# Fundamentals of AC machine windings

**Introduction to AC Machines:**

**Classification of AC Rotating Machines**

•**Synchronous Machines:**

•**Synchronous Generators:** A primary source of electrical energy.

•**Synchronous Motors:** Used as motors as well as power factor compensators (synchronous condensers).

•**Asynchronous (Induction) Machines:**

•**Induction Motors:** Most widely used electrical motors in both domestic and industrial applications.

•**Induction Generators:** Due to lack of a separate field excitation, these machines are rarely used as generators.

**Energy Conversion**

- Generators convert mechanical energy to electric energy.
- Motors convert electric energy to mechanical energy.
- The construction of motors and generators are similar.
- Every generator can operate as a motor and vice versa.
  - The energy or power balance is :
  - – Generator: Mechanical power = electric power + losses
  - – Motor: Electric Power = Mechanical Power + losses.

**Physical Arrangement Of Windings In Stator Are-**

- Pre-manufactured coils can be inserted into the stator slots one by one to form a three-phase distributed winding
- All the windings are in a single-speed three-phase motor design.

**Physical Arrangement Of Windings In Cylindrical Rotor Are-**

- The windings for rotor are less complex than stator.
- The windings of a rotor are less insulated.
- The size of the cylindrical rotor winding is small as it does not need to transmit huge current through it.

**Slots For Windings:**

The number of slots depends on how many phases of power are provided to the coil windings. A basic single-phase motor usually has **four slots** that contain two pairs of coil windings, each offset by 90 degrees; a basic three-phase motor has six slots with three pairs of coil windings, each pair offset by 120 degrees.

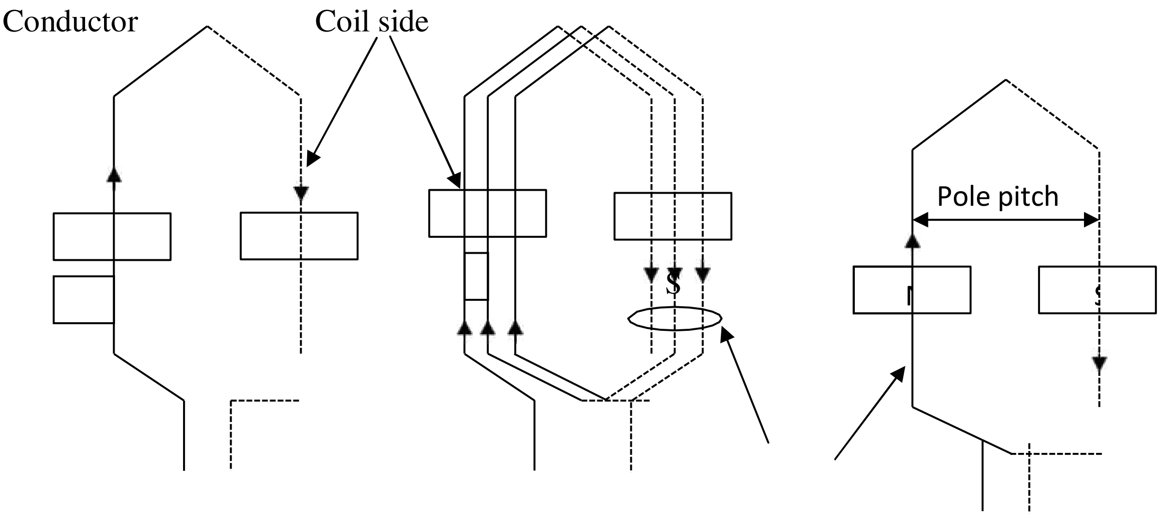
Winding Diagrams: (i) DC Winding diagrams (ii) AC Winding Diagrams

**Terminologies Used In Winding Diagrams:**

**Conductor:** An individual piece of wire placed in the slots in the machine in the magnetic field.

**Turn:** Two conductors connected in series and separated from each other by a pole pitch so that the emf induced will be additive.

**Coil:** When one or more turns are connected in series and placed in almost similar magnetic positions. Coils may be single turn or multi turn coils.




(a) Single turn Coil (b) 3 – turn coil (c) Multi turn coil

Fig. 1 Different types of winding coils representations

**Coil group:** One or more coil single coils formed in a group forms the coil group.

**Winding:** Number of coils arranged in coil group is said to be a winding.

**Pole Pitch:** Distance between the poles in terms of slots is called pole pitch.

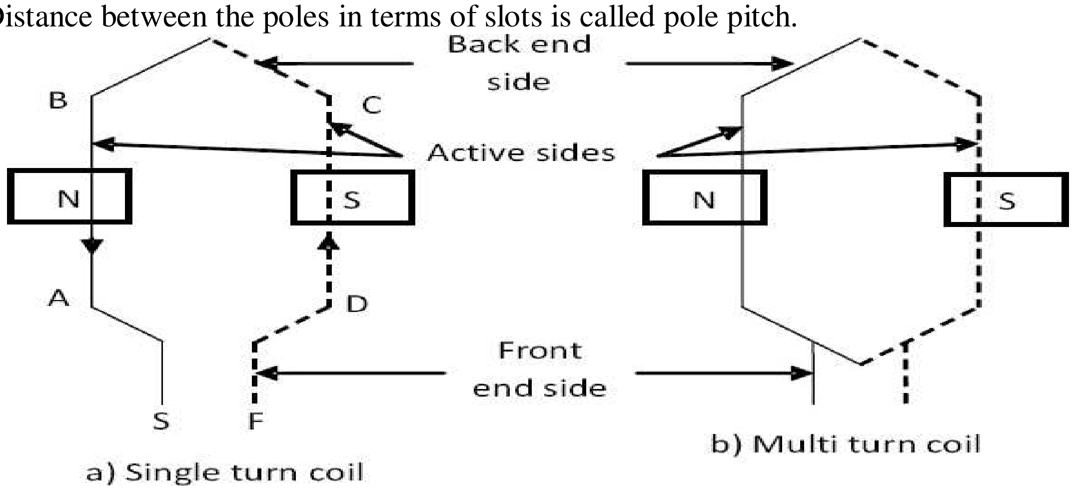



Fig. 2 Single and Multi turn coils

**Full Pitch winding:** If the coil pitch for a winding is equal to pole pitch the winding is called full pitch winding as shown in Fig .

**Chorded winding:** When the pitch of the winding is less than the full pitch or pole pitch then the winding is called short pitch winding or chorded winding.

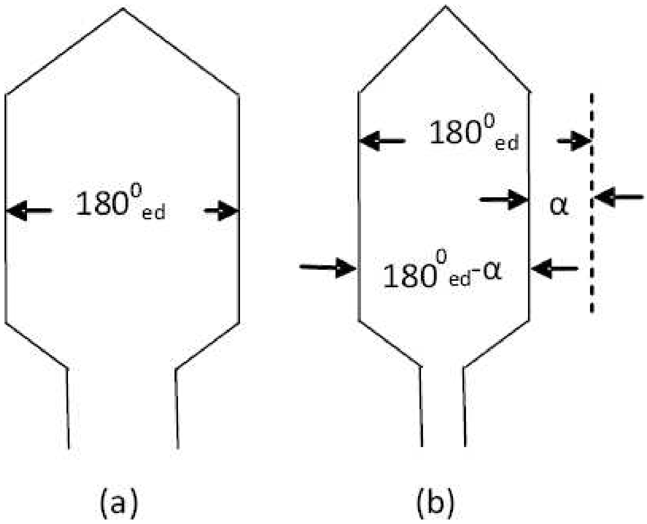



Fig. 3 Full pitched and short pitched coils

**Single layer winding:** Only one coil side placed in one slot.

**Double layer winding:** Two coil sides are placed in a single slot. Single and double layer windings are shown in Fig 4

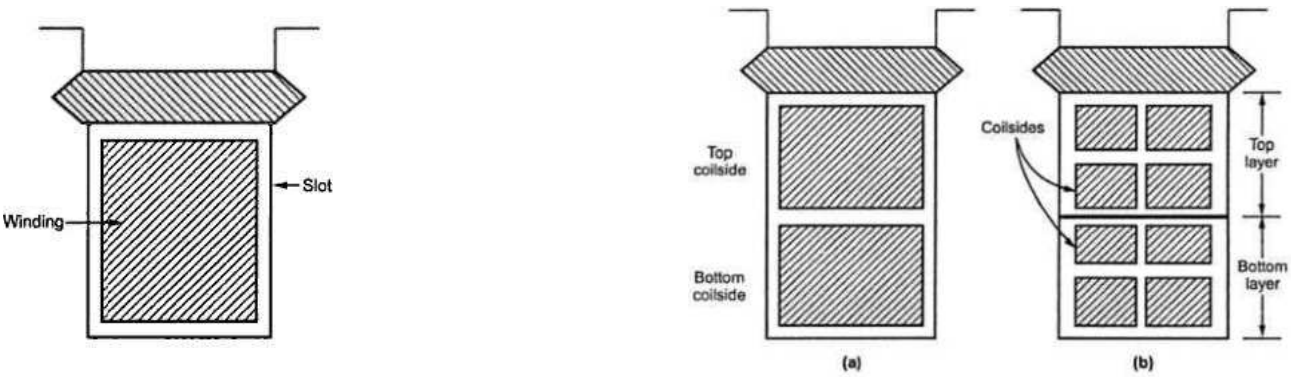
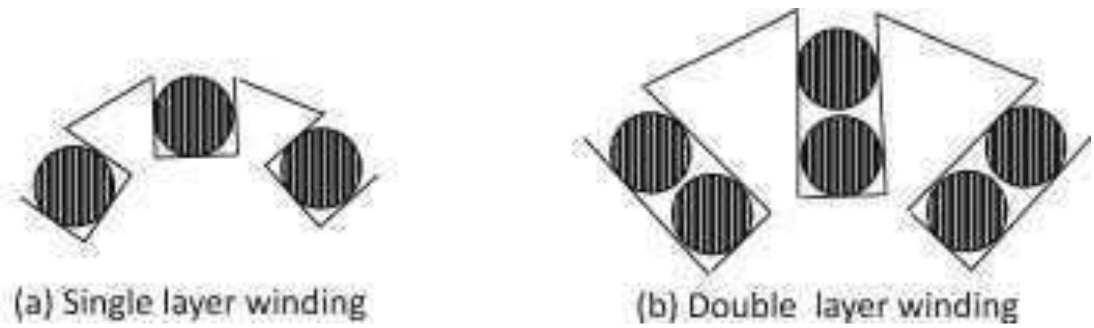



Fig.4 Single and double layer windings




**Classification of windings:** Closed type and open type winding

**Closed type windings:** In this type of winding there is a closed path around the armature or stator. Starting from any point, the winding path can be followed through all the turns and starting point can be reached. Such windings are used in DC machines.

**Open windings:** There is no closed path in the windings. Such windings are used in AC machines.

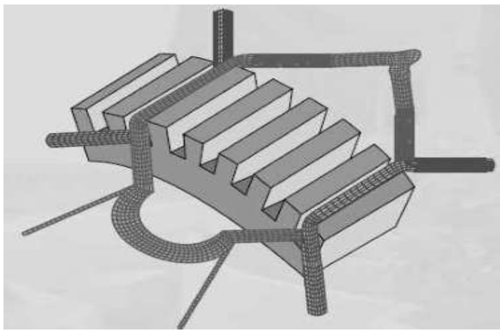


Fig. 5 Photographs of the windings and coils

**AC winding design:**

The windings used in rotating electrical machines can be classified as

**Concentrated Windings**

- All the winding turns are wound together in series to form one multi-turn coil
- All the turns have the same magnetic axis
- Examples of concentrated winding are
  - field windings for salient-pole synchronous machines
  - D.C. machines
  - Primary and secondary windings of a transformer

**Distributed Windings**

- All the winding turns are arranged in several full-pitch or fractional-pitch coils
- These coils are then housed in the slots spread around the air-gap periphery to form phase or commutator winding
- Examples of distributed winding are
  - Stator and rotor of induction machines
  - The armatures of both synchronous and D.C. machines

Armature windings, in general, are classified under two main heads, namely,

**Closed Windings**

- There is a closed path in the sense that if one starts from any point on the winding and traverses it, one again reaches the starting point from where one had started
- Used only for D.C. machines and A.C. commutator machines

**Open Windings**

- Open windings terminate at suitable number of slip-rings or terminals
- Used only for A.C. machines, like synchronous machines, induction machines, etc

**EMF Equation of a Synchronous Generator**

The generator which runs at a synchronous speed is known as the synchronous generator. The synchronous generator converts the mechanical power into electrical energy for the grid. The Derivation of EMF Equation of a synchronous generator is given below.

Let,

P be the number of poles

$\phi$ is Flux per pole in Webers

N is the speed in revolution per minute (r.p.m)

f be the frequency in Hertz

Zph is the number of conductors connected in series per phase

Tph is the number of turns connected in series per phase

Kc is the coil span factor

Kd is the distribution factor

Flux cut by each conductor during one revolution is given as P$\phi$ Weber.

Time taken to complete one revolution is given by 60/N sec

Average EMF induced per conductor will be given by the equation shown below

$$\frac{P\varphi}{60/N} = \frac{P\varphi N}{60} \quad \text{volts}$$

Average EMF induced per phase will be given by the equation shown below

$$\frac{P\varphi N}{60} \times Z_{ph} = \frac{P\varphi N}{60} \times 2T_{ph} \quad \text{and}$$

$$T_{ph} = \frac{Z_{ph}}{2}$$

$$\text{Average EMF} = 4 \times \varphi \times T_{ph} \times \frac{PN}{120} = 4\varphi f T_{ph}$$

The average EMF equation is derived with the following assumptions given below.

Coils have got the full pitch.

All the conductors are concentrated in one stator slot.

**Root mean square** (R.M.S) value of the EMF induced per phase is given by the equation shown below.

**$E_{ph}$ = Average value x form factor**

Therefore,

$$E_{ph} = 4\varphi f T_{ph} \times 1.11 = 4.44\, \varphi\, f\, T_{ph} \quad \text{volts}$$

If the coil span factor $K_c$ and the distribution factor $K_d$, are taken into consideration than the Actual EMF induced per phase is given as

$$E_{ph} = 4.44\, K_c K_d \varphi\, f\, T_{ph} \quad \text{volts} \ldots\ldots(1)$$

Equation (1) shown above is the EMF equation of the Synchronous Generator.

**Winding Factors (Coil Pitch and Distributed Windings)**

**Pitch Factor:**

A coil whose sides are separated by one pole pitch (i.e., coil span is 180° electrical) is called a full-pitch coil. With a full-pitch coil, the e.m.f.s induced in the two coil sides a in phase with each other and the resultant e.m.f. is the arithmetic sum of individual e.m.fs. However the waveform of the resultant e.m.f. can be improved by making the coil pitch less than a pole pitch. Such a coil is called short-pitch coil. This practice is only possible with double-layer type of winding The e.m.f. induced in a short-pitch coil is less than that of a fullpitch coil. The factor by which e.m.f. per coil is reduced is called pitch factor $K_p$. It is defined as:

$$K_p = \frac{\text{e.m.f. induced in short - pitch coil}}{\text{e.m.f. induced in full - pitch coil}}$$

Consider a coil AB which is short-pitch by an angle Θ electrical degrees as shown in Fig. (11). The e.m.f.s generated in the coil sides A and B differ in phase by an angle Θ and can

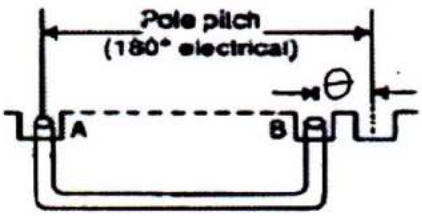
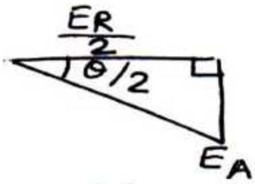



Fig. 11



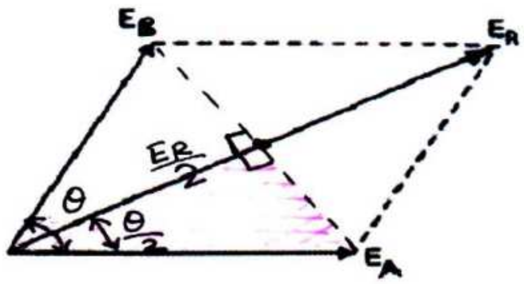




Fig. 12

Phasor sum

Since $E_A = E_B$. $\quad E_R = 2E_A \cos\theta/2$

$$\text{Pitch factor, } K_p = \frac{\text{e.m.f. in short - pitch coil}}{\text{e.m.f. in full - pitch coil}} = \frac{2E_A \cos\theta/2}{2E_A} = \cos\theta/2$$

$$\therefore \quad K_p = \cos\theta/2$$

arithmetic sum
$E_A + E_A$

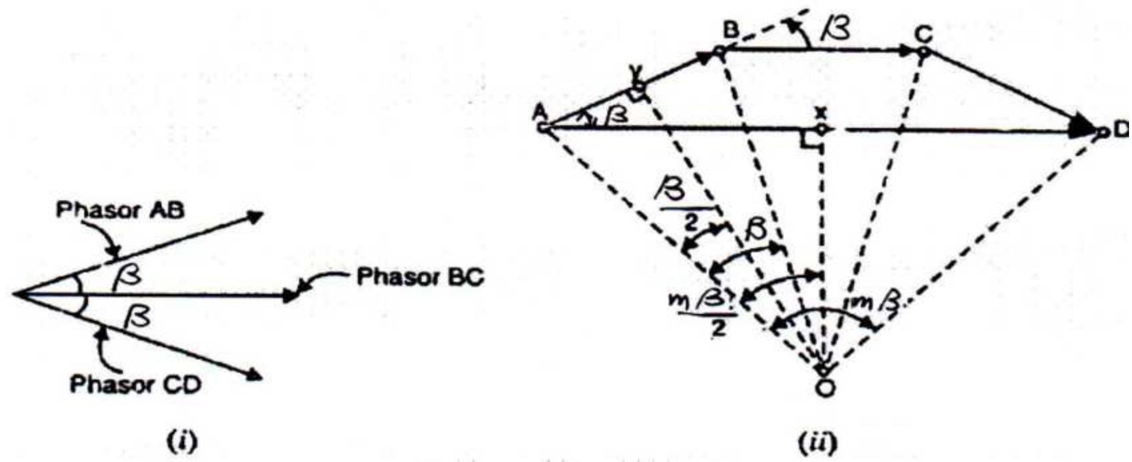
For a full-pitch winding, $K_p = 1$. However, for a short-pitch winding, $K_p < 1$.
Note that Θ is always an integer multiple of the slot angle β.

**Distribution Factor:**

Even though we assumed concentrated winding in deriving EMF equation, in practice an attempt is made to distribute the winding in all the slots coming under a pole. Such a winding is called distributed winding.

In concentrated winding the EMF induced in all the coil sides will be same in magnitude and in phase with each other. In case of distributed winding the magnitude of EMF will be same but the EMF s induced in each coil side will not be in phase with each other as they are distributed in the slots under a pole. Hence the total EMF will not be same as that in concentrated winding but will be equal to the vector sum of the EMF s induced. Hence it will be less than that in the concentrated winding. Now the factor by which the EMF induced in a distributed winding gets reduced is called distribution factor and defined as the ratio of EMF induced in a distributed winding to EMF induced in a concentrated winding.

Distribution factor Kd = EMF induced in a distributed winding/ EMF induced in a concentrated winding
= vector sum of the EMF / arithmetic sum of the EMF




Let
E = EMF induced per coil side
m = number of slots per pole per phase,
n = number of slots per pole
β = slot angle = 180/n
The EMF induced in concentrated winding with m slots per pole per phase = mE volts.
Fig below shows the method of calculating the vector sum of the voltages in a distributed winding having a mutual phase difference of β. When m is large curve ACEN will form the arc of a circle of radius r.
From the figure below AC = 2rsin β/2
Hence arithmetic sum = m2r sinβ/2
Now the vector sum of the EMF s = 2rsin mβ/2
Hence the distribution factor $K_d$ = vector sum of the EMF / arithmetic sum of the EMF
= (2r sin mβ/2) / (m x 2r sin β/2)
$K_d$ = ( sin mβ/2) / (m sin β/2)

In practical machines the windings will be generally short pitched and distributed over the periphery of the machine. Hence in deducing the EMF equation both pitch factor and distribution factor has to be considered. Hence the general EMF equation including pitch factor and distribution factor can be given as
EMF induced per phase = 4.44 f Tph x Kp$K_d$ volts
$E_{ph}$ = 4.44 KpKd f Tph vlolts
Hence the line Voltage $E_L$ = √3 x phase voltage = √3 Eph

**MMF Space Wave of a Single Coil (Air-gap MMF distribution with fixed current through winding)**

A cylindrical rotor machine with small air-gap as shown in Fig. 5.24(a) will be assumed here. The stator is imagined to be wound for two-poles with a single N-turn full-pitch coil carrying current i in the direction indicated. The figure shows some flux lines of the magnetic field set up. A north and corresponding south pole are induced on the stator periphery. The magnetic axis of the coil is from the stator north to the stator south. Each flux line radially crosses the air-gap twice, normal to the stator and rotor iron surfaces and is associated with constant mmf Ni. On the assumption that the reluctance of the iron path is negligible, half the mmf (Ni/2) is consumed to create flux from the rotor to stator in the air-gap and the other half is used up to establish flux from the stator to rotor in the air-gap. Mmf and flux radially outwards from the rotor to the stator (south pole on stator) will be assumed to be positive and that from the stator to rotor as negative.

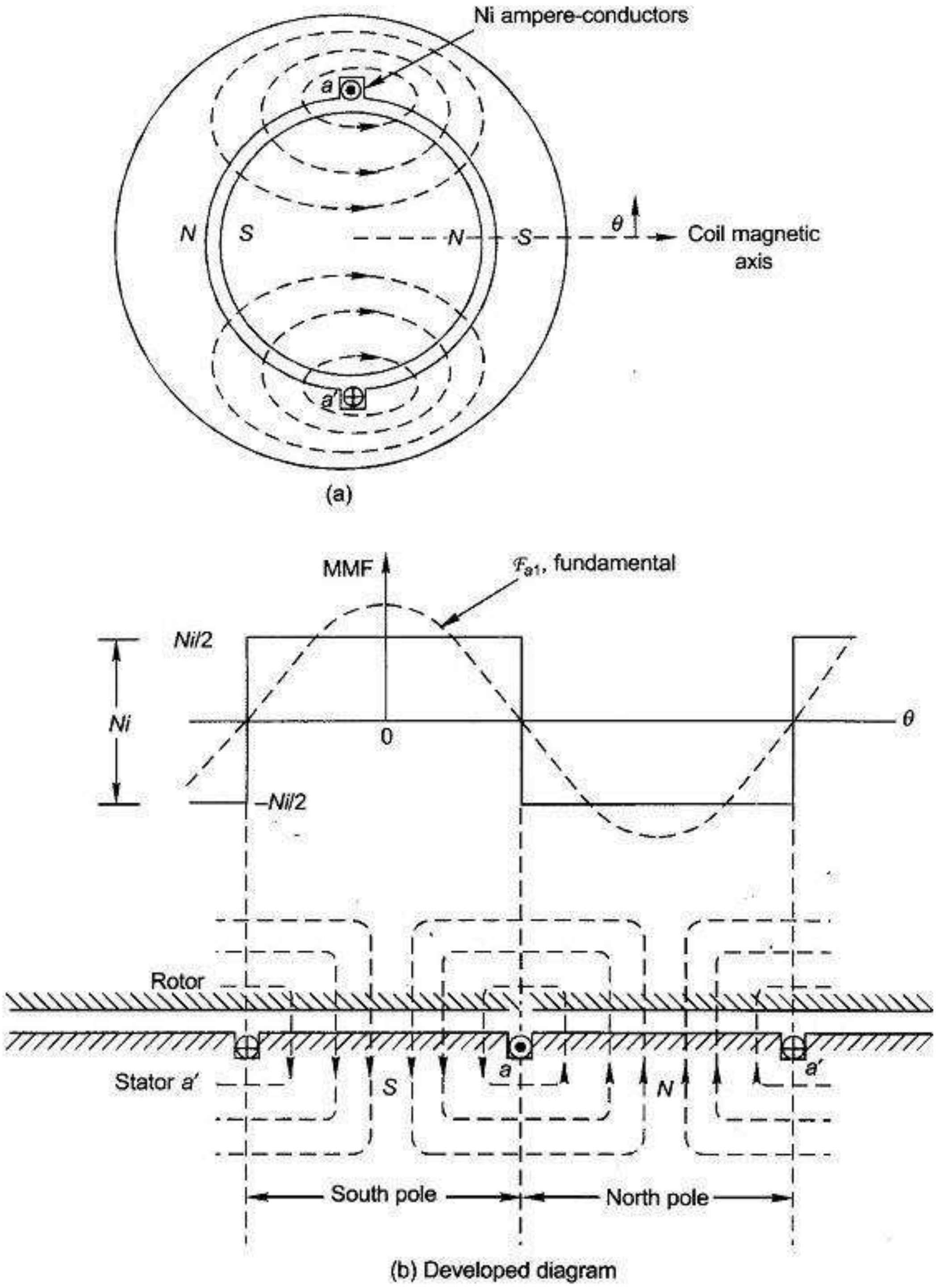



**Fig. 5.24** *Mmf space wave of a single coil*

The physical picture is more easily visualized by the developed diagram of Fig. 5.24(b) where the stator with the winding is laid down flat with the rotor on the top of it. It is seen that the mmf is a rectangular space wave wherein mmf of + Ni/2 is consumed in setting flux from the rotor to stator and mmf of – Ni/2 is consumed in setting up flux from the stator to the rotor. It has been imagined here that the coil-sides occupy a narrow space on the stator and the mmf changes abruptly from –Ni/2 to + Ni/2 at one slot and in reverse direction at the other slot. The mmf change at any slot is

$$Ni = \text{ampere-conductors/slot}$$

and its sign depends upon the current direction.

The mmf space wave of a single coil being **rectangular**, it can be split up into its fundamental and harmonics.

**MMF Space Wave of One Phase of a Distributed Winding:**

Consider now a basic 2-pole structure with a round rotor, with 5 slots/pole/phase (SPP) and a 2-layer winding as shown in Fig. 5.25. The corresponding developed diagram is shown in Fig. 5.26(a) along with the mmf diagram which now is a stepped wave obviously closer to a sine wave than the rectangular mmf wave of a single coil (Fig. 5.24(b)). Here since SPP is odd (5), half the ampere-conductors of the middle slot of the phase group a and a′ contribute towards establishment of south pole and half towards north pole on the stator. At each slot the mmf wave has a step jump of $2N_ci_c$, ampere-conductors where $N_c$ = coil turns (equal to conductors/layer) and $i_c$ = conductor current.

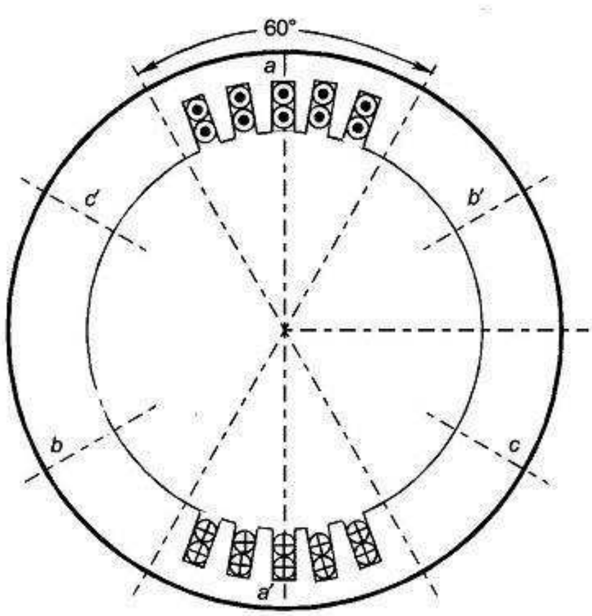



**Fig. 5.25** *A 3-phase, 2-pole structure with two-layer winding*

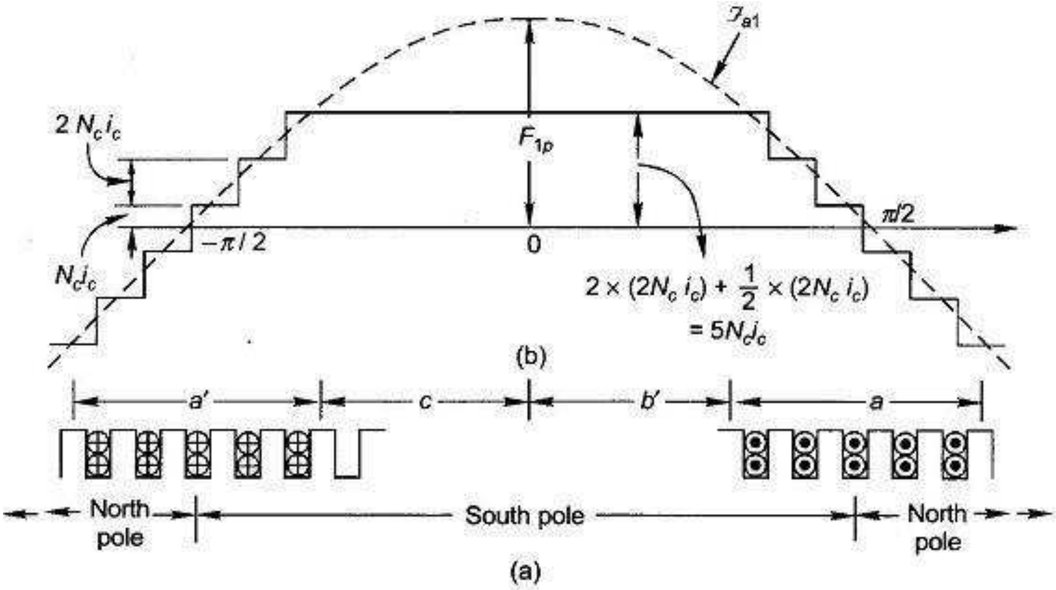



**Fig. 5.26** *Developed diagram and mmf wave of the machine of Fig. 5.25*

Now $F_{1p}$, the peak of the fundamental of the mmf wave, has to be determined. Rather than directly finding the fundamental of the stepped wave, one can proceed by adding the fundamentals of the mmf s of individual slot-pairs (with a span of one pole-pitch). These fundamentals are progressively out of phase (space phase as different from time phase) with each other by the slot angle $\gamma$. This addition is easily accomplished by defining the breadth factor $K_b$, which will be the same as in the case of the generated emf of a coil group.

**Harmonic Effect**

- The flux distribution along the air gaps of alternators usually is non-sinusoidal so that the emf in the individual armature conductor likewise is non-sinusoidal
- The sources of harmonics in the output voltage waveform are the non-sinusoidal waveform of the field flux.
- Fourier showed that any periodic wave may be expressed as the sum of a d-c component (zero frequency) and sine (or cosine) waves having fundamental and multiple or higher frequencies, the higher frequencies being called harmonics.

***Elimination or Suppressed of Harmonics***

Field flux waveform can be made as much sinusoidal as possible by the following methods:

1. Small air gap at the pole centre and large air gap towards the pole ends
2. ***Skewing:*** skew the pole faces if possible
3. ***Distribution*****:** distribution of the armature winding along the air-gap periphery
4. ***Chording:*** with coil-span less than pole pitch
5. ***Fractional slot winding***
6. ***Alternator connections:*** star or delta connections of alternators suppress triplen harmonics from appearing across the lines

Note: For Problems and winding diagrams, please go through the class note book

# Induction motors

AC s/m is universally adopted for all applications such as lighting purposes, heating purposes, industrial purposes.

Motor manufactures have tried to manufacture motors for industrial purposes as AC motors.

Industrial purposes the need of AC motors increases. thus the motor manufacture manufactures single AC & 3 phase AC motors.

An AC motor is connected to AC supply. a rotating magnetic field is set up in stator.

The speed of rotating magnetic field which revolues in the stator is called synchronous speed.

Such motors are called as synchronous motor.

AC motors are classified as

| Synchronous motors | asynchronous motor |
|---|---|
| Speed of rotating field is synchronous | Speed of rotating field is below the synchronous speed or other than synchronous speed. |

In case of dc motors, electrical power is conducted directly to the rotor part through brushes & commutation

Hence such motors are called a conduction motor.

In case of AC motor, Rotor does not recieve electric power by conduction but rotor recieves electric power by induction. Such motor recieves electric power by induction are called Induction motors.

An induction motor can be treated as a rotating transformer because the stator is stationary one can acts as primary winding which ~~[illegible]~~ takes the supply AC. the rotor is rotating one can acts as a secondary which recieves electrical power by induction to convert the mechanical power at the rotor.

Advantages of I.M

→ simple in construction, extremely rugged
→ cost is low & more reliable
→ It has high efficiency, brushes are absent for normal running conditions, hence frictional losses are reduced.
→ It maintains good power factor i.e 0.85
→ maintainance is minimum
→ It is inherently a self starting motor i.e it does not require any separate motor to start up from rest position.

No need to synchronise with the supply frequency.

Disadvantages

→ It Runs at Constant speed, speed Cannot be varied.

It is possible to control the speed, the efficiency of such motor losses or decreases

→ Its starting torque is low when Compared to dc shunt motor.

Principle of Induction motor

Induction motor is an ac motor, the supply is given to the stator which is stationary one, current will set up a magnetic field in stator. This field revolves in stator at synchronous speed. This rotating magnetic field will induces a Current in rotor by induction will set up a torque. The torque produced in rotor is rotating other than the synchronous ~~speed~~ speed.

Construction

It Consists of stator & Rotor

ⓐ Stator

It is a stationary one, which consists of number of stampings to recieve the windings as slots.

→ Stator Carries a 3-phase winding & is given 3 phase ac supply.

→ The speed of the Induction motor depends upon the number of poles

$P = 2n$ where $n$ is no. of stator slots per phase per pole the rotor slots are not parallel with the rotor shaft but are arranged as skewed the main advantages of provided skewed slots are

→ Reducing the magnetic hum
→ Reducing the locking tendency of rotor

i.e. Rotor teeth remains under stator teeth due to magnetic attraction.

In small motors, complete rotor core is placed in a mould & all the bars & end rings forms as one piece. The metal used here are alluminium alloy.

$\boxed{\text{phase - wound Rotor}}$

The rotor is provided with 3 phase supply consists of double layer, distributed winding.

→ Here rotor is wound as number of poles & is always wound 3 phase even, when stator is wound as 2 phase.

→ Here stator & rotor are 3 phase star connected manner.

Rotor

→ The three phases are star connected internally & are brought out connected to the 3 insulated slip rings mounted on the rotor shaft.

→ Here slip rings are used to increase the starting torque of I.m by provided or providing as external resistance connected to the each slip ring.

* rotors slots are not parallel to the rotor shaft they are provided slightly skewed.

Skewing of rotor helps

(I) Reduces magnetic humming

(II) Reduces lock tendency of rotor, it avoids of rotor teeth under stator teeth

Advantages of squirrel cage rotor

* construction is rugged
* Rotor bars completely welded so no burning of winding
* Long life + little maintenance
* High efficient

Disadvantages

* Rotor bars are permanently welded no starting resistances are added to control speed
* starting torque is low.
* Starting currents is very high i.e 6 to 7 times rated current.
* operates on low power factor.

(2) Phase wound rotor or slip ring rotor

* Here Rotor is wound for many poles.
* Rotor windings wound similar to stator winding for number of poles.
* Here rotor is wound for 3 phases. are connected to star manner.
* Other 3 windings are brought out + are connected to 3 insulated slip rings. mounted on shaft. with brushes.
* The windings through brushes are connected to 3 phase star connected rheostat.

Greater the number of poles, lesser the speed of I.M.

→ when stator is supplied with 3 phase supply a magnetic field is set up in stator, this field revolves in stator at synchronous speed having constant ~~[illegible]~~ magnitude.

$$N_S = \frac{120 * f}{P}$$

Rotor

$\boxed{\text{Rotor}}$

2 Types of rotor

(i) squirrel - cage rotor (ii) phase - wound rotor.

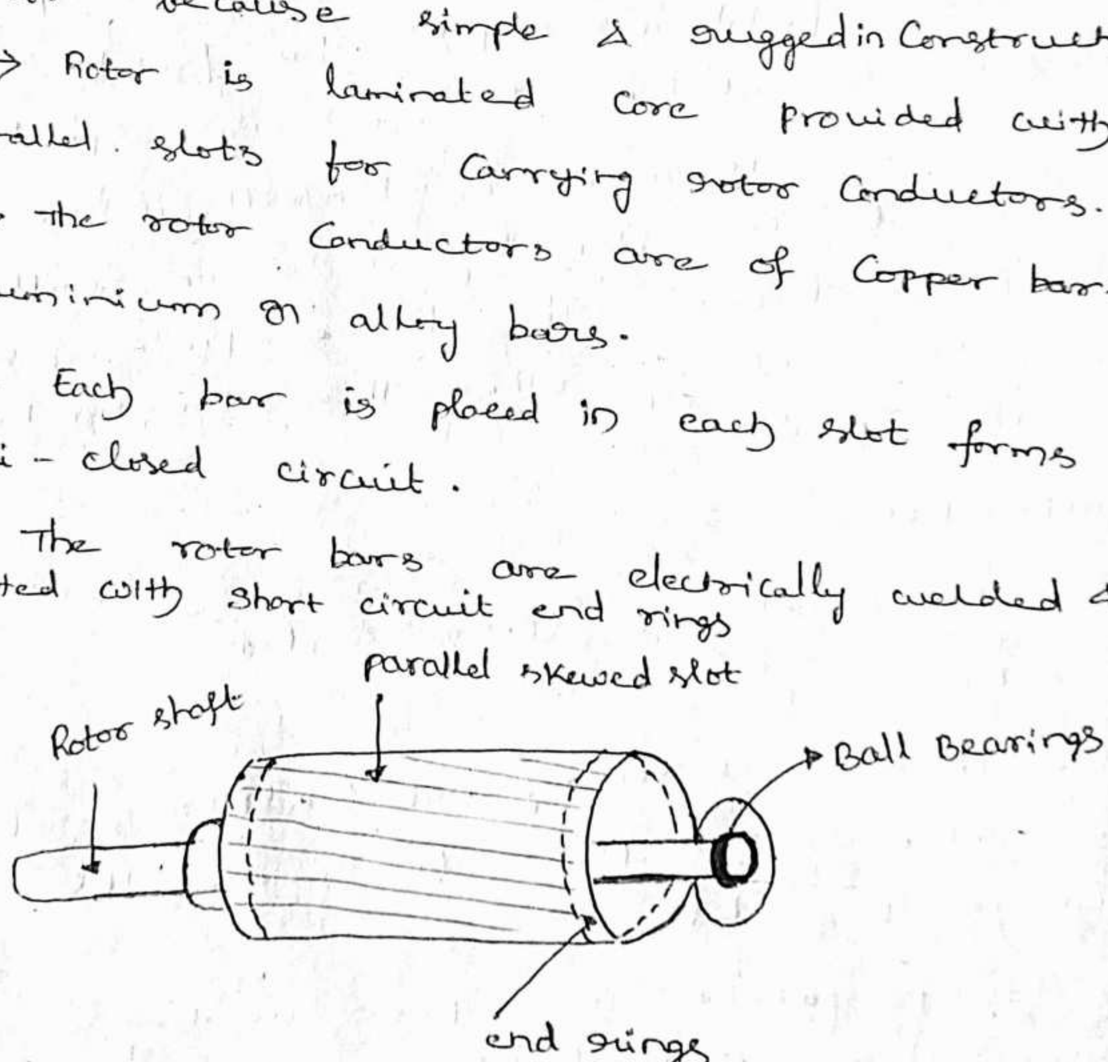
90% of I.M uses squirrel - cage type rotor because simple & rugged in construction.

→ Rotor is laminated core provided with parallel slots for carrying rotor conductors.

→ the rotor conductors are of copper bars or alluminium or alloy bars.

→ Each bar is placed in each slot forms a semi - closed circuit.

→ The rotor bars are electrically welded & is bolted with short circuit end rings




→ The rotor bars are ~~[illegible]~~ ~~[illegible]~~ completely short circuited and not possible to add any external resistance for starting purposes.

The rotor slots are not parallel with the motor shaft but are arranged as skewed. The main advantages of provided skewed slots are

→ Reducing the magnetic hum
→ Reducing the locking tendency of rotor
i.e. Rotor teeth remains under stator teeth due to magnetic attraction.

In small motors, complete rotor core is placed in a mould & all the bars & end rings forms as one piece. The metal used here are alluminium alloy.

$\boxed{\text{phase - wound Rotor}}$

The rotor is provided with 3 phase supply consists of double layer, distributed winding.

→ Here rotor is wound as number of poles & is always wound 3 phase even, when stator is wound as 2 phase.

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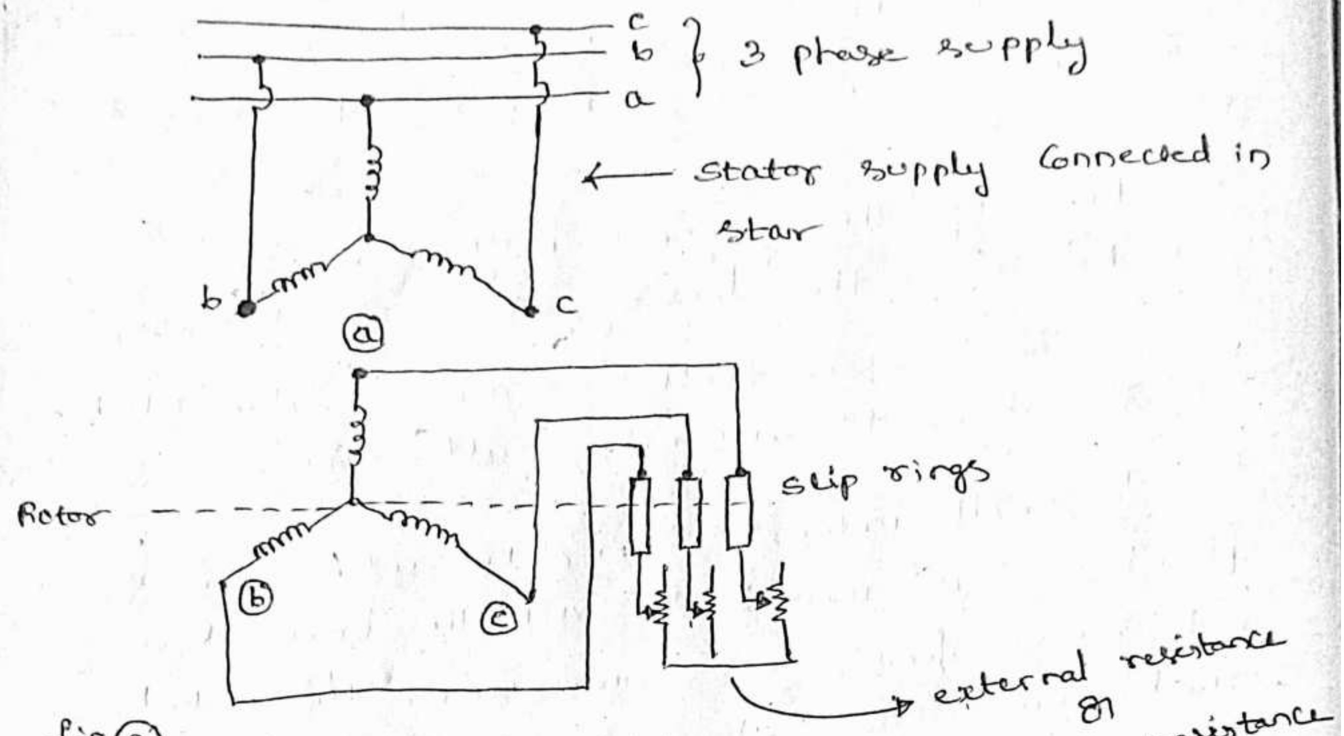



fig ⓐ

phase wound rotor provided with slip rings

During normal running conditions, the slip rings are short circuited automatically. i.e rotor is short circuited.

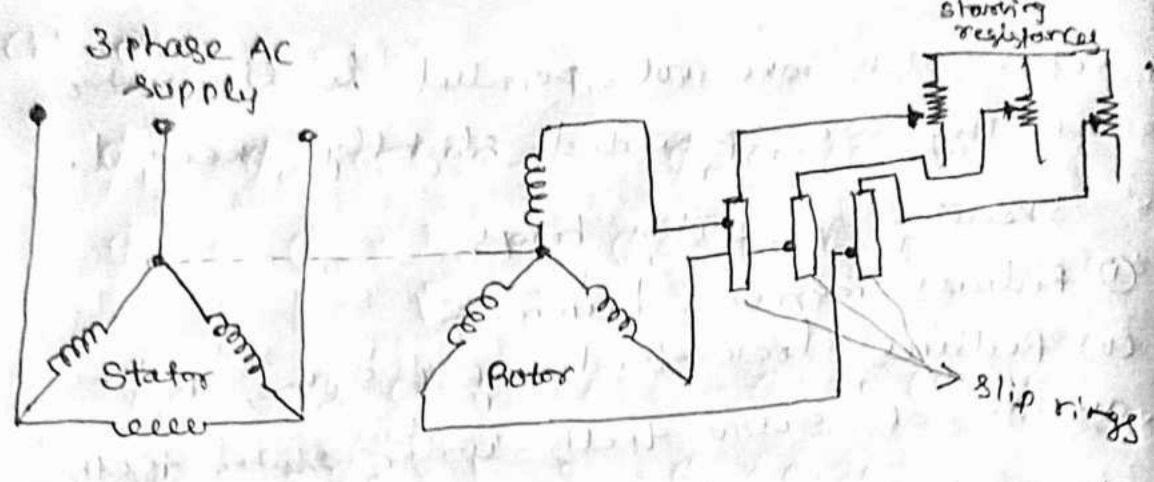
Production of Rotating field

When stator is provided with 2 phase supply or 3 phase supply, a rotating magnetic flux of constant magnitude is produced in stator.

Two supply

The principle of 2 phase, 2 pole stator consists of 2 identical windings each are spaced at 90° apart.

Let current flowing through each phase winding be sinusoidal. flux set up in each phase windings $\phi_1$ & $\phi_2$.




Advantages

* starting is high ~~torque~~ or starting ~~torque~~ is high due to internal resistances with slip rings.
* External resistances are to be added to rotor circuit
* requires little current to start
* speed can be easily controlled using External resistances.
* operates at high power factor.

Draw backs

* construction is complicated
* chance of burning of rotor windings
* frequent or Requires maintenance
* performance is low

General Principle of induction motor

In dc motors electric power is conductively give to armature (rotor). Hence dc motor is a conduction motor.

In case of AC motors, rotor recieve power by induction such motors recieves electric power are called induction motors.

when stator is connected to 3 phase supply stator windings carries ~~[illegible]~~ currents produces a rotating flux having Constant magnitude revolving in stator at synchronous speed.

* This revolving flux induces an emf in rotor Conductors by mutual induction.

* This induced emf set up a torque which tends to rotates the rotor other than the synchronous speed.

## Production of rotating field

if stator is provided with 2 phase or 3 phase supply, a rotating flux of constant magnitude is produced in stator.

Two phase supply:

Let phase ① & phase ② are 2 phase supply each are spaced at 90° apart.

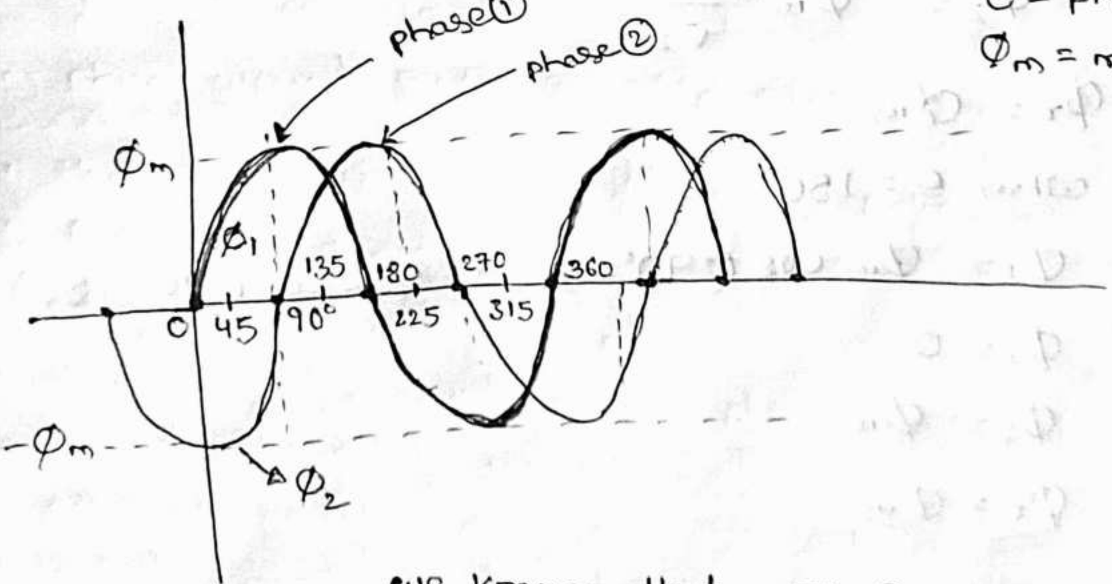
Let Current flowing through each phase is sinusoidal produces a sinusoidal flux in each windings $\phi_1$ & $\phi_2$.

Let $\phi_r$ be the resultant flux

$$\phi_r = \phi_1 + \phi_2$$

$\theta$ = phase angle

$\phi_m$ = max flux




we know that $\sin\theta = \cos(90-\theta)$

at

① $\theta = 0$, $\phi = \cos(90-0) = 0$

$\phi_1 = 0$, $\phi_2 = -\phi_m$

$$\phi_r = \sqrt{(\phi_1)^2 + (\phi_2)^2} = \sqrt{(-\phi_m)^2} = \phi_m$$

(b) when $\theta = 45°$

$$\phi_1 = \phi_m \cos(90-\theta) = \phi_m \cos(90-45)$$

$$\phi_m = \phi_m * \frac{1}{\sqrt{2}} \text{ or } \phi_m * 0.707$$

$$\phi_2 = -\phi_m \cos(90-\theta) = -\phi_m \cos(90-45)$$

$$\phi_2 = -\phi_m * \frac{1}{\sqrt{2}} \text{ or } -\phi_m * 0.707$$

$\phi_r = \phi_m$ $\qquad \phi_r = \sqrt{\phi_1^2 + \phi_2^2}$

(c) when $\theta = 90°$

$$\phi_1 = \phi_m \cos(90-\theta) = \phi_m \cos(90-90)$$

$$\phi_1 = \phi_m$$

$$\phi_2 = -\phi_m * 0 = 0$$

$$\phi_r = \sqrt{\phi_1^2 + \phi_2^2} = \sqrt{(\phi_m)^2 + 0} = \phi_m$$

(d) when $\theta = 135°$

$$\phi_1 = \phi_m \cos(90-\theta) = \phi_m \cos(90-135)$$

$$\phi_1 = \phi_m * \frac{1}{\sqrt{2}}$$

$$\phi_2 = \phi_m * \frac{1}{\sqrt{2}}$$

$$\phi_r = \phi_m$$

(e) when $\theta = 180°$

$$\phi_1 = \phi_m \cos(90° - \theta) = \phi_m \cos(90° - 180)$$

$$\phi_1 = 0$$

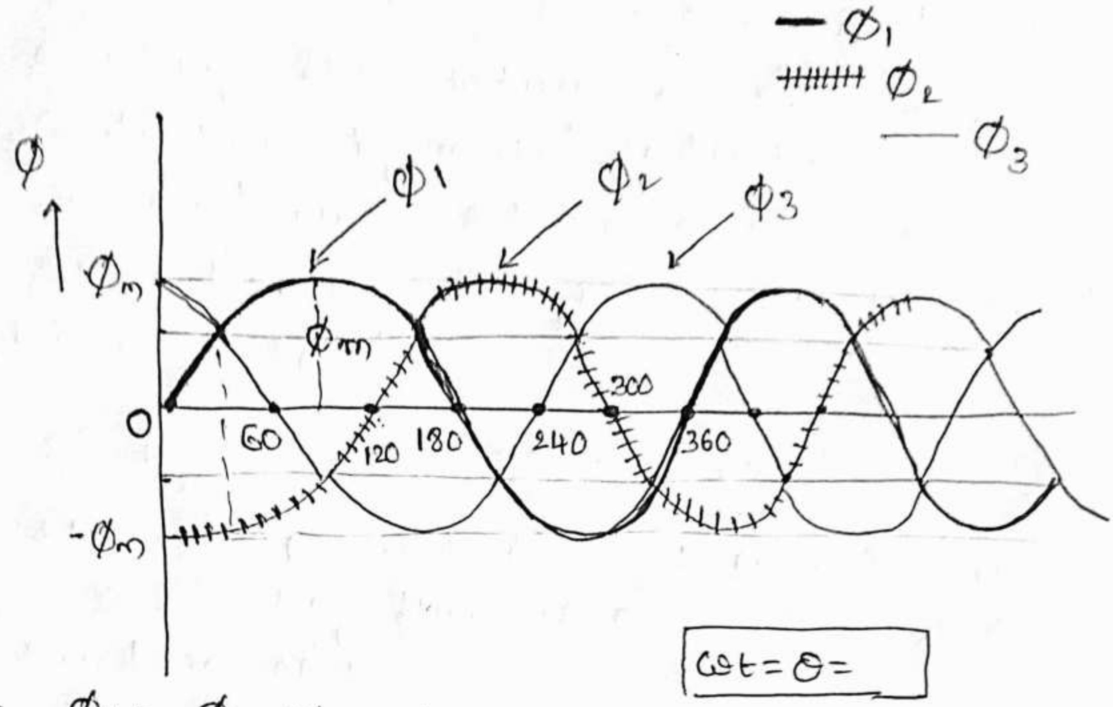
$$\phi_2 = \phi_m$$

$$\phi_r = \phi_m$$

from, we Conclude that resultant flux is Constant & is equal to $\phi_m$.

So $\phi_r \rightarrow$ resultant flux rotates at synchronous speed in stator $\quad N_s = 120f/P$.

For 3 phase supply




Let $\phi_1 = \phi_m \sin \omega t$

$\phi_2 = \phi_m \sin(\omega t - 120)$, $\phi_3 = \phi_m \sin(\omega t - 240)$

if $\theta = 0$

$\phi_1 = 0$, $\phi_2 = -\phi_m \sin(0 - 120) = -\phi_m * \left(-\frac{\sqrt{3}}{2}\right)$

$\phi_2 = \phi_m * \frac{\sqrt{3}}{2}$

$\phi_3 = \phi_m \sin(\omega t - 240) = -\phi_m * \frac{\sqrt{3}}{2}$

$$\phi_r = \sqrt{\phi_1^2 + \phi_2^2 + \phi_3^2} = \sqrt{0 + \left(-\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{3}{4} + \frac{3}{4} + 0 = \frac{6}{4} = \frac{3}{2}\phi_m$$

if $\theta = 60°$

## Rotor emf & Rotor reactance

$E_r = SE_2$
$X_r =$

Under Running Conditions

Let

$E_2$ = standstill rotor emf/phase
$X_2$ = rotor reactance/phase at standstill
$f_2$ = Rotor frequency at standstill

if rotor is at rest position (About to start)

$S = 1$

$f_2$ = stator supply frequency '$f$'

$E_2$ is maximum at stand still position

$$\left\{ \begin{array}{l} E_r = SE_2 = 1 * E_2 \text{ (maximum)} \\ X_r = SX_2 = 1 * E_2 \\ f_r = Sf_2 = 1 * f_2 = f \text{ (supply)} \end{array} \right\}$$

When rotor is running, it is due relative speed b/w ~~rotor~~ rotating stator flux & rotor conductor, $\phi$ is decreased, relative speed also decreased.

Hence for a slip 'S' rotor emf will be 'S' times the induced emf at standstill.

Under running Conditions

$E_r = SE_2$

$f_r = Sf_2$

$X_r = SX_2$

## Torque under running Conditions

$T \propto E_2 I_2 \cos\phi_2$ or

$T \propto \phi I_2 \cos\phi_2$ if V is constant $\phi = E_2$

under running Conditions the Torque is given as

$I \propto \phi \propto E_2$

$$T \propto E_r I_r \cos\phi_2$$

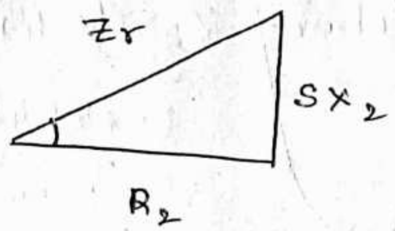
if supply voltage is Constant then

$\phi \propto E_r$ $\quad E_r = \phi$

$$T \propto \phi I_r \cos\phi_2$$

$E_r$ = Rotor emf / phase under running Conditions

$I_r$ = Rotor emf / phase " " "




$$I_r = \frac{E_r}{Z_r} = \frac{S E_2}{\sqrt{R_2^2 + (S X_2)^2}}$$

$$\cos\phi_2 = \frac{R_2}{Z_r} = \frac{R_2}{\sqrt{R_2^2 + (S X_2)^2}}$$

$$T \propto \frac{S \phi E_2 R_2}{R_2^2 + (S X_2)^2} = \frac{K \phi \cdot S \cdot E_2 R_2}{R_2^2 + (S X_2)^2}$$

at standstill $E_2 \propto \phi$

$$T = \frac{K_1 E_2 * S * E_2 R_2}{R_2^2 + (S X_2)^2} = \frac{K_1 * S * E_2^2 R_2}{R_2^2 + (S X_2)^2}$$

$$T = \frac{3}{2\pi N_S} * \frac{S E_2^2 R_2}{R_2^2 + (S X_2)^2} = \frac{3}{2\pi N_S} * \frac{S E_2^2 R_2}{Z_r^2}$$

At standstill rotor emf $E_2 \propto \phi$

$S = 1$

$$T = \frac{3}{2\pi N_S} * \frac{E_2^2 R_2}{Z_r^2} \qquad Z_r^2 = R_2^2 + X_2^2$$

(Q) λ connected I.M has standstill impedance (0.4+j4) Ω/phase & Rheostat impedance (6+j2) Ω/phase.

Induced emf of motor is 80V b/w slip rings at standstill when connected to normal supply voltage. Find (I) Rotor Current at standstill with Rheostat (II) when slip rings are short circuited & motor is running with a slip 3%.

sol

At standstill Conditions

Induced emf / phase $E_2 = 80/\sqrt{3} = 46.2V$

Stator impedance / phase $Z_2 = (0.4+j4) + (6+j2)$

$Z_2 = 6.4 + j6$

$= 8.77 \angle 43.16$

Rotor Current / phase $I_2 = \frac{E_2}{Z_2} = \frac{46.2}{8.77} = 5.27$

given $\phi = 43.16$

$P.f = \cos\phi = \cos(43.16) = 0.729$

(II) slip rings are short circuited under running Conditions.

Rotor induced emf / phase $= SE_2$

$E_r = SE_2 = 0.03 * 46.2 = 1.386V$

Rotor impedance / phase $Z_r = R_2 + j(SX_2)$

$= 0.4 + j(0.03*4)$

Rotor current / phase $\boxed{I_r = \frac{E_r}{Z_r}} = \frac{1.386}{0.4 + j0.12}$

Maximum Torque Condition under running Condition

under running Condition, Torque

$T \propto \frac{\phi E_2 * S * R_2}{R_2^2 + (X_2 S)^2}$ at standstill position $E_2 \propto \phi$

$T = \frac{K_1 * S * E_2^2 * R_2}{R_2^2 + (SX_2)^2}$

Let $Y = \frac{1}{T}$

$$Y = \frac{R_2^2 + (SX_2)^2}{KSE_2^2 R_2}$$

$$= \frac{R_2^2}{KSE_2^2 R_2} + \frac{(SX_2)^2}{KSE_2^2 R_2} = \frac{R}{SE_2^2 * K}$$

Torque under running condition

$$T = \frac{K\phi E_2 S R_2}{R_2^2 + (SX_2)^2} \Rightarrow \text{Let } Y = \frac{1}{T}$$

$$Y = \frac{R_2^2 + (SX_2)^2}{K\phi E_2 S R_2} = \frac{R_2^2}{K\phi E_2 S R_2} + \frac{(SX_2)^2}{K\phi E_2 S R_2}$$

$$Y = \frac{R_2}{K\phi S E_2} + \frac{S X_2^2}{K\phi E_2 R_2}$$

differentiate with respect to (S)

$$\frac{dy}{ds} = \frac{+R_2}{K\phi E_2} \frac{d}{dS}\left(\frac{1}{S}\right) + \frac{X_2^2}{K\phi E_2 R_2} = 0$$

$$= \frac{-R_2}{K\phi E_2 S^2} + \frac{X_2^2}{K\phi E_2 R_2} = 0$$

$$\frac{X_2^2}{K\phi E_2 R_2} = \frac{R_2}{K\phi E_2 S^2 \cancel{R_2}}$$

$$X_2^2 = \frac{K\phi \cancel{E_2} R_2^2}{\cancel{K\phi} E_2 S^2}$$

$$R_2^2 = S^2 X_2^2 = (SX_2)^2 \text{ or } R_2 = SX_2$$

Under running conditions, Torque is maximum if rotor resistance/phase $R_2$ = slip times the rotor reactance/phase

$$R_2 = SX_2 \Rightarrow \boxed{S = \frac{R_2}{X_2}}$$

$T_{st}$ & $T_{max}$ =

~~Tmax = KΦ E2 R2~~

Torque under running Condition

$$T = \frac{K\phi\, s E_2 R_2}{R_2^2 + (sX_2)^2} \longrightarrow (1)$$

if $R_2 = sX_2$ in above variation we get maximum torque

$$T_{max} = \frac{K\phi\, s E_2 (sX_2)}{(sX_2)^2 + (sX_2)^2} = \frac{K\phi\, s^2 E_2 X_2}{2(sX_2)^2} \text{ (or)}$$

$$T_{max} = \frac{K\phi\, s E_2 R_2}{R_2^2 + R_2^2} = \frac{K\phi\, s E_2 R_2}{2R_2^2}$$

$$T_{max} = \frac{K\phi\, s E_2 R_2}{2R_2^2} = \frac{K\phi\, s E_2}{2R_2}$$

(or)

$R_2 = sX_2$

$$\boxed{T_{max} = \frac{K\phi\, s E_2}{2sX_2} = \frac{K\phi\, E_2}{2X_2}}$$

$\Rightarrow$ if $s = \frac{R_2}{X_2}$ then maximum torque

$$T_{max} = \frac{K\phi\, s E_2 R_2}{R_2^2 + (sX_2)^2} = \frac{K_1 \phi \left(\frac{R_2}{X_2}\right) * E_2 * R_2}{R_2^2 + (sX_2)^2}$$

at stand-still $E_2 \propto \phi$

$$T_{max} = \frac{K_1 E_2^2 * \frac{R_2^2}{X_2}}{R_2^2 + \left(\frac{R_2}{X_2}\right)^2 \cdot X_2^2} = \frac{K_1 E_2^2 * \frac{R_2^2}{X_2}}{2R_2^2}$$

$$T_{max} = K_1 E_2^2 * \frac{R_2^2}{2R_2^2 X_2} = K_1 E_2^2 * \frac{1}{2X_2}$$

$$\boxed{T_{max} = \frac{K_1 E_2^2}{2X_2}}$$

$K_1 = 3/2\pi N_s$

Effect of change in supply voltage on starting torque

$T = K_1 \phi I_2 \cos\phi_2$

we know that


$$T_{st} = \frac{K_1 E_2^2 R_2}{R_2^2 + X_2^2}$$

$E_2 \propto V$ then starting torque

$$T_{st} = \frac{K_2 (V)^2 \times R_2}{R_2^2 + X_2^2}$$

Condition for maximum torque

Maximum torque occurs if rotor resistance $R_2$ = rotor reactance $X_2$

$R_2 = X_2$

Then

$$T_{st} = \frac{K_1 E_2^2 R_2}{R_2^2 + X_2^2} \qquad K_1 = \frac{3}{2\pi N_s}$$

if given supply voltage is constant them flux $\phi$ & induced emf $E_2$ both are constant then Tst is given

$V \propto \phi \propto E_2$ = Constant

$$T_{st} = \frac{K_3 R_2}{R_2^2 + X_2^2} = \boxed{\frac{K_3 R_2}{R_2^2 + X_2^2}}$$

page NO (3)

Problem

3-$\phi$ $\lambda$-connected I.M has stator to rotor turns ratio is 6.5. Rotor Reactance & Resistance 0.025 & 0.05 $\Omega$/phase. What would be the value of external resistance $R_E$ inserted in rotor circuit to obtain $T_m$ at starting & what is the starting current of rotor.

3

sol given stator to rotor turns ratio = 6.5

$$\boxed{1/K = \frac{\text{Stator}}{\text{Rotor}} = \frac{1}{6.5}}$$

$X_2 = 0.25$
$R_2 = 0.05$

$(\lambda \text{ to } \lambda)$

$$K = \frac{\text{Rotor}}{\text{Stator}} = \frac{1}{6.5}$$

Rotor emf $E_2$ at standstill per phase for $\lambda$

$$\boxed{E_2 = \frac{V_L}{\sqrt{3}} * K} = \frac{400}{\sqrt{3}} * \frac{1}{6.5} = 35.5\,V$$

$R_2 = X_2$
$R_2' = 0.25$
$R_E = R_2' - R_2$
$= 0.2$

$T_{st}$ is maximum if $R_2' = X_2$

$$R_2' = X_2 = 0.25\,\Omega$$

External Resistance $R_E$ or $R_E'$ per phase required is given

$$R_E' = R_2' - \cancel{X_2}\,R_2$$

$R_2 = X_2$
$R_2' = R_2 = 0.25$

or

$$\boxed{R_E = R_2' - \frac{R_2}{\cancel{X_2}}}$$

$$= 0.25 - 0.05 = 0.2\,\Omega$$

Rotor impedance / phase $Z_2 = \sqrt{(R_2') + X_2^2}$

$$= \sqrt{(0.25)^2 + (0.25)^2}$$

$$= 0.3535$$

Rotor Current / phase

$$I_2 = \frac{E_2}{Z_2} = \frac{35.5}{0.3535} = 100A$$

(1)

Problem

$V_P = 3000V$, $f = 50Hz$, $P = 6$ pole

$\lambda$ Connected I.m has, $\lambda$ - connected slip ring rotor with transformation ratio is 3.6.

$R_2 = 0.1\Omega/\text{phase}$, $X_2 = 3.61mH$. Neglect stator impedance. Find starting Current $I_1$ & starting torque $T_{st}$ on rated Voltage with slip rings short circuited.

given

Rotor emf/phase for Star Connected

$$E_2 = \frac{V_L}{\sqrt{3}} * K = 3000 * \frac{1}{3.6}$$

None

$$K = \frac{\text{rotor turns}}{\text{Stator turns}} = \frac{1}{3.6}$$

$$\boxed{3.6 = \frac{\text{Stator}}{\text{rotor}}} = \frac{1}{K}$$

also given rotor reactance $X_2$ is given in Hendry's

$$X_2 = 2\pi f * L = 2\pi * 50 * 3.61 * 10^{-3}$$

$$X_2 = 1.13\Omega$$

Change in rotor Rotor Resistance

$$\boxed{R_2' = \frac{R_2}{K^2}} = \frac{0.1 *}{\left(1/3.6\right)^2} = 1.3\Omega$$

$$\boxed{X_2' = \frac{X_2}{K^2}} = \frac{1.13}{\left(1/3.6\right)^2} = 14.7\Omega$$

2

$\frac{4\pi\phi N}{Z}$

$$T_{st} = \frac{K_1 * (V_p)^2 * R_2'}{(R_2')^2 + (X_2')^2}$$

Starting Current $\quad K_1 = \frac{3}{2\pi N_s'}$

$$I_{st} = I_1 = \frac{V}{\sqrt{(R_2')^2 + (X_2')^2}}$$

given I.m is connected in $\lambda - \lambda$

$$\boxed{V_p = \frac{V_L}{\sqrt{3}} = \frac{3000}{\sqrt{3}} = V} = 1732.05$$

$$I_{st} = \frac{V_p}{\sqrt{(R_2')^2 + (X_2')^2}} = \frac{1732.05}{\sqrt{(1.3)^2 + (14.7)^2}}$$

$I_{st} = 117.4V$

$$N_s = \frac{120 * f}{P} = \frac{120 * 50}{6} = 1000 \text{ rpm}$$

$$\boxed{N_s' = \frac{f}{P/2}}$$

$$\boxed{N_s' = \frac{\frac{1000}{20}}{\frac{P}{2}}} = \frac{\frac{50}{1}}{\frac{6}{2}\,3} = \frac{50}{3}$$

$$T_{st} = \frac{3}{2\pi(50/3)} * \frac{(1732.05)^2 * 1.3}{\sqrt{(1.3)^2 + (14.7)^2}}$$

$= 513$ Nm

(Q) $V_L = 1100V$, $f = 50Hz$ $\Delta$- connected I.m has stor connected slip ring rotor with phase t/formation ratio is 3.8.

Rotor

Resistance $R_2 = 0.012 \Omega/phase$

$X_2 = 0.25 \Omega/phase$.

Stator = K Rotor

Neglect stator impedance & magnetising currents. Determine.

(I) Rotor Current ($I_2$) at start with slip rings shorted

(II) Rotor power factor at start with slip rings shorted

(III) Rotor Current at 4% slip with slip rings shorted

(IV) Rotor Power factor at 4% " " "

(V) External Rotor Resistance ($R_E$) per phase required to obtain starting Current 100A in stator supply lines.

sol

given $\boxed{\Delta \text{ to } \lambda}$

$\Delta$- Connected I.M

in Delta Connected $V_p = V_L$ or $V_L = V_p$

$\Rightarrow$ Rotor emf at standstill per phase

$$\boxed{E_2 = V_p * K \text{ or } V_L * K}$$

$$K = \frac{\text{Rotor turns/phase}}{\text{Stator turns/phase}} = \frac{1}{3.8}$$

$$E_2 = \frac{1100}{3.8} = 289.5V$$

(i) Rotor Current per phase at start

$$I_2 = \frac{E_2}{Z_2}$$

Rotor impedance $Z_2$/phase $= \sqrt{R_2^2 + X_2^2}$

$= 0.2503\ \Omega$

$$I_2 = \frac{289.5}{0.2503} = 1157\text{A}$$

(ii) Rotor Power factor if slip rings are shorted.

$$\boxed{\phi_2 \text{ or } p.f = \frac{R_2}{Z_2}}$$

$p.f = VI\cos\phi$

$$\phi_2 \text{ or } p.f = \frac{0.012}{0.2503} = 0.048$$

(iii) Rotor Current at 4% slip

$$I_2 = \frac{E_r}{Z_r}$$

if Rotor is Condition is moving position then we consider the slip.

Rotor emf/phase $\boxed{E_r = SE_2}$

Rotor reactance/phase $\boxed{X_r = SX_2}$

Rotor impedance/phase $Z_r = \sqrt{R_2^2 + (SX_2)^2}$

or

$\sqrt{R_2^2 + (X_r)^2}$

slip $S = 4\% = \frac{4}{100} = 0.04$

$E_r = 0.04 * 289.5 = 11.58\text{V}$

$X_r = 0.04 * 0.25 = 0.01$

$Z_r = \sqrt{(0.012)^2 + (0.04 * 0.25)^2} = 0.0156\ \Omega$

$I_2 = \dfrac{11.58}{0.0156} = 742.8\,A$ $\qquad \dfrac{E_2}{E_1} = \dfrac{r}{s} = \dfrac{I_1}{I_2} = K$

(IV) Rotor Power factor with 4% slip

$$\boxed{\phi_2 \text{ or } p.f = \frac{R_2}{Z_r}} = \frac{0.012}{0.0156} = 0.769$$

(V) $\qquad \dfrac{I_2}{I_1} = K$

$$K = \frac{I_1}{I_2} \Rightarrow \boxed{I_2 = \frac{I_1}{K}}$$

where $I_1$ stator Current or starting Current $= 100\,A$

$$I_2 = \frac{100}{1/3.8} = 380\,A \text{ at standstill}$$

$$I_2 = \frac{E_2}{Z_2} \Rightarrow Z_2 = \frac{E_2}{I_2} = \frac{289.5}{380}$$

$$\boxed{Z_2 = 0.7618} \qquad \boxed{Z_2^2 = (R_2^1)^2 + X_2^2 \quad (R_2^1) = \sqrt{Z_2^2 - X_2^2}}$$

$$Z_2^2 = R_2^2 + X_2^2$$

$$\sqrt{Z_2^2 - X_2^2} = R_2^1 = 0.7169$$

External Resistance $R_E$ per phase required

$$\boxed{R_E = R_2^1 - R_2} = 0.7169 - 0.012 = 0.707$$

## Rotor torque & Breakdown Torque

Rotor torque at any slip can be expressed in terms of maximum torque

$$T = T_b \left[ \frac{2}{(S_b/s) + (s/S_b)} \right]$$

$S_b$ = breakdown or pull out slip

Q) Calculate Torque exerted by an 8 pole, 50Hz 3 $\phi$ I.m having 4% slip which develops max. torque of 150 kg-m at a speed of 660 rpm. Resistance / phase of rotor is 0.5 Ω.

sol

given maximum Torque $T_{max} = 150$ kg-m

Speed at maximum Torque = 660 rpm, P = 8

$\longrightarrow$ $N_S = \frac{120f}{P} = \frac{120 * 50}{8} = 750$ rpm

$\longrightarrow$ $S_b$ = breakdown slip = $\frac{N_S - N}{N_S} = \frac{750 - 660}{750}$

$S_b = 0.012$

also given original slip of I.m is 4%

$S = 0.04\%$ , $R_2 = 0.5$

$\longrightarrow$ Condition for maximum torque is

$R_2 = S_b X_2$

$X_2 = \frac{0.5}{0.012} = 4.167$

$T_{max} = \frac{K\phi S E_2 R_2}{R_2^2 + (S_b X_2)^2}$ if $R_2 = S_b X_2$

$\longrightarrow$ $T_{max} = \frac{K\phi S E_2}{2R_2} = \frac{K\phi S_b E_2}{2R_2}$

$= 0.12 K\phi E_2 \rightarrow (1)$

if slip 's' is 4% then

$T = \frac{K\phi E_2 * S * R_2}{R_2^2 + (SX_2)^2}$

$$T = \frac{K\phi E_2 * S * R_2}{R_2^2 + (SX_2)^2}$$

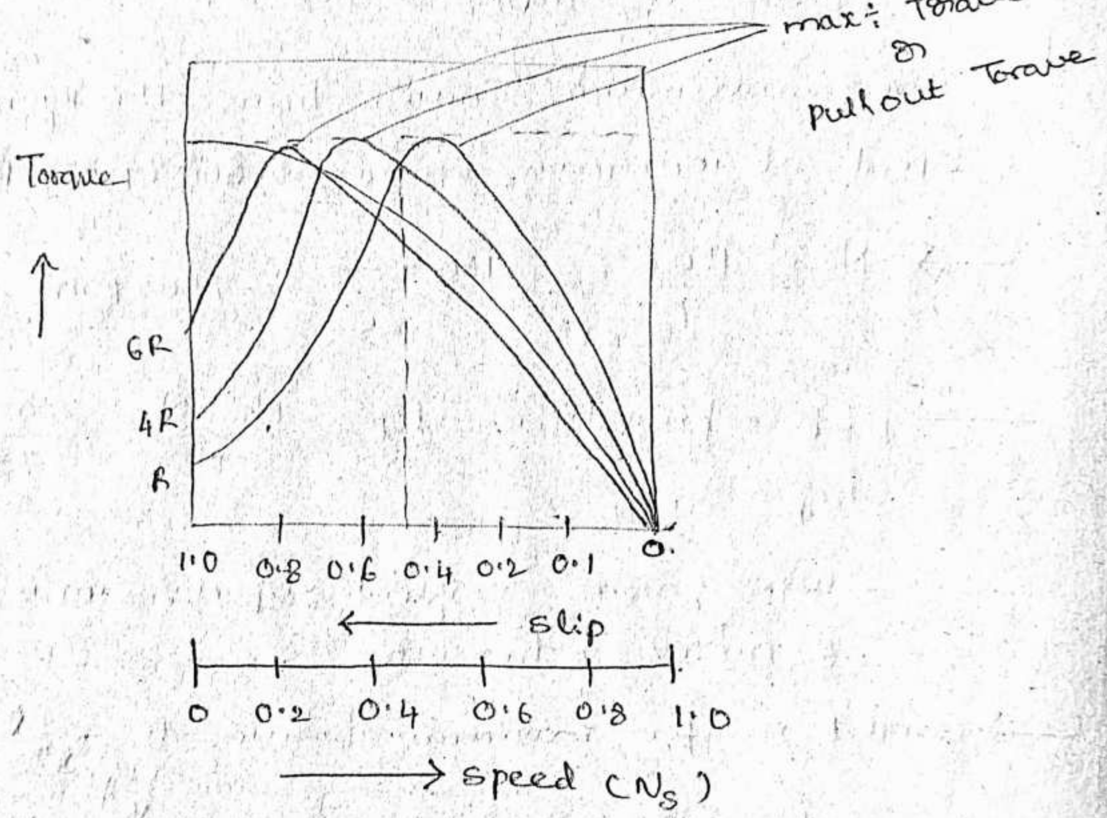
$$T = K\phi E_2 * \frac{0.04 * 0.5}{(0.5)^2 + (0.04 * 4.167)^2} = \frac{0.02 K\phi E_2}{0.2778}$$

$$\frac{T}{T_{max}} = \frac{T}{150} = \frac{0.02 K\phi E_2}{0.2778} * \frac{1}{0.12 K\phi E_2}$$

$$\frac{T}{150} = \frac{0.02}{0.2778 * 0.12} \Rightarrow T = \frac{0.02 * 150}{0.278 * 0.12}$$

$T = 90$ Kg-m

Relation between slip & Torque




from figure slip ranges from $s=0$ to $s=1$

→ if $s=0$, $T=0$

$$T = \frac{K\phi s E_2 R_2}{R_2^2 + (s x_2)^2} = 0$$

Curve starts from (0).

→ At normal speeds, $R_2$ is negligible, $sx_2$ is small

→ As slip (s) increases continously then

Torque becomes maximum. This torque is known as pullout Torque or breakdown Torque. or stalling Torque

[As load on motor increases, Torque of motor also increases & becomes max.

we know that maximum Torque occurs if $R_2 = sX_2$

$\therefore$ $\boxed{\text{slip 's'} = \frac{R_2}{X_2}}$

→ when Torque reaches max, as slip further increases, (as load on motor again increases), speed of motor decreases, Torque of motor also decreases, finally motor slows down & eventually stops.

$$T = K\phi sE_2 * \frac{R_2}{R_2^2 + (sX_2)^2}$$

if $K, \phi, E_2$ are constants, $R_2$ becomes negligible & $X_2$ becomes ~~negligible~~ small

$$T = \frac{s}{(sX_2)^2} = \frac{1}{s}$$

$$T = \frac{1}{s}$$

Effect of change in supply voltage on Torque & speed

we know that Torque under Running condition is given by

$$T = \frac{K\phi sE_2 R_2}{R_2^2 + (sX_2)^2}$$

At normal speeds $R_2$ is negligible, $sX_2$ is small

$$T = \frac{K \phi S E_2}{1}$$

if supply voltage 'V' is constant, $\phi$ is constant

Then $E_2$ is also constant

$E_2 \propto \phi \propto V$

$T \propto \phi S E_2$

$T \propto V * S * V \Rightarrow T \propto S V^2$

Torque is propotional square of voltage.

→ If stator voltage decreases 10%, Torque decreases by 20%, speed also decreases.

**Effect of changes in supply frequency on Torque & speed**

whenever a major disturbance occurs, first effected is supply frequency.

→ Change in frequency leads to change in motor speed. $(f \propto V)$

→ If supply frequency drops by 10%, motor speed decreases by 10%.

→ If motor is designed to operate 50 Hz is connected to 60 Hz supply frequency, Motor runs 20% faster than normal speed

→ for obtaining the normal speed we go for using the gears to control the speed.

→ If a 50 Hz motor is well operated at 60 Hz, its terminal voltage is raised by 120% of normal rating

→ A 60Hz motor will operates on 50Hz, its terminal voltage is reduced by 20% of normal Voltage

full-load Torque & maximum Torque

Let the full load slip be $S_f$

$T_f$ be the full load torque & $T_{max}$ be maximum torque

$$T_f \propto \frac{\phi S_f E_2 R_2}{R_2^2 + (S_f X_2)^2}$$

if rotor is at standstill position

$E_2 \propto \phi$

$$T_f = \frac{K S_f E_2^2 R_2}{R_2^2 + (S_f X_2)^2} \rightarrow ①$$

$$\left[ T_{max} \propto \frac{\phi E_2}{2X_2} \qquad T_{max} = \frac{K E_2^2}{2X_2} \right.$$

$$T_{max} = \frac{K E_2^2}{2X_2} \text{ or } \frac{K\phi E_2}{2X_2} \rightarrow ②$$

$$\frac{T_f}{T_{max}} = \frac{\dfrac{K S_f E_2^2 R_2}{R_2^2 + (S_f X_2)^2}}{K E_2^2 / 2X_2} = \frac{2 S_f R_2 X_2}{R_2^2 + (S_f X_2)^2}$$

divide both Numerator & denominator by $X_2^2$ we get

$$\frac{T_f}{T_{max}} = \frac{\dfrac{2 S_f R_2 X_2}{X_2^2}}{\dfrac{R_2^2 + S_f^2 X_2^2}{X_2^2}} = \frac{2 S_f R_2 / X_2}{(R_2/X_2)^2 + S_f^2} \rightarrow ③$$

let $a = R_2/X_2$ then equation ③

$$T_f / T_{max} = \frac{2 S_f a}{a^2 + S_f^2}$$

$$\frac{T_f}{T_{max}} = \frac{2a s_f}{a^2 + s_f^2}$$

if $a = S_{max}$ maximum slip then

$$\frac{T_f}{T_{max}} = \frac{2 S_{max} S_f}{S_{max}^2 + S_f^2}$$ divide numerator & denomin by $S_{max} S_f$

$$\frac{T_f}{T_{max}} = \frac{2}{\frac{S_{max}}{S_f} + \frac{S_f}{S_{max}}} = \frac{2}{\frac{S_{max}}{S_f} + \frac{S_f}{S_{max}}}$$

→ starting Torque & maximum torque

we know that starting torque

$$T_{st} \propto \phi E_2 \frac{R_2}{R_2^2 + X_2^2}$$

if rotor flux, induced emf are
Constants if supply voltage is constant

$$T_{st} \propto \frac{R_2}{R_2^2 + X_2^2} \longrightarrow (1)$$

we know that maximum torque

$$T_{max} \propto \frac{\phi E_2}{2X_2}$$ if supply voltage constant, $\phi$ & $E_2$ are also constant

$$T_{max} \propto \frac{1}{2X_2} \longrightarrow (2)$$

$$\frac{T_{st}}{T_{max}} = \frac{\frac{R_2}{R_2^2 + X_2^2}}{\frac{1}{2X_2}}$$

$$\frac{T_{st}}{T_{max}} = \frac{2R_2X_2}{R_2^2+X_2^2} = \frac{2R_2X_2}{X_2^2\left[\left(\frac{R_2}{X_2}\right)^2+1\right]}$$

$$\frac{T_{st}}{T_{max}} = \frac{2R_2/X_2}{(R_2/X_2)^2+1} = \frac{2a}{a^2+1}$$

Problem

A 746 KW 3 phase 50Hz, 16 pole I.m has rotor impedance $(0.02 + j0.15)$ at stand still. $T_f$ is obtained at 360rpm. calculate (i) Ratio of $T_{max}$ to $T_f$ (ii) Speed of $T_{max}$ (iii) Rotor resistance to be added to get maximum starting torque.

sol

(i) $$\frac{T_f}{T_{max}} = \frac{2aS_f}{a^2+S_f^2}$$

given $f = 50Hz$ $P = 16$

$$Z_2 = R_2 + jX_2 = 0.02 + j0.15$$

$T_f$ is obtained at speed $N = 360$ rpm

$$N_s = \frac{120f}{P} = 375 \text{ rpm}$$

$$S_f = \frac{N_s - N}{N_s} = \frac{375-360}{375} = 0.04$$

$$a = \frac{R_2}{X_2} = \frac{0.02}{0.15} = 0.133$$

$$\frac{T_f}{T_{max}} = \frac{2*0.133*0.04}{(0.133)^2+(0.04)^2} = 0.55$$

$$\frac{T_{max}}{T_f} = \frac{1}{0.55} = 1.818 \text{ N-m}$$

(ii) at maximum torque, slip is maximum

$$a = S_{max} = \frac{R_2}{X_2} = \frac{0.02}{0.15} = \frac{2}{15} = 0.133$$

$$N = N_S(1-S) = 375(1-0.133) = 325 \text{ rpm}$$

(iii) Maximum torque occurs

if $R_2 = X_2$

$R_2^1 = 0.15$

$$R_E = R_2^1 - R_2 = 0.15 - 0.02 = 0.13\,\Omega$$

Problem: $3\phi$ I.m having 6-pole $\lambda$-connected stator winding runs on 240V, 50Hz supply. Rotor Resistance & reactance are $0.12\,\Omega$ & $0.85\,\Omega$/phase. The ratio stator to rotor turns is 1.8. full load slip 4%. Calculate developed torque at full load, Tm & speed at Tmax.

Sol:

$$K = \frac{\text{rotor turns}}{\text{Stator turns}} = \frac{1}{1.8}$$

Rotor induced emf/phase $E_2 = \frac{V_L}{\sqrt{3}} * K = \frac{240}{\sqrt{3}} * \frac{1}{1.8}$

$E_2 = 77V$

$N_S = \frac{120*f}{P} = \frac{120*50}{6} = 1000$ per min

$N_S = \frac{N_S}{60} = \frac{1000}{60} = 16.666$

slip $S = 4\% = 0.04$

also $P = 6$, $f = 50Hz$, $R_2 = 0.12$, $X_2 = 0.85$

$$\boxed{N_S = \frac{120*f}{P*60}} = \frac{1000}{60} = \frac{50}{3}$$

$$\boxed{N_S = \frac{120f}{P*60} \text{ or } \frac{f}{P/2}}$$

$$N_S = \frac{50}{P/2} = \frac{50}{6/2} = \frac{50}{3}$$

$$T_f = K_1 * \frac{S_f E_2^2 R_2}{R_2^2 + (SX_2)^2} = \frac{3}{2\pi(N_S/60)} * \frac{S_f E_2^2 R_2}{R_2^2 + (SX_2)^2}$$

Condition for maximum torque is $R_2 = SX_2$

# synchronous alternator

## Armature Reaction

It is defined as the effect of armature mmf on the m.m.f of main field flux.

This effect is 2 ways

① Cross magnetising effect ② de-magnetising effect

↳ Due to these pattern distribution of main field flux changes.

Due to De-magnetising effect of armature reaction, main field flux becomes weakening or flux per pole decreases

The above 2 cases is for dc-generator.

In case of alternator these magnetising effect of armature reaction main field flux becomes strengthened or flux per pole increases

* If load is Resistive

Effect of armature reaction is cross magnetising

* if load is inductive, effect of armature reaction is de-magnetising
* if load is Capacitive, effect of armature reaction is magnetising.

Purely Resistive load ( p.f = unity )

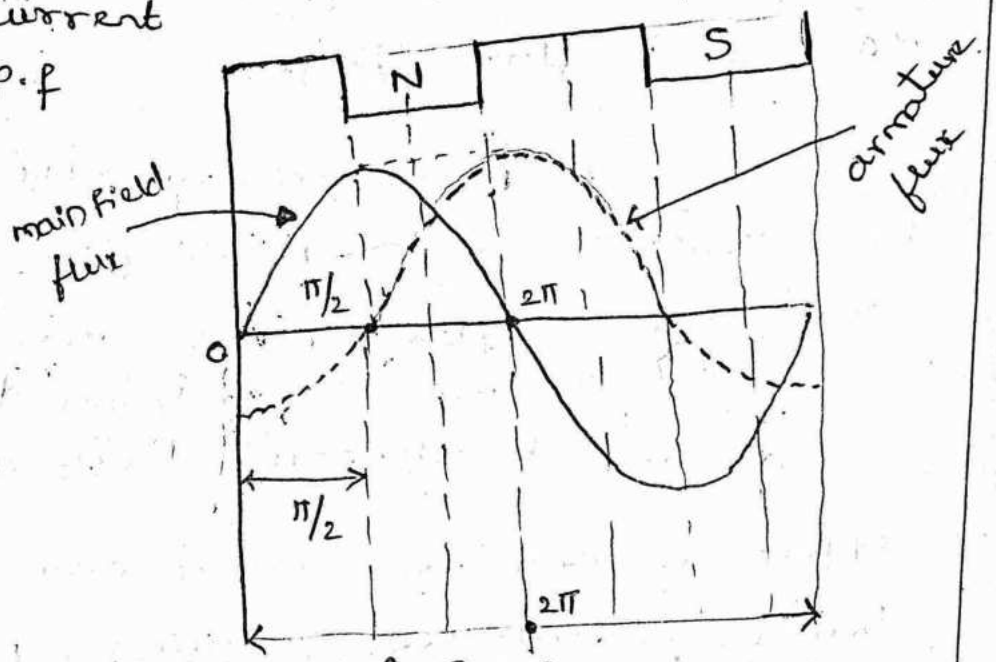
when 3 phase alternator is loaded, stator carries 3 phase currents which gives armature magnetic field. which revolves in synchronous speed.

The magnetic field depends on

→ position of poles

→ Load Current

→ Load p.f




fig① for unity load

Magnetic

→ from fig① Magnitude of armature MMF is constant with respect to time but it lags behind to the main field flux by 90° shown in fig①

Phase angle b/w the 2 fluxes is 90°. so the armature reaction is purely cross magnetising. Due to this flux at the leading pole tips decreases & at trailing pole tips flux increases. Totally main field flux gets distorted.

## purely inductive load ( P.f = 0, lagging)

P.f = 0 , if $\cos\phi$ is zero if $\phi = 90°$

$\cos\phi = P.f = 0$

$\cos(90) = 0 = P.f$

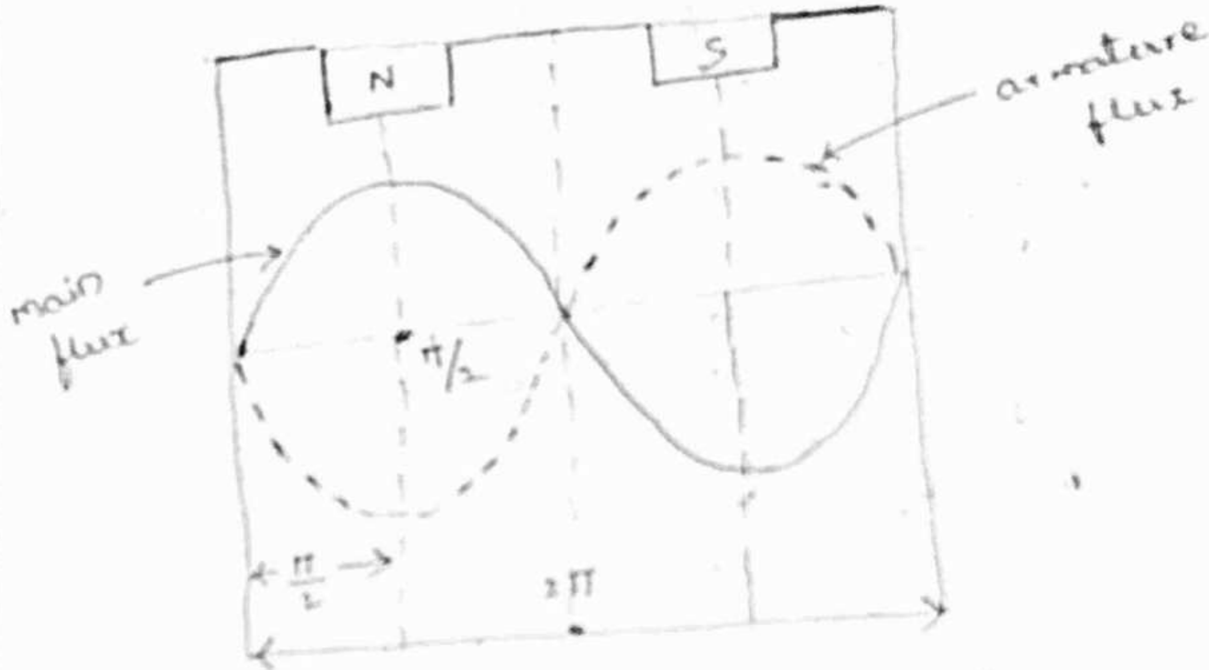



fig ② flux curves for P.f = 0 (lagging) load

In case of inductive load P.f = 0

if $\phi = 90°$ . ∵ $\cos(90) = 0$

$\cos\phi = P.f = 0$ , if $\phi = 90$

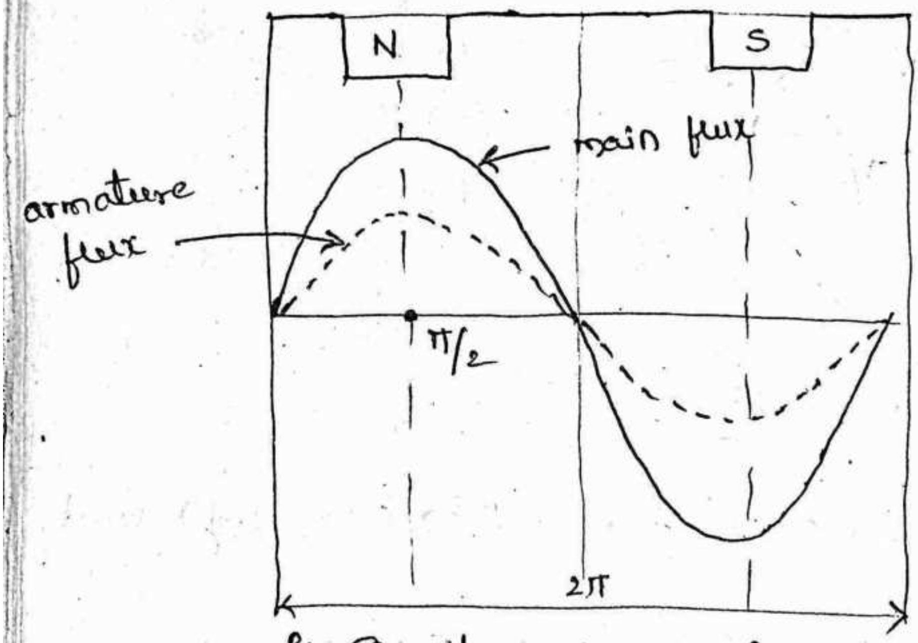
from fig ② the position of main field flux is same but position of armature flux shifted by 90°. So armature flux is direct opposition to main field flux. since Current lag behind the voltage by 90°. So armature reaction is purely de-magnetising due to this flux per pole decreases.

Purely Capacitive Load ( P.f = 0, leading )

If load is purely Capacitive, P.f = 0, if $\phi = -90°$

$\therefore$ $P.f = \cos(-90) = 0$

Here Current leads the voltage by $-90°$. Hence position of main field flux remains same but armature flux shifts by $90°$.




fig③ flux curves for leading p.f load

In this Case from fig③, Both main field flux armature flux are in phase to each other. So armature reaction is purely magnetising. The resultant main field flux per pole increases.

Voltage regulation of alternator

When an alternator is loaded, its terminal voltage decreases as load current ~~decreases~~ increases. The decrease in terminal voltage is due to

① Armature resistance & leakage reactance
② Armature reaction.

Voltage regulation definition

→ At Constant speed & constant field excitation, terminal voltage of alternator changes from no load to full load is termed as voltage regulation.*

Let $E_0$ → no load terminal
$V$ → Rated terminal voltage or full load terminal voltage } alternator

$$\% \text{ regulation} = \frac{E_0 - V}{V} * 100$$

→ Again regulation of alternator depends on load current & p.f of load.

→ Regulation is +ve for resistive & inductive loads

* Regulation is -ve for capacitive loads.

→ In case of small size alternators, loading the machine upto rated current delivers. Note down the full load terminal voltage & decrease load by keeping speed & excitation constant. Note down the no load voltage.

*** for testing large size alternators we go for adpoting Indirect methods.

1. EMF or synchronous impedance method
2. MmF or Ampere turn method
3. ZPF method or poteir method
4. ASA method

## 1. Synchronous impedance method

Also known as e.m.f method. These test involves 3 tests

(I) open circuit test (No load)
(II) short circuit test
(III) Measurement of armature Resistance.

### (I) Open circuit test

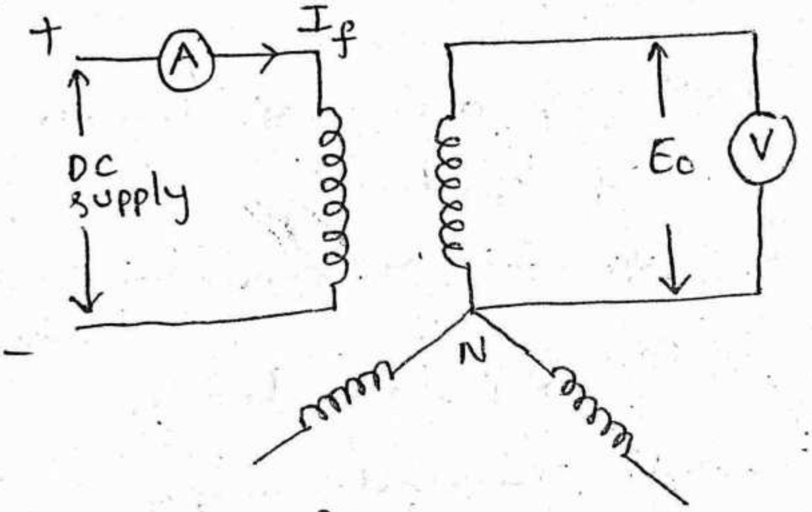
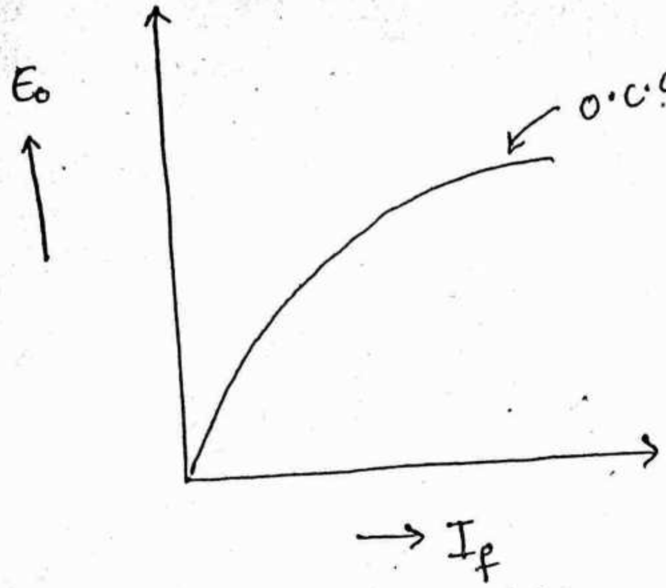



fig (1) O.C test

Also termed as no load test. By maintain the alternator speed constant & by varying field excitation, note down different values of induced emf.

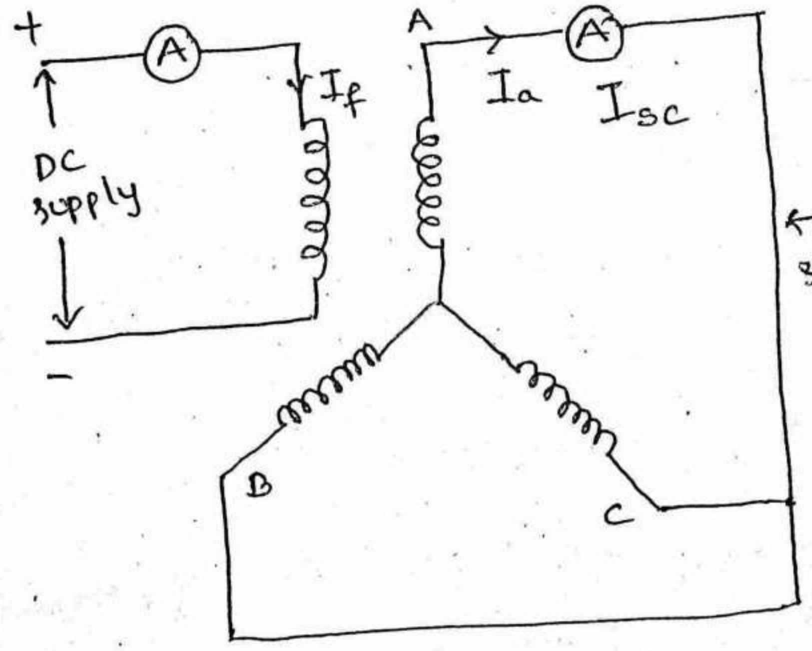
$I_f$ is continously increasing, untill the alternator developeds rated voltage. From this test we get the values of $I_f$ & $E_0$. Plot the $I_f$ vs $E_0$ graph. This graph is O.C.C graph




By increasing $I_f$ untill generator develops rated voltage by maintaining speed Constant

(II) Short circuit test

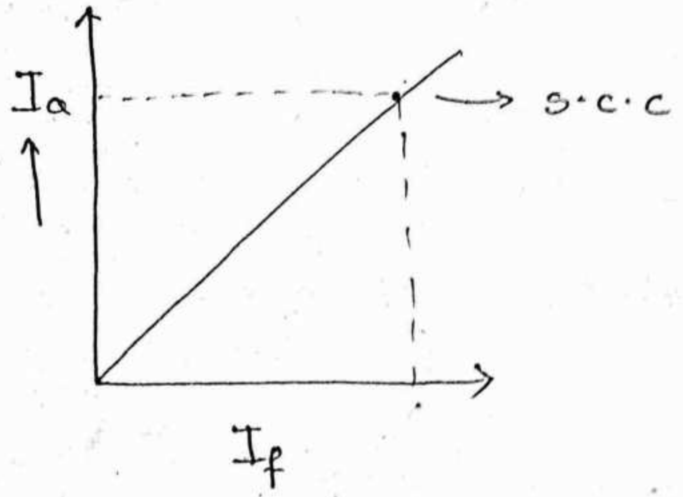
By keeping the field excitation at minimum position, allows to the alternator to run at rated speed on no load.



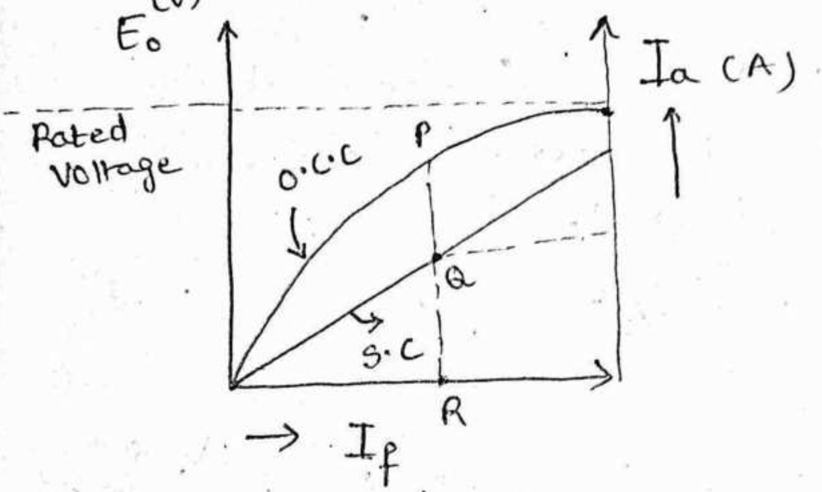

→ Alternator terminals are short circuited.

→ $I_f$ is adjusted untill the rated Current flows through the short circuited terminals. Note down value of short circuit Current ($I_{sc}$).

plot the graph $I_f$ vs $I_a$ characteristics




The above graphs O.C.C & S.C.C are drawn on common plot.




fig② shows O.C.C & S.C characteristics represented on common plot.

(iii) Measurement of Ra (armature resistance)

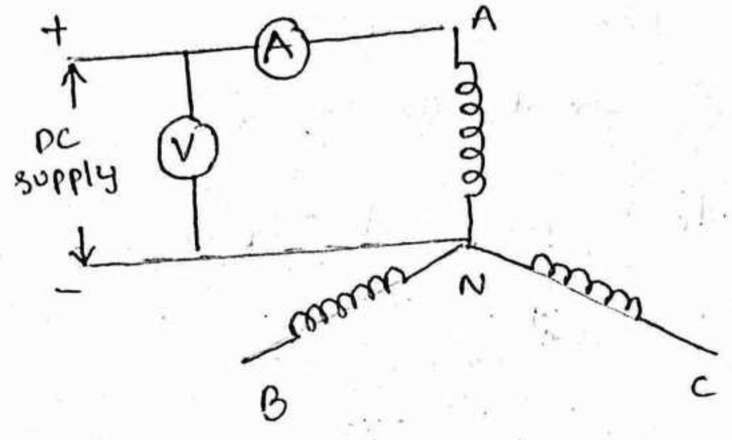
DC resistance of armature winding / phase is measured using Ammeter - Voltmeter or wheat stone bridge.

Here a small dc current is passed through any one of phase winding & note down the voltage drop across it.

$$\text{D.C resistance / phase} = \frac{\text{Voltage}}{\text{Current}}$$

Due to skin effect in a.c, effective or AC resistance is 1.5 times the d.c resistance

AC resistance = 1.5 times d.c resistance.




fig③ shows measuring armature resistance.

→ In addition an alternator posses both ohmic & leakage reactance. When armature winding carries alternating current, a fluxes set up in windings. A small amount of flux links to the armature winding without crossing the air gap. This is known as leakage flux. Hence an alternator supplies armature resistance drop (IR) & armature leakage reactance drop ($IX_L$).

Thus for inductive loads, armature reaction is de-magnetising as a result induced emf reduces & is given by considering an additional fictitious reactance $X_a$.

emf drop is considered as $IX_a$ due to armature reaction. when

when an alternator is loaded, current flows through armature winding supplies internal voltage drops are $IR_a$, $IX_L$ & $IX_a$

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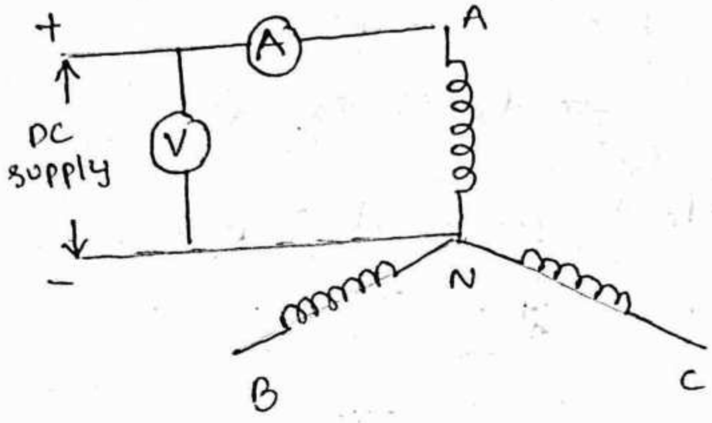
AC resistance = 1.5 times d.c resistance.

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when an alternator is loaded, current flows through armature winding supplies integral voltage drops are $IR_a$, $IX_L$ & $IX_a$

Let $E_0$ = No load induced emf / phase
$V$ = terminal voltage / phase

$$\therefore \; E_0 = V + IR_a + IX_L + IX_a$$

$IR_a$ is in phase with $I$.
$IX_L$ & $IX_a$ leads $I$ by $90^\circ$

for resistive load phasor diagram

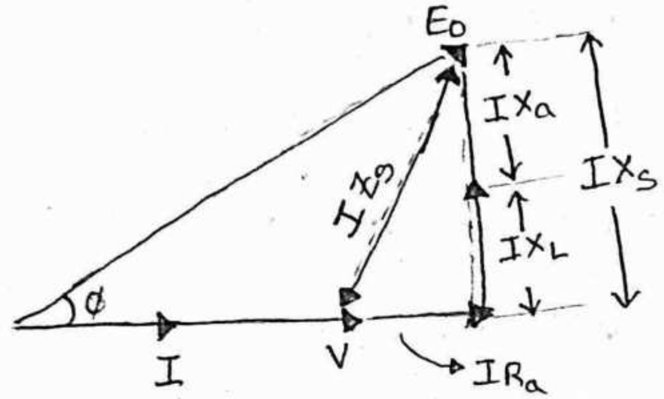
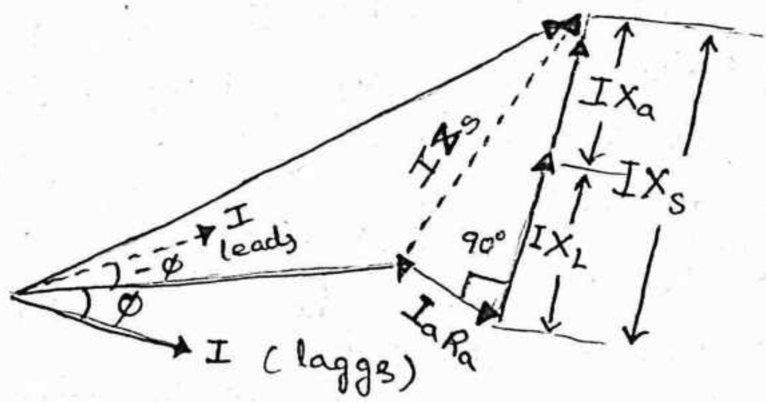



fig ① ⓐ phasor diagram resistive load.

for inductive load phasor diagram fig ① ⓑ




$\cos\phi$ = Load p.f

$$X_s = X_L + X_a$$

for capacitive load phasor diagram refer fig 1b by considering the dotted line current leads.

Let $X_S = X_L + X_a$,

$IX_S = I \cdot X_L + IX_a$

→ termed as synchronous reactance ($X_S$)

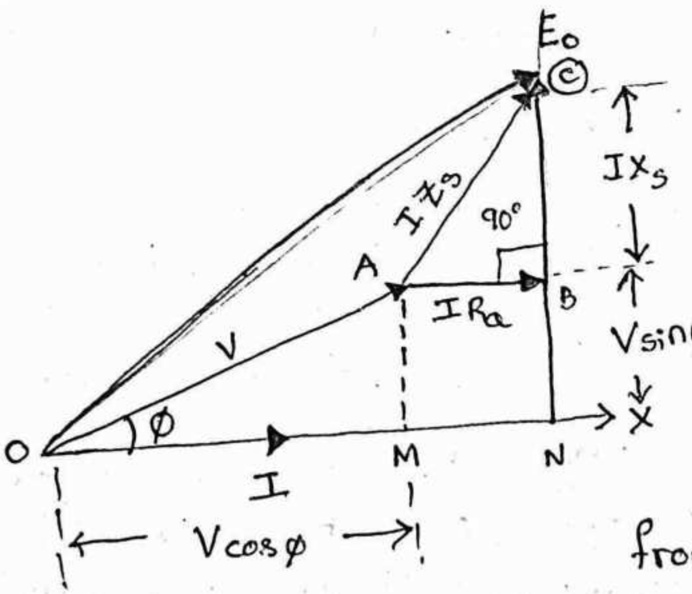
from phasor diagram fig 1 b

$$E_0 = V + IX_S + IR_a$$
$$= V + I(X_L + X_a) + IR_a$$
$$= V + I(R_a + X_S)$$

reactance x is in complex or imaginary from we write

$$E_0 = V + I(R_a + jX_S) = \boxed{V + IZ_S}$$
$$Z_S = R_a + jX_S$$

$Z_S$ → termed as synchronous impedance.




$BN = AM = V\sin\phi$

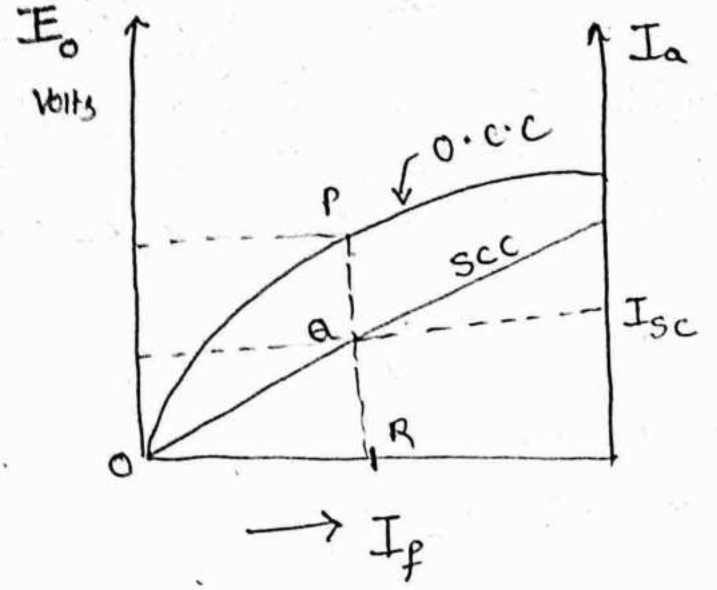
$OM = V\cos\phi$

$MN = AB = IR_a$

$BC = X_S I$

from right angle

$$OC^2 = ON^2 + CN^2$$
$$OC^2 = (OM + MN)^2 + (BN + BC)^2$$
$$E_0^2 = OC^2 = (V\cos\phi + IR_a)^2 + (V\sin\phi \pm IX_S)^2$$
$$E_0 = \sqrt{(V\cos\phi + IR_a)^2 + (V\sin\phi \pm IX_S)^2}$$




PR = No load induced emf/phase

OR = field Current

synchronous impedance $Z_S = \frac{PR}{QR} = \frac{E_0}{I_{sc}} = \frac{O.C}{S.C}$

$$Z_S = \sqrt{R_a^2 + X_S^2}$$

$R_a$ is known

$X_S$ is Computed

$$\% \text{ Reg} = \frac{E_0 - V}{V} * 100$$

Regulation at any desired load p.f can be determined.

Problems

if field excitation of 10A of alternator gives Current 15A on short circuit & terminal voltage 900V on open circuit. Find the internal voltage drop with load Current of 60A.

sol $I_f = 10A$, open circuit voltage $V_{oc} = 900V$

short circuit Current $I_{SC} = 15A$

internal voltage drop with 60A load Current is

$$V = I Z_S$$

$$Z_s = \frac{V_{oc}}{I_{sc}} = \frac{900}{15} = 60 \text{ ohms}$$

$$V = I * Z_s = 60 * 60 = 3600 \text{ V}$$

Problem A 500V, 50KVA single phase alternator has an effective resistance of 0.2 Ω. A field current 10A produces an armature current 200A on short circuit & an emf of 450V on open circuit. Calculate the full load regulation at 0.8 lag power factor.

Sol Rating of alternator = KVA = 50

Rated Voltage = V = 500V,

effective resistance $R_a = 0.2\,\Omega$, load p.f = 0.8 lag

$I_f = 10A$, $I_{sc} = 200A$, $E_0$ = open circuit voltage = $E_0 = 450V$

$$\boxed{\% \text{ Reg} = \frac{E_0 - V}{V} * 100}$$

$E_0$ = open circuit or no load voltage = 450V

V = terminal voltage = 500V

$$\boxed{E_0 = \sqrt{(V\cos\phi + IR_a)^2 + (V\sin\phi \pm IX_s)^2}}$$

We know that synchronous impedance $Z_s$

$$\boxed{Z_s = \frac{E_0}{I_{sc}} = \frac{\text{open circuit voltage}}{\text{short circuit current}}} = 2.25\,\Omega$$

$$\boxed{\begin{array}{l} + = \text{lag} \\ - = \text{lead} \end{array}}$$

$$Z_s = \sqrt{R_a^2 + X_s^2}$$

$$X_s = \sqrt{Z_s^2 - R_a^2} = 2.2411\,\Omega$$

$\cos\phi = p.f = 0.8$ lag

Rated Current $I = ?$

KVA rating $= 50$

Rated Voltage $V = 500V$

Rated Current $\boxed{I = \frac{VA * Rating}{Rated\ Voltage}}$

$$= \frac{50*1000}{500} = 100A$$

$$E_0 = \sqrt{(500*0.8 + 100*0.2)^2 + (500*0.6 + 100*2.241)^2}$$

$= 671.633V$

% full load regulation $= \frac{E_0 - V}{V} * 100$

$$= \frac{671.633 - 500}{500} * 100 = 34.3266\%$$

problem A 100KVA rating, 3000V, 50Hz, 3 phase Star Connected alternator has effective armature resistance $R_a = 0.2\Omega$. A field Current 40A produces a short circuit Current 200A & an O.C emf of 1040V (line value). Calculate full load percentage regulation at a power factor 0.8 lagging

sol Rating of alternator = 100 KVA = 100*1000 VA

Rated Voltage (line value) = 3000V, $R_a = 0.2$, p.f = 0.8 lag

field Current $I_f = 40A$, O.C Voltage $E_0 = 1040$ (line)

$I_{sc} = 200A$.

alternator is a 3 phase star connected, so line voltages are Converted into phase voltages.

→ Rated voltage / phase = $\frac{\text{line voltage}}{\sqrt{3}} = \frac{3000}{\sqrt{3}} =$

$V_P = 1732.051$ volts

→ open circuit voltage / phase = $\frac{V_{oc}}{\sqrt{3}} = \frac{V_{oL}}{\sqrt{3}}$

$V_\phi$

$V_{ph} = \frac{1040}{\sqrt{3}} = 600.44\,v$

→ Open Circuit Voltage / phase = $\frac{V_{o.c}}{\sqrt{3}} = \frac{1040}{\sqrt{3}} = 600.44$

→ synchronous impedance

$$Z_s = \frac{V_{o.c}}{I_{sc}} = \frac{600.44}{200} = 3.00\,\Omega$$

→ $Z_s = \sqrt{R_a^2 + X_s^2} \Rightarrow Z_s^2 = R_a^2 + X_s^2$

$$X_s^2 = \sqrt{Z_s^2 - R_a^2}$$

$$X_s^2 = \sqrt{(3.00)^2 - (0.2)^2} = 2.9933\,\Omega$$

→ Rated Current $\boxed{I = \frac{\text{VA rating}}{\sqrt{3} * \text{Rated voltage}}} = \frac{100 * 1000}{\cancel{1732.05}\ \sqrt{3} * 3000}$

$= 19.245\,A$

No load induced emf

$$E_0 = \sqrt{(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_s)^2}$$

$$= 1770.2352$$

full load voltage regulation $= \frac{E_0 - V}{V} * 100$

$$= \frac{1770.23 - 1732.051}{1732.051} * 100 = 2.205\%$$

## M.M.F method or Ampere turn method

EMF method sometimes referred as pessimistic method. It gives high regulation than actual values.

→ MMF method sometimes referred as Optimistic method, also termed as Rothert's Ampere turn method. this method gives low regulation than actual values.

→ In emf method armature reaction effect is substituted by additional armature reactance.

→ In MMF method leakage reactance of armature is substituted by additional armature reaction effect.

Thus an MMF method requires an emf method data. MMF method involves

① no load test & ② short circuit test

↳ this is similar to O.C test which is Obtained from EMF data. By drawing its open circuit characteristics.

In practical, alternator, $R_a$ & $X_L$ are usually small. During short circuit test, field excitation for full load current is the sum of excitation required to overcome leakage reactance drop & to overcome the de-magnetizing effect.

During short circuit conditions p.f is almost zero. Leakage resistance drop $IX_L$ may be regarded as additional armature reaction.

Thus the field current required to develop rated voltage on no load & same field current required to ~~[illegible]~~ circulate full load armature current during s.c test.

Let the field current develops rated voltage on no load is obtained from O.C.C as $E_0$ & field current develops rated voltage on full load is V.

$$\% \text{ Reg} = \frac{E_0 - V}{V} * 100$$

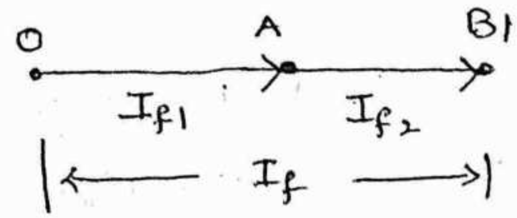
① when load is purely inductive

Let $OA = I_{f1}$ be the field (Ampere turn) current develops rated voltage on no load.

$OB = I_{f2}$ be the field (Ampere turn) current needed to produce full load current on short circuit.

Total field current $I_f = I_{f1} + I_{f2}$

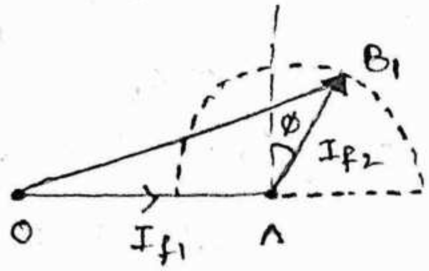
Armature reaction effect is purely de-magnetising




$OB = OA + AB1$

$I_f = I_{f1} + I_{f2}$

for an inductive load p.f is zero since I, ($\phi$) lags behind the voltage by 90°

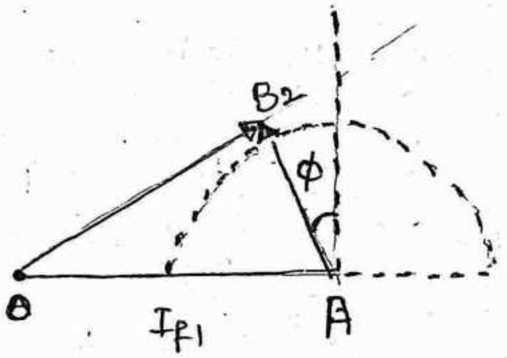



$\Rightarrow OB1 = OA + OB1$

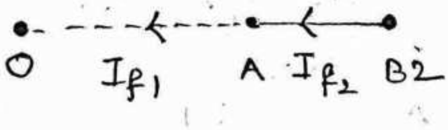
fig① if p.f is lagging at an $(90+\phi)$

Here field current $I_{f1}$ (OA) is laid horizontal then field current $I_{f2}$ (AB1) representing full load short circuit current is making at an angle $(90+\phi)$

② when load is purely Capacitive




fig② if p.f is leading at an angle $(90-\phi)$




$OB2 = OA - AB2$

field current ($I_{f2}$) required to circulate full load current is less than ($I_{f1}$) required to produce rated voltage

$AB_2 < OA$

Here armature reaction effect is purely magnetising.

(iii) when load is purely resistive (p.f = 1)

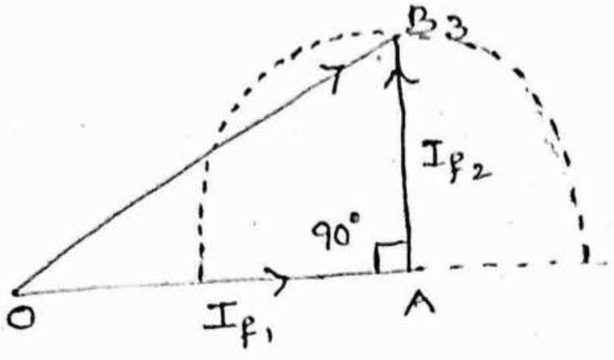
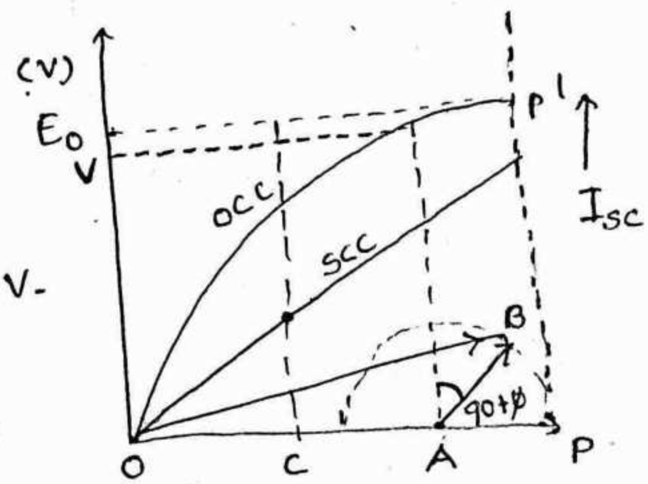



fig ③ p.f is unity for resistive load

If p.f is unity, armature reaction is purely magnetising as a result emf gets distortional. Hence $OB_3 = OA + AB_3$

general case




OA represents field current requires to produce rated voltage V.

OC represents field current required to produce full load current on short circuit.

Vector AB = OC drawn at angle 90 + ø with respect to OA (if p.f is lagging).

OB is total field current, with (O) as center and OB radius drawn an arc to x-axis at point 'p'

$$\% \text{ Reg} = \frac{E_0 - V}{V} \times 100$$

It should be noted that MMF gives regulation values less than actual values.

<u>Problem</u> No load excitation of alternator needed to give rated voltage is 80A. In a short circuit test, full load current flows through armature with field excitation is 50A. Calculate field excitation required to give full load current at 0.8 p.f lag & at rated terminal voltage.

<u>Sol</u> Let $I_{f1}$ be field current of 80A generates rated voltage at no load.

$$I_{f1} = 80A$$

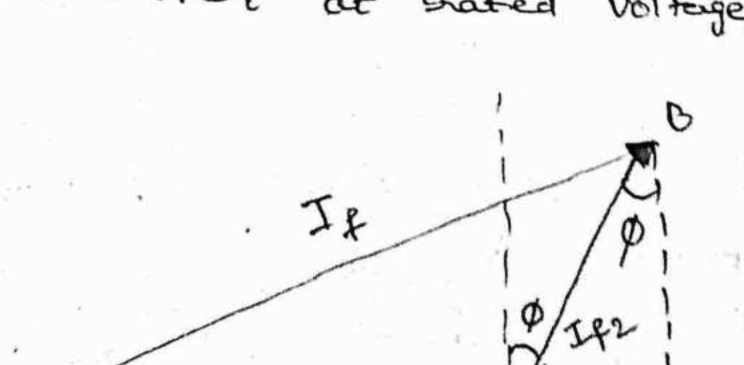
Let $I_{f2}$ be the field current of 20A required to circulate full load current during short circuit $I_{f2} = 50A$ at p.f = 0.8 lag

$$\cos\phi = 0.8$$

$$\phi = \cos^{-1}(0.8) = 36.87$$

$$\sin\phi = 0.6$$

Total field current required to deliver full load current at rated voltage is $I_f = I_{f1} + I_{f2}$




$$OB^2 = OC^2 + BC^2$$

$$= (OA + AC)^2 + BC^2$$

~~$OB^2 = OA^2 + AC^2 = OA^2 + BC^2$.~~

$= (I_{f1} + I_{f2} \sin\phi)^2 + (I_{f2} \cos\phi)^2$

$= (80 + 50 * 0.6)^2 + (50 * 0.8)^2 = 13700$ A

$OB = I_f = \sqrt{13700} = 117.05$

Problem

below are test results obtained on 6600V alternator

| | | | | | |
|---|---|---|---|---|---|
| O.C voltage | 3100 | 4900 | 6600 | 7500 | 8300 |
| field Current | 16 | 25 | 37.5 | 50 | 70 |

A field Current of 20A is required to circulate full load Current during short circuiting the armature. Calculate the MMF method, full load regulation at 0.8 p.f lag Neglect armature Resistance + leakage reactance.

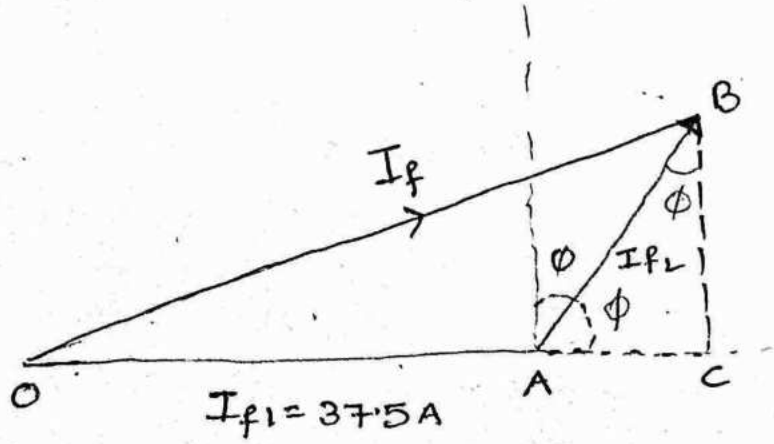
Sol

$I_{f2} = 20A$ required to circulate full load current, $\cos\phi = 0.8$.

$I_{f1} = ?$ which is obtained from O.C.C Curve.

Rated Voltage of alternator = 6600 V

$I_{f1} = 37.5$ required to generate 6600 V on no load



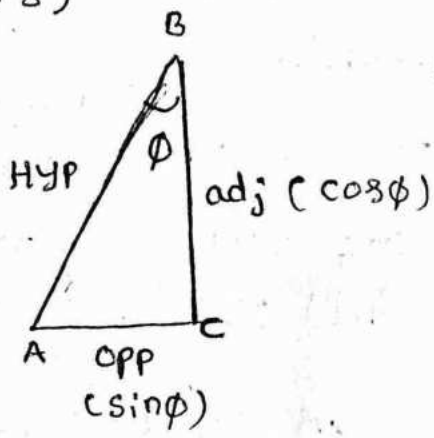
$OB^2 = OC^2 + BC^2$

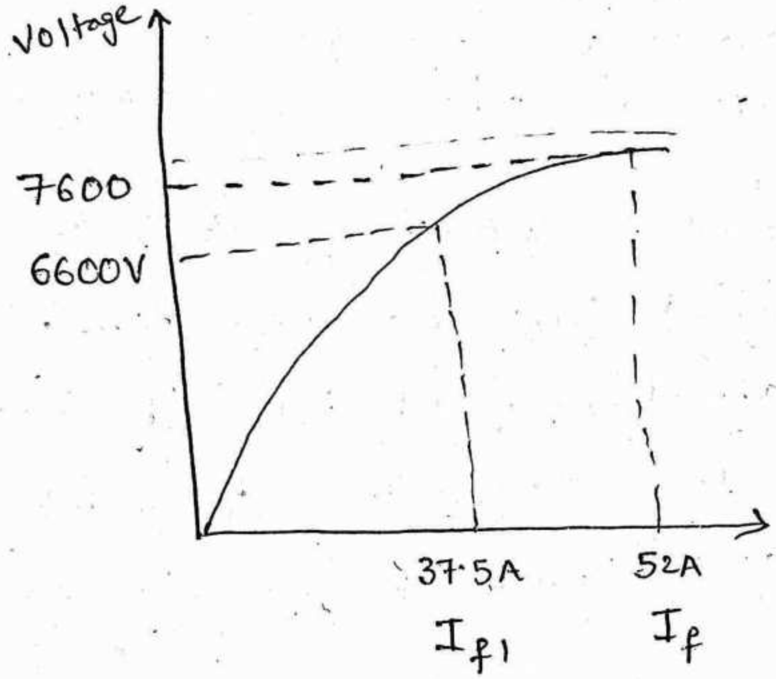
$= (OA + AC)^2 + BC^2$

$(I_{f1} + I_{f2}\sin\phi)^2 + (I_{f2}\cos\phi)$

$(37.5 + 20 * 0.6)^2 + (20 * 0.8)^2$

$I_f = \sqrt{2706.2} = 52A$




at $I_f = 52A$ assume the no load voltage as 7600 approximately. ($E_0 = 7600$)

At $I_{f1} = 37.5A$, rated voltage is 6600

$$\% \text{ Reg} = \frac{E_0 - V}{V} * 100 = \frac{7600 - 6600}{6600} * 100 = 15.152\%$$

Unit - 1

Synchronous Reactance $\rightarrow (X_s = X_L + X_a)$

Emf setup due to armature reaction mmf is always in quadrature with load Current I.

An emf induced in inductive coil & effect of armature reaction Considered as equivalent to reactance drop $IX_a$. This reactance drop $(IX_a)$ represents decrease in induced emf due to the effect of armature reaction.

where $X_a \rightarrow$ additional fictitious reactance added to the leakage reactance. The sum of leakage reactance $X_L$ & a fictitious reactance $(X_a)$ is synchronous in reactance $(X_s)$

$$X_s = X_L + X_a$$

Effective Resistance $(R_e = R_a)$

The effective resistance of armature winding is termed as armature resistance. It is denoted as $R_a/p$

It is measured by applying DC voltage & measure the Corresponding DC current flowing through armature winding.

Due to skin effect, effective resistance of AC is more than DC resistance.

$$R_{ac} > R_{dc}$$

Effective resistance under ac Condition is varied from 1·25 to 1·75 times the dc resistance

$$\boxed{R_{ac} = 1{\cdot}25 \times R_{dc}}$$

Unit -)

Synchronous impedance ($Z_s$)

If synchronous reactance is Combined with armature effective resistance, the quantity is called as synchronous impedance ($Z_s$).

$$Z_s = R_a + jX_s$$

Alternator on Load

If field excitation of alternator is adjusted to give normal voltage on NO load then a load is Connected to alternator. The terminal Voltage of alternator changes with load. It is due to

(I) Voltage drop of armature effective resistance ($IR_a$)

(II) Voltage drop of armature leakage reactance $IX_L$

(III) Voltage drop of armature reaction.

Phasor diagrams (for loaded alternator)

Let $IR_a$ be the voltage drop of armature $IX_L$ be the voltage drop of leakage reactance & $Z_s$ be the synchronous impedance.

phasor diagram for unity power factor

Unit - 1

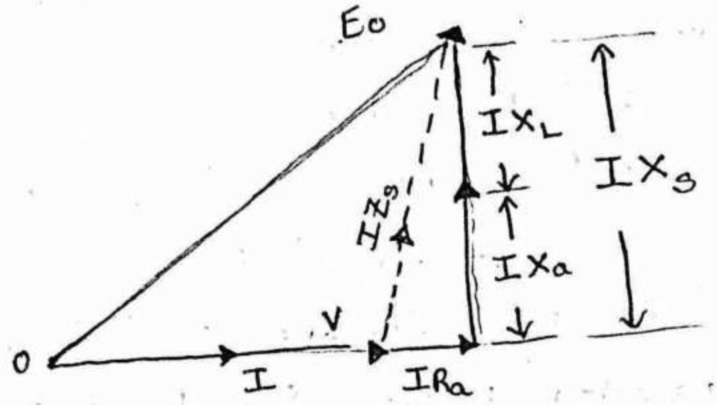
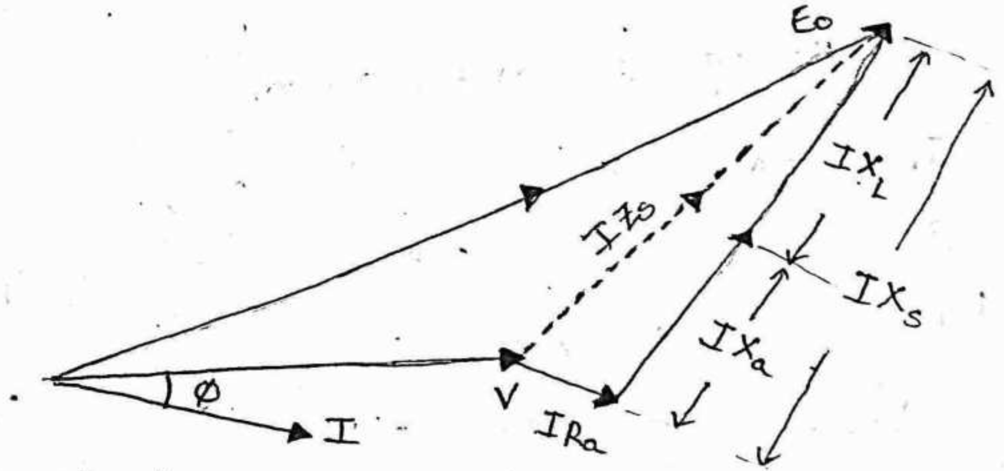


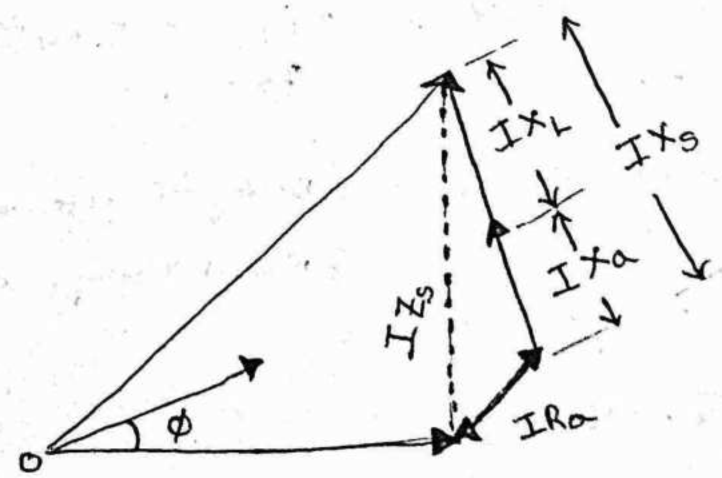

fig (a) At unity power factor.

from fig(a) Here, current is in phase with voltage so phase angle b/w V & I is 0




fig(b) at lagging power factor for inductive loads

Current lags behind the voltage by 90°. so Phase angle is +90°. P.f = 0




fig(c) at leading power factor for capacitive loads.

EMF problem

Rating of alternator is 3.5 MVA
Connection type of alternator is star
rated voltage in line value V = 4160 at f = 50Hz
field current is 200A, required to circulate full load current on short circuit. Calculate EMF method (ii) ampere turn method having full load regulation at 0.8 p.f lag. Assume Ra = 0 during s.c

| $I_f$ in Amp | 50 | 100 | 150 | 200 | 250 | 300 | 350 | 400 | 450 |
|---|---|---|---|---|---|---|---|---|---|
| EMF in volts | 1620 | 3150 | 4160 | 4750 | 5130 | 5370 | 5550 | 5650 | 5750 |

Sol Rating = 3.5 MVA = $3.5 * 10^6$

$I_f$ = 200A, $V_{oc}$ = 4750 in line,
rated voltage (V) = 4160 in line on full load

p.f = cos = 0.8, $\phi$ = 36.52°

$\rightarrow$ $V_{oc}/p = \frac{V_{oc}}{\sqrt{3}} = \frac{4750}{\sqrt{3}} = 2742.4\,V$

rated voltage in line V = 4160

$\rightarrow$ rated voltage in phase = $\frac{V}{\sqrt{3}} = \frac{4160}{\sqrt{3}} = 2401\,V$

$\rightarrow$ $Z_s = \frac{V_{oc} \text{ in phase}}{I_{sc}} = \frac{2742}{486} = 5.64\,\Omega/\text{phase}$

$I_{sc}$ or Rated Current = $\frac{\text{Rating of alternator}}{\sqrt{3}\text{ Rated Voltage on F.L}}$

$= \frac{3.5 * 10^6}{\sqrt{3} * 4160} = 486A$

Induced emf of alternator

$$E_0 = \left[(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_s)^2\right]^{1/2}$$

$$Z_s^2 = R_a^2 + X_s^2$$

$I_f \quad R_a = 0$

$$\sqrt{Z_s^2} = X_s$$


$$X_s = Z_s = 5.64\,\Omega/p$$

$\cos\phi = 0.8$, $\sin\phi = 0.6$

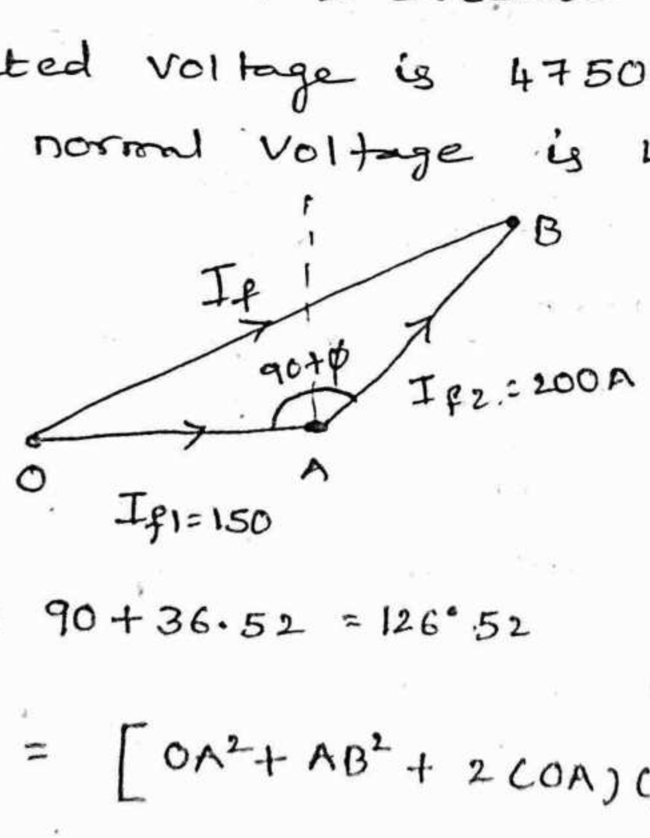
$$E_o = \left[(2401*0.8 + 486*0)^2 + (2401*0.6 + 486*5.64)^2\right]^{1/2}$$

$$= 4600$$

$$\%\ \text{reg up} = \frac{E_o - V}{V} * 100 = \frac{4600 - 2401}{2401} * 100 = 91.58$$

(11) from table in problem data in $I_{f_2} = 200\,A$ reaquired to circulate full load current have rated voltage is 4750.

The normal voltage is 4160 at $I_{f_1} = 150$




$$90 + \phi = 90 + 36.52 = 126^\circ.52$$

$$OB = \left[OA^2 + AB^2 + 2(OA)(AB)\cos(180^\circ - 126^\circ.5)\right]^{1/2}$$

$$= \left[(I_{f1})^2 + (I_{f2})^2 + 2(I_{f1})(I_{f2})\cos(53.5)\right]^{1/2}$$

$$OB = I_f = 313.35\,A$$

# ZPF - method & Potier method

EMF & MMF method gives regulation values higher & lower than actual values.

ZPF method gives approximate values very nearer to actual values.

In synchronous impedance method, armature reaction effect is substituted by an additional fictitious reactance ($X_a$).

In MMF method, armature leakage reactance ($X_L$) is substituted by an additional reaction.

Due to above armature reactions effect enormous results occur.

ZPF method is based on separating ($IX_L$) armature leakage reactance drop & effect of armature reaction.

This method uses first ~~[illegible]~~ 2 methods. Also it requires (i) no load curve (ii) full load ZPF curve. Reduction in voltage due to armature reactance is potier reactance.

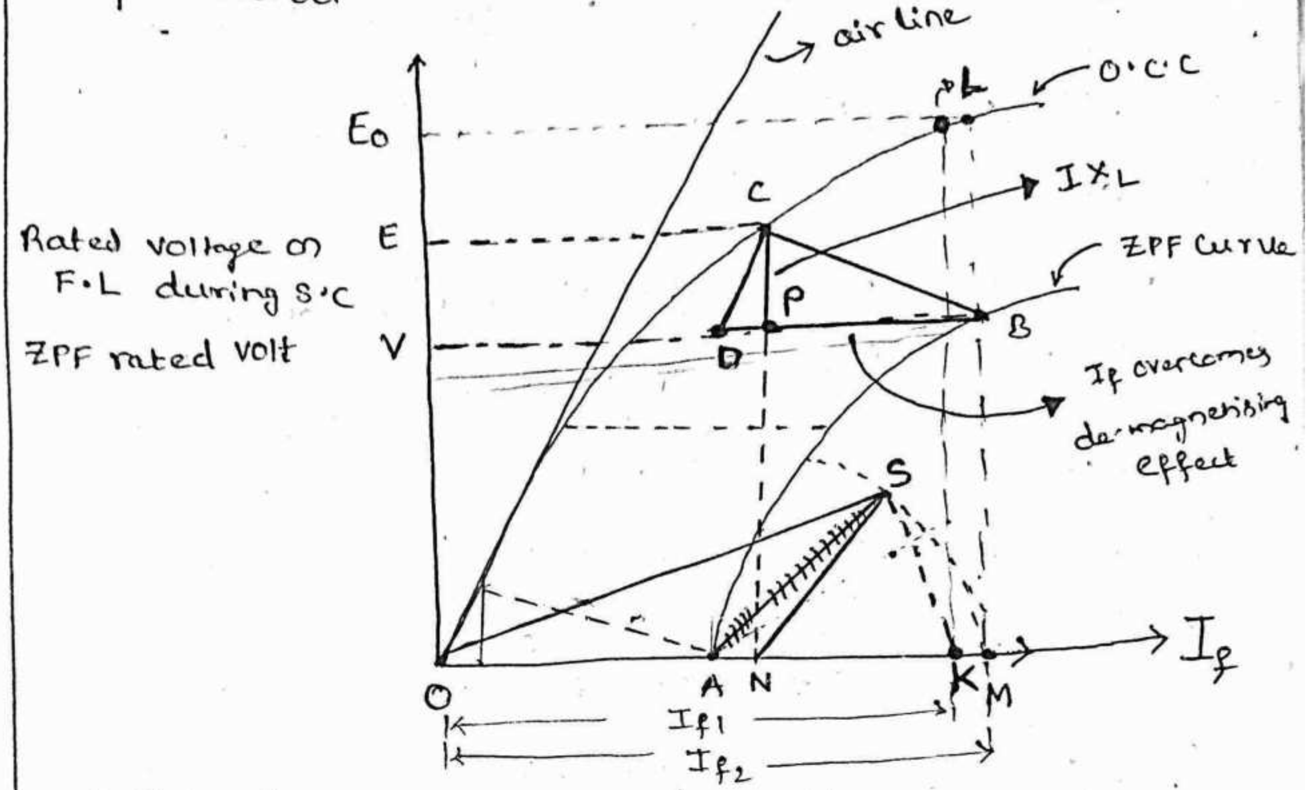
** ZPF lagging curve is obtained as ***

→ allow the alternator to run at constant speed.

→ Connect the alternator terminals with pure inductive loads.

→ Increase field current ($I_f$) untill the generator supplies full load current at ZPF at rated voltage.

→ first draw the O·C·C Curve obtained from emf method




→ plot the ZPF Curve (OM = field current required to deliver full load Current at ZPF at rated voltage. BM = rated voltage at ZPF Curve

→ A horizontal BD is drawn such that

BD = OA

where OA = $I_f$ required to circulate full load Current during S·C test

→ Draw Tangent OT from origin on O·C·C Curve. OT is air line

→ At point D, draw a line DC parallel to air line & this line intersect on O·C·C Curve as point 'C'.

→ Joined BC line from point 'C' on O·C·C to the point 'B' on ZPF Curve

→ Draw CP line perpendicular to line DB.

Traingle CDB is termed as potier traingle

→ line CP is voltage drop due to armature leakage reactance. $IX_L$.

→ PB gives $I_f$ requires to overcome the dermagnetising effect of armature at full load.

→ DP balancing the armature leakage reactance drop CP.

We have CN = E rated voltage on F.L by considering the drop $IX_L$

$$CN = CP + PN$$

But PN = BM $\quad = CP + BM$

$$CN = CP + BM$$

$$E = IX_L + V$$

$$E = V + IX_L$$

To determine $E_o$ at no load induced emf

$I_f$ required to generate voltage on NO load is $E_0$ = $I_f$ required to generate voltage on Full load E & $I_f$ required to overcome armature reaction is V.

$$I_f = E$$

$$E_0 = E + V$$

ON → field current required to generated voltage E on O.C.C curve during no load

→ At point 'N'

NS = BP is laid at angle $90+\phi$ with respect to ON then Joined OS.

→ With O as center with radius as OS draw an arc to meet x-axis at point 'K'.

OK = resultant field current.

→ ~~Joined~~ Draw a vertical line from point 'K' it touches the O.C.C at point L.

$\boxed{LK = E_0}$

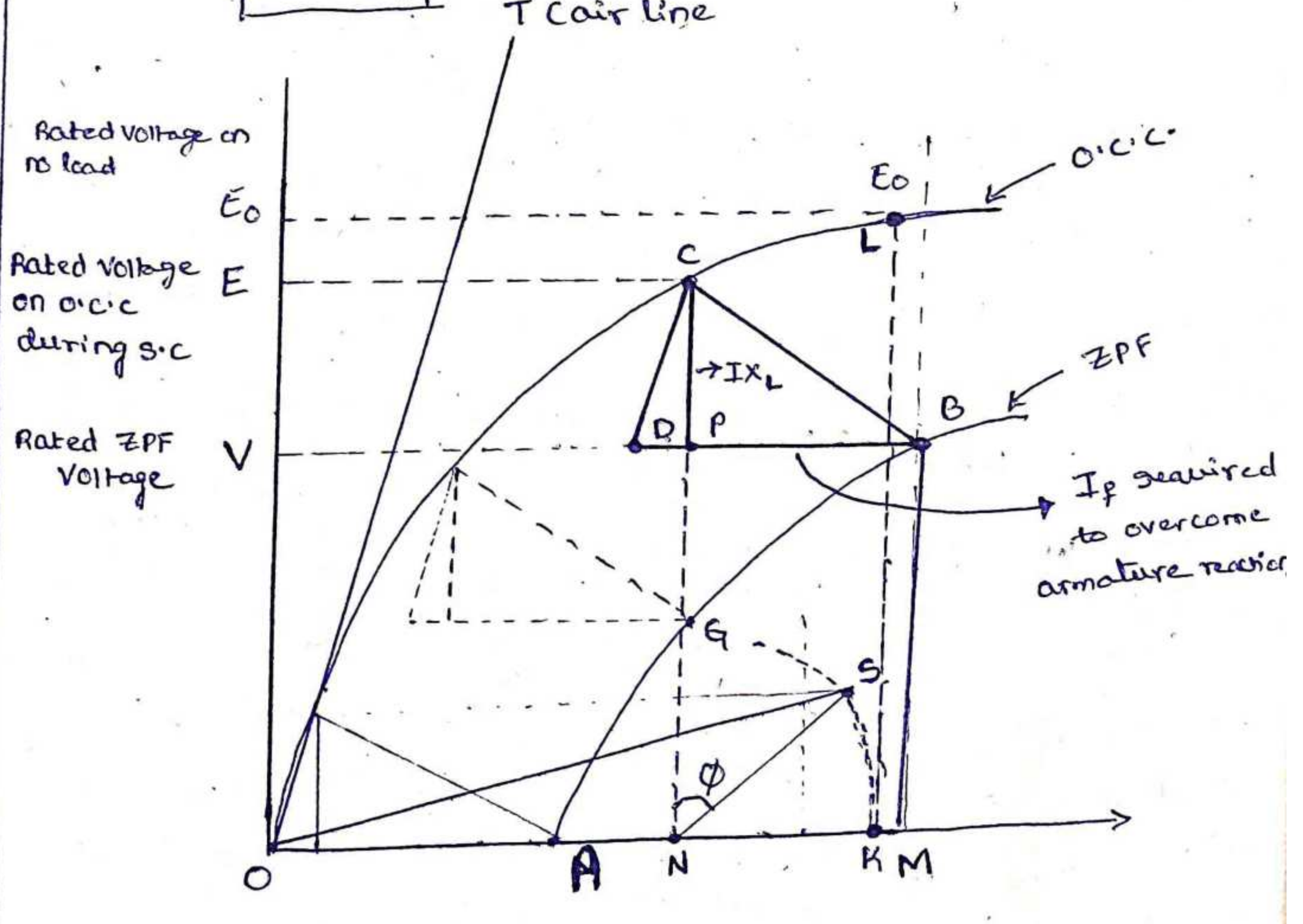
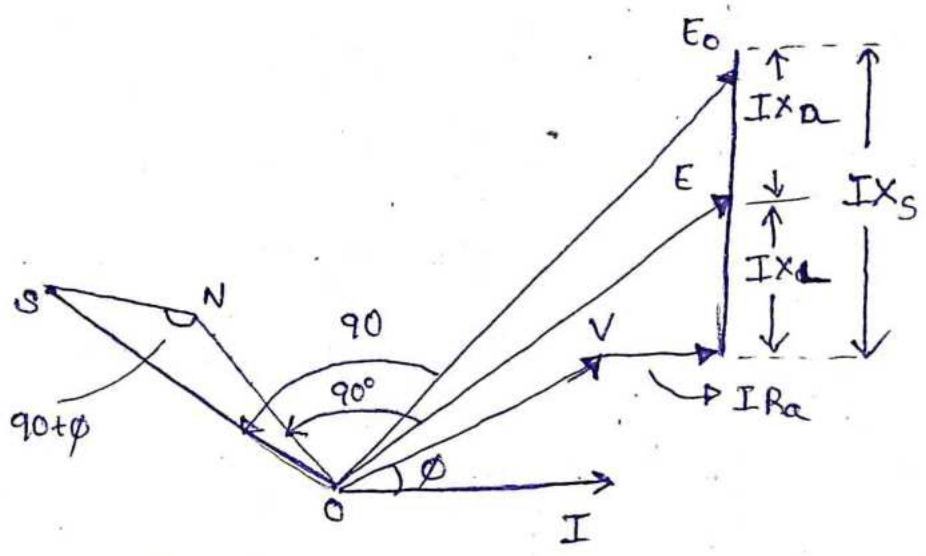



fig shows ZPF curves

The phasor diagram of loaded alternator is shown




V = rated voltage on ZPF curve
I = full load current
p.f = $\cos\phi$

$$E_0 = E + IX_a$$

$$\boxed{E^2 = (V + IX_L \sin\phi) + (IX_L \cos\phi)^2}$$

$$E = V + IR_a + IX_L$$

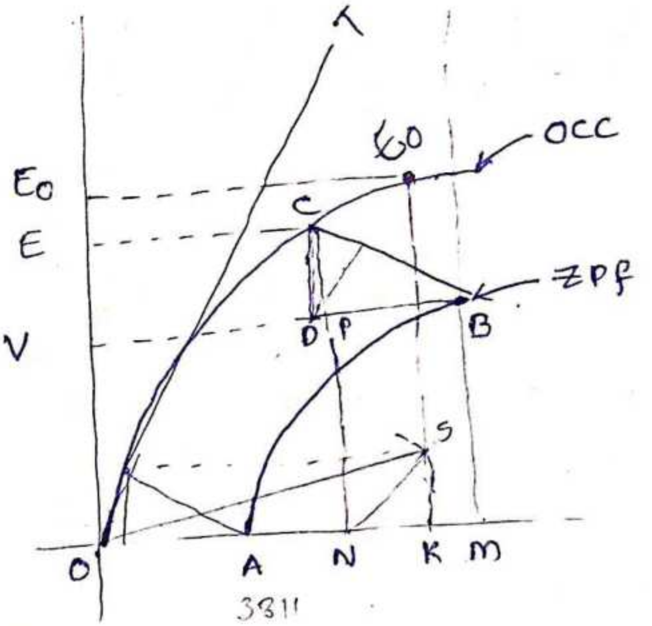
Problem

A 3 phase star connected alternator has 2.5 MVA rating gives 6.6 KV. the following data for O.C.C is

| $I_f$ (A) | 16 | 20 | 25 | 32 | 45 |
|---|---|---|---|---|---|
| Voltage terminal | 4400 | 5500 | 6600 | 7700 | 8800 |

With armature short circuited, 4 full load current flowing, field current is 18 A. when 3 machine is supplying full load current at ZPF at rated voltage, the field current is 45A.

Also at ZPF initial voltage 0 at $I_f = 18A$ ZPF final voltage 3811V at $I_f = 45A$ during S.C test or full load test, Find ① leakage reactance per phase (II) full load armature reaction in terms of equivalent field current (III) full load voltage regulation at 0.8 p.f lag, neglect Ra




Point (A)
$I_f = 18A$, $V_{ZPF} = 0$
Point (B)
$I_f = 45A$, $V_{ZPF} = 3811$

CP drawn perpendicular to DB
DC drawn parallel to OT
$CP = IX_L$

given data alternator is 3 phase star connected
we have $V_p = V_L/\sqrt{3}$. Rating of alternator = 6.6 KV
2.5 MVA

| | | | | | |
|---|---|---|---|---|---|
| $I_f$ = | 16 | 20 | 25 | 32 | 45 |
| $V_p$ = | $\frac{4400}{\sqrt{3}}$ | $\frac{5500}{\sqrt{3}}$ | $\frac{6600}{\sqrt{3}}$ | $\frac{7700}{\sqrt{3}}$ | $\frac{8800}{\sqrt{3}}$ |
| | 2540 | 3175 | 3811 | 4446 | 5081 |

Rated voltage of alternator $V = 6.6 * 1000$

→ Rated Current $I = \dfrac{\text{Rating of alternator}}{\sqrt{3} * V_{rated}}$

$$I = \frac{2.5 * 10^6}{\sqrt{3} * 6.6 * 1000} = 218.69\,A$$

→ $CP = I * X_L$

Let $CP = 462$ from measurement

(i) $X_L = \dfrac{462}{218.69} = 2.113\,\Omega$

CP = rated voltage on O.C.C − rated voltage on ZPF curve
$IX_L = E - V = 462V$

(ii) BP → field current required to overcome the armature reaction.

BP → ON − NM = 15 A by measurement

(iii) To find $E_0$

we have $CP = IX_L = E - V$

$$IX_L = E - V$$

$$\boxed{E = V + IX_L}$$

given p.f = 0.8 lag

$\cos\phi = 0.8$, $\phi = 36.86$, $\sin\phi = 0.6$

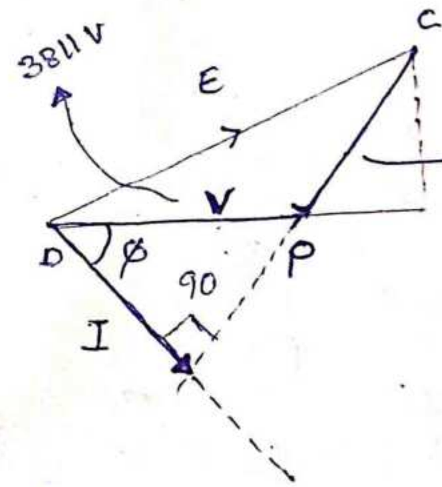
$CP = IX_L = 462V$ from phasor diagram

Rated voltage on ZPF curve = 3811

$$E = \sqrt{(V + IX_L \sin\phi)^2 + (IX_L \cos\phi)^2}$$

or

$$E = \sqrt{(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_L)^2}$$




$$E^2 = (3811 + 462 * 0.6)^2 + (462 * 0.8)^2$$

$$E = 4104.873V$$

$I_f$ = 28A is required to generate

Total field current = OS or OK = (28+15)

= 43A

$E_0 = 4965$

% Regulation = $\frac{E_0 - V}{V} * 100$

$= \frac{4965 - 3811}{3811} * 100 = 30.28\%$

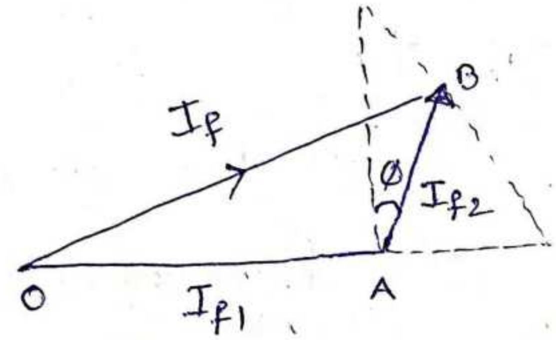
ASA- method → American standard Association

It is an improved one of MMF method. this method is more suitable for salient & non- salient

In MMF total field Current ( $I_f$) required to generate voltage ( $E_0$) on No load is obtained by adding field Currents $I_{f1}$ & $I_{f2}$.

$I_{f1}$ → field Current required to generate rated voltage on no load

$I_{f2}$ → field Current required to overcome armature reaction




$\phi$ = lagging

$\cos\phi = p.f$

The value of $I_f$ is somewhat smaller than actual value of $I_f$ required to generate $E_0$. This is due to the generated poles are unsaturated magnetically.

To get same amount of magnetic flux we need to increase the field excitation as a result poles becomes partially saturated.

This method calculates the additional field Current requires to saturate the generator poles.

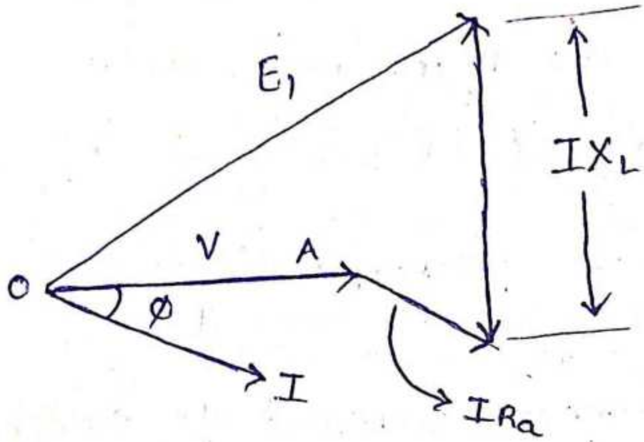
Let $I$ = Rated or full load Current

$V$ = Rated voltage

$R_a$ = armature Resistance

$X_L$ = armature leakage reactance

phasor diagram

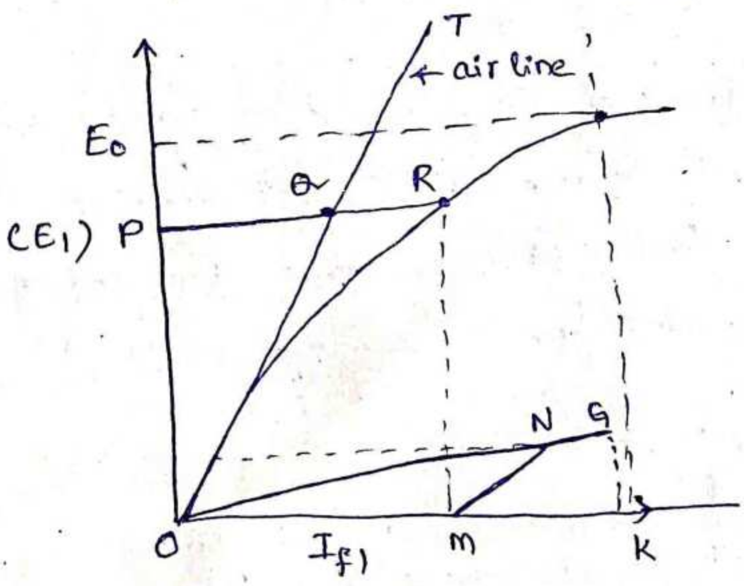



from phasor diagram ø = Load angle.

$$E_1 = V + IR_a + IX_L$$

$IX_L$ drop is Considered in ZPF method

→ Plot O.C.C Curve from no load or O.C test




$OP = E_1$

$MN = I_{f2}$

$OM = I_{f1}$

$ON = I_f = OM + MN = I_{f1} + I_{f2}$

$NG = QR$

$OK = E_0$

from origin 'O', draw a tangent passing through the surface of O.C.C Curve.

Let P be the point marked on Y-axis (voltage) such $OP = E_1$.

Extend a line from point P to O.C.C Curve. line intersect OT lines as 'Q' & intersects on O.C.C as 'R'.

→ length of RS indicates additional field current required to saturate poles of generator.

$$\% Reg = \frac{E_0 - V}{V} * 100$$

**Blondel's 2 reaction theory.** ($X_d$ & $X_q$)

Synchronous machine with non-salient pole (cylindrical rotor) has uniform air gap, hence reluctance of air gap is constant through out.

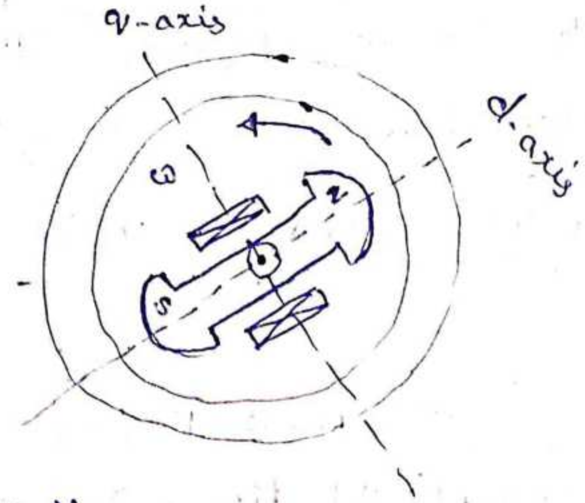
But in salient pole or projecting poles has non-uniform air gap, hence reluctance of the air gap is not constant.

Non salient pole machine posses only one axis either direct or pole axis.

But a salient pole machine has 2 axis
(I) field axis called direct axis (d-axis)
(II) axis passing through the centre of pole called quadrature or q-axis.

A salient pole with d-axis posses 2 mmf (I) field mmf & (II) armature mmf.

Whereas q-axis posses only one mmf i.e armature mmf on q-axis. Due to this reluctance is low along the pole & highly between the poles.




According to Blondel 2 reaction theory

① Armature Current is resolved into 2 components

$I_d$ perpendicular to $E_0$

$I_q$ along with $E_0$.

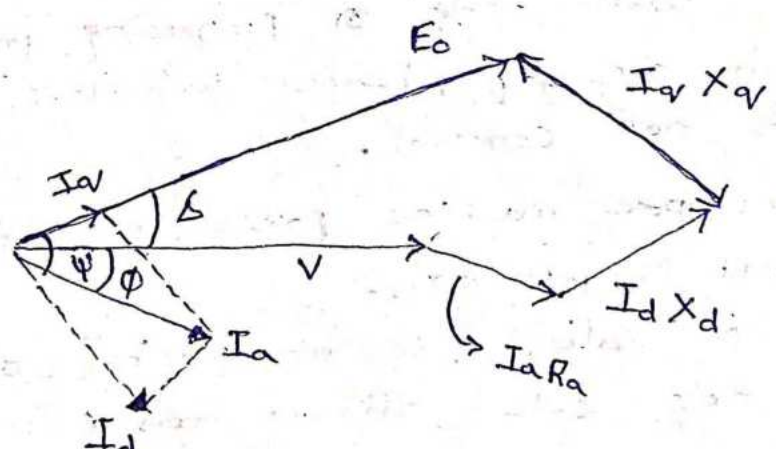
(II) for non salient pole alternator

$X_d = X_s$

for salient pole alternator

$X_d > X_q$.

$X_q$ ranges from 60 - 70 % of $X_d$.




fig① shows phasor diagram

V → Rated voltage

$I_a$ → armature Current

$I_d$ & $I_q$ Components of $I_a$

we have $I_d = I_a \cos(90° - \psi)$

$= I_a \sin(\psi)$

$I_q = I_a \cos\psi \quad (\psi = \delta + \theta)$

→ Voltage drop $I_a R_a$ is parallel to $I_a$

→ Voltage drops $I_d X_d$ & $I_q X_q$ are perpendiculars to current phasors $I_d$ & $I_q$.

Voltage drops $I_a R_a$, $I_d X_d$, $I_q X_q$ are vectorially added. The resultant is $E_0$.

from fig ①

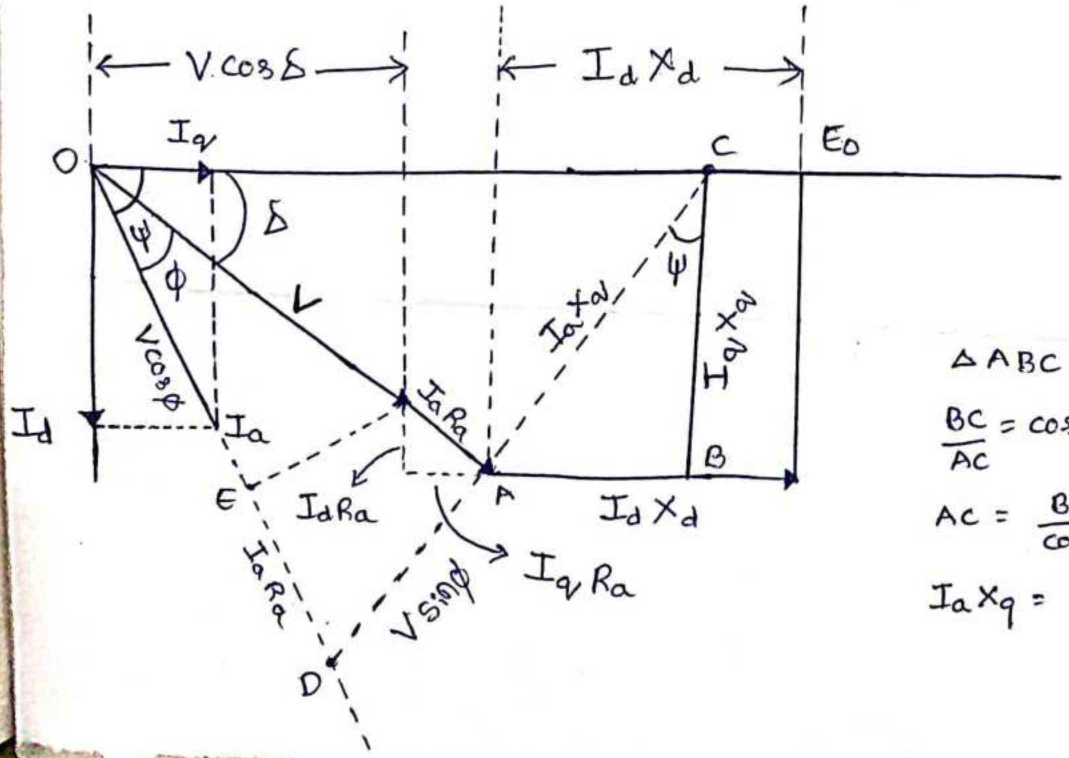
$$E_0 = V + I_a R_a + I_d X_d + I_q X_q$$

In above equation, $E_0$, $V$, $I_a$ & $R_a$ are known values. $X_d$ & $X_q$ values are getting from slip test.

Also current components $I_d$ & $I_q$, angles $\psi$ & $\delta$ are not known. These are calculated from $\Delta ODC$

$$\tan\psi = \frac{AD + AC}{OE + ED} = \frac{V\sin\phi + I_a X_q}{V\cos\phi + I_a R_a} \quad \text{generating action}$$

for motoring action $\tan\psi = \frac{AD - AC}{AD - ED} = \frac{V\sin\phi - I_a X_q}{V\sin\phi - I_a R_a}$




$\Delta ABC$

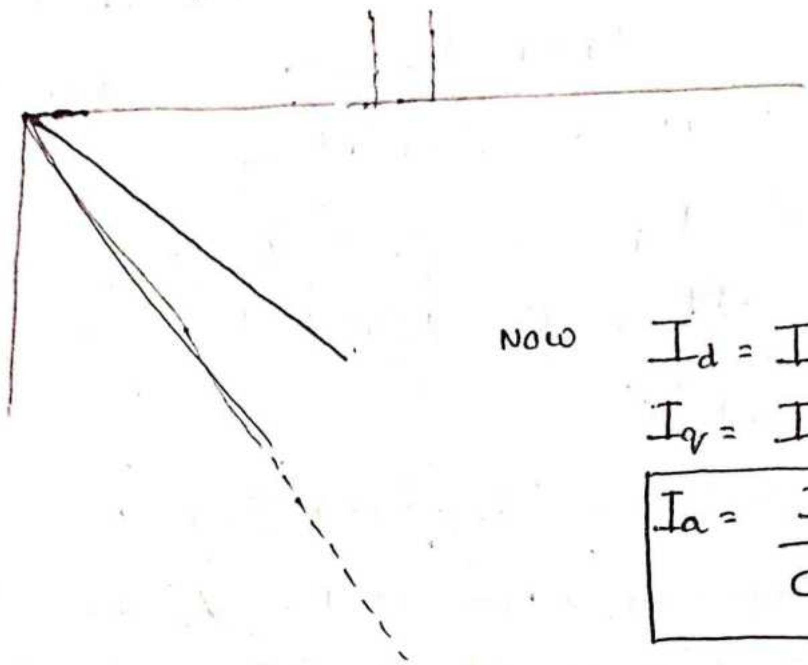
$$\frac{BC}{AC} = \cos\psi$$

$$AC = \frac{BC}{\cos\psi} = \frac{I_q X_q}{\cos\psi}$$

$$I_a X_q = \frac{I_q X_q}{\cos\psi}$$

Dotted line AC has drawn perpendicular to $I_a$ & BC is drawn perpendicular to $E_0$.

$\angle ACB$ is $\psi$.



Now $I_d = I_a \sin\psi$

$I_q = I_a \cos\psi$

$$\boxed{I_a = \frac{I_q}{\cos\psi}} \Rightarrow \cos\psi = \frac{I_q}{I_a}$$

in $\Delta ABC$

$$\frac{BC}{AC} = \cos\psi \Rightarrow AC = \frac{BC}{\cos\psi}$$

$$\frac{I_q X_q}{\cos\psi} = I_a X_q$$

From $\Delta ODC$.

$$\tan\psi = \frac{AD + AC}{OE + ED} = \frac{V\sin\phi + I_a X_q}{V\cos\phi + I_a R_a}$$

$\delta = \psi - \phi$ → for generating action

for motoring action ( $\delta = \phi - \psi$ )

$$\tan\psi = \frac{V\sin\phi - I_a X_q}{V\sin\phi - I_a R_a} = \frac{AD - AC}{AD - ED}$$

Excitation voltage is given as

$E_0 = V\cos\delta + I_q R_a + I_d X_d$ → for generator for motoring

$$E_0 = V\cos\delta - I_q R_a - I_d X_q$$

If we neglect armature resistance

$\psi = \phi + \delta$ generating

$\psi = \phi - \delta$ motoring

$$\psi = \phi \pm \delta$$

$$I_d = I_a \sin\psi = I_a \sin(\phi \pm \delta)$$

$$I_q = I_a \cos\psi = I_a \cos(\phi \pm \delta)$$

$$V\sin\delta = I_q X_q = I_a X_q \cos(\psi)$$

$$V\sin\delta = I_a X_q \cos(\phi \pm \delta)$$

$$= I_a X_q [\cos\phi \cos\delta \pm \sin\phi \sin\delta]$$

$$V = I_a X_q [\cos\phi \cot\delta \pm \sin\phi]$$

$$V = I_a X_q \cos\phi \cot\delta \pm I_a X_q \sin\phi$$

$$I_a X_q \cos\phi \cot\delta = V \pm I_a X_q \sin\phi$$

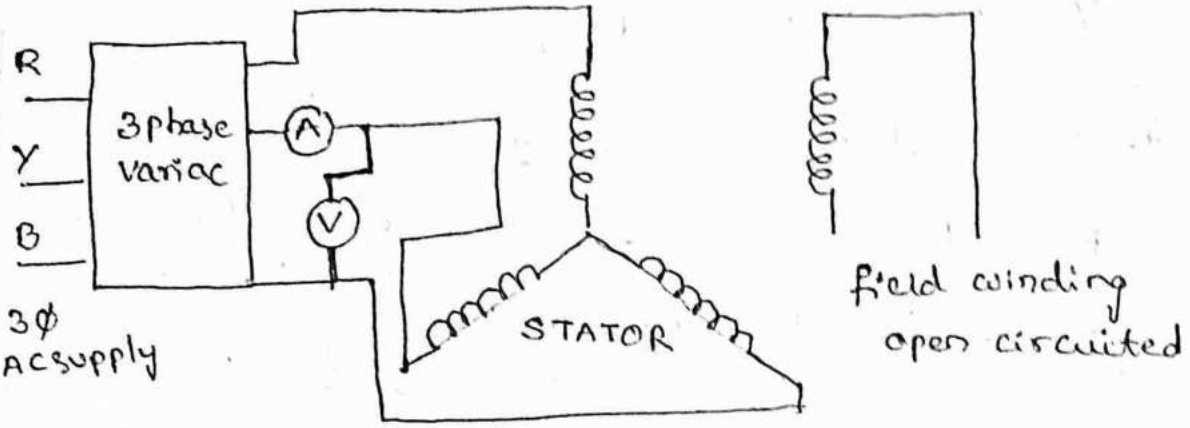
$$\cot\delta = \frac{V \pm I_a X_q \sin\phi}{I_a X_q \cos\phi}$$

$$\tan\delta = \frac{I_a X_q \cos\phi}{V \pm I_a X_q \sin\phi}$$

If $R_a$ is neglected in excitation voltage

$$E_0 = V\cos\delta \pm I_d X_d$$

## Determination of $X_d$ & $X_q$ by slip test




fig(a) shows experimental set up for salient pole machine.

→ Bring the alternator to near the synchronous speed. The rotor speed is less or more than synchronous speed.

→ Keep the field winding is open circuited.

→ Apply a small voltage to the stator terminals using 3 phase variac.

→ Stator windings carry 3 phase currents, a rotating magnetic field develops which is revolving in stator at synchronous speed.

→ Because of difference in rotor field speed & stator field speed (slip) an emf is induced in field winding of alternator.

→ As stator flux revolves, stator MMF aligns with field poles (direct axis), the reactance offered is $X_d$.

Stator MMF aligns with quadrature axis, the reactance offered is $X_q$.

→ This test is conducting of salient pole machine as a resultant length of air gap is not constant as a result magnetic reluctance is low therefore reactance $X_d$ & $X_q$ are unequal.

→ Because of unequal $X_d$ & $X_q$ current drawn from the supply tends oscillate between a maximum value & minimum value.

→ when current is at minimum value, the reactance offered is $X_d$ but voltage is maximum

when current is at maximum value, the reactance offered is $X_q$ but voltage is minimum.

→ Due to minimum & maximum values of reactance, the voltage drop of armature also varies.

$$X_d = \frac{\text{Maximum voltage/phase}}{\text{Minimum Current}}$$

$$X_q = \frac{\text{Minimum Voltage/phase}}{\text{Maximum Current}}$$

Problem A 1000KVA, 6600V, star connected synchronous generator has an $R_a = 0.5\,\Omega$, direct axis reactance $X_d = 8\,\Omega$, & quadrature reactance $X_q$ is $5\,\Omega$. It supplies rated voltage at 0.8 p.f lag. Calculate its no load induced emf.

Sol Rating of alternator = $1000 \times 1000$ VA

Rated Voltage $V = 6600$

$$V/P = \frac{6600}{\sqrt{3}} = 3810.51 \text{ Volts}$$

$R_a = 0.5$, $X_d = 8\,\Omega$, $X_q = 5\,\Omega$, p.f $= 0.8$ lag

→ full load current $= \dfrac{\text{Rated VA}}{\sqrt{3} * \text{Rated Voltage}}$

$$I = \frac{1000 * 1000}{\sqrt{3} * 6600} = 87.48\ A$$

$$I_a = I = I_{sc} = 87.48\ A$$

→ $$\tan\psi = \frac{V\sin\phi + I_a X_q}{V\cos\phi + I_a R_a}$$

$$\tan\psi = \frac{3810.5 * 0.6 + (87.48 * 5)}{3810.5 * 0.8 + (87.48 * 0.5)} = 0.881$$

$$\psi = \tan^{-1}(0.881) = 41.375^\circ$$

given $\phi = \cos^{-1}(0.8) = 36.869^\circ$

→ $$\boxed{\psi = \Delta + \phi}$$

$$\Delta = \psi - \phi$$

$$\Delta = 41.375 - 36.869 = 4.506^\circ$$

we have.

→ $\boxed{I_d = I_a \sin\psi} = 87.48 * \sin(41.375)$

$= 57.823\ A$

→ $\boxed{I_q = I_a \cos\psi} = 87.48 * \cos(41.375)$

$= 65.645\ A$

→ no load induced emf

$$\boxed{E_o = V\cos\Delta + I_q R_a + I_d X_d}$$

$= 3810.5 * \cos(4.506) + (65.645 * 0.5) + (57.823 * 8)$

$E_o = 4294.14\ V$

% Regulation $= \frac{E_0 - V}{V} * 100$

$= \frac{4294.141 - 3810.512}{3810.512} * 100 = 12.692$

ZPF Problem

Rating of alternator 30KVA Connected in star. The following are O.C, & full load ZPF data are, Rated voltage in line V = 400, 3 phase

| Exciting current ($I_f$) | 6 | 8 | 12 | 18 | 24 | 28 | |
|---|---|---|---|---|---|---|---|
| open circuit emf in line | 282 | – | 400 | 435 | 459 | 474 | |
| ZPF in line volts | – | 0 | – | – | – | 400 | |

Find armature reaction in Ampere turns & armature leakage reactance. Determine voltage regulation at full load 0.8 p.f. Neglect Ra

sol Rating = 30 * 1000 VA , Line voltage V = 400

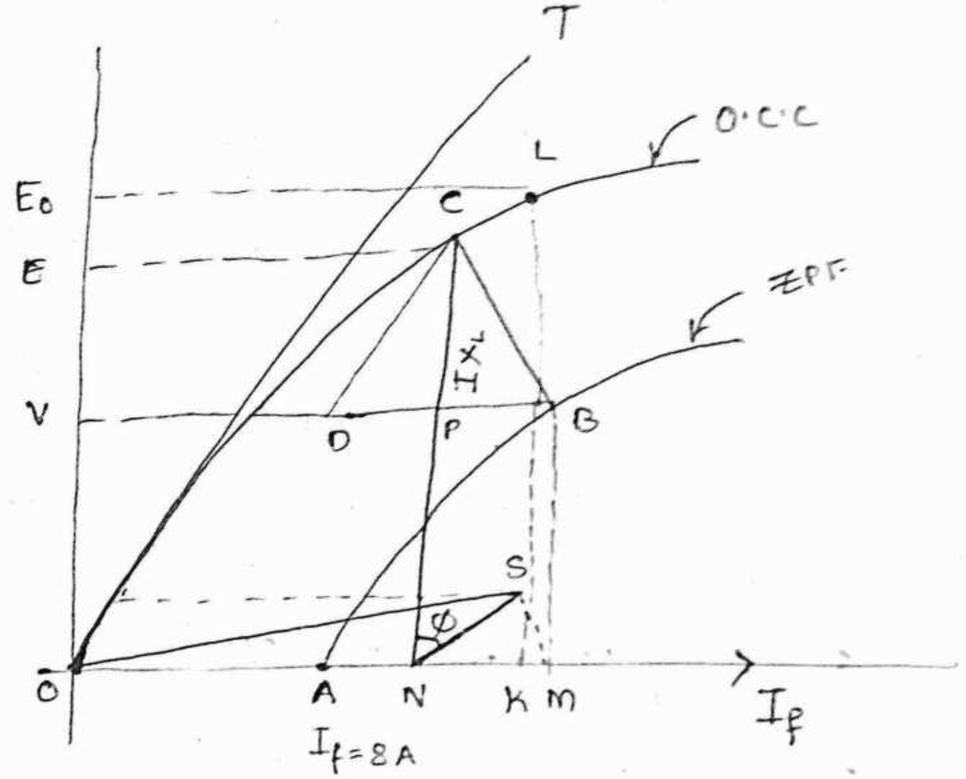
Rated voltage/p $= \frac{400}{\sqrt{3}} = 230.94V$

| $I_f$ in Amp | 6 | 8 | 12 | 18 | 24 | 28 | |
|---|---|---|---|---|---|---|---|
| O.C voltage in phase | $\frac{282}{\sqrt{3}}$ 162.8 | – | $\frac{400}{\sqrt{3}}$ 230.94 | $\frac{435}{\sqrt{3}}$ 251.15 | $\frac{459}{\sqrt{3}}$ 265 | $\frac{474}{\sqrt{3}}$ 273.66 | |

At $I_f = 8$, ZPF Voltage = 0

At $I_f = 28$, ZPF voltage = 400

draw ZPF Curves




point A = (8A, 0V)

point B = (28A, $400/\sqrt{3} = 230.94$ V)

BD = OA = 8A field Current

→ CD is drawn parallel to OT to cut on OCC at point C.

→ CP is drawn perpendicular DB.

$CP = IX_L$

→ $I = \dfrac{30 * 1000}{\sqrt{3} * 400} = 43.3\ A$

$CP = 43.3 * X_L$

$X_L = \dfrac{CP}{43.3}$

CP = 32.5 (its a actual measured value)

$X_L = \dfrac{32.5}{43.3} = 0.751\ \Omega$

BP = 7A by actual measurement

↳ $I_f$ required to overcome armature reaction.

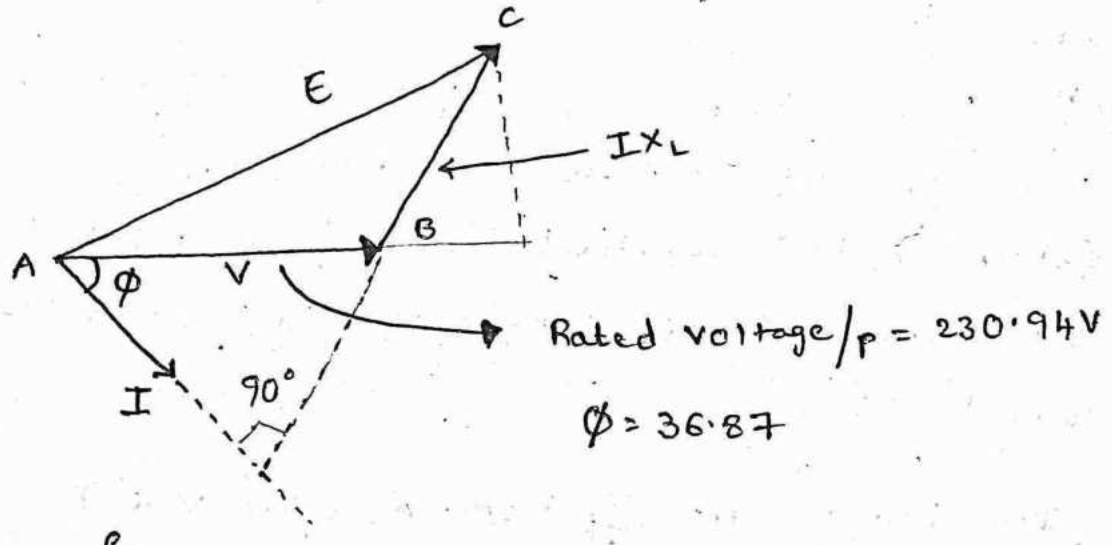
<u>To find $E_0$</u>

$$E = V + IX_L$$

$$p.f = \cos\phi = 0.8$$

$$\phi = \cos^{-1}(0.8) = 36.87$$

$$\sin\phi = 0.6$$




from phasor diagram

$$E = \left[(V + IX_L\sin\phi)^2 + (IX_L\cos\phi)^2\right]^{1/2}$$

$$= \left[(230.94 + 43.3 * 0.751 * 0.6)^2 + (43.3 * 0.751 * 0.8)^2\right]^{1/2}$$

$$= \sqrt{63396.2V} = 251.787\,V$$

from O.C.C, find excitation required to generate voltage 251.787 on no load is 15.5A from measurement.

$$I_{f1} = 15.5A$$

Also $I_{f2} = 7A$ at N.

Total field current required to generate voltage

$E_0 =$ (ON) (OA)

$I_f = I_{f1} + I_{f2}$

$= 15.5 + 7A = 22.5A$

(OK)

$I_f = 20.5A$ by actual measurement OK

→ ON = 15.5A, at point N, mark 126.87 angle.

→ NS = 7A = OA is drawn.

→ Joined OS.

→ With O as centre & radius as OS, drawn an arc cut at x-axis at a point 'K'.

→ Extend the vertical line from point 'K' to the curve O.C.C intersect at a point 'L'

→ Joined LK.

Corresponding to $I_f = 20.5A$ (OK line)

Corresponding voltage on O.C.C is $E_0 = 270$

$$\% \text{ Reg} = \frac{E_0 - V}{V} * 100 = \frac{270 - 230.94}{230.94} * 100 = 16.91\%$$

MMF method is used to compute voltage regulation

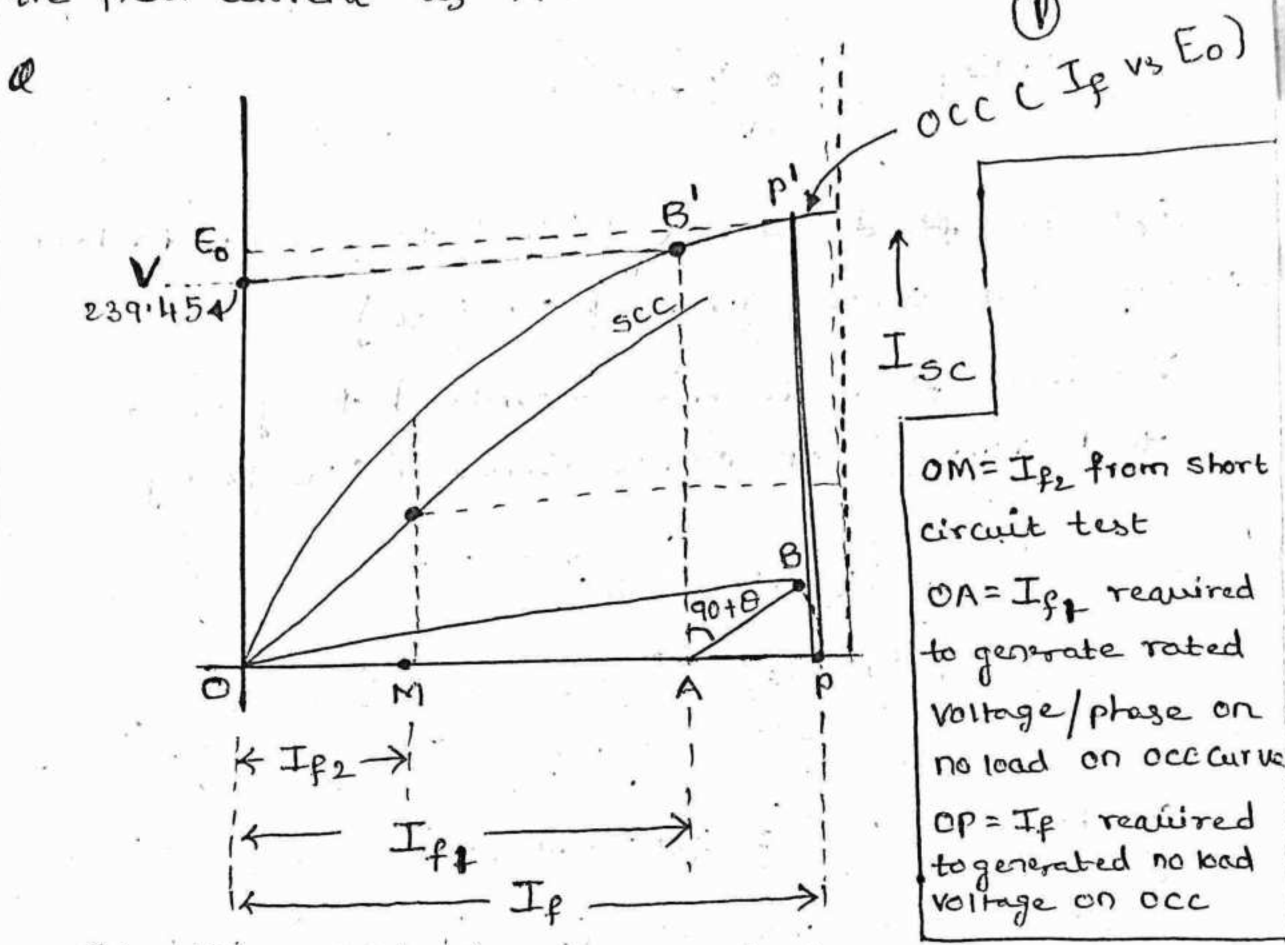
→ (I) Draw the O.C.C graph i.e $I_f$ Vs $E_0$. The data is obtained from O.C test.

→ (II) OM = $I_{f2}$ field current required to circulate full load current during short circuit test

→ (III) Let V be rated voltage per phase of alternator marked on O.C.C curve. Example is

$V_p = 415/\sqrt{3} = 239.4$

corresponding rated voltage per phase marks the field current as A




OA = $I_{f1}$, field required to generate rated voltage per phase (V) on no load curve.

→ (IV) Assume the load p.f is lagging

$\theta = 90+\theta$

→ (V) At point 'A' with an angle $90+\theta$ mark an angle.

Extended the line from point A to B. length of line AB = OM line.

→ (VI) Joined O & B, with length OB, with 'O' as Centre drawn an arc to cut at a point x-axis is marked as

→ (VII) Extend the line from point (P) to O.C.C curve as point $P'$.

OP = $I_f$ is the total field required to generate no load voltage $E_0$

Calculate perpentage regulation

$$\% \, Reg = \frac{E_0 - V}{V} * 100$$

$E_0 = I_f$ field current required to generate no load voltage on O.C.C curve [O.C test]

$V = I_{f1}$ field current required to generate rated voltage on O.C.C curve [O.C test]

## Unit - III

# Parallel Operation of Alternators

### Synchronisation of alternators

As load demand is within limit of alternator o/p, no problems is araised. However when the load demand is in excess, a single alternator cannot meet the load, so the necessary arises more number of generators.

Several alternators are employed it shares a common load. when alternators are connected in parallel. First keep the alternators in parallel & they made to share a common load.

the process of bringing 2 or more alternators to share a common load when operating in parallel with bus bars is termed as synchronisation

### Parallel operation advantages

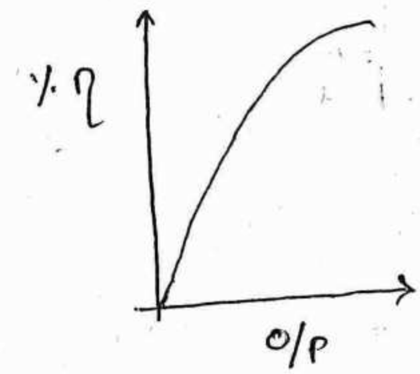
① Continunity of power supply

Single alternator cannot be guaranteed to provide continue power supply during fault conditions. This would completely interrupt the supply.

② Efficiency of operation



Efficiency of alternator increases with o/p.



Efficiency of alternator is maximum when supplying full or near full load.

Ex:- A 10KVA alternator supplies 6KVA load its efficiency is very low.

Using single alternator use 2 smaller units operating in parallel can share 3KVA & 3KVA equally.

③ smaller units running in parallel is more economically than single unit

④ Adding another unit to existing alternators in parallel provides future expansion.

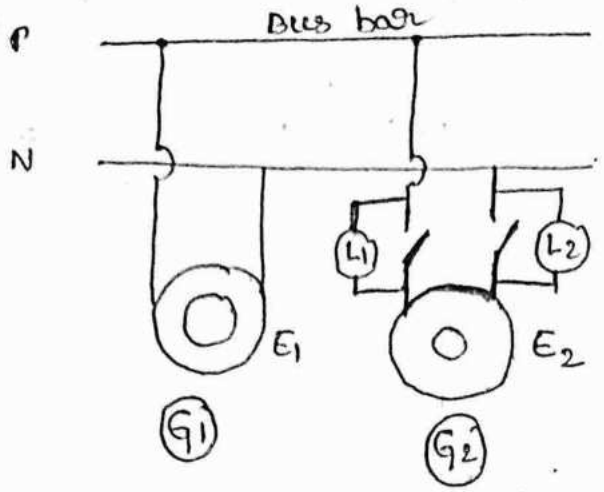
Conditions required for parallel operations

ⓐ for single phase alternators

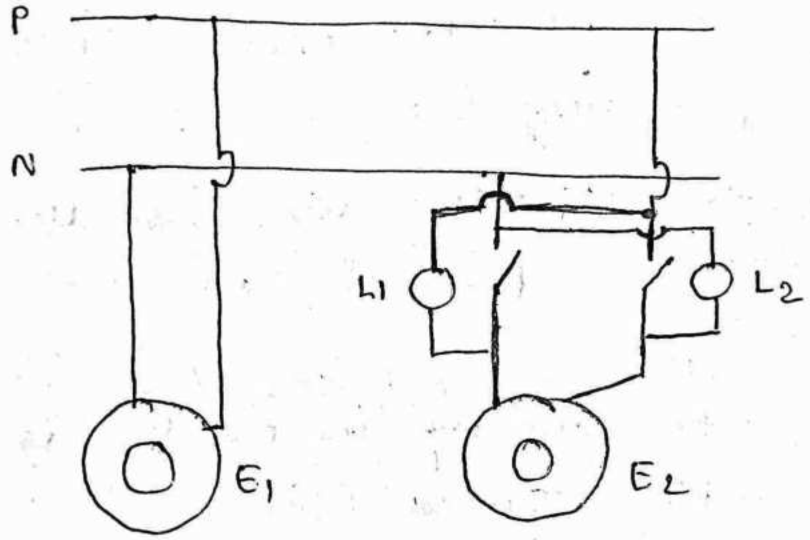
① terminal voltages of all generators must be similar

② frequencies of induced emf of all generators must be equal it is depending on the speed of prime movers.

③ Induced emf's $E_1$ & $E_2$ are in direct opposition to the local circuit but $E_1$ & $E_2$ are in phase with the external circuit.




fig(1) shows due to their unequal frequencies flicker will takes place in $L_1$ & $L_2$




fig(2) shows lamps flicker glow & dark during the phase differences in $E_1$ & $E_2$.

Lamps glow brighter if $E_1$ & $E_2$ are in phase with bus bar voltage

Lamps will dark if $E_1$ & $E_2$ are in phase opposition to bus bar voltage.

Synchronising is done at middle of any one of either dark or brighten lamp period.

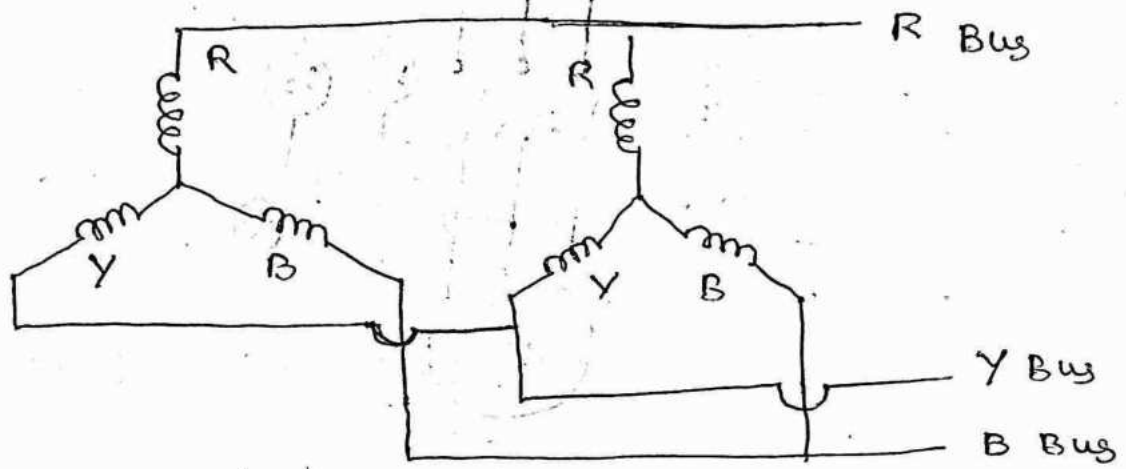
(b) For 3 phase alternator. 

(I) Terminal voltages of all generators must be equal,

(II) frequencies of generated emf's of all generators must be equal.

(III) phase sequence must be proper.

means similar phases must be joined




## Methods of synchronising alternators

If a new alternator is added to the existing alternator in parallel, the following practical methods are adopted for synchronising

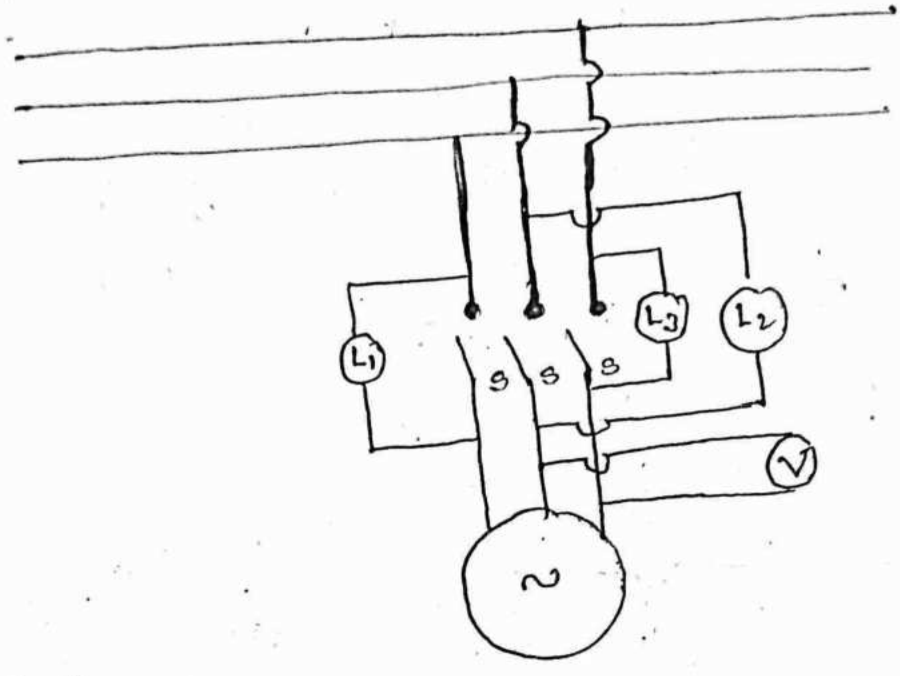
(I) Dark lamp method
(II) Bright lamp method
(III) Synchroscope method.

### (I) Dark lamp method

Assume that alternator 1 is already connected to bus bar & alternator 2 is to be ready to synchronised.

→ Initially alternator 2 synchronising switch kept open.

→ Bring the alternator 2 at rated speed & rated voltage. The generated emf of alternator 2 becomes equal to bus bar voltage, indicated by voltmeter.




→ If phase sequence of alternator voltages is same as that of bus bar voltages.

→ All lamps brighter at a time & dark out at a time.

If phase sequence is not proper, lamps go bright one after another & dark one after another. During this case, interchange any 2 terminals of the alternator.

→ Flickering of lamps takes place during phase sequences are correctly matched to the bus bar voltages. Flickering in lamps indicates the differences in frequencies of alternator & bus bar voltage. Reduce the speed of flickering by adjusting the speed of alternator.

All lamps go dark, switch 's' closed. Now 2nd alternator put in parallel with bus bar. They can share a common load.

This method of synchronising is termed as dark lamp method.

(II) Bright lamp method

Here the set up is pratically same as that of dark lamp method. Except $L_2$ & $L_3$ are ~~###~~ interchanged.

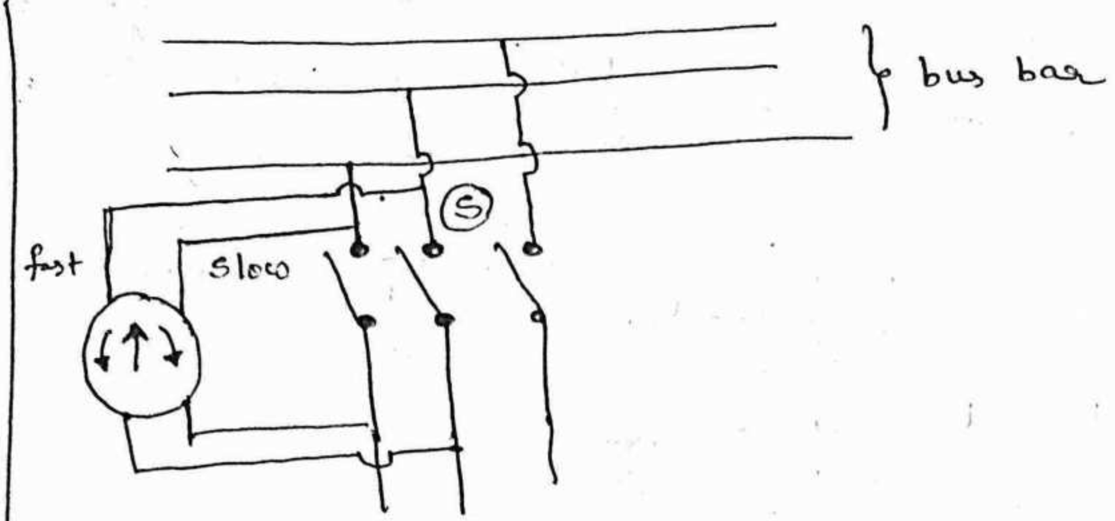
If phase sequences of alternator & bus bar voltages are similar, $L_1$, $L_2$ & $L_3$ lamps in ~~that~~ $L_1$ goes dark, lamps $L_2$ & $L_3$ gets bright

If frequencies are exactly same, lamp $L_1$ remain dark & lamps $L_2$ & $L_3$ remains bright. If $L_2$ & $L_3$ are bright & $L_1$ dark switch S is closed. This method is called bright lamp method.

(III) Synchroscope method

A synchronising switch (s) should be closed during the dark lamp method or bright lamp method. This is done if phase voltages & frequencies are exactly equal to bus bar voltages.

Here we cannot justify all lamps are go bright or dark exactly. This is overcomed by using synchronoscope method.




Here a pointer is used to estimate the difference in the frequencies.

If incoming alternator is slow, pointer move anticlock wise.

If incoming alternator is fast, pointer move clockwise.

If frequencies are exactly, the pointer becomes stable at the Center. $N = \frac{120f}{P}$

$f = \frac{NP}{120}$ N is speed of alternator

The pointer remain in steady state in 12 'O' clock position

Power flow equations in alternators

Power flow equation in non salient pole

Consider a 3 phase cylindrical rotor

Let V = terminal voltage

I = terminal or armature current } per phase

output power = VI

3 phase output power = 3VI

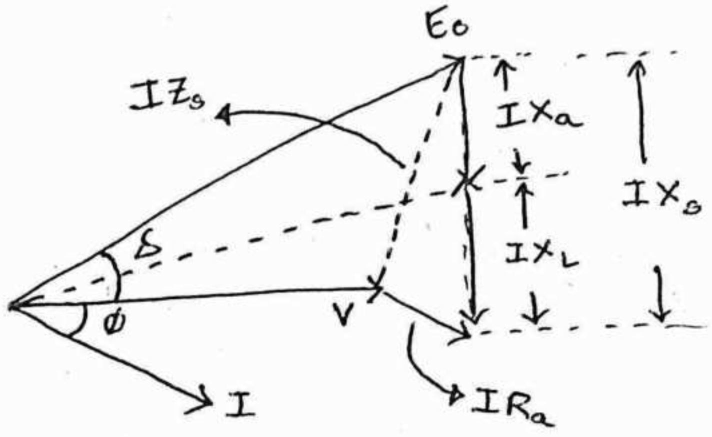
Let P = real or true power

Q = Reactive Power

∴ Complex power of 3 phase alternator

$$S = P + jQ = 3VI^*$$

$\downarrow$ Real, $\downarrow$ Reactive




$X_s = X_a + X_L$

$\delta$ = power angle or load angle

$\phi$ = p.f or phase angle

$$E_o = V + IR_a + IX_a + IX_L$$
$$= V + IR_a + IX_s$$
$$= V + IR_a + jIX_s \qquad Z_s = R_a + jX_s$$
$$E_o = V + I(R_a + jX_s)$$
$$= V + IZ_s$$

→ let take (V) as reference phasor

$V = V\angle 0^\circ$

$$I = \frac{E_o\angle\delta - V\angle 0^\circ}{Z_s\angle\phi_s} \Rightarrow I = \frac{E_o\angle\delta}{Z_s\angle\phi_s} - \frac{V\angle 0^\circ}{Z_s\angle\phi_s}$$

$$\phi_s = \tan^{-1}\left(\frac{X_s}{R_a}\right)$$

$$I = \frac{E_o\angle(\delta - \phi_s) - V\angle(0^\circ - \phi_s)}{Z_s}$$

Current Conjugate

$$I^* = \frac{E_0 \angle(\phi_s - \delta) - V\angle\phi_s}{Z_s}$$

$$\text{power} = 3VI^* \quad , \quad S = P + jQ$$

$$= \frac{3V}{Z_s}\left[E_0 \angle(\phi_s - \delta) - V\angle\phi_s\right]$$

$$I^* = \frac{3V}{Z_s}\left[E_0\{\cos(\phi_s - \delta) + j\sin(\phi_s - \delta)\} - V\{\cos\phi_s + j\sin\phi_s\}\right]$$

Real power $P$ = Real part of $S$

$$P = \frac{3V}{Z_s}\left[E_0 \cos(\phi_s - \delta) - V\cos\phi_s\right]$$

$$\boxed{P = \frac{3VE_0\cos(\phi_s - \delta)}{Z_s} - \frac{3V^2}{Z_s}\cos\phi_s}$$

Reactive power $Q$ = imaginary part of $S$

$$Q = \frac{3V}{Z_s}\left[E_0 \sin(\phi_s - \delta) - V\sin\phi_s\right]$$

$$\boxed{Q = \frac{3VE_0}{Z_s}\sin(\phi_s - \delta) - \frac{3V^2}{Z_s}\sin\phi_s}$$

Substitute $X_s = Z_s$ since $R_a$ is small practically its value $R_a = 0$. $\phi_s = 90°$

$\sin\phi_s = \sin(90) = 1$

Also $\cos(\phi_s - \delta) = \cos(90 - \delta) = \sin\delta$

$\cos\phi_s = \cos(90) = 0$ we get

6

$$P = \frac{3VE_0 \cos(90-\delta)}{Z_s} - \frac{3V^2}{Z_s}\cos(90)$$

$$\boxed{P = \frac{3VE_0 \sin\delta}{Z_s} - 0} \Rightarrow P = \frac{3VE_0 \sin\delta}{X_s}$$

similarly $\sin(\phi_s - \delta) = \sin(90-\delta) = \cos\delta$

$\sin\phi_s = \sin 90 = 1$

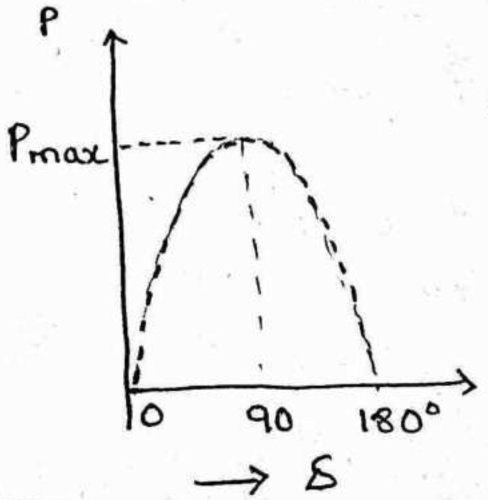
$$Q = \frac{3VE_0}{Z_s}\sin(90-\delta) - \frac{3V^2}{Z_s}\sin 90$$

$$\boxed{Q = \frac{3VE_0 \cos\delta}{Z_s} - \frac{3V^2}{Z_s} * 1} = \frac{3VE_0}{X_s}\cos\delta - \frac{3V^2}{X_s}$$

$\Rightarrow$ for P, if V, $E_0$ & $X_s$ are constant

we have

$P \propto \sin\delta$, where $\delta$ = load angle or power angle




with increase in load ($\delta$), power also increases. P becomes maximum at $\delta = 90°$ & P becomes zero at $\delta = 180°$

$$\frac{dP}{d\delta} = 0$$

$$P = \frac{3VE_0}{X_s} \sin\delta$$

$$\therefore \frac{dP}{d\delta} = \frac{3VE_0}{X_s} \cos\delta$$

$$\frac{dP}{d\delta} = 0 \quad \text{if} \quad \delta = 90$$

$$\cos\delta = \cos 90 = 0$$

consider Ra we have

$$P = \frac{3VE_0}{Z_s} \cos(\phi_s - \delta) - \frac{3V^2}{Z_s} \cos\phi_s$$

$$\frac{dP}{d\delta} = \frac{3VE_0}{Z_s} \sin(\delta - \phi_s) = 0$$

$$\sin(\delta - \phi_s) = 0$$

$$\boxed{\delta = \phi_s}$$

$\boxed{\text{o/p power is maximum when } \delta = \phi_s}$

To find the maximum o/p power

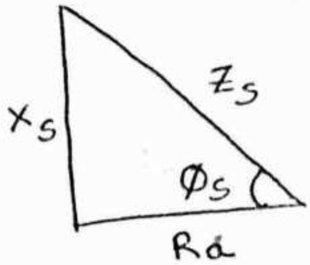
$$P = \frac{3VE_0}{Z_s} \cos(\phi_s - \delta) - \frac{3V^2}{Z_s} \cos\phi_s$$

if $\delta = \phi_s$ then P becomes

$$P = \frac{3VE_0}{Z_s} \cos(\phi_s - \phi_s) - \frac{3V^2}{Z_s} \cos\phi_s$$

$$= \frac{3V_0E_0}{Z_s}(1) - \frac{3V^2}{Z_s} \cos\phi_s$$

$\cos\phi_s = \frac{R_a}{Z_s}$




$P = \frac{3VE_0}{Z_s} - \frac{3V^2}{Z_s}\left(\frac{R_a}{Z_s}\right)$

$\cos\phi_s = \frac{R_a}{Z_s}$

$\sin\phi_s = \frac{X_s}{Z_s}$

$$\boxed{P = \frac{3VE_0}{Z_s} - \frac{3V^2 R_a}{Z_s^2}} = P_{max}$$

Reactive power o/p is given as

we have Reactive power with $R_a$

$$\boxed{Q = \frac{3VE_0}{Z_s}\sin(\phi_s - \delta) - \frac{3V^2}{Z_s}\sin\phi_s} \longrightarrow (1)$$

$Q = -\frac{3V^2}{Z_s}\sin\phi_s$ Now $\sin\phi_s = \frac{X_s}{Z_s}$

$Q = \frac{-3V^2}{Z_s} * \frac{X_s}{Z_s} = \frac{-3V^2 X_s}{Z_s^2}$

$\frac{dQ}{d\delta} = 0$

$\theta_s - \theta_s + 90$

$\frac{dQ}{d\delta} = \frac{3VE_0}{Z_s}\cos(\phi_s - \delta)$ ✓

if $\delta = \phi_s + 90$

$\sin(\phi_s - \phi_s - 90)$

$\sin(-90) = -1$

$\cos(\phi_s - \delta) = 0$

$\cos 90 = 0$

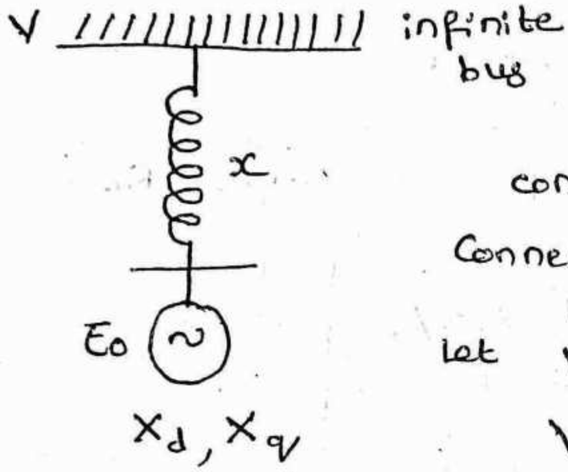
$\phi_s - \delta = \pm 90°$ or $\delta = \phi_s \pm 90°$

$Q_{max} = -\frac{3VE_0}{Z_s} - \frac{3V^2 X_s}{Z_s^2}$

## (b) Power flow in salient pole alternator

for cylindrical or non-salient pole alternator real power $P = \dfrac{3VE_0}{X_s} \sin\delta$

In case of salient pole rotors $X_d$ & $X_q$ are unequal because of non uniform air gap.




consider a salient pole alternator Connected to infinite bus bar.

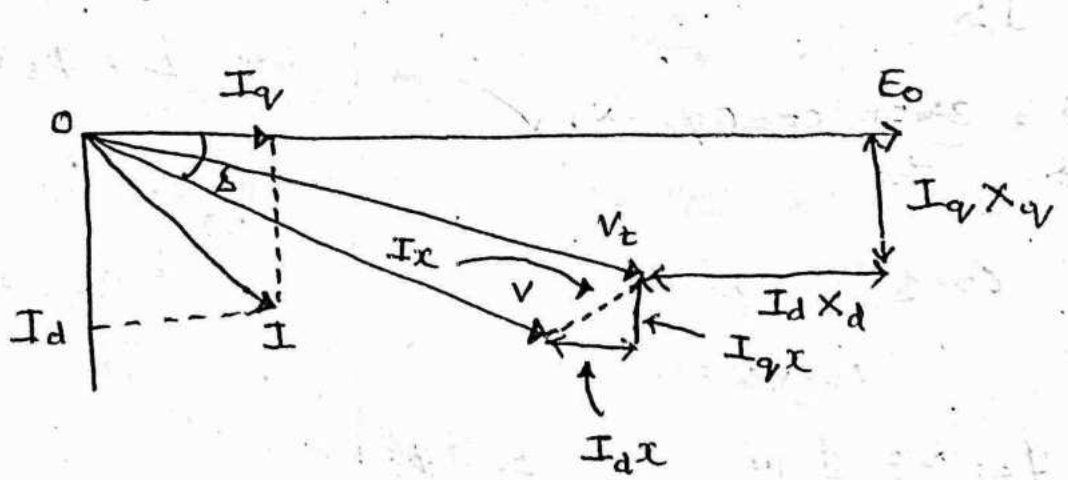
let $V$ = Bus bar voltage

$V_t$ = terminal voltage of the alternator

$E_0$ = No load induced emf

$x$ = reactance of line

let $x_d$ & $x_q$ are direct & quadrature reactances




phasor diagram

$I = I_d + I_q$

$V_t = V + I_d x + I_q x$ , $E_0 = V_t + I_d x_d + I_q x_q$

$I_d x$ & $I_d X_d$ are drawn leading $I_d$ by $90°$

similarly $I_q x$ & $I_q X_q$ are drawn leading $I_q$ by $90°$.

$\delta$ = load angle or power angle

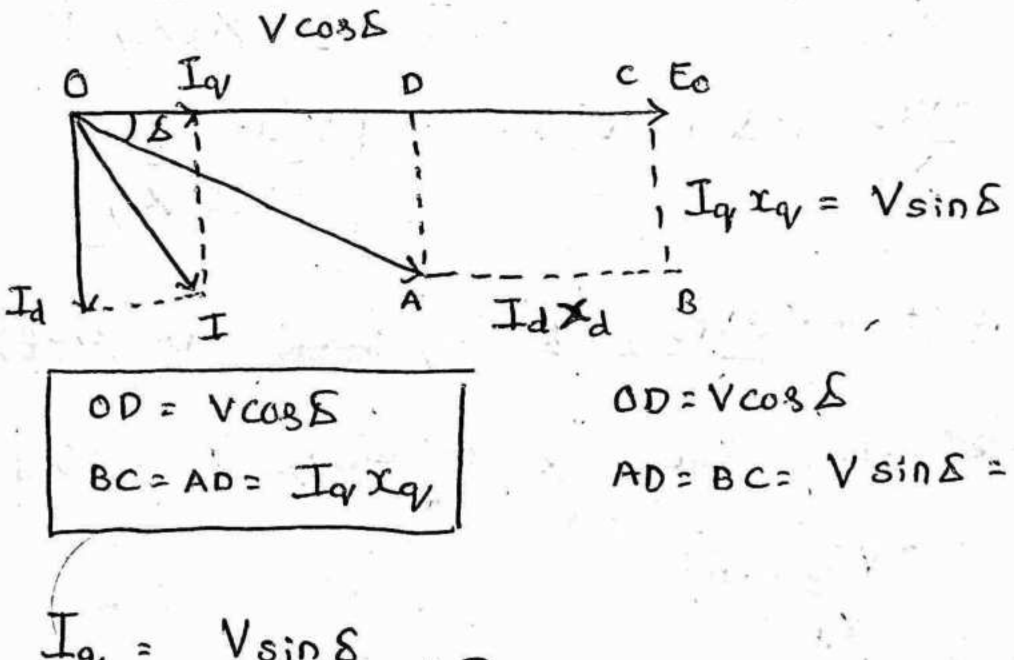
Let $X_d = x_d + x$ , $X_q = x_q + x$

$I_d x$ & $I_d X_d$ voltage drop combined to give $I_d X_d$.

$I_q x$ & $I_q X_q$ are voltage drop combined to give $I_q X_q$

$$I_d = \frac{E_0 - V\cos\delta}{X_d} \rightarrow (1)$$

$(E_0 = V\cos\delta + I_d X_d)$ OC = OD + DC




$$\boxed{\begin{array}{l} OD = V\cos\delta \\ BC = AD = I_q x_q \end{array}}$$

OD = $V\cos\delta$

AD = BC = $V\sin\delta = I_q X_q$

$$I_q = \frac{V\sin\delta}{X_q} \rightarrow (2)$$

Here $V\cos\delta$ is in phase with $I_q$

$V\sin\delta$ " " " with $I_d$

$$P = 3\left[I_d V\sin\delta + I_q V\cos\delta\right] \rightarrow (a)$$

Now $I_d = \dfrac{E_0 - V\cos\delta}{X_d}$, $I_q = \dfrac{V\sin\delta}{X_q}$

substitute $I_d$ & $I_q$ in equation (a) we get

$$P = 3V\left[\frac{E_0 - V\cos\delta}{X_d}\sin\delta\right] + 3V\left[\frac{V\sin\delta}{X_q}\cos\delta\right]$$

$$= \frac{3VE_0}{X_d}\sin\delta - \frac{3V^2\cos\delta\sin\delta}{X_d} + \frac{3V^2\sin\delta\cos\delta}{X_q}$$

$$= \frac{3VE_0}{X_d}\sin\delta - \frac{3V^2\frac{\sin 2\delta}{2}}{X_d} + \frac{3V^2\frac{\sin 2\delta}{2}}{X_q}$$

$$= \frac{3VE_0}{X_d}\sin\delta + 3V^2\frac{\sin 2\delta}{2}\left[\frac{1}{X_q} - \frac{1}{X_d}\right]$$

$$= \frac{3VE_0}{X_d}\sin\delta + 3V^2\frac{\sin 2\delta}{2}\left[\frac{X_d - X_q}{X_d X_q}\right]$$

$$P = \frac{3VE_0}{X_d}\sin\delta + 3V^2\sin 2\delta\left[\frac{X_d - X_q}{2X_d X_q}\right]$$

first term $\dfrac{3VE_0}{X_s}\sin\delta$ is same as that of salient pole. only $X_s$ is replaced by $X_d$

$$\frac{3VE_0}{X_d}\sin\delta$$

2nd term in above equation is termed as reluctance power.

we have Torque $T = \frac{P * 60}{2\pi N}$ in Nm

9.

Torque developed by sailent pole motor is

$$T = \frac{3VE_0 * 60}{2\pi N_s X_d} \sin\delta + \frac{3V^2 (X_d - X_q) * 60}{2\pi N_s (2 X_d X_q)} \sin 2\delta$$

in Nw-meters.

Problem

A 2500KVA, 8 pole alternator runs at 750rpm on 6.6 KV bus bars. If synchronous reactance is 20%, determine the synchronising power for 1° mechanical degree of replacement. Also Calculate Corresponding synchronising torque.

sol Rating of alternator = 2500 * 1000 VA

Rated voltage 6.6KV in line

Poles = 8, $N_s = 750$ rpm, $X_s = 20\%$.

→ Rated Voltage/phase = $\frac{V_L}{\sqrt{3}} = \frac{6600}{\sqrt{3}} = 3810.5$ V

→ full load Current or rated Current = $\frac{\text{rating of alternator}}{\sqrt{3} * \text{rated voltage}}$

$$I = \frac{2500 * 100}{\sqrt{3} * 6600} = 218.6933$$

→ Synchronous reactance drop = 20% = of $\frac{\text{Rated voltage}}{\text{phase}}$

$I X_s$ = ~~218.6933~~ 3810.5 * 0.2 = 762.1 V

→ Synchronous reactance drop / phase

$$X_s = \frac{I X_s}{\text{rated voltage/p}} = 762.1$$

$$I X_s = 762.1$$

$$X_s = \frac{762.1}{I} = \frac{762.1}{218.693} = 3.484$$

synchronous reactance drop / phase

$$\text{Total synchronising power} = \frac{3\alpha E_0^2}{X_s}$$

$\alpha$ in electrical degrees

assume $\boxed{E_0 = V}$

$$= \frac{3\alpha V^2}{X_s}$$

$\alpha = 1°$ mechanical in degree

$$\boxed{\begin{array}{lll} 1° \text{ electrical} & = 1° \text{ mechanical} * & \text{No. of poles per pair} \\ \theta_e & = \theta_m * (P/2) & \end{array}}$$

$$\theta_m = \frac{\theta_e}{\frac{P}{2}} \Rightarrow \frac{2\theta_e}{P} = \theta_m$$

$$\left[\theta_e = \frac{\theta_m * P}{2}\right]$$

$$\alpha = \frac{1 * 8}{2} = 4° \text{ electrical}$$

$$\alpha = 4 * \frac{\pi}{180°} \text{ radians} = \frac{\pi}{45}$$

$$\text{Total synchronising power} = \frac{3 * \frac{\pi}{45} * 3810.5^2}{3.4848} = 872.6 \text{ KW}$$

## Synchronising power, Current & synchronising torque

10

Any one of generator tends to step out from synchronism a torque is termed as synchronising torque.

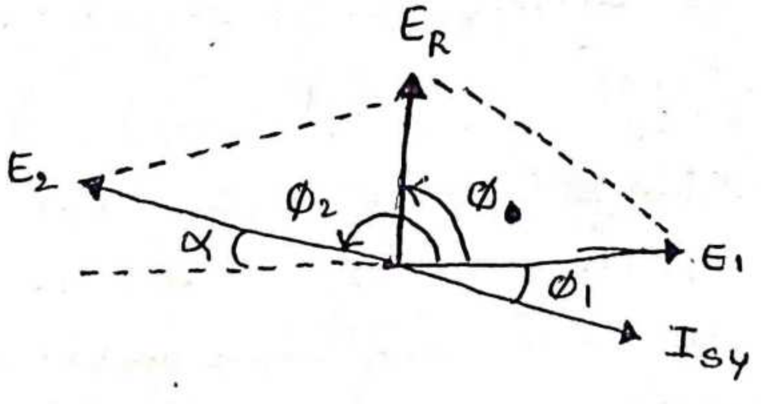
### ① Synchronising Current

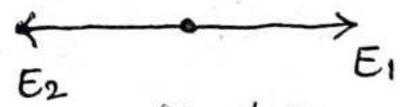
Considered 2 alternators terminal voltages are equal on no load as $E_1$ & $E_2$.

For parallel condition is achieved if magnitudes of $E_1 = E_2$ are equal.

For local armature circuit $E_1$ & $E_2$ are in phase opposition but for external circuit $E_1$ & $E_2$ are in phase.

If $E_1 = E_2$ there is no circulating current. Assume speed of 2nd alternator slightly reduces. Induced emf of 2nd alternator slightly changes by small angle




in phase opposition

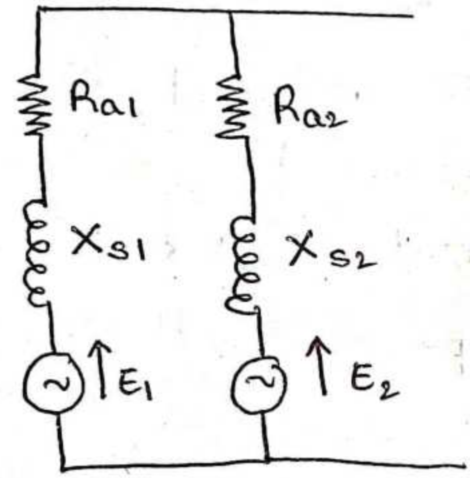
Any change in induced emfs of alternator creates a circulating currents in the local armature circuit. This current is called as synchronising current.

$$I_{sy} = \frac{E_R}{Z_s}$$ $E_R$ = difference or change in induced emf

$Z_s$ = Combined synchronous impedances of alternator ① & ②

$$Z_s = Z_{s1} + Z_{s2}$$

$$Z_s = (R_{a1} + jX_{s1}) + (R_{a2} + jX_{s2})$$




$I_{sy}$ synchronising current lags behind the $E_R$ by 90°

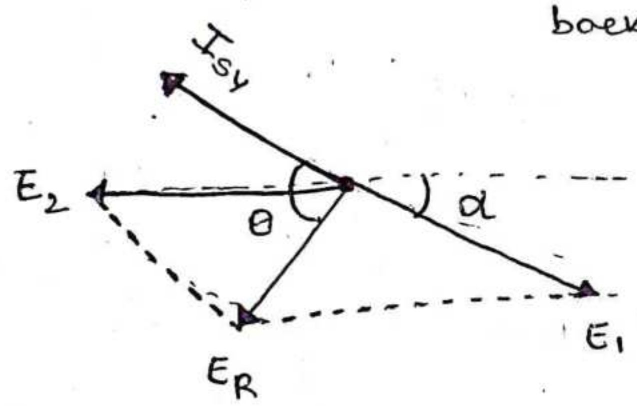
$$\boxed{\tan\theta = \frac{X_s}{R_a}}$$

$$X_s = X_{s1} + X_{s2}$$
$$R_a = R_{a1} + R_{a2}$$

In Practical armature resistances $R_{a1}$ & $R_{a2}$ are small. Hence synchronising current is in nature of generator current for alternator 1 & motoring current for alternator 2.

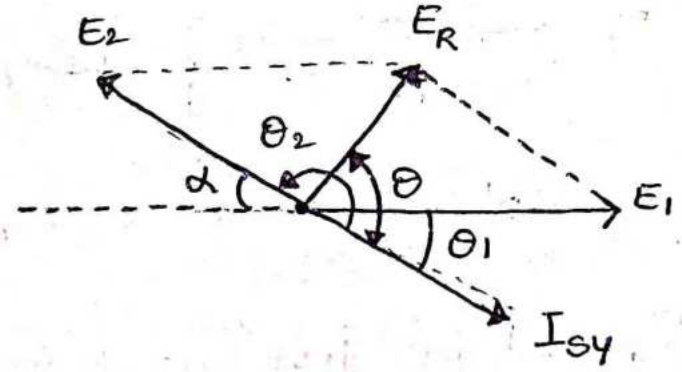
This synchronising current $I_{sy}$ gives a torque which speeds up the alternator 2 & retards the first alternator such that synchronism is again established. This torque is termed as synchronising torque.

Assume that speed of 1st alternator reduces shown in below figure, the resultant is induced emf $E_1$ falls back. Hence synchronizing current is in the nature of generating current for alternator (1) & motoring current for alternator (2).




Again synchronising current speeds up the alternator 1 & retards the 2nd generator until synchronism is achieved.

## (II) Synchronising Power




assume a alternator 2 emf slightly reduces due to reduced in speed. This creates induced emf's becomes unequal. This unequal emfs creates a circulating currents in local armature circuit. this current produces a torque which speeds up the alternator 2 & retards the first retards the speed of first alternator.

$$I_{sy} = \frac{E_R}{Z_s}$$

$$E_R = 2E\cos\left(90 - \frac{\alpha}{2}\right)$$

$$E_R = 2E\sin\frac{\alpha}{2}$$

$$E_R = 2E\left(\frac{\alpha}{2}\right) = E\alpha$$

Now synchronising power

$$I_{sy} = \frac{E\alpha}{Z_s}$$

Hence the first generator produces power is referred as synchronising power. It is denoted as $P_{sy}$.

→ Consider 2 alternators

Power supplied by each or first alternator } $E_1 I_{sy} \cos\theta_1$

Power ~~supplied~~ absorbed by 2nd alternator $E_2 I_{sy} \cos\theta_2$

Power supplied supplied by first alternator = power absorbed by 2nd alternator + Copper losses

$$E_1 I_{sy} \cos\theta_1 = E_2 I_{sy} \cos\theta_2 + \text{Copper losses}$$

if $\theta_1 = 0$

$\cos\theta_1 = 1$

$$P_{sy} = E_1 I_{sy}$$

$$P_{sy} = E_1 * \frac{E\alpha}{Z_s} \Rightarrow \text{put } E_1 = E$$

$$P_{sy} = \frac{E^2 \alpha}{Z_s} = \frac{E^2\alpha}{X_s} \quad \text{if } R_a \text{ is neglected}$$

12

→ Total synchronising power = $3 P_{sy}$

$$\boxed{P_{sy} = \frac{3E^2\alpha}{X_s}} \text{ watt}$$

$$\frac{E}{X_s} = I_{sc}$$

Total synchronising Power $P_{sy} = 3 * E\left(\frac{E}{X_s}\right)\alpha = 3EI_{sc}\alpha$

$$\boxed{P_{sy} = 3EI_{sc} * \alpha}$$

If the angle $\theta$ is not exactly $90°$ then more accurate form of expression is

Total synchronising Power $= 3 * \alpha * E * I_{sc} * \sin\theta$

## (iii) synchronising Torque

Sychronising Torque is produced due to synchronising Current which takes place during alternators stepping out of synchronism.

Let $T_{sy}$ denote synchronising torque

$$N_s = \frac{120f}{P} \qquad P = \omega * T \Rightarrow P = \frac{2\pi N_s}{60} * T$$

Power $P = \frac{2\pi NT}{60}$ if T is in Nw-meters

we have $P_{sy} = \frac{2\pi N_s T_{sy}}{60}$

$$\boxed{T_{sy} = \frac{P_{sy} * 60}{2\pi N_s}}$$

We have $3P_{sy} = \frac{3\alpha E^2}{X_s}$

$$\boxed{T_{sy} = \frac{3\alpha E^2 * 60}{2\pi N_s X_s}}$$

Problems A 10MVA, 50Hz, 3 phase alternator has an equivalent short circuit reactance of 20%. calculate synchronising power per mechanical degree of phase displacement when running in parallel are 10,000 V bus bars at 1500 rpm.

Sol Rating of alternator = 10MVA = 10,000 KW

bus-bar voltage = 10,000V in line, $\theta_m = 1°$

Speed $N_s = 1500$ rpm, $X_s = 20\% = 0.2$, $f = 50Hz$

Rated voltage/phase = $\frac{\text{line voltage}}{\sqrt{3}} = \frac{10,000}{\sqrt{3}} = 5773.5V$

→ Full load or Rated Current = $\frac{\text{alternator rating}}{\sqrt{3} * \text{Rated voltage in line}}$

$$I = \frac{10 * 10^6}{\sqrt{3} * 10,000} = 5773.5A$$

→ $\boxed{\text{Reactance drop} = 20\% \text{ of phase voltage}}$

$IX_s = 5773.5 * 0.2 = 1154.7V$

synchronous reactance/phase

$IX_s = 1154.7 \Rightarrow X_s = \frac{1154.7}{5773.5} = 0.2$

→ we know that $N_s = \frac{120f}{P} \Rightarrow P = \frac{120 * f}{N_s}$

No. of Poles } $P = \frac{120 * 50}{1500} = 4$

13

We have $\boxed{\theta_e = \theta_m * \frac{\text{no. of poles}}{2}}$

$\boxed{\alpha = \theta_e = 1^\circ * \frac{4}{2} = 2^\circ}$ in degree

$\theta_e = \frac{2 * \pi}{180}$ radians

$\boxed{\alpha = \theta_e = \theta_e^\circ * \frac{\pi}{180}}$

$\alpha = \theta_e = \frac{\pi}{90}$ radians

E = rated voltage per phase = V

⇒ ~~Total~~ synchronising power

$\cancel{\boxed{3P_{sy} = \frac{3 * \alpha * E^2 * 60}{2\pi N_s X_s}}}$

$P_{sy} = \frac{\alpha E^2}{X_s}$

$P_{sy} = \frac{\frac{\pi}{90} * (5773.5)^2}{0.2}$

$\cancel{E = \text{rated voltage/phase} = 5773.5V}$

$P_{sy} = 5817.7$ KW

$\cancel{3P_{sy} = \frac{3 * \frac{\pi}{90} * (5773.5)^2 * 60}{2\pi * 1500 * 0.2}}$

$\cancel{3P_{sy} = \frac{3 * \pi * (5773.5)^2 * 60}{90 * 2\pi * 1500 * 0.2} = 222.20 \text{ KW}}$

Problem

Rating of alternator = 5MVA = $5 * 10^6$ VA, Poles = 4
3 phase alternator runs at 1500 rpm.
Rated voltage in line = 3.3 KV
% of synchronous reactance $X_s$ = 25% = 0.25
Power factor = 0.8 lag. Determine (i) synchronising power (ii) Torque

Rated voltage/phase = $\frac{\text{line voltage}}{\sqrt{3}}$ = 1905·255 V

→ Full load current $I = \frac{\text{Rating of alternator}}{\sqrt{3} * \text{line voltage}}$

$$= \frac{5*10^6}{\sqrt{3}*3.3*1000} = 874.77 \text{ A}$$

Synchronous reactance drop

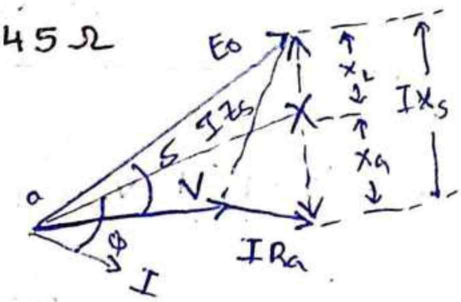
~~$(IX_s) = 0.25 * 874.77$~~

$$\boxed{IX_s = 0.25 * \text{Rated voltage/phase}}$$

$$IX_s = 0.25 * 1905.255 = 476.313 \text{ V}$$

$$X_s = \frac{476.313}{874.77} = 0.5445\,\Omega$$

→ To find $E_0$




$$E_0 = V + IR_a + IX_a + IX_L$$

$$= V + IR_a + IX_s$$

$$= V + I(R_a + X_s)$$

$$\boxed{E_0 = V + IZ_s}$$

$I = 874.77$

p.f ⇒ $\boxed{\cos\phi = 0.8}$

$\phi = 31.788$ , $\sin\phi = 0.526$

$$\boxed{\text{Voltage } V = V_p(\cos\theta + j\sin\theta)}$$

$V = (1905.255)\left[0.85 + i\,0.526\right]$

$V = 1619.4 + i(1002.16413)$ 14

$\Rightarrow E_0 = V + IZ_s$

$= V + I(R_a + jX_s)$ Neglect $R_a$

$= V + I(X_s)$

$= 1619.4 + i(1002.16) + i(476.313)$

$= 1619.4 + 1478.4(i)$

$E_0 = 2192.7 \angle \underline{42.4}$ Volts

$\delta - \phi$

Angle between $E_0$ & $V$ = $42.4 - 31.788$

$\delta = 10.612$

load angle = $\cos\delta = \cos(10.612) = 0.9828$

$\alpha = 1^\circ$ mechanical

$$\alpha = \boxed{\theta_e = \theta_m^\circ * \frac{\text{no. of poles}}{2}} = 1 * \frac{4}{2} = 2^\circ$$

$$\boxed{\alpha = \theta_e^\circ * \frac{\pi}{180} \text{ radian}}$$

$\alpha = 2^\circ * \frac{\pi}{180} = \frac{\pi}{90}$ radians.

$\alpha$ in degrees $= 2^\circ$

$\alpha$ in radians $= \frac{\pi}{90}$

$\cos(2) = 0.999$, $\sin\alpha = (0.03489)$

$$\boxed{\text{Total Synchronising Power} = \frac{3E_0V}{X_s}\cos\delta\sin\alpha}$$

$$= \frac{3 * 2193.875 * 1905.26}{0.544} * 0.9828 * 0.0349$$

$$= 789.938 \simeq 790 \text{ KW}$$

$$\text{Total synchronising torque} = \frac{\text{synchronising power} * 60}{2\pi N_s}$$

$$= \frac{790 * 60}{2\pi * 1500} = 5.03 \text{ K NW-m}$$

## Parallel operation of Alternators

### Conditions necessary for parallel operation

→ Terminal voltages of generators must be equal

→ The frequencies of generated emf's of all generators must be equal

→ Polarity of the voltages are must same

— x — x — x —

Consider 2 alternators, operating in parallel & sharing a Common load. Assume that both generators have same speed - load characteristics.

Let

$V$ = terminal voltage of each alternator

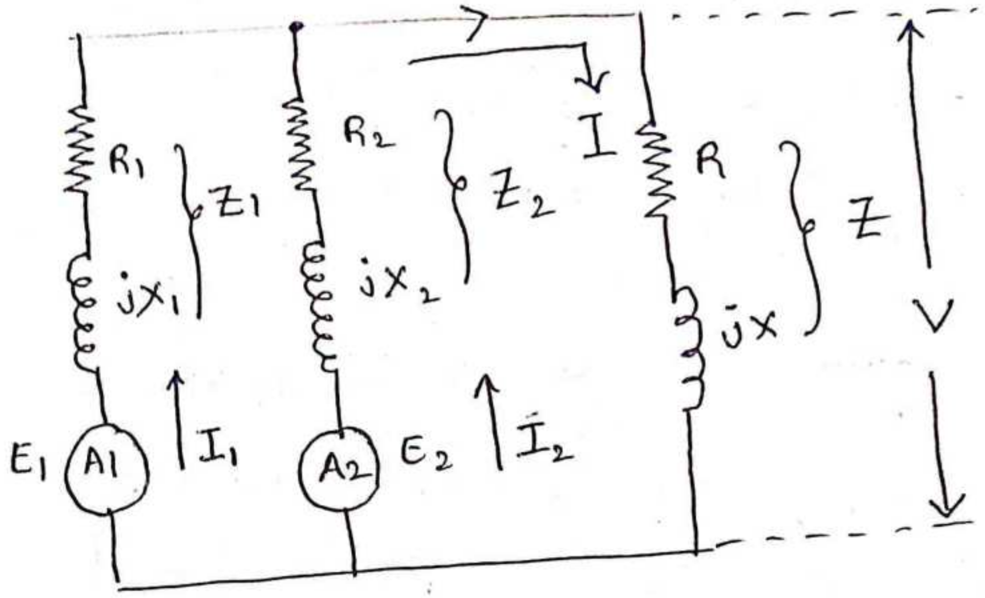
$Z$ = Load impedance

$I$ = Current delivered by 2 alternators to load

$Z_1, Z_2$ → synchronous impedance of alternator ① & alternator ②

$I_1, I_2 \rightarrow$ Current supplied by alternator ① & alternator ②.






$E_1, E_2 \rightarrow$ induced emf's of alternator ① & alternator ②

* For Alternator (A1)

$$E_1 = V + I_1(R_1 + jX_1) = V + I_1 Z_1$$

* For Alternator (A2)

$$E_2 = V + I_2(R_2 + jX_2) = V + I_2 Z_2$$

$$\therefore E_1 - E_2 = I_1 Z_1 - I_2 Z_2 \longrightarrow ①$$

* Total Current $I = I_1 + I_2$

terminal voltage or Bus bar voltage

$$V = IZ \longrightarrow ⓐ$$

→ Keep $V = IZ$ in $E_1$ & $E_2$ equation

$$E_1 = IZ + I_1 Z_1 = (I_1 + I_2)Z + I_1 Z_1$$

$$E_1 = I_1(Z + Z_1) + I_2 Z \rightarrow ②$$

similarly for

$$E_2 = IZ + I_2 Z_2 = (I_1 + I_2)Z + I_2 Z_2$$

$$E_2 = I_2(Z + Z_2) + I_1 Z \longrightarrow ③$$

from equation (2)

$$E_1 = I_1(Z+Z_1) + I_2 Z$$

$$I_2 Z = E_1 - I_1(Z+Z_1)$$

$$I_2 = \frac{E_1 - I_1(Z+Z_1)}{Z}$$

substitute $I_2$ value in equation (1)

$$E_1 - E_2 = I_1 Z_1 - Z_2\left[\frac{E_1 - I_1(Z+Z_1)}{Z}\right]$$

$$= I_1 Z_1 - \frac{Z_2 E_1}{Z} + \frac{Z_2 I_1 (Z+Z_1)}{Z}$$

$$E_1 - E_2 = I_1\left[Z_1 + \frac{Z_2(Z+Z_1)}{Z}\right] - \frac{Z_2 E_1}{Z}$$

$$E_1 - E_2 + \frac{Z_2 E_1}{Z} = I_1\left[Z_1 + \frac{Z_2(Z+Z_1)}{Z}\right]$$

$$\frac{Z(E_1-E_2) + Z_2 E_1}{\cancel{Z}} = I_1\left[\frac{Z Z_1 + Z_2(Z+Z_1)}{\cancel{Z}}\right]$$

$$I_1 = \frac{Z(E_1-E_2) + Z_2 E_1}{Z Z_1 + Z_2(Z+Z_1)} = \frac{Z(E_1-E_2) + Z_2 E_1}{Z Z_1 + Z Z_2 + Z_1 Z_2}$$

$$\boxed{I_1 = \frac{Z(E_1-E_2) + Z_2 E_1}{Z(Z_1+Z_2) + Z_1 Z_2}} \longrightarrow (4)$$

similarly for $I_2$ is obtained as

$$I_2 = \frac{Z(E_2-E_1) + Z_1 E_2}{Z(Z_1+Z_2) + Z_1 Z_2} \longrightarrow (5)$$

Total Current $I = I_1 + I_2$

$$= \frac{(E_1-E_2)Z + E_1Z_2}{Z(Z_1+Z_2)+Z_1Z_2} + \frac{(E_2-E_1)Z + E_2Z_1}{Z(Z_1+Z_2)+Z_1Z_2}$$

$$= \frac{\cancel{E_1Z} - \cancel{E_2Z} + E_1Z_2 + \cancel{E_2Z} - \cancel{E_1Z} + E_2Z_1}{ZZ_1 + ZZ_2 + Z_1Z_2}$$

$$I = \frac{E_1Z_2 + E_2Z_1}{Z(Z_1+Z_2)+Z_1Z_2}$$

Bus bar Voltage $V = IZ$

$$V = Z\left[\frac{E_1Z_2 + E_2Z_1}{Z(Z_1+Z_2)+Z_1Z_2}\right]$$

Circulating Current $I_c = \dfrac{E_1-E_2}{Z_1+Z_2}$

The above Current Concentrating around the local armature circuit.

Problems 2 single phase alternators operating in parallel & supply power to load having impedance $Z = 6+j2$. Open circuit induced emfs are $E_1 = 220\angle 0°$ & $E_2 = 220\angle 10°$ having Corresponding impedances $Z_1 = (0.5+j8)$ & $Z_2 = (0.3+j10)$.
Find (i) Common terminal Voltage (V)
(ii) Current o/ps of alternators
(iii) Power supplied by alternators

Sol - load impedance $Z = 6+j2$

$E_1 = 220\angle 0$, $E_2 = 220\angle 10°$

$Z_1 = 0.5+j8$ ; $Z_2 = (0.3+j10)$

$Z_2 + Z_1 = Z$

$Z = 0.8+j18 = 18.016\angle 87.455$

$Z_1Z_2 = (0.8$

$= (0.5+j8.0)(0.3+j10)$

$\Rightarrow Z_1Z_2 = 80.192 \angle 174.705^\circ$

$Z = 6+j2$

$= 6.3246 \angle 18.435$

$\Rightarrow E_1 = 220\angle 0^\circ = 220+0i$

$\Rightarrow E_2 = 220\angle 10^\circ = 216.658 + j38.202$

$\Rightarrow E_1 - E_2 = (220+0i) - (216.658 + j38.202)$

$E_1 - E_2 = 38.347 \angle -85^\circ$

(ii)

$$\boxed{I_1 = \frac{Z(E_1-E_2) + E_1Z_2}{Z(Z_1+Z_2) + Z_1Z_2}} = \frac{(38.347\angle -85)(6.3246\angle 18.43) + 220\angle 0\,(10.004\angle 88.282)}{(6.2346\angle 18.435)(18.018\angle 87.455) + 80.196\angle 174.706}$$

$$= \frac{242.542\angle -66.565 + 2201.1\angle 88.282}{113.957\angle 105.89 + 80.196\angle 174.706}$$

$$= \frac{2443.642\angle 21.717}{194.153\angle 280.596}$$

$$= \frac{1984.236\angle 85.303}{161.315\angle 133.506} = 12.300\angle -48.196$$

$I_1 = 8.199 - j9.1688$

$$\Rightarrow \boxed{I_2 = \frac{Z(E_2-E_1) + E_2Z_1}{Z(Z_1+Z_2) + Z_1Z_2}}$$

$$I_2 = \frac{1996.624\angle 98.46}{161.3133\angle 133.5} = 12.3773\angle -35.04$$

$I_2 = 12.3773\angle -35.04 = 10.1339 - j7.1064$

$I = I_1 + I_2$

$= (8.199 - j9.1688) + (10.1339 - j7.1064)$

$= (18.333 - j16.2752) = 24.51 \angle -41.6$

(III) Power O/Ps

Let $P_1$ & $P_2$ denote the power O/Ps

$\boxed{P_1 = V I_1 \cos\phi_1}$

$V = IZ$

$= 24.51 \angle -41.6 * 6.3246 \angle 18.435$

$V = 155.01 \angle -23.165$

$= (155.01 \angle -23.165) * 12.300 \angle -48.196$

$= (1906.5) \cos(48.196 - 23.165)$

$= (1906.5) * (0.9) = 1727.44$

$\boxed{P_2 = V I_2 \cos\phi_2}$

$= (155 * 12.3773) \cos(-35.04 - 23.165)$

$= 1877.424$

## Effect of change in excitation.

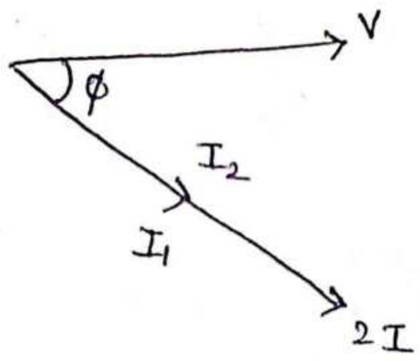
Considered 2 alternators operating in parallel sharing a common load.

Assume induced emfs $E_1$ & $E_2$, $I_1$ & $I_2$ are equal in magnitude & is in phase. As a result both generators operating at same power factor shares a common load. Reactive power supplied by 2 generators are equal.

Let $\phi$ be the angle

$I_1 = I_2 = I$

phasor diagram

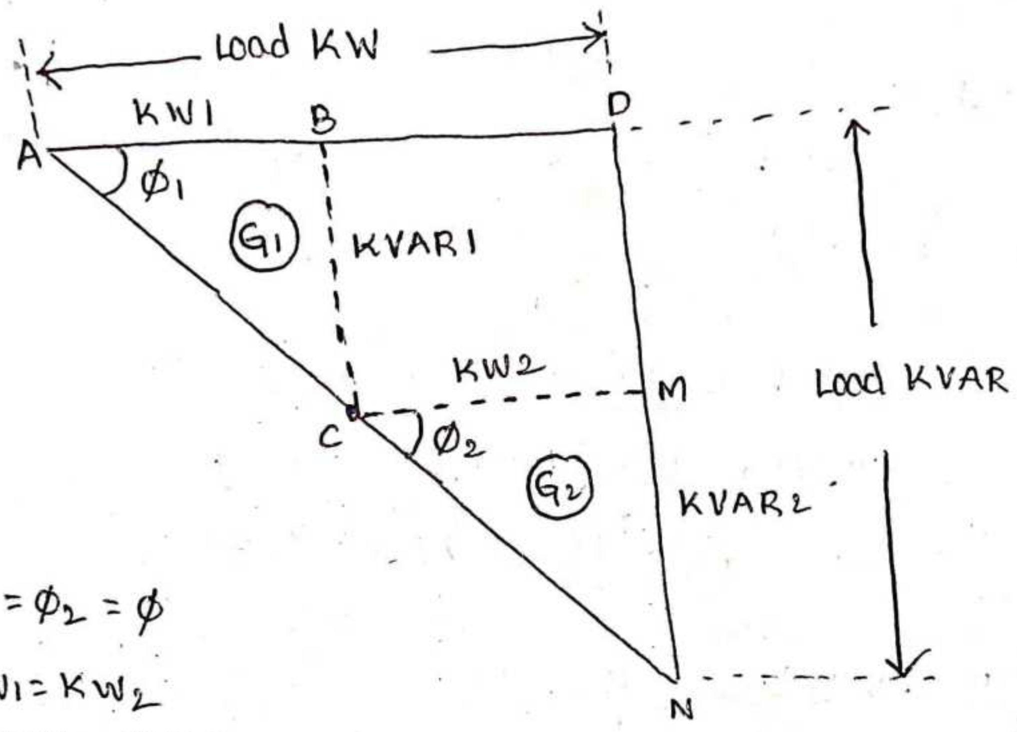



V = common terminal voltage

$I_1 = I_2 = I$

Load current

$I_1 + I_2 = 1+1 = 2I$




$\phi_1 = \phi_2 = \phi$

$KW_1 = KW_2$

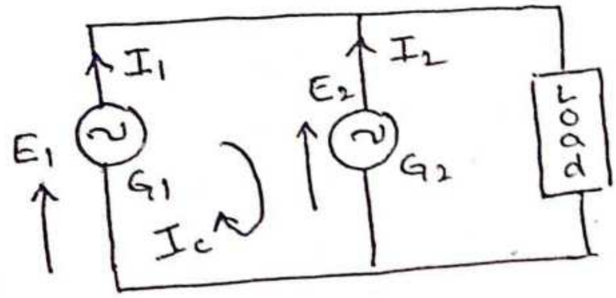
$KVAR1 = KVAR2$

$\cos\phi$ = p.f of each generator = Load p.f

Since $E_1$ & $E_2$ are equal & is opposition to local armature circuit, there is no circulating current.

circulating currents is induced if induced emf's are unequal. in local armature circuit

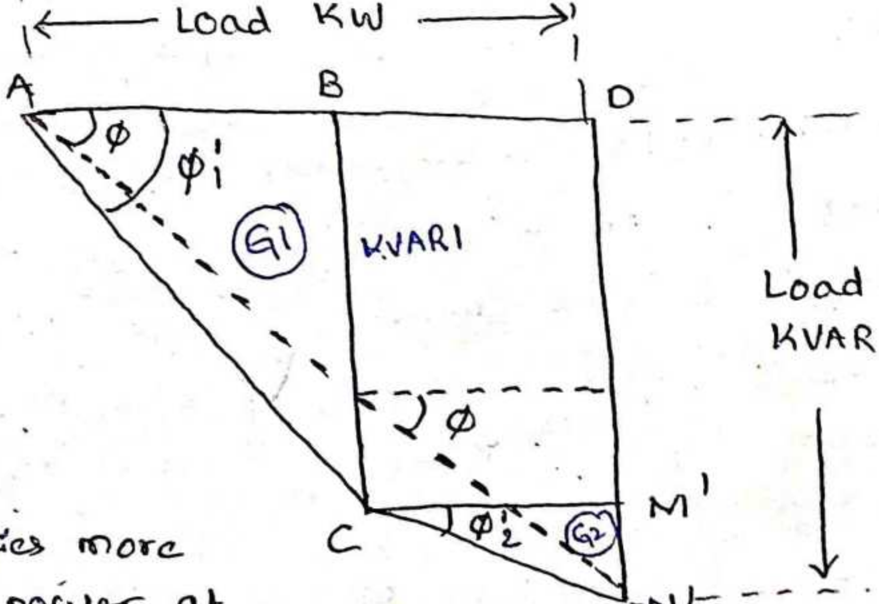
$$I_c = \frac{E_1 - E_2}{Z_1 + Z_2}$$




$I_c$ flows in same direction to $I_1$ but opposite to the direction $I_2$. As a result $I_1$ increases & $I_2$ decreases. $\phi_1$ ~~increases~~ decreases, $\phi_2$ increases. The resultant is that KW o/p of $G_2$ remains same.

Thus total load remains same if field excitation of alternator is changed but reactive power o/p of generator changes but not true power o/p.

Even though the field excitation changes KW o/p remains unchanged but power factor of power delivered changes.




$\phi_1' > \phi_1$

$\phi_2' < \phi_2$.

G1 supplies more reactive power at lower power factor &

G2 supplies ~~high~~ less reactive power at high p.f.

Effect of change in input power

Now we study the effect of change in i/p power to prime mover of any one of 2 alternators.

let $G_1, G_2$ are 2 generators operating in parallel share a common load having induced emf's $E_1$ & $E_2$ are equal & is in phase.

let the excitation & total load of 2 generators remains unchanged or Constant. If i/p to first generator ($G_1$) increases, speed of Generator ($G_1$) cannot be increased.

But induced emf $E_1$ advances with respect of $E_2$.. shown in phasor diagram

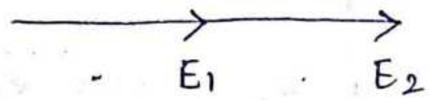



fig (a) $E_1 = E_2$ is in phase

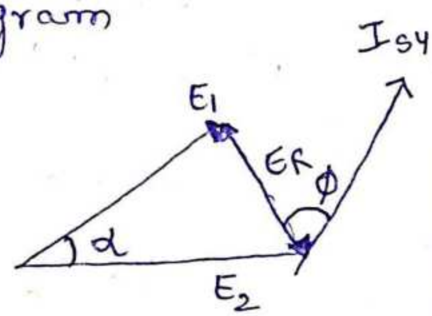
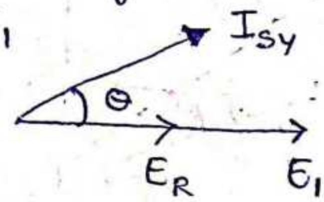



fig (b) $E_1 = E_2$

$E_1$ advances by $\alpha$

induced emf's becomes unequal as a result resultant emf $E_R$ is produced gives rise to synchronising Current $I_{sy}$.

Here $I_{sy}$ lags behind $E_R$ by 90° is in phase with $E_1$



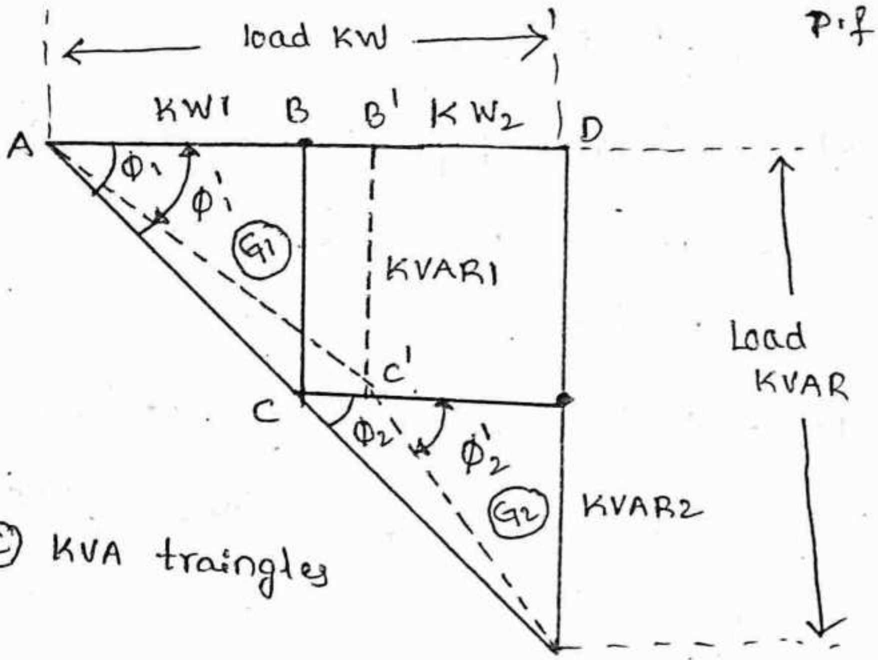

$I_{sy}$ lags $E_R$ by 90°

$E_R$ is in phase $E_1$.

→ Generator whose input power is more can share larger part of total load & other generator takes lesser part of load.

$G_1 \rightarrow$ i/p increases $\rightarrow$ $G_1$ shares more load

$G_2 \rightarrow$ i/p Constant $\rightarrow$ $G_2$ " less load.




p.f $\phi_1' < \phi_1$

$\phi_2' > \phi_2$

AB, BD KW o/ps

AB' is increased KW o/p

AB' > AB

B'D < BD

fig © KVA traingles

Even though the ($I_{sy}$) Synchronising produces reactive power supplied by generators remains same.

in fig © Generator G1 shares more load as KW1 & G2 shares less load B'D as KW2.

| Effect of change in excitation | Effect of change in i/p power |
|---|---|
| Even though the excitation of one generator changes KW o/p of generator remains same but reactive power o/p generator changes also P.f changes<br>$I_1 \uparrow, \phi_1 \downarrow, Q \uparrow$ $P = const$ | Even though i/p power of one generator increases or changes, KW o/ps of generators Varies but reactive power o/p of generator remains same.<br>Input $G_1$ increases, $P = \uparrow$, $Q = $ Constant. |

Problems

Two alternators working in parallel has loads (I) lights load 800 KW (II) 500 KW at 0.9 p.f lag (III) 1000 KW at 0.8 p.f lag (IV) 600 KW at 0.9 p.f lead.

One alternator is supplying 1000 KW at 0.95 p.f lag. Calculate the o/p, p.f of the other generator. (ii) find load p.f.

Sol for lighting loads p.f is almost unity

Total active power of 2 alternators is

800 + 500 + 1000 + 600 = 2900 KW

→ first alternator active power = 1000 KW

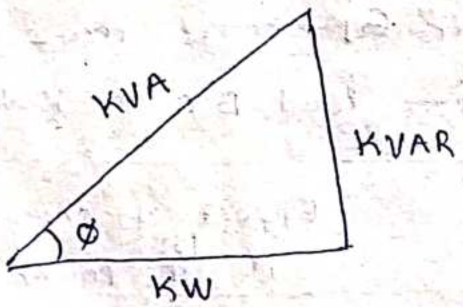
Total active power of generators = 2900 KW

$G_1 + G_2 = 2900$ KW

$1000 + G_2 = 2900$ KW

o/p of 2nd alternator $\boxed{G_2 = 2900 - 1000 = 1900 \text{ KW}}$

→ To calculate reactive component (KVA) of load




$\tan\phi = \frac{KVAR}{KW} = \frac{opp}{Adj}$

$\boxed{KVAR = KW * \tan\phi}$

For lag p.f KVA is +ve

lead p.f KVA is -ve

(I) for lights load of 800 KW

p.f = $\cos\phi = 1$

if $\phi = 0$ KVAR = $800 * \tan(0) = 0$

(II) 500 KW at 0.9 P.f lag

$$\boxed{KVAR = KW * \tan\phi}$$

$\cos\phi = 0.9$

$\phi = \cos^{-1}(0.9) =$

$\phi = 25.84$

$KVAR = 500 * \tan(25.84)$

$KVAR = 242.16$ (+ve)

(III) 1000 KW at 0.8 P.f lag

$$\boxed{KVAR = KW * \tan\phi}$$

$\cos\phi = 0.8$

$\phi = \cos^{-1}(0.8)$

$\phi = 36.86$

$= 1000 * \tan(36.86)$

$KVAR = 750$ (+ve)

(IV) 600 KW at 0.9 P.f leading

$$\boxed{KVAR = KW * \tan\phi}$$

$\cos\phi = -0.9$

$\phi = \cos^{-1}(-0.9)$

$\phi = 154.15$

$KVAR = 600 * (-0.48)$

$= -290.59$

Reactive power supplied by all loads

$\text{Total } KVAR = 0 + 242.161 + 750 - 290.59$

$= 701.568 \text{ KVAR}$

$\text{Total } KW = 2900 \text{ KW}$

→ Total load Power factor

$$\Rightarrow \boxed{\tan\phi_L = \frac{\text{Total KVAR}}{\text{Total KW}}} = \frac{701.568}{2900} = 0.2419$$

$\phi_L = \tan^{-1}(0.2419) = 13.56^\circ$

$\cos\phi_L = \cos(13.56) = 0.9719$

$\phi_L = 0.9719$

⇒ first alternator supplies 1000 KW at 0.95 p.f lag

$KVAR = KW * \tan\phi_L$ $\qquad \cos\phi = 0.95$

$= 1000 * \tan(18.19)$ $\qquad \phi = \cos^{-1}(0.95)$

$KVAR = 328.684$ $\qquad = 18.19$

⇒ Reactive power supplied by 2nd alternator

$KVAR2 = ?$

Total Reactive Power $KVAR = 701.568$

$$\boxed{KVAR1 + KVAR_2 = 701.568}$$

$$328.64 + KVAR2 = 701.568$$

$$KVAR2 = 701.568 - 328.64$$

$$\boxed{KVAR2 = 372.883}$$

→ P.f of the 2nd alternator

$$\boxed{\tan\phi_2 = \frac{KVAR2}{KW2}} = \frac{372.883}{1900} = 0.196$$

$$\phi_2 = \tan^{-1}(0.196) = 11.103$$

$$\Rightarrow \cos\phi_2 = \cos(11.103) = 0.98128$$

KW2 of 2nd alternator = 1900
KVAR2 of 2nd alternator = 372.883
p.f of 2nd alternator = 0.9812
Load p.f $\phi_L$ = 0.9719

Problems 2 same ratings of 3 phase alternator 2)

Connected in parallel supplying a load 1500 KW at 11 KV, 0.8 p.f lag.

Each machine has $X_s = 60$ (synchronous reactance, $R = 2.8$ (Resistance) / phase.

Power supplied by each machine is constant, excitation of 1st alternator is adjusted so that its armature current is 45 A. Calculate

(I) Armature current of other alternator

(II) P.f. of each alternator.

Sol

Load supplied = 1500 KW at 0.8 p.f lag

Bus bar voltage = 11 KV = 11000 V (line value)

Synchronous reactance $X_s = 2.8\ \Omega$ / phase

Resistance $R = 60\ \Omega$ / phase

⇒ Synchronous impedance of each alternator

$$Z_s = R + jX_s = 2.8 + j0.6\ / \text{phase}$$

Armature current of first alternator

$$I_1 = 45\,A$$

⇒ Total load current $\boxed{I_L = \dfrac{\text{load supplied in KW}}{\sqrt{3} * \text{bus bar volt} * \text{p.f}}}$

$$= \frac{1500 * 1000}{\sqrt{3} * 11000 * 0.8} = 98.4119\,A$$

$\boxed{I_L = \dfrac{\text{Load KW}}{\sqrt{3} * V * \text{p.f}}}$

Current supplied by each alternators = $\frac{\text{Total load current}}{2}$

$= \frac{98.412}{2} = 49.206A$

given p.f = 0.8

⇒ Active component $I_L = I_L \cos\phi$

$I_L = 98.412 * 0.8 = 78.7296$

⇒ Reactive component $I_L = I_L \sin\phi$

$= 98.412 * 0.6 = 59.0472A$

⇒ Active component current supplied by each alternator $= \frac{78.7296}{2} = 39.3648A$

⇒ Reactive component current supplied by each alternator $= \frac{59.0472}{2} = 29.5236A$

Change of excitation of 1st alternator

Current supplied by alternator ① i.e its armature current $I_a = 45A$

Active component of 1st alternator current

$I_1 = 39.364$

→ Reactive component of 1st alternator

$I_1 = \sqrt{(\text{Armature current})^2 - (\text{Active current})^2} = \sqrt{I_a^2 - I_1^2}$

$= \sqrt{(45)^2 - (39.368)^2}$

$= 21.804A$

$I_a = \sqrt{I_{AC}^2 + I_{RC}^2} \Rightarrow I_{RC}^2 = I_a^2 - I_{AC}^2$

let $I_2$ be the Current supplied by alternator ② 22

$I_a = I_{ac} + I_{re}$

Reactive Component of $I_2$

= (Total Current of load) − Reactive Component $I_1$ (active)

= $59.0472 - 21.0804 = \underline{\underline{37.24}}$

→ Active Component of $I_2 = 39.3648$

Armature Current of 2nd alternator

= $(\text{Reactive Component of } I_2)^2 =$

$(\text{Active Compent of } I_2)^2 - (\text{armature current 2nd alternator})^2$

⇒ Armature Current of 2nd alternator ($I_2$) $= \sqrt{\left(\text{Reactive } I_2\right)^2 + \left(\text{Active } I_2\right)^2}$

$= \sqrt{(37.24)^2 + (39.3648)^2}$

$= 54.191$

To calculate Power factors

$\cos\phi_1$ & $\cos\phi_2$ are p.f's of alternators ① & ② having Currents $I_1$ & $I_2$

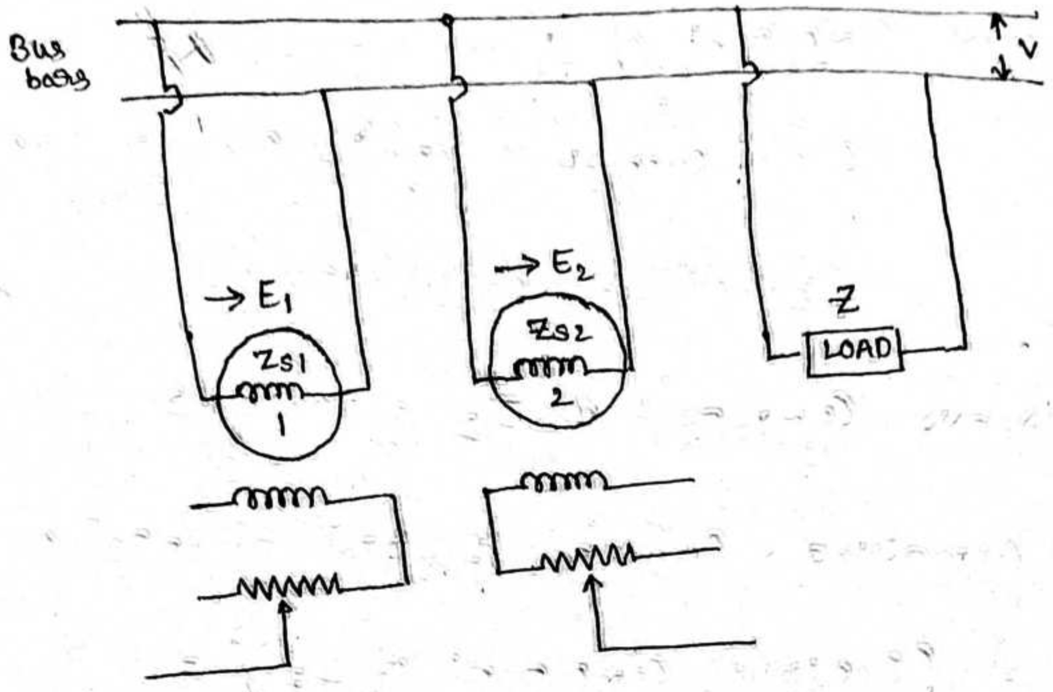
$$\boxed{\cos\phi = \frac{\text{Active Current Component}}{\text{Armature Current}}}$$

$\cos\phi_1 = \frac{39.3648}{45} = 0.8748$

$\cos\phi_2 = \frac{39.3648}{54.191} = 0.72641$

$I_{a2} = 54.191$
$\cos\phi_1 = 0.8748$
$\cos\phi_2 = 0.72641$

## Load sharing between two alternators




Considered 2 machines having same load-speed characteristics operating in parallel with common terminal voltage (V) & load impedance Z.

Let $E_1$ & $E_2$ are induced emfs of 2 machines $Z_{S1}, Z_{S2}$ are synchronous impedances of 2 machines

Let V be Common terminal voltage of 2 machines.

Terminal voltage V of machine ① & machine ② is

$$V = E_1 - I_1 Z_{S1} \rightarrow ①$$

$$V = E_2 - I_2 Z_{S2} \rightarrow ②$$

Also terminal Voltage of load is

$$V = IZ$$

$$V = (I_1 + I_2) Z \rightarrow ③$$

from equation ①

$$I_1 = E_1 - V / Z_{S1} \rightarrow ④$$

from equation ②

$$I_2 = \frac{E_2 - V}{Z_{S2}} \rightarrow ⑤$$

Adding (4) & (5)

$$I_1 + I_2 = \frac{E_1 - V}{Z_{s1}} + \frac{E_2 - V}{Z_{s2}}$$

$$= \frac{E_1}{Z_{s1}} - \frac{V}{Z_{s1}} + \frac{E_2}{Z_{s2}} - \frac{V}{Z_{s2}}$$

$$V \Rightarrow (I_1 + I_2) Z$$

$$I_1 + I_2 = \frac{V}{Z} \longrightarrow (6)$$

$$\frac{V}{Z} = \frac{E_1 - V}{Z_{s1}} + \frac{E_2 - V}{Z_{s2}} \Rightarrow \frac{E_1}{Z_{s1}} + \frac{E_2}{Z_{s2}} - V\left[\frac{1}{Z_{s1}} + \frac{1}{Z_{s2}}\right]$$

$$\frac{V}{Z} + V\left[\frac{1}{Z_{s1}} + \frac{1}{Z_{s2}}\right] = \frac{E_1}{Z_{s1}} + \frac{E_2}{Z_{s2}}$$

$$V\left[\frac{1}{Z} + \frac{1}{Z_{s1}} + \frac{1}{Z_{s2}}\right] = \frac{E_1}{Z_{s1}} + \frac{E_2}{Z_{s2}}$$

$$\boxed{V = \frac{\frac{E_1}{Z_{s1}} + \frac{E_2}{Z_{s2}}}{\frac{1}{Z} + \frac{1}{Z_{s1}} + \frac{1}{Z_{s2}}}} = \boxed{V = \frac{E_1 Y_1 + E_2 Y_2}{Y + Y_1 + Y_2}}$$

Load voltage

<u>Problem</u> Determine the terminal voltage & KW of each machine if $E_1 = 100$, $E_2 = 1100$, $Z = 3 + j4$, $Z_1 = Z_2 = 0.2 + j1$.

<u>Sol</u> Terminal voltage $V = \dfrac{E_1 Y_1 + E_2 Y_2}{Y + Y_1 + Y_2}$

$$Y_2 = Y_1 = \frac{1}{Z} = \frac{1}{3 + j4} = 0.1923 - 0.9615i$$

$$Y_0 = Y_0 = \frac{1}{Z_0 \text{ or } Z_0} = \frac{1}{3 + 4j} = 0.12 - 0.16i$$

Terminal voltage $V = \dfrac{E_1Y_1 + E_2Y_2}{Y + Y_1 + Y_2}$

$$V = \frac{40.83 - j201.915}{0.5046 - j2.083} = 96.11 \angle -2.311$$

$$I_1 = \frac{E_1 - V}{Z_1} = (E_1 - V)Y_1 = 5.457 \angle -34.64$$

$$I_2 = \frac{E_2 - V}{Z_2} = (E_2 - V)Y_2 = 14.525 \angle 15.45^\circ \times 0.9805 \angle -78.69$$

$$I_2 = 14.24 \angle -63.24 \text{ A}$$

KW o/p of machine (1) $= VI_1 = 96.11 \angle -2.311 \times 5.47 \angle -34.64$
$= 525.5 \text{ W}$

" " " machine (2) $= VI_2 = 1368 \text{ W}$

## Alternators on Infinite bus

The performance of alternator is not same when it operates separately on infinite bus.

An alternator is said to be on infinite bus if 2 or more generating units operated in parallel having same terminal voltage that is same as the common bus bar voltage.

A group of machines located at one place may treated as a single large machine. Also the machines connected to same bus may be grouped into one large machine. If adding or disconnection one machine which is in parallel would not affect the magnitude & phase of voltage & frequency.

Since an infinite bus is also a power s/m so that its voltage & frequency remains constant. 24

## characteristics of infinite bus

→ terminal voltage remain constant because incoming machine is too small to increase or decrease

→ frequency remains constant

→ $Z_s$ is very small since s/m has a large number of alternators in parallel.

→ In isolated operation, change of excitation leads to change the terminal of alternator, P.f depends on load only.

* If machines operating in parallel with an infinite bus its excitation is changed, P.f of machine changes but no change in terminal voltage.

* If excitation is increased, emf > bus bar voltage, the machine acts as a generator & supplies power to bus bars.

* If excitation is decreased, emf of generator < the bus bar voltage, the prime mover is replaced by mechanical load & acts a synchronous motor

Problems

Two 500 KVA alternators operate in parallel to supply following loads

(I) 250 KW at 0.9 p.f lag (II) 300 KW at 0.75 p.f lag (III) 150 KW at 0.8 p.f lag. One machine supplies 100 KW at 0.8 p.f lag. Calculate p.f of each machine.

Sol KVA of each alternator = 250 KVA.

Total active power (P) supplied by all alternators

= 250 + 300 + 150 ~~100~~ 700 KW

→ first machine supplies 100 KW = $P_1$

active power supplied 2nd alternator

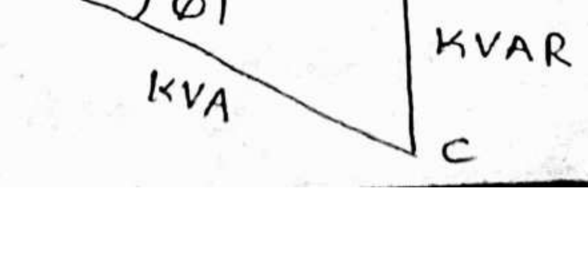
$P_2 = ?$

Total active Power (P) = $P_1 + P_2$

$700 = 100 + P_2$

$P_2 = 600$ KW

→ Reactive Power supplied by 1st machine




$\cos\phi_1 = 0.8$

$\phi_1 = \cos^{-1}(0.8)$

$\phi_1 = 36.87°$

$\tan\phi_1 = \tan(36.87) = 0.75$

$$\sin\phi_1 = \frac{KVAR}{KVA} = \frac{BC}{AC}$$

$$\cos\phi_1 = \frac{KW1}{KVA} = \frac{AB}{AC}$$

$$\frac{\sin\phi_1}{\cos\phi_1} = \frac{KVAR}{\cancel{KVA}} * \frac{\cancel{KVA}}{KW1}$$

$$\tan\phi_1 = \frac{KVAR}{KW1}.$$

Reactive Power supplied by first machine

$KVAR = KW1 * \tan\phi_1$

$KVAR = 100 * 0.75 = 75 KVAR$

→ Reactive power of total load

$$\boxed{KVA = KW * \tan\theta}$$

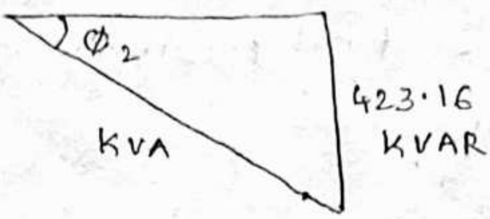
$$= KW1 \tan(\cos^{-1}(0.9)) + KW2 \tan[\cos^{-1}(0.8)] + KW3 \tan[\cos^{-1}(0.75)]$$

$$= 250 \tan(25.842) + 300 \tan^{-}[41.41] + 150 \tan(36.87)$$

$$= 498.16\ KVAR$$

Total KVA or reactive power supplied 2 machines = Reactive power of machine (1) + Reactive power 2nd machine

Reactive power of 2nd machine $= 498.16 - 75 = 423.16\ KVA$



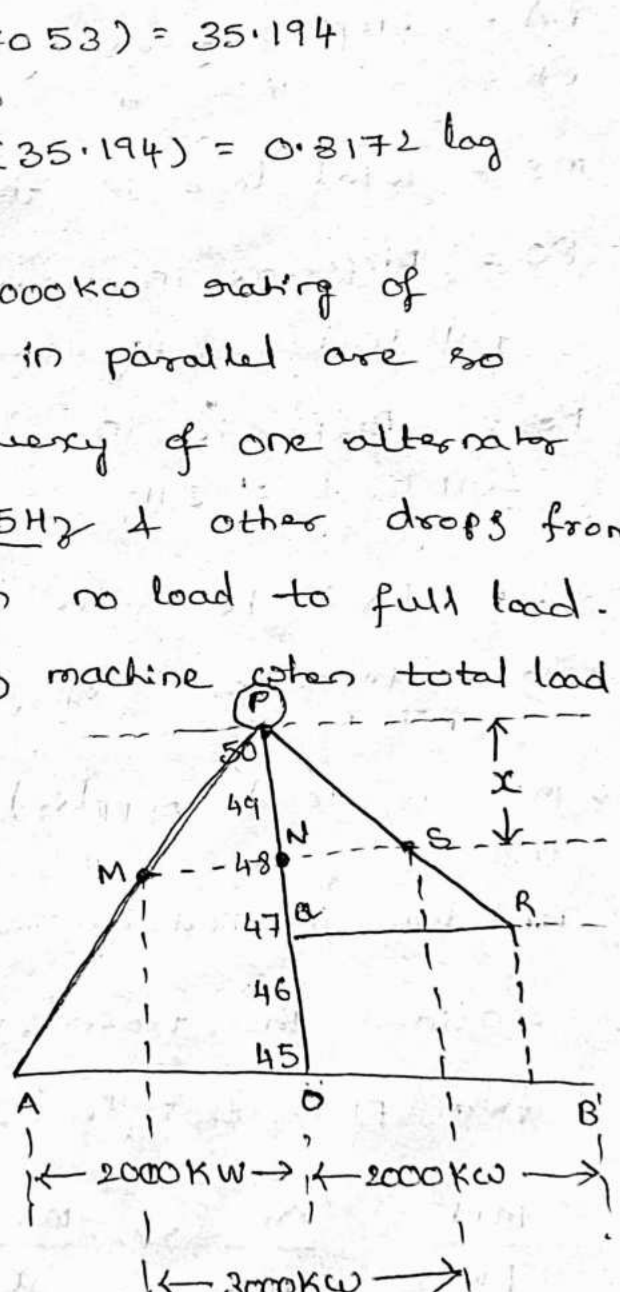

power factor of 2nd machine

$$\tan\phi_2 = \frac{KVAR}{KVA} = \frac{423.16}{600} = 0.7053$$

$$\phi_2 = \tan^{-1}(0.7053) = 35.194$$

$$P.f = \cos\phi_2 = \cos(35.194) = 0.8172 \text{ lag}$$

(a) Governor's of each 2000 kW rating of alternators running in parallel are so adjusted that frequency of one alternator drops from 50 to 45 Hz & other drops from 50 Hz to 47 Hz from no load to full load. Calculate load on each machine when total load is 3000 KW




sol Total load in s/m is 3000 KW

→ Each alternator have the capacity to supply 2000 KW

Decrease or difference in frequency of machine (A)

= normal frequency − final frequency

= 50 − 45 = 5 Hz

→ Decrease or difference in frequency of machine (B)

= normal frequency − final frequency

= 50 − 47 = 3 Hz

PA = Drop in frequency from 50 to 45 Hz
PR = " " " from 50 to 47 Hz
MS = total load in the s/m
PO = Difference in frequency from no load to full load = 5 Hz → 1st machine (A)
PQ = Difference in frequency from no load to full load = 3 Hz → 2nd machine (B)

→ Let '$x$' be rate of frequency drop in both alternator supplying total load is 3000 KW
PN = $x$

→ MN is load supplied by machine (A)
NS is " " " machine (B)
Total load supplied by 2 alternators = OA + OB = 2000 + 2000 = 4000 KW

→ consider the traingles PMN & PAO
from PMN traingle & PAO traingle

$$\frac{MN}{PN} = \frac{AO}{PO} \Rightarrow \frac{MN}{x} = \frac{2000}{5}$$

$$\boxed{MN = 400 * x}$$

→ consider the traingles PNS & PQR

$$\frac{NS}{PN} = \frac{QR}{PQ} \Rightarrow \frac{NS}{x} = \frac{2000}{3}$$

$N_S = \frac{2000}{3} * x$



$MN + NS = MS$

$MS = 400*x + \frac{2000}{3} * x$

$3000 = 400*x + \frac{2000}{3} * x$

$x = 2.8125$

→ load supplied by alternator (1)

$MN = 400*x = 400*2.8125 = 1125 kw$

→ load supplied by alternator (2)

$NS = \frac{2000}{3} * x \Rightarrow \frac{2000}{3} * 2.8125 = 1875 kw$

Actual frequencies of alternators supplies 3000kw is

$50 - x = 50 - 2.8125 = 47.1875 Hz$

(2) Two 2000 KW identical alternators operated in parallel. The governer of prime movers of 1st machine such that frequency drops from 50Hz on no load to 48 Hz on full load. 2nd machine speed drops from 50Hz to 47.5Hz.

(I) Calculate load shared by each machine of load 3000KW.

(II) what is the maximum load of unity p.f that can delivered without overloading each machine

Sol
Rating of each alternator = 2000 KVA

First Machine frequency drops on no load to full load 50 Hz - 48 Hz

2nd machine frequency drops from 50Hz to 47.5 Hz

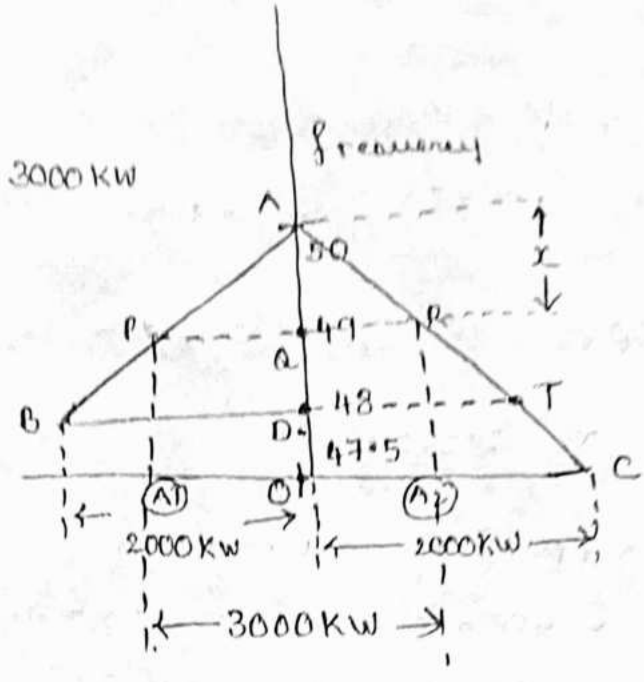
BD = 2000
DC = 2000 } KW

PR = load Line = 3000 KW

AB = $x$

AD = 2Hz

AO = 2·5Hz




Total load in the s/m is 3000KW

Each alternator have the capacity to supply = 2000 KW load

Assume the p.f supplied by alternators have unity p.f for the given load.

⇒ Decrease in frequency of machine (1) = nominal frequency - final frequency = 50-48 = 2Hz

Similarly for machine (2) = 50-47·5 = 2·5 Hz

⇒ AB = drop in frequency of machine (1)

= 50-48 ~~= 2Hz~~

AC ~~AD~~ = drop in frequency of machine (2)

= 50-47·5 = ~~2·5 Hz~~

⇒ AD = Difference in frequency from no load to full load = 2Hz

AO = Difference in frequency from no load to full load = 2·5Hz let (x) be the rate of frequency drop from no load to full load 28

① To find the load division

Considered traingles APQ & ABD

$$\frac{PQ}{AQ} = \frac{BD}{AD}$$

$$\Rightarrow \frac{PQ}{x} = \frac{2000}{AD} \Rightarrow \frac{PQ}{x} = \frac{2000}{2} \Rightarrow 1000x \rightarrow ①$$

$PQ = 1000x \longrightarrow ①$

Consider traingles AQR & AOC

$$\frac{QR}{AQ} = \frac{OC}{AO} \Rightarrow \frac{QR}{x} = \frac{2000}{2.5 Hz}$$

$$QR = \frac{2000}{2.5} * x = 800x \longrightarrow ②$$

$\Rightarrow$ Total load in the s/m = 3000KW

$PQ + QR = 3000 KW$

$1000x + 800x = 3000.$

$$x = \frac{3000}{1800} = 1.666 Hz$$

→ load supplied by machine ①

$PQ = 1000x = 1000 * 1.666 = 1666$ KW

load supplied by machine ②

$QR = 800x = 800 * 1.666 = 1333$ KW

→ Actual frequency of both alternators supplies 3000KW load: $50 - x = 50 - 1.666 = 48.334$

(11) To find maximum load at unity p.f

At point 'D' extend the line on AC and mark the point as 'T'.

load supplied by alternator = 2000 KW = DT

Consider the traingles ADT & AOC

$$\frac{DT}{AD} = \frac{OC}{AO}$$

$$\frac{DT}{2} = \frac{2000}{2.5}$$

$$DT = 1600 \text{ KW}$$

Total load supplied supplied

$= BD + DT \Rightarrow 2000 + 1600 = 3600$ KW

Unit - IV

# Synchronous MOTORS

## Theory of operation

### Introduction

If an alternator is supplied with an ac power, it capable of rotating as a motor & doing mechanical work.

If the mechanical power is supplied removed, with dc field remains energised & an ac supply is connected across the armature terminals, torque will developed & the alternator will at synchronous speed. Any change in mechanical load will not cause a speed changed (constant speed).

In case of dc motor field winding & armature winding requires a dc supply. But in case of synchronous motor field winding requires a dc supply & whereas armature winding is connected to 3 phase ac supply.

DC excitation may be provided either from dc s/m plant by using batterys or using dc generators.

For high speed motors Direct connected excitors are used with exciter armature mounted on the motor shaft.

For low speed machines belted exciters are employed

The main essential parts of 3 phase synchronous motor are ① laminated stator core with 3 phase armature winding.

33

(II) rotating field with damper windings & slip rings (III) brushes & holders

(IV) two end shields consists of bearings to support the rotor shaft.

Characteristic features of synchronous motor

→ It is not a self starting motor

→ It runs only at one speed that is synchronous speed at whatever load.

→ It can be operated for both lagging & leading p.f's.

Construction

It mainly consists of 2 parts

① Stator & (2) Rotor

→ also termed as armature, which is in cylindrical structure.

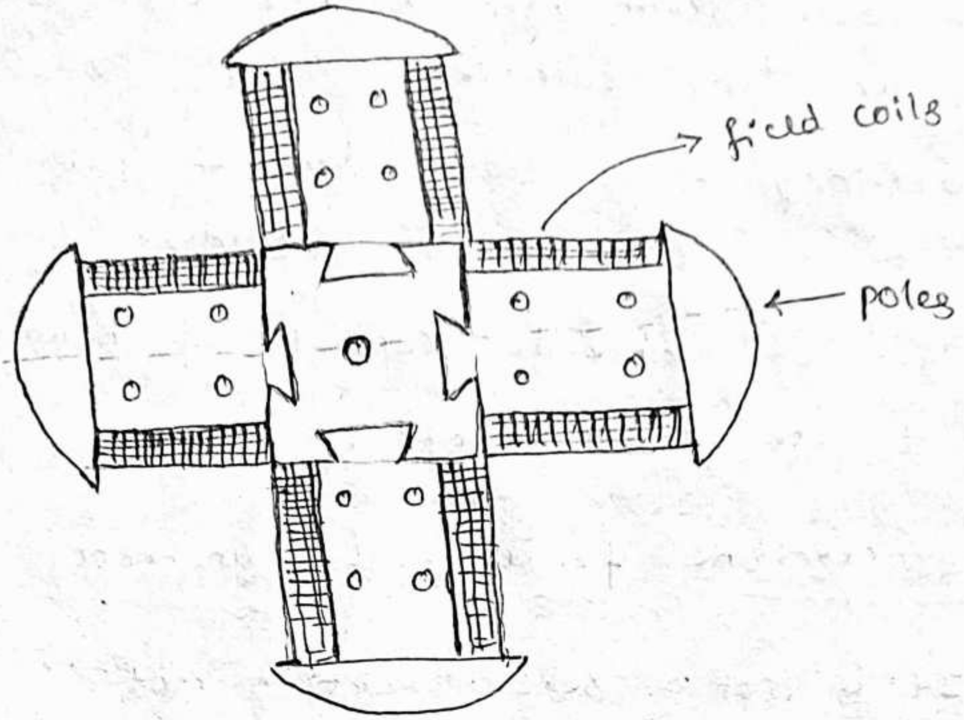
→ It is made up of cast steel or Iron or rolled steel.

→ Stator consists of slots which are arranged in the inner pheriphery.

→ Each slots carries 3 phase armature windings. which are connected in star manner.

→ The ends of windings are brought out & is connected to the power supply.

## Rotor




→ Rotor is Cylindrical in structure which is made up of cast steel.

→ the main poles are fixed to the rotor core.

→ The poles are projected out from the surface of the rotor.

→ Each pole carries a field coils wounded around the poles. All field coils are joired in series.

→ the ends of field coils are joined to 2 insulated Copper slip rings. mounted on the shaft of the motor.

→ Carbon brushes presses the slip rings, excitation Current is supplied to the motor through Carbon brushes via slip rings.

→ excitation is generally obtained from a dc source.

## principle of operation :

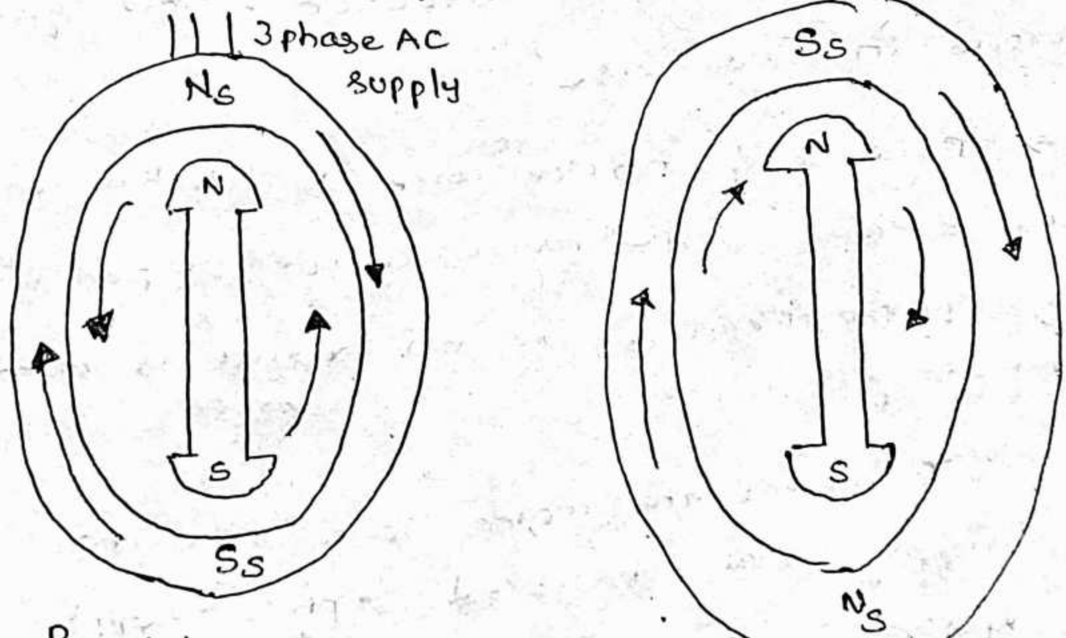


synchronous motor is inherently not a self starting motor because it does not develop a unidirectional torque.

When 3 phase supply is given to stator, a rotating magnetic field is set up in the stator revolving around the stator at synchronous speed.

$$N_s = \frac{120 f}{P} \qquad P = \text{no. of stator poles}$$

let us assume that there is one pair of poles assume the stator poles is rotating in clock-wise shown in figure




Repulsion
(Anticlockwise rotor)

Attraction
clockwise rotor

N & S → rotor poles
$N_S$ & $S_S$ → stator poles

Rotor is stationary field N & S poles remains stationary.

Stator poles ($N_s$ & $S_s$) revolves in the space at synchronous speed. If $N_s$ & N & $S_s$ & S there is a force of repulsion takes place as a result rotor tends to rotate in anticlockwise direction, torque also in anticlockwise direction.

→ After a half period stator interchanges a position of poles Then the position of poles $S_s$ & N & $N_s$ & S there is a force of attraction takes place as a result rotor tends in clockwise direction. This gives a torque in clockwise.

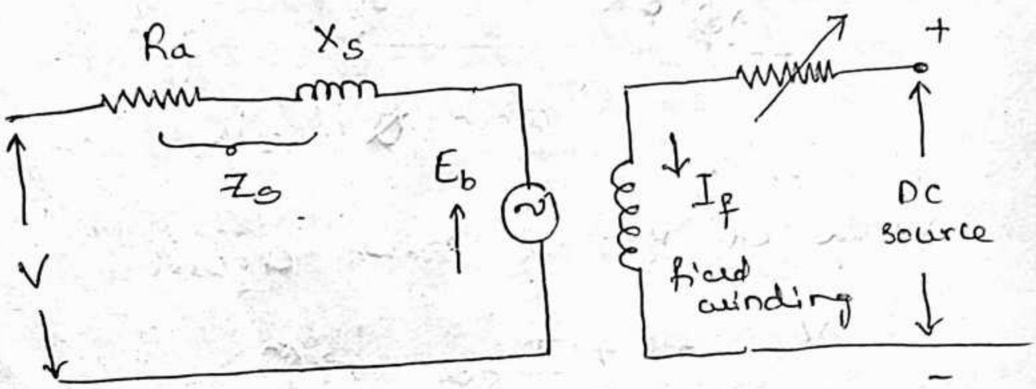
→ Because of anticlockwise & clockwise directions torques, the motor cannot respond easily to oppose torque. The net result is motor is stationory.

To start the motor we require 2 types of supply. one balanced 3 phase AC supply given to stator & a d.c excitation is supplied to field winding. Now both stator poles & rotor poles are rotating in same direction at same speed thus rotator poles get engaged with stator poles. Even if prime mover is removed the rotor continous to run at synchronous speed.

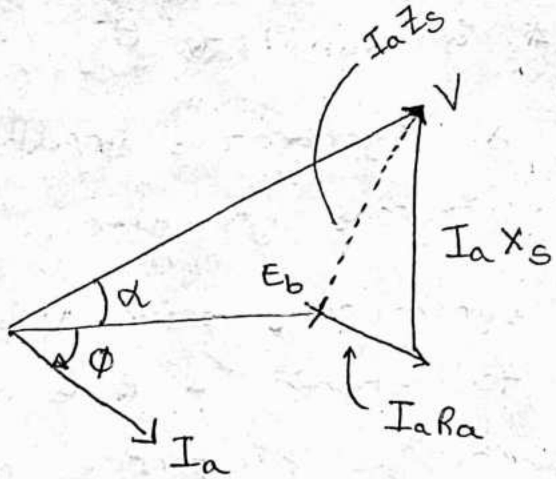
## Equivalent circuit

let $V$ = stator applied voltage
$E_b$ = back emf developed
$R_a$ = armature resistance
$X_s$ = synchronous reactance
$Z_s$ = " impedance

} per phase




equivalent circuit




$X_s = X_L + X_a$

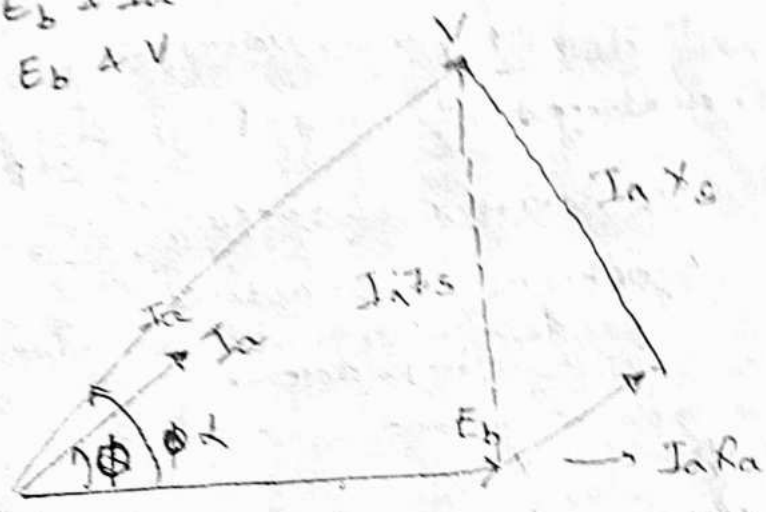
when $\phi$ is −ve

phasor diagram

The phasor diagram of synchronous motor is similar to alternator. Here armature reaction can be substituted by $X_a$ which is added to leakage reactance gives synchronous reactance $X_s$. shown in equivalent circuit

The phasor diagram of synchronous motor differs to synchronous alternator.

$\phi$ = $E_b$ & $I_a$
$\alpha$ = $E_b$ & V




when $\alpha$ is +ve

Voltage equation of stator circuit

$$V = -E + I Z_s \quad \rightarrow (1)$$

In this case excitation voltage acts as a source voltage & is equal to the sum of terminal voltage & synchronous impedance drop. above is for synchronous generator.

In case of synchronous motor V is source voltage applied to stator & E is an counter emf. Voltage equation of synchronous motor is shown in equation (1).

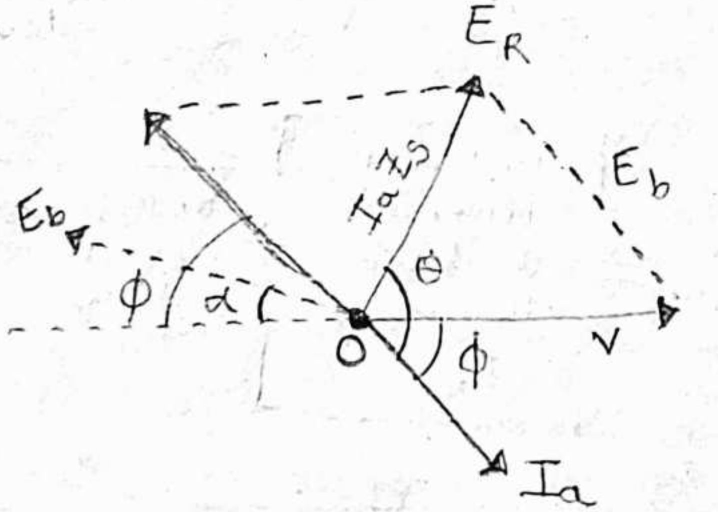
In 2nd case phasor diagram of synchronous motor differs from synchronous generator involves angle $\delta$ called power & torque angle

## Power flow equations in synchronous motor

we know that Voltage equation of syn motor

$$V = E_b + I_a Z_s$$

$$I_a = \frac{V - E_b}{Z_s} \qquad Z_s = R_a + jX_s$$




Angle $\theta$ by which $I_a$ lags behind $E_R$

$$\theta = \tan^{-1}\left(\frac{X_s}{R_a}\right)$$

Let the motor i/p $= V I_a \cos\phi$

$V$ = applied voltage / phase

Total i/p for star connected $P = \sqrt{3}\, V_L I_L \cos\phi$ → (1)

Mechanical power developed by rotor

$P_m = E_b I_a \cos(\alpha - \phi)$ per phase → (2)

out of i/p power some amount of power ($I_a^2 R_a$) is wasted in armature

Total i/p power for star connected (P) = armature Cu loss + Mechanical power ($P_m$)

$$P = P_m + I_a^2 R_a$$

Mechanical i/p power developed by rotor $P_m = P - I_a^2 R_a$

$$\boxed{P_m = \sqrt{3}\, V_L I_L \cos\phi - 3 I_a^2 R_a}$$

i/p power to stator

$P = V I_a \cos\phi$

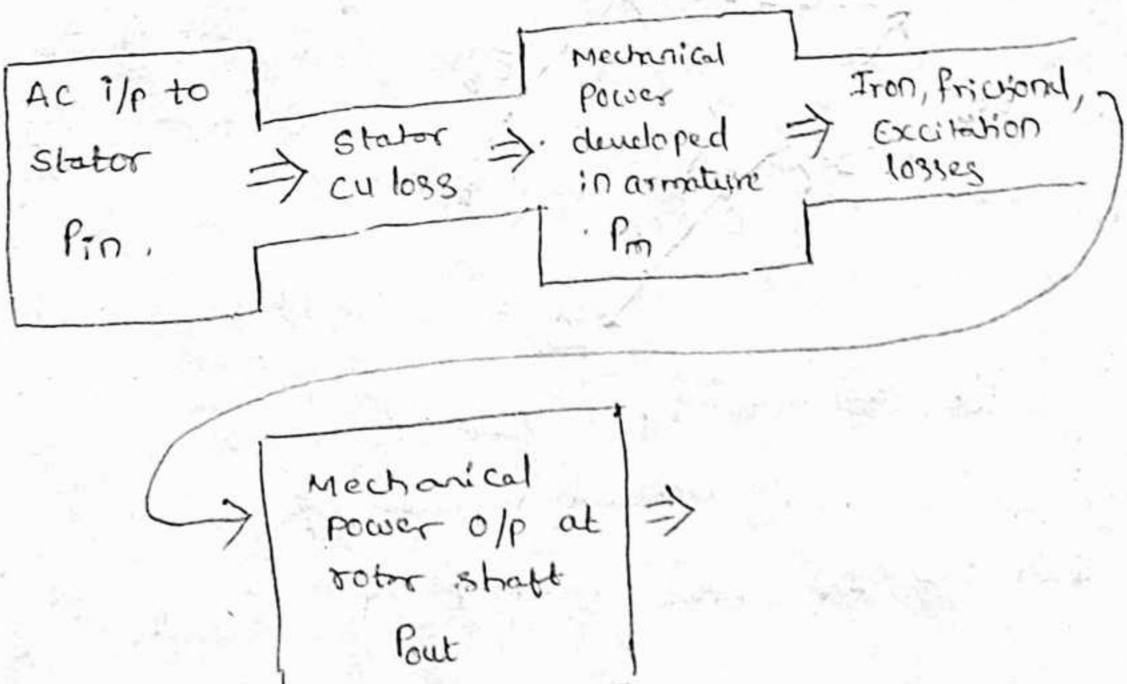
stator Armature Cu loss ($I_a^2 R_a$)

mechanical power ($P_m$)

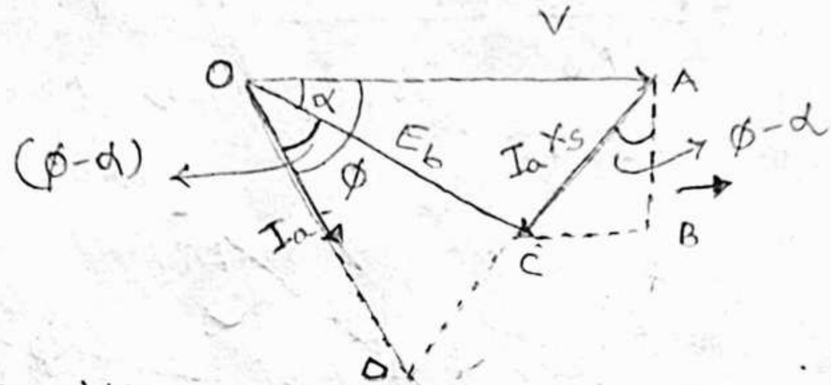
$P_m = E_b I_a \cos(\alpha - \phi)$

Iron, frictional, excitation losses

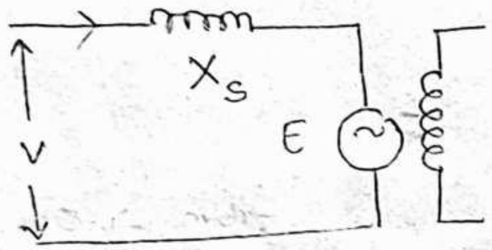
O/P power $P_{out}$



Power developed by synchronous motor 37




for small machines neglect the armature resistance hence equivalent circuit is given as




$\phi$ = angle b/w V & $I_a$

$\alpha$ = " b/w V & $E_b$

$\frac{OC}{OA} = \cos\alpha$

$AB = E_b \sin\alpha = I_a X_s \cos\phi$

motor i/p power per phase $P_{in} = V I_a \cos\phi$

$= I_a (V\cos\phi)$

from $\Delta$ OAD

$\cancel{\cos\phi = \frac{AD}{OD} \rightarrow \frac{AD}{V} \Rightarrow AD = V\cos\phi}$

$\frac{OD}{OA} = \cos\phi \Rightarrow \frac{OD}{V} = \cos\phi \Rightarrow OD = V\cos\phi$

$= I_a (OD)$

from $\Delta$ OCD

$\frac{OD}{OC} = \cos(\phi-\alpha) \Rightarrow \frac{OD}{E_b} = \cos(\phi-\alpha)$

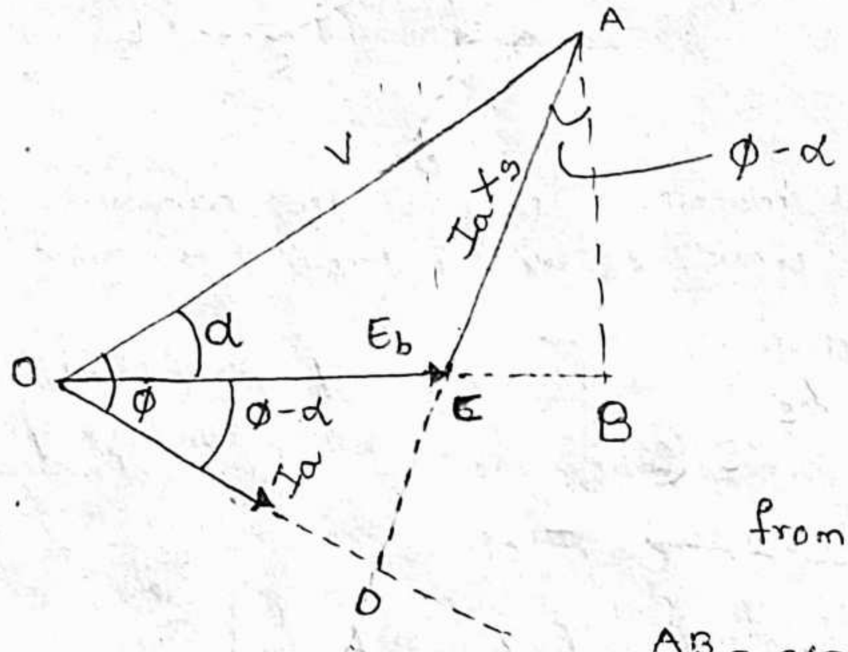
$OD = E_b \cos(\phi-\alpha)$

motor i/p power to stator/phase $= I_a E_b \cos(\phi-\alpha)$

from $\Delta$ ABC

$$\boxed{\cos(\phi-\alpha) = \frac{AB}{AC}}$$

$$\frac{AB}{I_a X_s} = \cos(\phi-\alpha) \Rightarrow AB = I_a X_s \cos(\phi-\alpha)$$




from $\Delta$ OAB

$$\frac{AB}{OA} = \sin\alpha$$

$$\frac{AB}{V} = \sin\alpha \Rightarrow AB = V\sin\alpha$$

$$\frac{V\sin\alpha}{I_a X_s} = \cos(\phi-\alpha)$$

motor i/p power to stator $= E_b I_a \cos(\phi-\alpha)$

$$P_{in} = \frac{E_b * V * I_a \sin\alpha}{I_a X_s}$$ if $\phi = 90°$

$$\boxed{P_{in} = \frac{E_b V}{X_s}\sin\alpha}$$

Total i/p 3 phase power to stator

$$P_{in} = \frac{3 V E_b}{X_s}\sin\alpha = P_m$$

## Synchronous motor with different excitations

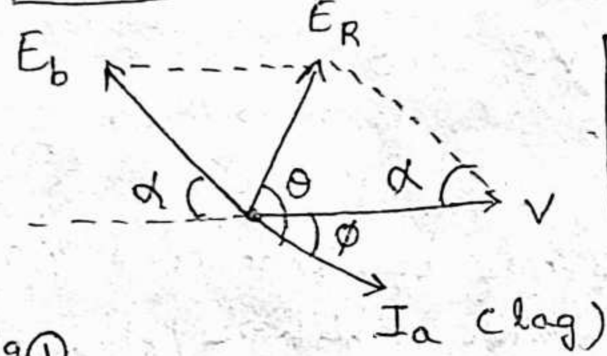
if $E_b = V$ synchronous motor is in normal excitation

$E_b < V$ is under excitation

$E_b > V$ is over excitation

$\theta-(\phi+\alpha)$
$\alpha = \theta-\phi$

$\boxed{\begin{matrix} E_b = V \\ E_b < V \end{matrix}}$ for lagging p.f is shown in fig ①



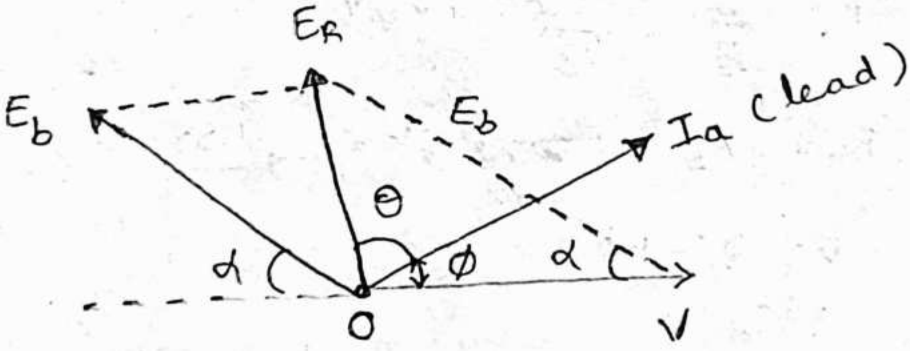

$$\boxed{E_b^2 = V^2 + E_R^2 - 2VE_R\cos(\theta-\phi)}$$

fig ①

↳ for normal excitation ($E_b = V$) & for under excitation ($E_b < V$)

$\boxed{\begin{matrix} V \text{ \& } I_a \text{ angle is } \phi \\ E_R \text{ \& } I_a \text{ '' is } \theta \end{matrix}}$

If $E_b > V$ then motor is said to be over excitation & it draws leading current shown in fig ②




$\theta = \phi + \alpha$

$\alpha = \theta - \phi$

fig ② over excitation $I_a$ leads voltage

$E_b > V$ for ~~lagging~~ leading p.f

$$E_b^2 = V^2 + E_R^2 - 2VE_R\cos(\theta+\phi)$$

$E_b > V$ for unity p.f shown in fig ③

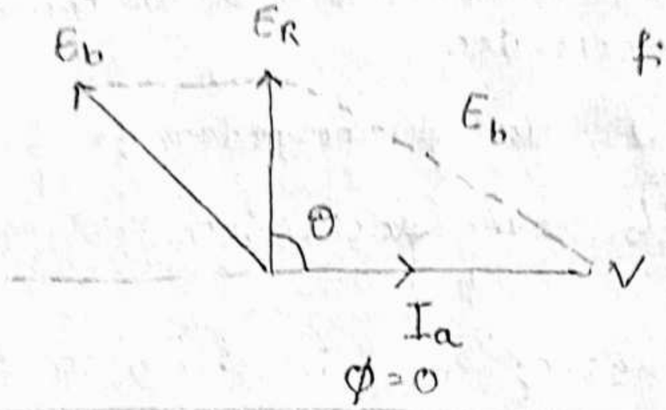



fig ③ $E_b > V$ at unity p.f

$$\boxed{E_b^2 = E_R^2 + V^2 - 2VE_R\cos\theta}$$

Problem  A 440V, 3 phase star connected synchronous motor has stator resistance / phase 0.1Ω & $X_s$/phase 4Ω. When loaded it takes 20A from supply mains. Determine back emf developed by armature. If p.f at which motor is operating ① 0.8 lag (ii) 0.8 leading (iii) unity

sol  $V_L = 440V$ , $I_a = 20A$

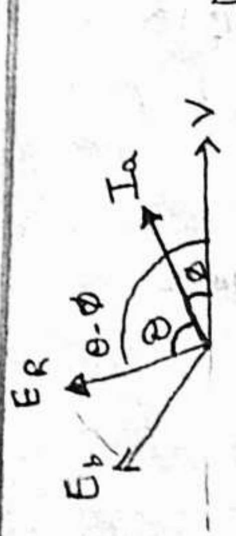
$$V_p = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254V$$

$$Z_s = R_a + jX_s = 0.1 + 4i = 4\angle 88.568$$

$$\theta = 88.568$$

(or)

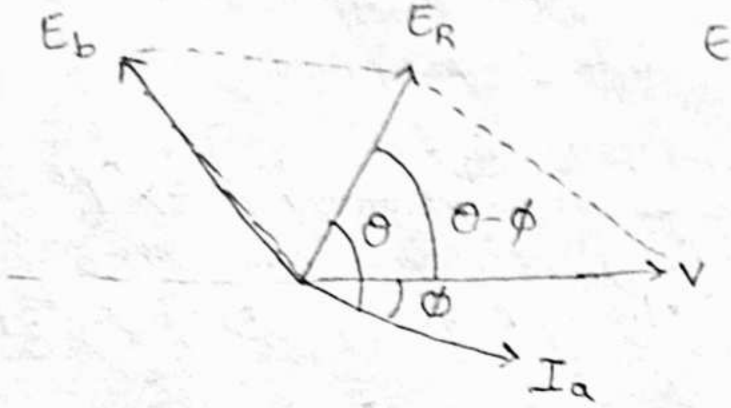
$$\theta = \tan^{-1}\left(\frac{X_s}{R_a}\right) = 88.568$$




$$I_a = \frac{E_R}{Z_s} \Rightarrow \boxed{E_R = I_a Z_s} = 20 \times 4 = 80V$$

when p.f is lag at 0.8






$E_b = V$ (?)

$E_b < V$

Angle b/w $E_R$ & V

$\theta-\phi$

$\theta - \phi$

$\theta = 88.568$

$\cos\phi = 0.8$

$\phi = \cos^{-1}(0.8) = 36.87$

Angle b/w $E_R$ & V is $\theta-\phi = 51.69°$

$$E_b = \sqrt{V^2 + E_R^2 - 2VE_R\cos(\theta-\phi)}$$

$$= \sqrt{(254)^2 + (80)^2 - 2(254 \times 80)\cos(51.69)}$$

$$= \sqrt{45,727.067} = 213.84$$

(ii) $\boxed{E_b > V}$ $\cos\phi = 0.8$ leading

Angle b/w $E_R$ & V = $\theta + \phi = 88.568 + 36.87$

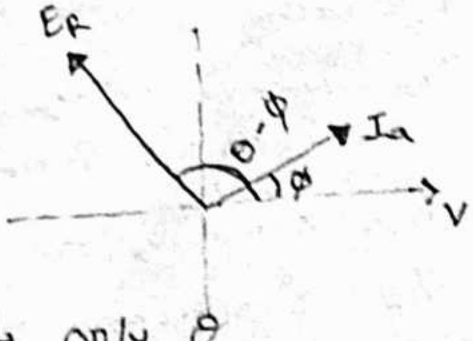
$\phi = \cos^{-1}(0.8) = 36.87$ $\Rightarrow 125.438°$

$E_b^2 = E_R^2 + V^2 - 2VE_R\cos(\theta+\phi)$

$= (80)^2 + (254)^2 - 2(80)(54)\cos(125.438)$

$E_b = \sqrt{94,480} = 307.38\,V$

$307.38\ (E_b) > V\ (254)$

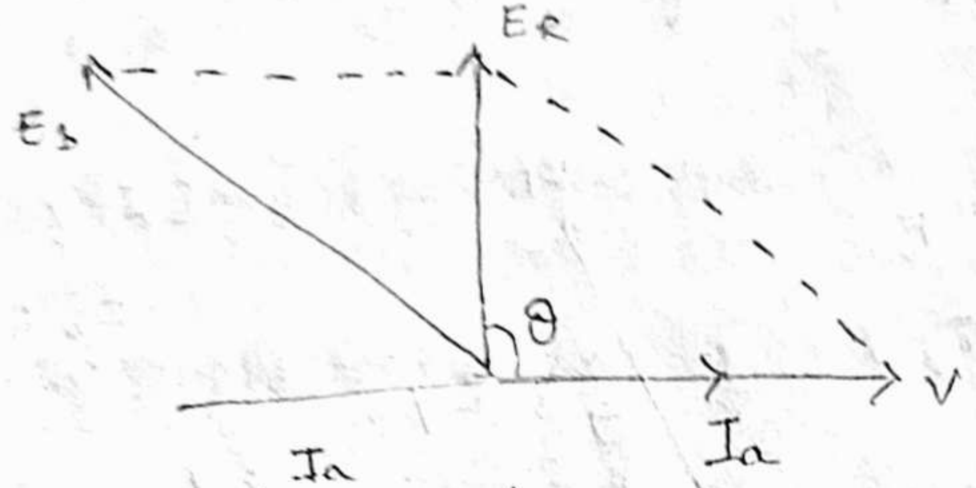



(iii) $E_b > V$, p.f = 1

Angle b/w $E_R$ & V is only $\theta$

$\cos\phi = 1 \Rightarrow \cos^{-1}(1) \Rightarrow \phi = 0$

$E_b > V$ p.f = unity




$\phi$ = angle b/w $I$ & V is 0, $\theta$ = angle $E_R$ & V

$$E_b^2 = E_R^2 + V^2 - 2 E_R V \cos\theta$$

$$= (80)^2 + (254)^2 - 2 * 80 * 254 \cos 88.5 68^\circ$$

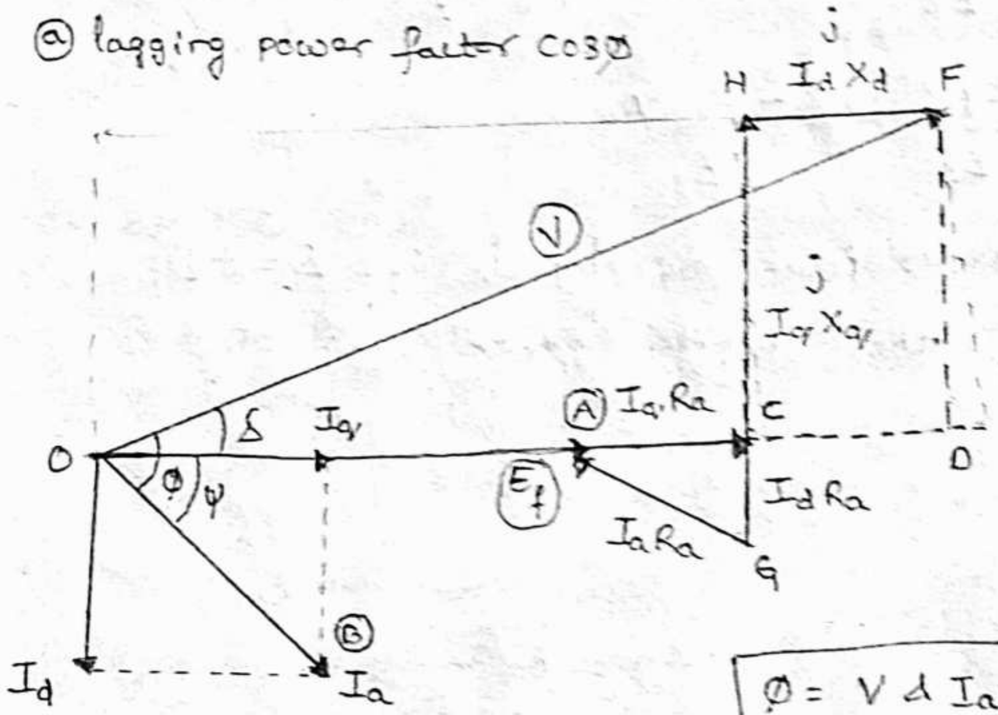
$$E_b = \sqrt{69,900} = 264.387$$

Phasor diagram of a salient pole synchronous motor

Voltage equation of salient synchronous motor

$$V = E_f + I_a R_a + jI_d X_d + jI_q X_q$$

ⓐ lagging power factor $\cos\phi$




Phasor diagram for lagging

$\phi = V \angle I_a$
$\psi = E_f \angle I_a$
$\delta = E_f \angle V$

$OA = E_f$, $AG = I_a R_a$, $GH = I_q X_q$, $HF = I_d X_d$

$OF = V$, $AC = I_q R_a$, $CD = HF = I_d X_d$

$OD = OA + AC + CD$

From $\Delta ODF$

$\cos\delta = \frac{OD}{OF} \Rightarrow \cos\delta = \frac{OD}{V} \Rightarrow \boxed{OD = V\cos\delta}$

$V\cos\delta = E_f + I_q R_a + I_d X_d$ → ①

$\frac{CH}{OF} = \sin\delta$

$GH = CG + CH$

$I_q X_q = I_d R_a + V\sin\delta$ → ②

$\phi = \psi + \delta$

$\psi = \phi - \delta$

$I_d = I_a \sin\psi = I_a \sin(\phi - \delta) \rightarrow (3)$

$I_q = I_a \cos\psi = I_a \cos(\phi - \delta) \rightarrow (4)$

$$I_a X_q \cos(\phi - \delta) = I_a R_a \sin(\phi - \delta) + V \sin\delta$$

$$I_a X_q \left[ \cos\phi \cos\delta + \sin\phi \sin\delta \right] = I_a R_a \left[ \sin\phi \cos\delta - \cos\phi \sin\delta \right] + V \sin\delta$$

$$\boxed{\begin{aligned} &I_a X_q \cos\phi \cos\delta + I_a X_q \sin\delta \sin\phi = \\ &\quad I_a R_a \sin\phi \cos\delta - I_a R_a \cos\phi \sin\delta + V \sin\delta \\ &\sin\delta \left( V - I_a R_a \cos\phi + I_a X_q \right) \end{aligned}}$$

$$V \sin\delta - I_a R_a \cos\phi \sin\delta - I_a X_q \sin\delta \; \overset{\sin\phi}{\cancel{\cos\phi}}$$

$$I_a X_q \cos\phi \cos\delta - \cancel{I_d R_d \cos\delta} \; I_a R_a \sin\phi \cos\delta$$

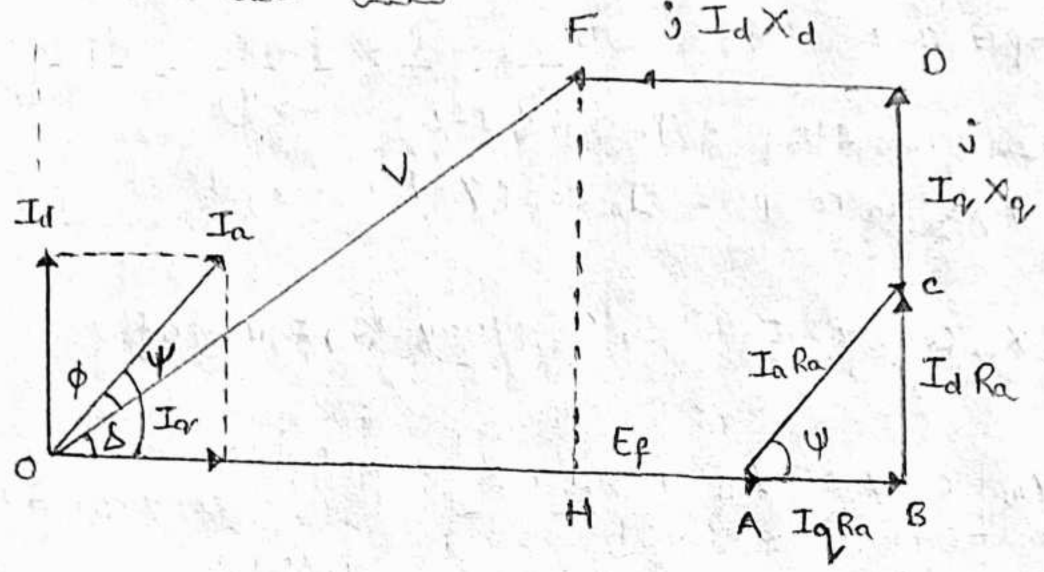
$$\Rightarrow \sin\delta \left( V - I_a R_a \cos\phi - I_a X_q \sin\phi \right)$$

$$= \cos\delta \left[ I_a X_q \cos\phi - I_a R_a \sin\phi \right]$$

$$\frac{\sin\delta}{\cos\delta} = \frac{I_a X_q \cos\phi - I_a R_a \sin\phi}{V - I_a R_a \cos\phi - I_a X_q \sin\phi}$$

$$\tan\delta = \frac{I_a X_q \cos\phi - I_a R_a \sin\phi}{V - I_a R_a \cos\phi - I_a X_q \sin\phi}$$

(b) leading power factor




$OA = E_f$ , $AC = I_aR_a$ $CD = I_qx_q$ $DF = I_dX_d$, $OF = V$,

$\delta$ = angle b/w $V$ & $E_f$

$\phi$ = angle b/w $V$ & $I_a$

$\psi$ = angle b/w $E_f$ & $I_a$

from $\Delta^{le}$ ABC

$$\frac{BC}{AC} = \frac{OPP}{HYP} = \sin\psi$$

$\Rightarrow BC = AC\sin\psi = I_aR_a$

$\Rightarrow$ NOW

$$OA = OH + AH = OH + (HB - AB)$$

$$E_f = V\cos\delta + I_dX_d + (-I_qR_a)$$

$$= V\cos\delta + I_dX_d - I_qR_a \rightarrow (1)$$

$\Rightarrow$ $HF = BD \Rightarrow BC + CD$

$$V\sin\delta = I_dR_a + I_qR_a \rightarrow (2)$$

$$\psi = \phi + \delta$$

$$I_d = I_a\sin\psi = I_a\sin(\phi+\delta) \rightarrow (3)$$

$$I_q = I_a\cos\psi = I_a\cos(\phi+\delta) \rightarrow (4)$$

from HAB

$HB = AH + AB$

$\boxed{AH = HB - AB}$

$\Rightarrow$ from $\Delta^{le}$ OHF

$\cos\delta = \frac{adj}{HYP} = \frac{OH}{AF}$

$\cos\delta = \frac{OH}{V}$

$\boxed{OH = V\cos\delta}$

from line BCD

$BD = BC + CD$

$\Rightarrow$ from $\Delta^{le}$ OHF

$\sin\delta = \frac{HF}{OF}$

$HF = V\sin\delta$

Substitute equation 3, 4 in equation (2)

47

$$V\sin\delta = I_aR_a\sin(\phi+\delta) + I_aX_q\cos(\phi+\delta)$$

$$= I_aR_a\left[\sin\phi\cos\delta + \cos\phi\sin\delta\right] + I_aX_q\left[\cos\phi\cos\delta - \sin\phi\sin\delta\right]$$

$$V\sin\delta = I_aR_a\sin\phi\cos\delta + I_aR_a\cos\phi\sin\delta + I_aX_q\cos\phi\cos\delta - I_aX_q\sin\phi\sin\delta$$

$$V\sin\delta + I_aX_q\sin\phi\sin\delta - I_aR_a\cos\phi\sin\delta = I_aR_a\sin\phi\cos\delta + I_aX_q\cos\phi\cos\delta$$

$$\sin\delta\left[V + I_aX_q\sin\phi - I_aR_a\cos\phi\right] = \cos\delta\left[I_aR_a\sin\phi + I_aX_q\cos\phi\right]$$

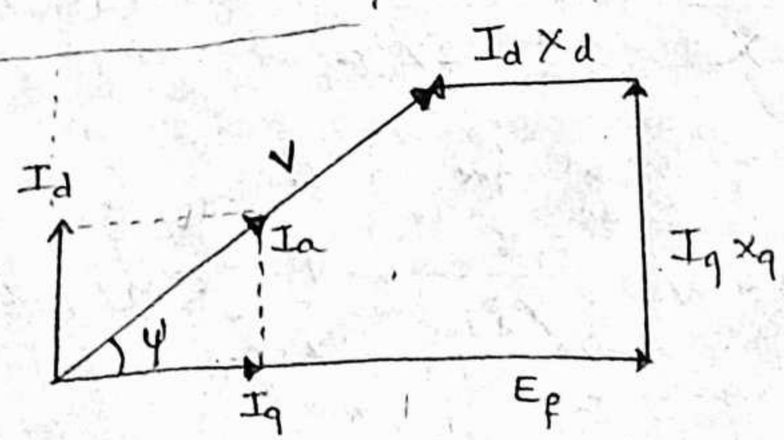
$$\frac{\sin\delta}{\cos\delta} = \boxed{\operatorname{Tan}\delta = \frac{I_aR_a\sin\phi + I_aX_q\cos\phi}{V + I_aX_q\sin\phi - I_aR_a\cos\phi}}$$

(c) unity power factor $\cos\phi = 1$ if $\phi = 0$

$\sin\phi = 0$ in lagging p.f

$$\boxed{\operatorname{Tan}\delta = \frac{I_aX_q}{V - I_aR_a}}$$

$\psi = \phi + \delta$



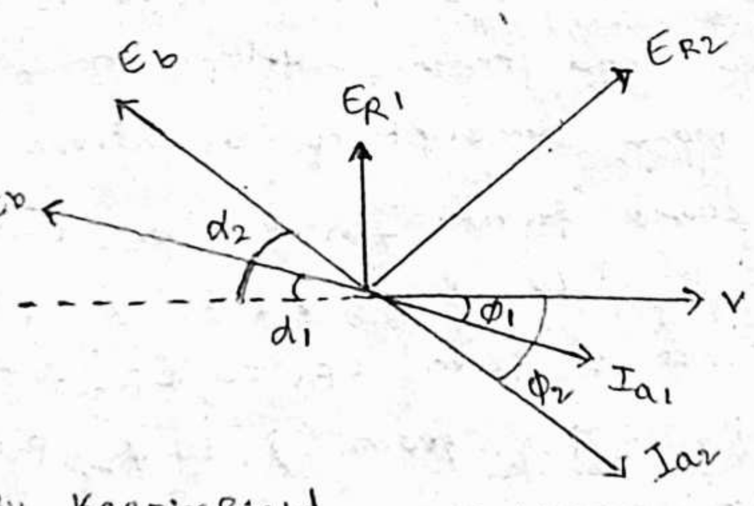
## Effect of change in load at Constant excitation



When the motor is on no load, rotor field slightly falls back in phase with stator field. If the motor is lightly loaded by keeping field excitation Constant, rotor falls back more in phase with stator field by a larger phase angle. Due to this more armature drawn by motor to develop torque in order to meet increasing load.

Effect of load in synchronous motor is explained in 3 ways of excitation
(1) normal excitation ($E_b = V$) (2) under excitation ~~$E_b < V$~~ $E_b < V$ (III) Over excitation $E_b > V$.

### (1) Normal excitation ($E_b = V$)




By keeping field excitation constant, allow the motor to run with small load, torque angle say $\alpha_1$ is small, similarly phase angle $\phi_1$ b/w V & $I_{a1}$ be small so p.f is high ($\cos\phi_1$)

Now the load on motor slightly increases torque angle increases from $\alpha_1$ to $\alpha_2$, to meet the extra load motor draws more armature Current say $I_{a2}$.

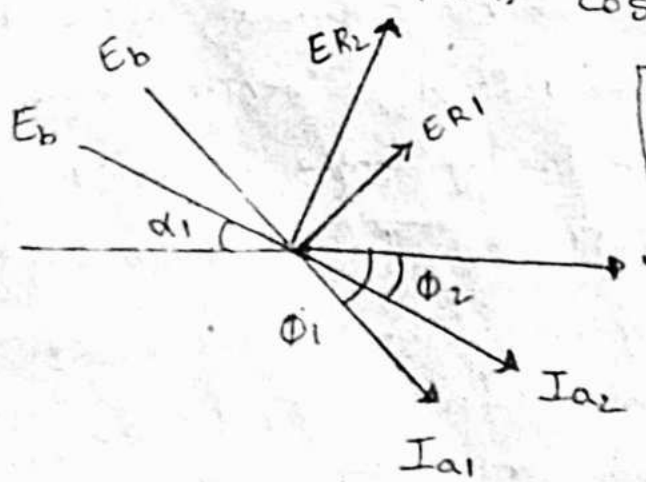
Angle between $I_{a2}$ & V say $\phi_2$ increases from $\phi_1$ to $\phi_2$, corresponding p.f decreases $\cos\phi_1$ to $\cos\phi_2$. Resultant Voltage $E_{R2}$ increases $E_{R1}$.

$$\boxed{\text{Load}\uparrow,\ \text{torque angle}\uparrow,\ I_a\uparrow,\ E_R\uparrow,\ \text{phase angle } \phi_1\uparrow,\ \text{p.f}\downarrow}$$

(2) under excitation ( $E_b < V$ )

Allow the motor to run with small load by maintaining Constant excitation, torque angle say $\alpha_1$ is small but $I_{a1}$ lags behind the voltage with with larger phase angle $\phi_1$. so larger the phase angle means lower the p.f say $\cos\phi_1$.

→ Now the load on motor slightly increasing, torque angle $\alpha_2$ increases from $\alpha_1$ to $\alpha_2$. To meet the extra load motor develops more torque by drawing more amount of armature current say $I_{a2}$ increases from $I_{a1}$ to $I_{a2}$. But the phase angle decreases $\phi_1$ to $\phi_2$. Resultant voltage increases from $E_{R1}$ to $E_{R2}$. Due to decrease in angle from $\phi_1$ to $\phi_2$ power factor increases from $\cos\phi_1$ to $\cos\phi_2$.




Load ↑, torque angle ↑, Ia ↑, ER ↑
But p.f ↑, Phase angle Ø ↓

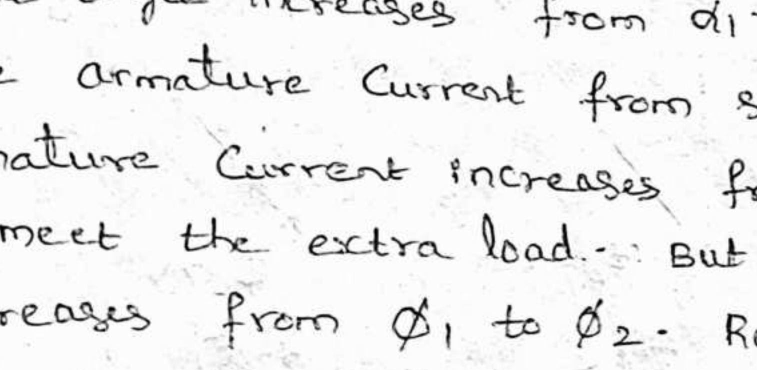
$E_b < V$ under excitation (lagging)

over excitation ( $E_b > V$ )

Allow the motor to run with small load by keeping excitation constant. Load angle $\alpha_1$ is small but current draw by motor say $I_{a1}$ leads the supply voltage at an angle say $\phi_1$. At light loads phase angle $\phi_1$ between $I_{a1}$ & V is larger. so power factor $\cos\phi_1$ is small.

Again add or increase some load on motor, torque angle increases from $\alpha_1$ to $\alpha_2$ by drawing more armature current from supply. Here Armature current increases from $I_{a1}$ to $I_{a2}$ to meet the extra load. But phase angle decreases from $\phi_1$ to $\phi_2$. Resultant p.f increases from $\cos\phi_1$ to $\cos\phi_2$.

So for under & over voltage excitation its power factor increases upto unity.




$E_b > V$ over excitation

Load ↑, torque angle ↑, $I_a$↑, $E_R$↑

but phase angle $\phi$ ↓, P.f ↑

Effect of variation on armature Current & power factor

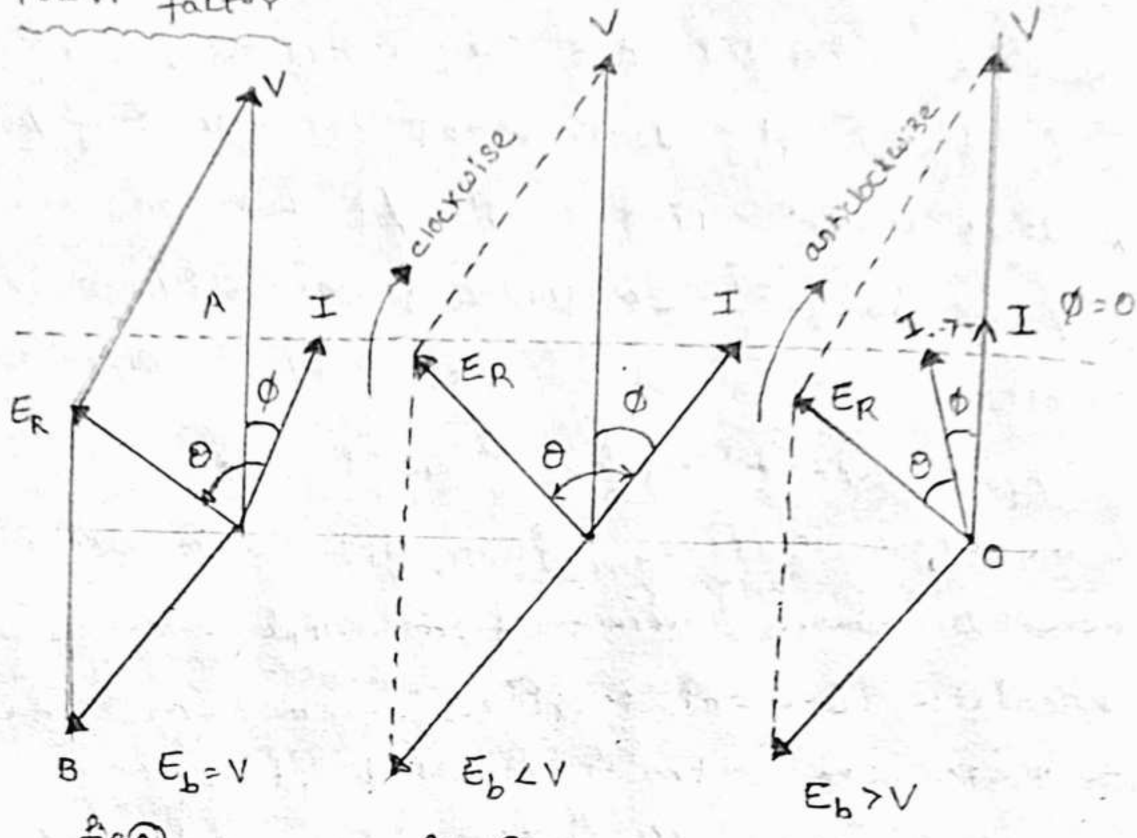



fig ⓐ fig ⓑ fig ⓒ

Consider the synchronous motor in which the mechanical load is Constant

fig ⓐ $E_b = V$ only for 100%. excitation. The armature ~~lags~~ Current I lags behind V by a small angle $\phi$. So angle $\theta$ with $E_R$ is fixed by stator

$$\tan\theta = \frac{X_s}{R_a}$$

fig ⓑ $E_b < V$ excitation is less than 100%.

Here $E_R$ is advanced clockwise by I. Magnitude of Current I increases but its phase angle $\phi$ increases as a result p.f decreases hence active component $I\cos\phi$ remains same but $I\sin\phi$ wattless Component decreases.

from fig ©

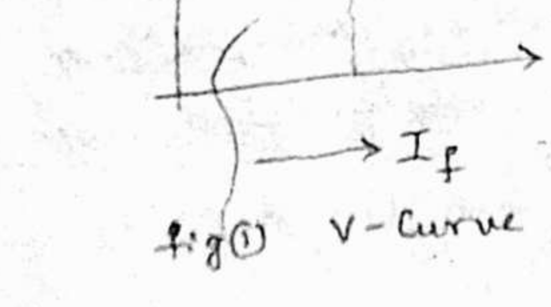


when $E_b > V$ indicates the overexcited motor. Here $E_R$ is pulled into anticlockwise to I. Now motor is drawing a leading current. The result I may be in phase with 'V' & $\phi = 0$ p.f becomes unity. So current draw by motor would be minimum.

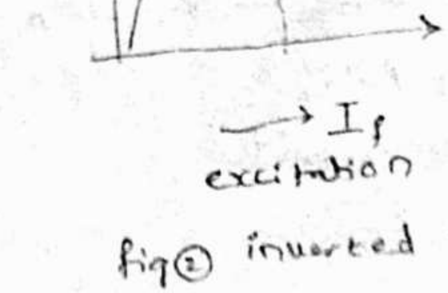
① Magnitude of $I_a$ varies with excitation the current has large value for both low & high excitation. Variation of armature current with field excitation is known as 'V' curves.

(2) when under excited motor runs with lagging p.f & over excited motor runs with leading p.f.

Variation of p.f with excitation is termed as inverted 'V' curves.




fig① V-Curve




fig② inverted 'V'

An over excited motor can be run with leading power factor. This is usefull for phase advancing purposes in industrial loads driven by induction motors.

Both transformers & induction motor draws lagging current from supply. If it is operated at light loads, it draws more reactive power & makes the p.f low.

Different torques a synchronous motor

① starting torque: It is developed when full voltage is applied stator winding. Also known as break away torque.

ⓑ Running torque Motor develops the torque during running conditions. It is determined by horse power & speed of driven machine. Horse power of motor indicate maximum torque.

ⓒ pull in torque

The amount of torque at which the motor will pull into step is called pull in torque.

When speed of synchronous motor is 2 to 5% below the synchronous speed it is acting as induction motor. After excitation is on, rotor pulls into synchronously with rotating stator field.

Ⓓ Pull out Torque



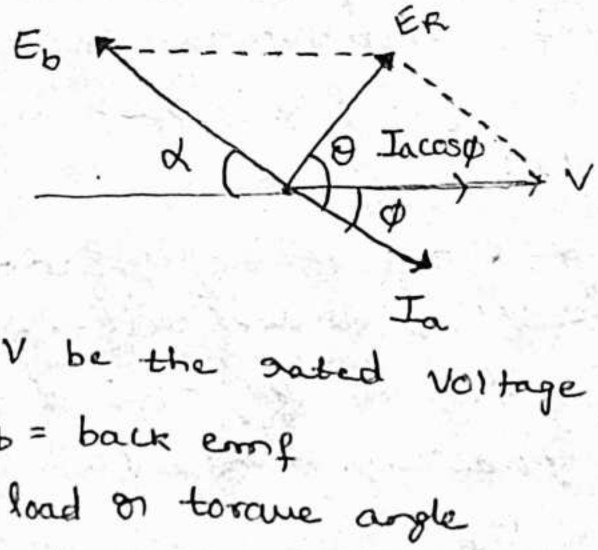
Maximum torque develops without pulling into step or synchronousim is called pull out torque.

Motor develops maximum torque when its ~~rotal~~ rotor is retarded by an angle of 90°. Any load increased, the motor pulled out from synchronism & stop.

V & inverted V Curves

if load on motor constant, load angle & torque angle remains same. change in excitation result change in back emf

Let the field excitation on motor is gradually increased by maintaining the load on motor constant.




$E_R$, V moves left side to $E_b$ if $I_f$ increases

Let V be the rated voltage / phase

$E_b$ = back emf

$\alpha$ = load or torque angle

$\phi$ = phase angle b/w V & $I_a$

$I_a$ = armature current drawn by motor.

$\theta$ is angle b/w $E_R$ & V, $\theta = \tan^{-1}\left(\frac{X_s}{R_a}\right)$

$E_R$ = Resultant voltage

Active Current Component = $I_a \cos\phi$

$P \cdot f = \cos\phi$

active power = $V I_a \cos\phi$

if load on motor is kept Constant

$I_a \cos\phi$ is constant

if $I_a \cos\phi$ is Constant, V is Constant

→ Let $I_f$ on motor gradually increase. $E_b$ developed by motor ~~decreases~~ increases. Resultant Voltage $E_R$ moves to left side. Here $I_a$ lags behind the $E_R$ by angle $\theta$.

→ If $E_R$ moves towards left, $I_a$ also moves towards left as a result angle $\phi$ progressively decreases.

→ For certain field excitation, $I_a$ is in phase with V then angle $\phi$ becomes zero

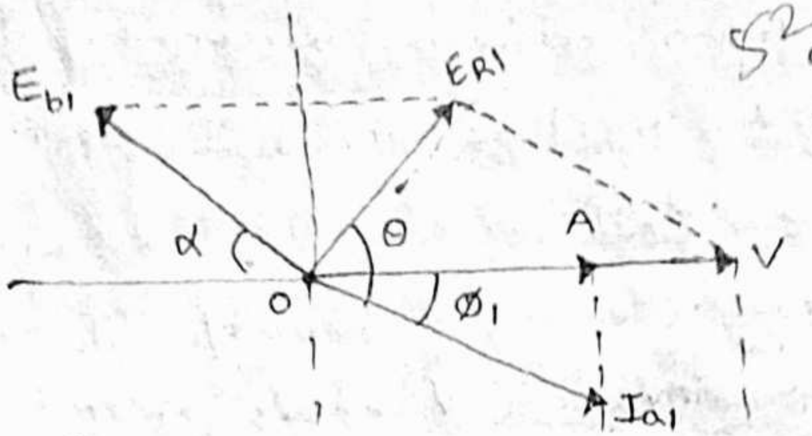
The p.f at which the motor operates as unity. The field excitation at which the motor operates as unity is termed as normal excitation. ($E_b = V$)

Any field excitation is less than normal excitation is termed as under excitation ($E_b < V$)

Any field excitation is more than normal excitation is termed as ~~under~~ over excitation ($E_b > V$)

→ when motor is ~~[illegible]~~ operated under excitation motor draws lagging current.

→ when motor is operated in overexcitation it draws a leading current

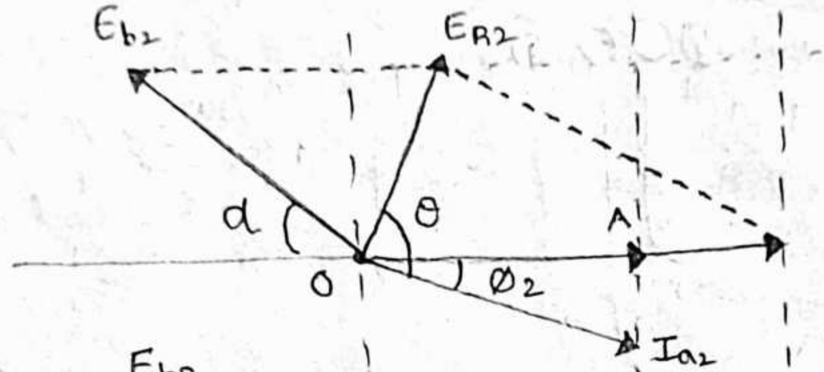



$OA = I_{a1}\cos\phi_1$

$E_b < V$

p.f $= \cos\phi$ (lag)

$\phi_1$

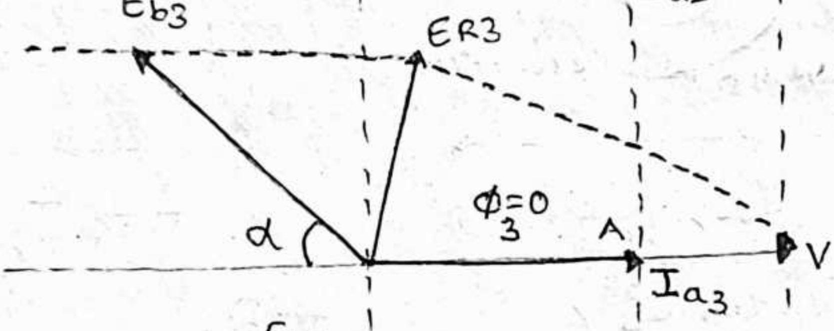



$E_{b2} > E_{b1}$

$I_{a2} < I_{a1}$

$OA = I_{a2}\cos\phi_2$

p.f $= \cos\phi_2$ is lagging

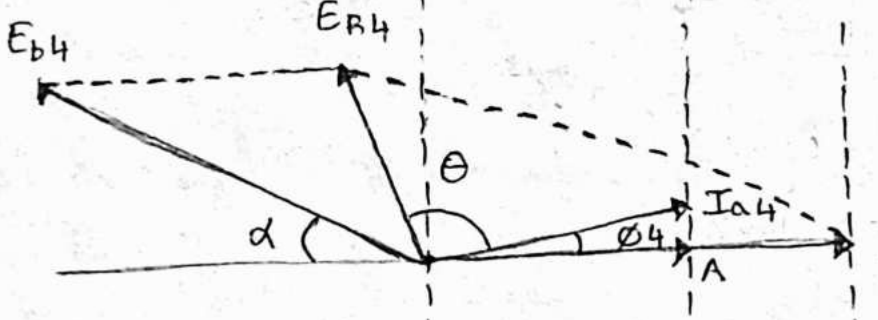



$E_{b3} > E_{b2}$

$I_{a3}$ is in phase with V

so $\phi = 0$ p.f $= 1$

$OA = I_{a3}$




$E_{b4} > E_{b3}$

$I_{a4}$ leads behind V

$\cos\phi_4$ (lead)

$OA = I_{a4}\cos\phi_4$

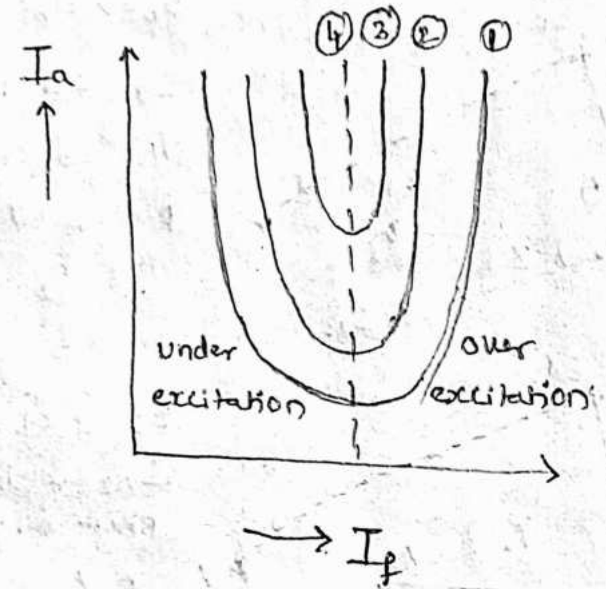
from phasor diagrams

If load on motor kept constant, field excitation increases, $I_a$, $E_b$, $E_R$ moves left side to $E_b$.

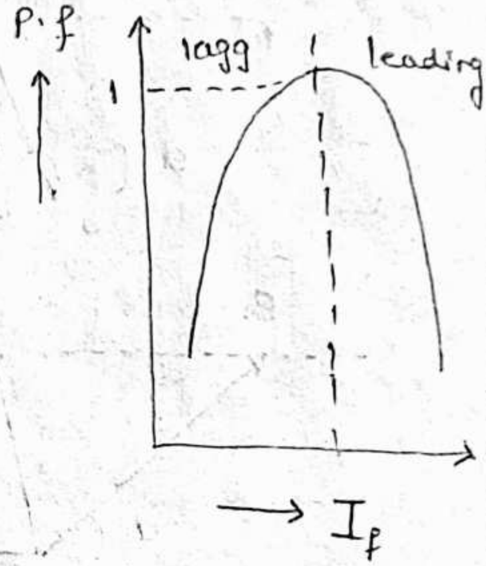
$I_f$ increases, $E_b$ increases corresponding $E_R$ & $I_a$ also increases also p.f increases from lagging to unity to leading.

Curves drawn between corresponding field current $I_f$ & armature current $I_a$ is known as V-curves fig(1)

similarly curves drawn b/w $I_f$ & p.f is known as inverted V curves. shown in fig(2)




fig(1) V-curves




fig(2)

(1) → no load

(2) → full load (50%)

(3) → full load (100%)

(4) → unity p.f

V curves can be drawn at different loads from no load to full load.

## Synchronous Condenser

An over excited synchronous motor is termed as synchronous condenser. During this case motor draws leading current from supply & operates at leading power factor.

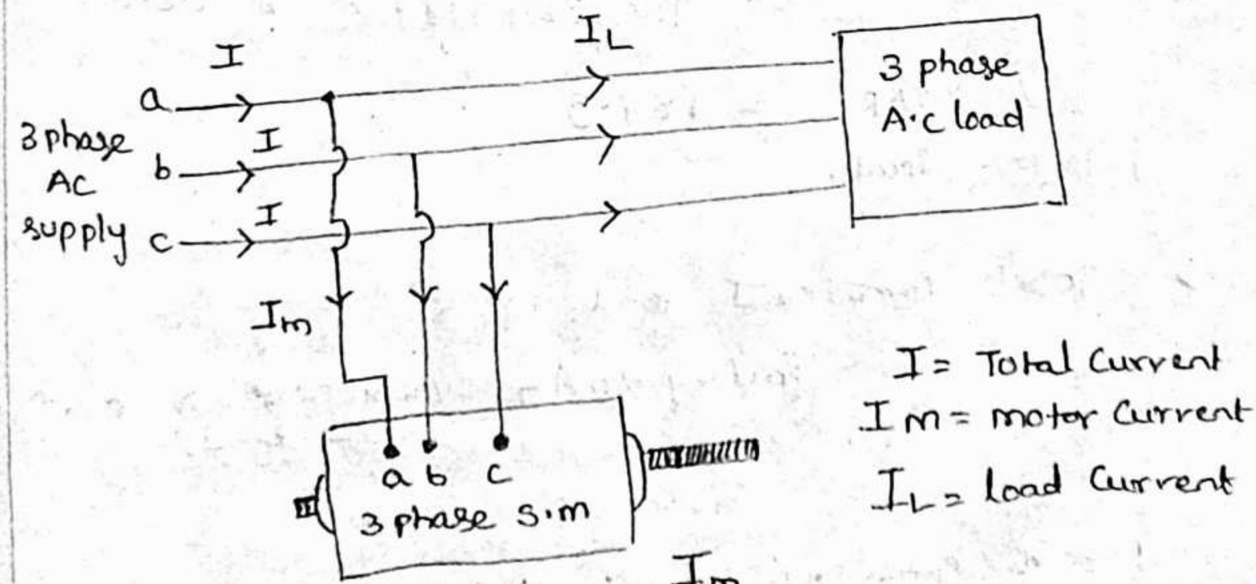
or

A synchronous motor running no load is also a (over excited motor) synchronous condenser. This has a capability to generate or absorb reactive power in lines.

In any power s/m most of the loads are reactive power loads like transformers, induction motors, current limiting reactors etc. These devices draws lagging current from supply and makes the power factor ~~[illegible]~~ is low.

If reactive component of power is quite large, same amount of power is supplied to the lines, to make the power factor high. Such leading power devices is a synchronous condenser. These can draws a leading current from the supply.

If the field winding of synchronous motor is overexcited by a d.c source then it supplies reactive power in opposite direction to the lagging devices load. This is used below 500KVAR is not at all economical.




$$I = I_L + I_m$$

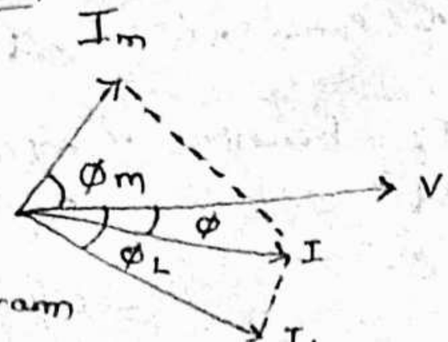



Fig synchronous motor & its phasor diagram

Problem A synchronous motor taking 50KW from mains Connected in parallel with factory load 250KW having 0.8 p.f lagging. The combined load has 0.9 p.f lagging. Find leading KVA supplied by motor & p.f at which it is operating.

Sol

Power consumed by synchronous motor = 50KW

" " " factory load = 250KW

Before installation of synchronous motor p.f is 0.8 lagging $\cos\phi_1 = 0.8$

After installing synchronous motor p.f is 0.9

$$\cos\phi_2 = 0.9$$

→ lagging Reactive (KVA) power drawn by factory load is at 0.8 p.f lagg

$$\tan\theta = \frac{KVAR}{KW}$$

$\cos\phi_1 = 0.8$

$$KVAR = \tan\theta_1 * KW$$

$\phi = \cos^{-1}(0.8)$

$$= \tan(36.87) * 250$$

$\phi = 36.87$

$$KVAR = 187.5$$

of factory load.

⇒ Total Combined load in KW is

$$= (\text{factory load} + \text{synchronous motor load}) \text{ in KW}$$

$$= 250 + 50 = 300\,KW$$

p.f of Combined load = 0.9 lag

→ lagging Reactive (KVA) Power of Combined load at 0.9 p.f is

$$\cos\phi_2 = 0.9 \Rightarrow \phi_2 = 25.842^\circ$$

KVAR of combined load = KW * Tan $\theta_2$

54

= 300 * Tan(25.842)

KVAR of Combined load = 145.3 KVA

Here p.f is increased from 0.8 to 0.9 by the synchronous motor

=> leading Reactive (KVA) of synchronous motor = $\boxed{\text{KVAR of factory load} - \text{KVAR of combined load}}$

= 187.5 − 145.3 = 42.2 KVAR (leading)

(b) To find the p.f of synchronous motor

$$\tan\theta_M = \frac{\text{leading KVAR of S.M}}{\text{Power consumed of S.M}}$$

$$= \frac{42.2}{50} = 0.844$$

$$\phi_M = \tan^{-1}(0.844) = 40.164$$

$$p.f = \cos\phi_m = \cos(40.164) = 0.7642$$

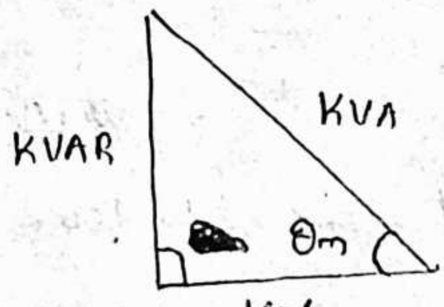
KVA taken by synchronous motor is

$$KVA = \sqrt{(KW)^2 + (KVAR)^2}$$

$$= \sqrt{(50)^2 + (42.2)^2}$$

$$= 65.428$$

KVA of synchronous motor



Problem A factory load 4000KW is connected to 11KV at 0.8 p.f lagging. An additional load of synchronous motor 1500 HP is connected in parallel factory load its p.f is 0.95 lagg. The efficiency of motor is 80%.

(1) Determine the KVA Capacity of motor

(2) p.f of the motor to operate

sol power Consumed by factory load = 4000 KW

p.f of factory load $\cos\phi_1 = 0.8$ lag

Capacity of synchronous motor is o/p of motor = 1500 HP

1 HP = 735.5 W

o/p of motor = 1500 * 735.5 W

= 1103.250 KW

also given efficiency of motor = 80%.

we know that $\eta = \frac{o/p}{i/p} \Rightarrow \boxed{i/p = \eta * o/p}$

$$\boxed{i/p = \frac{o/p}{\eta}} = \frac{1103.250}{0.8} = 1379.062 \text{ KW}$$

=> Total Combined load in KW = (motor load + factory load)

= 1379.062 + 4000 = 5379.062 KW

p.f of Combined load is 0.95 lagging

$\cos\phi_2 = 0.95$

=> lagging Reactive (KVA) power of factory load

KVAR = KW * $\tan\theta_1$

$\cos\phi_1 = 0.8 \Rightarrow \phi = 36.87$ 55

KVAR of factory load $= 4000 * \tan(36.87)$

$= 3000$ KVA

$\Rightarrow$ lagging reactive power of combined load ~~synchronous motor~~

KVAR of combined load ~~synchronous motor~~ $= \cancel{1379.062}\ 5379.062 * \tan\theta_2$

$= \cancel{1379.062}\ 5379.062 * \tan(18.195)$

$\cos\theta_2 = 0.95$.

$= 1768.026$ KVA

$\phi_2 \Rightarrow 18.195$

KVAR of Combined load $= 1768.026$ KVA

After installing the synchronous motor its p.f raises from 0.8 to 0.95 Lag

(1) $\Rightarrow$ leading reactive (KVA) power drawn by synchronous motor = KVAR of factory load − KVAR of combined load

leading KVAR syn motor $= 3000 - 1768.026 = 1231.974$

(ii) To find the p.f of synchronous motor

$$\tan\theta_m = \frac{\text{KVAR of s.m leading}}{\text{KW of s.m}}$$

$$= \frac{1768.026}{1379.062} = 1.2820$$

$$\theta_m = \tan^{-1}(1.2820)$$

$$p.f = \cos\theta_m = \cos\left[\tan^{-1}(1.2820)\right]$$

# Hunting in synchronous machine

we know that as the load on motor increases, rotor falls back in phase. Couple angle α also increases but motor continous to run at synchronous speed. It is because of magnetic interlocking b/w stator & the rotor poles.

If load on motor reduces, the rotor falls advance in phase, α (load or torque or power or coupling angle) decreases during this condition motor develops only electromagnetic torque.

During no load condition rotor poles coincides with the stator poles. This creates a magnetic locking b/w stator & rotor which tends to rotate the rotor at synchronous speed.

During load condition, if we increase the load in steps, magnetic interlocking b/w stator & rotor decreases to overcome this motor draws more current to develop torque their it maintains the rotor in synchronous speed. Here electric magnetic torque is equal to & opposite to load torque.

If a part of load is suddenly thrown off, motor need not develop much torque as a result, rotor falls advances in phase with coupling angle. The speed of the rotor is not decreased but its speed increases above the synchronous speed. To avoid these rotor is provided with damper winding. It is due to large inertia of rotor, the rotor tends to oscillate in new equilibrium position.

new equilibrium position.

56

The phenomenon of rotor tends to oscillate in new equilibrium position due to inertia is termed as hunting.

Causes of Hunting in synchronous motor.

(1) Sudden change in load
(2) Sudden change in field Current
(3) fault in supply s/m.
(4) Load torque Contains harmonics

Effects of huntings

(1) Make the machines to loose synchronism
(2) Causes a variations in supply
(3) creates a mechanical resonance in rotor
(4) increases mechanical stress in rotor shaft
(5) Increases mechanical losses as well as increasing the machine temperature

Minimising or Reduction of hunting

Hunting in synchronous machines can be reduced using

(1) Damper windings (2) use of flywheels

2nd method uses a flywheel provided with the prime mover. this increases the inertia of prime mover & helps in maintaining speed constant

using Damper windings

Damper windings is a low resistance copper bars embedded in the slots of salient pole machine. The windings is short circuited. when hunting takes place, an emf is induced in the damper winding. Due to relative motion b/w damper winding Conductors & rotating stator magnetic field a torque is produced. this torque opposes direction of rotor. Thus the oscillations in rotor reduces gradually.

hunting no

In absence of emf is induced in rotor damper winding. * Damper windings Can also help the synchronous motor as started as a induction motor.

## Methods of starting of synchronous motor

synchronous motor is not a self starting motor. Different starting methods are adopted to start the synchronous motor. These are

(1) using Damper windings (2) using pony motor (3) using a d.c motor.

### using Damper windings

Damper windings are used for high ratings of synchronous machines. these windings placed in the slots of rotor poles. All these windings are short circuited to carry currents to develop a torque.

when 3 phase A.C supply is given to stator, a rotating magnetic field is set up in stator. this rotating flux cuts the stationary damper winding Conductors. An emf is induced in the damper winding. This emf Induces a Current in the damper windings produces a torque. This torque tends to rotate rotor at near the synchronous speed.

Now DC excitation is applied to the field winding of rotor develops a rotor poles. The rotor also tends to rotate in the same direction of stator field at synchronous speed. Due to ~~stator~~ rotor poles coincides with stator fields poles a magnetic locking will takes place & Continously to run at synchronous speed.

thus a synchronous motor at the starting acts as induction motor & run as synchronous motor during d.c excitation.

② using pony motor

Pony motor is a small a.c & D.C motor driven at synchronous speed.

this is Connected to the rotor of synchronous motor through a belt.

when 3 phase supply given to stator, a rotating magnetic field is set up which can cuts the field winding of rotor. Here rotor is driven by a small ac motor at synchronous speed, the rotor develops rotor poles that creates a magnetic locking with the stator poles which can Continously run the rotor at synchronous speed.

The supply to the pony motor is disconnected it can Continuously run at the synchronous speed.

(3) using a DC motor

In this method, first run the synchronous motor as alternator which can be driven by d.c motor mounted on the shaft.

Here alternator is properly synchronised with the bus bar supply. If the external supply for the d.c motor is removed, the alternator continous to run at synchronous speed which take power from bus bar.

Here the synchronous motor started as slip ring induction motor since starting torque is high & damper winding does not form a short circuit. These slip rings are connected to external rheostat along with damper windings.

Problem

# Single phase and Special Machines

## Unit - 5

## Machines - III

### Single phase motors

These are also called as induction motors or AC motors. These are of 3 phase induction motors and single phase induction motor.

3 phase induction is inherently self starting motor whereas single phase is inherently NOT self starting. Single phase does not develop a unidirectional starting torque. To start the motor it is to be converted into 2 phase motor at starting.

If an A.C supply is given to single phase ac motor, the torque cannot put rotor into motion. However a slight push is given to rotor, the rotor begins to run in that direction. So single phase doesn't develop the starting torque. That's why it is inherently not self starting.

### Construction

Its construction is similar to 3 phase induction motor.

① Its stator is provided with 1- phase AC supply. Also it consists of centrifugal switch which is used to separate the winding during starting purpose.

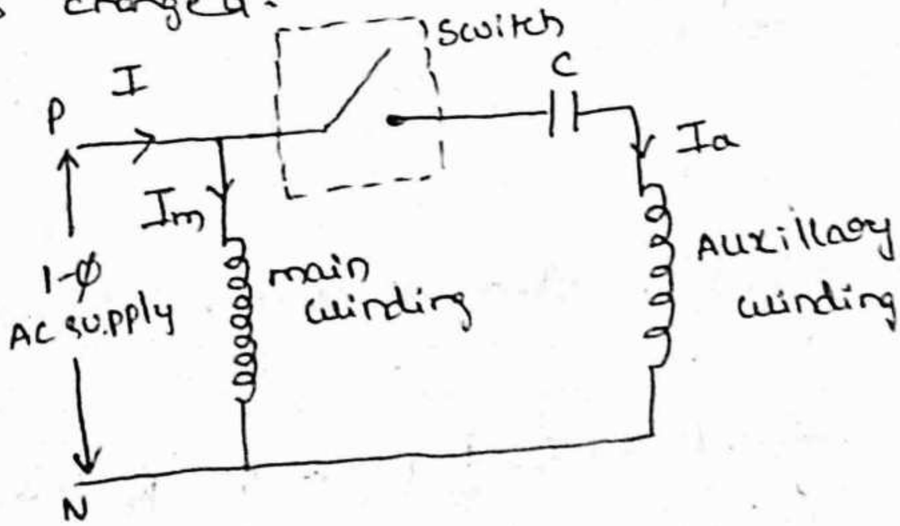
Stator is Provided with slots in the inner pheriphery to provide a low resistance stator Conductors.

Here the rotor is invariably a squirrel Cage type. To start the motor, we to Convert the single phase into 2 phase motor it requires an auxillary winding called starting winding is mounted on stator.

Here the auxillary winding is a high resistance winding placed in upper slots of stator. provided with a centrifugal switch.

The centrifugal switch separates auxillary winding from main winding during starting if rotar speeds upto 75% of rated speed.

Here the auxillary winding is Connected in parallel with main winding. To change the direction of rotor, the polarities of auxillary is changed.




Here the stator uses a distributed windings.

working

1-Ø phase Induction motor is inherently not self starting motor.

When the power to stator is on, stator develops an alternating flux because of ac currents. This alternating flux links with rotor conductors induces an emf in rotor conductors. Due to the stator flux & rotor induced currents a torque will be produced.

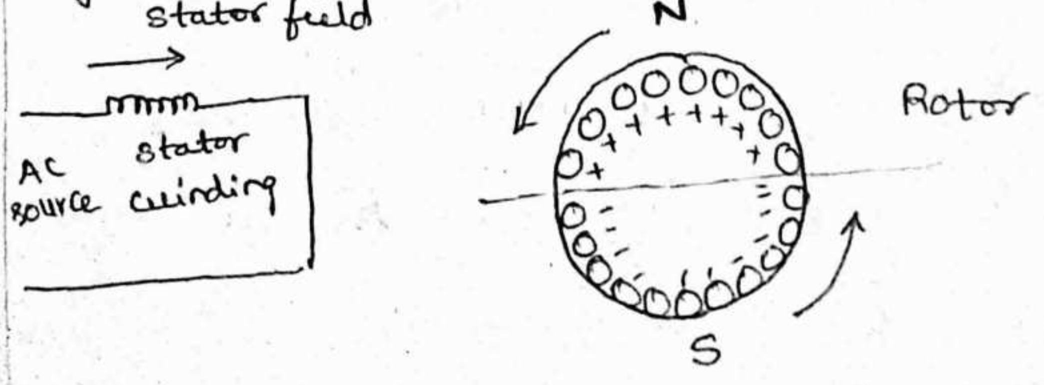
Rotor currents develops the N & S polarities such with respected to stator N & S pole.

Here rotor conductors in the upper ½ comes under stator N pole, rotor conductors in lower ½ comes with stator pole.

Rotor under N pole of stator develop a torque to rotate the rotor one direction

Rotor under S pole of stator develops a torque tends to rotate in another direction.

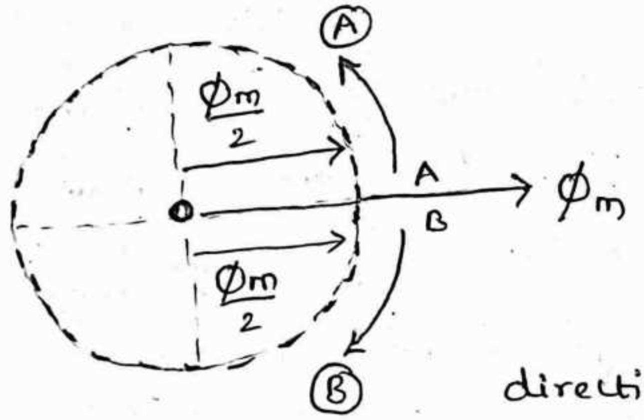
The 2 torques are equal & opposite then net torques becomes cancel out. Their fore net torque becomes zero so rotor remains stationary.



The analysis of single phase can be made on 2 theories. (1) Double field (2) cross field theory.

## Double Revolving field theory

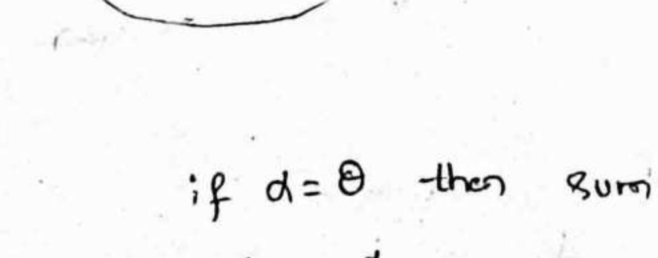
stator has a large no. of slots provided with distributed windings generates a alternating flux. The net amount of flux is splitted into 2 equal halfs having opposite directions at synchronous speed.




Considered 2 fluxes, each has $\phi_m/2$ maximum value.

Let flux phase OA & OB rotates in opposite directions at synchronous speed

$N_S = \frac{120f}{P}$. If OA has moved by making some angle say $\theta$, OB also moved by making some angle $-\theta$.




Let $\phi = \phi_m \cos\alpha$

if $\alpha = 0$, then sum of fluxes

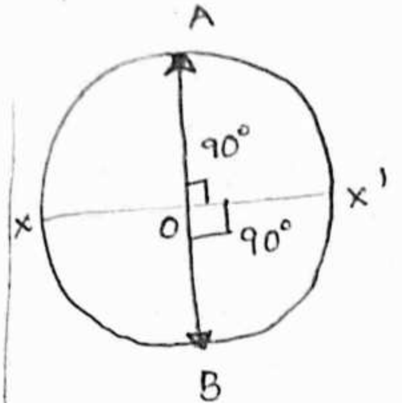
$$\phi = \frac{\phi_m \cos(0)}{2} + \frac{\phi_m \cos(0)}{2}$$

$$\boxed{\phi = \phi_m}$$

if $\alpha = \theta$ then sum of fluxes is

$$\phi = \frac{\phi_m}{2}\cos\theta + \frac{\phi_m}{2}\cos\theta = \phi_m \cos\theta$$

if $\theta = \alpha$, $\phi = \phi_m \cos\alpha$



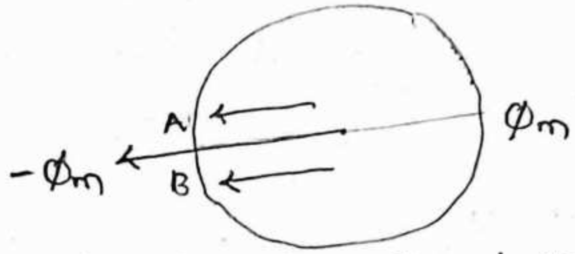

if $\alpha = 90°$ then projections of OA + OB are in direct opposition

$$\phi = \phi_m \cos 90$$

$$\phi = \frac{\phi_m}{2}\cos 90 + \frac{\phi_m}{2}\cos 90 = 0$$

sum of fluxes on x-axis is zero.

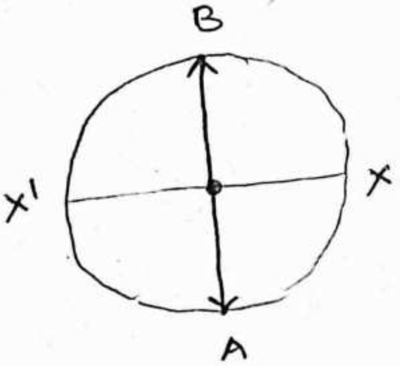
if $\alpha = 180°$




phasors OA + OB are in phase and each is equal to $-\frac{\phi_m}{2}$ $\quad \phi = \phi_m \cos(180) = -\phi_m$

sum of fluxes $\phi = -\frac{\phi_m}{2} - \frac{\phi_m}{2} = -\phi_m$

if $\alpha = 270°$




$$\phi = \phi_m \cos\alpha$$

if $\alpha = 270$

$$\phi = \phi_m \cos(270) = 0$$

phasor projections on x-axis is 0 since $\phi = 0$

if $\alpha = 360°$

$$\phi = \phi_m \cos\alpha$$

$$\phi = \phi_m \cos(360) = \phi_m$$

$$\phi = \frac{\phi_m}{2} + \frac{\phi_m}{2} = \phi_m$$

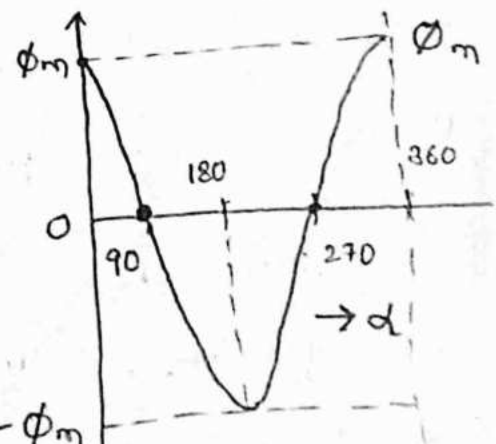



fig shows instaneous value of flux, against $\alpha$ the curve is indicated.

At slip (s) = 1, $T_f$ & $T_b$ are equal in magnitude but $T_R$ becomes zero at $s=1$.

power $P_g = \omega * T_g$.

Power developed by rotor $P_g = \left(\frac{1-s}{s}\right) I_2^2 R_2$

$$T_g = \frac{P_g}{\omega} = \frac{\left(\frac{1-s}{s}\right) I_2^2 R_2}{\frac{2\pi N}{60}} = \frac{60 * \left(\frac{1-s}{s}\right) I_2^2 R_2}{2\pi N}$$

if 60 is assumed constant

$$T_g = \frac{\left(\frac{1-s}{s}\right) I_2^2 R_2}{2\pi N} = \frac{\cancel{N}}{sN_s} * \frac{I_2^2 R_2}{2\pi \cancel{N}}$$

$$T_g = \frac{1}{2\pi N_s} * \frac{I_2^2 R_2}{s} = K * \frac{I_2^2 R_2}{s}$$

$$K = \frac{1}{2\pi N_s}$$

forward torque $T_f = K * \frac{I_2^2 R_2}{s}$ synch wat

backward torque $T_b = -\frac{K * I_2^2 R_2}{(2-s)}$ synch wat

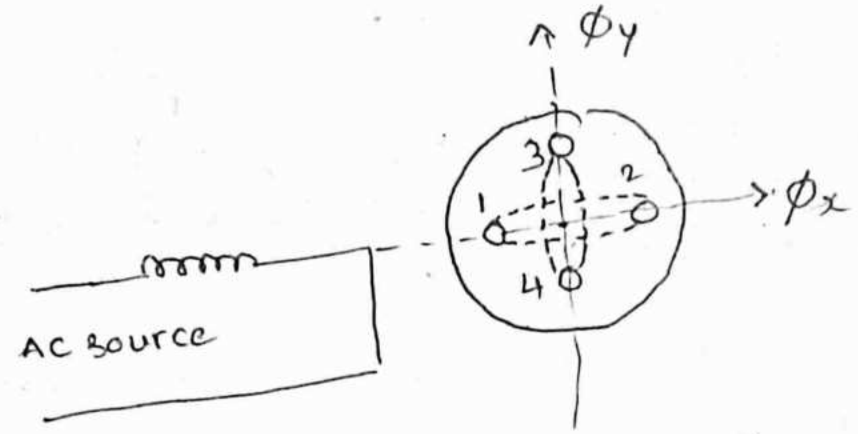
$$T_R = T_f + T_b$$

$$T_R = K * I_2^2 R_2 \left[\frac{1}{s} - \frac{1}{(2-s)}\right]$$

Rough

$$= \frac{(2-s) - s}{s(2-s)} = \frac{2-2s}{s(2-s)} = \frac{2(1-s)}{s(2-s)}$$

## cross field theory




According to this theory, stator flux is divided into 2 perpendicular components. one Along the axis of stator winding & other perpendicular to it.

Considered the above the fig, coils 1-2 & 3-4, flux $\phi_y$ is perpendicular to coils 1-2, whereas flux $\phi_x$ is along the axis of coils 3-4.

The flux $\phi_x$ linking the coil 3-4 induces an emf will set up a current in the coil 3-4. The current in the coil 3-4 will set up an mmf is in opposition to mmf setting up the flux $\phi_x$. Thus phase displacement b/w 2 mmf's is 180°. Hence there is no torque.

If a slight initial push is given to rotor when power is on, flux $\phi_x$ cuts across coil 1-2 a dynamically induced emf is set up in the coil Due to interaction b/w flux $\phi_y$ & current in coil 3-4 & between flux $\phi_x$ & current in coil 1-2, a torque is produced. This torque speeds up the rotor. hence rotor continous to run in initial push direction.

## Methods of starting induction motor

Single phase induction motor is inherently not self starting motor. If rotor is pushed in any direction by hand, it would continue to run in that direction.

The most common method of starting single phase motor is by converting temperorily into 2 phase motor.

Depending on suitable mechanism, single phase motors are classified as 2 types

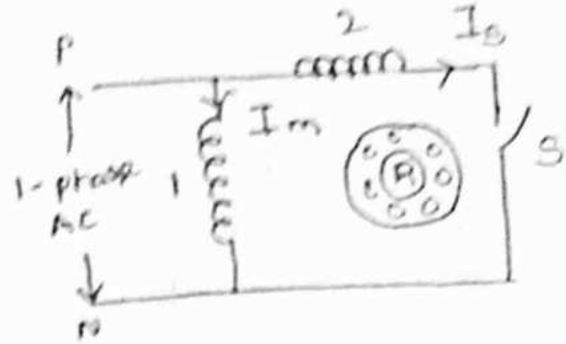
① split phase motors

- ⓐ Resistance split phase motor
- ⓑ Capacitor split phase motor
  - ① Capacitor start induction motor
  - ⅱ Capacitor start capacitor run motor

② shaded pole motor

Split phase motors

① Resistance split phase motor




1 → main winding
2 → auxillary winding
S → Centrifugal switch
R → Squirrel Cage rotor.

①Stator Consists 2 windings ① main winding & (ii) Auxillary or starting winding.

Main winding has low resistance & high Inductance path where as auxillary winding has high resistance & low inductance. 2 windings are connected in parallel.

The phase displacement b/w 2 windings is 90°. There is a Centrifugal switch Connected in series with auxillary winding. Here Rotor is of squirrel Cage type.

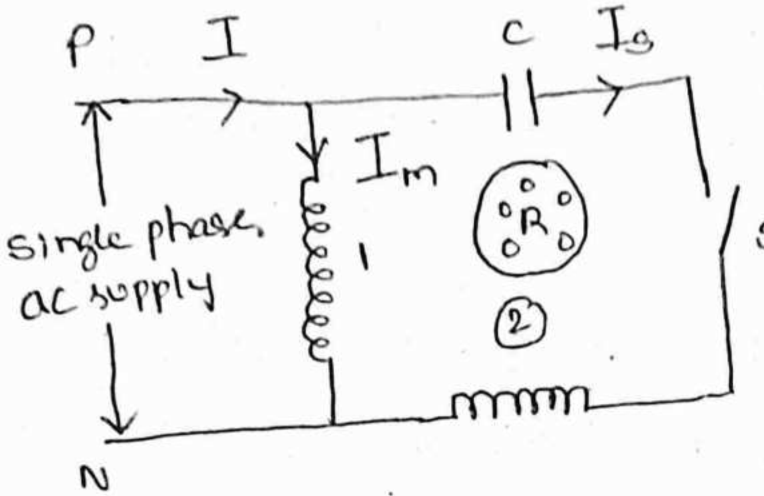
If a supply is given to stator from single phase a.c source, Current $I_m$ & $I_s$ flows in main & auxillary windings creates large phase angle b/w them. A rotating field is setup in motor develops driving torque.

If rotor reaches 75% of synchronous speed the Centrifugal switch opens out. & separates the auxillary winding & the motor continous to run as single phase motor.

The direction of rotation can be changed by reversing the Connections of auxillary winding.

These motors used in Centrifugal Pumps, fans, blowers, etc. Range is 0.5 HP - $1/_{20}$ HP

## (B) Capacitor - start motor




1 → main winding
2 → auxillary or starting winding.
3 → S centrifugal switch
R → squirrel cage rotor
C → electrolytic Capacitor. is used only for starting purpose.

Construction is similar to split phase motor. Here an electrolytic Capacitor 'C' is Connected in series with the auxillary winding through switch 'S'. Starting Procedure is similar to split phase motor. When rotor runs 75% of synchronous speed Centrifugal switch opens by separating the auxillary winding. Because of Resistance to reactance of 2 windings is different & a large phase angle, the motor develops high starting torque Compared to split phase motor.

It available range is 1/8 HP to 3/4 H.P

## Capacitor - start Capacitor - run motor

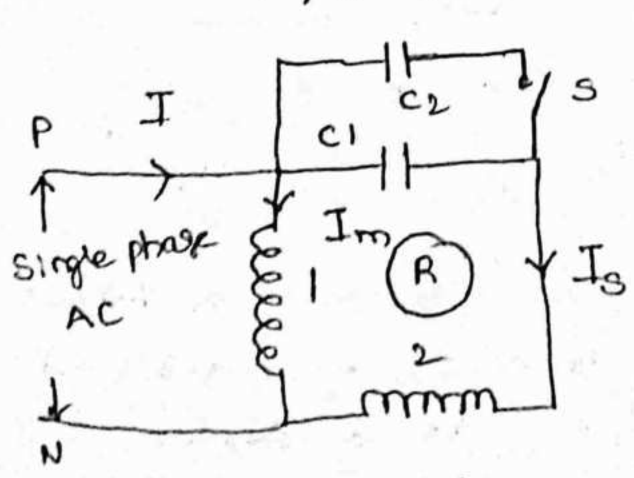
This is also similar to Capacitor start induction motor. Here stator carries 2 windings ① main and ② auxillary or starting winding. Both Auxillary & main windings remain in operation.

Here 2 Capacitors $C_1$ & $C_2$ are used in parallel with auxillary or starting winding. Here $C_2$ is connected in series with the centrifugal switch. and is parallel with capacitor $C_1$.

$C_1$ & $C_2$ are designed to develop high starting torque. When motor speed reaches 75% of synchronous centrifugal switch 'S' opens and disconnects the capacitor $C_2$.

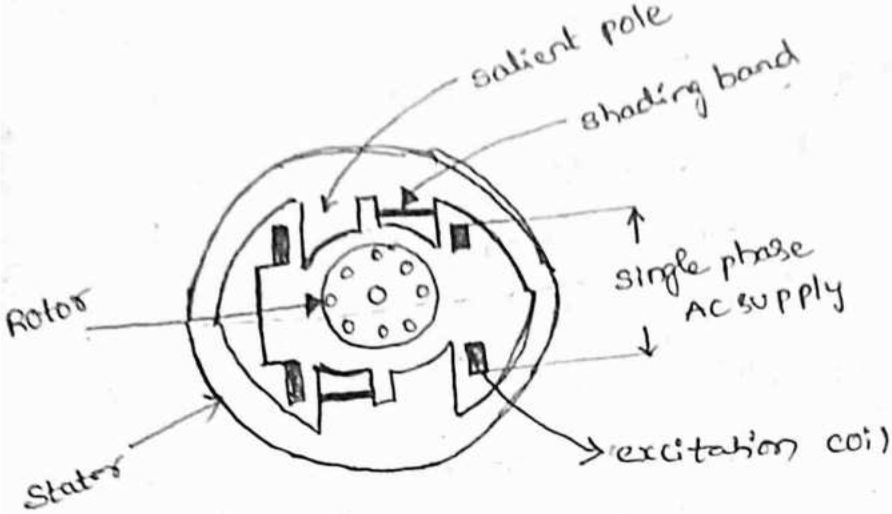
$C_1$ is designed for continous duty during running it improves powerfactor & efficiency of motor. As a result starting torque of motor is high.

It is widely used in Compressors, blowers, Conveyors, Centrifugal Pumps. Its ranges is from 1/8 HP to 3/4 HP




1 → main winding
2 → auxillary winding
$C_1$ & $C_2$ are Capacitors
S → Centrifugal switch
R → Squirrel Cage rotor

## 2) Shaded pole motor



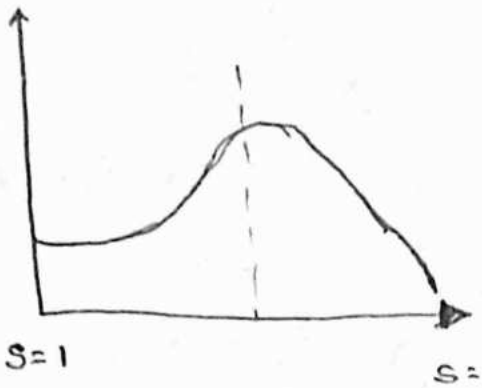

It Consists of a squirrel cage rotor and a stator Consists of salient poles.

Each pole of stator wounded by a excitation coil. It is supplied from a single phase AC source.

Each salient pole in stator is splitted into 2 equal halves. Each half pole of salient is Called as shaded pole. Each half pole is wrapped with copper band called as a shading band shown in above figure. These shading band forms as short circuit.

If single phase supply is given to stator, a rotating alternating flux is set up. This flux Links with shading band induces an emf which inturns induces a Current in the shading band because shading band forms as short circuit.

According to lenz's law direction of induced emf & the direction of Current in shaded band must oppose each other. As a result flux in shaded pole reduces. This gives a a small starting torque.




Limitations

* The starting torque of motor is low
* Efficiency is low
* There is no chance to change the direction of rotor by reversing

Applications

* Domestic toys * small fans, hair dyers, also where some applications requires low starting torque

Single phase series motor

A series motor which can work on both AC + DC supply. Such motor is also called universal motor.

Construction

It consists of a field winding mounted on salient pole fixed to the yoke.

The outer part of armature is provided with slots where armature windings are accomodiated. All the armature windings are connected to the commutator and is connected to the brushes.

Here field + armature windings are connected in series to carry same current.

If motor is provided with dc supply, a flux will set up in field winding and set up a current in armature winding. Due to interaction b/w field flux & current in armature a torque will develops.

If motor is provided with AC supply, ac currents flows in both field & armature windings. But the direction of currents reverse. This results a pulsating torque is converted into unidirectional torque due to ~~[illegible]~~ commutator. thus an series motor works on AC supply also.

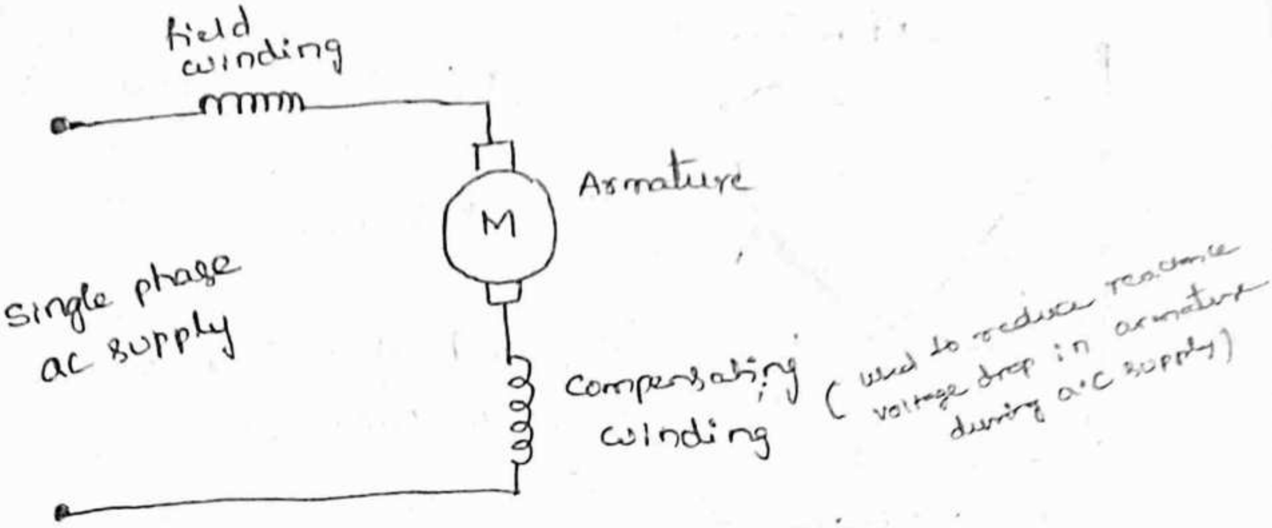
Demerits of AC series motor

* starting torque is low
* Power factor is low
* Efficiency is low
* Severe sparking at brushes results power loss
* Hysteries & eddy current losses is high to reduce both stator & rotor are fully laminated

— x — x —

Sparking at brushes is eliminated using a high resistance carbon brushes & interpoles are used.

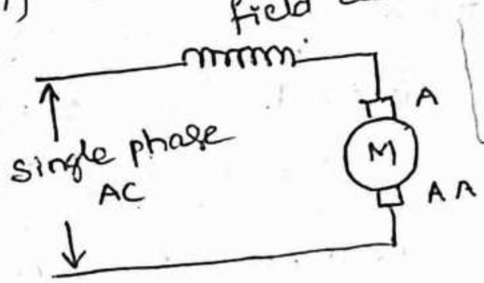
Low power factor in AC series motor is minimised by using compensating windings. This winding is connected in series with armature winding.




MMF Produced by Compensating winding balances the mmf Produced by the armature. Hence it neutralises. this Compensating winding is a low reactance winding.

## universal motor (UM)

It is a series motor which works on both AC 4 DC supply. But this motor does not have Compensating winding.



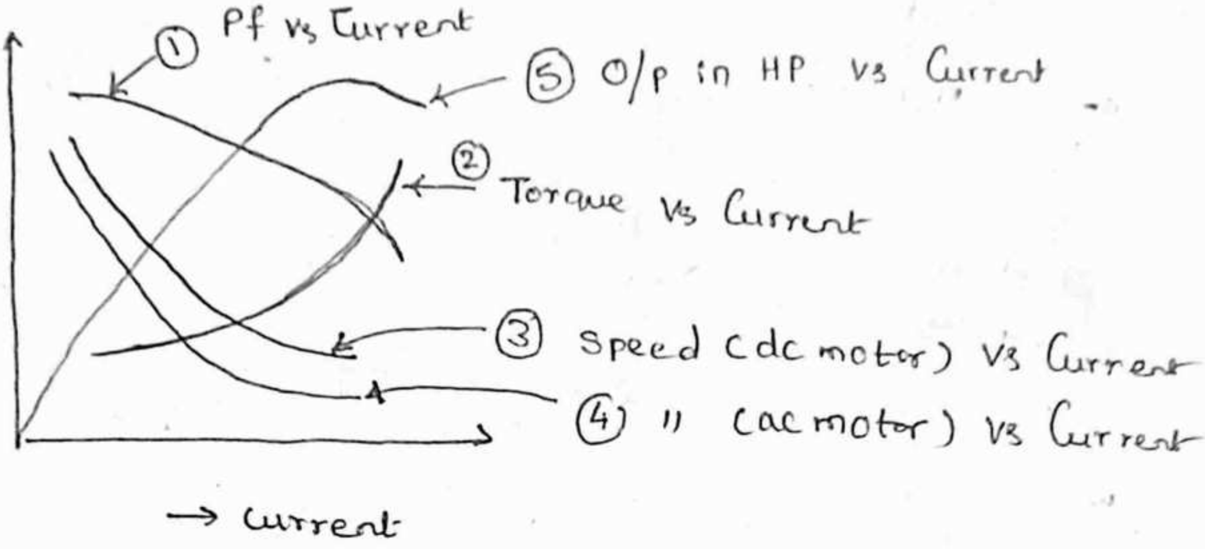

U.M
- non compensated → 2 salient pole & laminated Stator
- Compensated type → Stator core & Compensated winding → Similar to split phase

The operating speed is quite high from 7000 - 20,000 rpm. this is small in size 4 less weight. The no load speed of motor is limited to a safe value by Considerable amount of windage & frictional loss.

Also load torque is quite small. Commutation Problems are absent because armature winding carries a small Current.

## Applications

* vaccum cleaners, hair dyers, drill machines shavers, food mixers, sewing machines etc.




characteristics of universal motor.

Single phase synchronous motor (unexcited - type)

A revolving magnetic field produced in synchronous motors from a single phase source by use of same method in single phase Induction motors. It is a self starting motor.

Here also Consists of 2 windings ① main field winding ② auxillary or starting winding with a Capacitor.

Main winding is fed to single phase ac supply. An auxillary winding of high resistance is employed.

single phase synchronous motors are Constant speed machines. These motors are simple in Construction but it does not require dc excitation. thus we called as unexcited motor.

These are classified as 2 types

① Reluctance motor (II) Hysteresis motor

# Reluctance motor

single synchronous reluctance motor is same as single squirrel cage type Induction motor

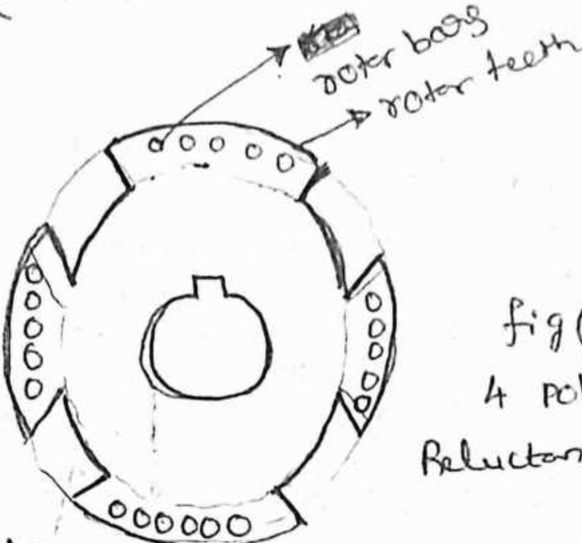



fig ① 4 pole Reluctance motor

* It has 2 windings
  main & auxillary or starting stator windings.

* The rotor of reluctance motor is basically a squirrel cage with some rotor teeth is removed at appropriate places. to get desired number of salient poles.

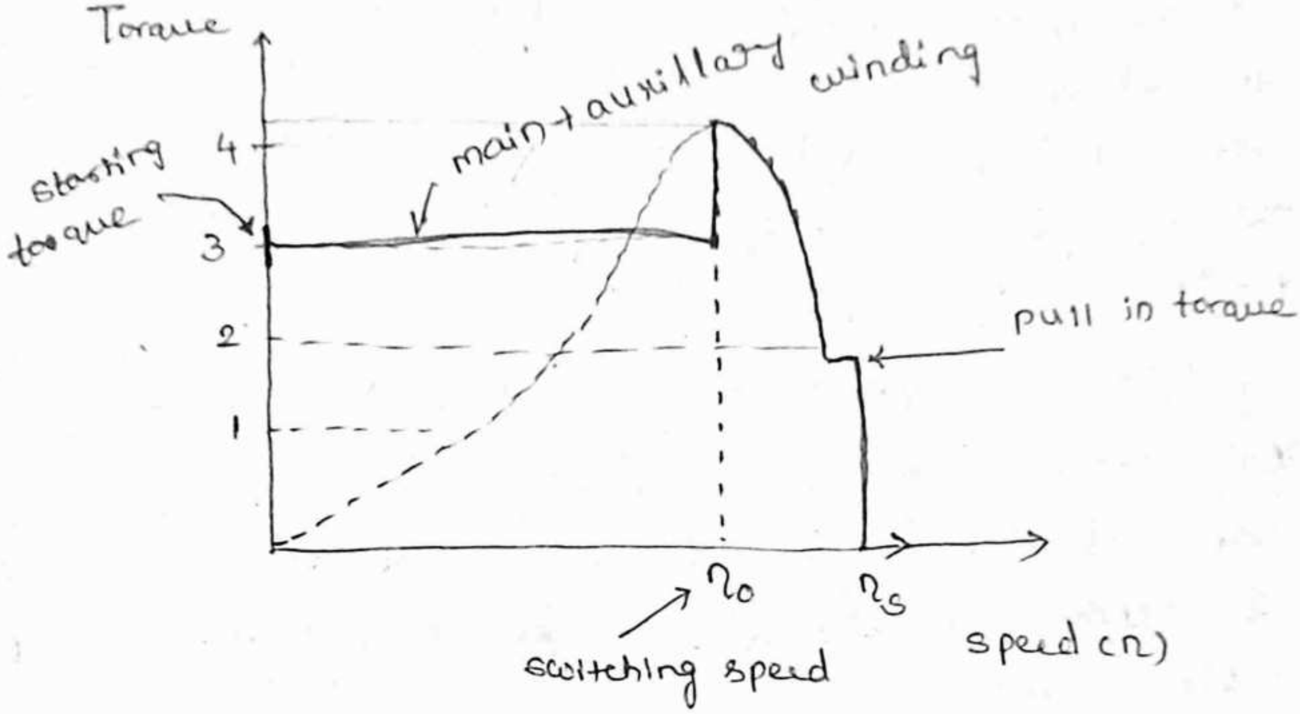
Fig ① shows 4 pole reluctance motor here teeth are removed at four locations to produce 4 pole salient pole machine. The remaining teeth can carry copper bars and forms as short circuit. to the end rings.

The motor is starting by providing single phase supply to stator, motor starts as induction motor. As motor reaches 75% of synchronous speed centrifugal switch opens & disconnects the auxillary winding.

When rotor speed reaches nearer the synchronous speed a reluctance torque is produced. This torque pulls the rotor into synchronism. It is depending load inertia.

If rotor reaches synchronous speed then induction torque disappears & rotor continous at synchronous speed.

torque - speed characteristics of reluctance motor




Starting torque depends on position of rotor. The reluctance torque depends on torque angle $\alpha = 45°$. The value is in b/w 300 - 400 % of full load. If rotor speeds close to synchronous speed, rotor develops reluctance torque which can keep the motor into synchronism.

The motor operates at constant speed over 200% of its full load torque. If loading increases above 200% of pull load torque, the motor looses synchronism & rotor runs as Induction motor. This is due to starting of Reluctance motors are undergoes cogging. Cogging in reluctance motors are minimised by skewing the rotor.

## Advantages of Reluctance motor

→ Construction is simple
→ low cost
→ maintainence is low
→ no slip rings
→ no brushes
→ no d.c field winding

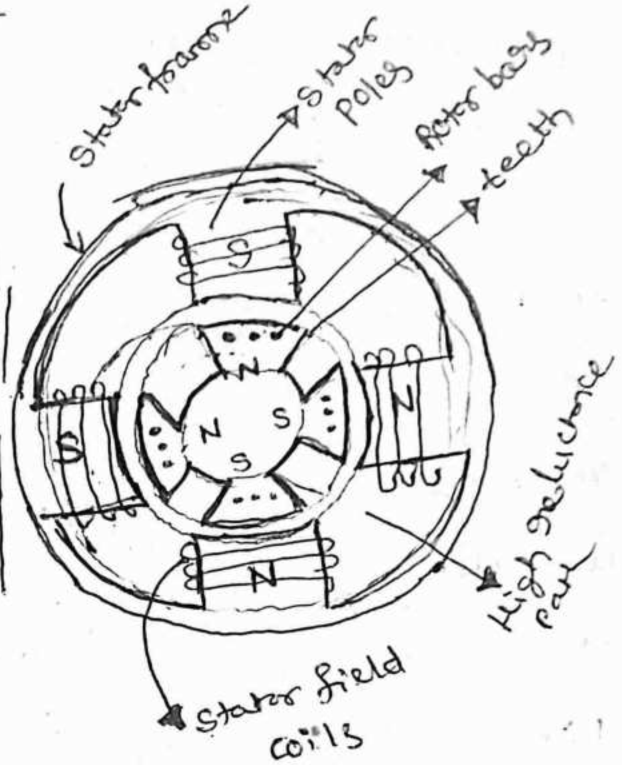



fig Reluctance motor

## Disadvantages

* Power factor is low
* O/P of reluctance motor is greatly reduced due to absence of field winding
* The size of reluctance motor is larger than synchronous motor.

## Applications

* electric clock timers
* used in signalling devices
* recording instruments
* used in phonographs
* also used for Constant speed applications.

## Hysterises motor

Basically synchronous motor with uniform air gap without field winding.

It can operate either on single phase & 3 phase supply.

Torque developed in this is due to hysteresis & eddy Currents induced in rotor.

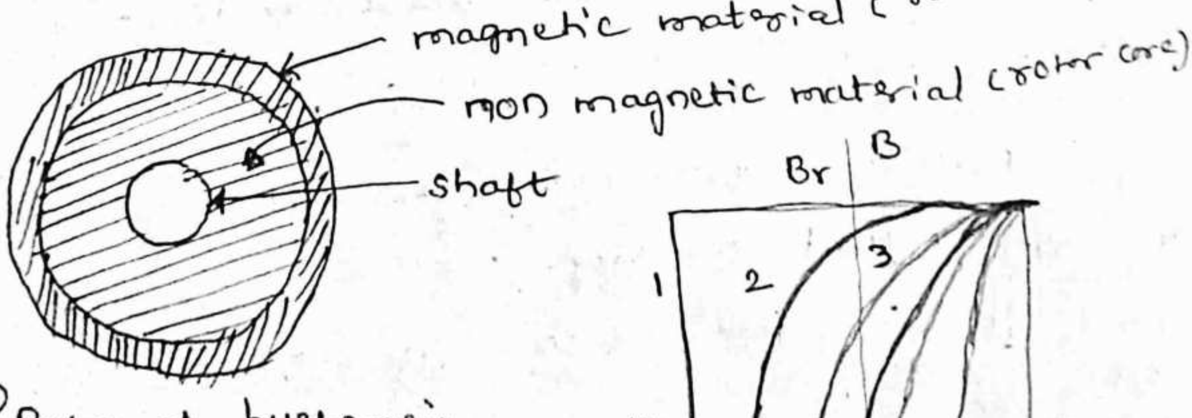
## Construction of Hysteresis motor

Stator is Connected to either single phase or 3 phase supply. If 3 phase supply is given to stator it produces a uniform rotating field. If single phase supply stator winding is provided with split capacitor type with an auxillary winding is used to produce uniform field.

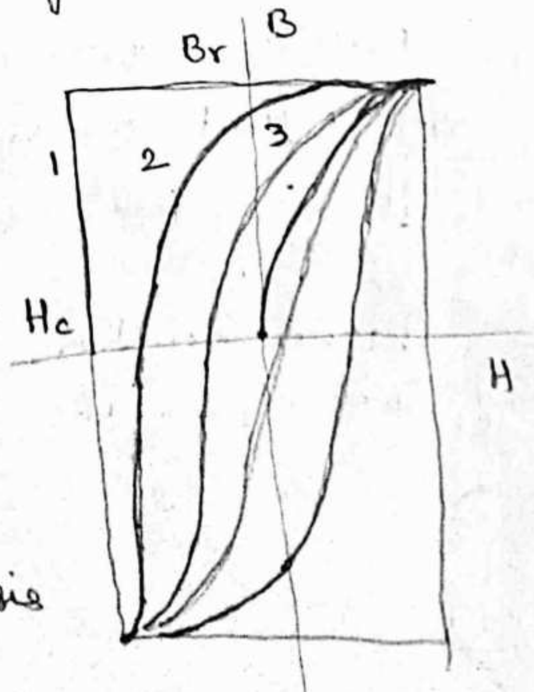
## Rotor Construction

The rotor of hysteresis motor is made of magnetic & non magnetic materials which is a smooth cylinder and it does not carry any field winding.

In small motor a solid ring is used which is made up of special magnetic material such as chrome or co-bolt steel having very large hysteresis loop.




fig① Rotor of hysteresis motor




fig② various hysteresis loop

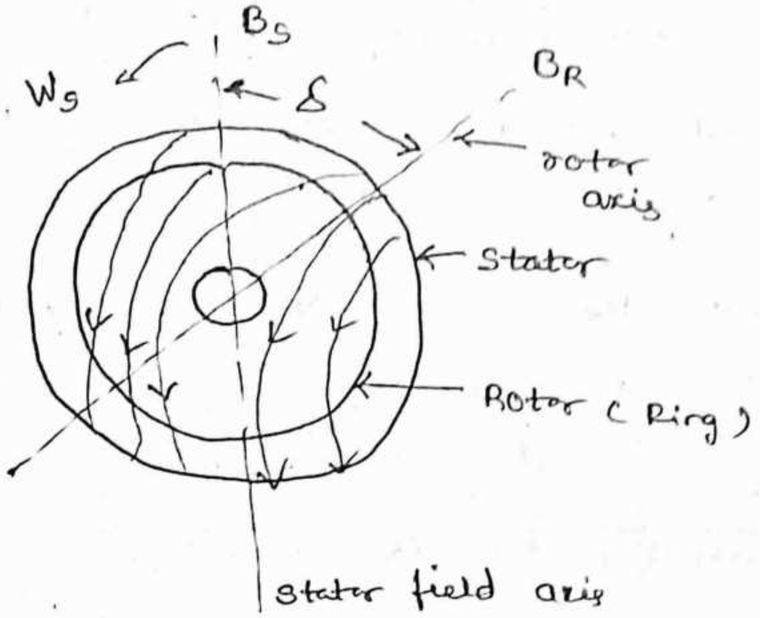



fig ③ Magnetic field in hysterises motor

if either 3 phase or single phase AC supply is given to stator, a rotating magnetic is set up in stator.

This magnetic field magnetizes the rotor ring which is made of magnetic material and develops a poles in it.

The flux in the rotor ring lags behind the stator flux, it is due to hysteresis loss in rotor.

Consider above fig ③

Let $B_S \rightarrow$ stator magnetic field

$B_R \rightarrow$ Rotor " "

$\delta \rightarrow$ angle b/w stator field axis & rotor field axis

The angle $\delta$ is responsible for the production of torque. $\delta$ depends on shape of Hysteresis loop.

An ideal material would have rectangular loop shown in loop 1 fig ①.

loop 3 is for ordinary steel, which is not suitable for hysterises motor

loop ② is approxiametly similar to loop ①
since the materials here are used for hysteresis motor hard cobolt.

Torque in the rotor is due to magnetic field in rotor also produce additional currents called eddy currents.

eddy current loss is given by

$$P_e = K_e f_2^2 B^2$$

$P_e$ = eddy current loss
$K_e$ = " " constant
$f_2$ = frequency of rotor eddy currents
$B$ = flux density

$$\boxed{f_2 = s f_1}$$

where $s$ = slip, $f_1$ = stator frequency

$$P_e = K_e s^2 f_1^2 B^2$$

Torque is given

$$\tau = \frac{P_e}{s\omega_s} \Rightarrow K^1 s$$

where $K^1 = \frac{K_e s^2 f_1^2 B^2}{s\omega_s} = \frac{K_e s f_1^2 B^2}{\omega_s}$

Torque due to hystrises loss

Hystrises loss is given by

$$P_h = K_h f_2 B^{1.6} = K_h s f_1 B^{1.6}$$

Torque is given by

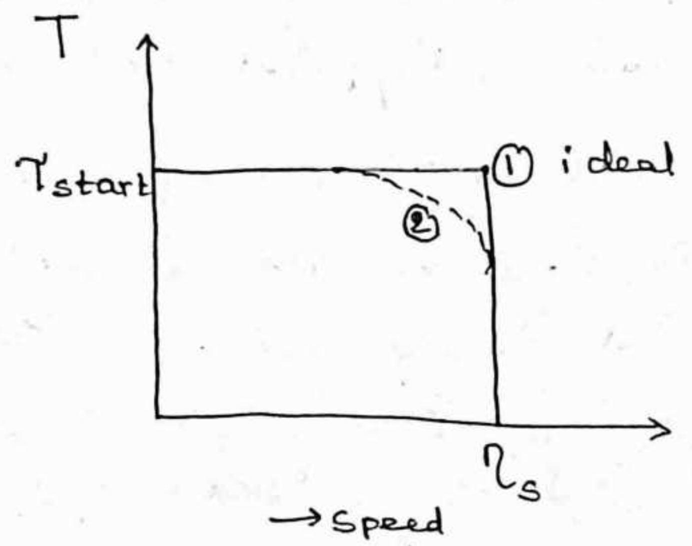
$$\tau = \frac{P_h}{s\omega_s} = \frac{K_h f_1 B^{1.6}}{\omega_s} = K^{11}$$

$T_e = K's = \frac{1}{N}$  $K' = \frac{K_e f_1^2 B^2}{\omega_s}$

eddy Current Torque is proportional slip

ass torque decreases speed increases because slip decreases. If motor speed N reaches $N_s$ then slip becomes zero & Torque $T_e$ is zero. Here the Machines Continous to run as Permanent magnet motor.

Torque speed characteristics




ideal characteristics is due to presence of harmonics. Torque developed by an induction motor is zero at synchronous speed.

Where as for hysteresis motor torque is Constant at all speed including $N_s$.

Applications

→ It is also used low noise devices

→ Also used sound reproduction equipments like record players, tape recorders

→ Also used in electric clocks.

## Stepper motors

Also known as stepping motor. The rotor moves through a definite angle for each pulse applied to the stator.

The pulse can adjusted by selecting suitable number of stator phases & rotor poles.

Speed of motor depends on the number of pulses actuating the rotor.

Stepper motors are used where incremental motion is required. These are used in printers, plotters, tapdrives, numerically controlled devices.

Stepper motor mainly consists of stator & rotor. A set of train pulses are applied to the stator develops a torque on rotor to move certain angle.

These are classified as 2 types

(a) variable reluctance (b) Permanent magnet

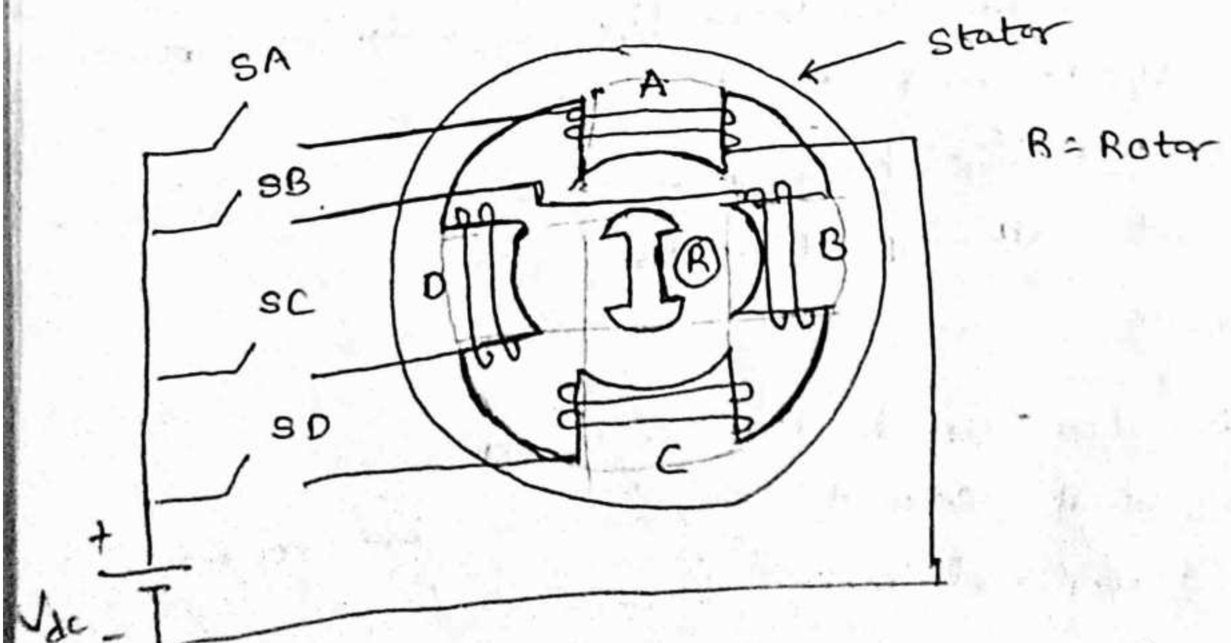



fig (1) 4 phase 2 pole Variable reluctance motor

The principle is based on flux linkages. Here flux lines occupies the low reluctance path. such that stator & rotor allignment minimises the magnetic reluctance.

Here both Stator & Rotor are provided with same number of teeth. Also the alignment of stator teeth & rotor teeth must be perfect so as to reduce the reluctance.

If stator is excited by by unidirectional pulse Current, it produces a torque and at every instant of incident pulse the rotor exhibits oscillatory behaviour. Here there must be adequate damping provided in motor.

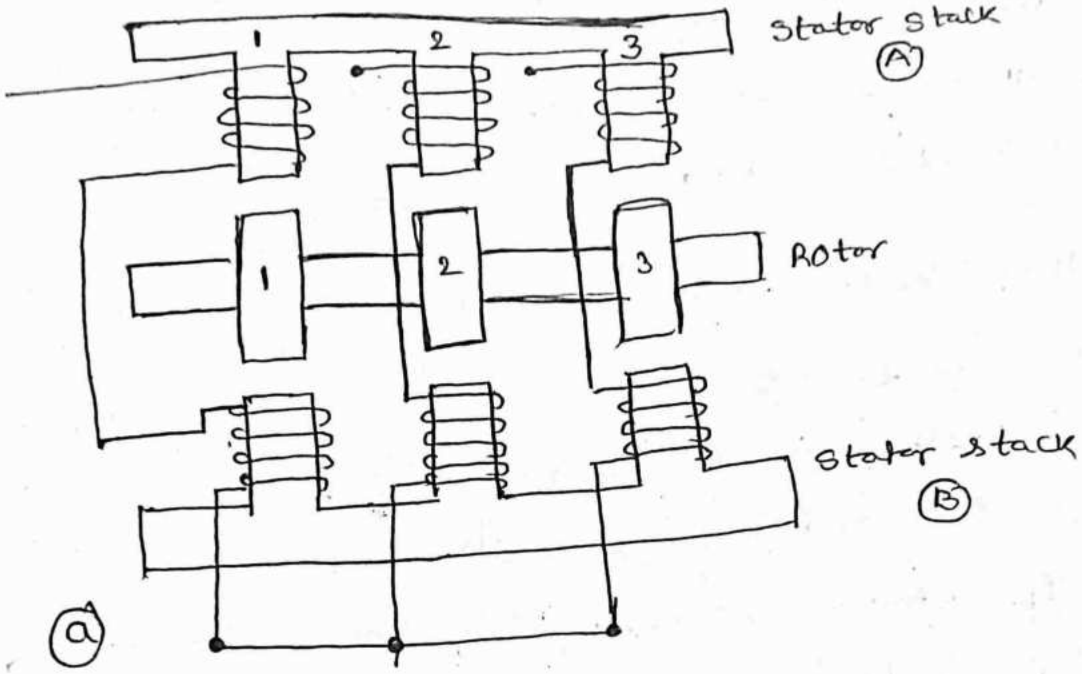
In Practical a stator has several stacks also Called stator phases. These stator phases are energised sequentially by appling pulse voltages. For every voltage pulse each of stator phase energises, the result is rotor moves forward rapidly with some angle.

Here rotor angle depends on the pulse applied to energise the stator.

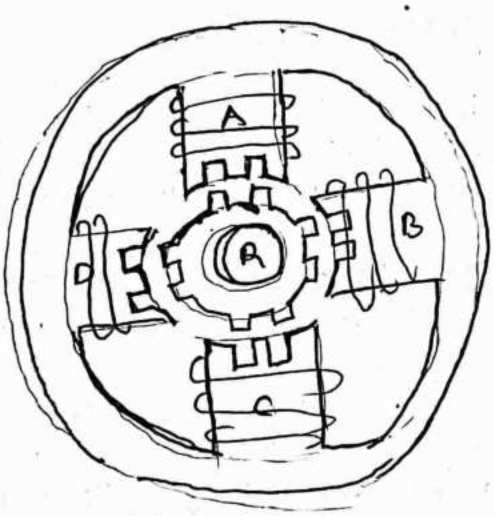
if $T$ = number of teeth

$n_S$ = no. of stacks

then angular displacement $\beta = \dfrac{360^\circ}{n_S * T}$




(a)




ABCD are stator stacks, each stack has 2 teeth, Each stator phase teeth is facing to rotor also, have 2 teeth

(b)

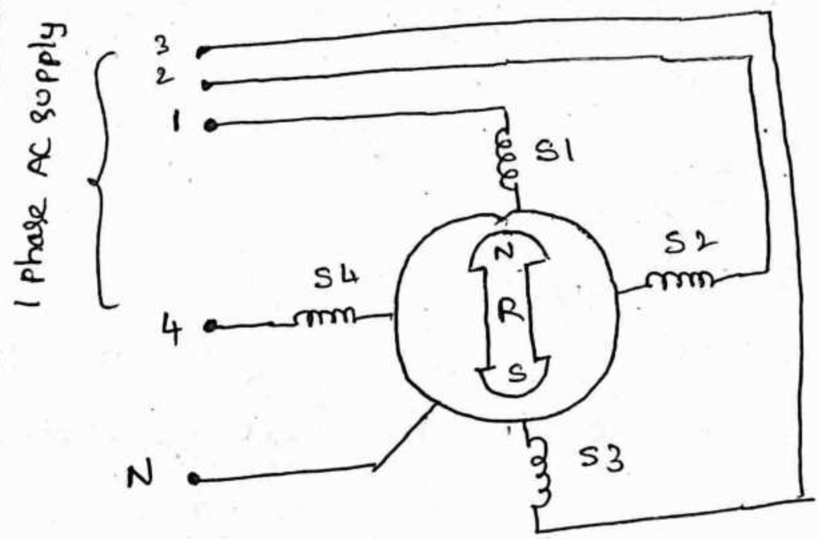
fig (a) Variable reluctance stepper motor, fig (b) Variable reluctance with stator & rotor teeth arrangement.

If stepper motor is loaded, the pulse voltages are applied to stator stack at a finite rate, the rotor follows command and drives the load to next position. The maximum stepping rate is around 1200 pulses per second.

(b) Permanent magnet stepper motor

In this type of motor, stator carries several windings. These windings are wound for any number of poles.

The rotor has definite number of permanent magnet poles. The rotor is a salient pole type which is made up of ferrite material & they are project out of the rotor core.




S1, S2, S3, S4 are stator windings

R -> salient pole rotor

1, 2, 3, 4, N are supply terminals.

Here all the stator windings S1, S2, S3, S4 are excited from single phase ac source.

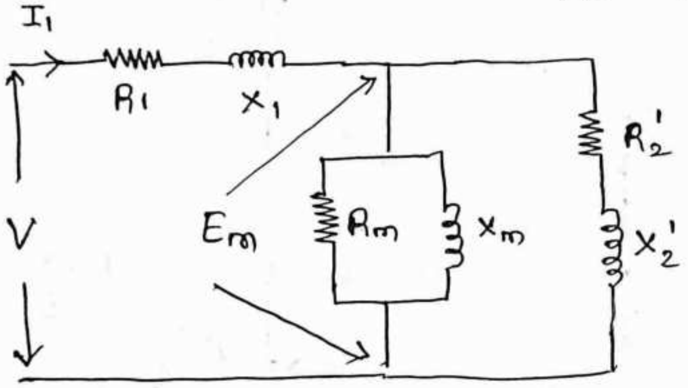
Here each winding get excited one after other sequentially.

Due to interaction in stator fluxes & rotor fluxes, a torque is developed in rotor to rotate. This continues untill the stator axis & rotor axis come in same allignment. For each excitation of stator winding, the rotor moves certain angle, for each excitation of stator. Desired angle can getting

## Equivalent circuit of single phase I.M

By Conducting No load & Blocked rotor test by drawing its equivalent circuit.

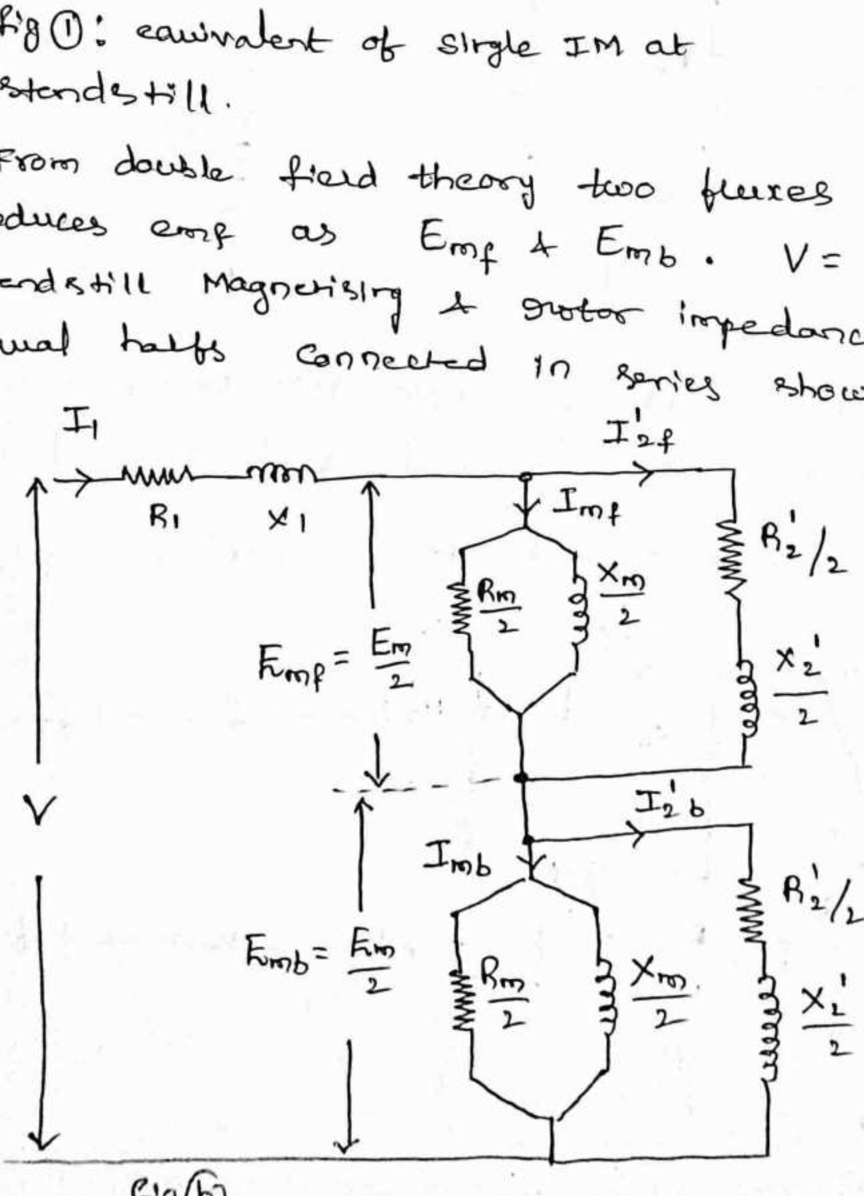
Equivalent circuit of I.M depends on double field revolving theory. Let stator impedance $Z_1 = R_1 + jX_1$ & impedance of rotor is $R_2 + jX_2$. Where $R_2$ & $X_2$ are equal to half of standstill Resistance $= R_2'/2$ & standstill reactance $= \frac{X_2'}{2}$.




fig① shows if rotor is Completely blocked looks like transformer with endary short circuited.

fig①: equivalent of single IM at standstill.

* From double field theory two fluxes in stator winding induces emf as $E_{mf}$ & $E_{mb}$. $V = E_{mf} + E_{mb}$. At standstill Magnetising & rotor impedances are divided into 2 equal halfs Connected in series shown in fig(b)




fig(b) shows Equivalent of single phase IM at standstill based on double field theory.

fig(b)

When rotor is running at speed N with respect to forward field the slip is 'S' & same rotor is running with respect to backward field the slip is (2−S). Then the equivalent circuit under normal running conditions is fig ⓒ.

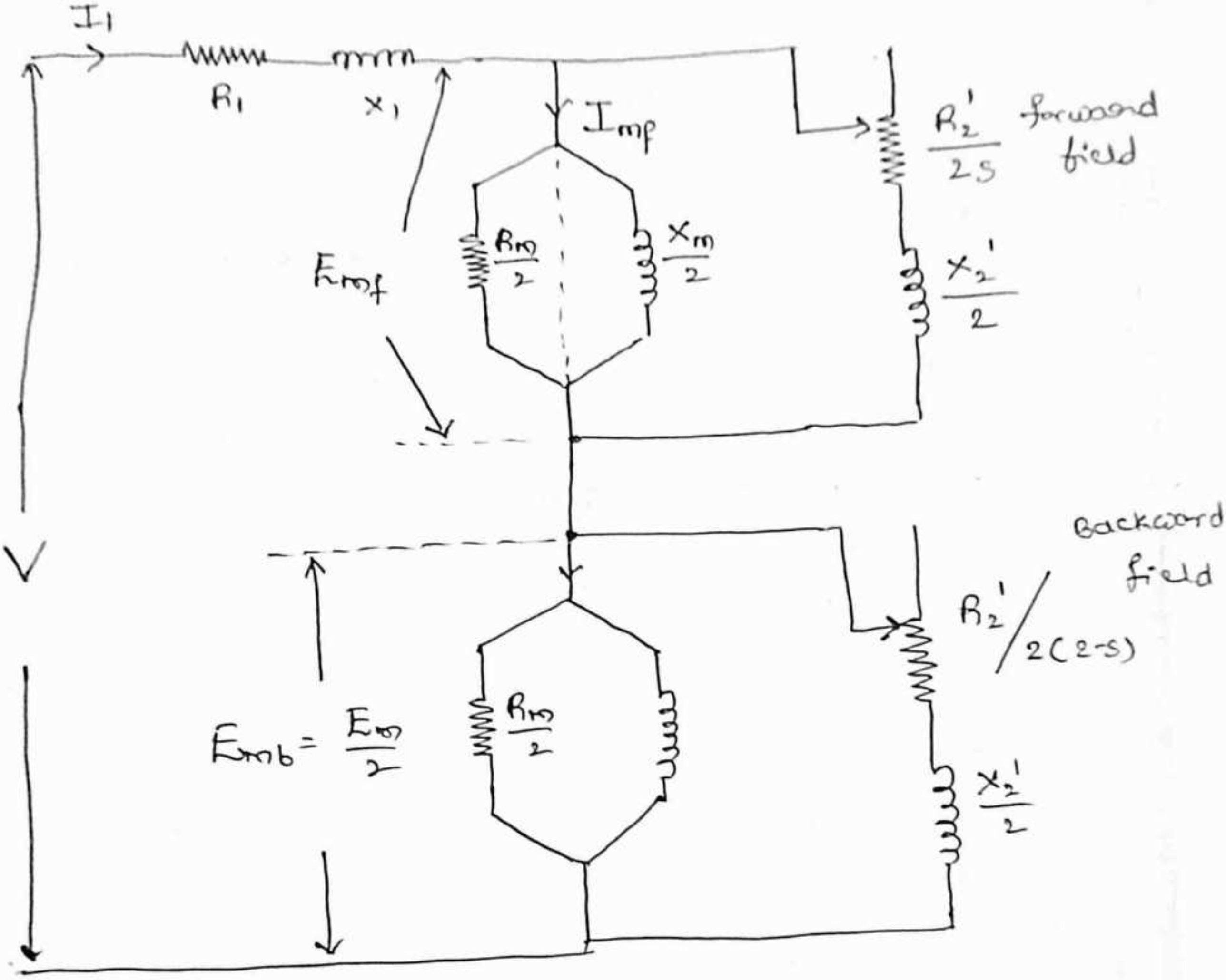



fig ⓒ: Equivalent circuit of single I.M under normal running condition with forward & backward slip

If core losses is neglected then circuit is fig ⓓ

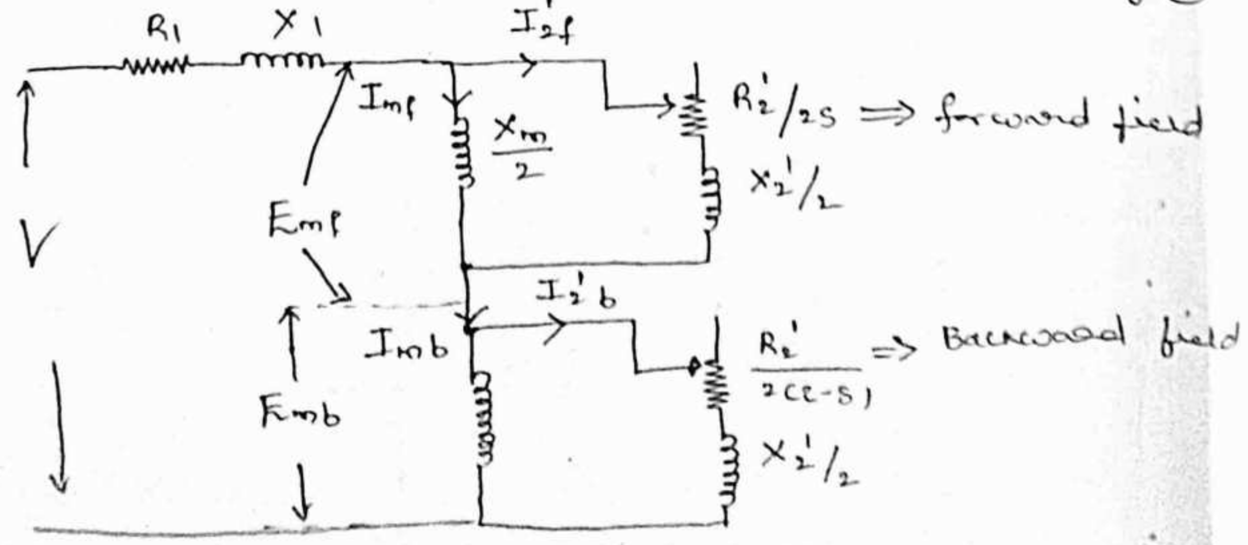



fig ⓓ Equivalent circuit under running condition without core

Stator impedance $Z_1 = R_1 + jX_1$

Total equivalent impedance $Z_{eq} = Z_{01} = Z_T = Z_1 + Z_f + Z_b$

$\Rightarrow$ forward field impedance $Z_f = \left(\frac{R_2'}{2S} + \frac{jX_2'}{2}\right) \| \frac{jX_m}{2}$

$$\boxed{Z_f = \frac{\frac{jX_m}{2}\left[\frac{R_2'}{2S} + \frac{jX_2'}{2}\right]}{\frac{R_2'}{2S} + j\left[\frac{X_m + X_2'}{2}\right]}}$$

$\Rightarrow$ Backward field impedance $Z_b = \left(\frac{R_2'}{2(2-S)} + \frac{jX_2'}{2}\right) \| \frac{jX_m}{2}$

$$\boxed{Z_b = \frac{\frac{jX_m}{2}\left[\frac{R_2'}{2(2-S)} + j\frac{X_2'}{2}\right]}{\frac{R_2'}{2(2-S)} + j\left[\frac{X_m + X_2'}{2}\right]}}$$

$\Rightarrow$ Motor Current $I_1 = \frac{V}{Z_{eq}} = \frac{V}{Z_1 + Z_f + Z_b}$

$\Rightarrow$ power factor $\cos\phi = \frac{R_{eq}}{Z_{eq}} = \frac{R_1 + R_f + R_b}{Z_1 + Z_f + Z_b}$

$R_f = \frac{R_2'}{2S}$, $R_b = \frac{R_2'}{2(2-S)}$

$\Rightarrow$ forward emf $E_{mf} = I_1 * Z_f$

backward emf $E_{mb} = I_1 * Z_b$

$\Rightarrow I_{2f}' = \frac{E_{mf}}{\sqrt{(R_2'/2S)^2 + (X_2'/2)^2}}$, $I_{2b}' = \frac{E_{mb}}{\sqrt{\left(\frac{R_2'}{2(2-S)}\right)^2 + \left(\frac{X_2'}{2}\right)^2}}$

$\Rightarrow$ Gross power developed in forward field

$$P_{gf} = (I_{2f}')^2 * \left(\frac{R_2'}{2S}\right) \text{ watt}$$

$$P_{gb} = (I_{2b}')^2 * \left(\frac{R_2'}{2(2-S)}\right) \text{ watt}$$

⇒ Torque developed in forward field

$$T_f = K * (I'_{2f})^2 * \frac{R_2'}{2S} = K = \frac{1}{2\pi N_s} \text{ or } \frac{P_{gf}}{\omega_s}$$

Torque developed in Reverse field

$$\boxed{N_s = N_s/60}$$

$$T_b = \frac{P_{gb}}{\omega_s} = K * (I_2'b) * \frac{R_2'}{2(2-S)}$$

$$K = \frac{1}{\frac{2\pi N_s}{60}} = \frac{9.55}{N_s}$$

$$\boxed{T_b = \frac{9.55}{N_s} * (I_2'b) * \frac{R_2'}{2(2-S)}}$$

$$\boxed{T_f = \frac{9.55}{N_s} * (I_2'f) * \frac{R_2'}{2S}}$$

Total Nettorque $T = T_f - T_b = \frac{9.55}{N_s} * \frac{R_2'}{2}\left[\frac{I_2'f}{S} - \frac{I_2'b}{2-S}\right]$

—— x —— x ——

Experiment Calculations for equalivent I.M

No load test ⇒

| $V_0$ | $I_0$ | |
|---|---|---|
| 220 | 4.2 | |

Blocked test

| $V_{sc}$ | $I_{sc}$ | $W_{sc}$ |
|---|---|---|
| 58 | 6.7 | 351 |

⇒ $R_1 = R_{ac}$

$R_{ac} = 1.5 R_{dc}$ ⇒ $R_1 = R_{ac} = 1.5 R_{dc}$

⇒ from no load test we know $V_0$ $I_0$

Magnetising reactance $X_m = \frac{V_0}{2I_0}$

S.C parameters

$Z_{sc} = Z_{01}$

$R_{sc} = R_{01}$

$X_{sc} = X_{01}$

$$\boxed{Z_{sc} = \frac{V_{sc}}{I_{sc}}}$$

$$\boxed{Z_{sc} = Z_{01}}$$

$$\boxed{R_{sc} = R_{01} = \frac{W_{sc}}{I_{sc}^2}}$$

$X_{01} = X_{sc} = \sqrt{Z_{01}^2 - R_{01}^2}$

or

$X_{sc} = X_{01} = \sqrt{Z_{sc}^2 - R_{sc}^2}$

$R_{01} = R_2' + R_1$ ⇒ $\boxed{R_2' = R_{01} - R_1}$

$X_{01} = 2X_1$ , $X_1 = X_2' = \frac{X_{01}}{2} = \frac{X_{sc}}{2}$

$$Z_F = \frac{\frac{jX_m}{2}\left[\frac{R_2'}{2S} + \frac{jX_2'}{2}\right]}{\frac{R_2'}{2S} + j\left(\frac{X_m + X_2'}{2}\right)} \; ; \; Z_b = \frac{\frac{jX_m}{2}\left[\frac{R_2'}{2(2-S)} + \frac{jX_2'}{2}\right]}{\frac{R_2'}{2(2-S)} + j\left(\frac{X_m + X_2'}{2}\right)}$$

$\Rightarrow Z_T = Z_{01} + Z_F + Z_B$

$$I_1 = \frac{V}{Z_T}$$

$\Rightarrow$ calculating total Current $I_1$, we need to find $I_F$ & $I_B$

$$E_{2F} = I_1 * Z_f \; ; \; E_{2b} = I_1 * Z_b$$

$E_{2F}$ $\rightarrow$ forword emf ; $E_{2b}$ $\rightarrow$ backward emf

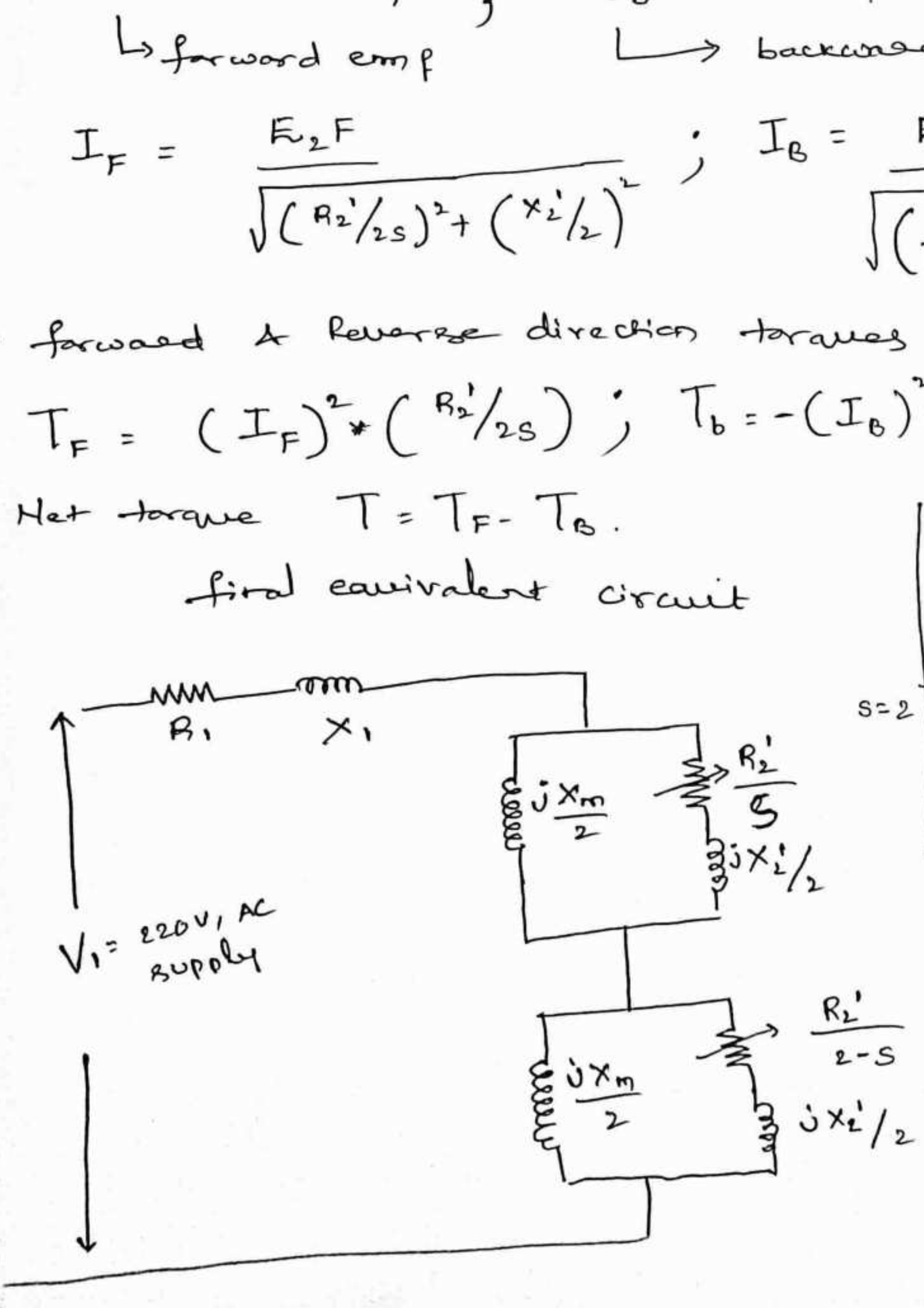
$$I_F = \frac{E_{2F}}{\sqrt{(R_2'/2S)^2 + (X_2'/2)^2}} \; ; \; I_B = \frac{E_{2b}}{\sqrt{\left(\frac{R_2'}{2(2-S)}\right)^2 + \left(\frac{X_2'}{2}\right)^2}}$$

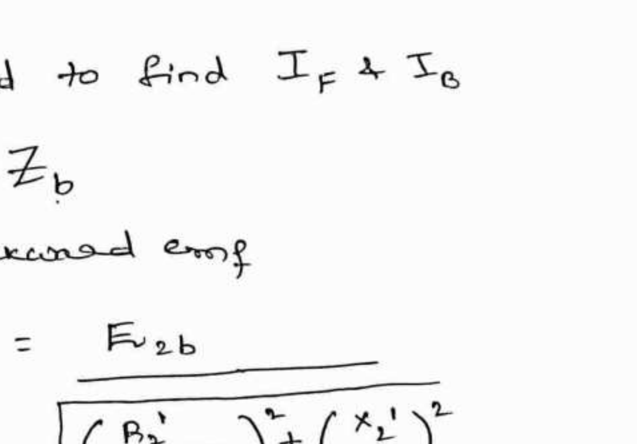
$\Rightarrow$ forward & Reverse direction torques

$$T_F = (I_F)^2 * (R_2'/2S) \; ; \; T_b = -(I_B)^2 * \left(\frac{R_2'}{2(2-S)}\right)$$

$\Rightarrow$ Net torque $T = T_F - T_B$.

final equivalent circuit




Torque - slip Curves.

# BLDC motors

* Also known as Brushless DC-motor. It is electronically commutated DC-motor.
* Speed & torque of this motor is controlled by producing current pulses to the rotor winding
* In this motor, permanents magnets rotates around a fixed armature to produce a large amount of torque over wide a speed ranges.
*

## Working of BLDC motor

⇒ In brushed motors, an armature which is rotating one consists of permanent magnets which creates a magnetic field when power is switched on.

⇒ For continues rotation of armature, this brushes changes the polarity of poles by reversing the current.

⇒ Same principle is employed for BLDC motors also. The only difference is BLDC motors shaft consist of position feedback controller.

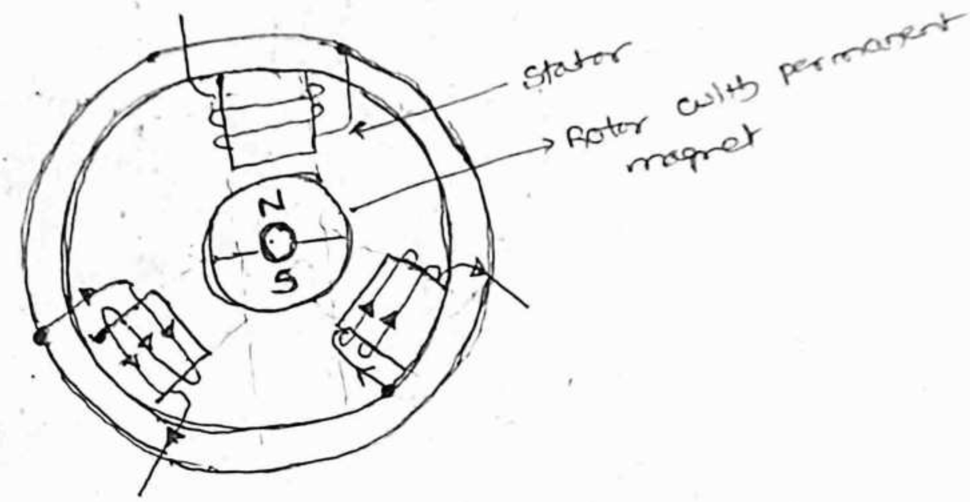
## Construction of BLDC motor

It also consists of 2 parts stator & rotor.

* the rotor consists of permanent magnets which continously rotating. These magnets revolve in stator.

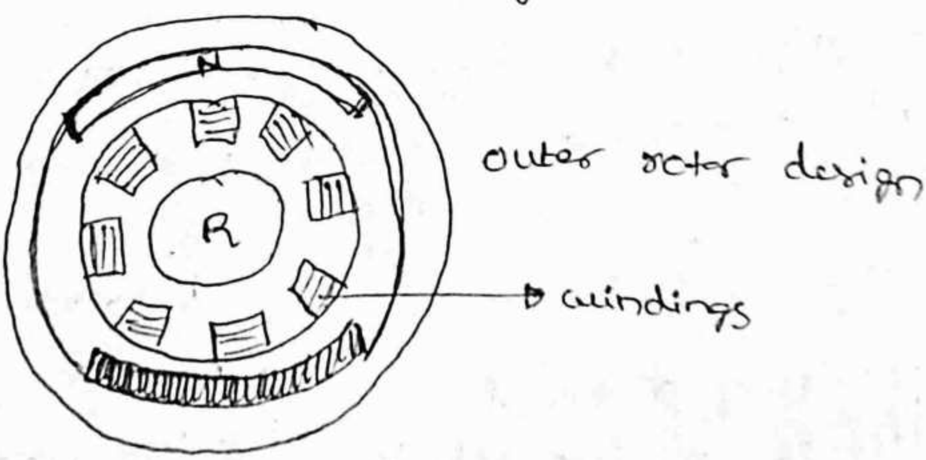
* Rotator which is stationary are consists of stator windings.
* A solid state circuit performs controlling & distribution of power to motor.

Types of BLDC motors




2 types of BLDC motors

(1) outer rotor motor (2) Inner rotor motor
↳ winding sourrounds the rotor




In this design, the rotor is completely sourrounded by windings in the core of motor. The magnets in the rotor traps the heat of motor.

* Such motors operates at lower current
* Cogging phenomenon is low.

Inner rotor Design

surrounds

↳ stator winding, rotor & stator winding

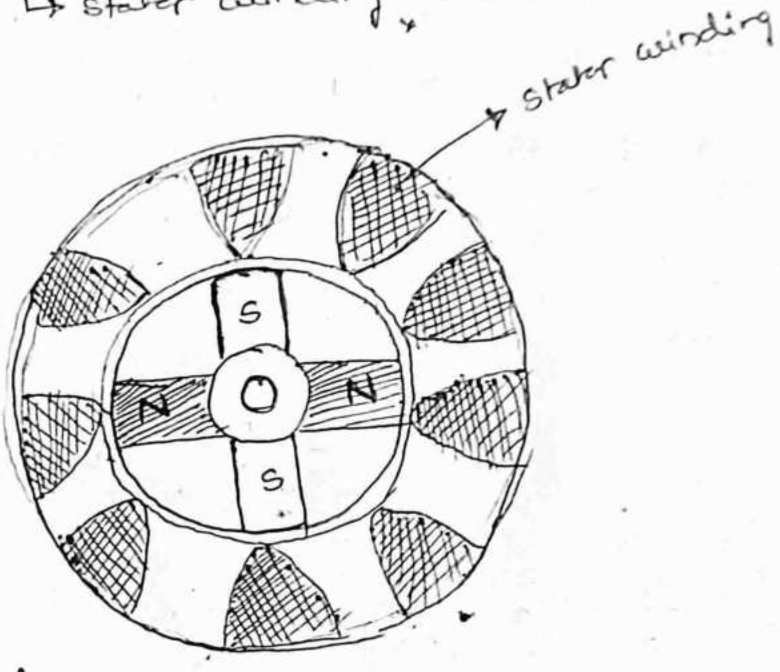



fig: Inner motor design

* Rotor is located in the Center of Stator winding.
* Here rotor magnets do not provide heat insulation.
* such rotors dissipates heat outside.
* such motors produces high amount of torque.

BLDC motor

| Advantages | Disadvantages |
|---|---|
| (1) velocity & speed depends on current but not voltage | (1) cost is very high |
| (2) No brushes means, frictional losses is less, no sparking | (2) High power is required |
| (3) produces less noise | (3) During low power, heat generates weakes the magnetic field. |
| (4) Accelerate & decelerate is easily due to low rotor inertia. | |
| (5) less maintenance | |