frac{dH_z}{dx}$$

$$= \frac{\gamma}{h^2} \cdot C \cdot \frac{m\pi}{a} \cdot \sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right) \cdot y \cdot e^{(j\omega t - \gamma z)}$$

$$H_y = -\frac{\gamma}{h^2} \cdot C \cdot \frac{n\pi}{b} \cdot \cos\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{(j\omega t - \gamma z)}$$

TE Modes in rectangular waveguide:

a) TE Mode:

- When we substitute $m=0, n=0$ values in E_x, E_y, H_x and H_y Equations, All field components are vanished

b) TE_{01} Mode:

When $m=0, n=1$, E_x and H_y values exist and $E_y=0$ and H_x Values become zero. Hence TE_{01} mode exist

c) TE_{10} Mode:

When $m=1, n=0$ E_z and H_x Values Exist and $E_x=0$ & $H_y=0$. Therefore TE_{10} mode Exist

d) TE_{11} Mode:

When $m=1$ and $n=1$ all field components i.e., E_x, E_y, H_x and H_y exist. The remaining all higher Order modes also exist.

Dominant Mode:

The mode which has highest cuttoff wavelength and lowest cuttoff frequency is called Dominant mode

$$\lambda_{C_{mn}} = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}}$$

for TE_{01} mode $\Rightarrow \lambda_{c_{01}} = \frac{2ab}{\sqrt{a^2}} = 2b$

for TE_{10} mode $\Rightarrow \lambda_{c_{10}} = \frac{2ab}{\sqrt{b^2}} = 2a$

for TE_{11} mode $\Rightarrow \lambda_{c_{11}} = \frac{2ab}{\sqrt{a^2+b^2}}$

$\Rightarrow$ $\lambda_{c_{10}}$ has the maximum value since 'a' is larger in dimension, so TE_{10} is the dominating mode in the Rectangle wave guide

$\Rightarrow$ The Other Expressions for β, V_p, V_g and λ_g remains the same for TM waves.

$$\beta = \frac{2\pi}{\lambda_g} = \sqrt{\omega^2\mu\epsilon - \omega_c^2\mu\epsilon}$$

$$* \quad V_p = \frac{c}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$* \; V_g = c \cdot \sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$

$* \; \lambda_g = \dfrac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$

Wave impedance ($Z_1 Z_1$) in TM and TE Waves:

Wave impedance is defined as the ratio of the strength of electric field in One transverse direction to the strength of the Magnetic field along the Other transverse direction. i.e.,

$$Z_1 Z_1 = \frac{E_x}{H_y} = \frac{-E_y}{H_x} \quad \text{(or)}$$

$$Z_1 Z_1 = \frac{\sqrt{E_x^2 + E_y^2}}{\sqrt{H_x^2 + H_y^2}}$$

(i) **Wave impedance for a TM wave in rectangular Waveguide :-**

$$Z_1 Z_1 = Z_1 TM = \frac{E_x}{H_y} = \frac{\frac{-\gamma}{h^2} \cdot \frac{\partial E_{Z_1}}{\partial x} - \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_{Z_1}}{\partial y}}{\frac{-\gamma}{h^2} \cdot \frac{\partial H_{Z_1}}{\partial y} - \frac{j\omega\epsilon}{h^2} \cdot \frac{\partial E_{Z_1}}{\partial x}}$$

for a TM wave $H_{Z_1} = 0$ and $\gamma = j\beta$

$$Z_1 TM = \frac{\frac{-\gamma}{h^2} \cdot \frac{\partial E_{Z_1}}{\partial x}}{\frac{-j\omega\epsilon}{h^2} \cdot \frac{\partial E_{Z_1}}{\partial x}} \Rightarrow \frac{\gamma}{j\omega\epsilon}$$

$$= \frac{j\beta}{j\omega\epsilon} = \frac{\sqrt{\mu\epsilon}\sqrt{\omega^2-\omega_c^2}}{\omega\epsilon}$$

$$= \frac{\sqrt{\mu\epsilon}\cdot\omega\sqrt{1-(\omega_c/\omega)^2}}{\omega\epsilon}$$

$$= \sqrt{\frac{\mu}{\epsilon}}\sqrt{(1-\left(\frac{f_c}{f}\right)^2}$$

$$= \sqrt{\frac{\mu}{\epsilon}}\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

$\mu_0 = 4\pi\times10^{-7}$

$\epsilon_0 = \frac{1}{38\pi}\times10^{-9}$

for air $\sqrt{\frac{\mu}{\epsilon}} = 377\,\Omega = \eta$

$$Z_{TM} = \eta\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

→ Since λ_0 is less than λ_c for wave propagation

$Z_{TM} < \eta$

→ This shows that wave impedance for a TM wave is always less than free space impedance in rectangular waveguide:

2 Wave impedance of TE waves in rectangular waveguide:

$$Z_{Z_1} = Z_{TE} = \frac{E_x}{H_y} = \frac{-\frac{\gamma}{h^2}\cdot\frac{\partial E_{z_1}}{\partial x} - \frac{j\omega\mu}{h^2}\cdot\frac{\partial H_z}{\partial y}}{-\frac{\gamma}{h^2}\frac{\partial H_{z_1}}{\partial y} - \frac{j\omega\epsilon}{h^2}\cdot\frac{\partial E_{z_1}}{\partial x}}$$

for TE wave $E_{z_1} = 0$

$$Z_{TE} = \frac{-\frac{j\omega\mu}{h^2}\cdot\frac{\partial H_{z_1}}{\partial y}}{-\frac{\gamma}{h^2}\cdot\frac{\partial H_{z_1}}{\partial y}} = \frac{j\omega\mu}{j\beta} \quad (\because \gamma = j\beta)$$

$$= \frac{\omega\mu}{\sqrt{\mu\epsilon}\sqrt{\omega^2-\omega_c^2}} = \sqrt{\frac{\mu}{\epsilon}}\cdot\frac{1}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}$$

$$= \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\eta}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$$

$\therefore Z_{TE} > \eta$ as $\lambda < \lambda_c$ for wave propagation.

⇒ This shows that wave impedance for TE wave is always greater than free space impedance

power losses in waveguide:

→ As the electromagnetic wave propagates through a waveguide the wave intensity gets Attenuated because of losses in the waveguide

→ There are three types of losses in the waveguide which causes attenuation of transmitted signal.

1. power loss in dielectric filling.
2. power loss in waveguide walls.
3. Misaligned waveguide sections.

1) power loss in dielectric filling:

⇒ When the waveguide is completely filled with a low loss dielectric $(\sigma < \omega\epsilon)$, the attenuation constant α in the waveguide due to dielectric loss is,

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$$

W.k.T $\eta = \sqrt{\frac{\mu}{\epsilon}}$

Therefore for free space

$$\Rightarrow \alpha = \frac{\sigma\eta}{2}$$

⇒ The attenuation in waveguide α_g for TE_{mn} and TM_{mn} mode is given by

for TE_{mn} mode,

$$\alpha_g = \frac{\alpha}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\alpha_g = \frac{\sigma h}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

for TM_{mn} mode

$$\alpha_g = \alpha\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\alpha_g = \frac{\sigma h}{2}\sqrt{1-\left(\frac{f_c}{f}\right)^2}.$$

2. power loss in waveguide:

⇒ In a waveguide, the wave is propagated by Reflections from walls, the tangential component of electric field and normal component of magnetic field across the walls vanished. Due to this the Average power in the waveguide is dissipated.

⇒ The attenuation in waveguide α_g is given by

$$\alpha_g = \frac{P_L}{2P_{tr}}$$

where P_L = power loss per unit length

P_{tr} = power transmitted through the waveguide

Circular Waveguide: (Unit III)

Defination: A Hallow metallic tube of uniform circular cross section for travelling EM wave by the successive reflection from the inner walls of tube is called as circular waveguide.

⇒ Manufacturing of circular waveguide is easy compared to that of rectangular Wave guide polarized.

⇒ It converts linear walls into Circular walls

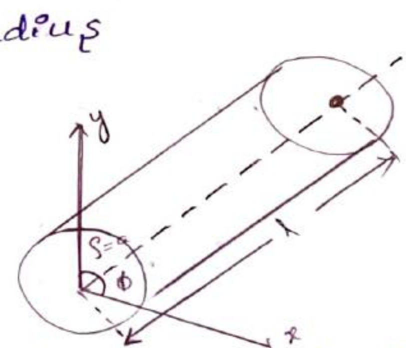
⇒ It is suitable for long distance communication

⇒ It uses cylindrical coordinate System

⇒ circular waveguide is basically a tubular, circular conductor with a inner radius 'ρ' and length 'l'.

⇒ Here we consider inner radius

$$\rho = a$$



$$\phi \to 0 \text{ to } 2\pi$$
$$\rho \to 0 \text{ to } a$$
$$z_1 \to 0 \text{ to } \infty$$

⇒ The HelmHolz Equation for TE and TM wave travelling in direction of a waveguide are

$$\nabla^2 H_{z_1} = 0$$
$$\nabla^2 E_{z_1} = 0$$

Nature of fields: Nature of fields describes that how the microwave can travel in circular waveguide, What are the modes available to propagate, In which mode it will follow in which path.

Propagation of TE wave in Circular Waveguide :-

$\Rightarrow$ for TE wave $E_{z_1} = 0$, $H_z \neq 0$

In TE Mode the electric field lines are entirely transverse to the direction of the propagation, where as magnetic field has a component along the direction of propagation.

The maxwell Equation

$$\nabla^2 H_z = -\omega^2 \mu \epsilon H_z \longrightarrow (1)$$

Expanding $\nabla^2 H_z$ in cylindrical co-ordinates.

$$\frac{\partial^2 H_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial H_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 H_z}{\partial \phi^2} + \frac{\partial^2 H_z}{\partial z_1^2} = -\omega^2 \mu \epsilon H_z \longrightarrow (2)$$

We know that $\frac{\partial^2}{\partial z_1^2} = \gamma^2$

$$\frac{\partial^2 H_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial H_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 H_z}{\partial \phi^2} + (\gamma^2 + \omega^2 \mu \epsilon) H_{z_1} = 0 \longrightarrow (3)$$

We know that $\gamma^2 + \omega^2 \mu \epsilon = h^2$

$$\frac{\partial^2 H_{z_1}}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial H_{z_1}}{\partial \rho} + \frac{1}{\rho}\frac{\partial^2 H_z}{\partial \phi^2} + h^2 H_z = 0 \longrightarrow (4)$$

The above Equations is second Order partial differential equation whose solution can be calculated by Using Variables and Separate method

Calculated by Using Variables and Separable method

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$$HZ_1 = PQ \longrightarrow (5)$$

Where P is a function of ρ only

Q is a function of ϕ only

Substitute equ (5) in equ (4)

$$\frac{\partial^2 (PQ)}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial (PQ)}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 (PQ)}{\partial \phi^2} + h^2 PQ = 0.$$

- Multiplying above equation with $\frac{\rho^2}{PQ}$, we get

$$\frac{\rho^2}{P} \cdot \frac{\partial^2 P}{\partial \rho^2} + \frac{\rho}{P} \frac{\partial P}{\partial \rho} + \frac{1}{Q} \frac{\partial^2 Q}{\partial Q^2} + h^2 \rho^2 = 0 \longrightarrow (5)$$

The terms in the above equation (1), (2) & (4) are function of ρ only and the term (3) is a function of ϕ only

Let $\frac{1}{Q} \frac{\partial^2 Q}{\partial \rho^2} = -h^2$, Where h^2 is a constant

- Then Equation (6) becomes

$$\frac{\rho^2}{P} \cdot \frac{\partial^2 P}{\partial \rho^2} + \frac{\rho}{P} \frac{\partial (PQ)}{\partial \rho} + (\rho^2 h^2 - n^2) = 0$$

Multiplying throughout by 'P' then

$$\rho^2 \frac{\partial^2 P}{\partial \rho^2} + \rho \frac{\partial P}{\partial \rho} + P (\rho^2 h^2 - h^2) = 0 \longrightarrow (8)$$

The above equation is similar to the Bessel's Equation of the form

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - x^2)y = 0 \longrightarrow (9)$$

Whose solution is $y = C_n J_n(x)$

When $J_n(x)$ represents n^{th} Order based function C_n is Constant. Where To bring Eq (8) is Similar to Eq (9) Replace 'ρ' with 'ρh'

$$(\rho h)^2 \frac{\partial^2 P}{\partial(\rho h)^2} + (\rho h)\frac{\partial P}{\partial(\rho h)} + P(\rho^2 h^2 - h^2) = 0$$

Therefore, the solution of this equation is given by

$$P = C_n J_n(\rho h)$$

also $$\frac{1}{\phi} = \frac{\partial Q^2}{d\phi^2} = -h^2$$

The general solution of this equation is given by

$$\phi = A_n \cos\phi + B_n \sin\phi$$

Therefore the complete solution becomes

$$H_{Z_1} = P Q_1$$

$$H_{Z_1} = C_n J_n(\rho h)\sqrt{A_n^2 + B_n^2}\ \cos\left(n\phi + \tan^{-1}\frac{A_n}{B_n}\right)$$

$$= C_n J_n(\rho h)\ C_n' \cos n\phi'$$

Where $C_n' = \sqrt{A_n^2 + B_n^2}$; $n\phi' = n\phi + \tan^{-1}\frac{A_n}{B_n}$

$$H_{Z_1} = C_0 J_n(\rho h)\cos(n\phi) \quad [\because \text{Where } C_0 = C_n C_n'')$$

if we consider a sinusoidal variation along 'Z_1'

$$H_{Z_1} = C_0 J_n(\rho h)\cos(n\phi)e^{-\gamma Z_1} \longrightarrow (10)$$

Boundary Conditions:

Now applying the Boundary conditions, w.k.T all 96 along the surface of the circular waveguide at $\rho = a$, $E\phi = 0$ for all values of varying b/w 0 to 2π

$$\text{i.e } \left.\frac{\partial H_{z_1}}{\partial \rho}\right|_{\rho=a} = 0$$

This implies

$$\frac{\partial H_{z_1}}{\partial \rho} = C_0 \cdot J_n'(\rho h) \cos n\phi \, e^{-\gamma z_1}$$

where $\rho = a$

$$\left.\frac{\partial H_{z_1}}{\partial \rho}\right|_{\rho=a} = C_0 J_n'(ah) \cdot \cos n\phi \, e^{-\gamma z_1} = 0$$

$$= J_n'(ah) = 0 \longrightarrow (11)$$

Here the prime denotes differentiation w.r.to ah is denoted by P'_{mm} = the m^{th} root of this equation

Which are the eigen values given by $P'_{mm} = ah \longrightarrow (12)$

i.e various root values are given by

n/m	1	2	3
0	3.832	7.016	10.173
1	1.841	5.331	8.536
2	3.054	6.706	9.969
3	4.201	8.015	11.346

Table P'_{nm} Values for TE_{nm} mode in circular waveguide

The Equation (10) Reduces to

$$H_z = C_0 \cdot J'_n(ah) \cos n\phi \cdot e^{-\gamma z}$$

* This equation represents all possible solution of the H_z for TE_{nm} waves in circular waveguide

Since J_n are Oscillator functions, the $J'_n(ah)$ are also Oscillatory functions.

* Here the first subscript 'n' represents the number of full cycles of field variation in One revolution through 2π radians of ϕ. The Second subscript 'm' represents the number of zeros of $E\phi$ i.e $J'_n(ab)$ along the radial of the waveguide but the zeros on the axis is excluded if it exists.

* The permissible values of h is given by

$$h = \frac{P'_{nm}}{a} \quad (\because \text{from equ } (12))$$

* The Various field components $E\rho$, $E\phi$, E_z, $H\phi$ can be obtained by using the cylindrical coordinates by Using Maxwell curl equation

$$E_\rho = \frac{-j\omega\mu}{h^2} \cdot \frac{1}{\rho} \cdot \frac{\partial H_z}{\partial \phi}$$

$$-E_\phi = \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_z}{\partial \rho}$$

$$E_z = 0$$

$$H_\rho = \frac{-\gamma}{h^2} \cdot \frac{\partial H_z}{\partial \rho}$$

$$H_\phi = \frac{-\gamma}{h^2} \cdot \frac{1}{\rho} \cdot \frac{\partial H_z}{\partial \phi}$$

$H_z = C_0 \cdot J_n(\rho h) \cos n\phi \cdot e^{-\gamma z}$ 27

Where $h^2 = \gamma^2 + \omega^2\mu\epsilon$

Substituting H_z in above equations with $h = \frac{P'_{nm}}{a}$

* The complete field equations for TE_{mn} make in circular Wave guide

$$E_\rho = \frac{-j\omega\mu}{h^2} \cdot \frac{1}{\rho} \cdot \frac{\partial H_z}{\partial \phi}$$

$$= \frac{-j\omega\mu}{\frac{P'_{nm}}{a}} \cdot \frac{1}{\rho} \cdot \frac{\partial}{\partial \phi} [C_0 J_n(\rho h) \cdot \cos n\phi \, e^{-\gamma z}]$$

$$= \frac{-j\omega\mu \cdot a}{P'_{nm}} \cdot \frac{1}{\rho} \cdot C_0 J_n(\rho h)(-\sin n\phi)(n)\, e^{-\gamma z}$$

$$= C_{0\rho} J_n(\rho h) \sin n\phi \cdot e^{-\gamma z} \longrightarrow (1)$$

$$= C_{0\rho} J_n\left(\frac{P'_{nm}}{a} \cdot \rho\right) \sin n\phi \cdot e^{-\gamma z} \longrightarrow (1)$$

Where $C_{0\rho}$ is constant

$$E_\phi = \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_z}{\partial \rho}$$

$$= \frac{j\omega\mu}{\left(\frac{P'_{nm}}{a}\right)^2} \cdot \frac{\partial}{\partial \rho} (C_0 \cdot J_n(\rho h) \cdot \cos n\phi \cdot e^{-\gamma z})$$

$$= \frac{j\omega\mu}{P'^2_{nm}} a^2 \cdot C_0 \cdot J'_n\left(\frac{P'_{nm}}{a} \cdot \rho\right) \cos n\phi \cdot e^{-\gamma z}$$

$$E_\phi = C_{0\phi} J'_n\left[\frac{P'_{nm}}{a} \cdot \rho\right] \cos n\phi \, e^{-\gamma z} \longrightarrow (2)$$

$$E_z = 0 \longrightarrow (3)$$

$$H\rho = \frac{-\gamma}{h^2} \cdot \frac{\partial H_z}{\partial \rho}$$

$$= \frac{-\gamma}{h^2} \cdot \frac{\partial}{\partial \rho}\left[C_0 . J_n(\rho h) . \cos n\phi . e^{-\gamma z} \right)$$

$$= \frac{-\gamma}{h^2} . C_0 J_n'(\rho h) \cos n\phi . e^{-\gamma z}$$

$$= \frac{-\gamma . a^2}{P_{nm}'^2} C_0 . J_n'(\rho h) \cos n\phi . e^{-\gamma z}$$

$$= -\frac{C_0 \Phi}{Z . z} . J_n'\left(\frac{P_{nm}'}{a} . \rho\right) . \cos n\phi . e^{-\gamma z} \longrightarrow (4)$$

$$H\phi = \frac{C_0 \Phi}{Z_1 Z} J_n \left(\frac{P_{nm}'}{a} . \rho\right) \sin n\phi . e^{-\gamma z} \longrightarrow (5)$$

$$H_z = C_0 . J_n \left(\frac{P_{nm}'}{a} . \rho\right) \cos n\phi . e^{-\gamma z} \longrightarrow (6)$$

Where $Z_1 Z = \frac{E\rho}{H\phi} = \frac{-E\phi}{H\rho}$ $\quad \therefore m = 0, 1, 2, 3 \ldots\ldots$

$n = 0, 1, 2, 3 \ldots\ldots$

Propagation of TM modes in Circular waveguide:

When Tm wave is propoltional in circular wave guide the electric and magnetic field components are as follows

$$H_z = 0 , \quad E_z \neq 0$$

* In Tm mode magnetic field lines are entirely transverse to the direction of the propagation. Where as the electric field has a component along the direction of propagation.

$\nabla^2 E_z = -\omega^2 \mu \epsilon E_z \longrightarrow (1)$

Expanding $\nabla^2 E_z$ in cylindrical coordinates

$$\frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial E_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu \epsilon E_z \longrightarrow (2)$$

We know that $\frac{\partial^2}{\partial z^2} = \gamma^2 \longrightarrow (3)$

Cut off wavelength in circular waveguide:

The cut off wavelength is that mode for which the mode propagation constant 'γ' is vanishes

$\gamma = \alpha + j\beta = 0$

$$\beta = \sqrt{\omega^2 \mu \epsilon - \omega_c^2 \mu \epsilon}$$

$$= \sqrt{\omega^2 \mu \epsilon - \left(\frac{2\pi f_c}{1/\sqrt{\mu \epsilon}}\right)^2}$$

$$= \sqrt{\omega^2 \mu \epsilon - \left(\frac{2\pi}{c/f_c}\right)^2}$$

$$= \sqrt{\omega^2 \mu \epsilon - \left(\frac{2\pi}{\lambda_c}\right)^2}$$

$$= \sqrt{\omega^2 \mu \epsilon - h^2}$$

Where $h = \frac{P'_{nm}}{a}$ for a TE wave and $h = \frac{P_{nm}}{a}$ for TM wave. Therefore for TE wave, the cut off wavelength is given by

$$\lambda_c = \frac{2\pi}{h} = \frac{2\pi}{\left(\frac{P'_{nm}}{a}\right)} = \frac{2\pi a}{P'_{nm}}$$

λ_c will be maximum when P'_{nm} is minimum

Similarly for TM wave $\lambda_c = \frac{2\pi}{ah}$

where $h = \frac{P_{nm}}{a}$

$$\boxed{\lambda_{cm} = \frac{2\pi a}{P_{nm}}}$$

The formulas for phase velocity, group velocity and guided wavelength remains same for a circular waveguide as rectangular waveguides

$$v_p = \frac{\omega}{\beta} = \frac{v_g}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

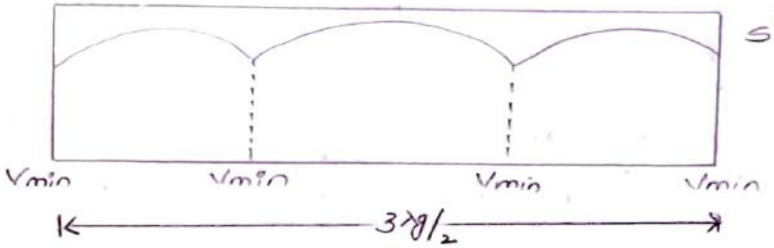
$$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$Z_{zTE} = \frac{\omega\mu}{\beta} = \frac{\eta}{\sqrt{1-(\lambda_0/\lambda_c)^2}} \qquad \left[\because \eta = \sqrt{\frac{\mu}{\epsilon}}\right]$$

$$Z_{zTM} = \frac{\beta}{\omega\mu} = \eta \cdot \sqrt{1-(\lambda_0/\lambda_c)^2}$$

Cavity Resonators:

When One end of the waveguide is terminated in a shorting plate. There will be reflections and hence standing waves will Results when another shorting plate is kept at a distance of a "Multiple of $\lambda g/2$" then the hallow space so formed can support a signal which bounces back and forth between the shorting plates. This Results in Resonance and hence the hallow space is called cavity and the Resonator as the "Cavity Resonator".



The waveguide section can be Rectangular or Circular

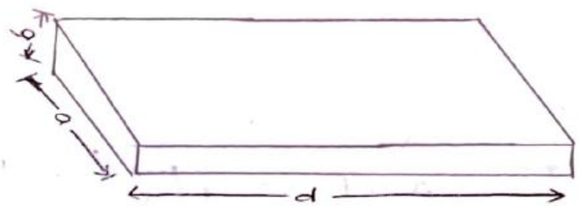
- 

fig: Rectangular Cavity Resonator

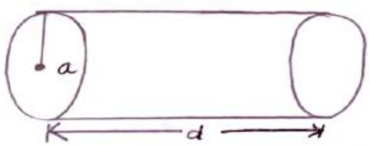


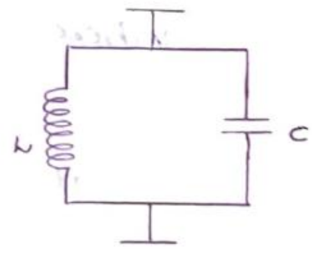
fig: Circular Cavity Resonator

* The microwave cavity resonator is similar to a tuned circuit at low frequency having a resonant freq.

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

* The cavity resonator can resonate at only one particular frequency just as a parallel resonant circuit

$$d = 3\lambda_g/2$$



* for a given resonator and mode a, b, m and n are constants. Therefore λ_c is also fixed and λ_0 will also

have a constant value. Equal to f_0, which is the resonant frequency of the cavity resonator

Rectangular Cavity Resonator:

In a rectangular waveguide section if the short circuit is placed at the two ends. The result configuration is called a rectangular cavity resonator in which the signal bounces back and forth between the opposite walls.

Expression for f_0:

W.K.T $\quad h^2 = \gamma^2 + \omega^2 \mu \epsilon = A^2 + B^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

$$\omega^2 \mu \epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \gamma^2$$

for wave propagation

$$\gamma = j\beta$$

$$\gamma^2 = j^2 \cdot \beta^2 = -\beta^2$$

$$\omega^2 \mu \epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \beta^2$$

⇒ If a wave has to exist in cavity resonator there must be a phase change corresponding to a given guide wavelength

$$\beta = \frac{2\pi}{\lambda g} \Rightarrow \beta = \frac{p \cdot \pi}{d}$$

Where p is constant

d is length of resonator

at $f = f_0$, $\omega = 2\pi f_0 = \omega_0$

$$\omega_0^2 \mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2$$

$$\omega_0 = \frac{1}{\sqrt{\mu\epsilon}} \sqrt{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]}$$

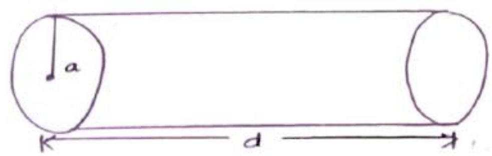
$$f_0 = \frac{1}{2\pi\sqrt{\mu\epsilon}} \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]^{1/2}$$

$$f_0 = \frac{c}{2} \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]^{1/2}$$

General mode of propagation in a cavity resonator is TE mnp (or) TMmnp for both TE and TM. The Resonant frequency is the same in a rectangular cavity resonator.

Circular Cavity Resonator:

In Circular cavity resonator circular end plates are used to short both ends.



In circular cavity resonator 'a' if the radius of the waveguide, 'd' is the length of the waveguide.

Resonance frequency:

for a circular waveguide section, W.K.T

$$h^2 = \gamma^2 + \omega^2 \mu\epsilon \longrightarrow (1)$$

$$h^2_{nm} = \left(\frac{P_{nm}}{a}\right)^2 \longrightarrow (2)$$

h^2_{nm} from equation (1) & (2)

$$\omega^2 \mu\epsilon + \gamma^2 = \left(\frac{P_{nm}}{a}\right)^2$$

$$\omega^2\mu\epsilon = \left(\frac{P_{nm}}{a}\right)^2 - \gamma^2$$

for wave propagation $\gamma = j\beta$, and

for resonance $\beta = \frac{p\pi}{d}$, and

$$\omega = \omega_0$$

$$\omega_0^2\mu\epsilon = \left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2$$

$$f_0 = \frac{1}{2\pi\sqrt{\mu\epsilon}}\left[\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]^{1/2}$$

$$f_0 = \frac{c}{2\pi}\sqrt{\left[\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]}$$

This is for TM_{nmp} mode

Similarly for TE_{nmp} mode, $f_0 = \frac{c}{2\pi}\sqrt{\left(\frac{P'_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$

Applications:

They can be used as

* Tuned circuits
* In UHF tubes, Klystron Amplifier / Oscillator, Cavity magnetron
* In Duplexers of Radars.

Field expressions for TM_{nmp} and TE_{nmp} modes in a rectangular cavity resonator :(unit - III)

TM Waves:

The field expression can be obtained by using wave equation and Bending condition

NOTE: It is same as field components for TM wave What ever earlier we derive

$$E_z = C \sin\left(\frac{m\pi}{a}\right)x \ \sin\left(\frac{n\pi}{b}\right)y \ e^{j\omega t} \cdot e^{-\gamma z}$$

Here the wave is propagating along the two positive z- direction. Similarly, When the wave is propagating along the negative z- direction.

$$E_z = C \cdot \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t} \cdot e^{+\gamma z}$$

the wave propagates $\gamma = j\beta$, Adding the

We also know the wave propagates $\gamma = j\beta$, Adding the field components of two travelling wave i.e, One is positive 'z' direction and the Other in the Negative 'z' direction. We Obtain

$$E_z = C \cdot \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{j(\omega t \mp \beta z)} \longrightarrow (1)$$

* Let A^+ be the Amplitude constant for the wave propagating in positive z-direction and A^- be the Amplitude constant for the wave propagating in Negative z- direction

$$\text{then}, \ E_z = \left(A^+ e^{-j\beta z} + A^- e^{j\beta z}\right]$$

To make E_z Vanish at $z=0$ and $z=d$, choose

$$A^+ = A^- = A$$

then Equation (1) Reduces to

$$E_z = CA\left[e^{-j\beta d} + e^{j\beta d}\right] \cdot \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t}$$

But $e^{-i\theta} + e^{i\theta} = 2\cos\theta$

$$e^{-j\beta d} + e^{j\beta d} = 2\cos\beta d$$

With these value the above Equation become

$$E_z = C \cdot 2A\cos\beta d \cdot \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t}$$

$E_z = 0$, all along the surface of the Resonator

$$0 = C \cdot 2A\cos\beta d \cdot \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t}$$

Since $C \neq 0$, $\sin\left(\frac{m\pi}{a}\right)x$ and $\sin\left(\frac{n\pi}{b}\right)y \neq 0$ and $A \neq 0$

only $\cos\beta d = 0$

$$\beta d = p\pi$$

$$\boxed{\beta = \frac{p\pi}{d}}$$

$$E_z = c' \cdot \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \; \cos\left(\frac{p\pi}{d}\right)z \cdot e^{-j\omega t - \gamma z}$$

Where $c' = 2cA$

$$E_z = c'\sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \quad \cos\left(\frac{p\pi}{d}\right)^2 \cdot e^{+j\omega t - \gamma z}$$

for TE Wave :-

We can derive the field components for TE wave by using wave equation and Boundary condition

$$H_z = C \cdot \cos\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}$$

Since $\gamma = j\beta$ for a wave propagating along the +ve z-direction

$$H_z = C \cdot \cos\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \; e^{j(\omega t - \beta z)}$$

When wave is propagating along the -ve direction

$$H_z = C \cdot \cos\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \; e^{j(\omega t - \beta x)}$$

the amplitude constant along the $+z$ direction is represented by A^+ and that along the $-z$ direction by A^-. Adding the two travelling waves to obtain the field of standing wave.

$$H_z = (A^+ . e^{-j\beta z} + A^- . e^{+j\beta z}) \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \, e^{j\omega t}$$

To make E_y vanish at $z=0$ and $z=d$ we must take

$A^+ = -A^-$ or $A^- = -A^+$ and also w.k.T

$$E_y = -\frac{\gamma}{h^2} \frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x}$$

for TE waves $E_z = 0$,

$$E_y = \frac{j\omega\mu}{h^2} . \frac{\partial H_z}{\partial x}$$

$$E_y = \frac{j\omega\mu}{h^2}\left(\frac{\partial}{\partial x} (A^+ . e^{-j\beta z} + A^- e^{+j\beta z}) . \left(-\left(\frac{m\pi}{a}\right)\right) \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y\right] e^{j\omega t} \longrightarrow (3)$$

$\sin\left(\frac{m\pi}{a}\right)x$ and $\cos\left(\frac{n\pi}{b}\right)y \neq 0$

Therefore only $A^+ e^{-j\beta z} + A^- e^{j\beta z} = 0$

To make $E_y = 0$ choose $A^- = -A^+$, putting these value in Equation (3) then

$$0 = (A^+ e^{-j\beta z} - A^+ e^{j\beta z}) = A^+ [e^{-j\beta z} - e^{j\beta z}]$$

$$= -2jA^+ \sin\beta z$$

Since $A^+ \neq 0$ Only $\sin\beta d = 0$ with $z = d$

$$\beta = \frac{p\pi}{d} \quad \text{where } p = 1, 2, 3, 4 \ldots\ldots$$

$$H_z = -2j \cdot A^+ \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \cdot \sin\left(\frac{p\pi}{d}\right)z \cdot e^{j(\omega t - \gamma z)}$$

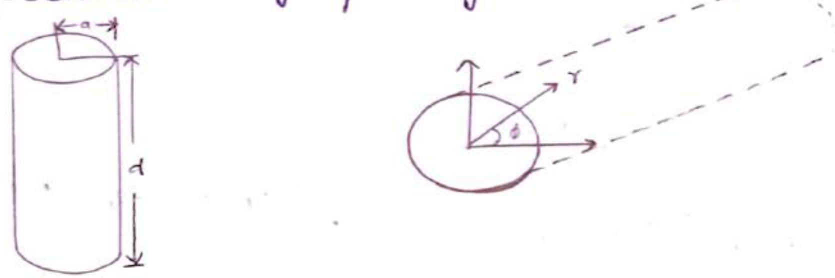
putting $-2jA^+ = c$, another constant

$$H_z = c\left(\cos\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot \sin\left(\frac{p\pi}{d}\right)z \cdot e^{j(\omega t - \gamma z)}\right).$$

Field Expression for TE_{mnp} and TM_{mnp} modes in Circular cavity Resonator:

for TE wave:

Consider a TE mode to be propagating in a circular cavity resonator by placing shorting plates at each end



d is length of waveguide and a is radius

In circular waveguide, for TE mode of propagation

$$H_z = A' J_n(\rho h) \cos(n\phi) e^{j\omega t - \gamma z}$$

Where $c'_n = \sqrt{(A'_n)^2 + (B'_n)^2}$

and $n\phi' = n\phi - \tan^{-1}\frac{B_n}{A_n}$

Now the wave travelling in $+z$ direction is given by

$$H_z = C_n \cdot J_n(\rho h)\cos n\phi \, e^{j(\omega t + \beta z)}$$

Let A^+ be the Amplitude component of the wave in positive z direction and A^- be the Amplitude component in Negative z direction. 33

$$(A^+ e^{-Bz} + A^- e^{Bz}) \cos n\phi = H_z$$

* Since H_z cannot be made Equal to zero, only E_ϕ & E_ρ can be made Equal to zero, Therefore to make E_ϕ and E_ρ Vanish at $z=0$, we choose $A^- = -A^+$

i. The factor $-n^+ e^{-Bd} + A e^{-Bd}$ becomes equal to $-2jA^+ \sin\beta d$

ii. To make $\sin\beta z$ Vanish

$$\beta d = p\pi \quad \text{where } p = 1, 2, 3 \ldots$$

$$\beta = \frac{p\pi}{d}$$

$$\Rightarrow H_z = -2jA\, J_n(\rho h) \cos(n\phi) \sin\left(\frac{p\pi}{d}\right) z \cdot e^{j(\omega t - \gamma z)}$$

i.e., $H_z = c \cdot J_n(\rho h) \cos(n\phi) \sin\left(\frac{p\pi}{d}\right) z \cdot e^{j\omega t - j\beta z}$.

Where $c = -2jA$

i.e., $H_z = c \cdot J_n(\rho h) \cos(n\phi) (A^+ e^{-j\beta z} + A^- e^{j\beta z})$

Quality factor for a Cavity Resonator:

The Quality factor (Q) at any Resonant or Anti resonant circuit is a measure of frequency selection and is design by the equation

$$Q = \frac{\omega_0 \omega}{\rho}$$

Where ω = Maximum energy stored

ρ = Average power loss

ω_0 = Resonant frequency.

$$Q = 2\pi \cdot \frac{\text{Maximum energy stored per cycle}}{\text{Energy dissipated per cycle.}}$$

* The Q of a perfect or ideal cavity Resonator is infinate since in a perfect conductor forming the cavity, p would be zero and also once energised it would resonate for ever.
* The Cavity Resonator that is Resonant at one frequency only if there is more than one resonant frequency, there will be different value of Q for the various values of frequencies.
* Normally coupling loops are used to couple the energy in and out of a cavity resonator.
* This coupling has the effect of an imperfectly reflecting walls and so is the finite termination or load of the cavity, this would also changes the value of Q. This Q that makes into account the coupling b/w the cavity and coupling paths is known as the loaded Q_L, so Q_L can be given by

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$$

Where Q_0 = Q at an unloaded Cavity

Q_{ext} = Q due to extended ohmic losses.

* There are three types of couplings, Resulting, incritically coupled, Under coupled and Over Coupled cases. An unloaded resonator can be represented by a series or

parallel resonant circuit

* The Unloaded Q is then given by

$$Q_0 = \frac{\omega_0 L}{R}$$

* The Q_{ext} can be written as,

$$Q_{ext} = \frac{Q_0}{K} = \frac{\omega_0 L}{KR}$$

Which; K = Coupling co-efficient of the cavity

power transmission in rectangular waveguide:

* The power transmitted through a waveguide and the power loss in the waveguide walls can be calculated by means of complex poynting theorem
* The power transmitted P_{tr} through a waveguide is given by $P_{tr} = \oint P.d.s = \oint \frac{1}{2}(E \times H^*)\, ds$

for a lossless dielectric, the time-Avg power flow through a rectangular waveguide is given by

$$P_{tr} = \frac{1}{2Z_z}\int_s |E|^2 ds = \frac{Z_z}{2}\int_s |H|^2 ds$$

Where

$$Z_z = \frac{E_x}{H_y} = -\frac{E_y}{H_x}$$

$$|E|^2 = |E_x|^2 + |E_y|^2$$

$$|H|^2 = |H_x|^2 + |H_y|^2$$

* for TM_{mn} mode, the Avg power transmitted through a rectangular waveguide of dimensions a and b is

$$P_{tr} = \frac{1}{2\eta\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} \int_0^b \int_0^a |E_x|^2 + |E_y|^2 \, dx\, dy.$$

for TEmn modes, $Z_z = \dfrac{\eta}{\sqrt{1-(\lambda/\lambda_c)^2}}$

$$P_{tr} = \frac{\sqrt{1-(\lambda/\lambda_c)^2}}{2\eta} \int_0^b \int_0^a |E_x|^2 + |E_y|^2 \, dx\,dy.$$

An Air filled Rectangular Waveguide of dimension $a = 8$ cm and $b = 4$ cm, Operates in dominant TE_{10} mode, then find

a. The cut off frequency

b. Group Velocity of waveguide of a frequency at 3.75 GHz.

c. The guided wave length at the same frequency.

Sol. Given that

$a = 8\text{cm} \Rightarrow 8 \times 10^{-2}$ m

$b = 4\text{ cm} \Rightarrow 4 \times 10^{-2}$ m

$m = 1, \quad n = 0$

a. cut off frequency is

$\therefore c = 3 \times 10^8$

$$\lambda_{C_{mn}} = \frac{2ab}{\sqrt{m^2b^2 + n^2a^2}}$$

$$\lambda_{c_{10}} = \frac{2ab}{\sqrt{b^2}} = 2a = 2 \times 8 \times 10^{-2} \text{ m}$$

$$f_c = \frac{c}{\lambda_c} = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 8 \times 10^{-2}} = 1.87 \text{ GHz}$$

b. Group Velocity:

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$$V_g = c\sqrt{1-(f_c/f)^2}$$

$$= 3.46 \times 10^8 \text{ m/s}$$

c. Guided Wavelength:

$$\lambda_g = \frac{\lambda}{\sqrt{1-(f_c/f)^2}} = 9.2 \text{ cm}$$

When the dominant mode is propagated in an Air filled rectangular waveguide, the guide wave length for a frequency of 9 GHz is 4 cm calculate the width of the guide.

Sol: free space wavelength is

$$\lambda = \frac{c}{f} = \frac{3 \times 10^{10}}{9 \times 10^9} = 3.33 \text{ cm}$$

$$\lambda_g = 4 \text{ cm}$$

W.K.T $$\lambda_g = \frac{\lambda}{\sqrt{1-(\lambda/\lambda_c)^2}}$$

$$4 = \frac{3.33}{\sqrt{1-\left(\frac{3.33}{\lambda_c}\right)^2}}$$

$$\lambda_c = 6.028 \text{ cm}$$

* In rectangular waveguide TE_{10} is a dominating mode

$$\lambda_{c10} = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}} = 2a$$

$2a = 6.028$ cm $\quad (2a = 2b)$

$a = 3.014$ cm $\quad b = 1.5$ cm

* Determine the cut off wavelength for the Dominant mode is a rectangular wave guide or breadth 10cm for a 2.5 GHz signal propagated in the waveguide in the Dominant mode. calculate the Guide wavelength, Group and phase velocity.

⇒ In a rectangular waveguide the dominant mode is the TE_{10} mode λ_c for $TE_{10} = 2a = 2 \times 10 = 20$cm

$f = 2.5$ GHz

$$\lambda = \frac{c}{f} = \frac{3\times10^{10}}{2.5\times10^{9}} = 12 \text{ cms}$$

$$\lambda g = \frac{\lambda}{\sqrt{1 \cdot (\lambda/\lambda_c)^2}} = 1.5\text{cm}$$

$$v_p = \frac{c}{\sqrt{1-(\lambda/\lambda_c)^2}} = \frac{3\times10^{10}}{\sqrt{1-\left(\frac{12}{20}\right)^2}} = 3.75\times10^{10} \text{ cm/sec}$$

$$v_p \cdot v_g = c^2$$

$$v_g = \frac{(3\times10^{8})^2}{3.75\times10^{10}} = 2.4\times10^{10} \text{ cm/sec}$$

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A rectangular waveguide with dimensions of 3×2 cm operates in the T_{mn} mode at 10 GHz. Determine the characteristics of wave impedance.

Sol

$$Z_{TM} = \eta \sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

η is the intrinsic impedance $= 120\pi$

$$\lambda_{cn} = \frac{2ab}{\sqrt{m^2 b^2 + n^2 a^2}} = \frac{2ab}{\sqrt{a^2 + b^2}} = \frac{12}{\sqrt{9+4}}$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^{10}}{10 \times 10^9} = 3\,\text{cm}$$

$$Z_{TM} = 120\pi \sqrt{1 - (\lambda_0/\lambda_c)^2}$$

$$= 120\pi \sqrt{1 - (3/3.328)^2}$$

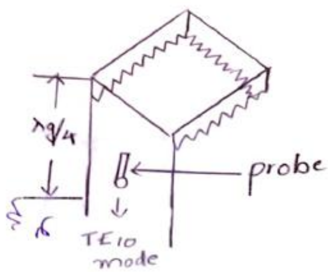
$$\boxed{Z_{TM} = 161.618\,\Omega}$$

Microwave components

The rectangular and circular waveguide cavity resonators etc. that were discussed in previous chapters are also microwave components. In this chapter we study other components like waveguide junctions, joints, corner, drives, posts and screws, directional couplers, ferrite device, phase shifters, filters etc.

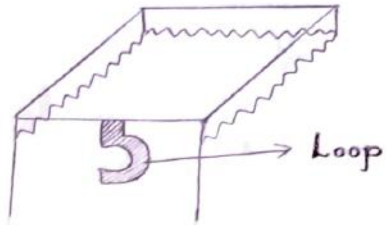
Coupling probes and coupling loops :-

* When a short antenna in the form of a probe or a loop is inserted into a wave guide, it will radiate and if it is correctly placed. The wanted mode will be setup.
* The dominate mode in Reset TE10. The probe is placed at a distance of $\lambda g/4$ from the shorted end of waveguide and the center of broader dimension of the waveguide because at that point E-field is maximum



Coupling loop

The coupling loop placed at the center of shorted end plate of the waveguide can also be used to launch TE10 mode i.e., coupling is achieved by means of a load antenna located in a plane perpendicular to the plane of the probe.

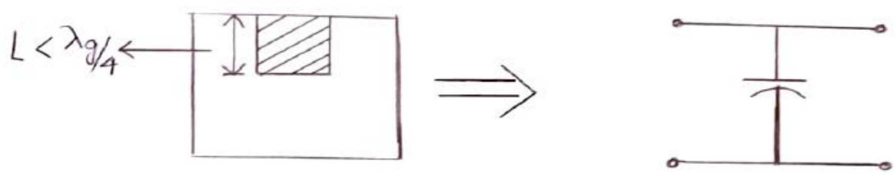


posts and tuning screws :

When a metallic cylindrical post is introduced into the broader side of wave guide, it produce capacitivity, resistivity and Inductivity.

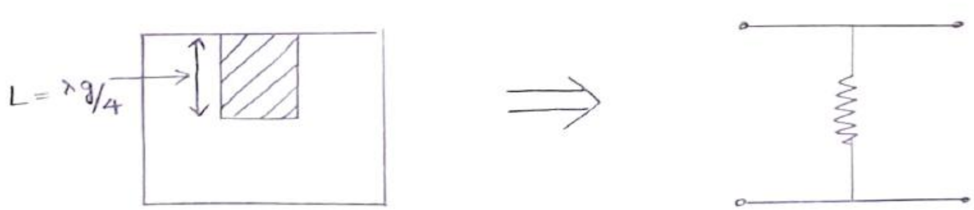
Case (i):

If the post extend only a short distance ($< \lambda g/4$) into a waveguide, it behave capacitively and this capacitive suspectance increases with depth of penetration.



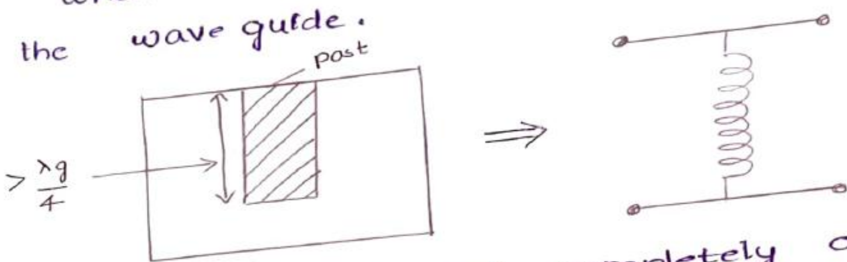
Case-(ii):

When the post extend upto depth of $> \lambda g/4$, The wave-guide behave like a inductively, and inductive suspectance decreases when the post is more furthur away from the centre of the waveguide

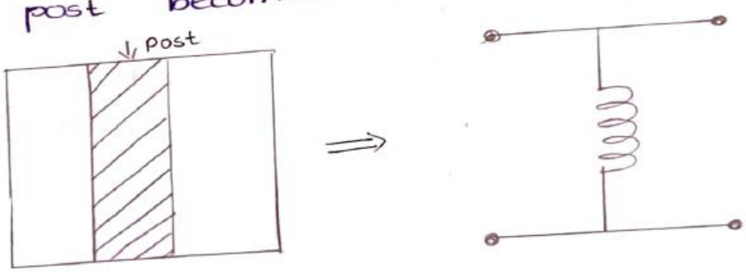


ase (iii):

When the post extended upto depth of $> \lambda g/4$, The waveguide behave like a inductively, and inductive suspectance decreases when the post is move furthur away from the centre of the wave guide.

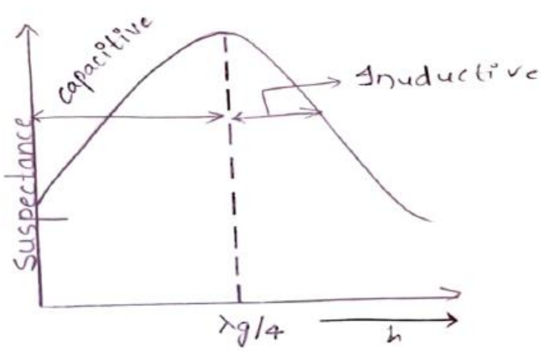


⇒ When the post is extended completely across the wave-guide, The post becomes inductive.

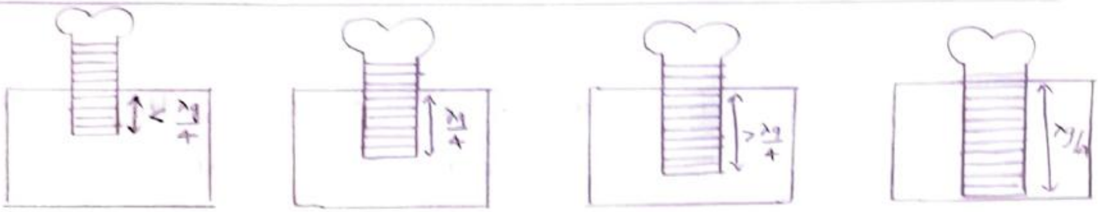


Suspectance Vs penetration (h):

* The amount of suspectance decreases as the diameter of the post is reduce.



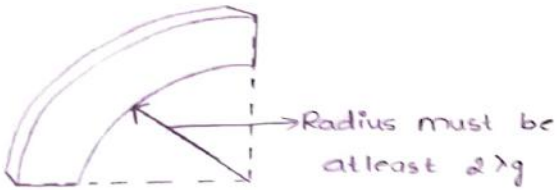
*** An Adjustable post is known as "screw" or "slug".



(a) capacitive (b) Resistive (c) Inductive (d)

Waveguide bends, corners, transitions and twists :-

* Waveguide bend, corners and twist's are useful for changing the direction of the guide by the desired angle
* The bends can be H-bend or E-bend
* If the bend is in the direction of the wide dimension The H-lines are effected, this bend is called H-bend
* If the bend is in the direction of Narrow dimension. The E-lines are effected this bend is called E-bend
* The bending radius must be atleast $2\lambda g$ to avoid SWR's greater than 1.05.

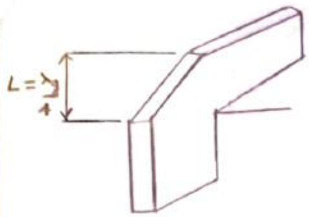


H- bend

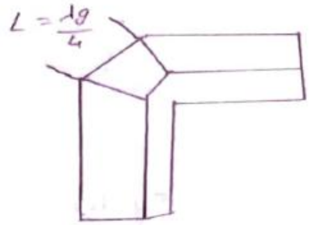
E- bend

corner :

* corners can be E-corners, H-corners
* corner is in the small dimension in H-corner
* corner is in the big dimension (length) in the E-corner



H-corner



E-corner

In order to minimise the reflections, the mean length 'L' must be an odd number of quarter wavelength, so that reflected wave from both ends of the waveguide are completely cancelled.

$$L = (2n+1)\frac{\lambda g}{4} \; ; n = 0, 1, 2, 3 \ldots$$

Transitions:

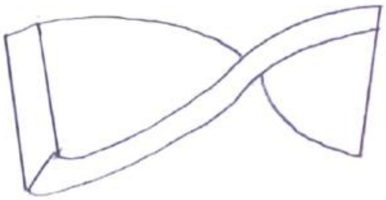
* Transitions are also called as taper.
* These are required to join two waveguide sections that have different shapes for their cross sectional areas
* Ex: A circular to rectangular waveguide transition

Circular - Rec

* Rectangular to circular transition

Rec to Circular

* Waveguide twist such as 90° and 45° twists are helpful in converting vertical to horizontal polarization or vice versa



90° twist

Waveguide Irises:

In any waveguide system, when there is a mismatch there will be reflections. suspectance appearing across the guides causing mismatch need to be cancelled by introducing another suspectance of the same magnitude but in opposite nature. Irises or windows are made use for the purpose

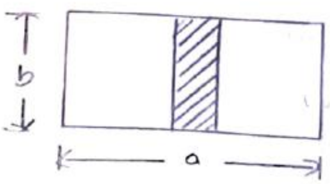
Microwave attenuators:

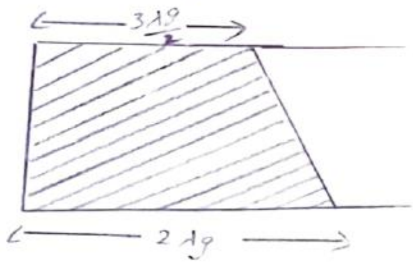
for perfect matching some times we require that the micro power in a waveguide be absorbed completely without any reflections for this we make use of attenuators

* Attenuators are commonly used for measuring power gain or loss in dB's
* Attenuators can be clasified as fixed or Variable type

fixed attenuators:

They are used where fixed amount of attenuation is to be provided if such attenuators absorb all the energy entering into it, we call it as a waveguide terminators



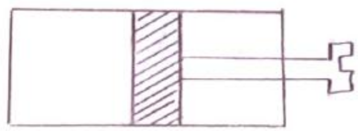


Fixed attenuator

Variable attenuator:

Variable attenuators provides continuous or step wise Variable attenuation.

* for rectangular waveguides, these attenuators can be flap type or Vaue type for circular waveguide rotary type is used



Maximum attenuation

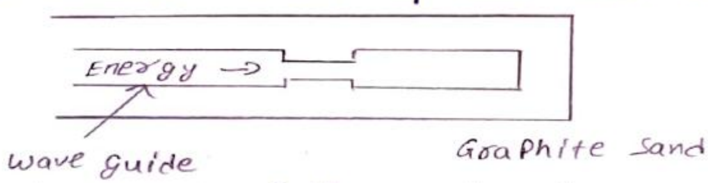
minimum attenuation

Phase shifters:

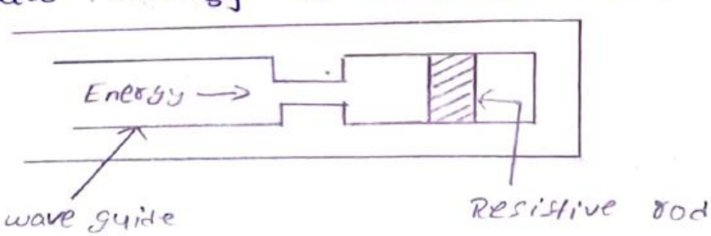
Wave guide terminations:

In a waveguide system it is not possible to attach a fixed resistive load as a termination.

1. A Graphical sand at the end of the waveguide can dissipate energy to achieve SWR's of less than 1.01.



2. Alternatively a resistive rod placed at a point in the waveguide where electric field strength is maximum can also dissipate energy to achieve SWRs

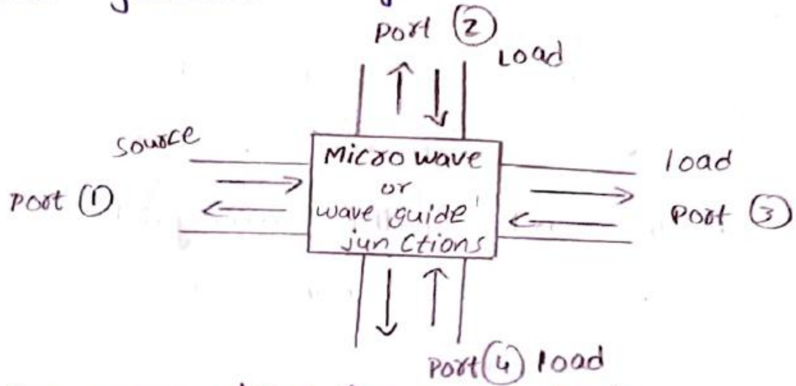


3. A Wedge of resistive material in the form of a taper where that can be act as a termination.

4. A permanent metal plate needed at the end of the waveguide may be employed for complete reflections.

Waveguide Junctions:

* In a microwave system, many times it becomes too necessary to split all microwave energy into particular directions. This is achieved by Waveguide Junctions.
* The same junction may be used to continue two or more signals



Scattering parameters: (s-parameters)

* Low frequency circuit can be described by two port n/w's and their parameters as Z, Y, H, ABCD etc.
* In a similar way at microwave frequency, microwave Junction can be described by s-parameters
* from above fig. When i/p from microwave source is applied to port ①, apart of

$$\boxed{b_1 = s_{11} a_1}$$

Where S_{i1} = Reflection co-efficient of 1st line
i = Source connected to ith line
1 = reflection from 1st line.

Hence, the contribution of the outward travelling wave in the ith line is given by

$$\boxed{b_i = S_{i1} a_1}$$

Case (ii):

Let all n lines be terminated in an impedance other than Z_0. Then, there will be reflections into junctions from energy line and hence the total contribution to the outward travelling wave in the ith line is given by

$$b_i = b_1 + b_2 + b_3 + \cdots\cdots + b_n$$

$$b_i = S_{i1}a_1 + S_{i2}a_2 + S_{i3}a_3 + \cdots\cdots + S_{in}a_n$$

$i = 1$ to n since i can be any value from 1 to n

therefore

$$b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 + \cdots\cdots + S_{1n}a_n$$

$$b_2 = S_{21}a_1 + S_{22}a_2 + S_{23}a_3 + \cdots\cdots + S_{2n}a_n$$

$$\vdots$$

$$b_n = S_{n1}a_1 + S_{n2}a_2 + S_{n3}a_3 + \cdots\cdots + S_{nn}a_n$$

In matrix form

$$\underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix}}_{\substack{\text{matrix (b) corresponding} \\ \text{to reflected waves}}} = \underbrace{\begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ S_{31} & S_{32} & \cdots & S_{3n} \\ \vdots & \vdots & & \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix}}_{\substack{\text{scattering coloumn} \\ \text{matrix (S) of order } n \times n}} \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix}}_{\substack{\text{matrix a corresponding} \\ \text{to Incident waves} \\ \text{or Input}}}$$

Output: $[b]_{n\times 1} = [S]_{n\times n}\,[a]_{n\times 1}$

a's $\xrightarrow{}$ rep i/p to particular port

b's $\xrightarrow{}$ rep o/p of various port

S_{ij} corresponds to scattering coefficients resulting due to i/p at ith port and o/p taken at jth port.

S_{ii} denotes how much of power is reflected back from the junction into the ith port when i/p power is applied at the ith port itself

Properties of Scattering matrix :

1. [S] is always a square matrix of order (n×n)

2. [S] is symmetric matrix i.e

$$[S] = [S]^T$$

$$\Rightarrow S_{ij} = S_{ji}$$

3. [S] is a unitary matrix i.e

$$[S][S]^* = [I]$$

Where $[S]^*$ is complex conjugate of [S]

[I] is unit matrix of the ~~square~~ same order as the [S]

4. The sum of the product of each term of any row/column multiplied by the complex conjugate of the corresponding terms of any other row (or column) is zero

i.e ; $\sum_{i=1}^{n} S_{ik} S_{ij}^* = 0$ for $k \neq j$

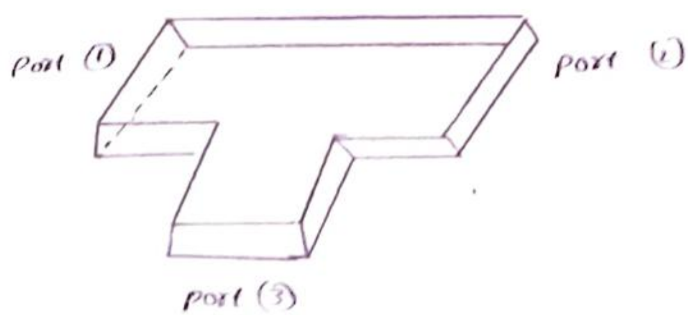
Microwave T- junctions :

A T- junction is an intersection of three waveguides in the form of english alphabet "T". There are several types of Tee junctions

1. H - plane Tee junction
2. E plane Tee junction
3. E - H plane Tee junction (Hybrid T junction)
4. magic Tee junction
5. Rat race junction

H - plane Tee junction:

* A - H plane Tee junction is formed by cutting rectangular slot along the width of a waveguide and attaching another waveguide, The side arm called the H-arm.



* All three arms of H-plane Tee lie in the plane of magnetic field. The magnetic field divides itself into the arms. so these is also called as current junction
* The properties of a H-plane tee can be completely

defined by its [S] matrix. The Order of scattering matrix is 3x3. since there are three possible i/p and 3 possible Outputs

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

* Now we determine the s-parameters by applying the properties of [S].

* Because of plane of symmetry of the junction

1. Scattering co-efficients S_{23} and S_{13} must be equal

$$S_{13} = S_{23}$$

2. from symmetric property $S_{ij} = S_{ji}$

$$S_{12} = S_{21} \longrightarrow (a)$$

$$S_{13} = S_{31} \longrightarrow (b)$$

$$S_{23} = S_{32} = S_{13} \longrightarrow (c)$$

3. We assume port ③ perfectly matched to the Junction

$$S_{33} = 0$$

With these properties [S] matrix becomes

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix}$$

i.e., We have four unknowns

(7)

4. from the Unitary property

$$[S][S]^* = [I]$$

i.e., $\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_1C_1 = S_{11}S_{11}^* + S_{12}S_{12}^* + S_{13}S_{13}^* = 1$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \longrightarrow (1)$$

$R_2C_2 = S_{12}S_{12}^* + S_{22}S_{22}^* + S_{13}S_{13}^* = 1$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \longrightarrow (2)$$

$R_3C_3 = S_{13}S_{13}^* + S_{13}^*S_{13} = 1$

$$|S_{13}|^2 + |S_{13}|^2 = 1 \longrightarrow (3)$$

R_3C_1 : $S_{13}S_{11}^* + S_{13}S_{12}^* = 0 \longrightarrow (4)$

from Equ (3)

$$2|S_{13}|^2 = 1$$

$$\Rightarrow S_{13} = \frac{1}{\sqrt{2}} \longrightarrow (5)$$

from Equ (1) and (2)

$$|S_{11}|^2 = |S_{22}|^2$$

$$S_{11} = S_{22} \longrightarrow (6)$$

from equ ④

$$S_{13}\ (S_{11}^* + S_{12}^*) = 0$$

$$S_{13} \neq 0 \Rightarrow S_{11}^* + S_{12}^* = 0$$

$$\Rightarrow S_{11}^* = -S_{12}^*$$

$$\text{or} \quad S_{11} = -S_{12} \longrightarrow ⑦$$

$$\text{or} \quad S_{12} = -S_{11}$$

from Equ ①

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$$

$$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1$$

$$2|S_{11}|^2 = \frac{1}{2}$$

$$S_{11} = \frac{1}{2}$$

$$S_{12} = -\frac{1}{2} \quad \text{and} \quad S_{22} = \frac{1}{2}$$

$$[S] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

We know that $[b] = [S][a]$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

i.e.,

$$b_1 = \frac{1}{2}a_1 - \frac{1}{2}a_2 + \frac{1}{\sqrt{2}} a_3 \longrightarrow (8)$$

$$b_2 = -\frac{1}{2} a_1 - \frac{1}{2} a_2 + \frac{1}{\sqrt{2}} a_3 \longrightarrow (9)$$

$$b_3 = \frac{1}{\sqrt{2}} a_1 + \frac{1}{\sqrt{2}} a_2 \longrightarrow (10)$$

Case (i):

Input is given at port (3) and no i/p at port (1) & (2)

$$\Rightarrow a_3 \neq 0 \quad , \quad a_1 = a_2 = 0$$

Substituting in above Equ 8, 9, & 10 we get

$$b_1 = \frac{a_3}{\sqrt{2}}$$

$$b_2 = \frac{a_3}{\sqrt{2}} \quad \text{and} \quad b_3 = 0$$

Let P_3 be the power i/p at port 3. Then this power divides equally between port (1) and port (2) in phase i.e., $P_1 = P_2$

$$\text{but} \quad P_3 = P_1 + P_2 = P_1 + P_1 = 2P_1 = 2P_2$$

The amount of power coming out of port (1) or port (2), due to i/p at port (3)

$$\Rightarrow 10\log_{10}\left(\frac{P_1}{P_3}\right) = 10\log_{10}\left(\frac{P_1}{2P_1}\right) = 10\log_{10} 1/2$$

$$\Rightarrow -10\log_{10}^{2} = -10\,(0.3010) = -3\text{dB}$$

Hence the power coming out of port ① or port ② is 3dB down with respect to input power at port ③. Hence H-plane tee is called as 3-dB "Splitter".

case(ii):

$$a_1 = a_2 = a \; , \; a_3 = 0$$

$$b_1 = \frac{a}{2} - \frac{a}{2} + \frac{a_3}{\sqrt{2}} = 0$$

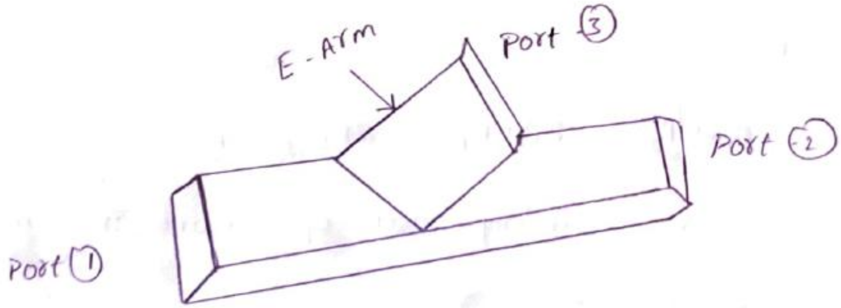
$$b_2 = -\frac{a}{2} + \frac{a}{2} + \frac{a_3}{\sqrt{2}} = 0$$

$$b_3 = \frac{a_1}{\sqrt{2}} + \frac{a_2}{\sqrt{2}} = \frac{a}{\sqrt{2}} + \frac{a}{\sqrt{2}} = \sqrt{2}a$$

i.e., the o/p at port ③ is addition of the two i/p's at port ① and port ② and these are added in phase

E-plane tee junction :-

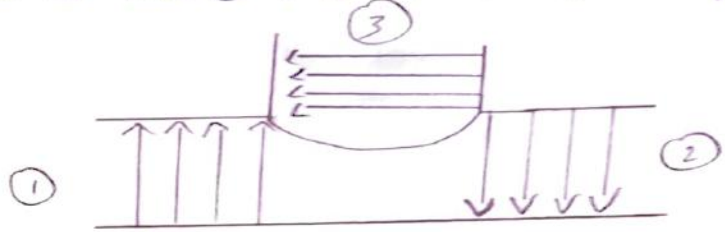
A rectangular slot is cut along the broader dimension of a main waveguide and a side arm is attached.



in the above figure port ① and ② are collinear arms and port - ③ is E-arm.

* When TE_{10} mode is made propagate into port ③, The

two Output ports ① and ② will have a phase shift of 180°. since electric field lines changes their direction when they connect at port ① and ②, it is called E-plane Tee



* The scattering matrix of an E-plane tee can be used to describe it's properties.
* In E-plane tee number of ports is ③ The order of scattering matrix is 3

1. [S] is a 3×3 matrix

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

2. The Scattering co-efficients

$$S_{32} = -S_{13}$$

Since o/p at port ① and ② are Out of phase by 180° with an i/p at ③

$$S_{23} = -S_{13}$$

3. If port ③ is perfectly matched

$$S_{33} = 0$$

4. from the symmetric property $S_{ij} = S_{ji}$

$$S_{12} = S_{21} \; ; \quad S_{13} = S_{31} \; ; \quad S_{23} = S_{32} = -S_{13}$$

With the above properties, [s] becomes

$$[s] = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{12} & s_{22} & -s_{13} \\ s_{13} & -s_{13} & 0 \end{bmatrix}$$

5. from Unitary property

$$[s][s]^* = [I]$$

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{12} & s_{22} & -s_{13} \\ s_{13} & -s_{13} & 0 \end{bmatrix} \begin{bmatrix} s_{11}^* & s_{12}^* & s_{13}^* \\ s_{12}^* & s_{22}^* & -s_{13}^* \\ s_{13}^* & -s_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1C_1 \Rightarrow |s_{11}|^2 + |s_{12}|^2 + |s_{13}|^2 = 1 \longrightarrow (1)$

$R_2C_2 \Rightarrow |s_{12}|^2 + |s_{22}|^2 + |s_{13}|^2 = 1 \longrightarrow (2)$

$R_3C_3 \Rightarrow |s_{13}|^2 + |s_{13}|^2 = 1 \longrightarrow (3)$

$R_3C_1 \Rightarrow s_{13}s_{11}^* - s_{13}s_{12}^* = 0 \longrightarrow (4)$

from equ (1) & (2)

$$s_{11} = s_{22} \longrightarrow (5)$$

from Equ (3)

$$s_{13} = \frac{1}{\sqrt{2}} \longrightarrow (6)$$

from equ (4)

$$s_{13}(s_{11}^* - s_{12}^*) = 0$$

$$s_{13} \neq 0$$

$$s_{11} = s_{12} = s_{22} \longrightarrow (7)$$

from equ ①

$$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1$$

$$S_{11} = \frac{1}{2}$$

$$[S] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$[b] = [S][a]$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$b_1 = \frac{1}{2} a_1 + \frac{1}{2} a_2 + \frac{1}{\sqrt{2}} a_3$$

$$b_2 = \frac{1}{2} a_1 + \frac{1}{2} a_2 - \frac{1}{\sqrt{2}} a_3$$

$$b_3 = \frac{1}{\sqrt{2}} a_1 - \frac{1}{\sqrt{2}} a_2$$

Case (i):

$$a_1 = a_2 = a, \quad a_3 = 0$$

Substituting in above equ

$$b_1 = \frac{a}{2} + \frac{a}{2} = a$$

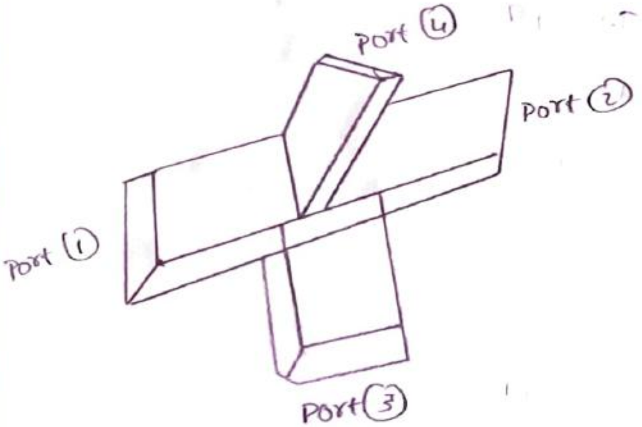
$$b_2 = \frac{a}{2} + \frac{a}{2} = a$$

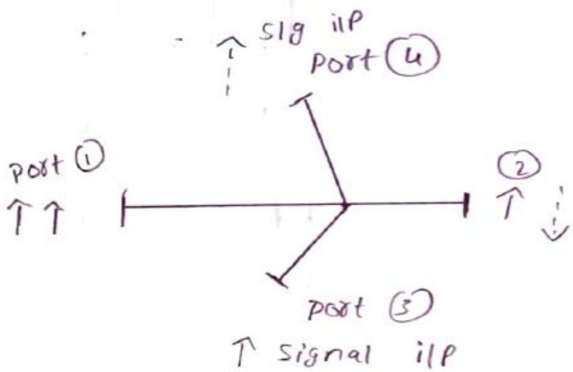
$$b_3 = 0$$

i.e equal i/p's are at port ① and ② results no o/p at port ③

E-H plane Tee Junction:

* E-H plane Tee junction also called as hybrid Tee junction and magic tee junction.
* In E-H plane Tee junction rectangular slots are cut along the both width and breadth of a main wave-guide and side arms are attached
* port ① and port ② are collinear ports, port ③ is the H-arm and port ④ is the E-arm





⇒ In E-H plane Tee junction we have four ports. so, we get 4×4 scatering matrix

1. $$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

2. Because of H-plane Tee section

$$S_{23} = S_{13}$$

3. Because of E-plane Tee Junction.

$$S_{24} = -S_{14}$$

4. port ③ and port ④ are perfectly matched i.e.,

$$S_{33} = S_{44} = 0$$

5. from symmetric property $S_{ij} = S_{ji}$

$$S_{12} = S_{21} \; ; \; S_{13} = S_{31} \; ; \; S_{23} = S_{32}$$

$$S_{34} = S_{43} \; ; \; S_{24} = S_{42} \; ; \; S_{41} = S_{14}$$

6. Because of geometry of the junction an input at port ③ cannot come out at port ④ since they are isolated ports and Vice Versa.

$$S_{34} = S_{43} = 0$$

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix}$$

7. from Unitary property $[S][S]^* = I$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1 C_1$: $|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \longrightarrow (1)$

$R_2 C_2$: $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \longrightarrow (2)$

$R_3 C_3$: $|S_{13}|^2 + |S_{13}|^2 = 1 \longrightarrow (3)$

$R_4 C_4$: $|S_{14}|^2 + |S_{14}|^2 = 1 \longrightarrow (4)$

from equ (3) and (4)

$$S_{13} = \frac{1}{\sqrt{2}} \longrightarrow (5)$$

$$S_{14} = \frac{1}{\sqrt{2}} \longrightarrow (6)$$

from equ (1) and (2)

$$S_{11} = S_{22} \longrightarrow (7)$$

$R_4 C_1 \Rightarrow S_{14} \cdot S_{11}^* - S_{14} S_{12}^* = 0$

$$S_{14} (S_{11}^* - S_{12}^*) = 0$$

$$S_{14} \neq 0$$

$$\Rightarrow S_{11}^* - S_{12}^* = 0$$

$$\Rightarrow S_{11}^* = S_{12}^*$$

$$\Rightarrow S_{11} = S_{12} \longrightarrow (8)$$

from equ (1)

$$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$2|S_{11}|^2 = 0$$

$$S_{11} = 0$$

$$S_{11} = S_{22} = S_{12} = S_{21} = 0$$

$S_{11} = S_{22} = 0$, means port ① and ② are also perfectly matched to the junction. Hence in a four port junction, if any two ports are perfectly matched to the junction, then the remaining two ports are automatically matched to the junction. Such a junction where all the four ports ~~meet~~ are perfectly matched to the junction is called magic tee.

Scattering matrix of magic tee is Obtained by substituting the scattering coefficients

$$[S] = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

$$[b] = [S][a]$$

$$\Rightarrow b_1 = \frac{a_3}{\sqrt{2}} + \frac{a_4}{\sqrt{2}}$$

$$\Rightarrow b_3 = \frac{a_1}{\sqrt{2}} + \frac{a_2}{\sqrt{2}}$$

$$\Rightarrow b_2 = \frac{a_3}{\sqrt{2}} - \frac{a_4}{\sqrt{2}}$$

$$\Rightarrow b_4 = \frac{a_1}{\sqrt{2}} - \frac{a_2}{\sqrt{2}}$$

Case (i):

$$a_3 \neq 0 \; ; \; a_1 = a_2 = a_4 = 0$$

$$b_1 = \frac{a_3}{\sqrt{2}} \quad ; \quad b_2 = \frac{a_3}{\sqrt{2}}$$

$$b_3 = 0 \quad ; \quad b_4 = 0$$

This is the property of H - plane tee

Case (ii) :

$$a_1 \neq 0 \quad ; \quad a_2 = a_3 = a_4 = 0$$

$$b_1 = 0 \; ; \; b_2 = 0 \; ; \; b_3 = \frac{a_1}{\sqrt{2}} \; , \; b_4 = \frac{a_1}{\sqrt{2}}$$

When the power is fed into port ①, nothing came out of port ② even through they are collinear ports. hence port ① and ② are called isolated ports.

Case (iii) :

$$a_4 \neq 0 \quad ; \quad a_1 = a_2 = a_3 = 0$$

$$b_1 = \frac{a_4}{\sqrt{2}} \quad ; \quad b_2 = -\frac{a_4}{\sqrt{2}}$$

$$b_3 = b_4 = 0$$

This is the property of E- plane tee.

Case (iv) :

$$a_3 = a_4 = a \; ; \qquad a_1 = a_2 = 0$$

$$b_1 = \frac{2a_3}{\sqrt{2}} \quad ; \quad b_2 = b_3 = b_4 = 0$$

* When we apply equal inputs at port ③ and result in an output at port ①

Case 5:

$$a_1 = a_2 \; ; \quad a_3 = a_4 = 0$$

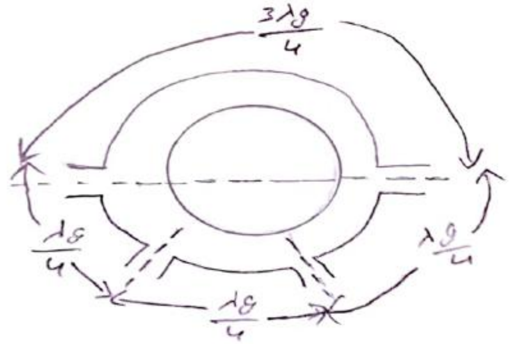
$$b_1 = b_2 = b_4 = 0 \; ; \quad b_3 = \frac{2a_1}{\sqrt{2}}$$

Applications of Magic tee :-

* Measurement of Impedance
* It is Used as a duplexer
* It is Used as a mixer

Rat race junction:

* Rat race junction also called as a hybrid ring



* Rat race junction is a four port junction
* The four ports / arms are connected in the form of an ring and these ports are separated by proper electrical lengths to sustain a standing waves.
* It is necessary that the mean circumference of the total race be "$1.5\lambda g$" and each of the four ports be separated from its neighbour by a distance of $\lambda g/4$
* When power is fed into at port ① it split equally in two direction i.e., in clock wise and Anti clock wise

direction. At port ② and ④ these powers combine in phase but at port ③ cancellation occurs due to $\lambda g/2$ phase diffe...

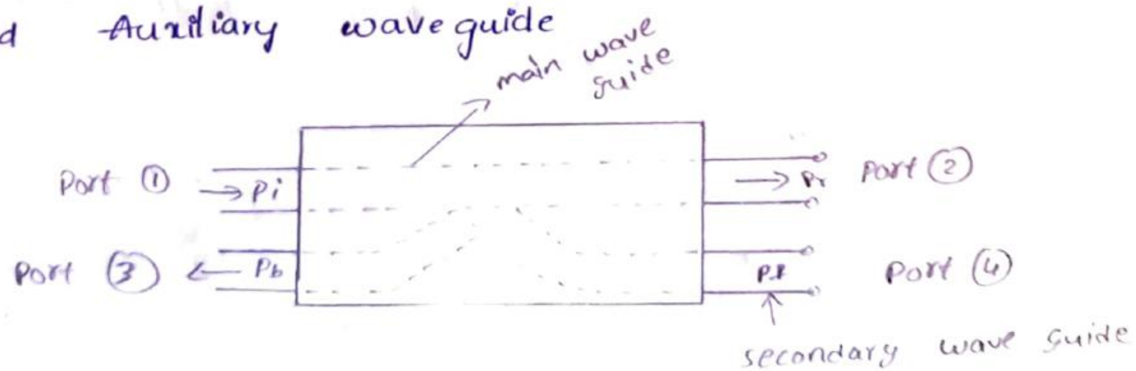
* Similar reasons, any i/p applied at port ② is equally divided between port ③ and ④ but no o/p at port ①
* Scattering matrix of rat race junction or hybrid ring can be written as

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

Directional Couplers:

Directional couplers are flanged, built in waveguide assemblies which can samples a small amount of microwave power for measurement purpose

* They can be designed to measure incident, reflected, SWR values.
* In it's most common form directional couplers are four port wave guide junctions consists of main waveguide and Auxiliary wave guide



phase differ

P_i = incident power at port ①

P_r = Received power at port ②

P_f = forward power / coupled power at port ④

P_b = back power at port ③

Properties of Ideal directional Coupler:

1. A portion of power travelling from port ① to port ② is coupled to port ④ but not to port ③
2. A portion of power travelling from port ② to port ① is coupled to port ③ but not to port ④
3. A portion of power incident on port ③ is coupled to port ④ but not to port ① and portion of power incident on port ④ is coupled to port ① but not port ②

* The performance of directional coupler is usually defined in terms of two parameters.

1. Coupling factors:

It is defined as the ratio of the incident power 'P_i' to the forwarded power 'P_f' measure in dB

$$C = 10\log_{10}\left(\frac{P_i}{P_f}\right) dB$$

Directivity:

The directivity of a D.C is defined as the ratio of forward power 'P_f' to the back power 'P_b' expressed in dB

$$\boxed{D = 10\log_{10}\left(\frac{P_f}{P_b}\right) dB}$$

Typical D.C, the values of $C = 20\,dB$
$D = 60\,dB$

$$C = 20 = 10\log_{10}\left(\frac{P_i}{P_f}\right)$$

$$\frac{P_i}{P_f} = 10^2 = 100 \longrightarrow (1)$$

$$D = 60 = 10\log_{10}\left(\frac{P_f}{P_b}\right)$$

$$\frac{P_f}{P_b} = 10^6$$

$$P_b = \frac{P_f}{10^6} = \frac{P_i}{10^8}$$

$$\boxed{P_b = \frac{P_i}{10^8}}$$

i.e., P_b is very small value, $\frac{1}{10^8}(P_i)$ power coming out of port (3)

Isolation:

It is defined as the ratio of the incident power P_i to the back power P_b expressed in dB

$$I = 10\log_{10}\left(\frac{P_i}{P_b}\right) dB$$

$$I = 10\log_{10}\left(\frac{P_f}{P_f}\cdot\frac{P_i}{P_b}\right)$$

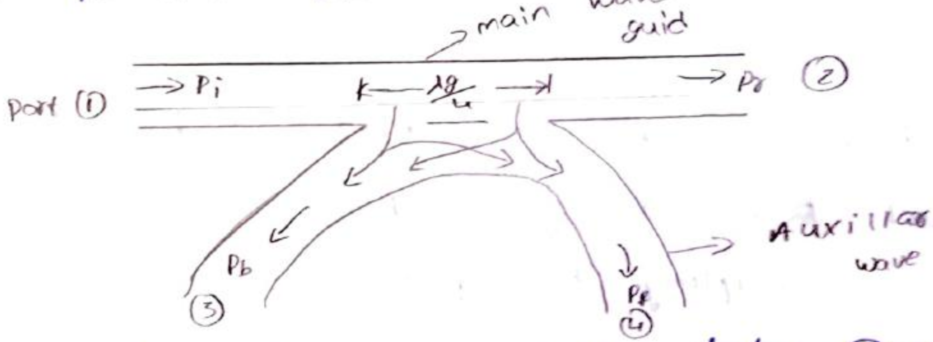
$$= 10\log_{10}\left(\frac{P_f}{P_b}\cdot\frac{P_i}{P_f}\right)$$

$$= 10\log_{10}\left(\frac{P_f}{P_b}\right) + 10\log_{10}\left(\frac{P_i}{P_f}\right)$$

$$\boxed{I = D + C}$$

Two-hole Directional Coupler :-

* It Consists of two waveguides One main Waveguide and Second One is Auxilary Waveguide With two tiny holes common b/w them. The two holes are at a distance of $\frac{\lambda g}{4}$



* The two large leackages at the holes (1) and (2) both are in phase at the position of 2nd hole and hence they are add up contributing to P_f. But the two leakages are Out of phase by 180° at position of the 1st hole and therefore they are cancel each other by making $P_b = 0$.
* The magnitude of the power coming out of 2 holes depends upon the diamension of the two holes. since the distance b/w holes is $\lambda g/4$, P_b is made zero.

Scattering matrix of a directional Coupler :

1. DC is a four port n/w. Hence [S] is a 4×4 matrix

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

2. In a directional coupler, all four ports are perfectly matched to the junction.

i.e., $S_{11} = S_{22} = S_{33} = S_{44} = 0$

3. from Symmetric property

$$S_{ij} = S_{ji}$$

$S_{21} = S_{12}$; $S_{23} = S_{32}$; $S_{13} = S_{31}$; $S_{24} = S_{42}$; $S_{34} = S_{43}$; $S_{41} = S_{14}$.

4. Ideally back power is zero ($P_b = 0$) i.e.,

There is no coupling b/w port ① & ③

$$S_{13} = S_{31} = 0$$

Similarly

$$S_{24} = S_{42} = 0$$

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}$$

5. $[S][S]^* = [I]$

$R_1C_1 \Rightarrow |S_{12}|^2 + |S_{14}|^2 = 1 \longrightarrow (1)$

$R_2C_2 \Rightarrow |S_{12}|^2 + |S_{23}|^2 = 1 \longrightarrow (2)$

$R_3C_3 \Rightarrow |S_{23}|^2 + |S_{34}|^2 = 1 \longrightarrow (3)$

$R_4C_4 \Rightarrow |S_{14}|^2 + |S_{34}|^2 = 1 \longrightarrow (4)$

$R_1C_3 \Rightarrow S_{12}\, S_{23}^* + S_{14}\, S_{34}^* = 0 \longrightarrow (5)$

from

* from equ ① and ②

$$S_{14} = S_{23} \longrightarrow ⑥$$

* from equ ② and ③

$$S_{12} = S_{34} \longrightarrow ⑦$$

Let's assume S_{12} is real and positive $= P$

$$S_{12} = S_{34} = P = S_{34}^{*} \longrightarrow ⑧$$

Equ ⑤

$$S_{12} S_{23}^{*} + S_{14} \cdot S_{34}^{*} = 0$$

$$P\, S_{23}^{*} + S_{14} P = 0$$

$$P\,(S_{23}^{*} + S_{23}) = 0 \qquad (S_{14} = S_{23})$$

$$P \neq 0$$

$$S_{23}^{*} + S_{23} = 0 \longrightarrow ⑨$$

To satisfy above equ. S_{23} must be imaginary.

Let

$$S_{23} = jq$$

$$S_{23}^{*} = -jq$$

$$S_{12} = S_{21} = S_{34} = S_{43} = P$$

$$S_{14} = S_{41} = S_{23} = S_{32} = jq$$

$$[S] = \begin{bmatrix} 0 & P & 0 & jq \\ P & 0 & jq & 0 \\ 0 & jq & 0 & P \\ jq & 0 & P & 0 \end{bmatrix}$$

Q A Signal of power 32 mw is fed into one of the Collinear ports of a lossless H-plane tee. Determine the powers in the remaining ports when Other ports are terminated by means of matched load.

Sol Let the collinear port be port ① to which the signal of 32mw is fed, then the Other ports are terminated with matched loads

$$a_2 = a_3 = 0 \quad \text{and} \quad a_1 = 32\,mw$$

W.K.T S-matrix for H-plane tee

$$[S] = \begin{bmatrix} 1/2 & -1/2 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$[b] = [S][a]$$

$$b_1 = \frac{a_1}{2} - \frac{a_2}{2} + \frac{a_3}{\sqrt{2}} = \frac{a_1}{2} = 16\,mw$$

$$b_2 = 16\,mw$$

$$b_3 = \frac{32\,mw}{\sqrt{2}}$$

Q A 90w power source is connected to the i/p of a DC with C=0 and D=35dB. Then find P_f & P_b

$$C = 20 = 10\log_{10}\left(\frac{P_i}{P_f}\right) \qquad \because 10^2 = \frac{P_i}{P_f}$$

$$P_f = \frac{P_i}{100} = 9W \; ; \; D = 35 = 10\log\left(\frac{P_f}{P_b}\right) \; ; \; \frac{P_f}{P_b} = 10^{3.5}$$

$$P_b = \frac{P_f}{10^{3.5}} = 284.6\,\mu w$$

Ferrite Devices:

* ferrites are non-metallic materials with resistivity (ρ) nearly 10^4 times greater than the metals and with dielectric constant (ϵ_r) around 10-15 and relative permiabilities of order of 1000.
* ferrites are Oxide based compounds having general composition of the form $MeO \cdot Fe_2O_3$ i.e mixer of metal Oxide & ferric Oxide
* ferrites have atoms with large number of spinning electrons resulting in strong magnetic properties. Because of this properties, ferrites find applications in a number of microwave devices to reduce reflected power for modulation purposes and in switching circuit
* ferrites have One more property which is useful at microwave frequency i.e., non-reciprocal property

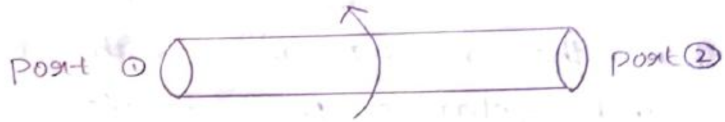
Non-reciprocal property:

When two circularly polarised waves are rotating clockwise and other one anticlockwise are made to propagate through ferrite, the ferrite material react differently to the two rotating fields, there by presenting different effective permeabilities to the both waves and ϵr_2, μr_2, ρ_2 for right circularly polarised wave

faraday rotation:-

faraday demonstrated that the plane of polarization of linearly polarized wave is rotated with distance when it passes through ferrite materials along the direction of applied magnetic field.

A two port ferrite device is shown in figure



When a wave is transmitted from port ① to port ② it undergoes rotation in the anticlockwise direction. Even if the same signal is allowed to propagate from port ② to port ①, it will undergoes rotation in the same direction

<u>Microwave devices which make use of faraday rotation:</u>

We discuss three important device which make use of faraday rotation.

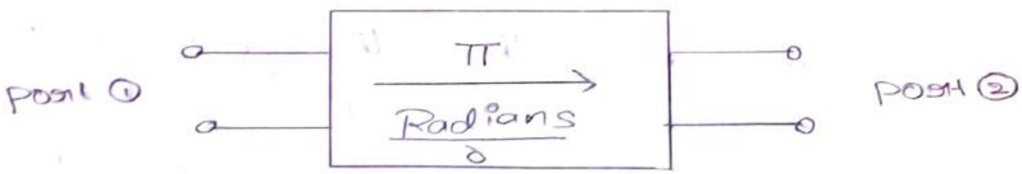
a) Gyrator

b) Isolator

c) circulator

<u>Gyrator:</u>

It is a two port device that has a relative phase difference of 180° for transmission from port ① and ② and no phase for transmission from port ② to port ①



The construction of Gyrator consists of a piece of circular waveguide carrying the dominant TE_1 mode with transition to standard rectangular waveguide with dominant mode (TE_{10}) at both ends.

* A thin circular ferrite rod tapered at both ends is located inside the circular waveguide and the waveguide is surrounded by a permanent magnet which generates dc magnetic field for proper operation of ferrite.
* To the i/p end a 90° twisted rectangular waveguide is connected.

Operation:

* When a wave enters into port ① its plane of polarization rotates by 90° because of twist in the waveguide. It again undergoes faraday rotation through 90°, because of ferrite rod. and the wave which is comes out of port ② will have a phase shift of 180° compared to the wave entering port ①
* But when the same wave enters at port ②, it undergoes faraday rotation through 90° in the same anticlockwise direction. Because of the twist, this wave gets rotated back by 90° comes out of port ① with 0° phase shift.

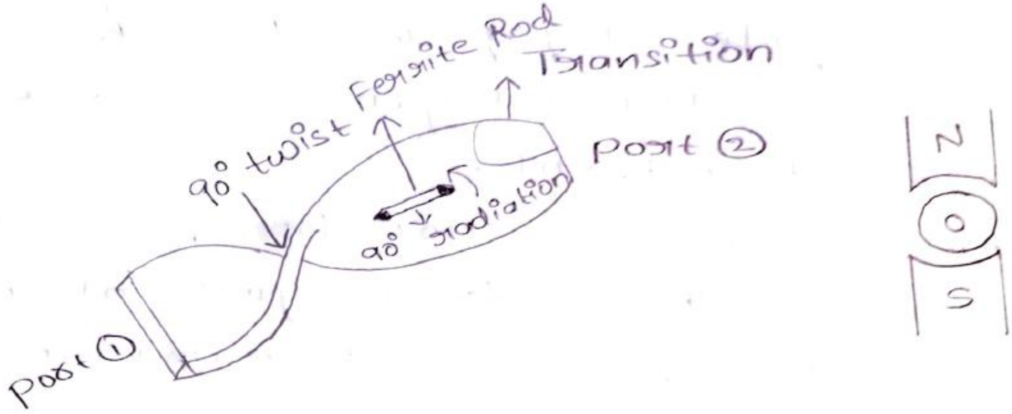
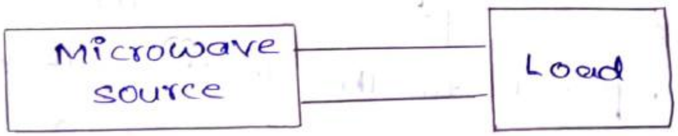


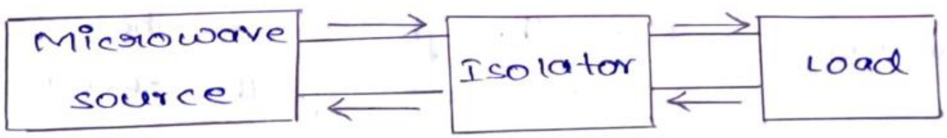
fig:- Gyrator

Isolator:

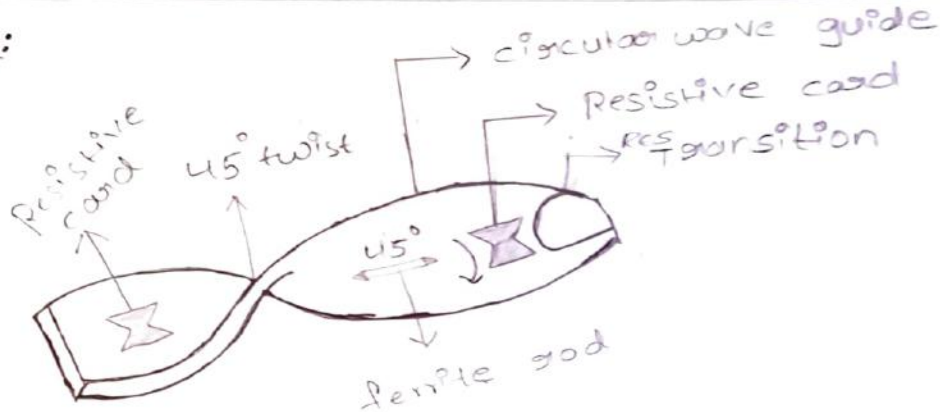
An isolator is a 2-port device which provides very small attenuation for transmission from port ① to port ② but provides maximum attenuation for transmission from port ② to port ①

Port ①





Construction:



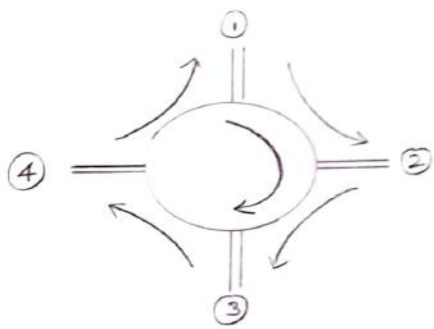
* Isolator consists of a pieces of circular wave Guide carrying dominant TE_{11} mode with transition to the standard rectangular waveguide with dominant mode (TE_{10}) at both ends.
* A thin ferrite rod tapered at both ends is located inside the circular Waveguide.
* The waveguide is surrounded by a permanent magnet which generates dc magnetic field for proper Operation of ferrite.
* To the input end a 45° twisted rectangular waveguide is connected.
* A resistive card is placed along the larger dimension of the rectangular waveguide, so as to absorb any wave whose plane of polarization is parallel to the plane of resistive card. The resistive card does not absorb any wave whose plane of polarization is perpendicular to its own plane.

Operation:

* A wave passing from port ① through the resistive card and it is not attenuated. After coming out of the card, the wave gets shifted by 45° because of twist in anticlockwise direction and then by another is in clockwise direction because of ferrite rod and hence comes out of port ② with the same polarization as at the port ① with out any attenuation.
* But a TE_{10} wave feed from port ② gets a pass from the resistive card place near to the port ② since plane of polarization of the wave is ⊥'r to the plane of the resistive card. Then the wave gets rotated by 45° in clockwise direction due to the faraday rotation, and furthur gets rotated by 45° in clock wise direction due to the twist in waveguide
* Now plane of polarization of the wave will be parallel with that resistive card and hence the wave will be completely absorbed by the resistive card and the o/p at port ① will be zero. This power is dissipated in the form of heat.

Circulator:

A Circulator is a four port microwave device which has a peculiar property that each terminal is connected to the next clockwise terminal only i.e., port ① is connected

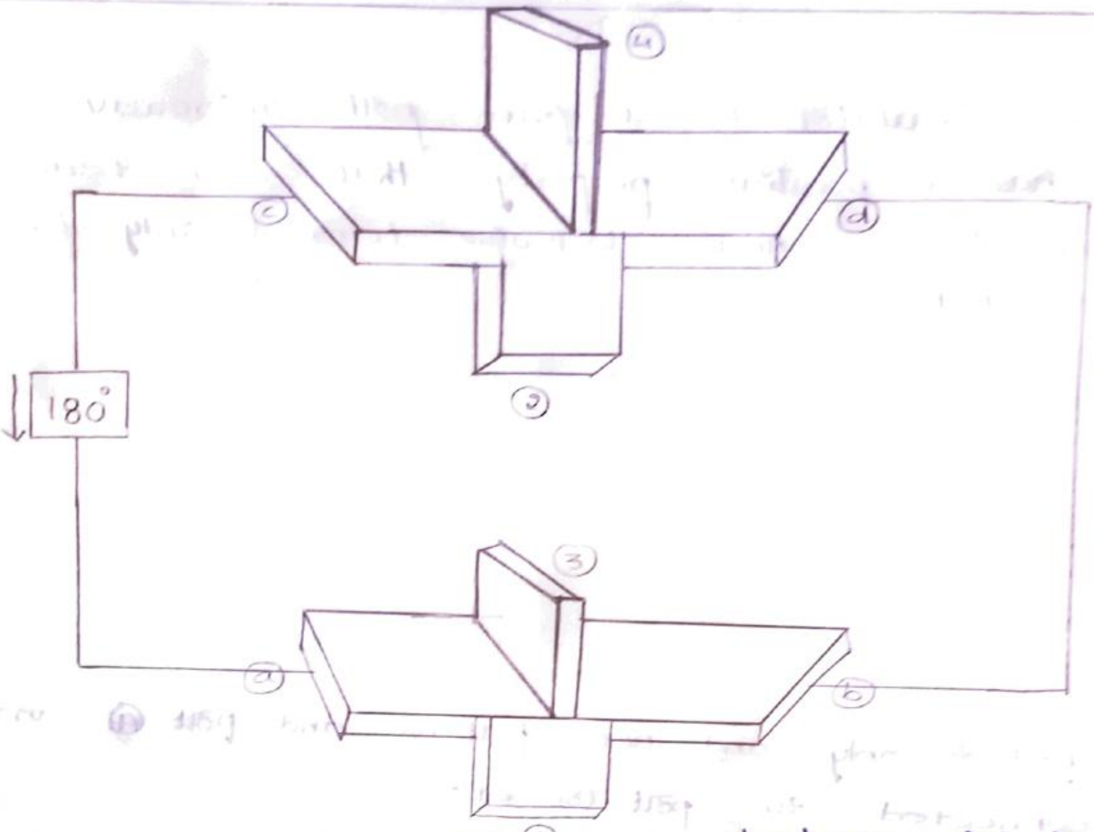


to port ② only and not port ③ and port ④ and port ② is connected to port ③ etc

* There is no restriction on the number of ports, four ports most commonly used.
* They are useful in parametric amplifiers, tunnel diode, amplifiers and duplexers.

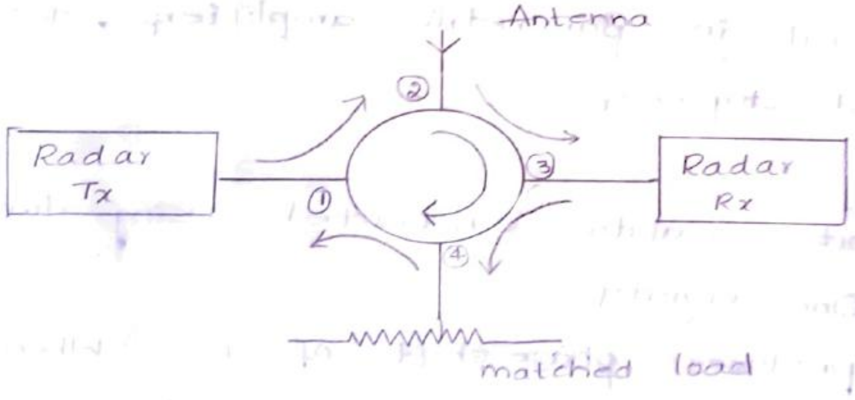
construction:

* The four port circulator constructed using two magic tees and One Gyrator
* The Gyrator produces phase shift of 180° when wave traveled from port ① to port ② and there is no phase shift when wave travells from port ② to port ①.



Applications:

1. A circulator can be used as a duplexer for a radar antenna system.



2. Two three port circulators can be used in tunnel diode or parametric Amplifier.

Unit- III ①

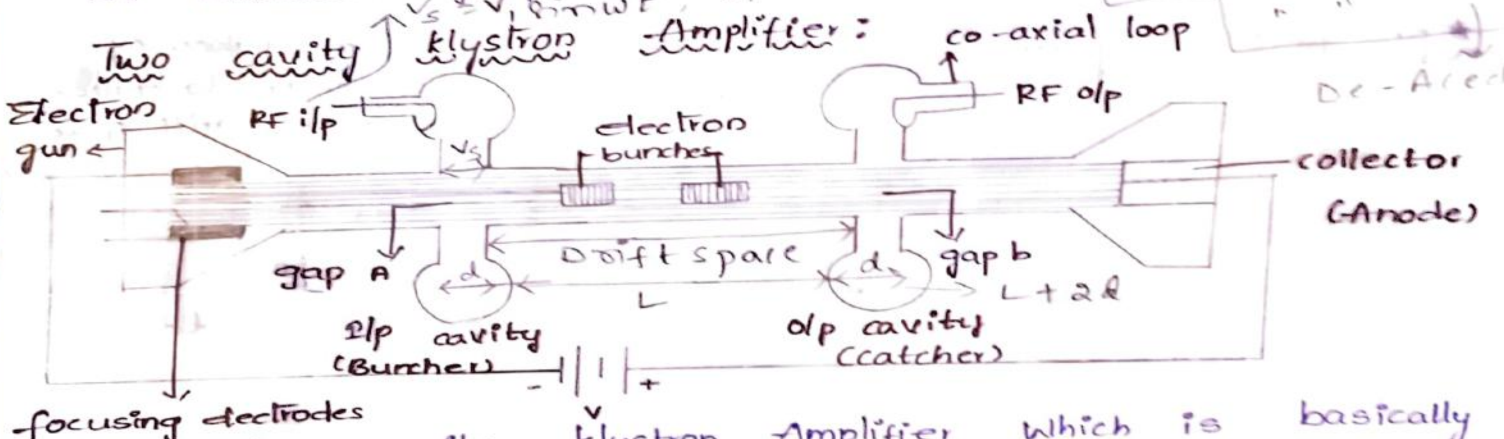
Classification of Microwave tubes:

(1) O-type tubes (original (or) linear type tubes)
(2) M-type tubes

O-type tubes:

Klystron: A klystron is a vaccum tube that can be used either as a generator (or) as a amplifier of power at microwave frequencies

Two cavity klystron Amplifier:

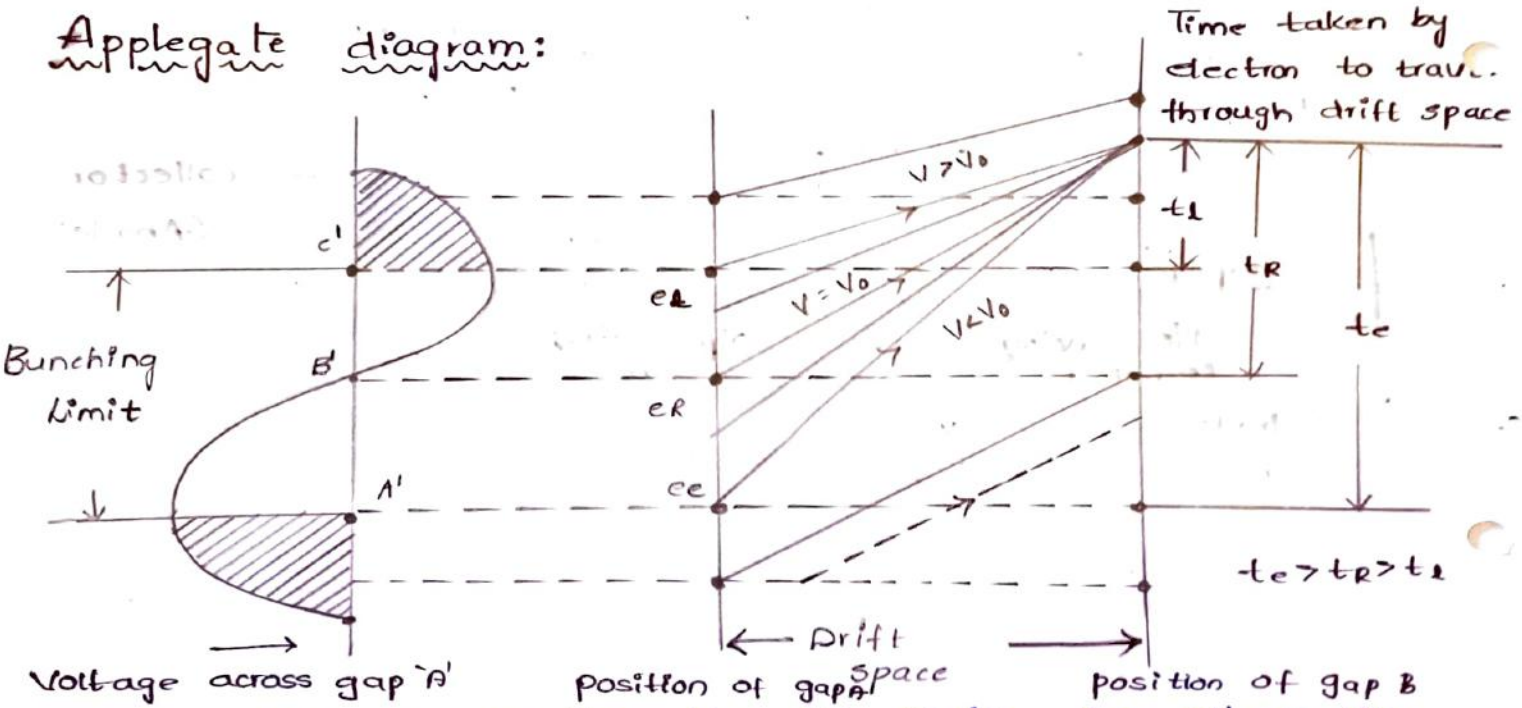


* A two cavity klystron Amplifier which is basically a "velocity modulated tube".
* Here a high velocity electron beam is passed focused and set down along a glass tube through an input cavity (buncher), a field free drift space and an o/p cavity (catcher) to a collector electrode (Anode).
* The Anode is kept at a positive potential with respect to cathode.

* The electron beam passes through a gap "A" consisting of two grids of the buncher cavity separated by a very small distance and two other grids of the catcher cavity with a small gap "B".

* The i/p & o/p are taken from the tube via "resonant cavities" with the aid of coupling loops

Applegate diagram:



* At point B on the i/p RF cycle, the alternating voltage is zero and going positive.

* At this instant, the electric field across gap "A" is zero and an electron which passes through gap "A" at this instant is uneffected by RF signal.

* Let this electron be called the Reference electron "e_R" which travels with an unchanged velocity $V_0 = \sqrt{\frac{2eV}{m}}$ where V = Anode to cathode voltage

$eV_0 = \underbrace{\frac{1}{2} m V_0^2}_{K.E}$

* At point "c" of the input RF cycle an electron which leaves gap A later than reference electron e_R called the "late electron "e_L" " is subjected to maximum positive RF voltage and hence travels towars gap 'B' with an increased velocity *[and hence travels towards gap 'B' with an increased]* $(V > V_0)$ and this electron tries to overtake the reference electron
* Similarly an "early electron (e_e)" that passes the gap 'A' slightly before the reference election 'e_R' is subjected to a maximum Negative field. Hence this early electron is declarated and travels with a reduced velocity V_0 $(V < V_0)$. This electron 'e_e' falls back and reference electron 'e_R' catch up with the early electron 'e_e'
* Therefore the velocity of the electron varies in accordance with RF input voltage, resulting in "velocity modulation" of the election beam
* As a result of these actions, the electrons in the bunching limit (b/w points 'A' & 'C') gradually

bunch together as they travel down the drift space, from gap 'A' to gap 'B'.

* The pulsing stream of electrons passes through gap 'B' and excite Osillations in the o/p cavity.
* The density of electrons passing the gap B vary cylindrically with time, that is the electron beam contains an "ac" current and is "current modulated".
* The "drift space" converts the velocity modulation into current modulation
* Bunching occurs only once per cycle centered around the reference electron 'e_R'.
* With proper design (optimum gap width's, Anode to cathode voltage, drift space, length .... etc) a little RF power applied to the buncher cavity result in large beam currents at the catcher cavity with a considerable power gain

Performance characteristics:

1) frequency : 250 MHz to 100 GHz (6 GHz Nominal)
2) power : 10 kW - 500 kW (30 MW pulsed)

3) power gain : 15dB - 70dB (60dB Nominal)

4) Band width : limited (10-60 MHz)

5) Noise figure : 15 - 20 dB

6) Theoritical efficiency : 58% (30-40% Nominal)

Applications:

(i) As power o/p tubes

- In UHF TV transmitters
- In troposphere scatter transmitters
- Satellite communication ground stations
- Radar transmitters

(ii) As power Oscillators (5-50 GHz) if used as a klystron Oscillator.

Mathematical Analysis of a klystron Amplifier:

⇒ Let dc Voltage b/w cathode and Anode $= V_0$

Velocity of the electron $= v_0$

Drift space length $= L$

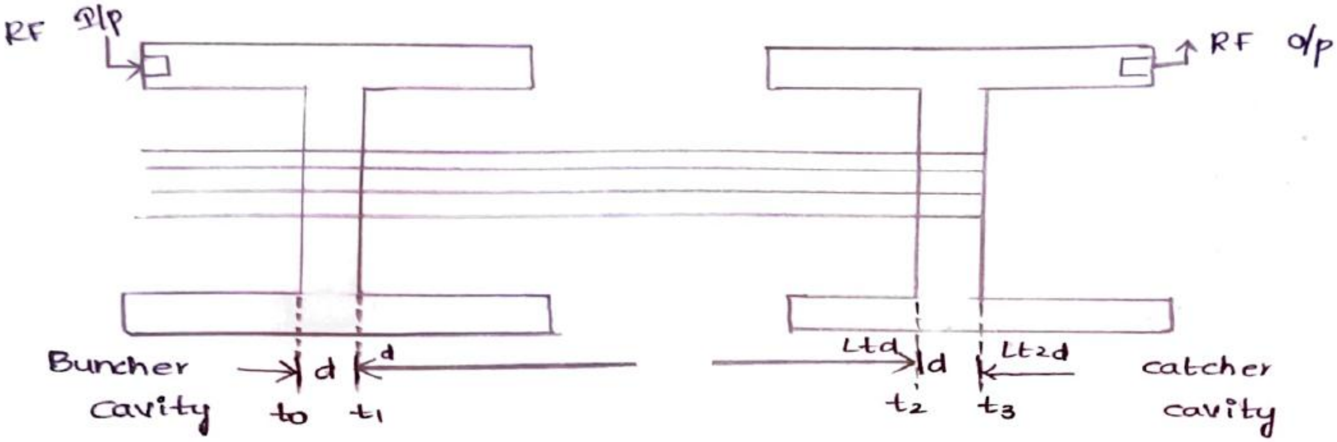
RF i/p signal to be amplified by klystron $= V_s$

Then

$\acute{e}$ velocity $$v_0 = \sqrt{\frac{2e\overset{\text{voltage}}{V_0}}{m}} = 0.593 \times 10^6 \sqrt{V_0} \ (m/sec) \longrightarrow (1)$$

and $$V_s = V_1 \sin \omega t \longrightarrow (2)$$

hence $V_1 =$ Amplitude of the signal & $V_1 << V_0$ is assumed.



* The energy of the electron at the time of leaving bunches cavity is given by

$$\frac{1}{2} m v_1^2 = e(V_0 + V_1 \sin\omega t_1)$$

$$v_1 = \sqrt{\frac{2e(V_0 + V_1 \sin\omega t_1)}{m}}$$

$$v_1 = \sqrt{\frac{2eV_0}{m}} \sqrt{1 + \frac{V_1}{V_0} \sin\omega t_1}$$

$$v_1 = V_0 \left(1 + \frac{V_1}{V_0} \sin\omega t_1\right)^{1/2} \longrightarrow (3)$$

⇒ Expanding binomially and neglecting higher powers of $\sin\omega t$, We get

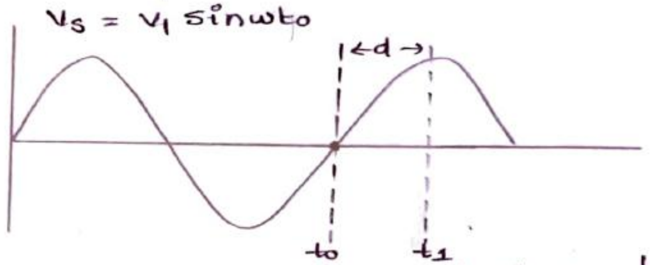
$$\boxed{V_1 = V_0 \left(1 + \frac{V_1}{2V_0} \sin\omega t_1\right)} \longrightarrow (4)$$

Eq (4) is the Equation for "Velocity modulation".

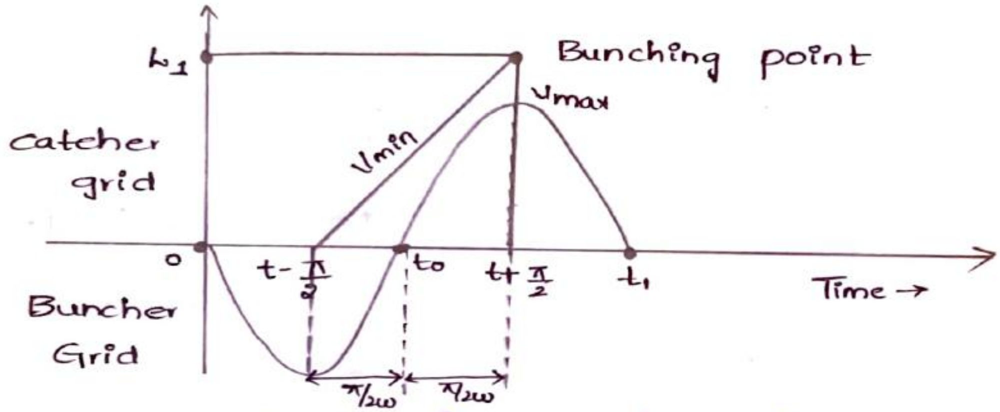
Here $\omega t_1 = \omega t_0 + \frac{\theta_g}{2}$

Here θ_g = phase angle of the RF i/p voltage during which the electron is accelerated.

So $\theta_g = \omega t = \omega(t_1 - t_0) = \frac{\omega d}{V_0} \longrightarrow (5)$



Bunching process of the electron beam:



* Maximum velocity occurs at $+\frac{\pi}{2}$

So $V_1(max) = V_0\left(1 + \frac{V_1}{2V_0}\right) \longrightarrow (6)$

* Minimum velocity occurs at $-\frac{\pi}{2}$

So $V_1(min) = V_0\left(1 - \frac{V_1}{2V_0}\right) \longrightarrow (7)$

* If the distance in the drift space (L) at which the bunching occurs from the buncher grid at time 't_1' is 'L_1'

So $L_1 = V_0(t_1 - t_0) \longrightarrow (8)$

* The distance L_1 at $t+\frac{\pi}{2} = V_{max}(t_1 - t - \frac{\pi}{2}) \rightarrow (9)$

* The distance L_1 at $t-\frac{\pi}{2} = V_{min}(t_1 - t + \frac{\pi}{2}) \rightarrow (10)$

Here
$$\left.\begin{aligned} t-\pi/2 &= t_0 - \frac{\pi}{2\omega} \\ t+\pi/2 &= t_0 + \frac{\pi}{2\omega} \end{aligned}\right\} \longrightarrow (11)$$

So equ (9) becomes L_1 at $t_{+\pi/2} = V_0\left(1+\frac{V_1}{2V_0}\right)\left(t_1 - t_0 - \frac{\pi}{2\omega}\right)$

$$L_1 = V_0(t_1 - t_0) + V_0\left[-\frac{\pi}{2\omega} + \frac{V_1}{2V_0}(t_1 - t_0) - \frac{V_1}{2V_0}\left(\frac{\pi}{2\omega}\right)\right] \longrightarrow (12)$$

Equ (10) becomes

$$L_1 \text{ at } t_{-\frac{\pi}{2}} = V_0\left(1 - \frac{V_1}{2V_0}\right)\left(t_1 - t_0 + \frac{\pi}{2\omega}\right)$$

$$L_1 = V_0(t_1 - t_0) + V_0\left[+\frac{\pi}{2\omega} - \frac{V_1}{2V_0}(t_1 - t_0) - \frac{V_1}{2V_0}\left(\frac{\pi}{2\omega}\right)\right] \longrightarrow (13)$$

If the distance has to be same for the $-\frac{\pi}{2}, 0, \frac{\pi}{2}$ bunches. 'L_1' for all three should be equal to $V_0(t_1 - t_0)$

$$\therefore L_1 = V_0(t_1 - t_0)$$

Sub in equ (13)

$$\frac{\pi}{2\omega} - \frac{V_1}{2V_0}(t_1 - t_0) - \frac{V_1}{2V_0}\left(\frac{\pi}{2\omega}\right) = 0$$

$$-\frac{V_1}{2V_0}(t_1 - t_0) = -\frac{\pi}{2\omega} + \frac{V_1}{2V_0}\left(\frac{\pi}{2\omega}\right)$$

$$-(t_1 - t_0) = \left[-\frac{\pi}{2\omega} + \frac{V_1}{2V_0}\left(\frac{\pi}{2\omega}\right)\right]\frac{2V_0}{V_1}$$

$$-(t_1 - t_0) = -\frac{\pi V_0}{\omega V_1} + \frac{\pi}{2\omega}$$

As $\frac{V_0}{V_1}$ is very high, $\frac{\pi}{2\omega}$ can be neglected

$$-(t_1 - t_0) = -\frac{\pi V_0}{\omega V_1}$$

$$\text{So} \quad t_1 - t_0 \cong \frac{\pi V_0}{\omega V_1}$$

$$\therefore L_1 = V_0 (t_1 - t_0) = V_0 \left(\frac{\pi V_0}{\omega V_1}\right)$$

⇒ Bunching occurs as the RF signal changes from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ i.e., π for a value of $\pi = 3.682$

optimum bunching occurs and

$$L_{max} = 3.682 \, \frac{V_0 V_0}{\omega V_1} \longrightarrow (14)$$

⇒ If beam coupling coefficient of i/p cavity is "β" and is given by

$$\beta = \frac{\sin\left(\frac{\theta_g}{2}\right)}{\theta_g/2}$$

here $\theta_g = \frac{\omega d}{V_0}$ (Average gap transit angle) from equ (5)

Then L_{max} is given by

$$\boxed{L_{max} = 3.682 \, \frac{V_0 V_0}{\omega \beta V_1}} \longrightarrow (15)$$

Output power (P_{out}):

⇒ At the catcher cavity RF voltage $= V_2 \sin \omega t_2$

⇒ Energy given by the electron to the bunch

$$= (-e) V_2 \sin \omega t_2$$

$$= -eV_2 \sin \omega t_2$$

⇒ The Average energy given to the RF field in a cycle

$$P_{avg} = \frac{1}{2\pi} \int_{\omega t_1 = 0}^{\omega t_2 = 2\pi} (-eV_2 \sin \omega t_2) \, d\omega t_1 \longrightarrow (16)$$

⇒ In the field free space b/w cavities, the transit time for velocity modulated electron is given by

$$\tau = t_2 - t_1 = \frac{L}{v_1} = \frac{L}{v_0 \left[1 + \frac{V_1}{2V_0} \sin \omega t_1\right]} \quad \text{from equ (4)}$$

$$\tau = \frac{L}{v_0} \left[1 - \frac{V_1}{2V_0} \sin \omega t_1\right]$$

Multiplying by 'ω' on both sides

$$\omega\tau = \omega(t_2 - t_1) = \frac{\omega L}{v_0} \left[1 - \frac{V_1}{2V_0} \sin \omega t_1\right] \longrightarrow (17)$$

In this Equ $\frac{L}{v_0} = \tau_0$, the transit time without RF voltage 'V_1' is buncher cavity $\frac{\omega L}{v_0} = \omega\tau_0 = \theta_0 = 2\pi N$, is the transit angle without RF voltage V_1 in buncher cavity.

Here N = No of electron transit cycle in drift space

⇒ The Bunching parameter 'x' of a klystron is defined by $X = \frac{V_1}{2V_0}\theta_0 \longrightarrow (18)$

Which is a dimensionless quantity and propotional to i/p P_0

⇒ Eq (16) can be rewritten by Using Eq (17)

(i.e., $\omega\tau = \omega t_2 - \omega t_1$)

$$P_{avg} = \frac{-eV_2}{2\pi}\int_0^{2\pi} \sin(\omega t_1 + \omega\tau)\cdot d\omega t_1$$

Sub Equ (17)

$$P_{avg} = -\frac{eV_2}{2\pi}\int_0^{2\pi} \sin\left[\omega t_1 + \theta_0\left(1 - \frac{V_1}{2V_0}\sin\omega t_1\right)\right] d\omega t_1 \longrightarrow (19)$$

This is a "Bessel function" and its solution is given by

$$P_{avg} = -eV_2\, J_1(X)\cdot\sin\theta_0 \longrightarrow (20)$$

Here $J_1(X)$ = Bessel function of the first Order for the argument $X = \frac{V_1}{2V_0}\theta_0$, from equ (18)

⇒ for 'N' electron transit cycles

Energy transfered $= P_{avg} = -N eV_2 J_1(X)\sin\theta_0$

Here $N_e = I_0$; Output Current

So Energy transferred $P_{avg} = -I_0 V_2 J_1(X) \sin\theta_0$

→ Max value of $J_1(X) = 0.58$ for $X = 1.84$

(from Bessel function tables)

→ for Max energy transfer

$$P_{avg} = -I_0 V_2 (0.58) \sin\theta_0$$

Here $\sin\theta_0 = -1$; $\theta_0 = 2n\pi - \frac{\pi}{2}$

$$P_{avg} = +I_0 V_2 (0.58)$$

∴ The Output power

$$\boxed{P_{out} = P_{max} = (0.58) I_0 \cdot V_2} \longrightarrow (21)$$

Input power (P_{in}):

→ The Input power is basically the dc i/p given by $\boxed{P_{in} = I_0 \cdot V_0} \longrightarrow (22)$

Efficiency: (η):

$$\text{Efficiency } (\eta) = \frac{P_{out}}{P_{in}} = \frac{0.58 I_0 \cdot V_2}{I_0 \cdot V_0} = 0.58 \frac{V_2}{V_0}$$

$$\eta = 0.58 \frac{V_2}{V_0} \longrightarrow (23)$$

→ As V_2 is always lessthan V_0, The max efficiency that can be attained is "0.58 (or) 58 %."

Reflex klystron:

⇒ The reflex klystron is a single cavity Variable frequency microwave generator of low power and low efficiency.

Applications:

1) In Radar receivers.
2) Local Oscillator in Microwave receivers.
3) signal source in Microwave generator of Variable frequency.
4) portable Microwave links
5) pump Oscillator in parametric Amplifier.

structure:

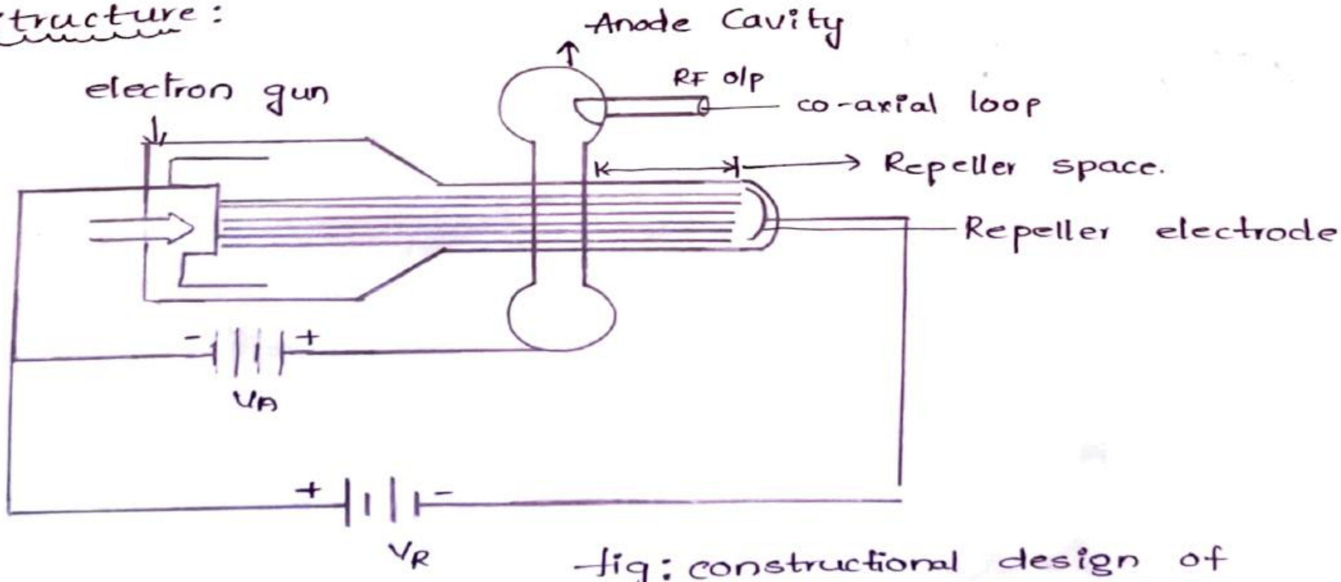


fig: constructional design of reflex klystron

⇒ It consists of an electron gun, a filament surrounded by cathode and a focusing electrode at cathode potential.

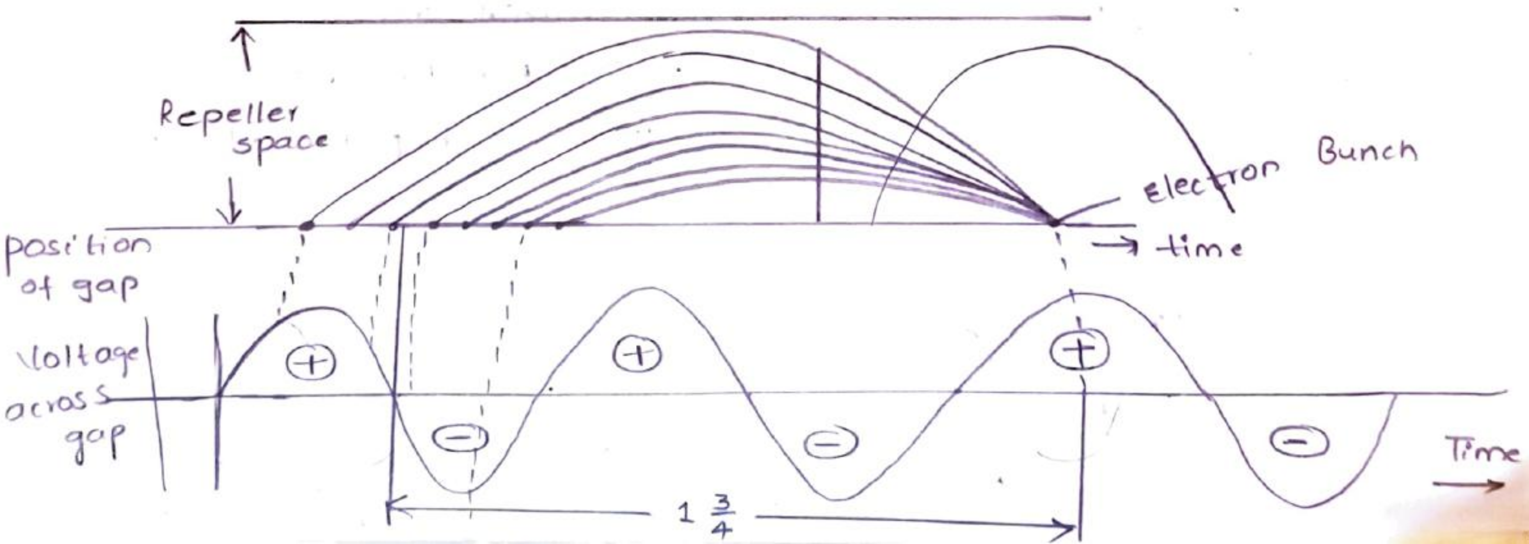
⇒ The electron beam is accelerated towards the anode cavity.

⇒ After passing the gap in the cavity electrode travels towards a repeller electrode which is at a high negative potential (V_R).

⇒ The electrons never reach the repeller because of the negative field and returned back towards the gap

⇒ Under sustainable conditions, the electrons give more energy to the gap than they took from the gap on their forward journer and oscillations are sustained.

Applegate diagram:



→ It is assumed that the Oscillations are setup in the tube initially due to noise (or) switching transients and these Oscillations are sustained by device Operation. This can be explained by using applegate diagram.

→ The RF Voltage that is produced across the gap by the cavity Oscillations act on the electron beam to cause "Velocity modulation".

→ 'e_R' is a reference electron that passes through the gap when the gap voltage is '0' and going -Ve. 'e_R' is uneffected by the gap Voltage. This Moves towards repeller electrode and gets reflected by the Negative Voltage on the repeller. It returns and passes through the gap for a second time.

→ The early electron 'e_e' that passes through the gap before the reference electron 'e_R' experiences a max +Ve voltage across the gap and this electron is accelerated. It moves with greater Velocity and penetrates deep into repeller space. The return time for early electron 'e_e' is greater as the depth of penetration into the repeller Space is more. Hence 'e_e' and 'e_R' appear at the gap for the Second time at the Same instant.

→ The late electron 'e_l' that passes the gap better than reference electron 'e_R' experiences a max -ve voltage and moves with a retarding velocity. The return time is shorter as the penetration into repeller space is less and catches up with 'e_R' and 'e_e' electrons forming a "Bunch".

⇒ Bunches occur only once per cycle centered around the reference electron 'e_R' and 'e_e' electrons forming a "Bunch".

→ Bunching occurs only once per cycle concentrated around the reference electron 'e_R' and the bunches transfer maximum energy to the gap to get sustained oscillations.

⇒ for oscillations to be sustained, the time taken by the electrons to travel into the repeller space and back to the gap, called "transit time" (T).

⇒ It appears that the best time for electrons to return to the gap is at the 90° point of sine wave gap voltage returning of electrons after

$$1\frac{3}{4} \text{ (or) } 2\frac{3}{4} \text{ (or) } 3\frac{3}{4} \text{ ------ etc}$$

→ In general, the optimum 'transit time (T)' should be

$$T = n + \frac{3}{4}$$

Here n = Any integer

This depends on repeller Voltage & Anode Voltages

Output (or) Operating characteristics:

(i) Voltage characteristic:

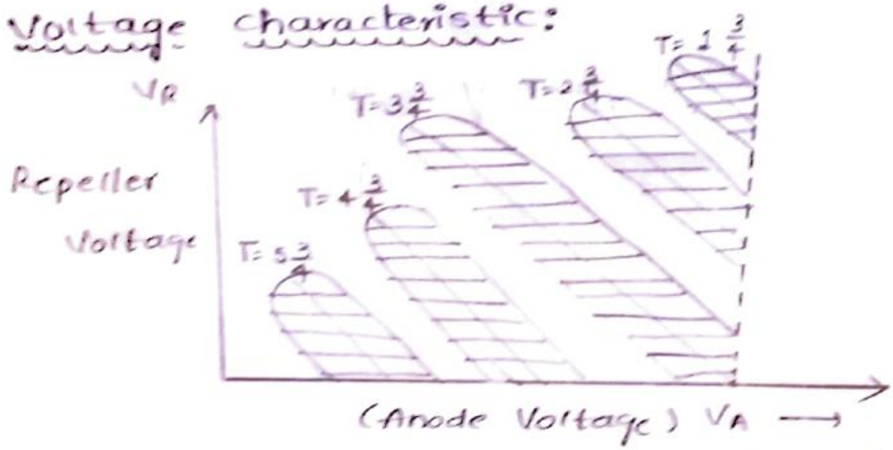


fig: Voltage characteristics of Reflex klystron

→ Oscillations can be obtained only for specific combinations of anode and repeller voltages that give a favourable transit time

$$T = n + \frac{3}{4}$$

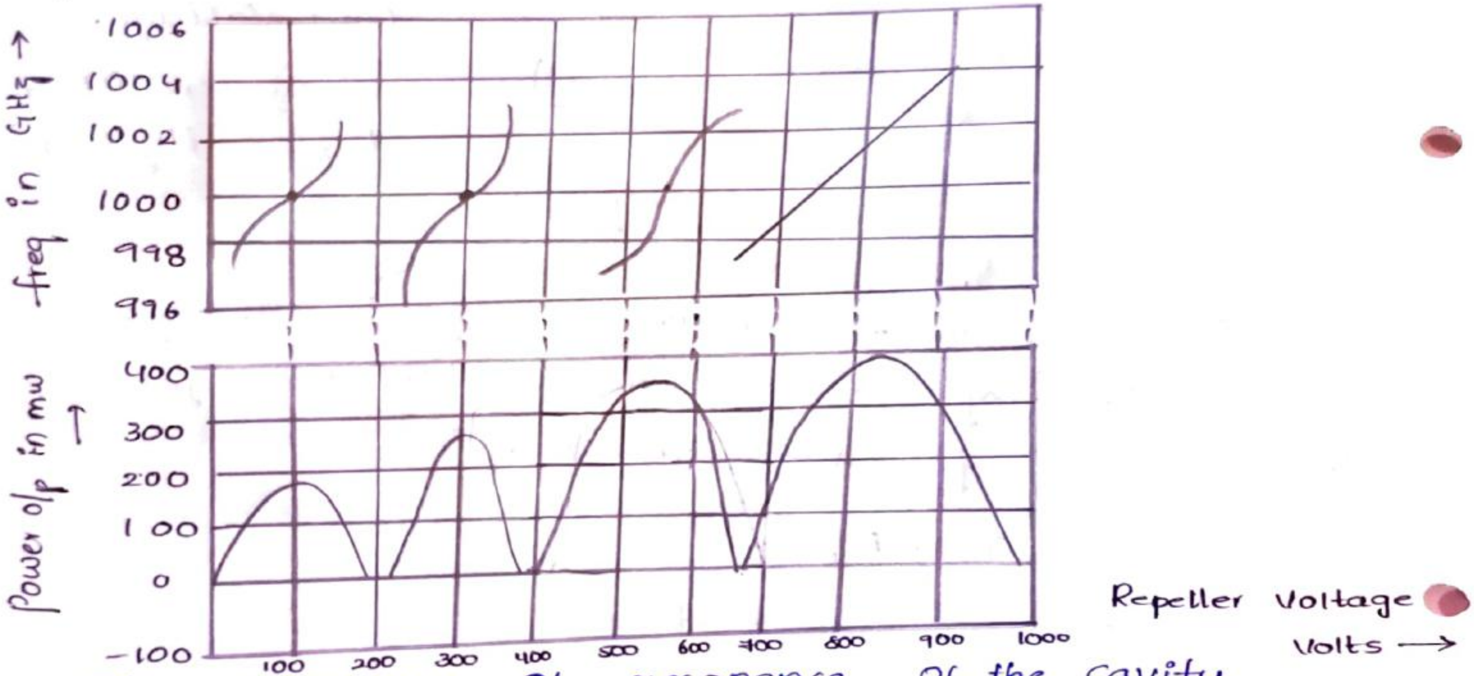
Here T = Transit time
$n = 1, 2, 3, 4 \ldots\ldots$

→ The shaded areas show possible Oscillation combination and heavy lines show optimum combinations, each value of $n = 1, 2, 3 \ldots$ is said to correspond to a different modes for the reflex klystron Oscillator. These modes called as "modes of Oscillation".

→ The earlier the mode, the larger the o/p power, Which is a advantage. But the Voltages required are also higher.

→ The modes corresponding to n=2 (or) n=3 are most widely used.

Repeller Voltage and frequency characteristics:



* The frequency of resonance of the cavity decides the frequency of Oscillation. Variations of repeller Voltage (V_R) slightly changes the frequency. This makes it possible to use reflex klystron as a "Voltage tuned Oscillator" (or) "frequency modulated Oscillator".

Mathematical theory of bunching:

V_0 = Electron gun anode voltage — fig

$V_1 \sin\omega t$ = RF voltage at cavity gap

V_R = Repeller voltage with respect to cathode

s = distance b/w cavity gap and repeller electrode

v_0 = velocity of electron in gun

v_1 = velocity due to RF voltage in addition to the electron accelerating voltage "V_0"

t_0 = time for electron entering cavity gap at $x=0$

t_1 = time for same electron leaving cavity gap at $x=d$.

t_2 = time for same electron returned by retarding field at $x=d$.

Calculations:

We know $\frac{1}{2}mv_0^2 = eV_0$

at time 't_0' $v_0 = \sqrt{\frac{2eV_0}{m}} \longrightarrow (1)$

at time 't_1' $\frac{1}{2}mv_1^2 = e(V_0 + V_1 \sin\omega t_1)$

$$v_1^2 = \frac{2e}{m} V_0 \left(1 + \frac{V_1}{V_0}\sin\omega t_1\right)$$

$$v_1 = \sqrt{\frac{2eV_0}{m}} \sqrt{\left(1 + \frac{V_1}{V_0}\sin\omega t_1\right)}$$

By sub equ ①, We get

$$\nu_1 = \nu_0 \left(1 + \frac{V_1}{2V_0} \sin\omega t_1\right) \longrightarrow ②$$

Now the Voltage is $V_0 + V_1 \sin\omega t_1$, Here $V_1 << V_0$

$\Rightarrow$ Voltage b/w repeller & Anode $= V_R - (V_0 + V_1 \sin\omega t_1)$.

$$\cong V_R - V_0$$

$\Rightarrow$ Retarding electrostatic field b/w repeller and anode is given by

$$E = -\left(\frac{V_R - V_0}{s}\right) \longrightarrow ③$$

$\therefore$ force on electron $= -eE = e\left(\frac{V_R - V_0}{s}\right) \longrightarrow ④$

Also, force on electron = mass × Acceleration

$$= m \cdot \frac{d^2x}{dt^2} \longrightarrow ⑤$$

Equate Eq ④ & Eq ⑤

$$m \cdot \frac{d^2x}{dt^2} = \frac{e}{s}(V_R - V_0)$$

$$\frac{d^2x}{dt^2} = \frac{e}{ms}(V_R - V_0) \longrightarrow ⑥$$

Integrating Equ. ⑥ once, we get

$$\frac{dx}{dt} = \frac{e}{ms}(V_R - V_0)t + c \longrightarrow ⑦$$

at $t = t_1$, $\frac{dx}{dt} = \nu_1$

so $\nu_1 = \frac{e}{ms}(V_R - V_0)t_1 + c$

$$C = v_1 - \frac{e}{ms}(V_R - V_0)t_1$$

sub 'c' in equ (7)

$$\frac{dx}{dt} = \frac{e}{ms}(V_R - V_0)t + v_1 - \frac{e}{ms}(V_R - V_0)t_1$$

$$\frac{dx}{dt} = \frac{e}{ms}(V_R - V_0)(t - t_1) + v_1 \longrightarrow (8)$$

Integrating Once again

$$x = \frac{e}{2ms}(V_R - V_0)(t - t_1)^2 + v_1 t + c_1 \longrightarrow (9)$$

at $x=0$ i.e at the point of return from repeller space $t = t_2$,

$$0 = \frac{e}{2ms}(V_R - V_0)(t_2 - t_1)^2 + v_1 t_2 + c_1$$

$$c_1 = \frac{-e}{2ms}(V_R - V_0)(t_2 - t_1)^2 - v_1 t_2$$

sub 'c_1' in eq (9)

$$x = \frac{e}{2ms}(V_R - V_0)\left[(t - t_1)^2 - (t_2 - t_1)^2\right] + v_1(t - t_2)$$

again with $t = t_1$; $x = 0$

i.e $\frac{-e}{2ms}(V_R - V_0)(t_2 - t_1)^2 - v_1(t_2 - t_1) = 0$

$(t_2 - t_1)$ is the round trip transit time is given by

$$(t_2 - t_1) = \frac{-2ms\,v_1}{e(V_R - V_0)} \longrightarrow 10$$

→ The transit angle 'w_t' is defined as transit angle at time 't'

$$\omega(t_2 - t_1) = \frac{-2msv_1\omega}{e(V_R - V_0)}$$

→ The transit angle 'ω_t' is defined as transit angle at time 't' $\omega(t_2 - t_1) = \frac{-2msv_1\omega}{e(V_R - V_0)}$

$$\omega t_2 = \omega t_1 - \frac{2msv_1\omega}{e(V_R - V_0)} \longrightarrow (11)$$

from equ (2) $v_1 = v_0 \left(1 + \frac{V_1}{2V_0} \sin\omega t_1\right)$ sub in eq (11)

$$\omega t_2 = \omega t_1 - \frac{2ms\omega}{e(V_R - V_0)} v_0 \left(1 + \frac{V_1}{2V_0} \sin\omega t_1\right) \longrightarrow (12)$$

Let $\frac{-2ms\omega v_0}{e(V_R - V_0)} = \omega T_0^1 x = \theta_0^1 \longrightarrow (13)$

Here θ_0^1 = Round trip dc transit angle of centre of bunch electron

Let $\frac{V_1}{2V_0} \theta_0^1 = x' \longrightarrow (14)$

Here x' = bunching parameter

$$\boxed{\therefore \omega t_2 = \omega t_1 + \theta_0^1 \left(1 + \frac{V_1}{2V_0} \sin\omega t_1\right)} \longrightarrow (15)$$

Relation b/w repeller Voltage (V_R) and accelerating voltage:

$\Rightarrow$ Consider Equ (15), when $V_1 << V_0$

(i.e., for the centre bunch of electrons which is unaffected by RF

$\therefore \omega t_2 = \omega t_1 + \theta_0^1$ from equ (15) when $V_1 << V_0$

$\Rightarrow$ Also from maximum transfer of energy, the modes are $1\frac{3}{4}$ cycles apart

i.e., $2\pi(n-\frac{1}{4})$ here $n-\frac{1}{4} = \frac{3}{4}$; $1\frac{3}{4}$ etc

$$2\pi(n-\frac{1}{4}) = 2\pi n - \frac{\pi}{2}$$

$\therefore$ The Optimum value of $\theta_0^1 = (2\pi n - \frac{\pi}{2}) \longrightarrow (16)$

from Eq (13)

$$\theta_0^1 = \frac{-2ms\omega \vartheta_0}{e(V_R - V_0)}$$

$$\vartheta_0 = \frac{-e(V_R - V_0)}{2ms\omega}\theta_0^1$$

$$\vartheta_0^2 = \frac{e^2(V_R - V_0)^2(\theta_0')^2}{4\omega^2 m^2 s^2} \longrightarrow (17)$$

We know that $\frac{1}{2}m\vartheta_0^2 = eV_0$

$$V_0 = \frac{m}{2e}\vartheta_0^2 \quad \text{sub Eq (17)}$$

$$V_0 = \frac{m}{2e}\,\frac{e^2(V_R - V_0)^2(\theta_0')^2}{4\omega^2 m^2 s^2} \quad \text{Here } \theta_0' = 2\pi n - \frac{\pi}{2}$$

$$\frac{V_0}{(V_R-V_0)^2} = \frac{m}{2e} \frac{e^2}{4\omega^2 m^2 s^2} (2\pi n - \frac{\pi}{2})^2$$

$$\boxed{\frac{V_0}{(V_R-V_0)^2} = \frac{1}{8} \cdot \frac{1}{\omega^2 s^2} \cdot \frac{e}{m} (2\pi n - \frac{\pi}{2})^2} \longrightarrow (18)$$

Eq (18) provides the relationship b/w 'V_R' & 'V_0'.

Expression for change in frequency due to repeller voltage variation : (Electronic tuning of reflex klystron)

Equ (18) can be rewritten as

$$(V_R - V_0)^2 = \frac{8ms^2 V_0}{e(2\pi n - \frac{\pi}{2})^2} \cdot \omega^2$$

differentiating V_R w.r.t 'ω'

$$2(V_R - V_0) \frac{dV_R}{d\omega} = \frac{16 m s^2 V_0}{e(2\pi n - \frac{\pi}{2})^2} (\omega)$$

$$\frac{dV_R}{d\omega} = \frac{8ms^2 V_0 \omega}{e(2\pi n - \frac{\pi}{2})^2} \cdot \frac{1}{(V_R - V_0)}$$

We can resubstitute the value of $(V_R - V_0)$ as

$$(V_R - V_0) = \sqrt{\frac{8ms^2 V_0 \cdot m^2}{e(2\pi n - \frac{\pi}{2})^2}}$$

Then

$$\frac{dV_R}{d\omega} = \frac{8ms^2 V_0 \omega}{e(2\pi n - \frac{\pi}{2})^2} \times \sqrt{\frac{e(2\pi n - \frac{\pi}{2})^2}{8ms^2 V_0 \omega^2}}$$

$$= \sqrt{\frac{8ms^2 V_0}{e}} \cdot \left(\frac{1}{2\pi n - \frac{\pi}{2}}\right)$$

$$\frac{dV_R}{d\omega} = \frac{s}{(2\pi n - \frac{\pi}{2})} \sqrt{\frac{8mV_0}{e}}$$

$$\boxed{\frac{dV_R}{dt} = \frac{2\pi s}{(2\pi n - \frac{\pi}{2})} \sqrt{\frac{8mV_0}{e}}} \longrightarrow (19)$$

This is a very useful relationship for "Electronic tuning" of reflex klystron.

Efficiency of reflex klystron:

→ Similar to klystron amplifier, Max power is transferred to the o/p when the returning bunched electrons arrive at the cavity and when the field is at antinode power output.

→ DC power supplies by beam voltage 'V_0' is

$$P_{dc} = V_0 I_0 \longrightarrow (20)$$

→ AC power delivered $P_{ac} = -I_0 V_2 \cdot J_1(x') \sin\theta_0^1$.

→ As the current flows in -ve direction, the negative sign becomes +ve and $\sin\theta_0^1$ is '1' and V_2 is V_1 being single and same cavity.

So $P_{ac} = I_0 \cdot V_1 \cdot J_1(x') \longrightarrow (21)$

Here $x_0' = \frac{V_1}{2V_0}\theta_0^1$ Here $\theta_0^1 = 2n\pi - \frac{\pi}{2}$

So $\frac{V_1}{V_0} = \frac{2x'}{(2n\pi - \frac{\pi}{2})}$

In Equ (21) sub 'V_1'

$$P_{ac} = I_0\left[\frac{2x' \cdot V_0}{(2n\pi - \frac{\pi}{2})}\right] J_1(x')$$

$$P_{ac} = \frac{2V_0 \cdot I_0 \cdot x' \cdot J_1(x')}{(2n\pi - \frac{\pi}{2})} \longrightarrow (22)$$

Efficiency $\eta = \frac{P_{ac}}{P_{dc}} = \frac{2V_0 \cdot I_0 \cdot x' \cdot J_1(x')}{V_0 \cdot I_0 \, (2n\pi - \frac{\pi}{2})}$

$$\boxed{\eta = \frac{2x' \cdot J_1(x')}{(2n\pi - \frac{\pi}{2})}} \longrightarrow (23)$$

$\Rightarrow$ The factor $x' . J_1(x')$ reaches a max value of 1.252 at $x' = 2.408$ and $J_1(x') = 0.52$. The max power o/p is obtained when $n = 2$ (or) $1\frac{3}{4}$ mode.

$\therefore$ Max Theoritical efficiency is

$$\eta_{(max)} = \frac{2(2.408)(0.52)}{2(2)\pi - \pi/2} = 22.78\% \longrightarrow (24)$$

for practical values however around 20%.

Power Output in terms of Repeller Voltage (V_R):-

(Effect of repeller voltage on power o/p):

We know $P_{out} = \dfrac{2V_0 I_0 x' \cdot J_1(x')}{(2n\pi - \frac{\pi}{2})}$ from equ (22)

Here $(2n\pi - \frac{\pi}{2}) = \theta_0^1 = \omega T_0^1 = \dfrac{-2msw v_0}{e(V_R - V_0)}$

⇒ Here -ve is not taken, as electron bunch travels in the reverse direction. so

$$\theta_0^1 = \frac{2msw v_0}{e(V_R - V_0)} \quad \text{--- From eqn (13)}$$

$$P_{out} = \frac{2V_0 I_0 x' J_1(x')}{\left(\frac{2msw v_0}{e(V_R - V_0)}\right)}$$

$$= \frac{2V_0 I_0 x' J_1(x')}{2msw v_0} \times e(V_R - V_0)$$

$$= \frac{V_0 I_0 x' J_1(x')}{\omega \cdot s} \times (V_R - V_0) \frac{e}{m v_0}$$

$$= \frac{V_0 I_0 x' J_1(x')}{\omega \cdot s} (V_R - V_0) \frac{e}{m\sqrt{\frac{2eV_0}{m}}}$$

Here $v_0 = \sqrt{\dfrac{2eV_0}{m}}$

$$P_{out} = \frac{V_0 I_0 x' J_1(x')}{\omega s} (V_R - V_0) \sqrt{\frac{e}{2mV_0}}$$

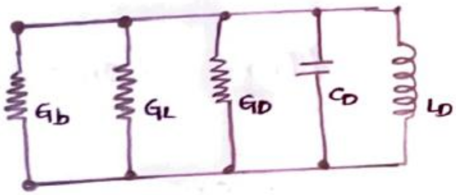
The max. value of $x' \cdot J_1(x') = 1.25$

Maximum Output power

$$P_{max} = \frac{1.25\ V_0 I_0 (V_R - V_0)}{\omega s}\sqrt{\frac{e}{2mV_0}} \longrightarrow (25)$$

Equivalent circuit of Reflex klystron :

It consists of the klystron in parallel with beam loading conductance (G_b) copper cavity losses (G_D), load conductance (G_L) and parallel tuned ckt



→ The electronic Admittance is given by

$$Y_e = \frac{I_2}{V_2}$$

Here I_2 = O/p current

V_2 = O/p voltage

i.e., $Y_2 = \frac{2I_0 J_1(x')\, e^{-j\theta_0'}}{V_1 e^{-j\pi/2}}$ $\quad \therefore I_2 = 2I_0 J_1(x') e^{-j\theta_0'}$

$V_2 = V_1 e^{-j\pi/2}$

putting $v_1 = \frac{2V_0 x'}{\theta_0'}$

$$Y_e = \frac{I_0}{V_0}\frac{\theta_0'}{x'} J_1(x')\, e^{j(\frac{\pi}{2} - \theta_0')}$$

→ Oscillations will take place when the net conductance is less than zero (-ve resistance)

$$Y_e = G_e + jB_e$$

for Oscillation, $|-G_e| \geq G$

Here $G = \frac{1}{R_{sh}} = G_D + G_L + G_b$, R_{sh} = shunt resistance

Performance characteristics of Reflex klystron :

1) Frequency Range : 4G to 200GHz
2) Output power : 1MW to 2.5W
3) Theoritical efficiency : 22.78%
4) practical efficiency : 10% to 20%
5) Tuning range : 5GHz at 2W to 30GHz at 10MW.

Significance, Types & characteristics of slow wave structures :-

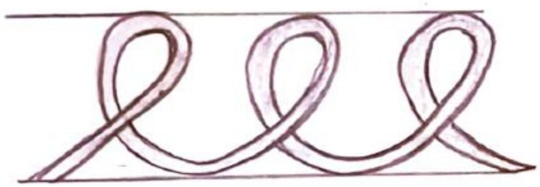
Significance:

slow wave structures are special type of circuits used in Microwave tubes to reduce the wave velocity in a certain direction so that a prolonged interaction b/w the electron beam and the signal may take place.

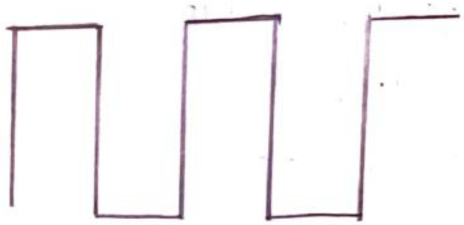
In Ordinary wave guides the phase velocity of a wave is greater than velocity of light in a vaccum. In both TWT and magnetron the slow wave structure changes the phase velocity of the Microwave signal in such a way that microwave signal and electron beam will move side by side in same place and as a result a prolonged interaction b/w the electron beam and the RF field (microwave) take place.

The RF field may propagate through the slow wave structure in both forward & backward direction. Thus "Travelling wave tube" (TWT) falls under "forward slow wave structure" and "Backward wave Amplifier (BWA) (or) "Backward wave Oscillator (BWO)" falls under Backward slow wave structure.

* Types of slow wave structures:

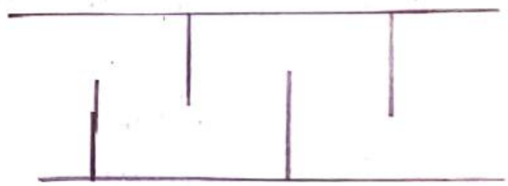


(i) Helical line

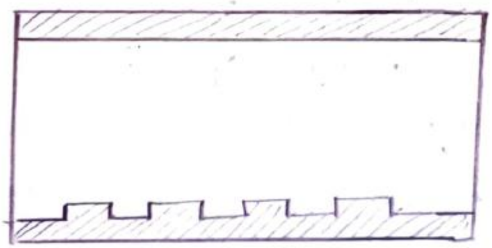


(2) folded back line

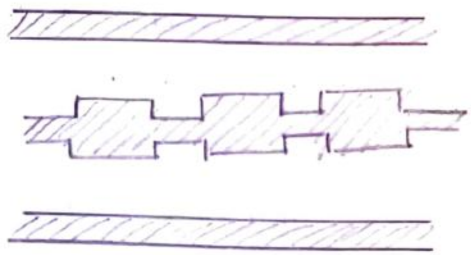
(3) Zig-Zag line



(4) Inter digital line



(5) Corrugated line



(6) Co-axial line with corrugated conductor.

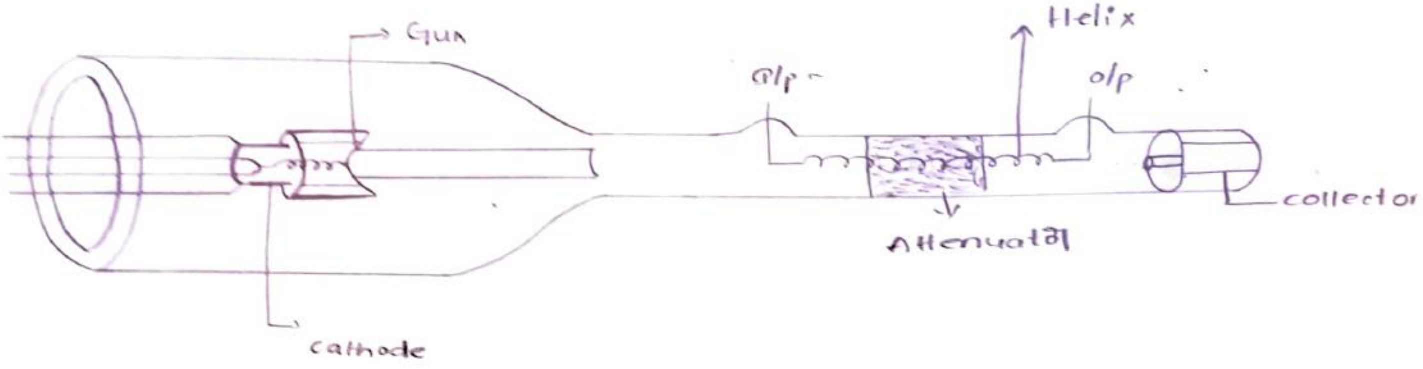
Characteristics of slow wave structures:

1) It is having larger gain over a wide bandwidth
2) These are used in microwave tubes to reduce the wave velocity in a certain direction so that the electron beam and the signal wave can interact.
3) The gain-bandwidth product is limited
4) It have the property of "periodicity" in the axial direction.

Travelling wave Tube (TWT):

* It is a new type of tube which has displayed considerable promise as a broadband amplifier, proposed by 'pierce' and others in 1946.
* The TWT makes use of a distributed interaction b/w an electron beam and a travelling wave
* Klystrons are essentially narrow band devices as they utilize cavity resonators to velocity modulate the electron beam over a narrow gap where as TWT's are broadband devices in which there are no cavity resonators

Structure of TWI :



* It has an electron gun as used in klystrons, which is used to produce a narrow constant velocity electron beams.
* The electron beam is in turn passed through the center of a long axial Helix
* A Magnetic focussing field is provided to prevent the beam from spreading and to guide it through the centre of the helix. Helix is a loosely wound thin conducting helical wire, which acts as a slow wave structure.
* The signal to be amplified is applied to the end of the ~~**~~ helix adjacent to the electron gun. The amplified signal appears at the o/p or other end of the helix under appropriate operating conditions.

Performance characteristics of TWT:

1. frequency of Operation : 0.5 GHz to 95 GHz
2. power outputs : 5mw (10-40 GHz) - Low power TWT
   250 kW (CW) at 3GHz - High power TWT
   10 MW (pulsed) at 3GHz - High power TWT
3. Efficiency : 5% to 20% (30% with depressed collector)
4. Noise figure : 4-6 dB (Low power TWT 0.5G to 16 GHz)
   25 dB (High power TWT at 40 GHz)

Applications of TWT:

1. Low noise RF Amplifier is broad band Microwave receivers.
2. Repeater amplifier in wide band communication links and co-axial cable
3. Due to long tube life, TWT is used as a power o/p tube in communication satellites
4. continous wave high power TWT's are used in troposcatter links (due to long power and larger bandwidth's.)
5. Air borne and shipborne pulsed high power radars ECM ground based radars use a TWT

Amplification process in TWT :-

→ The electrons are entering the helix at zero field are not effected by the signal wave, those electrons entering the helix at the accelerating field are accelerated and those entering the helix at the retarding field are decelerated.

⇒ As the electrons travel furthur along the helix, they begin forming bunch centered about those electrons that enter the helix during the zero field and collected at the collector end. The bunching shifts the phase by "$\pi/2$".

⇒ Since the electrons is greater than the axial wave velocity, more electrons are in the retarding field than in the accelerating field. And a great amount of energy is transferred from the beam to the electromagnetic field. The Amplification of the signal wave is accomplished.

→ The bunch becomes more compact and a larger amplification of the signal voltage occurs at the end of the helix

→ The attenuator placed near the center of the helix reduces all the waves travelling along the helix to nearly zero so that the reflected waves from the mismatched loads can be prevented from reaching the input and causing Oscillation

→ The bunched electrons emerging from the attenuator induce a new electric field with the same frequency. The field in turn induces a new amplified Microwave signal on the helix

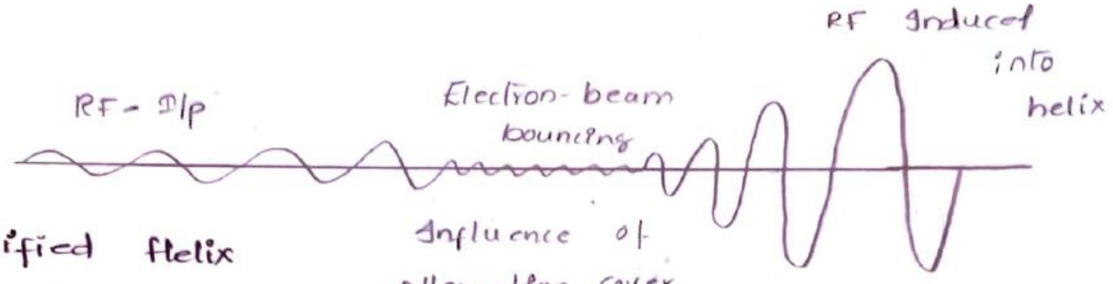


fig: Amplified Helix signal

Comparison of klystron Amplifier & TWT Amplifier:

Klystron Amplifier	TWT Amplifier
1) Linear beam (or) O-type device	1) Linear beam (or) O-type device
2) Uses resonant cavities for i/p & o/p circuits	2) uses non-resonant wave circuits
3) Narrow band device	3) Wide band device.

Gain Consideration in TWT:

The o/p power gain is defined as

$$A_p = 10\log\left|\frac{\text{Output Voltage}}{\text{Input Voltage}}\right|^2 = -9.54 + 47.3NC \text{ dB}$$

⇒ The first term -9.54 dB represents a loss due to the fact that the input wave divides into three ways of equal magnitude and only one of these ways is amplified

⇒ N is the length of the interaction region in wavelength. $\rightarrow N = \frac{l}{\lambda_e}$

Here l = length of the slow wave structure in meters

here $\lambda_e = \frac{2\pi v_0}{\omega}$ here $v_0 = \sqrt{\frac{2eV_0}{m}}$

and the factor 'c' is the gain parameter of the ckt

$$C = \left(\frac{I_0 Z_0}{4V_0}\right)^{1/3}$$

here I_0 = dc beam current

V_0 = dc beam voltage

Z_0 = characteristic impedance of helix.

MAGNETRON:

→ "Magnetron was invented by "Hull" in 1921 and an improved high power magnetron was developed by "Randall" and "Boot" around 1939. Magnetrons provide microwave oscillations of very high peak power.

→ There are three types of Magnetrons

1> Negative Resistance type
2> cyclotron frequency type
3> travelling wave (or) Cavity type

1) Negative Resistance type :- Makes use of -ve resistance b/w two anode segments but have low efficiency and are useful only at low frequencies (< 500 MHz)

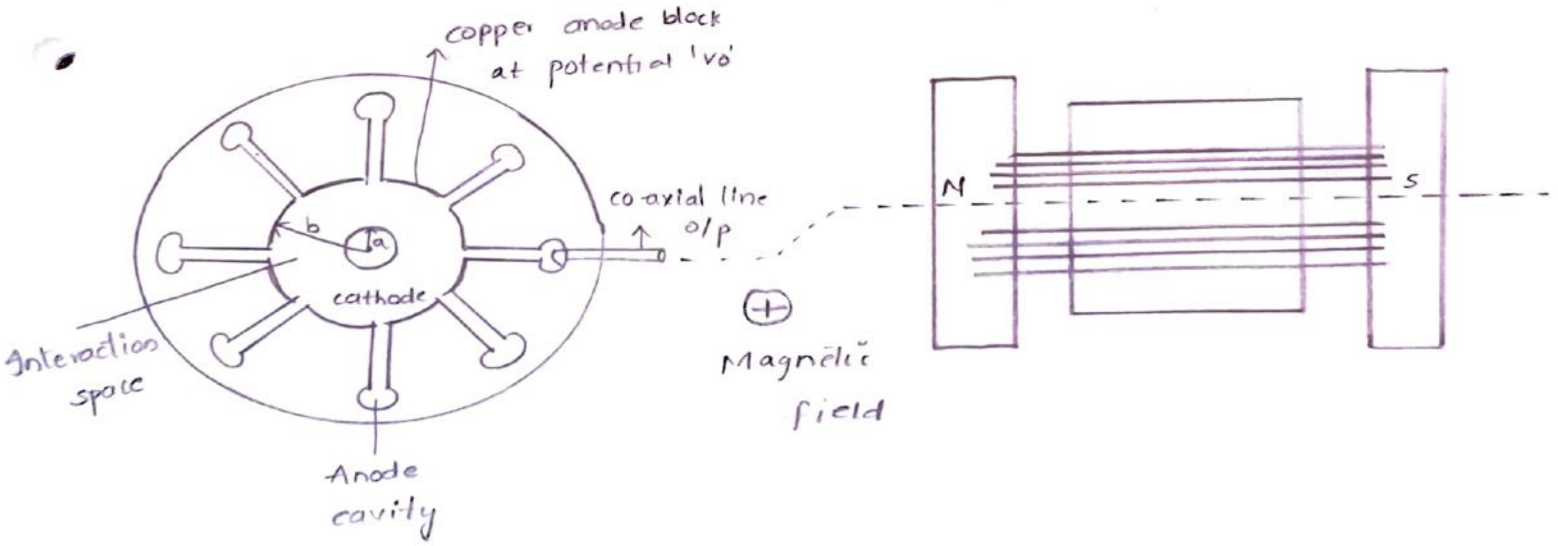
(2) Cyclotron frequency magnetrons:

depends upon synchronism b/w an alternating component of electric and periodic Oscillator of electrons in a direction parallel to this field. These are useful only for frequencies greater than 100MHz.

3) Cavity Magnetrons:

depends upon the interaction of electrons with a rotating electro-magnetic field of constant angular velocity. These provide Oscillations of very high peak power and hence are very useful in radar applications

8-Cavity Cylindrical travelling wave Magnetron:



The cavity magnetron as shown in figure has 8 cavities, that are tightly coupled to each other.
We know, in general that a N-cavity tightly coupled system will have N-modes of operation each of which is uniquely characterised by a combination of freq and phase oscillation relative to adjacent cavity.

→ In addition of these modes must be self consistent so that the total phase shift around the rig of cavity resonators is "$2n\pi$" where 'n' is an integer.

→ If "ϕ_v" represents the relative phase change of the ac electric field across adjacent cavities, then

$$\phi_v = \frac{2\pi n}{N} \quad \text{where} \quad n = 0, \pm1, \pm2, \pm\left(\frac{N}{2}-1\right), \pm\frac{N}{2}$$

i.e $\frac{N}{2}$ mode of resonance can exist if 'N' is an even number

If $n = \frac{N}{2}$ then $\phi_v = \pi$

→ This mode of resonance is called π-mode

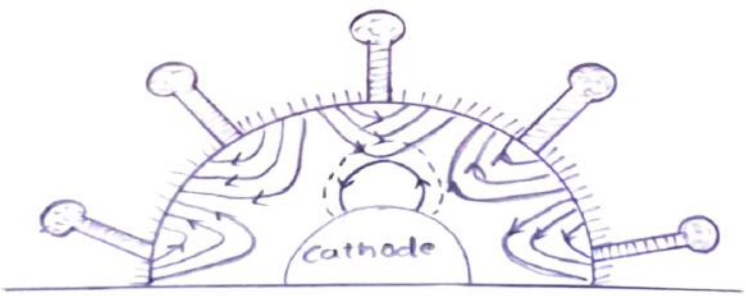


fig: π- mode of magnetron.

We know $\phi_V = \frac{2\pi n}{N}$, if $n=0$, $\phi_V = 0$

⇒ This is the "zero mode" i.e., there will be no RF electric field b/w Anode and cathode (fringing field) and is of no use is magnetron Operation

⇒ To Understand the Operation of cavity magnetron, we must first look at how the electrons behave in the presence of closed electric and magnetic fields

Mathematical Analysis:

[Hull cutoff & Hartree conditions of magnetron]

⇒ Let the cathode and Anode radius be 'a' & 'b' respectively and 'ϕ' is the angular displacement of the electron bends being a crossfield device electric and magnetic fields, are perpendicular to each Other and

⇒ force acting on electron is $F = BeV$

⇒ In the direction of 'ϕ' the force ~~constant~~ component '$F\phi$' is given by

$$F_\phi = eB v_\rho$$

Here v_ρ = Velocity in the direction of the radial distance 'ρ' from centre of cathode Cylinder.

$\Rightarrow$ Torque in ϕ direction is $T\phi = \rho F\phi = e\rho v_\rho . B \rightarrow (1)$

$\Rightarrow$ Angular momentum = Angular velocity $\times$ moment of inertia

$$= \frac{d\phi}{dt} \times m\rho^2$$

$\Rightarrow$ Time rate of angular momentum $= \frac{d}{dt}\left(\frac{d\phi}{dt} \times m\rho^2\right) \longrightarrow (2)$

Which gives the torque in ϕ- direction

by equating eq (1) & (2)

$$\frac{d}{dt}\left(\frac{d\phi}{dt} \times m\rho^2\right) = e\rho v_\rho . B$$

i-e $\quad 2m\rho \frac{d\phi}{dt} + m\rho^2 \frac{d^2\phi}{dt^2} = e\rho v_\rho . B \longrightarrow (3)$

We know that $\quad v_\rho = \frac{d\rho}{dt}$

$$\int \rho v_\rho = \int \rho \frac{d\rho}{dt} \quad \text{Here } \int \rho . \frac{d\rho}{dt} = \frac{\rho^2}{2}$$

$$\text{so } \int \rho . v_\rho = \frac{\rho^2}{2}$$

Integrating Equ (3) w.r.t "t"

$$2 m\rho\phi + m\rho^2 \frac{d\phi}{dt} = eB\left(\frac{\rho^2}{2}\right)$$

for a particular direction 'ϕ', $m.\rho.\phi$ can be

through of as a constant

$$m\rho^2 \frac{d\phi}{dt} + c = eB\left(\frac{\rho^2}{2}\right) \longrightarrow (4)$$

Now applying boundary conditions (i.e at surface of the cathode $\rho = a$ and $\frac{d\phi}{dt} = 0$ being zero angular velocity at emission. We can determine the value of constant "c".

$$0 + c = eB\left(\frac{a^2}{2}\right)$$

$$c = \frac{eBa^2}{2} \quad \text{sub in equ } 4$$

then

$$m\rho^2 \frac{d\phi}{dt} + \frac{eBa^2}{2} = \frac{eB\rho^2}{2}$$

$$m \cdot \rho^2 \cdot \frac{d\phi}{dt} = \frac{eB}{2}(\rho^2 - a^2)$$

$$\frac{d\phi}{dt} = \frac{eB}{2m}\left(1 - \frac{a^2}{\rho^2}\right)$$

When $\rho = a$ (i.e at cathode), $\frac{d\phi}{dt}$ approaches "zero".

When $\rho >> a$, $\frac{d\phi}{dt}$ approaches $(\omega)_{max}$

i.e., $\left(\frac{d\phi}{dt}\right)_{max} = (\omega)_{max} = \frac{eB}{2m} = \frac{eB_c}{2m} \longrightarrow 5$

Here $B = B_c$ is the cut-off magnetic flux density

=) We know from conservation of energy that potential energy of electron = kinetic energy of electron

$$eV_0 = \frac{1}{2}mv^2$$

$$eV_0 = \frac{m}{2}\left(v_\rho^2 + v_\phi^2\right) \longrightarrow 6$$

Here v_ρ and v_ϕ are components in 'ρ' and 'ϕ' direction in cylindrical co-ordinates

$v_\rho = \frac{d\rho}{dt}$ and $v_\phi = \rho \frac{d\phi}{dt}$, sub in Equ ⑥

$$eV_0 = \frac{m}{2}\left[\left(\frac{d\rho}{dt}\right)^2 + \rho^2\left(\frac{d\phi}{dt}\right)^2\right]$$

from Equ ⑤ $\frac{d\phi}{dt} = (\omega)_{max}\left(1 - \frac{a^2}{\rho^2}\right)$

$$\therefore eV_0 = \frac{m}{2}\left[\left(\frac{d\rho}{dt}\right)^2 + \rho^2(\omega)^2_{max}\left(1 - \frac{a^2}{\rho^2}\right)^2\right] \longrightarrow ⑦$$

At anode $\rho = b$, $\frac{d\rho}{dt} = 0$, sub in Equ ⑦

$$\therefore eV_0 = \frac{m}{2}\left[b^2(\omega)^2_{max}\left(1 - \frac{a^2}{b^2}\right)^2\right] \longrightarrow ⑧$$

also $(\omega)^2_{max} = \left(\frac{eB_c}{2m}\right)^2$ from Equ ⑤

Here 'B_c' is the cut-off value of the magnetic flux density. sub $(\omega)^2_{max}$ value in Equ ⑧

$$eV_0 = \frac{m}{2} b^2 \left(\frac{eB_c}{2m}\right)^2 \left(1 - \frac{a^2}{b^2}\right)^2$$

$$eV_0 = \frac{e^2 B_c^2 b^2}{8m}\left(1 - \frac{a^2}{b^2}\right)^2$$

$$8mV_0 = eB_c^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2$$

$$B_c^2 = \frac{8mV_0}{eb^2\left(1 - \frac{a^2}{b^2}\right)^2}$$

$$B_c = \frac{\left(\frac{8mV_0}{e}\right)^{1/2}}{b\left(1 - \frac{a^2}{b^2}\right)}$$

Since $b >> a$ and $\frac{a^2}{b^2}$ can be Neglected

$$B_c = \frac{1}{b}\sqrt{\frac{8mV_0}{e}} \longrightarrow (9)$$

⇒ At this cut-off value of magnetic field, the electron grazes the anode and the value of this magnetic field can be Obtained by the knowledge of anode voltage.

⇒ Equ (9) is called " Hull's cut-off voltage Equation"

performance characteristic of magnetron:

Power Output : In excess of 250 kw (pulsed mode)

10 mw (UHF Band)

2 mw (X - band)

8 kw (at 95 GHz)

frequency: 500 MHz to 12 GHz

Duty Cycle: 0.1 %

Efficiency: 40 % to 70 %

Applications of Magnetron:

1. pulsed radar is the single most Important application with large pulse powers.
2. Voltage tunable Magnetrons (VTM) are used in sweep Oscillators in telemetry and in Missile applications.

3. fixed frequency, CW (continous wave) Magnetrons are used for industrial heating and Microwave Ovens.

4. (5000 MHz - 2.5 GHz) frequency range, 300W to 10KW power outputs at efficiency (η) of 50%.

MICROWAVE SOLID STATE DEVICES

Semi Conductor

→ By Using microwave tubes, the amplification, Oscillation, switching, limiting (or) frequency multiplication basically employed Velocity modulation theory.

→ for achieving the above function, low noise, high frequency, high bandwidth, lesser switching time and other improvements in the performance characteristics must be taken

⇒ In this purpose several semiconductor microwave devices are devolped

⇒ All the solid state devices employ negative resistance char's rather than velocity modulation of their Operation.

Classification of Solid state Microwave devices

solid state Microwave devices can also be classified as

(i) Based on their electrical behaviour

(ii) Based on their construction.

(i) Based on their electrical behaviour we have

- Non-linear reactance type
- Non-linear resistance type.
- Negative resistance type
- controllable impedance type

(ii) Based on their construction we have

- point contact diodes
- schottky Barrier diodes
- Metal Oxide semi conductor devices
- Metal insulation devices

Applications:

1) Modern Communication Systems
2) Radars
3) Navigation
4) Medical and Biological equipment
5) Industrial electronic products.

Transfer Electronic Devices [TED's] :-

⇒ We know solid state devices are having "Negative Resistance Characteristics".

→ In a Negative resistance the current and voltage are out of phase by 180° the voltage drop across it is -ve and a power of $-I^2R$ is generated by the power supply associated with the -ve resistance.

→ So simply "positive resistances" absorb power (passive devices) and "Negative resistances" generate power (Active devices).

→ TED's are bulk devices having no Junction (or) no gate

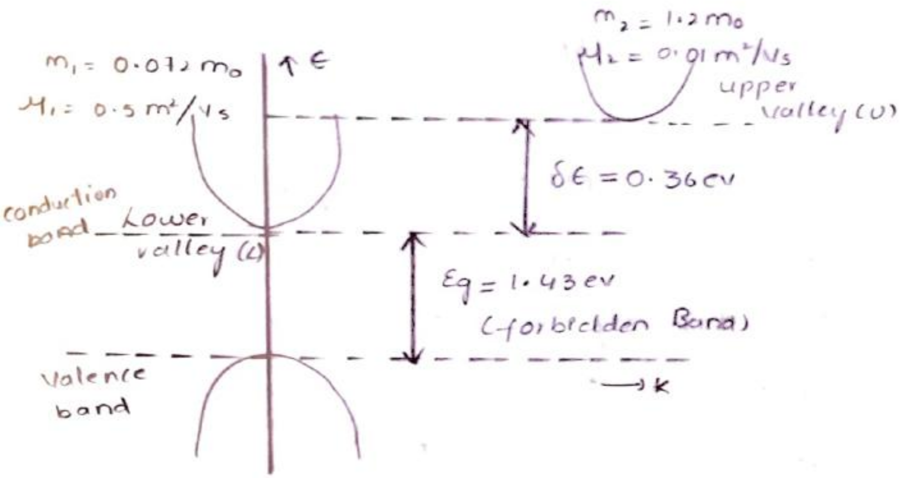
⇒ TED's are fabricated from compound semi conductors such as GaAs (Gallium Arsinic), InP (Indium phospate), CdTe (cadmium Telluride).

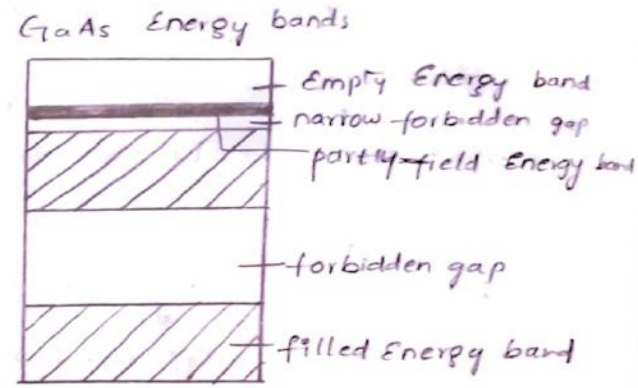
→ TED's Operate with "Hot electrons" whose energy is verymuch greater than thermal energy.

GunDiode :-

⇒ Gunn effect diodes are named after "J.B. Gunn" (1963), whose discovered "periodic fluctuations of current" passing through the n-type "GaAs" specimen when the applied voltage exceeded a certain critical value (2-4kv/cm)

⇒ Gunn effect can be explained on the basis of two valley theory of "Ridley Watkins-Hilsum (RWH) theory" of the transferred electron Mechanism.





- Here m = mass of electron
  μ = mobility

⇒ In figure, the Curvature of the two Valley's in the conduction band also called as "Sub bands" are different, so that an electron in L-valley has a similar effective mass ($m_1 = 0.072 m_0$) than one in the U-valley ($m_2 = 1.2 m_0$)

⇒ The different effective masses means different mobilities for the L-valley ($\mu_1 = 0.5 m^2/Vs$) and the U-valley ($\mu_2 = 0.01 m^2/Vs$) respectively.

⇒ The ratio of density of states in the U-valley to that in the L-valley is about "60". Thus the U-valley has a very high density of states compared with the lower valley location

⇒ As the applied field is increased, the electrons gain energy from it and move upward from L-valley to U-valley.

⇒ As the electrons transfer to this upper valley, their mobility decreases and the effective mass is increased thus decreasing the current density "J" and hence the Negative differential conductivity produced.

→ As the transfer of electron is taking place, the current density should be given by

$J = a\epsilon = en_0\mu\epsilon$ $\quad n =$ Carrier Concentration

$\mu =$ mobility

here $\mu = \frac{n_1\mu_1 + n_2\mu_2}{n_0}$ = The Avg. mobility of electrons

$$J = a\epsilon = en_0\left(\frac{n_1\mu_1 + n_2\mu_2}{n_0}\right)\epsilon$$

$$J = e(n_1\mu_1 + n_2\mu_2)\epsilon$$

As the applied field is raised even higher almost all the electrons in the L-valley are transferred to U-valley and the current density will be given as $J = a\epsilon = en_2\mu_2\epsilon$

Here

$n_2 =$ Carrier Concentration

$\mu_2 =$ Mobility in U-valley.

→ As a J (Vs) ϵ curve is Obtained similar to VI characteristics of p-n junction diode, for the voltage controlled bulk Negative conductance GaAs sample as follows

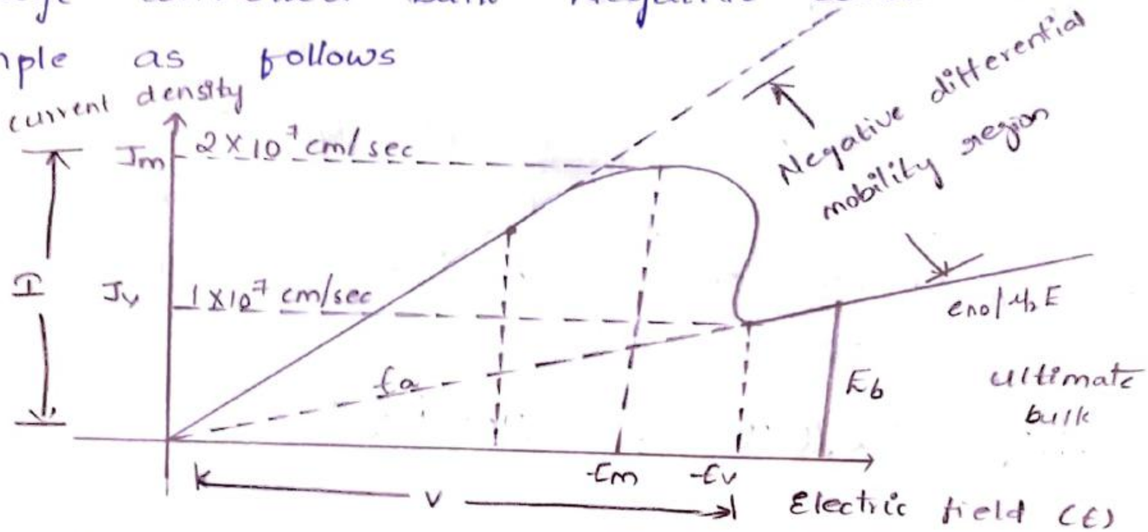


fig:- $J\epsilon$ i.e VI characteristics of Gunn diode.

Here J_m = Maximum Current density

J_v = Valley Current density

ϵ_m = Maximum electric field required before the onset of -ve conductance region

ϵ_v = Electric field corresponding to J_v

ϵ_a = Max. electric field for which $J = a\epsilon$ is valid

ϵ_b = electric field for which $J = en_2\mu_2\epsilon$ holds.

L - Valley mobility $\mu_L = \dfrac{V_d}{\epsilon}$ $\quad$ V_d = electron drift velocity

U - Valley mobility $\mu_U = \dfrac{V_d}{\epsilon}$ $\quad$ f = frequency

(Neutral) Mobility $\mu_n = \dfrac{dV_d}{d\epsilon}$ $\quad$ L = device length

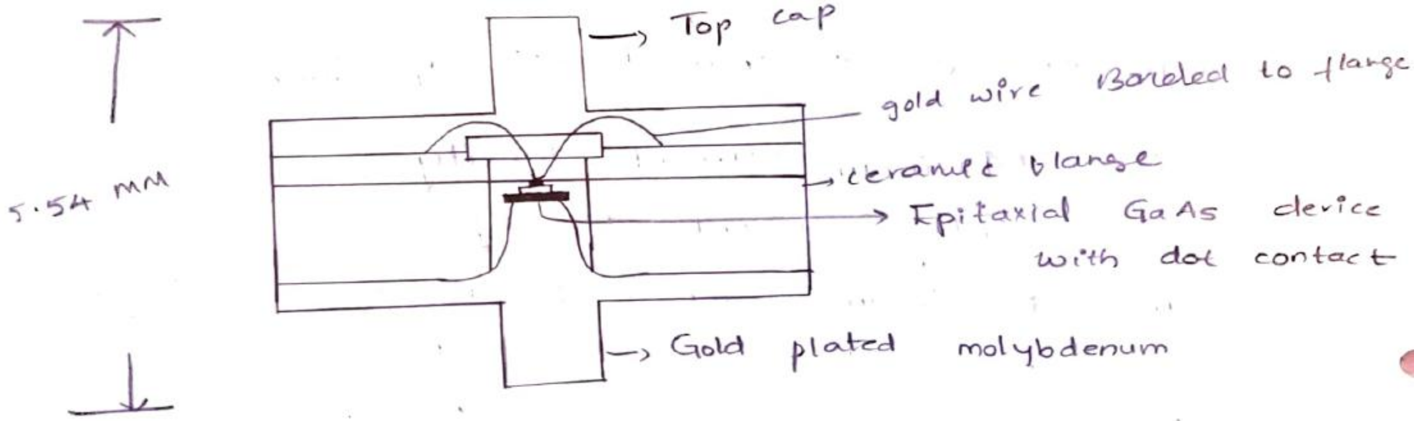
here $V_d = f \cdot L$

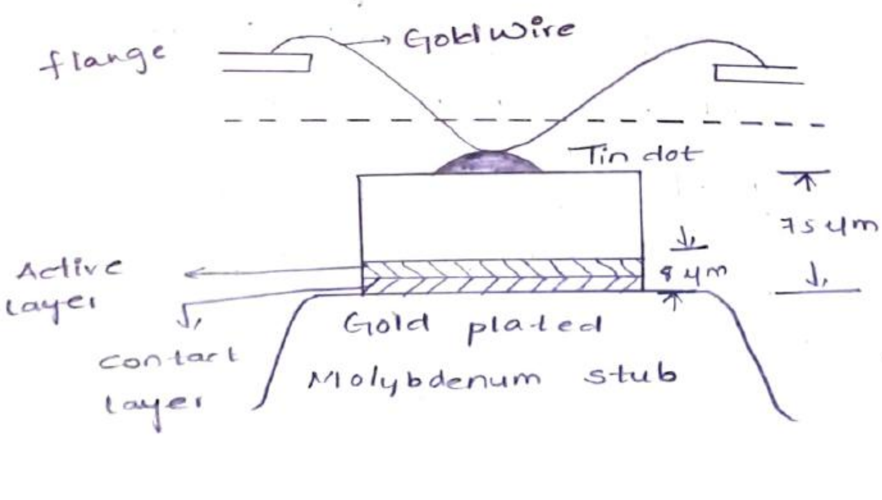
finally $J \propto$ terminal Current i.e., $J \propto I$
$E \propto$ Applied Voltage i.e., $E \propto V$

⇒ Hence this JE char's represents the VI char's of Gunn diode

⇒ The region of the characteristic b/w 'E_m' and "E_v" where current density (I) decreases with increasing electric field (V) one of the "Negative differential resistivity (NDR)".

Operating diagrams of Gunn Diode :





Typical Characteristics of Gunn diode:

Voltage : 10-12V

Basic Current : 250 mA

pulsed power : 5W (5-12 GHz)

Efficiency : 2% to 12% (at 1.5W CW to 50mW CW)

CW power : 25 mW to 250mW - xband (5-15 GHz)

[continous wave power] 100 mw at (18 - 26.5 GHz)

40 mW at (26.5 - 40 GHz)

Applications of Gunn diode:

1) In RADAR transmitters (police Radar, CW doppler radar)

2) pulsed Gunn diode Oscillator used in transponders for air traffic control (ATC) and in industry telemetry systems.

3) Broad band linear Amplifier

4) fast combinational and sequential logic circuits

5) Low and medium power Oscillator in Microwave receivers

6) As pump sources in par Amplifier.

Basic Modes of Operation (Gunn Oscillator modes):

→ Depending on the material parameter and Operating condition, a gunn effect Oscillator can be made to Oscillate in any of the four frequency modes.

1) Transit time domain mode
2) Delayed (or) Inhibited domain mode
3) Quenched domain mode
4) Limited space charge Accumulator (LSA) mode.

① Transit time domain mode : ($f_L = 10^7$ cm/sec)

⇒ This mode also called as "Gunn Mode".

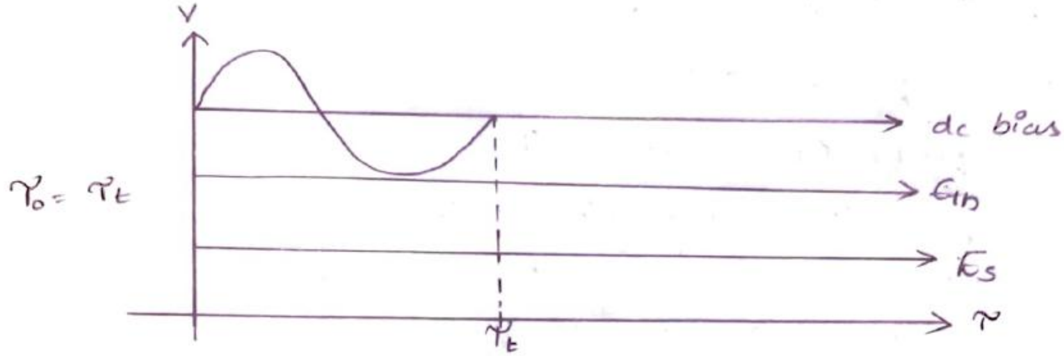
⇒ Here the transit time taken by dipole domain to travel from cathode to Anode is transit time of device. The fundamental frequency in MHz is given by $f = \frac{V_d}{L}$

V_d = drift Velocity
L = device Length

Here $V_d = f \cdot L = 10^7$ cm/sec

⇒ When $V_d = V_s$ (sustaining Velocity), the high field domain is stable

→ In this case the Oscillation period = Transit time i.e $\tau_0 = \tau_t$. This is shown in the figure.

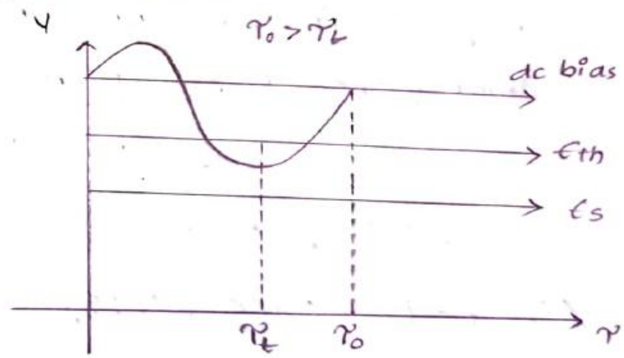


⇒ Efficiency is below 10% because domain arrives at the anode at a lower current level.

⇒ This mode doesn't require any external circuits for its operation.

⇒ It is a low power, low efficiency mode and requires that the operating frequency be lesser than 30 GHz.

② Delayed (or) Inhibited domain mode :-



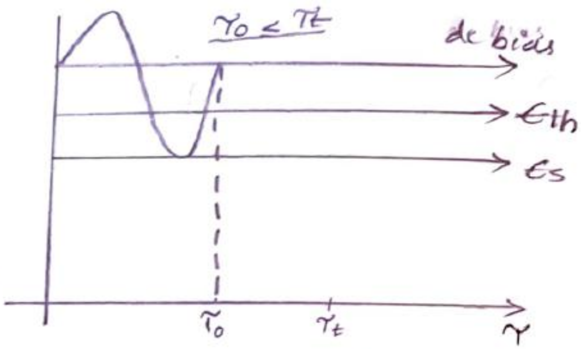
⇒ Here, Oscillation period is greater than transit time
$\tau_0 > \tau_t$.

⇒ This delay inhibited mode has an efficiency of 20% approximately

⇒ Hence the operating frequency can be equal (or) less than that in Gunn mode (or) transit time mode.

Quenched domain mode ($f_L > 2\times10^7$ cm/sec) :

If the bias field drops below sustaining field (E_s) during the -ve half cycle the domain collapses before it reaches the anode. i.e., the domain disappears somewhere in the sample itself.



=) Hence the domain does not travel all the way to anode and there by the Operating frequency will be higher than that of delayed domain mode.

So, this depends on external circuit.

=) When the bias field swings back above thresold value (V_{th}) a new domain will be formed and the process repeats

=) Hence, in this mode the domain is quenched before it reaches the anode

=) Therefore Oscillation Occurs at the frequency of resonant circuit rather than at transit time frequency. Efficiency is 13%.

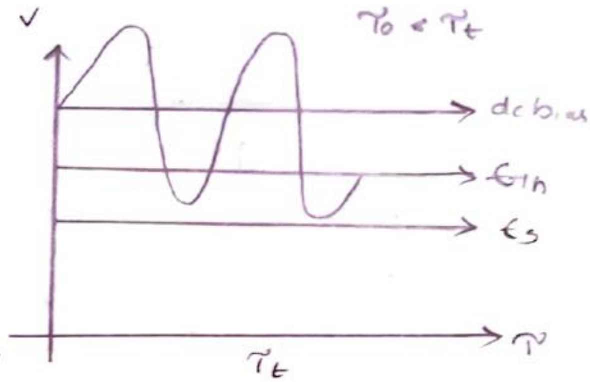
Limited Space Charge Accumulation Mode :

($f_L > 2\times10^7$ cm/sec).

This is most important mode of Operation for Gunn Oscillator as this mode gives high power with high efficiency about 20%.

⇒ In this mode the domain is not allowed to form at all.

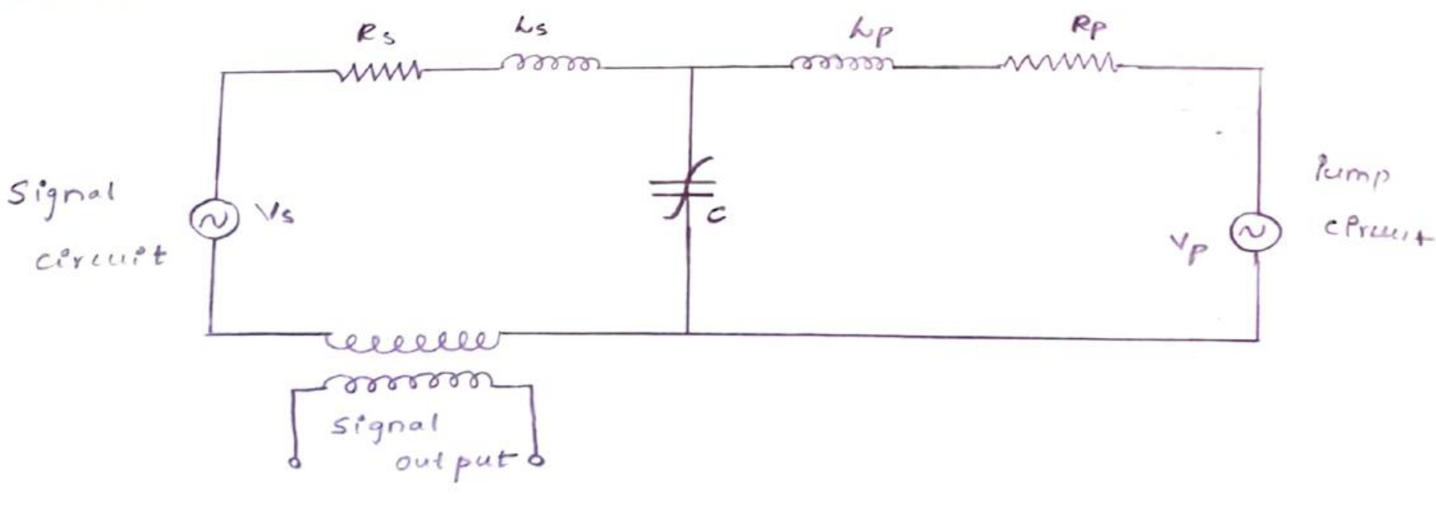
⇒ The frequency and RF voltage are so choosen that the domains don't have sufficient time to form while the field is above threshold as a result most of the domains are maintained in the Negative conductance state during a large part of the voltage cycle



5

Parametric Amplifiers:

⇒ In 1950's the requirement of extremely low noise amplification for the radio telescope, space-probe tracking and tropospheric scatter receiver gives rise to an interest in the minds of scientists, is the reason behind the development of parametric amplifier, It uses a device whose reactance is varied in such a manner that amplification results



→ The power supply by the pump signal is (P_p)

$P_p = -\frac{1}{2} V^2 f_p (\delta c)$ here V = Voltage
f_p = pump frequency
δc = change in Capacitance

⇒ Thus power is supplied by the pump when 'δc' is negative and this power is proportional to pump frequency. When 'δc' is positive, power would be absorbed by the pump.

⇒ If 'v' is not zero at the same time when the freq of the pump source is exactly twice that of the Original, then the amplifier is said to be "degenerate". The whole ~~basi~~ logic sequence is explained graphically in below figure

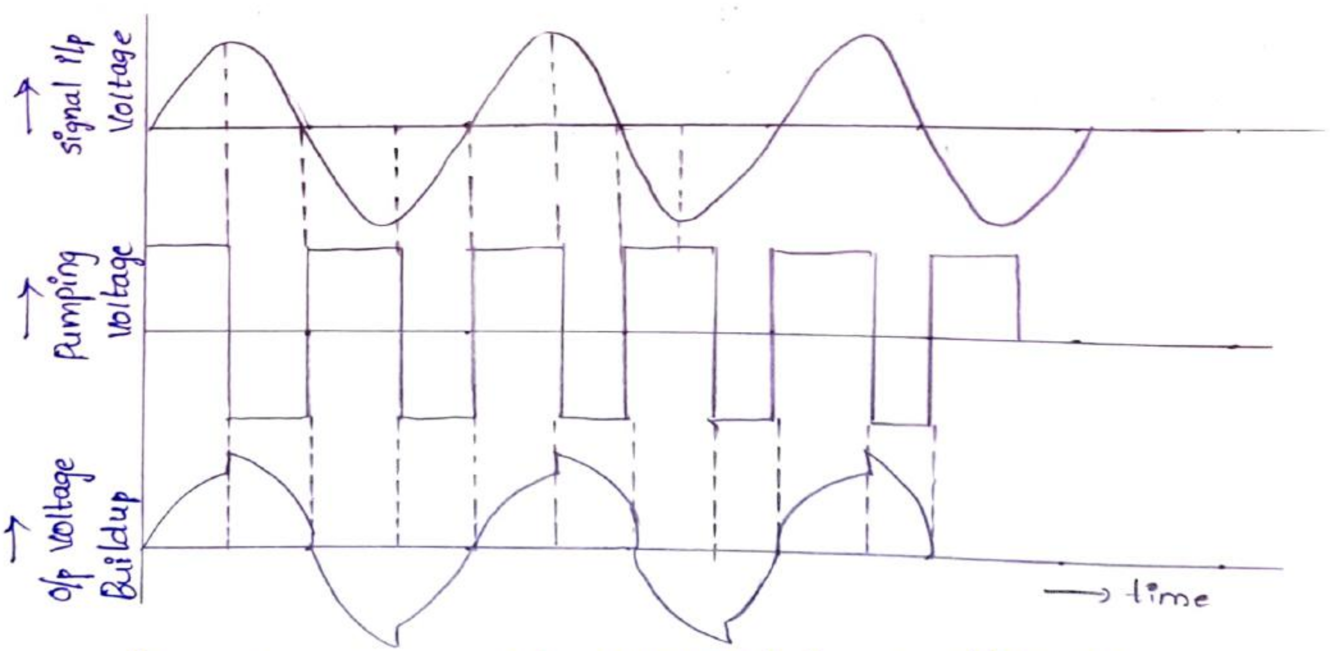


fig : Sequence of parametric amplification

It is difficult to maintain the correct relationship b/w the pump and the signal waveform in practice, owing to lack of control over the signal frequency.

⇒ However, Greater flexibility in Operation & improved power gain is Obtained in "non-degenerate mode", Here a second tuned circuit known as an "idler circuit" contains no energy source but it does allow pump to work at a frequency other than twice the signal frequency by modifying voltage 'v' across the variable capacitor 'c'

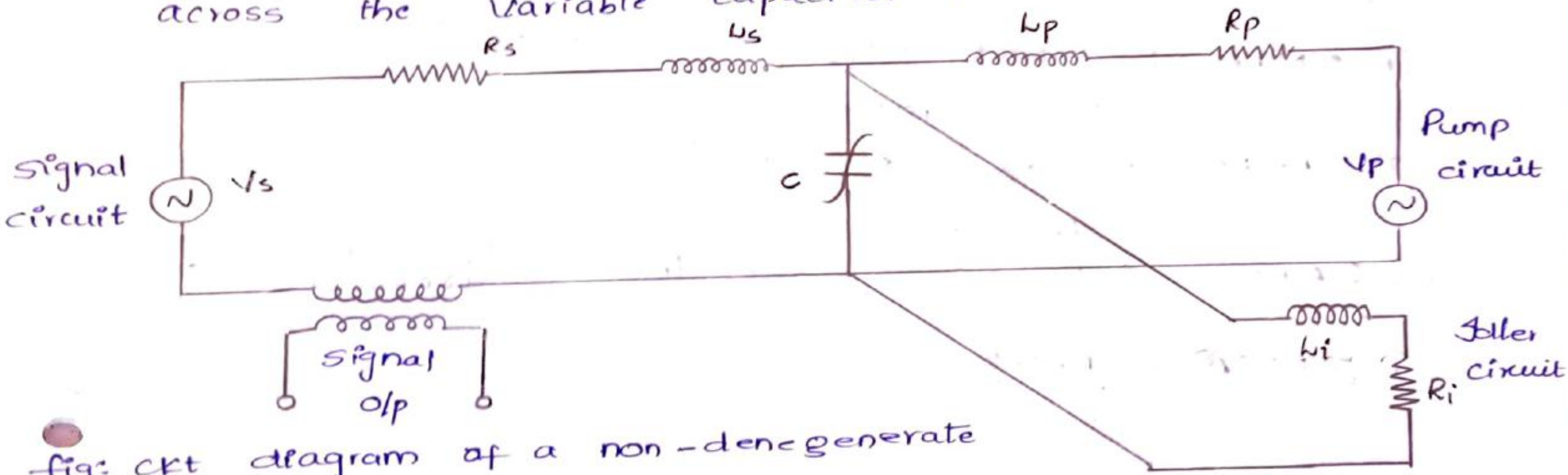


fig: ckt diagram of a non-denegenerate type parametric Amplifier.

⇒ At the zero point of the variable signal, if the pump source applied on its own, idler and signal circuits are excited into Oscillator at their respective resonant frequencies

⇒ The pump signal frequency $\omega_p = \omega_i - \omega_s$

When $\omega_p >> \omega_s$

then idler frequency

$\omega_i > \omega_s$ then amplifier is known as "up-converter".

$\omega_i < \omega_s$ then amplifier is known as "down-converter".

Introduction to Avalanche Transit time Devices:

⇒ It is possible to make a microwave diode exhibit negative resistance by having a delay b/w voltage & current in an avalanche together with transit time through the material. such devices are called "Avalanche transit time devices".

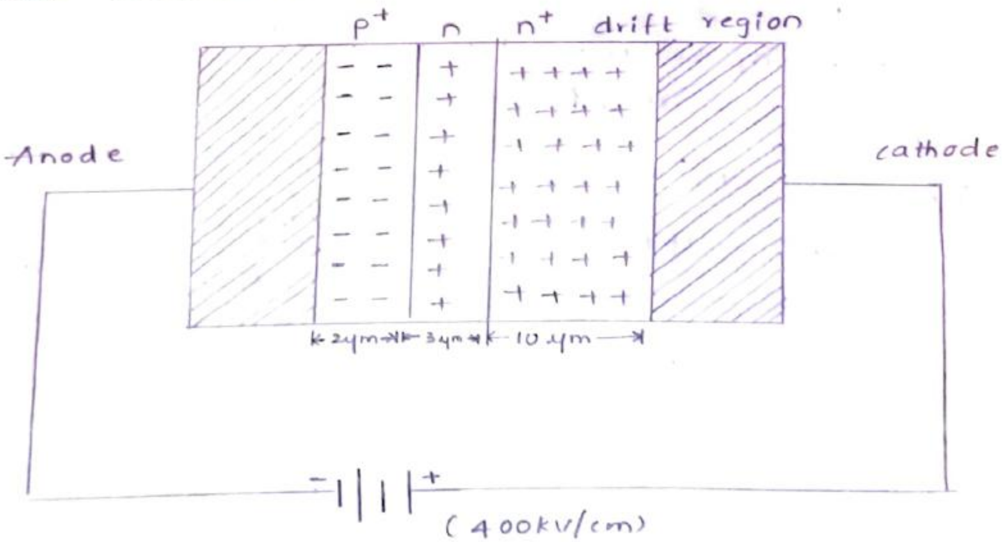
⇒ There are three distinct modes of Avalanche devices

(1) IMPATT: Impact Ionization Avalanche Transit time device.

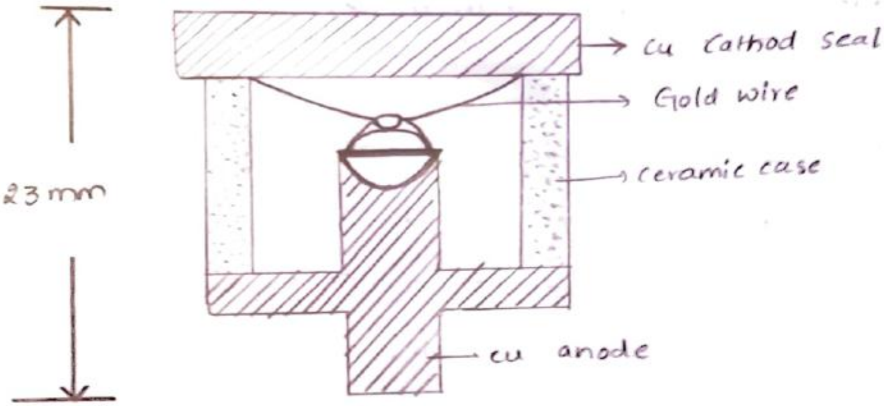
(2) TRAPATT: Trapped plasma Avalanche triggered transit device

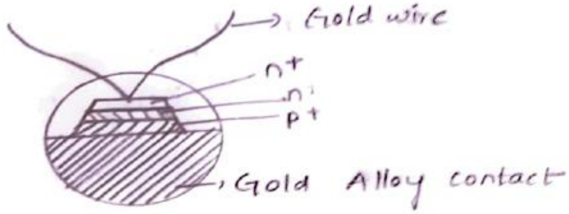
(3) BARITT : Barrier injected transit time device

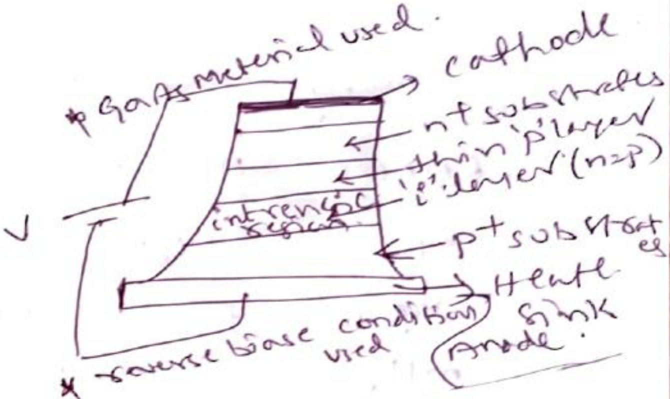
① IMPATT DEVICE:



Construction:







Performance characteristics:

Theoritical Efficiency : 30%.

practical Efficiency : < 30%.

frequency : 1 to 300 GHz

Max o/p power for single diode : 5W at x Band to 0.5W at 30GHz.

Several diodes : 40W at x Band

pulsed power : 4KW

Applications

1- Microwave generator
2- Modulated Output Oscillators
3- Receiver local Oscillators
4- Amplifier's pump
5- Alarm Network's
6- low power microwave transmitter
7- FM telecommunication transmitter
8- police radar

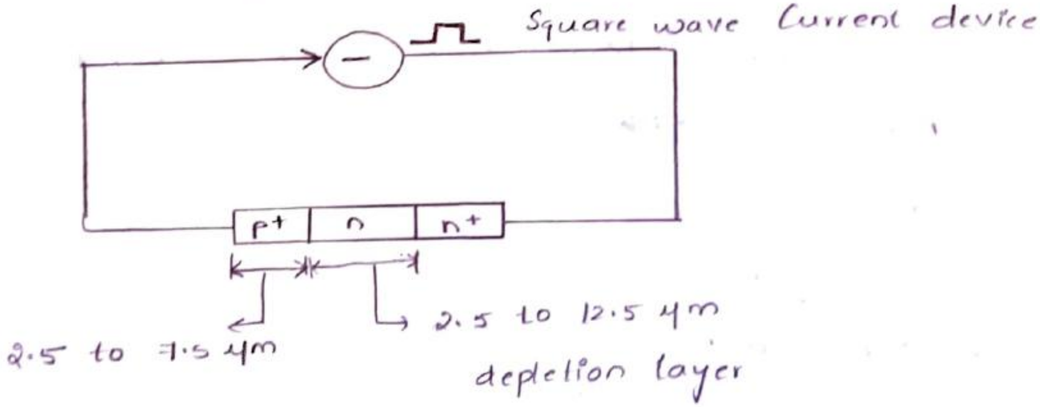
9 - CW (continous wave) doppler radar transmitter.

Disadvantages:

1. It is very noisy because avalanche is a noisy process
2. Noise figures for IMPATT being 30dB are not as good as klystron / Gunndiode / TWT Amplifier.
3. Tuning range is not as good as Gunn diodes.

TRAPATT DIODE:

-) It is a high efficiency microwave generator capable of operating from several hundred MHz to several GHz
-) It is typically $p^+ - n - n^+$ si (or) GaAs structures



Specifications:

CW power : 1 to 3W b/w 8GHz to 0.5GHz

pulse power : 1.2 kW at 1.1 GHz

Operating voltage : 60 - 150V

Efficiency : 15 to 40%.

Noise figure : >30 dB

Frequency : 3 to 50 GHz

Applications :

1 - In low power doppler radars

2 - local Oscillator for radars

3 - Microwave beacon landing system

4 - Radio Altimeter

5 - phased Array Radar.

3 - BARITT diode:

→ It is a low noise Microwave Oscillator, But it is used as a amplifier rather than an Oscillator

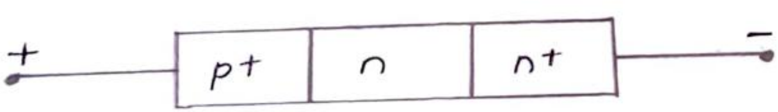


fig : Structure of BARITT DIODE

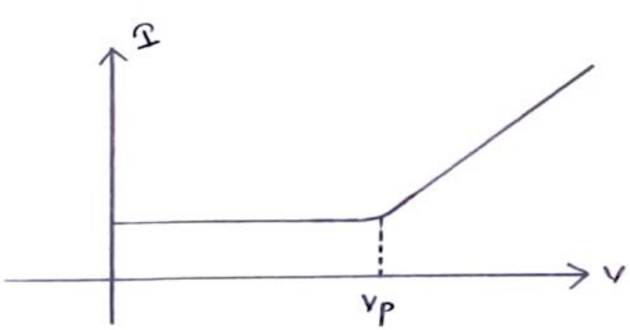


fig: VI char's of BARITT diode

here Vp = punch through Voltage.

- It is a $p^+ n n^+$ structure
- It is a low noise microwave Oscillator
- It is equivalent to a pair of diodes connected back-to-back. One diode is forward biased and another is reverse biased.
- It can be used up to X-band microwave frequencies
- It is a narrow band devices
- It is a low efficiency diode
- It is a low power device
- It's power output is less than '1mw'.
- Noise figures as low as 9dB at 6.35 GHz with 15dB gain are possible
- BARITT diodes are used as amplifiers rather than Oscillator.